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Danish Atomic Energy Commission
Research Establishment Risø

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in a Magnetically Driven Shock Tube

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Abstract

An investigation was made of the possible effect of the boundary layer on the lack of separation observed between the shock and the current sheath in a magnetically driven shock tube operating under discharge conditions of high voltage and low pressure. Since the calculated and the observed maximum displacement thickness in the pressure range investigated was around 1 mm only, it was concluded that the presence of a viscous layer at the wall cannot be of importance. Instead, the observed lack of separation is most likely due to the leakage of the compressed gas through the current-sheath, and to the diffusive nature of the driving current.

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1. INTRODUCTION

In magnetically driven shock tubes operating under discharge conditions of high voltages and low pressures, i. e. conditions where high speed shocks are produced, a lack of separation between the shock and the current-sheath is often observed¹⁾. As a possible analogy a lack of sufficient separation between the shock and the contact surface was observed by previous investigators in conventional diaphragm-type shock tubes operating at low initial pressures^{2, 3)}. They attributed the observed lack of separation, and the associated non-uniformity of the flow, to the mass leakage of the flow through the viscous boundary layer at the wall. Since the reported phenomena show the same tendencies as those observed in the magnetically driven shock tubes, the possible influence of the boundary layer should not be overlooked.

In this report, we first present a brief review of the boundary layer effect in conventional shock tubes; this is followed by an examination of its possible influence on our experiments. Based on the calculated as well as on the experimentally indicated displacement thickness, it is concluded that the effect of boundary layer cannot be of great importance under our experimental conditions. Instead, the observed lack of separation is to a great extent caused by the leakage of the mass through the current-sheath and by the diffusive nature of the driving current.

2. BRIEF REVIEW OF THE BOUNDARY LAYER EFFECT IN CONVENTIONAL SHOCK TUBES

In an idealized shock tube, one assumes all the gas initially occupying the region between the position x_g reached by the shock and the diaphragm to be compressed into region 2 between the shock and the contact surface, see fig. 1. Accordingly, the separation distance, Δ , between the shock and the contact surface, and the corresponding test time, τ , is given by

$$\tau = \Delta/u_2 = x_g/V_g(\eta-1), \quad (1)$$

where u_2 is the flow velocity of the compressed gas, V_g is the shock speed, and

$$\eta = \rho_2/\rho_1 \quad (2)$$

is the compression ratio. ρ_2 and ρ_1 are the mass densities of the gas downstream and upstream of the shock respectively. For a given gas, η is a function of V_S . Consequently, for a given shock speed, one should expect the test time τ to increase proportionally with the distance x_S travelled by the shock.

In an experiment conducted in a shock tube operating at low initial pressures (~ 1 Torr) Duff first noticed that at a given initial pressure p_1 , contrary to expectations, there is a limiting value of the test time τ_m , i. e. τ cannot be increased by increasing x_S indefinitely²⁾. He attributed this effect to the mass leakage through the viscous boundary layer at the wall. Subsequently, Roshko³⁾ made some further experiments and obtained quite good agreement with Duff's earlier results. He also showed analytically that the maximum test time τ_m is related to the shock Mach number, M_S , and the initial state of the gas by

$$\tau_m \sim p_1 D^2 / G(M_S) . \quad (3)$$

In the above expression

$$D = 4A/L \quad (4)$$

is the hydraulic diameter with A as the cross sectional area of the tube and L the wetted perimeter. The function $G(M_S)$ is given explicitly by

$$G(M_S) = \frac{1}{Z_2} \frac{T_2}{T_1} \frac{(\eta-1)^2}{\eta} , \quad (5)$$

where T_1 and T_2 are the temperature upstream and downstream of the shock respectively. Z_2 is the compressibility factor for the downstream gas in region 2, thus

$$p_2/p_1 = \frac{1}{Z_2} \frac{kT_2}{m} . \quad (6)$$

For helium, Z_2 is related to the degree of first and second ionization α and ξ by

$$\frac{1}{Z_2} = (1 + \alpha + \xi)_2 . \quad (7)$$

Assuming the existence of an equilibrium state in region 2⁴⁾, using the result of Fucks and Artmann for α_2 and ξ_2 ⁵⁾, we have calculated $G(M_S)$ in

the range of shock Mach number M_S covered in our experiment. (The corresponding current-shock speed u_2 is from 1.76 to 7.30 cm/ μ s). The result is shown in fig. 2. It should be mentioned that average values of Z_2 , η and T_2/T_1 are used in calculating $G(M_S)$ by interpolating the data of reference 5 between $p_1 = 0.5$ and 1 Torr.

