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Probabilistic Production Simulation Including CHP Plants

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Abstract

A probabilistic production simulation method is presented for an energy system containing combined heat and power plants. The method permits incorporation of stochastic failures (forced outages) of the plants and is well suited for analysis of the dimensioning of the system, that is, for finding the appropriate types and capacities of production plants in relation to expansion planning.

The method is in the tradition of similar approaches for the analysis of power systems, based on the load duration curve. The present method extends on this by considering a two-dimensional load duration curve where the two dimensions represent heat and power.

The method permits the analysis of a combined heat and power system which includes all the basic relevant types of plants, viz., condensing plants, back pressure plants, extraction plants and heat plants.

The focus of the method is on the situation where the heat side has priority. This implies that on the power side there may be imbalances between demand and production. The method permits quantification of the expected power overflow, the expected unserved power demand, and the expected unserved heat demand.

It is shown that a discretization method as well as the double Fourier series may be applied in algorithms based on the method.

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1. INTRODUCTION

An important element in the planning of future heat and power production systems is the dimensioning of the system, that is, finding the appropriate types and capacities of production plants. The planning problem in this respect then consists of obtaining a balance between security of supply and a seldom used reserve capacity.

Traditionally, the dimensioning problem for power-only systems has been analyzed by using so-called probabilistic production simulation. In this method the power demand is represented by a probability distribution, and each power plant is represented by its capacity and forced outage rate, i.e. the probability of not being able to produce. This information is combined in a simulation, which gives results regarding the expected production of each power plant and the expected power demand that cannot be met because of failures in the production system. The method is often referred to as the Baleriaux-Booth method (see Baleriaux et al. (1967), Booth (1972)).

However, due to the large extent of combined heat and power production (CHP) already existing or being planned in many countries, in particular in Denmark, the heat production has to be considered as well. Therefore, work has been done to extend the classical method (see Søndergren (1994), Søndergren and Ravn (1996)).

Similar to the ideas in the classical method of probabilistic production simulation, the combined heat and power demand is here represented by a two-dimensional probability distribution, where the two dimensions are power demand and heat demand. The CHP plants are represented by their power and heat capacities and forced outage rates.

The idea of using two-dimensional probability distributions has been applied independently within the analysis of power-only systems (see Ahsan et al. (1983), Noyes (1983), Rau et al. (1982), Rau et al. (1983), Schenk et al. (1985)). The specific application has been the analysis of two interconnected power systems.

While both interconnected power systems and CHP systems may be analyzed using two-dimensional probability distribution representations, it turns out that a CHP system has its unique features. For power-only systems there is a positive probability of unsatisfied power demands, due to forced outages. Similarly for CHP systems there is a positive probability of unsatisfied power and heat demands. However, in addendum, an overflow power production may occur, assuming that the heat production is attempted satisfied. This is due to the problem of simultaneously satisfying both heat and power demands from the same plants.

In more general terms, there is in the CHP systems a trade-off between trying to satisfy power and heat demands and trying to avoid overproduction of power and heat. Therefore, in order to analyze such systems it is necessary to develop concepts and methods that are specifically directed towards the systems' characteristics.

In the present work we extend the results of Søndergren and Ravn in three directions: First, we include in the analysis the important extraction type CHP unit. Second, we eliminate the difficulty that the so-called loading order influences the results of the analysis. Third, we introduce the application of Fourier series in the calculations.

The report is organized as follows: In Section 2 we introduce the basic concepts in the two-dimensional analysis of CHP systems. Section 3 gives the main theoretical results. The section confines itself mainly to an analysis of the situation where the production is organized with the heat side having priority. Section 4 discusses implementation aspects, describing both discretization and analysis of analytical representations, the latter in particular by application of the double Fourier series. Finally, Section 5 brings the conclusions of the work.

2. THE CONCEPTS IN TWO-DIMENSIONAL ANALYSIS

In this section we describe the main concepts that are necessary for the two-dimensional analysis. This includes the description of the system (units and demands), the convolutions, and the results in terms of expected unserved energies and expected overflow energies. The exposition largely follows Søndergren (1984), Søndergren and Ravn (1996), but includes also extraction units.

