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MECHANICAL VELOCITY SELECTOR, NEUTRON FLUX AND Q-RANGE FOR THE SMALL ANGLE NEUTRON SCATTERING FACILITY AT RISØ

Ian Heilmann and Jørgen Kjems

Abstract. A mechanical velocity selector (MVS) was placed at the beam hole at the end of the neutron guide in the "Neutron House" at Risø. A 240 cm long evacuated tube with "pin hole" collimation was mounted after the velocity selector in order to simulate the geometry of the proposed small angle neutron scattering (SANS) instrument. At the end of the tube, corresponding to the sample position, count rates and absolute fluxes were measured with a monitor and Au foil activation, respectively. Likewise, Au foil flux measurements were performed immediately before and after the MVS. At the peak position $k = 1.9$ Å^{-1} ($\lambda = 3.3$ Å, $E = 7.2$ meV) of the cold source reactor spectrum and at bandwidth $\Delta k/k = 0.20$ the flux at the sample position was measured to $\phi = 1.0 \cdot 10^6$ n/cm^{2}sec. This is about 5 times lower than the corresponding flux obtained at the D11A SANS facility at ILL, Grenoble. A lower limit of momentum transfer for the proposed SANS instrument is estimated to $Q_{min} \sim 1 \cdot 10^{-3}$ Å^{-1}.

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CONTENTS

I. INTRODUCTION

The proposed Small Angle Neutron Scattering (SANS) facility at Risø will be situated in the Neutron House in connection with the neutron guide tube from the cold source in the DR3 reactor. A mechanical velocity selector (MVS), which already is in existence, will be used for the monochromation, and it was thought desirable to test the MVS as well as to measure the neutron flux at the "sample position" by means of a set-up which simulates the geometry of the SANS instrument.

II. THE EXPERIMENTAL SET-UP

Figure 1 shows the experimental set-up. (1) denotes the end of the 22 m long neutron guide tube coming from the DR3 reactor. The aperture of the neutron guide has a width of 2.5 cm. (2) denotes heavy concrete shielding with the beam hole (3) confined to 2.5×2.5 cm^2. The space between (1) and (2) is shielded and contains the monochromator (M) for the triple axis spectrometer TAS 7. The MVS (4) is placed in front of the beam hole (3) and can be rotated around a vertical axis (A). (5) is a 240 cm long evacuated brass tube with 1.3×1.3 cm^2 "pin holes" in both ends so that (6) corresponds to the sample position of the SANS instrument. A N_2-cooled Be filter can be inserted at (F) in the beam path. The filter effectively cuts off neutrons of energy higher than $E = 5.1$ meV ($k = 1.60$ Å^{-1}).

III. THE MECHANICAL VELOCITY SELECTOR (MVS)

The MVS consists of a horizontal rotating cylindrical drum with narrow slits in it through which the neutrons pass. The slits form an angle of 1.27° with respect to the axis of the drum and moreover the angle between the beam direction and the drum axis may be varied by rotating around the vertical axis (A). The resulting tilt angle between the slits and the beam direction will be denoted by θ. When the drum is rotating with constant rate it allows neutrons of wavevectors in a band Δk around an optimum value k to be transmitted. A detailed description of the transmission and resolution properties of the MVS based on square and triangular neutron distributions is given in Appendix A. An alternative resolution calculation based on Gaussian distributions is presented in Appendix B, while Appendix C gives the dimensions of the MVS. The drum is driven by an electrical motor and the revolution rate of the drum may be monitored by means of pulses from a small magnet in the wheel. The calibration between the potentiometer setting of the motor and the revolution rate was measured and is plotted in Fig. 2. Open circles show present results while closed circles show result from a previous test.

IV. MEASUREMENT WITH DETECTOR OR MONITOR

A boron plast mask with a hole of 2 mm$^{\phi}$ was mounted at the end of the evacuated tube and a standard LND ^{3}He neutron detector was placed immediately after (position 6 in Fig. 1). The velocity selector was set to ν = 83.3 rev/sec (Pot. setting = 8.00) and the count rate as function of tilt angle θ of the velocity selector was measured. The setting of θ is done by a digitized motor drive and Fig. 3 shows results of measurements done without and with the Be-filter inserted (open and closed squares, respectively). In order to convert the tilt reading (in digs) into the actual tilt angle θ the cut-off at $k = 1.60$ Å^{-1} of the Be filter was used. Using Eq. A1 we obtain $\theta = 2.47^{\circ}$ at $k = 1.60$ Å^{-1}. Moreover, the TAS 7 monochromator was used to determine k (and thereby θ through Eq. A1) at tilt = 9.0 dig by measuring the dip in transmitted intensity when the monochromator picks up that particular wavelength. (The method can be used only when the bandwidth $\Delta k/k$ of the velocity selector is small and comparable to the bandwidth of the monochromator, i.e. for high values of θ, see Fig. A3). Thus, we obtained at tilt = 9.0 dig, $k = 0.99$ Å^{-1} and $\theta = 4.00^{\circ}$.The value of k at the peak of the measured spectrum in Fig. 3 is now easily found, $k = 2.05(10)$ Å^{-1} and corresponds to the peak of the cold source spectrum. Finally, the bandwidth may be calculated using the results of Appendix A or B. For the present geometry the bandwidth is $\Delta x = \frac{\Delta k}{k} = 0.40/\theta(\text{deg})$, according to Eq. A2. At the peak of the spectrum of Fig. 3 we thus obtain $\Delta k/k = 0.21$.

In the following measurements the neutron detector was replaced by a standard LND fission monitor chamber. The 2 mm$^{\emptyset}$ mask was removed and the flux was measured directly through the 1.3×1.3 cm^2 hole at the end of the evacuated tube. The circles in Fig. 3 show the result thus obtained. This spectrum reflects directly the reactor spectrum, since the 1/v dependence of the monitor efficiency cancels the v dependence of the bandwidth ($\Delta k/k \propto \frac{1}{\theta} \propto k \propto v$, see Eq. A2).

Figure 4 shows results similar to those of Fig. 3, obtained at $\nu = 43.2$ rev/sec (pot. set. = 3.9). The calibration of the θ- and k-axes was done in a manner similar to that of Fig. 3. The maximum point of the spectrum is here found to be $k = 1.80(20)$ Å^{-1}, within the uncertainty equal to the maximum point found in Fig. 3. The bandwidth at this point is $\Delta k/k = 0.40/1.14 = 0.35$. The ratio $\frac{0.35}{0.21} = 1.7$ between the bandwidths at the maximum points of Fig. 3 and Fig. 4 is seen to be consistent with the ratio $\frac{11.5}{6.3} = 1.8$ between the respective count rates.

V. FLUX MEASUREMENTS WITH Au ACTIVATION

A reliable determination of absolute neutron fluxes is achieved by Au foil activation. One utilizes the process

$$^{197}\mathrm{Au} + n \rightarrow {}^{198}\mathrm{Au} \xrightarrow{2.7d} {}^{198}\mathrm{Hg} + \gamma + \beta$$

A known amount m of ^{197}Au is irradiated by the neutrons of flux density ϕ during the time t_e. By measuring the γ activity from the $^{198}\mathrm{Au} \rightarrow {}^{198}\mathrm{Hg}$ decay a time t after the irradiation has taken place, ϕ may be calculated, using the half time τ = 2.7 days and the neutron capture cross section σ. The γ activity at time t after the irradiation ($t_e << \tau$) is:

$$A(t) = N\ \sigma\ \phi\ t_e\ \frac{1}{\tau}\ln 2\ e^{-t/\tau \ln 2} \qquad \frac{\text{disintegrations}}{\text{sec}}$$

and thus

$$\phi = \frac{A(t)e^{t/\tau\ \ln 2}\tau}{N\ \sigma\ t_e\ \ln 2}\ \mathrm{n/cm^2\ sec}$$

where N is the number of ^{197}Au nuclei.