From eq. (2) one observes that the limiting test time τ_m is proportional to the initial pressure p_1 and to the square of the tube diameter as indicated by Duff's earlier experiment. Furthermore, one notices that τ_m decreases rapidly with the shock Mach number M_S , since $G(M_S)$ increases rapidly with M_S .

3. ESTIMATION OF THE BOUNDARY LAYER EFFECT IN OUR EXPERIMENT

The following phenomenon was observed in the magnetically driven shock tube used in our experiment. When the discharge voltage is kept constant, the current-sheet speed increases as the discharge pressure decreases. According to the predication of eq. (3), one would expect the separation distance to be smaller at lower pressures than at higher pressures. Since this trend was observed experimentally (see fig. 3, ref. 1), the possible influence of the boundary layer effect cannot be overlooked.

A direct check of the theoretical prediction, as given by eq. (3), is not feasible, because an experiment of this kind involves a considerable elongation of the electrodes and of the duration of the driving current. Instead, we shall resort to the following indirect means.

The importance of the boundary layer depends on the ratio between the displacement thickness δ^* ⁷⁾ and the hydraulic diameter D . Since in our case $D = 2d$, where d is the gap between the electrodes, from eq. (4), we have this ratio as $\delta^*/2d$. According to Mirels⁸⁾, for a laminar flow, δ^* at a distance l behind the shock is given by

$$(-1)\delta^* = \left(\frac{l}{p_1}\right)^{1/2} f(T_1, M_S) , \quad (8)$$

where the function $f(T_1, M_S)$ can be shown to depend on the free stream property of the compressed gas and on two other functions related to the velocity and the static enthalpy profile of the boundary layer. These latter two functions are tabulated by him with respect to the compression ratio η .

Assuming the existence of an equilibrium state in the free stream region behind the shock, and interpolating between the tabulated data of reference 8,

we can express $f(T_1, M_S)$ explicitly as a function of the shock Mach number M_S , or, as shown in fig. 3, a function of the current-sheet speed u_2 .

Once more it should be mentioned that in evaluating the property of the free stream flow in the compressed region, the same average process was applied as used in the computing of $G(M_S)$.

The detailed calculation is rather involved and is not worth presenting here, but it should be mentioned that a Prandtl number $Pr = 0.67$ is used, and that in evaluating the viscosity coefficient, μ_2 , a relationship

$$\mu_2/\mu_1 = (T_2/T_1)^{0.647} \tag{9}$$

is used instead of the more accurate Sutherland formula¹⁰⁾. From the figure we notice that in the range of the current-sheet speed encountered, the maximum value of $\delta^* \approx 6 \times 10^{-2} \text{ cm}^{1/2}$. Since the observed separation distance in the pressure range in question never exceeds 4 cm^6 , even if we assume that it occurs at the lowest discharge pressure $p_1 = 0.1 \text{ Torr}$, we obtain $(-1) \delta^*/2d \approx 0.1$ from eq. (8).

The above calculation, however, is based on a viscosity law that is only valid for a neutral gas, a more correct way of evaluating the viscosity coefficient μ_2 might be to assume the gas in region 2 to be fully ionized and to take the Coulomb interaction into account.