2.1 Description of the units

The system considered is a combined heat and power (CHP) system. A CHP system consists of several units each producing power and/or heat.

The units in the system can be divided into two groups: units which only have one type of generation, either heat or power, and those which have a combined generation, both heat and power (CHP units). The former group consists of heat units, which generate only heat, and condensing units, which generate only power. Within the group of CHP units we consider back pressure units and extraction units.

Each unit is characterized by its

- working area
- forced outage rate (FOR), r
- position in the loading order list of all units in the CHP system

The relation between the possible heat and power generations for a unit is represented by the working area. The working areas of the four types of units are sketched in Figure 1. The shape of the working area defines the possible combinations of heat and power generations.

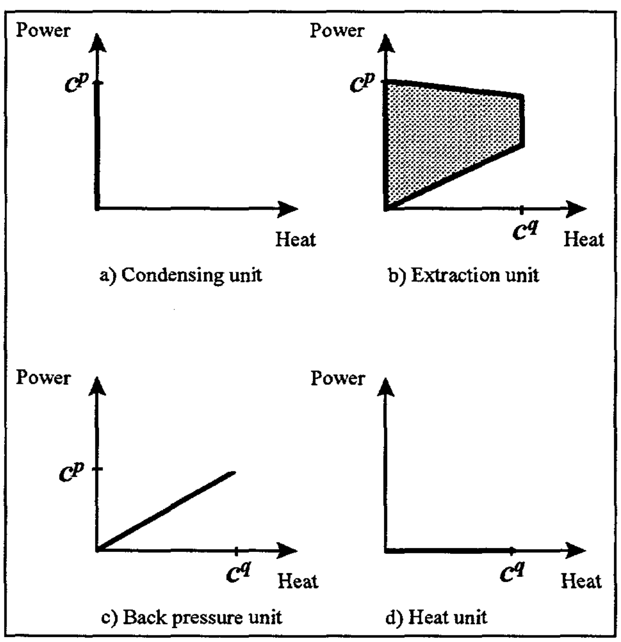

Figure 1 : Working area for production units.

For a heat unit the working area is characterized by the capacity $c^q\ [MJ/s]$ for heat, and for a condensing unit the working area is characterized by the capacity $c^p\ [MW]$ for power. It is seen that a heat unit has zero power capacity, c^p=0, and that a condensing unit has zero heat capacity, c^q=0.

For back pressure units, the back pressure line is defined with a slope, $c^m = c^p / c^q$, where c^q and c^p are capacities for heat $[MJ/s]$ and power, $[MW]$, respectively.

Formally, we define c^m=0 for heat units and c^m=∞ for condensing units.

For extraction units there is some freedom in the choice of possible combinations of heat and power. This is in contrast to the back-pressure unit, where the combinations are fixed at the back-pressure line.

The working area of the extraction unit is a polygon, defined by four lines. Two lines limit the heat production q to the interval $0 \le q \le c^q$. The back-pressure line with slope $c^m = c^p / c^q$ defines the lower limit of power production to any given heat production. The line with slope $-c^v$ (observe that c^v is positive) defines the upper limit of power production to any given heat production. The combinations (q, p) of heat and power that are within the working area are therefore seen to satisfy the relations,

$$0 \le q \le c^q \tag{1}$$

$$c^m q \le p \le c^p - c^v q \tag{2}$$

The relations are seen to imply that $0 \le p \le c^p$.

As seen, the back-pressure and extraction units introduce a dependency between the production of heat and power. This dependency is fundamental to the method presented below since it introduces difficulties which are absent in the power-only systems. In fact, if the system contained only condensing units and heat units, then the power-only analysis could be readily extended to include also the heat production.

Each of the units is assigned a forced outage rate, i.e. the probability of non-availability of the unit.

The N units are assigned to generate in the loading order. Traditionally the loading order, or priority list, reflects the economy of the units. The generation costs of the combined units are usually lower than the heat only and the power only units, and therefore they might be placed first in the loading order.

For the analysis of the two-dimensional case it will be found desirable to use other loading orders that better reflect the character of the problem (see Section 3).

In the sequel lower indexes i will usually indicate unit number, e.g., r_i, c_i^q, c_i^p, and c_i^m.