For thermal neutrons (E = 25 meV) σ = 98 barn. The present measurements were done at the peak of the spectrum ($k \sim 1.9\ \mathrm{Å}^{-1}$) where E $\sim$ 7 meV. Therefore, we use $\sigma_c = \sqrt{\frac{25}{7}} \cdot 98$ barn = 185 barn, according to the 1/v dependence of the cross section. The mass of the irradiated Au foils was m = 0.100 g and thus

$$\phi = \frac{A(t)}{t_e}\ e^{t/\tau \cdot \ln 2} \times 5.967 \cdot 10^6\ \mathrm{n/cm^2\ sec}$$

The activity A(t) was measured at the health physics department, and in the following we make use of the relation 1 μC = $2.7 \cdot 10^4$ disintegrations/sec.

Au foil activation was done at ν = 43.2 rev/sec at three positions: A: in front of the beam hole, before the velocity selector, B: immediately after the velocity selector and C: at the end of the flight tube. For these measurements, the setting of the tilt angle θ was discovered to be in error, i.e. away from the peak position $\theta = 1.14^{o}$. The C position measurement was therefore repeated with the correct setting of θ. In addition, a measurement at the C position was done at $\nu = 83.3$ rev/sec. and $\theta = 1.92^{o}$. The results are listed in Table 1

$\nu = 43.2$ rev/sec. C2: $k = 1.90$ Å^{-1}, $\Delta k/k = 0.35$

Position	t_e (sec)	t(sec)	A(t) (μC)	ϕ (n/cm^{2}sec)
A	120	55800	$7.55 \cdot 10^{-2}$	$1.6 \cdot 10^{8}$
B	300	55800	$8.08 \cdot 10^{-3}$	$7.0 \cdot 10^{6}$
C1	1200	55800	$6.26 \cdot 10^{-3}$	$1.4 \cdot 10^{6}$
C2	1200	∿0	$1.27 \cdot 10^{-2}$	$2.3 \cdot 10^{6}$

$\nu = 83.3$ rev/sec. $k = 1.90$ Å^{-1}, $\Delta k/k = 0.21$

C	1200	3600	$5.5 \cdot 10^{-3}$	$1.0 \cdot 10^{6}$

Table 1. Results of flux measurements by means of Au foil activation. Horizontal divergence at sample position $\sigma_H = 13$ min.

The horizontal divergence of the beam at the sample position (C) is $\sigma_H = 13$ min. (Appendix A). This is consistent with the Gaussian treatment in Appendix B, where for $\sigma_1 = \sigma_3 = 20$ min one finds $\sigma_H = 14$ min.

VI. DISCUSSION

The measured neutron fluxes at the sample position (C2 and C) may be compared to the fluxes obtained at the D11A SANS instrument at ILL, Grenoble, usually considered as the best obtainable today. According to "Neutron Beam Facilities at HFR, ILL", ILL Grenoble, (1974) p. 52 the sample position flux here has been found to be $\phi = 2.4 \cdot 10^{6}$ n/cm^2 sec for $k = 1.96$ Å^{-1} ($\lambda = 3.2$ Å),

$\Delta k/k = 0.10$ and an entrance collimation corresponding to the present one. From this we may conclude that the ILL flux is a factor of about 5 higher than what will be obtainable at the RISØ SANS instrument. An improvement of the RISØ flux might be obtained by inserting sections of neutron guides in the 2 m long flight path between the filter (F) and the beam hole (3), (see Fig. 1). The effect will be largest in the wavelength range where the critical reflection angle θ_c exceeds the angular divergence at the sample.

In Fig. 4 the crosses show a spectrum similar to that shown with open circles, but with atmospheric pressure instead of vacuum inside the flight tube (5), (see Fig. 1). The reduction in intensity is about 15%, roughly consistent with the attenuation expected from the scattering from N_2 molecules in atmospheric air.

From the recorded spectra and the dimensions of the proposed SANS instrument we can roughly estimate a lower limit of scattering vector. For practical purposes the lower limit of k is $k \sim 1.0$ Å^{-1} as seen from Figs. 3 and 4. With a distance between sample and detector equal to 400 cm and a detector element of ~ 0.6 cm, the minimum scattering angle 2θ is $\sim 0.6/400 = 0.0015$. We thus estimate a minimum value of scattering vector $Q_{min} = 2\pi\theta/\lambda = k\theta \sim 8 \cdot 10^{-4}$ Å^{-1}. The value of Q_{min} may be used to estimate an upper limit of size in real space which may be "seen" by the neutrons. This estimate depends on the type of sample. For a periodic structure a peak in the scattering occurs at $Q = \frac{2\pi}{d}$, d being the period, and obviously $d_{max} = \frac{2\pi}{Q_{min}} = 8000$ Å.

For dilute solutions of particles (f.inst. macromolecules) interference occurs only between waves scattered within the same particle and the diffraction pattern is that of a single particle, averaged over all orientations. Especially simply results obtain for spherical particles. Fig. 5 is reproduced after B. Jacrot: "The Study of Biological Structures by Neutron Scattering from Solutions", Rep. Prog. Phys. 39, 911-953 (1976). It is seen from the figure that a condition for observing the structure is roughly $QR/2\pi \sim 0.3$, yielding $R_{max} = 0.3 \frac{2\pi}{Q_{min}} = 3000$ Å. It should be mentioned, however, that the above estimates are ideal limits, since we have ignored the extent of the direct beam on the detector surface. Depending on collimation, the actual value of Q_{min} may be somewhat larger than that stated above.

SUMMARY

The results obtained in the present investigation may be summarized in the following way:

1. The mechanical neutron velocity selector described and tested in this experiment has proven to be well-suited for monochromation in connection with the small angle neutron scattering facility to be constructed in the neutron house at Risø.

2. The neutron flux at the sample position of the SANS instrument is measured to $\phi = 1.0 \cdot 10^6$ n/cm^2/sec at $k = 1.9$ Å^{-1} and $\Delta k/k = 0.20$, which is about a factor of five less than the fluxes obtainable on the SANS facility at the high flux reactor at ILL, Grenoble.

3. Based on the dimensions of the proposed SANS instrument, detector properties and the cold source spectrum a lower limit of scattering vector has been estimated to $Q_{min} \approx 10^{-3}$ Å^{-1}. This allows for studies of structural features up to several thousand Ångstrøms.

