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Forskningscenter Risø, Roskilde

Publication date:
1982

Document Version
Publisher's PDF, also known as Version of record

Risø-M-2312

THE INFLUENCE OF BEAM BOUNDARIES AND VELOCITY REDUCTION ON PIERCE INSTABILITY IN LABORATORY PLASMAS

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Abstract. The influences of the beam-plasma boundary and of weak nonlinearities on the Pierce instability are investigated. It is shown that the finite width of the beam has negligible influence on both the stability of the system and growth rate. In the nonlinear regime the wavelength decreases and enhancement of the wave potential close to the beam inlet boundary is observed. The relationship between this effect and the formation of double layers is discussed.

UDC 533.9:621.039.61

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January 1982
Risø National Laboratory, DK-4000 Roskilde, Denmark

ISBN 87-550-0796-1
ISSN 0418-6435

Risø Repro 1982

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1. INTRODUCTION

It has been known for almost forty years that an electron beam penetrating throughout a plasma between two short-circuited conductive plates and completely neutralized is nonoscillatory unstable [1]. This so-called Pierce instability results from the interaction of the electrons in the beam (giving rise to the beam current), with the current in the external circuit maintaining the same potential at the two plates. The Pierce instability in such systems acts as a current-limiting mechanism. The virtual cathode creation [4] is also governed by the Pierce instability. Recently [2,3], the double-layer formation was attributed to the instability although experiments on the electron-beam-excited double layers [5,6] have been performed for more than a decade.

However, the complete physical picture of the Pierce instability, especially with respect to the real physical conditions under which double layers are formed, has not yet been obtained. The presence of the external magnetic fields and the finite transverse dimensions of the beam can alter the plasma dispersion characteristics significantly. Apart from that, the saturation mechanism of the instability and the transition from a multiwavelength field in long systems (with the length of several wavelengths of the Pierce field) to the double layer, which is a single peak of the electric field, has not yet been clarified.

This article has been prepared in connection with the current experiments on double layers produced by an electron beam at Risø Q-machine. This electron beam was of a diameter much less than the longitudial characteristic length (the distance of the double layer from the cathode), and could be regarded neither as infinitely wide [3], nor as wide as the plasma penetrated by it [5,7]. This paper consists of two parts: In the first the Pierce instability in a transversely bounded beam-plasma system will be discussed, and in the second the nonlinear stage in the development of the instability will be investigated. The possible saturation mechanisms and final state will also be analyzed in this part.

2. PIERCE INSTABILITY OF FINITE-WIDTH BEAM-PLASMA SYSTEMS

The presence of the boundaries in the beam-plasma systems gives rise to new eigenmodes of the oscillations, namely the surface wave modes [8,9], and to new mechanisms of energy transfer leading to instabilities [10]. The surface waves are confined to a narrow layer around the boundary, exponentially decreasing with distance from it.

2.1. Basic equations

We consider a plasma slab occupying the space ($0 < x < 2a$) penetrated by an electron beam of the same width with velocity, u, parallel to the z-axis. The plasma and beam are assumed to be homogeneous; n_{pe_o}, n_{be_o} are the electron plasma- and beam densities, respectively. The system is completely neutralizd by the plasma ions with the density n_{pi_o} ($= n_{be_o} + n_{pe_o}$). The slab is surrounded by quiet plasma, extending to $x = \pm\infty$; the electron and ion densities are n_{se_o} and n_{si_o}, respectively, and equal tc each other. The external magnetic field, B_o, is oriented along the z-axis.

We shall use the standard set of linearized hydrodynamic equations, describing a cold, collisionless plasma, the same assumption as in [1,7] and [11] to some extent. Assuming all the variables to be proportional to $e^{-i(\omega t - k_x \cdot x - k_z \cdot z)}$,

$$- i(\omega - k_z u) \cdot \vec{v}_{be} = \frac{e}{m}(\vec{E} + \vec{v}_{be} \times \vec{B}_o + \vec{u} \times \vec{B})$$

$$-i\omega \cdot \vec{v}_\alpha = \frac{e}{m}(\vec{E}+\vec{v}_\alpha x \vec{B}_O)$$

$$-i(\omega-k_z \cdot u) \cdot n_{be} = -n_{be_o} \cdot \nabla \cdot \vec{v}_\alpha$$

$$-i\omega \cdot n_\alpha = -n_{\alpha o} \cdot \nabla \cdot \vec{v}_\alpha \qquad (1)$$

α = pe, pi, se, si

Where the subscripts pe, pi, se, and si correspond to the plasma electrons and ions, and the surrounding electrons and ions, respectively. Introducing (1) to the Maxwell equation:

$$\frac{1}{\varepsilon_o}(e_e \cdot n_{be} + e_e \cdot n_{pe} + e_e \cdot n_{pe} + e_i \cdot n_{pi}) = \nabla((1-||\varepsilon||)\vec{E})$$

$$0 < x < 2a \qquad (2)$$

$$\frac{1}{\varepsilon_o}(e_e \cdot n_{se} + e_i \cdot n_{si}) = \nabla((1-||\varepsilon||)\vec{E})$$

$x > 2a$

$x < 0$

We obtain the expression for the dielectric tensor:

$$||\varepsilon|| = \left\| \begin{array}{ccc} \varepsilon_1+\phi_1 & i(\varepsilon_2+\phi_2) & -\frac{n_x}{n_z} \cdot \phi_1 \\ -i(\varepsilon_2+\phi_2) & \varepsilon_1+\phi_1 & i\frac{n_x}{n_z} \cdot \phi_2 \\ o & o & \varepsilon_3 \end{array} \right\| \qquad (3)$$

the components of which for $0 < x < 2a$ (in the beam) are:

$$\varepsilon_1 = 1 - \frac{\omega_{be}^2}{(\omega-k_z u)^2-\Omega_e^2} - \frac{\omega_{pe}^2}{\omega^2-\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2}$$

$$\varepsilon_2 = 1 - \frac{\omega_{be}^2 \quad \Omega_e}{(\omega-k_z u)((\omega-k_z u)^2-\Omega_e^2)} - \frac{\omega_{pe}^2 \Omega_e}{\omega(\omega^2-\Omega_e^2)}$$

$$\varepsilon_3 = 1 - \frac{\omega_{be}^2}{(\omega-k_z u)^2} - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{p1}^2}{\omega^2}$$

$$\phi_1 = \frac{k_z u}{\omega} \cdot \frac{\omega_{be}^2}{(\omega-k_z u)^2-\Omega_e^2}$$

$$\phi_2 = \frac{k_z u}{\omega} \cdot \frac{\omega_{be}^2 \quad \Omega_e}{(\omega-k_z u)((\omega-k_z u)^2-\Omega_e^2)}$$

and for $x < 0$, $x > 2a$ (in the surrounding plasma):

$$\varepsilon_1 = 1 - \frac{\omega_{se}^2}{\omega^2-\Omega_e^2} - \frac{\omega_{si}^2}{\omega^2}$$

$$\varepsilon_2 = - \frac{\omega_{se}^2 \quad \Omega_e}{\omega(\omega^2-\Omega_e^2)}$$

$$\varepsilon_3 = 1 - \frac{\omega_{se}^2}{\omega^2} - \frac{\omega_{si}^2}{\omega^2}$$

$$\phi_1 = \phi_2 = 0$$

With the standard notation, ω_α being the plasma frequency of the particle species α, and Ω_e, the electron gyrofrequency. For the sake of simplicity, the ion gyrofrequency was assumed equal to zero, implying either a weak magnetic field, B_o, or a relatively thin beam, with 2a of the order less than the ion Lamor radius (this last assumption is in agreement with the experimental conditions at the Risø Q-machine).