2.2 Load Probability Density Function, LPDF

Based on, e.g. recorded data, such as illustrated in Figure 2, the heat and power load can be classified and represented as shown in Figure 3.

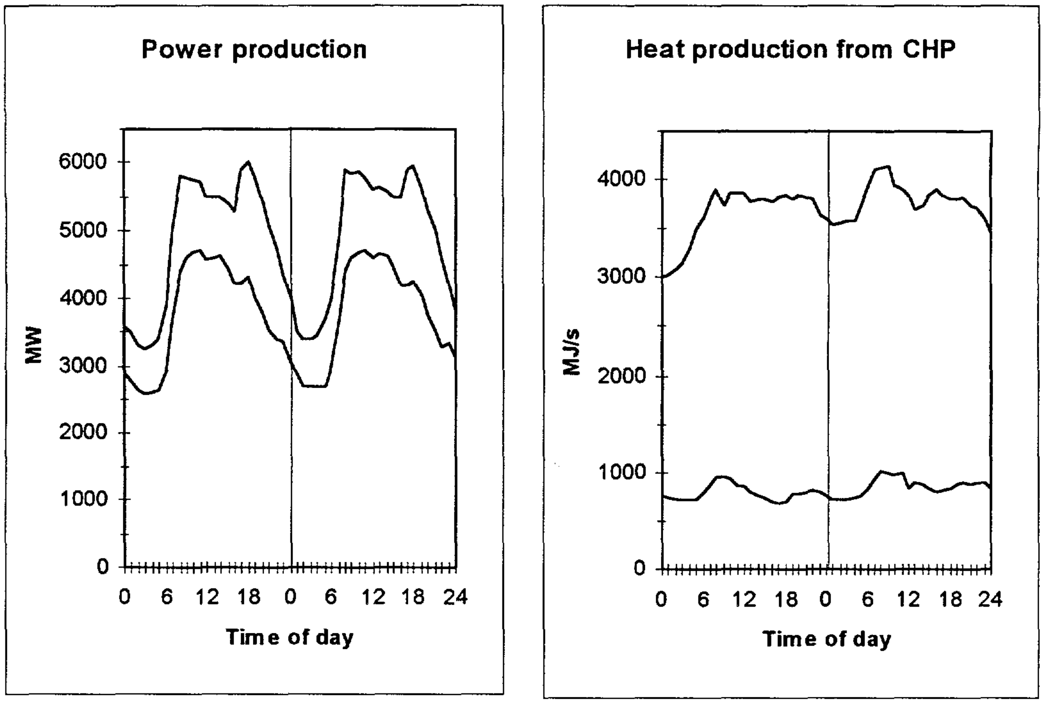

Figure 2 : Examples of CHP production. Upper curves : Winter. Lower curves : Summer.

This figure illustrates the relative frequency with which particular combinations of power and heat occurred during the period considered (typically one year), viz., the load probability density function, LPDF. For planning purposes the LPDF must be derived by forecasting.

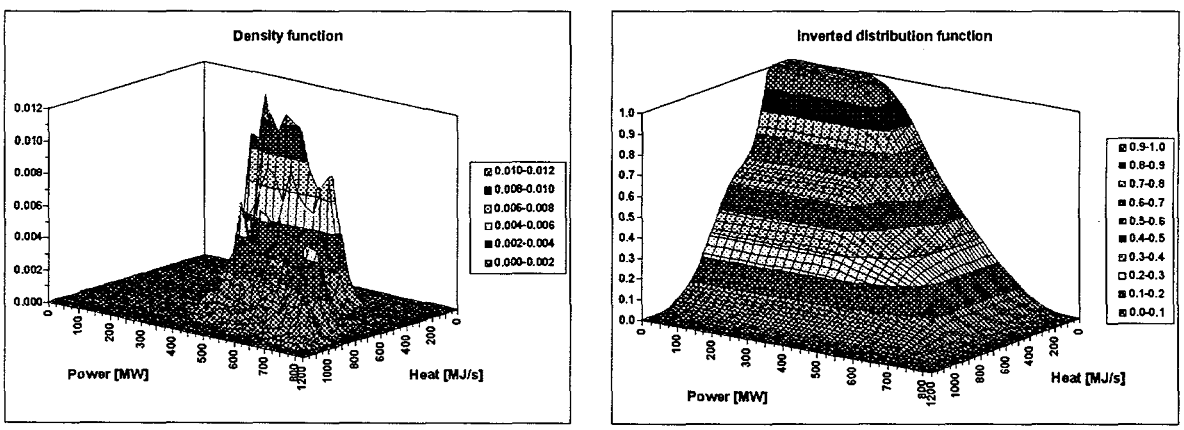

Figure 3 : Density function and Inverted distribution function.