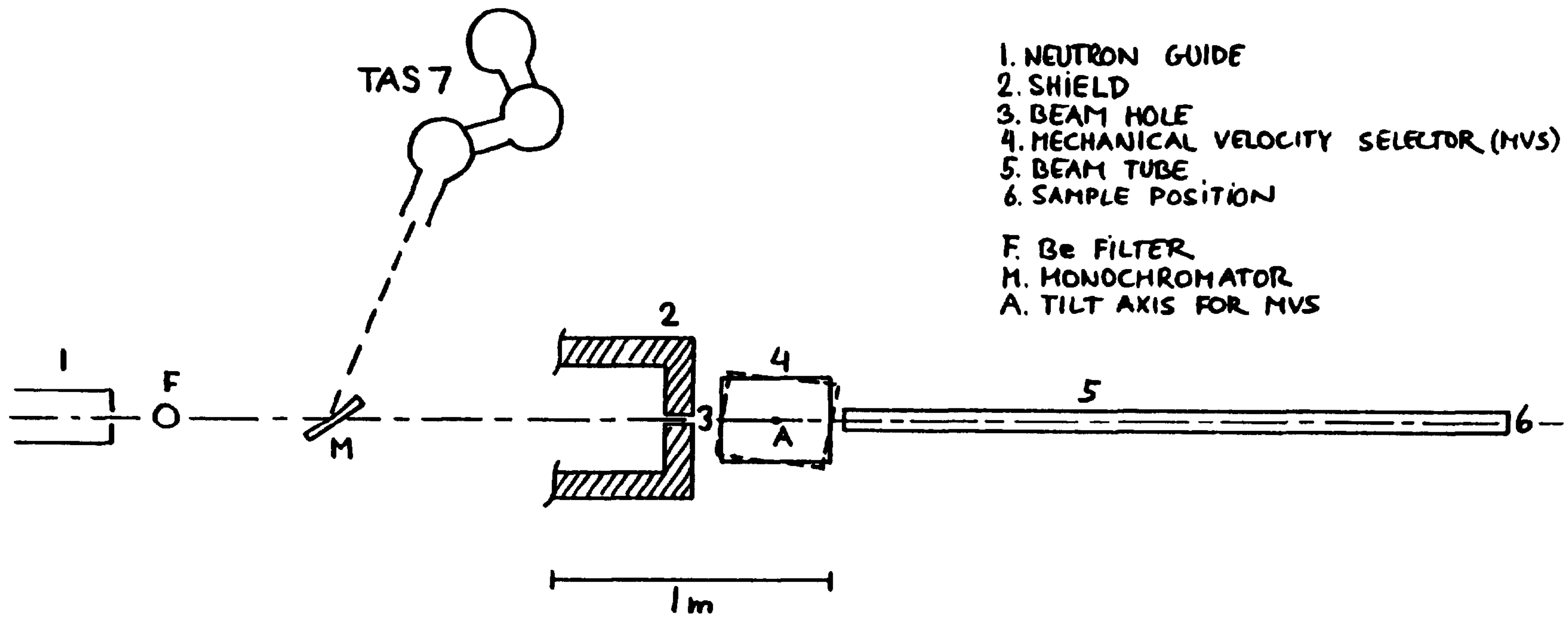


FIG. 1 EXPERIMENTAL SET-UP

FIG. 2 CALIBRATION OF MECHANICAL VELOCITY SELECTOR

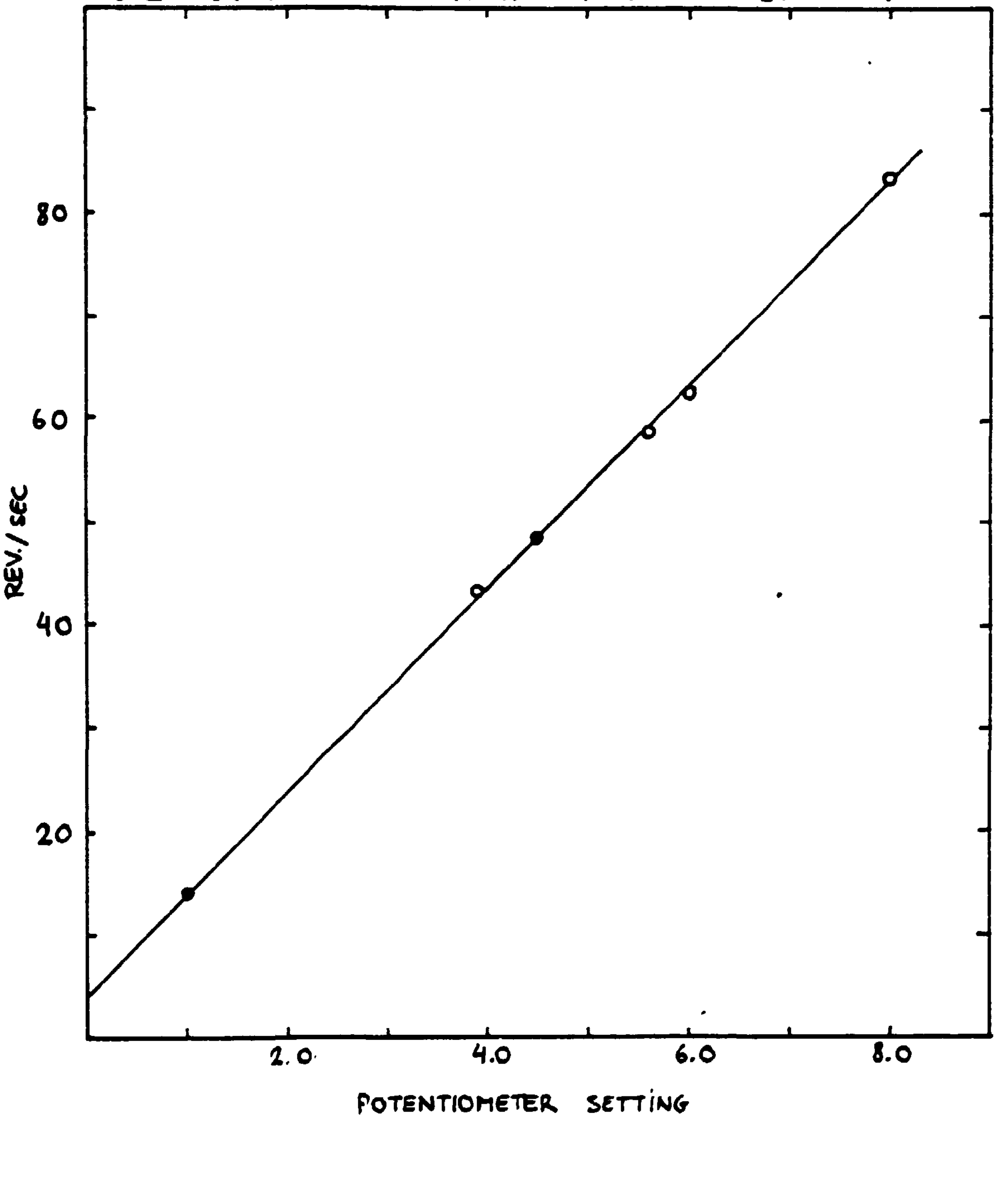


FIG. 3

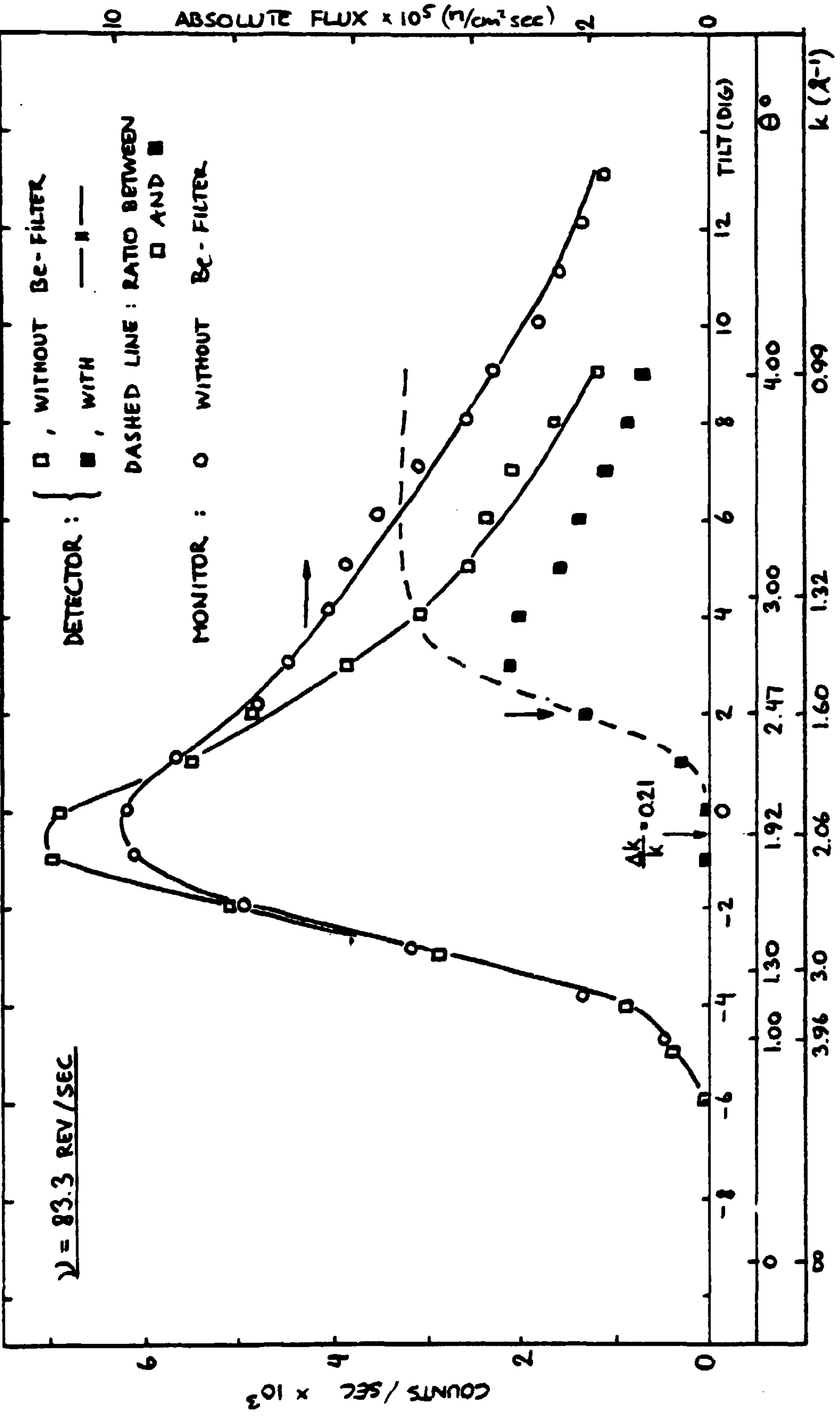
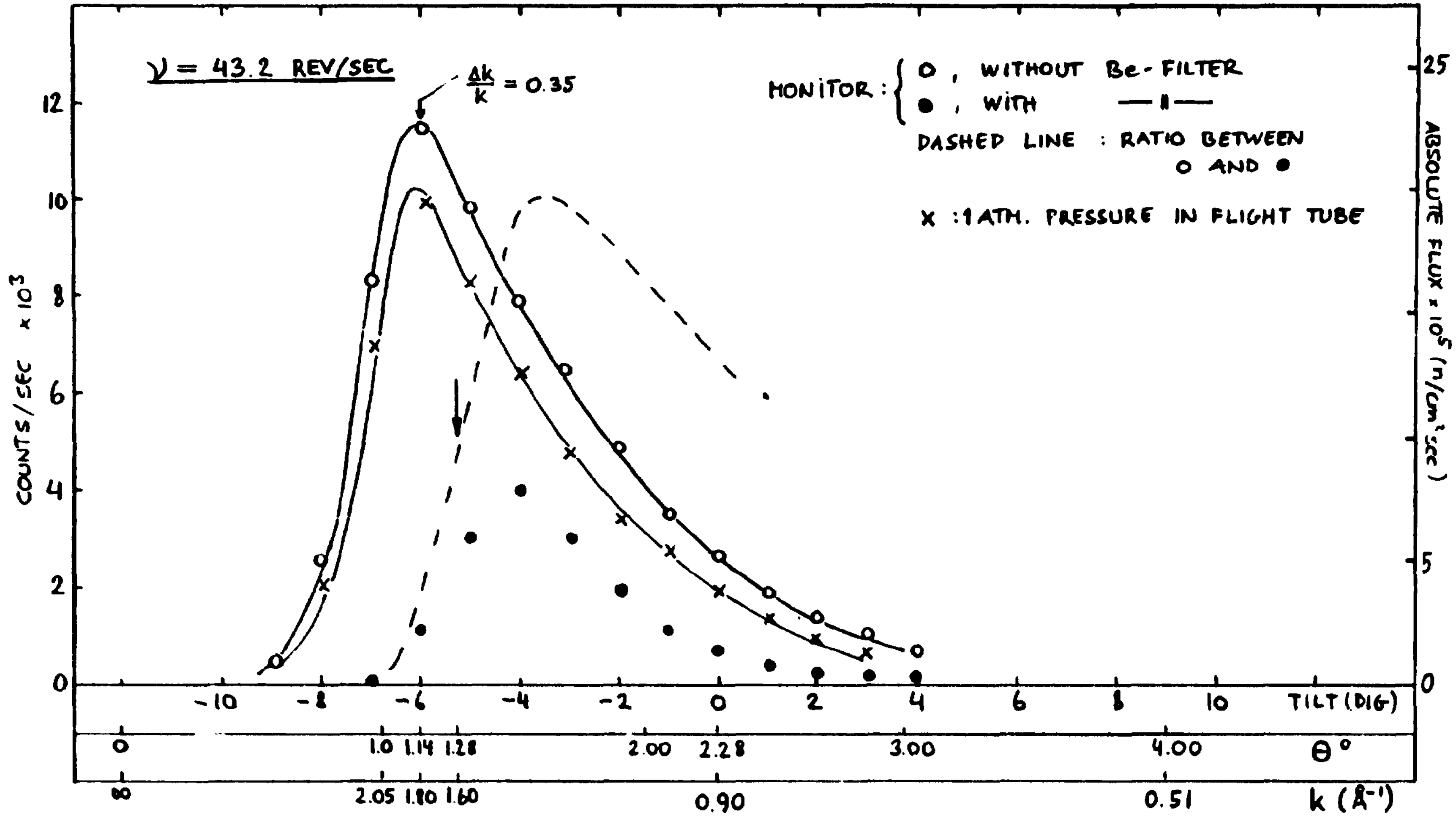

ν = 83.3 REV/SEC
DETECTOR : □ , WITHOUT Be-FILTER
■ , WITH —"—
DASHED LINE : RATIO BETWEEN □ AND ■
MONITOR : ○ WITHOUT Be-FILTER
ABSOLUTE FLUX × 10^5 (n/cm² sec)
COUNTS / SEC × 10^3
TILT (DIG)
θ°
k (Å⁻¹)
Δk/k = 0.21

FIG. 4



ν = 43.2 REV/SEC
Δk/k = 0.35
MONITOR : O , WITHOUT Be-FILTER
● , WITH —"—
DASHED LINE : RATIO BETWEEN O AND ●
X : 1 ATM. PRESSURE IN FLIGHT TUBE
COUNTS/SEC x 10³
ABSOLUTE FLUX x 10⁵ (n/cm² sec)
TILT (DIG)
θ°
k (Å⁻¹)

The first point is that $I(Q)$ is given by the shape of the particle and by its internal structure. We have seen that, with some limitations, one can separate the two contributions and then reduce the complexity of the analysis. We shall first deal with a more simple case.