The boundary conditions describes the beam injection at $z = 0$:

$$v_{be}\ (z = 0) = 0$$
$$n_{be}\ (z = 0) = 0 \tag{6}$$

the existance of two perfectly conducting plates at $x = 0$ and $x = L$:

$$E_x\ (x = 0) = E_x\ (x = L) = E_y\ (x = 0) = E_y\ (x = L) = 0 \tag{7}$$

and the short circuiting of the two plates through the outer circuit yields the result:

$$\int_0^L E_z \cdot dz = 0 \tag{8}$$

In the contrast with [1-3, 11] we cannot strictly assume the wave field to be purely electrostatic, although we shall adopt the assumption:

$$\frac{c|\vec{k}|}{\omega} >> 1 \tag{9}$$

because the electrostatic, or quasi-electrostatic, waves alone cannot satisfy boundary condition (7) in the case of transversely bounded beams [12]. The equation describing the electromagnetic field [13]:

$$\vec{k} \cdot (||D|| \cdot \vec{E}) = 0 \tag{10}$$

$$D_{ij} = k^2\ \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \cdot \varepsilon_{ij}$$

gives the wellknown dispersion relation:

$$\det\ ||D|| = 0 \tag{11}$$

In the region $0 < x < 2a$ Eqs. (10) and (11) must have one solution corresponding to the Pierce field [1, 11]. Adopting $k_x = 0$, this solution is recovered. It has $E_x = E_y = 0$, and (10) reduces to the equation for the longitudinal (electrostatic) waves.

$$k_z \cdot \varepsilon_3 \cdot E_z = 0 \tag{12}$$

which allows three modes, with $k_{z_o} = 0$ and $k_{z_{1,2}}$ found from $\varepsilon_3(k_z,\omega) = 0$:

$$k_{z_{1,2}} = \frac{1}{u}\left(\omega \pm \frac{\omega_{be}}{\sqrt{1-\frac{\omega_{pe}^2+\omega_{pi}^2}{\omega^2}}}\right) \tag{13}$$

This Pierce field will "leak" to the quiet plasma regions $x < 0$, $x > 2a$. However, it can be shown that it will grow exponentially with the distance from the beam boundaries. So, it is necessary to include as well the solution with $k_x \neq 0$ and with k_z given by (13). These are the electromagnetic waves, having two modes for each choice of k_z:

$$k_{x_{1,2}}^2 = -k_{z_{1,2}}^2 \cdot \left(1-\frac{\omega}{ck_{z_{1,2}}}(\varepsilon_1+\phi_1-\frac{\varepsilon_2}{\varepsilon_1}(\varepsilon_2+\phi_2))\right) \tag{14}$$

$$E_{x_{1,2}} = \frac{k_{x_{1,2}}}{k_{z_{1,2}}} \cdot E_z \tag{15}$$

$$E_{y_{1,2}} = i \cdot \frac{k_{x_{1,2}}}{k_{z_{1,2}}} \cdot \frac{\varepsilon_1}{\varepsilon_2+\phi_2} \cdot E_z$$

E_z-arbitrary

In the limit $\frac{ck_z}{\omega} >> 1$, (14) reduces to

$$k_{x_{1,2}}^2 = - k_{z_{1,2}}^2$$

Outside of the beam, four modes for each choice of k_z (13) are possible. In the limit $ck_z/\omega >> 1$, these modes are:

1° electrostatic wave

$$k_{x_{1,2}}^2 \equiv \kappa_{x_{1,2}}^2 = - \frac{\varepsilon_3}{\varepsilon_1} k_{z_{1,2}}^2 \tag{16}$$

$$\frac{E_{x_{1,2}}}{E_z} = \frac{\kappa_{x_{1,2}}}{k_{z_{1,2}}} , \quad E_{y_{1,2}} = 0 \tag{17}$$

2° electromagnetic wave

$$k_{x_{1,2}}^2 = - k_{z_{1,2}}^2 \tag{18}$$

$$\frac{E_{x_{1,2}}}{E_z} = \frac{k_{x_{1,2}}}{k_{z_{1,2}}}, \quad \frac{E_{y_{1,2}}}{E_z} = i\frac{k_{x_{1,2}}}{k_{z_{1,2}}} \cdot \frac{\varepsilon_1 - \varepsilon_3}{\varepsilon_2} \tag{19}$$

The component of the Pierce field with $k_z = 0$ will leak in the x-direction as the "ordinary" electromagnetic wave with $E_x = E_y = 0$ and

$$k_{x_o}^2 = \frac{\omega^2}{c^2} \cdot \varepsilon_3 \tag{20}$$

2.2. Boundary conditions and the electric field structure

According to (13)-(20), the electric field in the region $0 \leqslant x \leqslant 2a$ can be written in the form:

$$\vec{E} = e^{-i\omega t}\Bigg[(\alpha_0^{(o)} + \alpha_1^{(o)} e^{ik_{z_1}z} + \alpha_2^{(o)} e^{ik_{z_2}z})\vec{e}_z$$

$$+ e^{ik_{z_1}z}\left(\alpha_1^{(+)} e^{ik_{x_1}x}\left\|\begin{matrix} i \\ \dfrac{-\varepsilon_1}{\varepsilon_2+\phi_2}(k_{z_1}) \\ 1 \end{matrix}\right\| + \alpha_1^{(-)} e^{ik_{x_1}x}\left\|\begin{matrix} -i \\ \dfrac{\varepsilon_1}{\varepsilon_2+\phi_2}(k_{z_1}) \\ 1 \end{matrix}\right\|\right) +$$

$$+ e^{ik_{z_2}z}\left(\alpha_2^{(+)} e^{ik_{x_2}x}\left\|\begin{matrix} i \\ \dfrac{-\varepsilon_1}{\varepsilon_2+\phi_2}(k_{z_2}) \\ 1 \end{matrix}\right\| + \alpha_2^{(-)} e^{-ik_{x_2}x}\left\|\begin{matrix} -i \\ \dfrac{\varepsilon_1}{\varepsilon_2+\phi_2}(k_{z_2}) \\ 1 \end{matrix}\right\|\right)\Bigg]$$

$$+ \text{c.c.} \tag{21}$$

where α denotes arbitrary constants of integration. For reasons of symmetry, $E_z\ (x = 0) = E_z\ (x = 2a)$, and the following conditions need be satisfied:

$$\alpha_{1,2}^{(+)} \cdot e^{ik_{x_{1,2}}\cdot a} = \alpha_{1,2}^{(-)} \cdot e^{-k_{x_{1,2}}\cdot a} \tag{22}$$

The Pierce field corresponds to the terms with the superscripts 0, and the rest is the electromagnetic surface wave.