At the discharge condition $V_0 = 13 \text{ kV}$ and $p_1 = 1 \text{ Torr}$ from the spectroscopic data, we may take the average value of $T_e = 2.67 \times 10^4 \text{ K}$ and $n_e \approx 10^{17}$ and obtain $\mu_2 = 8.28 \times 10^{-5} \text{ C. G. S. unit}$ according to the Spitzer formula¹¹⁾. Since it is about two orders of magnitude smaller than the calculated value based on eq. (9), we should expect that the maximum ratio $(-1) \delta^*/2d$ will be reduced to about two per cent when the Coulomb interaction is present. The calculation presented here is certainly very crude, but it may give a reasonable order of magnitude estimate, as can be seen from the differential interferogram (presented in fig. 4) that the indicated boundary layer thickness does not seem to exceed 1 mm.

4. CONCLUSION

Phenomenologically, insufficient separation between the shock front and the boundary of the driving medium was observed both in magnetically driven shock tubes and in conventional diaphragm-type shock tubes running at low initial pressures.

Since the displacement thickness is not sufficiently developed at the point of observation, both with regard to calculations and to experiments, the observed lack of separation in magnetically driven shock tubes cannot be caused by the presence of the viscous boundary layer at the wall. To a large extent the phenomenon can be attributed to the flow leakage in the compressed region downstream of the shock. However, the leakage is not through the wall boundary layer, but through the driving medium (the current sheath) itself. Furthermore, due to the diffusive nature of the driving current in most of our experiments (especially those corresponding to high speed shocks) the shock has insufficient time to emerge from the current sheath.

NOTES AND REFERENCES

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- 7) The displacement thickness δ^* is defined as

$$\delta^* \equiv \int_0^{\infty} \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy \sim \left(1 - \frac{\rho_w u_w}{\rho_e u_e}\right) \delta,$$
 where δ is the boundary layer thickness. Flow parameters with subscripts w and e refer to those evaluated at the wall and at the free stream respectively. $\delta^* < 0$ implies a mass leakage through the boundary layer.
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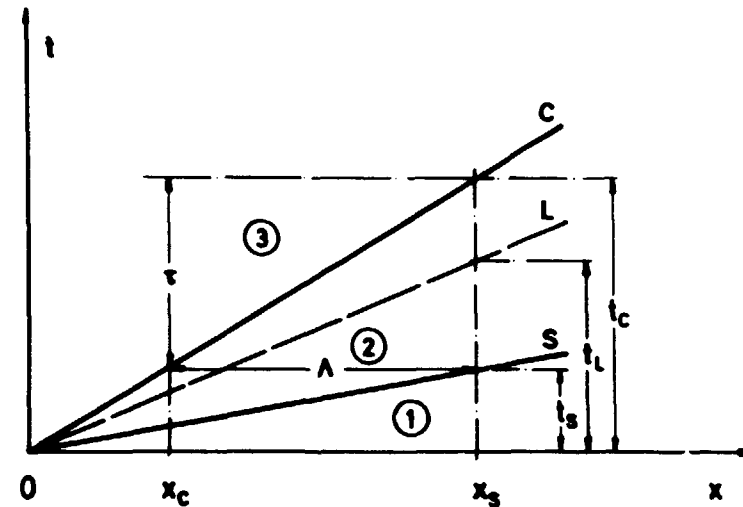


Fig. 1. Idealized trajectories of the current sheet \overline{OC} , the luminous front \overline{OL} and the shock \overline{OS} in the $x-t$ diagram. Their arrival times at a fixed location are denoted by t_c , t_L and t_s respectively. A is the separation distance; $\tau = t_c - t_s$ is the test time. In a pressure driven shock tube the diaphragm is assumed to be located at the origin.