3.1.2.1. Spherical particle. In this case $\rho(r)$ is a scalar and (3.2) can be inverted, leading to

$$\rho(r)=\frac{1}{2\pi^2 r}\int_0^\infty QA(Q)\sin(Qr)\,\mathrm{d}Q \tag{3.23}$$

where $A(Q)$ is a real number, obtained from the square root of $I(Q)$. The only ambiguity is its sign. But the points where $A(Q)$ goes to zero are experimentally known and signs simply oscillate at each zero. Especially simple is a sphere of radius R with uniform density ρ. Then

$$I(Q)=9(\rho-\rho_S)^2\,V^2\left(\frac{\sin QR-QR\cos QR}{Q^3R^3}\right)^2. \tag{3.24}$$

This function is represented in figure 10. As we can see from figure 5 the features of figure 10 dominate the scattering by quasi-spherical objects. This will be illustrated later by examples.

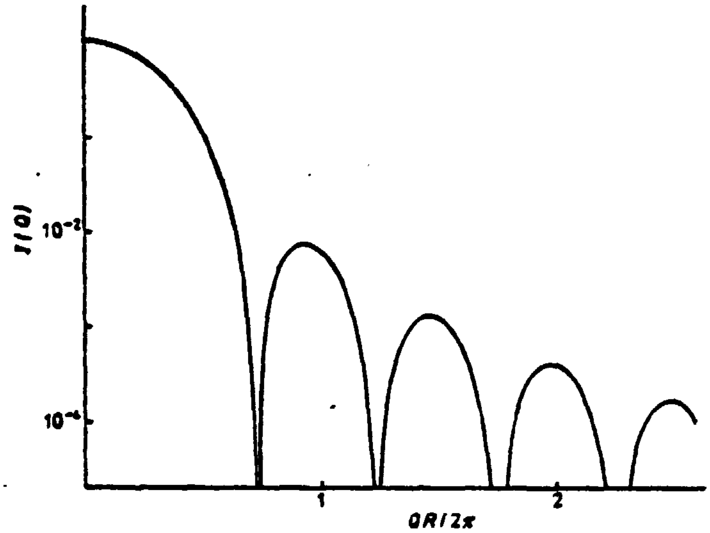


Figure 10. Intensity plotted against Q for a solid sphere of radius R. The analytical expression is given in relation (3.24).

3.1.2.2. The Fourier transform of the intensity. Following Porod (1951), we introduce

$$P(r)=\frac{1}{2\pi^2 r}\int_0^\infty QI(Q)\sin(Qr)\,\mathrm{d}Q \tag{3.25}$$

where $P(r)$ is the probability of an interatomic distance r inside the particle. So this function goes to zero for a value which gives the maximum distance between atoms in the particle. This is illustrated in figure 11. Apart from this important fact,

Fig. 5

APPENDIX A

Resolution properties of the velocity selector

The upper part of Fig. A1 shows a section of the velocity selector. The slits have a width a and a length L, and form an angle θ relative to the beam axis (dashed line). θ is the sum of the intrinsic tilt angle of the slits in the drum and the angle between the drum axis and the beam axis. The angular velocity of the drum is ν revolutions/sec.

In the lower part of Fig. A1 OA denotes a wavevector parallel to the beam axis with the optimum value of k to be transmitted through the velocity selector. The condition for optimal transmission is most easily seen in a frame moving along with the slits at velocity $v = 2\pi R\nu$, R being the radius of the drum. In this frame the wavevector OA' is turned an angle κ/k relative to OA, where $\kappa = m/\hbar v$, and the transmission condition is simply $\kappa/k = \theta$, or

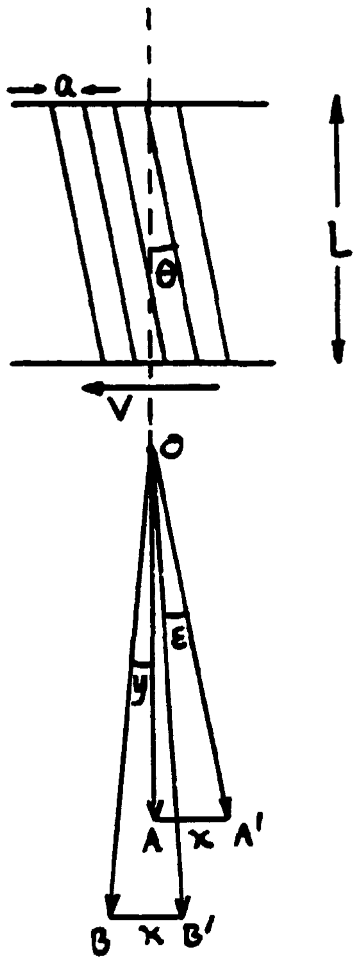


Fig. A1

$$k = \frac{2\pi Rm}{h}\,\frac{\nu}{\theta} \equiv c\cdot\frac{\nu}{\theta} \tag{A1}$$

For the present velocity selector R = 8.3 cm and thus c = 0.000829 A^{-1} sec.

We now want to calculate wavelength (-number) resolution of the velocity selector, i.e. we want to determine the bandwidth Δk around k which passes through the velocity selector.

In the following we assume that the neutrons entering the velocity selector have a square angular distribution of full width σ_H. In Fig. A1 the vector OB denotes a wavevector $k(1+x)$ deviating in length $(k\cdot x)$ and in direction (y) from OA. In the moving reference frame OB is turned into OB' and forms an angle $\varepsilon \approx \frac{\kappa}{k} x + y$ with respect to OA'. Only if $|\varepsilon| < \delta = a/L$ will this ray have a chance to pass the velocity selector, and assuming the transmission probability $P(x,y)$ to be triangular we obtain

$$P(x,y) = \begin{cases} 0 & \text{for } |\varepsilon| > \delta \\ 1 - \frac{|\varepsilon|}{\delta} & \text{for } |\varepsilon| < \delta \end{cases} \qquad \varepsilon(x,y) \approx \frac{\kappa}{k} x + y$$

According to the assumption that the angles y are evenly distributed between $-\frac{1}{2}\sigma_H$ and $\frac{1}{2}\sigma_H$ (square distribution) the wavelength distribution independent of y becomes

$$f(x) = \int_{y=-\frac{1}{2}\sigma_H}^{\frac{1}{2}\sigma_H} P(x,y)\,dy \qquad \text{(A2)}$$

The function $f(x)$ has a maximum for $x = 0$ and we want to determine the full width at half maximum (FWHM) Δx of $f(x)$ since this gives the relative bandwidth $\Delta k/k$ transmitted by the velocity selector. By introducing new variables $\alpha = \frac{\kappa}{k}/\delta$ $(= \theta/\delta)$, $\beta = \frac{1}{2}\sigma_H/\delta$, $x' = \alpha x$ and $y' = y/\delta$ we obtain

$$f(x') = \delta \int_{-\beta}^{\beta} T(x',y')\,dy' \qquad \text{(A3)}$$

where

$$T(x',y') = \begin{cases} 0 & \text{for } x' + y' > 1 \\ 1 - (x'+y') & \text{for } x' + y' < 1 \end{cases}$$