In the free plasma region $x < o$, the field has the following structure:

$$\vec{E} = e^{-i\omega t}\Bigg[\beta^{(o)} e^{k_{x_0}x} \cdot \vec{e}_z$$

$$+ e^{ik_{z_1}z}\left(\beta_1^{(1)} e^{-i\kappa_{x_1}x}\left\|\begin{matrix} -i\sqrt{\dfrac{\varepsilon_3}{\varepsilon_1}} \\ 0 \\ 1 \end{matrix}\right\| + \beta_1^{(2)} e^{-ik_{x_1}x}\left\|\begin{matrix} \dfrac{\varepsilon_1-\varepsilon_3}{\varepsilon_2} \\ 1 \end{matrix}\right\|\right) +$$

$$+e^{ik_{z2}z}\left(\beta_2^{(1)}e^{i\kappa_{x2}x}\left\|\begin{matrix} i\sqrt{\dfrac{\varepsilon_3}{\varepsilon_1}} \\ 0 \\ 1 \end{matrix}\right\| + \beta_2^{(2)}e^{ik_{x2}x}\left\|\begin{matrix} i \\ -\dfrac{\varepsilon_1-\varepsilon_3}{\varepsilon_2} \\ 1 \end{matrix}\right\|\right)\Bigg]$$

$$+\ \text{c.c.} \tag{23}$$

Where the exponentially growing terms have been removed. The field at $x > 2a$ is symmetrical with this one.

The terms with the superscripts 0, 1, 2 correspond to the "leaking" of the Pierce field, the potential, and the electromagnetic surface waves, respectively.

The boundary conditions of the beam plasma boundary $x = o$:

$$\begin{aligned} E_y(0+) &= E_y(0-) \\ E_z(0+) &= E_z(0-) \end{aligned} \tag{24}$$

$$(\varepsilon_1+\phi_1)E_x+i(\varepsilon_2+\phi_2)E_y - \frac{k_x}{k_z}\phi_1 E_z\Big|_{0+} = \varepsilon_1 E_x+i\,\varepsilon_2 E_y\Big|_{0-}$$

together with (22) lead to:

$$\alpha^{(\pm)}_{1,2} = \frac{\alpha^{(o)}_{1,2}}{\psi_{1,2}}\cdot\frac{-e^{\mp ik_{x_{1,2}}\cdot a}}{2\,\mathrm{sh}(-ik_{x_{1,2}}\cdot a)}\cdot\frac{\varepsilon_2+\phi_2}{\varepsilon_1}\Big|_{k_{z_{1,2}}}\cdot\frac{\varepsilon_1-\varepsilon_3}{\varepsilon_2}\Big|_{x=0-}$$

$$\beta^{(1)}_{1,2} = \frac{\alpha^{(o)}_{1,2}}{\psi_{1,2}}$$

$$\psi_{1,2} = 1-\sqrt{\frac{\varepsilon_3}{\varepsilon_1}}\Big|_{x=0-}+\mathrm{cth}(-k_{x_{1,2}}\cdot a)\cdot\frac{\varepsilon_2+\phi_2}{\varepsilon_1}\Big|_{k_{z_{1,2}}}\cdot\frac{\varepsilon_1-\varepsilon_3}{\varepsilon_2}\Big|_{x=0-}$$

$$\tag{25}$$

The boundary conditions of the beam injection at $z = 0$ (6) and the equipotentiality of the plates can be met for the Pierce field [1, 11], leading to:

$$\alpha_1^{(o)} = \frac{A}{k_{z_1}}$$

$$\alpha_2^{(o)} = \frac{-A}{k_{z_2}}$$

$$\alpha_o^{(o)} = -\frac{\omega^2}{(\omega-k_z u)^2} \cdot (\alpha_1^{(0)}+\alpha_2^{(0)}) \tag{26}$$

$$= A \cdot \frac{-\omega^2}{(\omega-k_z u)^2} \cdot \left(\frac{1}{k_{z_1}} - \frac{1}{k_{z_2}}\right)$$

(A being an arbitrary real constant) and to the following dispersion relation:

$$\frac{-i\omega^2}{(\omega-k_z u)^2}\left(\frac{1}{k_{z_1}} - \frac{1}{k_{z_2}}\right) + \frac{1}{k_{z_1}^2} \cdot e^{ik_{z_1} \cdot L} - \frac{1}{k_{z_2}^2} \cdot e^{ik_{z_2} \cdot L}$$

$$= \frac{1}{k_{z_1}^2} - \frac{1}{k_{z_2}^2} \tag{27}$$

Eq. (27) has a purely imaginary solution $\omega = i\alpha$, with a possible marginally stable choice of system length. This length becomes

$$L_{st} = n\pi \cdot \frac{u}{\omega_{be}} \text{ (n is an arbitrary integer)} \tag{28}$$

This is true only in the simplest case of no background electrons and imobile ions [1]; otherwise [11] the system is always nonoscillatory unstable. One can readily see that the constant component of the Pierce field $e^{\gamma t} \cdot \alpha_o^{(o)} \sim \gamma^2$ is not present in the marginally stable case (28). It appears, however, if the system is made longer than L_{st}, in order to cancel the surplus of $\int_0^L E_z dz$. Using the expression $\omega = i\gamma$, $\gamma \ll \min(\omega_{be}, \Omega_e)$, (21) and (23) can be simplified, yielding for $0 < x < 2a$ (in the beam):

$$\vec{E} = e^{\gamma t} \cdot \left[\left(\frac{1}{2}\alpha_o^{(o)} + \alpha_1^{(o)} e^{ik_{z_1} \cdot z}\right)\vec{e}_z - \right.$$

$$-\frac{\alpha_1^{(o)}}{2\mathrm{ch}(-ik_{x_1} \cdot a)}\left(e^{ik_{x_1}(x-a)} \left\| \begin{matrix} i \\ i \cdot \dfrac{-\gamma\Omega_e}{\omega_{be}^2} \cdot \zeta \\ 1 \end{matrix} \right\| + e^{-ik_{x_1}(x-a)} \cdot \left\| \begin{matrix} -i \\ i \dfrac{\gamma\Omega_e}{\omega_{be}^2} \cdot \zeta \\ 1 \end{matrix} \right\| \right)$$

$$\left. + \text{c.c.} \right], \quad 0 < x < 2a \tag{29}$$

and for $x < 0$ (free plasma):

$$\vec{E} = e^{\gamma t} \left[\frac{1}{2}\alpha_o^{(o)} e^{k_{x_o} \cdot x} \cdot \vec{e}_z + \right.$$

$$+\zeta \cdot \alpha_1^{(o)} \cdot \mathrm{th}(-ik_{x_1} \cdot a) \cdot e^{i(k_{z_1} z - \kappa_{x_1} x)} \cdot \left\| \begin{matrix} \dfrac{-i \cdot \omega_{se}^2}{\omega_{be}^2 \left(1 + \dfrac{\omega_{si}^2}{\gamma^2} + \dfrac{\omega_{se}^2}{\gamma^2 + \Omega_e^2}\right)} \\ 0 \\ \dfrac{\gamma \cdot \omega_{s_e}}{\omega_{be}^2 \sqrt{1 + \dfrac{\omega_{si}^2}{\gamma^2} + \dfrac{\omega_{se}^2}{\gamma^2 + \Omega_e^2}}} \end{matrix} \right\|$$

$$+\zeta\cdot\alpha_1^{(o)}\cdot th(-ik_{x_1}\cdot a)\cdot e^{i(k_{z_1}\cdot z-k_{x_1}\cdot x)}\left\|\begin{matrix} i\dfrac{\gamma^2}{\omega_{be}{}^2} \\ \\ i\,\dfrac{\gamma\cdot\Omega_e}{\omega_{be}{}^2} \\ \\ -\dfrac{\gamma^2}{\omega_{be}{}^2}\end{matrix}\right\| + c.c.\Bigg]$$

$$\zeta \equiv \frac{1+\dfrac{\omega_{pe}^2}{\omega_{be}^2}}{1+\dfrac{\omega_{pi}^2}{\gamma^2}} \tag{30}$$