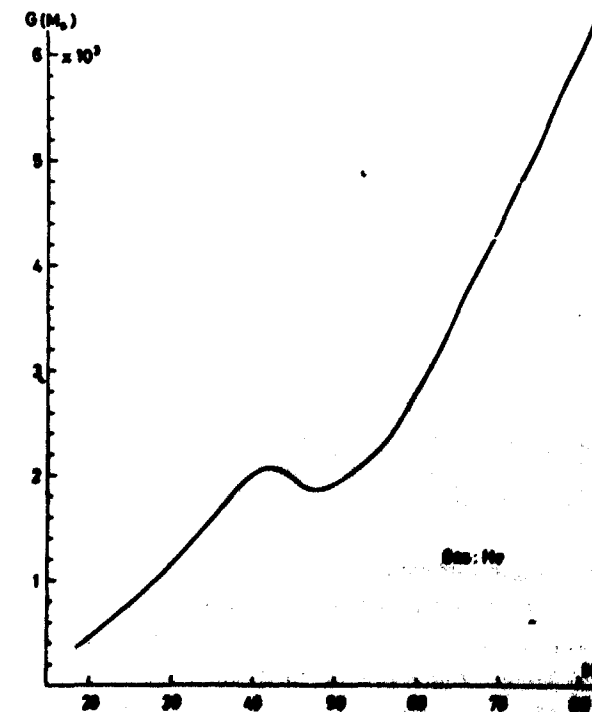


Fig. 2. Variation of the function $G(M_s)$ versus the shock Mach number M_s .

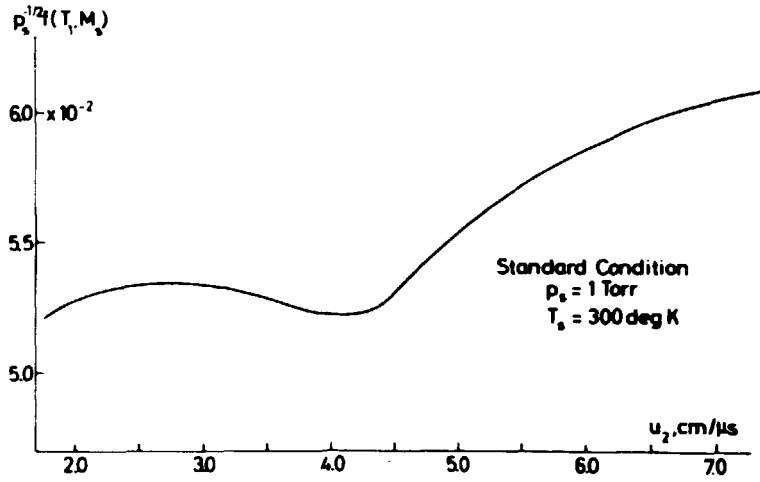


Fig. 3. Variation of the function $f(T_s, M_s)$, eq. (8), with respect to the current sheet speed u_2 . p_s is a reference pressure.

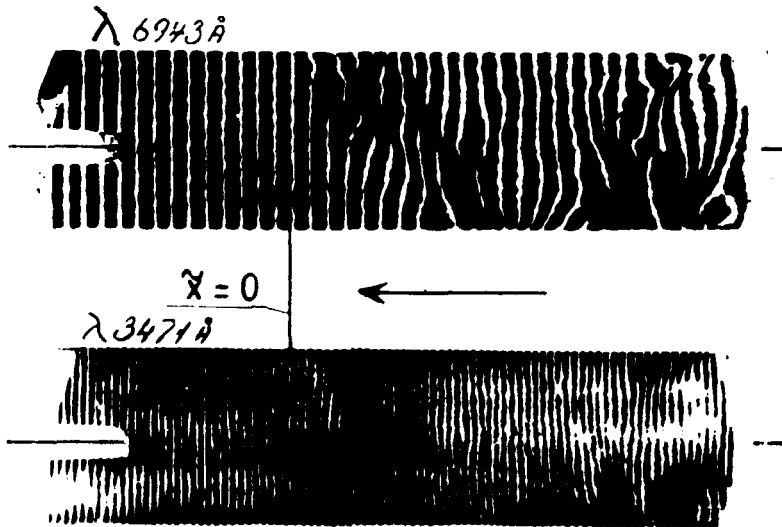


Fig. 4. Two-wavelength differential-interferograms of the plasma region behind the shock front. The tip of the probe is at an axial position $x = 35 \text{ cm}$ from the end insulator plate. Discharge condition: Voltage, $V_0 = 13 \text{ kV}$, pressure $p_1 = 0.5 \text{ Torr}$. Light source, a Q-switched ruby laser, pulse duration 30 nsec. Direction of motion is indicated by the arrow. 2 cm gap between the electrodes.