For each value of x' $f(x')$ is proportional to the section of the area of triangle $T(x',y')$ confined by the integration limits $y' = \pm\beta$. The FWHM $\Delta x'$ ($= \alpha\Delta x$) of $f(x')$ depends on the value of β as illustrated in Fig. A2. For $1 \leq \beta$ the hatched area in (a) represents $\frac{1}{\delta}$ $f(x' = 0) = 1$ and clearly the solution of $f(\frac{1}{2}\cdot\Delta x') = \frac{1}{2}f(x' = 0)$ yields $\Delta x' = 2\beta$ as shown in (b). For $\beta < 1$ $\frac{1}{\delta}f(x' = 0) = 2\beta - \beta^2$, and it can easily be found that for $0.4 \leq \beta \leq 1$ $\Delta x' = 2(1 + \beta - \sqrt{2\beta-\beta^2})$ (c and d) and for $\beta \leq 0.4$ $\Delta x' = 1 + \frac{1}{2}\beta$ (e and f). Figure A3 shows $\Delta x'$ as function of β, (solid curve) and the inset gives the bandwidth $\Delta k/k = \Delta x = \Delta x'/\alpha$ for the three regions of the parameter $\beta = \frac{1}{2}\sigma_H/\delta$.

If there is no collimation except that provided by the neutron guide and the MVS, the value of σ_H is equal to two times the critical reflection angle $\theta_c = 36.8/k$ (Å^{-1}) (min) of the Ni-coated walls of the guide tube. However, in the present set-up (see Fig. 1) the apertures at (1), (3) and (6) squeeze down the value of σ_H below $2\theta_c$. Thus, we obtain

$$\sigma_H \simeq \frac{\frac{1}{2}(1.3+2.5)}{490} = 13 \text{ min},$$

considerably smaller than typical values of $2\theta_c$. In addition, it should be mentioned that in this case the collimation modifies the assumed square distribution, making it more triangular. A resolution calculation based on Gaussian distributions is presented in Appendix B, and the result of $\Delta k/k$ versus β is included in Fig. A3 for comparison (dashed curve). It is seen that the results of the two treatments agree to within $\sim$ 15%. In the present case $\delta = 20$ min (see Appendix C) and thus $\beta = 0.33$. Both treatments yield

$$\frac{\Delta k}{k} = 1.2\,\frac{\delta}{\theta} = \frac{0.40}{\theta\,(\text{deg})} \qquad \text{(A2)}$$

and thus for fixed ν $\Delta k/k$ is proportional to k (see Eq. A1). The minimum bandwidth at $k = 1.9$ Å^{-1} can easily be found from Eqs. A1 and A2 and the maximum rotation frequency $\nu_{max} = 120$ rev/sec. One finds $(\Delta k/k)_{min} = 0.13$.

Finally, we want to consider the total transmitted intensity $I = \int f(x)dx \approx f(0)\Delta x$. In order to optimize the geometry of the velocity selector i.e. the ratio $\delta = a/L$, we consider the ratio $\frac{I}{\Delta k/k} = \frac{I}{\Delta x} = f(0)$ as function of δ. From Fig. A2 we obtain

$2\delta/\theta_H$	$\frac{I}{\Delta k/k}/\frac{1}{2}\sigma_H = 2f(0)/\sigma_H$
$0 < 2\delta/\theta_H < 1$	$2\delta/\theta_H$
$1 < 2\delta/\theta_H$	$\frac{2\delta}{\theta_H}(2\beta-\beta^2) = 2 - \sigma_H/2\delta$

The result is plotted in Fig. A4. We see immediately that δ should not be smaller than $\sim \frac{1}{2}\theta_H$, and a somewhat larger value of δ is preferable. If the ratio δ/θ_H is large θ should be increased to maintain a reasonable bandwidth and thereby ν must be increased to keep the wavelength fixed (see Eq. A1). In conclusion, for given values of k and $\frac{\Delta k}{k}$ a fast revolving drum with wide slits is preferable for a slow revolving drum with narrow slits.

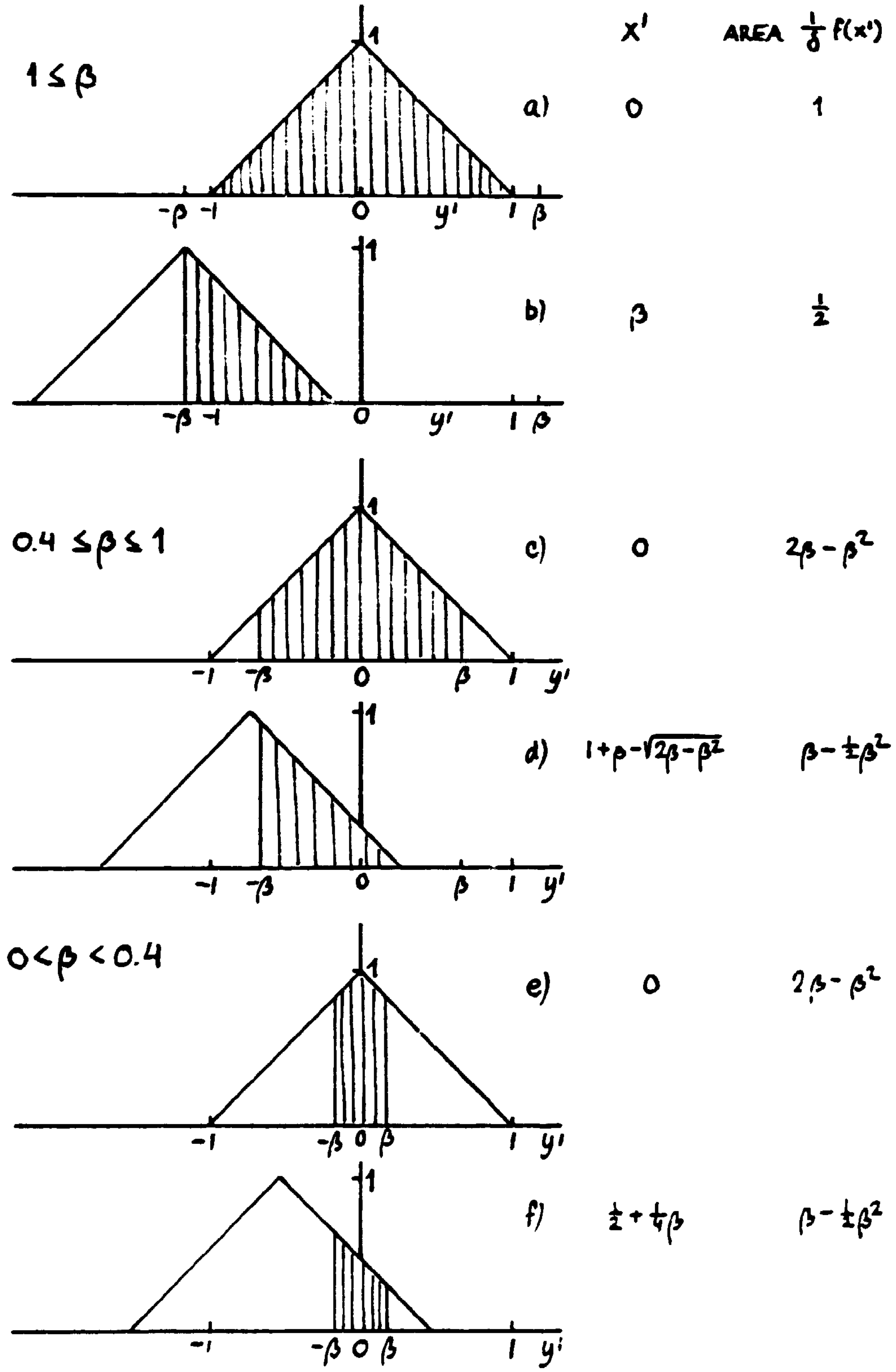

x'
AREA $\frac{1}{\delta} f(x')$
$1 \le \beta$
a) 0 1
$-\beta$ -1 0 y' 1 β
b) β $\frac{1}{2}$
$-\beta$ -1 0 y' 1 β
$0.4 \le \beta \le 1$
c) 0 $2\beta - \beta^2$
-1 $-\beta$ 0 β 1 y'
d) $1+\beta-\sqrt{2\beta-\beta^2}$ $\beta - \frac{1}{2}\beta^2$
-1 $-\beta$ 0 β 1 y'
$0 < \beta < 0.4$
e) 0 $2\beta - \beta^2$
-1 $-\beta$ 0 β 1 y'
f) $\frac{1}{2} + \frac{1}{4}\beta$ $\beta - \frac{1}{2}\beta^2$
-1 $-\beta$ 0 β 1 y'
1