One can see that the z-component of the surface-wave electric field has the same structure as the Pierce field, apart from the exponential decay with distance from the beam boundary, $x = 0$. In the marginally stable case, $\gamma = 0$ (28), the boundary conditions (6) - (8) are also satisfied for the surface-wave component of the field. Extending the system slightly, conditions (6)-(8) are violated. The violation of (8) will be more important because one can see from (30), that the surface waves component parallel to the conductive plates:

$$E_{||}(x=0,L) \sim \sin\frac{\gamma\cdot x}{u}\cdot e^{\frac{-\omega_{se}\cdot x}{u}}\cdot E_z(x=0,L) \tag{31}$$

and (7) is still well satisfied, provided $\gamma \ll \omega_{se}$. In analogy with the Pierce case (26) violation of (8) can be overcome by the introduction of E_{aux} - an auxiliary z-oriented electric field independent of z and with the same x-vari tion as the surface wave component:

$$E_{aux} = \vec{e}_z \cdot e^{i\omega t} \cdot \frac{-\omega^2}{(\omega - k_z u)^2} \cdot \frac{\alpha_1^{(o)}}{2} \cdot \frac{ch(-ik_{x_1} \cdot (x-a))}{ch(-ik_{x_1} \cdot a)} + c.c.$$

$$0 < x < 2a \qquad (32)$$

and the analogous expressions for $x < 0$, $x > 2a$. E_{aux} is not an eigenmode, and one would expect it to propagate out of the regions $x = 2a$, assuming its value at each thin layer $(x, x+dx)$ to be forced boundary condition. However, the group velocity for electromagnetic waves with the low frequency, $\omega = i\gamma$, is almost equal to zero [13], and E_{aux} will not spread. Besides that, its amplitude is proportional to $\gamma^2/\omega_{be}^2 << 1$, and in most of the cases it can be neglected.

The Pierce field and x- and z-components of the surface wave field can be described by an electric potential (see(12)-(19/)). On the other hand, the electromagnetic y-component of the surface wave is proportional to $\gamma\Omega_e/\omega_{be}^2$, and can be neglected, unless the magnetic field is very strong. The structure of this potential is plotted in Fig. 1. One can see from (12)-(19) that it is confined within the beam, and only a small portion $\sim \gamma/\omega_{be}$ will "leak" to the free plasma, so it can be neglected. The characteristic length in the x-direction, ℓ_x, scales as:

$$\frac{\ell x}{\ell_z} \sim \frac{sh\frac{x}{\ell_z}}{ch\frac{a}{\ell_z} - ch\frac{x}{\ell_z}} \qquad (33)$$

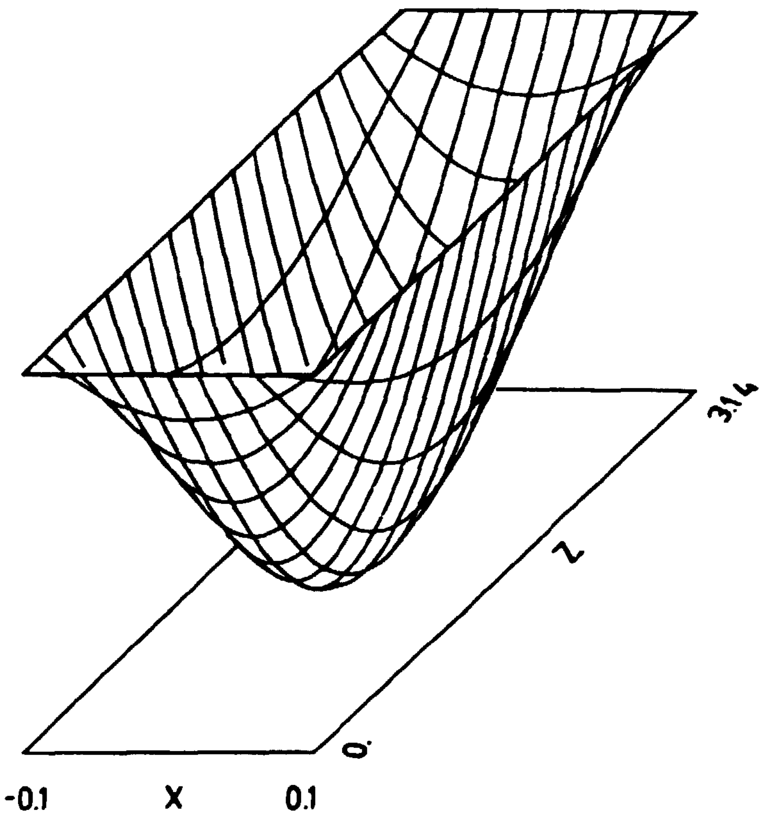

Fig. 1. Electric potential (in arbitrary units) for the marginally stable case $\gamma = 0$. The length of the system is equal to the critical length, $L = \pi \cdot (u)/(\omega_{be})$, while the width of the beam is $2a = (1)/(5\pi) \cdot L$.

Where ℓ_z is the characteristic length in the z-direction, and for $z \ll \ell_z$ we have $\ell_x \ll \ell_z|_{x\neq o}$ i.e. the potential well has much steeper "walls" in the transverse than in the longitudinal direction.

From the results above it can be concluded that the presence of the beam boundaries and the external magnetic field do not affect the stability of the system. The z-component of the wave field is the same as in the one-dimensional case [1]. Apart from its x-dependence, the growth-rate governed by (8) is the same.

3. THE NONLINEAR STAGE AND SATURATION OF THE PIERCE INSTABILITY

It as been proposed [3,4] that the virtual cathode and/or double layer represent the final (saturated) stage of the growth of the (negative) potential well of the Pierce field (29). However, the actual mechanism of the transition from the linear to saturated regime has been unclarified. The authors of [14] claim that in the process of forming the electron and ion holes the front of the beam, being strongly unstable, gives rise to particle trapping during the transition time. This occurs before the beam front has reached the other end of the system. These phenomena, supposed to be connected with the formation of double layers, can exist only in the kinetic regime. In the case of penetration of the heavy ion background by an electron beam, which is treated here, the characteristic time of the leectric potential growth $1/\gamma$, is much larger than the beam transition time [1], and the hydrodynamic description is applied. Besides that, it will be shown below that the group velocities of the linear and the nonlinear response are less than or equal to the beam velocity. As the conditions in front of the wave front do not affect the wave, the physics of the process is the same during the transition time and after the beam front has reached the other boundary. Thus, no special treatment of the transient phenomena is necessary [18]. The nonlinear development of the instability and the saturated state were treated in [15, 16]. The authors of [15] recognise the saturation mechanism in beam deceleration, treating the weak nonlinear case (retaining the quadratic nonlinear terms only). Their analysis is somewhat insufficient, because only slightly supercritical systems of the length:

$$L = L_C + \delta \quad , \quad L_C = \frac{\pi u}{\omega_{be}} \, , \quad \delta \ll L_C$$

are regarded. However, the suppression of the wave field at the distance $x > L_C$, leading to the double layer and/or virtual cathode formation (being a single potential jump) is left out.