The occurrence depicted in Figure 3 will be considered as a two-dimensional stochastic variable. Thus, the graph in Figure 3 is interpreted as representing a joint probability density function of the two loads. Therefore, let the two loads, heat and power, be represented as random variables $X^q[MJ/s]$ and $X^p[MW]$, respectively. Thus, $f_0(x^q, x^p)$ represents their joint load probability density function LPDF; that is, $f_0(x^q, x^p)$ represents the probability that X^q takes the particular value x^q and X^p takes the particular value x^p .

Figure 3 also shows the inverted distribution function. Observe that this function, according to the tradition of probabilistic power planning, is inverted relative to the statistical tradition. Thus, the inverted distribution function F_0 , is defined as,

$$F_0(q, p) = \int_q^\infty \int_p^\infty f_0(x^q, x^p) dx^p \, dx^q \tag{3}$$

From Figures 2 and 3 it is readily observed that the two variables are not stochastically independent, nor are they totally correlated.

Marginal frequency functions for heat and power are shown in Figure 4. Inverted marginal distribution functions for heat and power are shown in Figure 5.

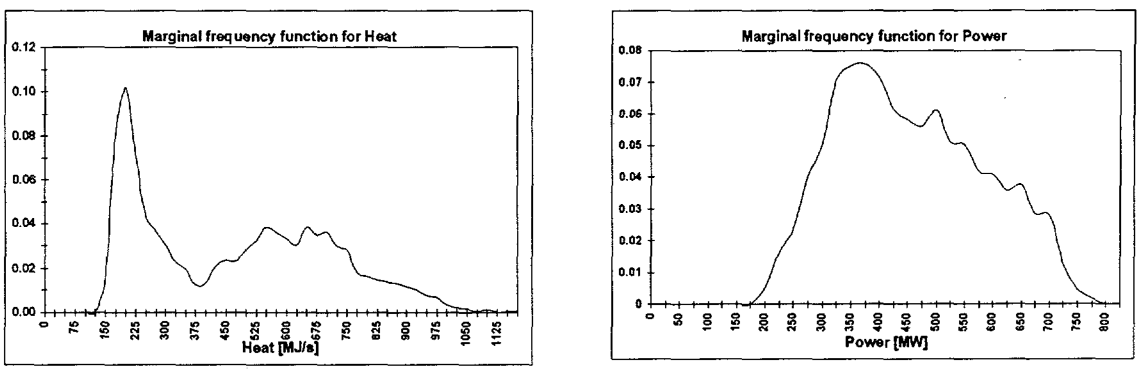

Figure 4 : Marginal frequency functions for heat and power.

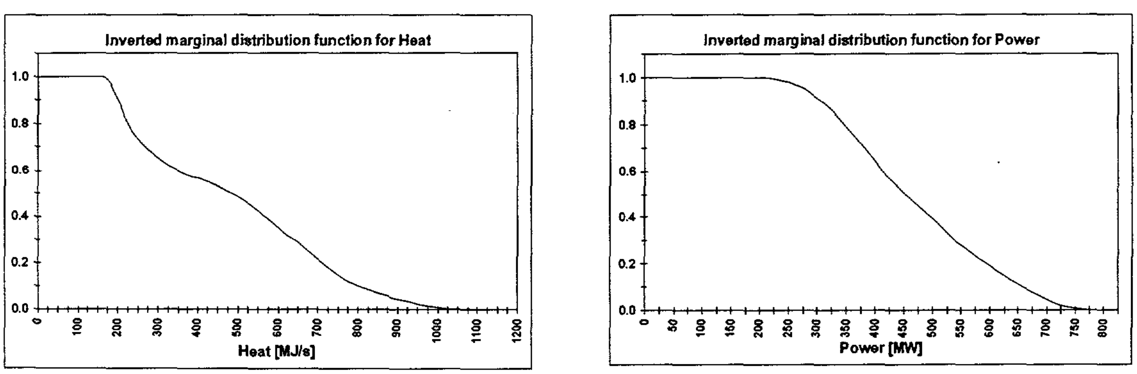

Figure 5 : Inverted marginal distribution functions for heat and power.