FIG. A2

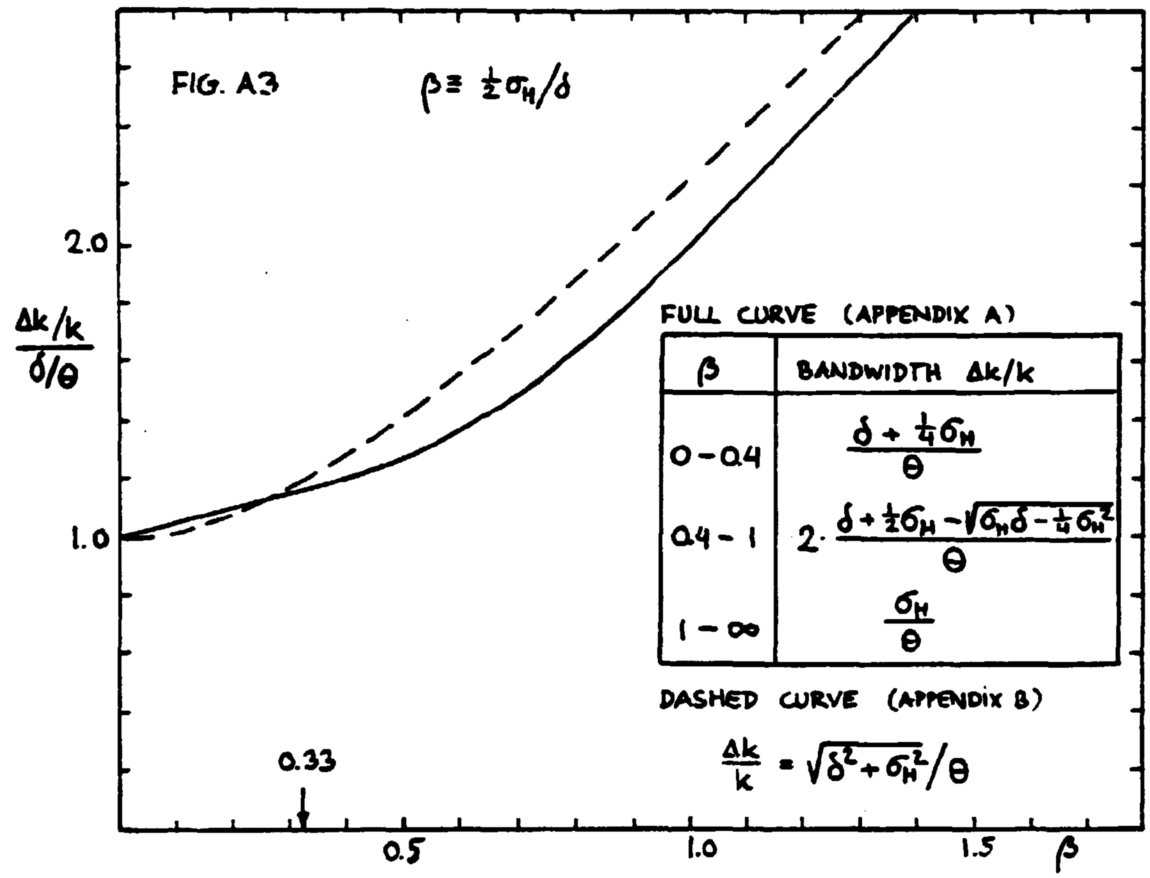


FIG. A3
$\beta \equiv \frac{1}{2}\sigma_H/\delta$
$\frac{\Delta k/k}{\delta/\theta}$
2.0
1.0
FULL CURVE (APPENDIX A)
β	BANDWIDTH $\Delta k/k$
0 – 0.4	$\frac{\delta + \frac{1}{4}\sigma_H}{\theta}$
0.4 – 1	$2 \cdot \frac{\delta + \frac{1}{2}\sigma_H - \sqrt{\sigma_H\delta - \frac{1}{4}\sigma_H^2}}{\theta}$
1 – ∞	$\frac{\sigma_H}{\theta}$
DASHED CURVE (APPENDIX B)
$\frac{\Delta k}{k} = \sqrt{\delta^2 + \sigma_H^2}/\theta$
0.33
0.5
1.0
1.5
β

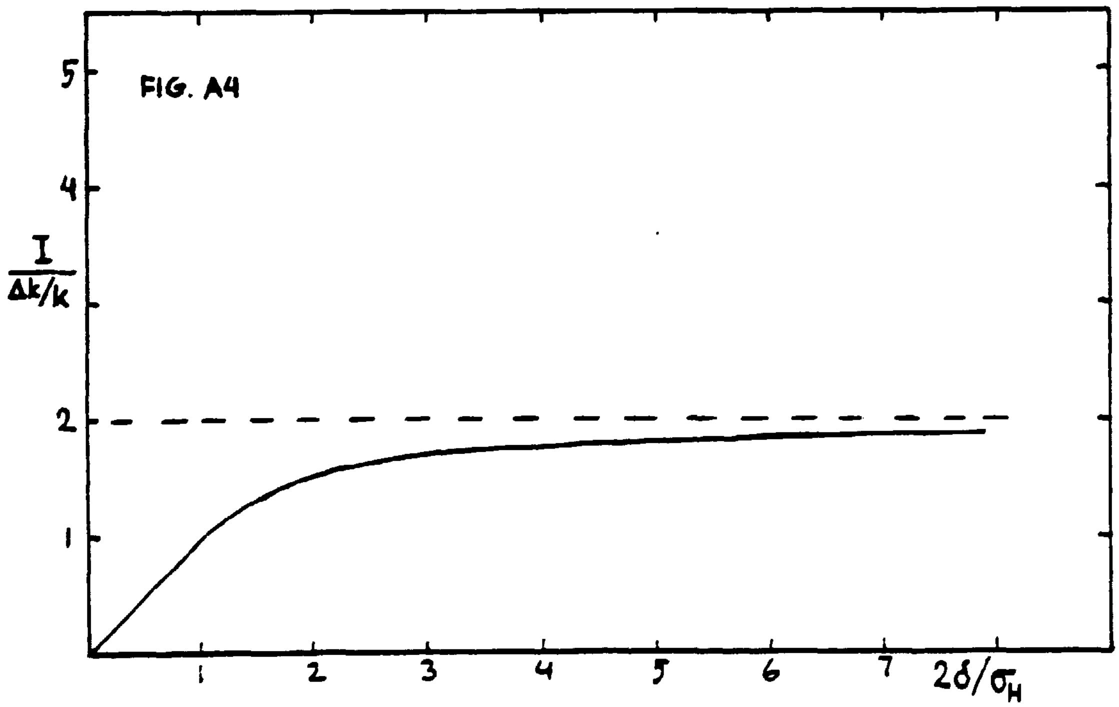


FIG. A4
$\frac{I}{\Delta k/k}$
5
4
2
1
1
2
3
4
5
6
7
$2\delta/\sigma_H$

APPENDIX B

Resolution calculation using Gaussian distributions

We consider a system consisting of three elements, a neutron guide which also acts as a collimator, a velocity selector and a collimator.