In this section, electron beam deceleration caused by build up of the electric potential is calculated, taking into account second-order nonlinearities. The nonlinear development of the potential is obtained regarding the interaction of the perturbed (decelerated) beam with the plasma, in the next step of the calculations. For the sake of simplicity, the condition (8)

$$\int_0^L E \cdot dz = 0$$

is not taken into account, i.e. the far boundary is assumed to be free.

3.1. Deceleration of the beam

Following [1,15,16] we shall use the hydrodynamical description of the cold, collisionless plasma, consisting of heavy ions penetrated by an electron beam of infinite width, injected at $z = 0$, with velocity u, along the z-axis, and of desity, n_{be_o}. The equation for the first- and second-order perturbations, i.e. the linear and the lowest order nonlinear response in the quasi-electrostatic limit are:

$$\frac{\partial v_1}{\partial t} + v_1\frac{\partial v_1}{\partial z} + v_o\frac{\partial v_1}{\partial z} = \frac{e}{m}E_1$$

$$\frac{\partial n_1}{\partial t} + n_o\frac{\partial v_1}{\partial z} + v_1\frac{\partial n_o}{\partial z} + v_o\frac{\partial n_1}{\partial z} + n_1\frac{\partial v_o}{\partial z} = 0$$

$$\frac{\partial E_1}{\partial z} = \frac{en_1}{\varepsilon_o} \tag{34}$$

$$\frac{\partial v_2}{\partial t} + v_2\frac{\partial v_o}{\partial z} + v_o\frac{\partial v_2}{\partial z} = \frac{e}{m}E_2 - v_1\frac{\partial v_1}{\partial z}$$

$$\frac{\partial n_2}{\partial t} + n_o\frac{\partial v_2}{\partial z} + v_2\frac{\partial n_o}{\partial z} + v_o\frac{\partial n_2}{\partial z} + n_2\frac{\partial v_o}{\partial z} = -\frac{\partial}{\partial z}(n_1 v_1)$$

$$\frac{\partial E_2}{\partial t} = \frac{en_2}{\varepsilon_o} \tag{35}$$

Where v_o, n_o are the beam velocity and density, while subcripts 1, 2 correspond to the first- and second-order perturbations, respectively.

In the first approximation, assuming:

$$n_o = n_{be_o}, \quad v_o = u \tag{36}$$

Eq. (34) with boundary conditions (6)-(8) give the linear Pierce field:

$$E_1 = e^{-i\omega_o t}\left(\frac{\alpha_o}{2}+\alpha_1 \cdot e^{ik_o \cdot z}\right) + c.c.$$

$$k_o = \frac{1}{u}(\omega_o+\omega_{be})$$

$$\alpha_o = -\frac{\omega_o^2}{(\omega_o-k_o u)^2} \cdot \alpha_1 \quad , \quad \alpha_1 = \frac{A}{k_o} \tag{37}$$

A being an arbitrary real constant. The frequency, ω_0, is purely imaginary, found from the dispersion relation (27).

Substituting (36) and (37) in (35) and applying the Laplace transformation in time and space, with the initial and boundary values:

$$v_2(t=0) = v_2(z=0) = n_2(t=0) = n_2(z=0) = 0 \tag{38}$$

we obtain:

$$ik\cdot\varepsilon(\omega,k)\cdot E_2 = \frac{k}{\omega}\cdot J_{NL}+E_2(z=0) \tag{39}$$

where $\varepsilon = 1 - (\omega^2{}_{be})/((\omega-ku)^2)$ and J_{NL} is the nonlinear current [17]:

$$J_{NL} = -\frac{m}{e}\cdot i\omega(1-\varepsilon)\int_0^\infty\!\!\int_0^\infty dtdz\cdot e^{i(\omega t-kz)}\cdot v_1\frac{\partial v_1}{\partial z} + \frac{\omega}{\omega-ku}\cdot\int_0^\infty\!\!\int_0^\infty dtdz\cdot e^{i(\omega t-kz)}\cdot\frac{en_1v_1}{\varepsilon_0} \tag{40}$$

The second-order perturbations of the velocity and the electron density, using (35) and (39), can be expressed in the form:

$$v_2 = \frac{e}{m}\cdot\frac{1}{(\omega-ku)^2-\omega_{be}^2}\cdot\left(\frac{\omega-ku}{k}\cdot E_2(z=0) - \frac{m}{e}\cdot i(\omega-ku)\int_0^\infty\!\!\int_0^\infty dtdz\cdot e^{i(\omega t-kz)}\cdot v_1\frac{\partial v_1}{\partial z} + \int_0^\infty\!\!\int_0^\infty dtdz\cdot e^{i(\omega t-kz)}\cdot\frac{en_1v_1}{\varepsilon_0}\right) \tag{41}$$

$$n_2 = \frac{\varepsilon_o}{e} \cdot \frac{1}{(\omega-ku)^2-\omega_{be}^2} \cdot \Bigg(\omega_{be}^2 \cdot E_2(z=0) \; -$$

$$- \, ik \cdot \frac{m}{e} \cdot \omega_{be}^2 \iint_{\infty}^{\infty} dtdz \cdot e^{i(\omega t-kz)} \cdot v_1 \frac{\partial v_1}{\partial z}$$

$$- \, ik \cdot i(\omega-ku) \cdot \iint_{\infty}^{\infty} dtdz \cdot e^{i(\omega t-kz)} \cdot \frac{en_1 v_1}{\varepsilon_o} \Bigg) \tag{42}$$