2.3 Equivalent Load Probability Density Function, ELPDF

Now we analyze what happens when we attempt to satisfy the load by inserting the first unit in the priority list. Assume for a moment that the unit is not an extraction unit. This unit has capacity $(c_1{}^q, c_1{}^p)$. Assuming furthermore that the unit produces at full capacity all the time (i.e., it has no outages and thus the forced outage rate is not zero, i.e., r_1=0), we get the equivalent load probability density function, ELDPC, for the remaining load, f_1 as ,

$$f_1(x^q, x^p) = f_0(x^q + c_1{}^q, x^p + c_1{}^p) \tag{4}$$

Thus, the equivalent load density function f_1 is obtained by figuratively "moving" all points representing probability mass in the direction of $(-c_1{}^q, -c_1{}^p)$ when inserting unit 1. This represents the load remaining to be served by the other N-1 units. This process is repeated for all N units.

Now, as the units have a FOR, $r_n > 0$ they are not available all the time but only with probability $(1 - r_n)$. Considering the equivalent load and the outages as independent stochastic variables, the recursive formula for the derivation of the ELPDF is obtained as,

$$f_n(x^q, x^p) = (1 - r_n) f_{n-1}(x^q + c_n^q, x^p + c_n^p) + r_n f_{n-1}(x^q, x^p) \tag{5}$$

If all units considered are assumed to produce at capacity (c^q, c^p) then for some n, points with positive loads will move such that they may end up with either negative equivalent power load or negative equivalent heat load (or both). Therefore, it might be desirable to reduce the generation of a unit to a level less than (c^q, c^p).

We can specify this by introducing a capacity reduction factor s. With this, the capacities (c^q, c^p) in the convolution described in (5) are substituted by production levels other than the capacities.

The selection of appropriate production levels is complicated. In particular it is not even clear what production of full capacity of an extraction unit should be interpreted to mean, cf. Figure 2. We must therefore state more explicitly what combination (q, p) of heat and power will be applied during the convolution of the unit. We shall describe this in more detail in Section 3.

2.4 Overflow and Trade-off

A distinctive feature of the two-dimensional analysis is that relative to the one-dimensional case we have to introduce the concept of overflow energy. This is a direct consequence of the dependency between power and heat production described in Section 2.1.

Consider the insertion of unit n (a back-pressure unit, for simplicity) and the derivation of the ELPDF, f_n. We refer to Figure 6.

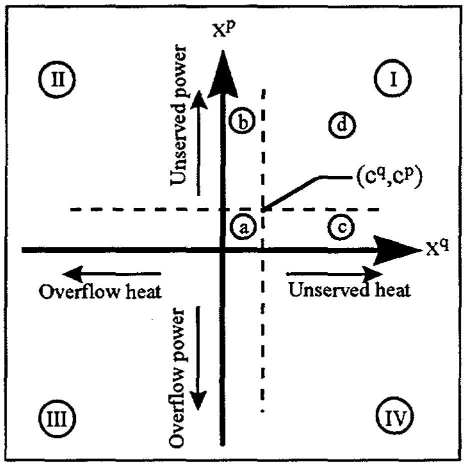

Figure 6 : Various regions in relation to overflow.

If all points representing probability mass of f_{n-1} are moved in the direction $\left(-c^{q},-c^{p}\right)$, only one point (namely, $\left(x^{q},x^{p}\right)=\left(c^{q},c^{p}\right)$) is moved to $\left(x^{q},x^{p}\right)=(0,0)$. Points in region a in Figure 6 are moved to quadrant III which implies that both heat and power overflows are obtained. By overflow is meant that more energy is generated than demanded. Points in region b are moved to quadrant II where heat overflow is obtained while some unserved energy for power still remains. Points in region c are moved to quadrant IV where power overflow is obtained while some unserved energy for heat still remains. Points in region d are moved to points in quadrant I.