The corresponding probabilities for the transmission through each element are given by

$$P_1(\gamma_1) = \exp\{-\ell n2 \left(\frac{\gamma_1}{\sigma_1}\right)^2\} \qquad \text{(first collimator, or guide)}$$

$$P_2(\gamma_1,\Delta k) = \exp\{-\ell n2 \left(\frac{\frac{\Delta k}{k}\cdot\theta-\gamma_1}{\sigma_2}\right)^2\} \qquad \text{(MVS)}$$

$$P_3(\gamma_1) = \exp\{-\ell n2 \left(\frac{\gamma_1}{\sigma_3}\right)^2\} \qquad \text{(second collimator)}$$

where γ_1 denotes the horizontal angle between the actual neutron path and the most probable one, θ is the angle between the slits in the velocity selector and the beam. The simple relation between the most probable wavevector k, the revolution frequency ν and θ is

$$k = c\cdot\frac{\nu}{\theta} ,$$

where c = 0.0475 A^{-1}·deg·sec for the velocity selector used here (see Appendix A).

The combined transmission probability is simply the product

$$P_H(\gamma,\Delta k) = \exp\{-\ell n2[\left(\frac{\frac{\Delta k}{k_o}\cdot\theta_o-\gamma_1}{\sigma_2}\right)^2 + \left(\frac{\gamma_1}{\sigma_1}\right)^2 + \left(\frac{\gamma_1}{\sigma_2}\right)^2]\}$$

By integrating over γ_1 or Δk, respectively, one obtains the following expressions for the final bandwidth and horizontal angular distribution.

$$P_H(\gamma_1) = \sqrt{\frac{\pi}{\ell n2}}\,\frac{\sigma_2 k_o}{\theta_o}\,\exp\{-\ell n2\,(\frac{\gamma_1}{\sigma_H})^2\}$$

$$\sigma_H^2 = \frac{\sigma_1^2\ \sigma_3^2}{\sigma_1^2+\sigma_3^2}$$

$$P(\Delta k) = \sqrt{\frac{\pi}{\ell n2}}\left(\frac{\sigma_H^2\sigma_2^2}{\sigma_H^2+\sigma_2^2}\right)^{\frac{1}{2}}\exp\{-\ell n2\,\frac{(\frac{\Delta k}{k}\theta)^2}{\sigma_2^2+\sigma_H^2}\}$$

The total intensity can be calculated by another integration and gives

$$I = (\frac{\pi}{\ell n2})^{3/2}\,\frac{\sigma_1\sigma_2\sigma_3}{\sqrt{\sigma_1^2+\sigma_3^2}}\,\frac{k}{\theta}\cdot\frac{\sigma_{1v}\sigma_{3v}}{\sqrt{\sigma_{1v}^2+\sigma_{3v}^2}}\cdot\phi$$

where ϕ is the flux in the guide in units of n/sec/cm^2/steradian/Å^{-1} and σ_{iv} are the vertical collimations which are trivial to include since the vertical divergence is uncorrelated with the wavelength distribution.

Expressed in terms of the FWHM of the distributions, we can summarize the results in the following way

Horizontal divergence $\sigma_\gamma = \sigma_H = (\frac{\sigma_1^2\sigma_3^2}{\sigma_1^2+\sigma_3^2})^{\frac{1}{2}}$

Vertical divergence $\sigma_v = (\frac{\sigma_{1v}^2\sigma_{2v}^2}{\sigma_{1v}^2+\sigma_{2v}^2})^{\frac{1}{2}}$

Bandwidth $\sigma_k = \frac{k}{\theta}\sqrt{\sigma_2^2+\sigma_H^2}$

The ratio between the total intensity and the bandwidth is useful for considerations of the optimum velocity selector design and is given by

$$R = \frac{I}{\sigma_k} = \phi \left(\frac{\pi}{\ell n2}\right)^{3/2} \frac{\sigma_1\sigma_2\sigma_3\sigma_v}{\sqrt{\sigma_2^2\sigma_3^2+\sigma_3^2\sigma_1^2+\sigma_1^2\sigma_2^2}}$$

which immediately gives the criterion that $\sigma_2 > \sigma_H$ provided that the desired bandwidth can be obtained with the maximum frequency ν.

APPENDIX C

Dimensions of the velocity selector

Fig. C1 shows a cross section of the drum of the velocity selector. The square hole denotes the beam hole.

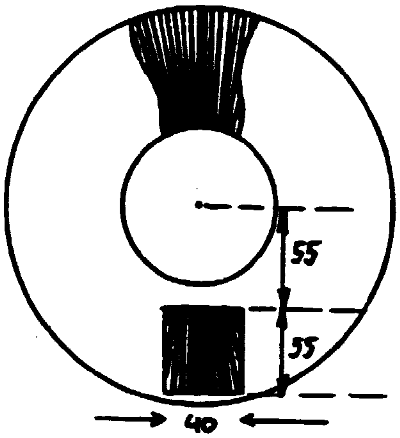


Fig. C1

The number of plates is 156 and at the centre of the beam hole the radius is $R = 55 + \frac{1}{2} \cdot 55$ mm = 83 mm. The thickness of the plates is 1 mm and the slit width at the centre of the beam hole therefore is

$$a = \frac{2\pi}{156} \cdot 8.3 - 1 = 2.3 \text{ mm}$$

The length of the drum is L = 400 mm and thus

$$\delta = \frac{2.3}{400} \cdot \frac{180}{\pi} \cdot 60 \text{ min} = 20 \text{ min.}$$

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MECHANICAL VELOCITY SELECTOR, NEUTRON FLUX AND Q-RANGE FOR THE SMALL ANGLE NEUTRON SCATTERING FACILITY AT RISØ

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Department or group

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Abstract

A mechanical velocity selector (MVS) was placed at the beam hole at the end of the neutron guide in the "Neutron House" at Risø. A 240 cm long evacuated tube with "pin hole" collimation was mounted after the velocity selector in order to simulate the geometry of the proposed small angle neutron scattering (SANS) instrument. At the end of the tube, corresponding to the sample position, cou nt rates and absolute fluxes were measured with a monitor and Au foil activiation, respectively. Likewise, Au foil flux measurements were performed immediately before and after the MVS. At the peak position $k = 1.9$ Å^{-1} ($\lambda = 3.3$ Å. $E = 7.2$ meV) of the cold source reactor spectrum and at bandwidth $\Delta k/k = 0.20$ the flux at the sample position was measured to $\phi = 1.0 \cdot 10^6$ n/cm^{2}sec. This is about 5 times lower than the corresponding flux obtained at the D11A SANS facility at ILL, Grenoble. A lower limit of momentum transfer for the proposed SANS instrument is estimated to $Q_{min} \sim 1 \cdot 10^{-3}$ Å^{-1}.

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