Using the expression (37) for the first-order electric field, assuming the frequency to be purely imaginary ($\omega_o = i\gamma$), we obtain:

$$\iint_{\infty}^{\infty} dtdz \cdot e^{i(\omega t-kz)} \cdot v_1 \frac{\partial v_1}{\partial z} =$$

$$= \left(\frac{e}{m}\right)^2 \cdot \frac{-ik_o}{\omega-2\omega_o} \cdot \Bigg(\frac{1}{k-k_o} \cdot \frac{\alpha_1}{\omega_o-k_o u} \cdot \frac{1}{2}\left(\frac{\alpha_o}{\omega_o} - \frac{\alpha_o^*}{\omega_o^*}\right) +$$

$$+ \frac{1}{k-2k_o} \cdot \frac{\alpha_1^{\,2}}{(\omega_o-k_o u)^2} + \frac{1}{k-(k_o-k_o^*)} \cdot \left|\frac{\alpha_1}{\omega_o-k_o u}\right|^2 \Bigg) + c.c. \tag{43}$$

$$\iint_{\infty}^{\infty} dtdz \cdot e^{i(\omega t-kz)} \cdot \frac{en_1 v_1}{\varepsilon_o} =$$

$$= \frac{e}{m} \cdot \frac{\omega_{be}^2}{\omega-2\omega_o} \cdot \frac{-k_o}{(\omega_o-k_o u)^2} \cdot \Bigg(\frac{1}{k-k_o} \cdot \alpha_1 \cdot \frac{1}{2}\left(\frac{\alpha_o}{\omega_o} - \frac{\alpha_o^*}{\omega_o^*}\right) +$$

$$+ \frac{1}{k-2k_o} \cdot \frac{\alpha_1^{\,2}}{\omega_o-k_o u} - \frac{1}{k-(k_o-k_o^*)} \cdot \frac{|\alpha_1|^2}{(\omega_o-k_o u)^*} \Bigg) + c.c. \tag{44}$$

Consequently, the nonlinear current J_{NL} is purely growing, with twice the growth rate of the first-order field. In order to satisfy the boundary condition, the constant of integration E_2 ($z=0$, t) will have the same time variation, i.e. E_2 $(z=0) \sim (\omega-2\omega_o)^{-1}$

It can be seen from (41)-(44) that the second-order perturbation can be expressed as:

$$E_2(z,t) = \sum_n E_2^{(n)} \cdot e^{i(2\omega_o t-k_n \cdot z)} \tag{45}$$

Where the summaration is performed over all possible wave numbers. There are five possible modes:

$$\begin{aligned} k_1 &= 0 \\ k_2 &= k_o-k_o^* = i \cdot \frac{2\gamma}{u} \\ k_3 &= k_o = \frac{1}{u}(i\gamma+\omega_{be}) \\ k_4 &= 2k_o \\ k_5 &= k(2\omega_o) = \frac{1}{u}(2i\gamma+\omega_{be}) \end{aligned} \tag{46}$$

with the analogous complex-conjugates.

One can see that all the modes (45) have the group velocity equal to u, except the third one, whose group velocity is $\frac{1}{2}u$. The first two modes have no z-oscillatory character, and the corresponding terms of $v_2(z,t)$, $n_2(z,t)$ can be recognized as the perturbations of the beam velocity and density, respectively, while the other terms are higher space harmonics. Integrating (41) and (42) and adopting E_2 ($z=0$) in agreement with boundary conditions (38), the perturbation of the beam velocity and density can be written in the form:

$$v_2^{(o)} = u \cdot 2\left(2\pi \cdot \frac{e}{m} \cdot \frac{|\alpha_1|}{u \cdot \omega_{be}}\right)^2 \cdot e^{2\gamma t} \cdot (e^{-2\gamma z/u}-1)$$

$$n_2^{(o)} = n_{be_o} \cdot \left(\frac{2\gamma}{\omega_{be}}\right)^2 \cdot \left(2\pi\frac{e}{m} \cdot \frac{|\alpha_1|}{u \cdot \omega_{be}}\right)^2 \cdot e^{2\gamma t} \left(e^{-2\gamma z/u} - 1\right) \tag{47}$$

One can see that the density perturbation is negligably small. The surplus of electrons, obtained by the beam deceleration is "stored" as the Pierce field related density perturbation rather than becoming "free" background electrons. The perturbed beam velocity profile is plotted in Fig. 2.

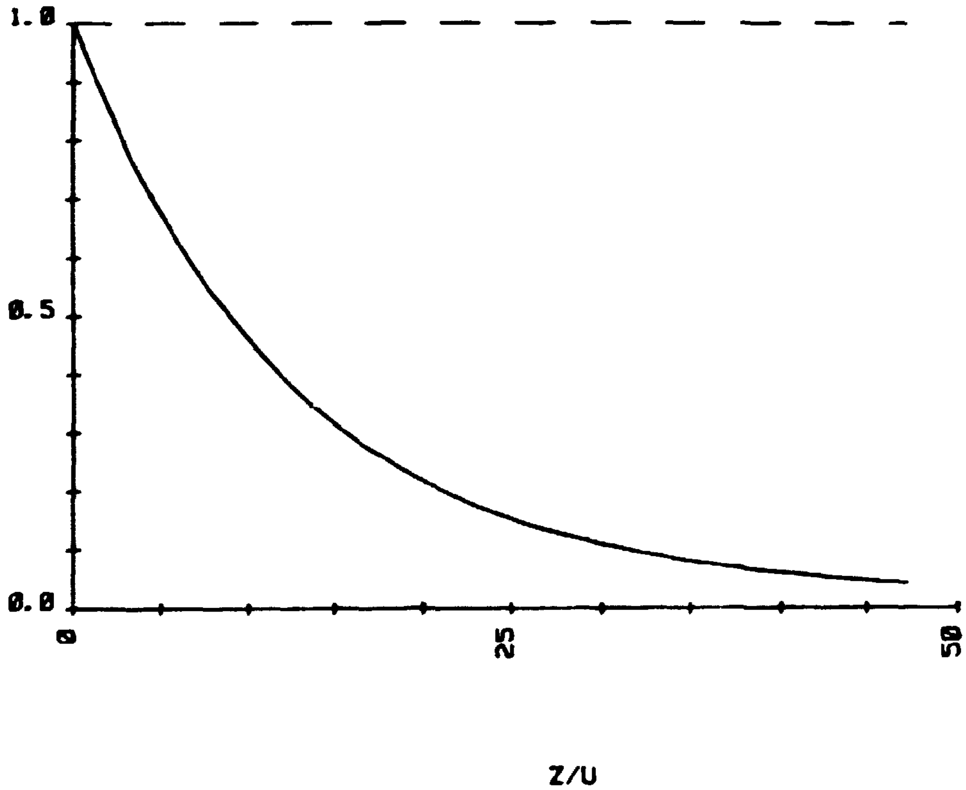

Fig. 2. Velocity profile, $(v_o)/(u)$, versus the distance from the beam inlet, $z \cdot (\omega_{be})/(u)$. The growth rate is taken to be $\gamma = 4.10^{-2}.\omega_{be}$. The dashed line corresponds to the unperturbed beam, while the solid line is for $t-t_o = -0.25$.