By reduction of the production level, cf. the previous subsection, it will be possible to avoid moving to quadrants II, III or IV where overflow is obtained.

The overflow obtained is mathematically well defined through (5). Overflow is simply represented by points $\left(x^{q},x^{p}\right)$ with $x^{q}<0$ or $x^{p}<0$ (or both), having $f_{n}\left(x^{q},x^{p}\right)>0$, i.e., there is a positive probability that either x^{q} or x^{p} (or both) are negative.

The physical and operational interpretation of overflow can vary according to the circumstances. For instance, power overflow might imply that the surplus power is delivered as rotational energy (implying an increase in frequency) or it might imply that power is exported to neighboring systems. A heat overflow might imply an increasing temperature in the water of the district heating system or the storage of heat in a storage tank.

Now a key point in the two-dimensional system is that overflow is not necessarily undesirable. This differs from the power-only system where the production level adjustment is always used to attain either an exact fulfillment of the load or, if this is not possible, to have unserved energy.

To understand why this is so, it should be noted that there is a trade-off between the following situations: "Unserved heat and power energy" versus "power (or heat) overflow but less unserved heat (or power) energy".

In the calculations in (5) it is therefore necessary to define a strategy for production level adjustment. The convolution of the units can be done by choosing one of the following criteria (possibly others):

- no overflow (in the sequel denoted no-overflow)
- heat overflow permitted but not power overflow (denoted q-overflow)

- power overflow permitted but not heat overflow (denoted *p*-overflow or heat priority)
- both heat and power overflows permitted (denoted *qp*-overflow)

For any criterion it may be necessary to define loading orders and production adjustments strategies in order that the results of the convolution is well defined.

Which criterion to choose depends on the system in question (e.g. economical, operational and institutional aspects) and the purpose of the analysis.

As noted in the Introduction we shall in this paper assume that the heat-side determines the trade-off. That is, we consider only the *p*-overflow case. The criterion may also be denoted as a heat priority criterion.

We may describe this criterion in more detail. It implies a hierarchical decision structure for production adjustment (dispatch), i.e. for choice of heat and power production. For any given unit the heat production is first determined, so that the remaining heat demand will be minimized, and so that there is no heat overflow. For the given heat production on the unit the power production is then determined. On a heat unit there is no power production. On a back-pressure unit the size of power production is implied by the size of the heat production. Therefore, only in the case of extraction and condensing units there is a freedom of choice of power production. In general, the size of the power production on the two units will be chosen in order to minimize power overflow as well as unserved power. See further below.

2.5 Energy Quantities

From knowledge of the ELPDFs, $f_n(x^q, x^p)$, for the equivalent load density function after unit n has been added we can derive the marginal density functions, $f_n^{\,Q}(x^q)$ and $f_n^{\,P}(x^p)$ for heat and power, respectively, as

$$f_n^{\,Q}(x^q) = \int_{-\infty}^{\infty} f_n(x^q, x^p)dx^p \tag{6}$$

$$f_n^{\,P}(x^p) = \int_{-\infty}^{\infty} f_n(x^q, x^p)dx^q \tag{7}$$

The expected energy generation of unit n in terms of heat and power can now be found as

$$EE_n^{\,q} = \tau\int_{-\infty}^{\infty}\left(f_{n-1}^{\,Q}(x^q) - f_n^{\,Q}(x^q)\right)\cdot x^q dx^q \tag{8}$$

$$EE_n^{\,p} = \tau\int_{-\infty}^{\infty}\left(f_{n-1}^{\,P}(x^p) - f_n^{\,P}(x^p)\right)\cdot x^p dx^p \tag{9}$$

where τ is the time interval considered.