3.2. Nonlinear development

The destortion of the Pierce field due to the nonlinearities can be investigated further by introducing the perturbed beam-related quantities (47) into the initial set of equations (34). The perturbation of the beam density will be neglected, and the z-variation of the beam velocity:

$$v_o = u + v_2^{(o)} = u\left[1 + e^{2\gamma(t-t_o)} \cdot (e^{-2\gamma z/u} - 1)\right]$$

$$t_o \equiv -\frac{1}{\gamma}\ln\left(2\pi\cdot\sqrt{2}\cdot\frac{e}{m}\frac{|\alpha_1|}{u\cdot\omega_{be}}\right) \tag{48}$$

will be assumed weak compared with the variation of the Pierce field. In the case of the weak nonlinearity, the field should be much different than the unperturbed one, which behaves as $e^{-\gamma z/u}\cdot\sin(\omega_{be}z/u)$, while v_o varies like $e^{-(2\gamma z)/(u)}$, and above WKB assumption is valid, provided $\gamma \ll \omega_{be}$. Neglecting the terms: $(\partial v_o)/(\partial z)$, $(\partial^2 v_o)/(\partial z^2)$, the equation for the electron density reduces to:

$$\frac{\partial^2 n_1}{\partial t^2} + 2v_o\frac{\partial^2 n_1}{\partial z\partial t} + v_o^2\frac{\partial^2 n_1}{\partial z^2} + \frac{\partial v_o}{\partial t}\frac{\partial n_1}{\partial z} + \omega_{be}^2 n_1 = v_o^2\left.\frac{\partial n_1}{\partial z}\right|_{z=0} \tag{49}$$

Using the Laplace transformation in z (with $n_1\ (z=0)=0$) and introducing the new variable, $\tau = e^{\gamma(t-t_o)}$, Eq. (49) can be written in the form:

$$\frac{\partial^2 n_1}{\partial \tau^2} + P\cdot\frac{\partial n_1}{\partial t} + Q\cdot n_1 = \frac{v_o^2}{\gamma^2\tau^2}\left.\frac{\partial n_1}{\partial z}\right|_{z=0} \tag{50}$$

$$P = \frac{1}{\gamma\tau} \cdot (\gamma+2iku+2iku\cdot\xi\cdot\tau^2)$$

$$Q = \frac{1}{\gamma^2\tau^2} \cdot ((iku)^2+\omega_{be}^2+2iku(\gamma+iku)\cdot\xi\tau^2 + (iku)^2\xi^2\tau^2)$$

$$\xi = e^{-2\gamma z/u}-1$$

which is further simplified to:

$$\frac{d^2U}{d\tau^2} + \kappa^2(\tau)\cdot U = H \qquad (51)$$

$$U = n_1\cdot e^{\frac{1}{2}\int P\cdot d\tau}$$

$$= n_1\cdot\tau^{(\gamma+2iku)/2\gamma}\, e^{iku\xi\tau^2/2\gamma}$$

$$\kappa^2(\tau) = Q - \frac{1}{2}\frac{dP}{d\tau} - \frac{1}{4}P^2$$

$$= \frac{1}{\gamma^2\tau^2} \cdot \Big[\omega_{be}^2+\frac{\gamma^2}{4}+\tau^2\cdot(\frac{5}{2}iku+\frac{\gamma}{4})\frac{d\xi}{dz} +$$

$$+ \tau^4\left(ik\xi - \frac{1}{4}\frac{d\xi}{dz}\right)\cdot\frac{d\xi}{dz}\Big]$$

$$H = \frac{v_o^2}{\gamma^2 \tau^2} \cdot \frac{\partial n_1}{\partial z}\Bigg|_{z=0,t} \cdot e^{\frac{1}{2}\int P \cdot d\tau}$$

The solution of (51) can be expressed in terms of the confluent hypergeometric functions. Note that in the case:

$$\tau \cdot \xi(z=z_o) = 1 (\Longleftrightarrow e^{2\gamma(t-t_o)} (e^{-2\gamma z_o/u} - 1) = 1)$$

(48) leads to the negative beam velocity:

$$v_o(z) < 0 \quad \text{for} \quad z > z_o$$

i.e. the adopted perturbation method is applicable only if $\tau \cdot \xi(z) < 1$ ($\Rightarrow z < z_o$).

Consequently, Eq. (51) can be simplified by neglecting the terms proportional to $(d\xi)/(dz) \cdot \tau^2$ and the higher-order terms (being much less than unity in the region of validity of (51). The simple solution is then obtained:

$$U = C_1 U_1 + C_2 U_2 -$$

$$- U_1 \int_o^\tau \frac{d\tau' \cdot H \cdot U_2}{U_1 \frac{dU_2}{d\tau'} - U_2 \frac{dU_1}{d\tau'}} + U_2 \int_o^\tau \frac{d\tau' \cdot H \cdot U_1}{U_1 \frac{dU_2}{d\tau'} - U_2 \frac{dU_1}{d\tau'}} \tag{52}$$

Where U_1, U_2 are two linearly independent solutions of the homogeneous equation corresponding to (51):

$$U_{1,2} = \tau^{(\gamma \pm 2i\omega_{be})/2\gamma} \tag{53}$$

Returning to the original function, n_1, and the variable t, we have:

$$n_1(t,k) = e^{-i(k\cdot\phi(t)-\omega_{be}\cdot t)} \cdot$$

$$\cdot \left(C_1(k) + \frac{1}{2i\omega_{be}} \cdot \int_{-\infty}^{t} dt' \cdot v_o^2(z,t) \cdot \frac{\partial n_1}{\partial z}\bigg|_{z=0,t'} \cdot e^{i(k\phi(t')-\omega_{be}t')} \right)$$

$$+e^{-i(k\cdot\phi(t)+\omega_{be}t)} \cdot$$

$$\cdot \left(C_2(k) - \frac{1}{2i\omega_{be}} \int_{-\infty}^{t} dt' \cdot v_o^2(z,t) \cdot \frac{\partial n_1}{\partial z}\bigg|_{z=0,t'} \cdot e^{i(k\phi(t')+\omega_{be}t')} \right)$$

$$\phi(t) \equiv u \cdot \left(t + \frac{1}{2\gamma} \cdot e^{2\gamma(t-t_o)} \cdot (e^{-2\gamma z/u} - 1) \right) \qquad (54)$$

The constants of integration C_1, C_2 can be found from the initial conditions. For $t \to -\infty$, expression (54) should coincide with the unperturbed Pierce field (37), which leads to:

$$n_1(t,k) = \frac{\varepsilon_o}{e} \cdot e^{\gamma t} \cdot \left(\frac{k_o \alpha_1}{k-k_o} - \frac{k_o^* \alpha_1^*}{k+k_o^*} \right) \qquad (55)$$

$$k_o \equiv \frac{1}{u}(i\gamma + \omega_{be})$$

Comparing (55) and (54) in the limit $t \to -\infty$, one readily obtains:

$$C_1(k) = \frac{1}{k-k_o} \cdot \frac{\varepsilon_o}{e} k_o \alpha_1$$

$$C_2(k) = -\frac{1}{k+k_o^*} \cdot \frac{\varepsilon_o}{e} k_o^* \alpha_1^* \qquad (56)$$

Finally, the electric potential can be calculated using:

$$\phi(z,t) = \frac{1}{2\pi} \int_{C_k} \frac{dk \cdot e^{ikz}}{k^2} \cdot \left(\frac{en_1}{\varepsilon_o} - \frac{\partial \phi}{\partial z}\bigg|_{z=0} - \phi\,(z=0) \right) \quad (57)$$

with the boundary conditions:

$$\phi(z=0) = 0$$

$$\frac{\partial \phi}{\partial z}\bigg|_{z=0} = \frac{e}{\varepsilon_o} \cdot \frac{u^2}{\omega_{be}^2} \cdot \frac{\partial n_1}{\partial z}\bigg|_{z=0}$$

The potential (57) can be split into two parts: the contribution of the poles $k = k_o$ and $k = k_o^*$ (which are inherent in n_1) leads to a component with a wave-like z-variation:

$$\phi_1(z,t) = e^{\gamma t} \cdot \frac{i\alpha_1}{k_o} \cdot e^{ik_o\left(z - \frac{u\xi(z)}{2\gamma} \cdot e^{2\gamma(t-t_o)}\right)} \quad (58)$$

and the contribution of the second-order pole, $k^2 = 0$, giving a time-oscillating responce proportional to $e^{i\omega_{be} \cdot t}$, which is not of interest, and a timegrowing one:

$$\phi_o(z,t) = \frac{e}{\varepsilon_o} \cdot \frac{1}{\omega_{be}^2} \cdot \left[u^2 \cdot \frac{\partial n_1}{\partial z}\bigg|_{z=0} \cdot z + \right.$$

$$+ i\frac{\omega_{be}}{2} \cdot \int_{-\infty}^{t} dt' \cdot v_o^2(z,t) \cdot \frac{\partial n_1}{\partial z}\bigg|_{z=0} \cdot$$

$$\left. e^{i\omega_{be}(t-t')} \cdot (z - \phi(t) + \phi(t')) \right] + c.c. \quad (59)$$

The boundary value of the derivative of the density perturbation $(\partial n_1)/(\partial z)|_{z=0,t}$, which enters (59) can readily be found from

$$\left.\frac{\partial n_1}{\partial z}\right|_{z=0,t} = \frac{1}{2\pi} \int_{C_k} dk \cdot e^{ikz} \cdot ik \cdot n_1(t,k) \Big|_{z=0}$$

$$= \frac{1}{2\pi} \int_{C_k} dk \cdot ik \cdot e^{-i(ku-\omega_{be})t} \cdot \left(\frac{\varepsilon_o}{e} \cdot \frac{k_o \alpha_1}{k-k_o} + \right.$$

$$\left. + \frac{u^2}{2i\omega_{be}} \cdot \int_{-\infty}^{t} dt' \left.\frac{\partial n_1}{\partial z}\right|_{z=0,t} \cdot e^{i(-i\gamma+ku-\omega_{be})t'} \right) +$$

$$+ \text{c.c.}$$

$$= -\frac{1}{2} \cdot \frac{\varepsilon_o}{e} \cdot e^{\gamma t} \cdot k_o^2 \alpha_1 + \text{c.c.} \qquad (60)$$

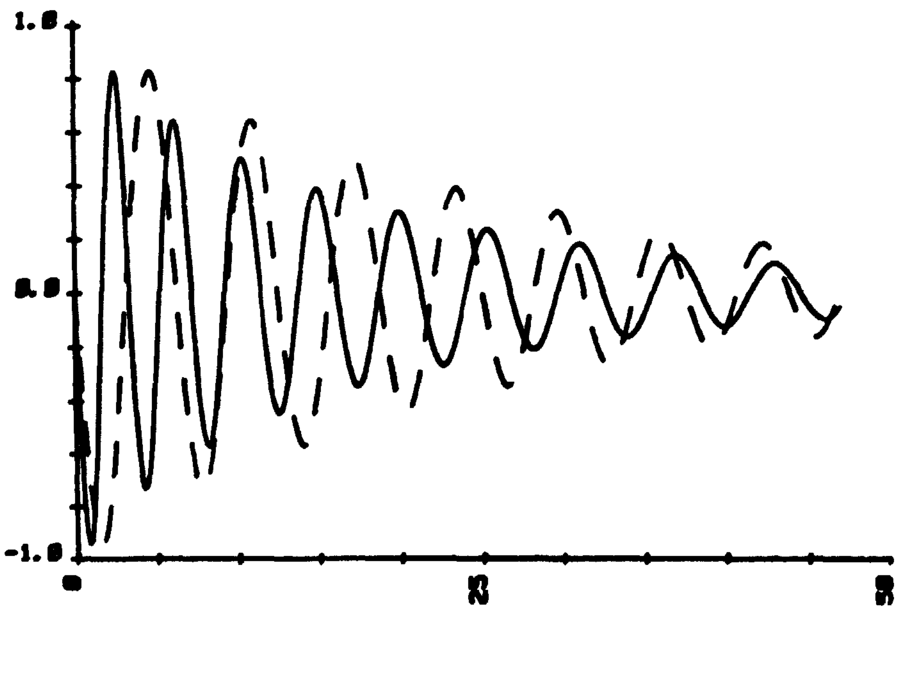

Fig. 3. Potential $\phi_1(z,t)$ in arbitrary units versus the distance $z.(\omega_{be})/(u)$. All the parameters are the same as in Fig. 2. The curves are normalized to have identical values for the first potential minimum.

Introducing (60) in (59) we find that $\phi_o(z,t)$ scales as $(\gamma^2)/(\omega_{be}^2)\cdot z$. It represents the nonlinear analogue to the component $\alpha_o \cdot z$ in the linear case (37); in most of the cases it can be neglected, being small compared with ϕ_1. The potential $\phi_1(z,t)$ (58) is plotted in Fig. 3. One can see that in the case of a beam that is almost stopped (solid line), the wave structure of the potential is retained, but it is more heavily damped at larger distances from the beam inlet which is at $z = 0$. This damping is due to the increase of the imaginary part of the effective wave number in the region $z << u/2\gamma$. Expanding $\xi(z)$ into series, the wave number in this region can be expressed as:

$$k_{eff}(z << \frac{u}{2\gamma}) = k_o(1 + e^{2\gamma(t-t_o)}) \qquad (61)$$

(at larger values of z, $k_{eff}(z) = k_o$). This leads to the enhancement of the potential near $z = 0$, and also to a certain "compression" due to the decrease of the wavelength. If the far boundary was kept fixed the condition (8) $\int_0^L E.dz = 0$, would be violated by the effects of the wave compression, forcing the system deeper into the unstable regime. The subsequent increase of the growth rate, γ, gives rise to the reduction of the characteristic decay length u/γ, leading to the further suppression of the potential at the larger values of z.

The above results are obtained by assuming immobile ions. This assumption is valid on a time scale short compared with the characteristic ion motion time. On a longer time scale, however, the potential in Fig. 3 may be deformed by the effects of particle trapping. For instance, the trapping of ions in the potential minimum nearest to $z = 0$ could broaden this potential well and eventually lead to the formation of a stationary double layer, as was observed in [3].

ACKNOWLEDGEMENT

This work was carried out while the author was visiting Risø National Laboratory. The hospitality and support of this institution are gratefully acknowledged. The author would also like to thank H.L. Pécseli and J.J. Rasmussen for help in his work including valuable discussions.

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Title and author(s)	Date January 1982
The influence of beam boundaries and velocity reduction on Pierce instability in laboratory plasmas Dušan Jovanović +	Department or group Physics Dept. Group's own registration number(s)
34 pages + tables + illustrations	

Abstract

The influences of the beam-plasma boundary and of weak nonlinearities on the Pierce instability is investigated. It is shown that the finite width of the beam has negligible influence on both the stability of the system and growth rate. In the nonlinear regime the wavelength decrease and enhancement of the wave potential close to the beam inlet boundary is observed. The relationship between this effect and the formation of double layers is discussed.

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