When all N units have been loaded, the expected unserved energies for heat, EUE^q and for power, EUE^p are obtained as

$$EUE^q = \tau\int_0^{\infty} f_n^{\,Q}(x^q)\cdot x^q dx^q \tag{10}$$

$$EUE^p = \tau\int_0^{\infty} f_n^{\,P}(x^p)\cdot x^p dx^p \tag{11}$$

The expected heat and power overflow energies, EOE^q and EOE^p can be determined in a way similar to the expected unserved energy. The expected heat overflow EOE^q is calculated as the integral of all negative values of the equivalent heat load, x^q, multiplied by the probability that X^q takes the value x^q. The probability that X^q takes the value x^q is expressed by the marginal probability density function, $f^Q(x^q)$. The expected power overflow EOE^p can be calculated similarly. We may write this as

$$EOE^q = -\tau \int_{-\infty}^{0} f_N{}^Q(x^q) \cdot x^q dx^q \tag{12}$$

$$EOE^p = -\tau \int_{-\infty}^{0} f_N{}^P(x^p) \cdot x^p dx^p \tag{13}$$

Thus, for each criterion (cf. the previous section) we define four characteristic energy quantities, EUE^q, EUE^p, EOE^q, and EOE^p.

The quantities may or may not be uniquely defined with reference to the criterion chosen. For no-overflow, for instance, values of EUE^p have to be traded against values of EUE^q, cf. Søndergren (1994). This is why in general also a loading order and a production adjustment (dispatch) strategy need be specified in relation to a criterion. As we shall show in Subsection 3.4, the values EOE^q, EOE^p, EUE^q, and EUE^p for the heat priority criterion are defined in a natural way without such specification. (However, the quantities $EE_n{}^q$ and $EE_n{}^p$ are not.)

Finally, observe that in addition to these energy quantities, the concept of loss of load probability (LOLP), known from the traditional power-only analysis, may be extended to the CHP analysis. We omit this in the present work.

3. THE BASIC METHOD FOR POWER AND HEAT

In this section we present the essential elements of the method for two-dimensional probabilistic analysis. In order to limit the extent of the discussion, we confine ourselves to the criterion of heat priority. The main result is the demonstration of how to obtain the values *EUE* and *EOE*.

It turns out that the concept of outage pattern will be expedient for the subsequent analysis. The concept is defined in Subsection 3.1, and used in Subsections 3.2 and 3.3, while it is shown in Subsection 3.4 how to come from outage patterns to ordinary convolutions. Subsection 3.4 constitutes the main result of the theoretical development.

3.1 Outage patterns

Consider one demand point with associated probability mass on the original two-dimensional load probability density function F_0.

Assume that units are used with given capacities (c_i^q, c_i^p). We observe that by the convolutions this point will generate two points on F_1, corresponding to unit number one being on or off, respectively. Further, there will be four points on F_2 (disregarding that in general two or more points may coincide). In general there will be 2^i points on F_i and 2^N points after convolution of all N units.

Now define an *outage pattern* as a sequence of N zeros and ones. For example, if $N = 5$, then one outage pattern is $(0,1,1,0,0)$. This is interpreted to mean that units 1, 4 and 5 are off, while units 2 and 3 are on. Similarly $(0,0,0,0,0)$ and $(1,1,1,1,1)$ correspond to all units being off and on, respectively.

As a motivation we make the following observation: For a specific outage pattern the original point in relation to F_0 generates one point on F_1. Its location is identical with that of F_0 if the outage pattern specifies a zero for unit 1, and the location is shifted by (c_i^q, c_i^p) if the outage pattern specifies a one. Continuing this way, we see that one point is generated on F_N by one outage pattern (some of the points are possibly identical). Finally observe that the above-mentioned 2^N points with associated probability masses may be seen as generated by the 2^N possible outage patterns. We may therefore use outage patterns to reach the same results as we do by using convolutions. As we shall see, outage patterns will be expedient for analytical purposes.

3.2 Illustration without extraction units

As a simple illustration of the above, we now consider a system without extraction units. We consider this case, as it is simpler than a system with extraction units. Assume a given outage pattern so that four units are available. Recall the relation $c_i^{\,m} \equiv c_i^{\,p} \,/\, c_i^{\,q}$ for back pressure units, $c_i^m \equiv \infty$ for condensing units, and $c_i^m = 0$ for heat units.

Consider three different loading orders. One, *A*, in which the units are loaded in a sequence of increasing c^m-values. One, *C*, in which the units are loaded in a sequence of decreasing c^m - values. And one, *B*, with an arbitrary sequence.

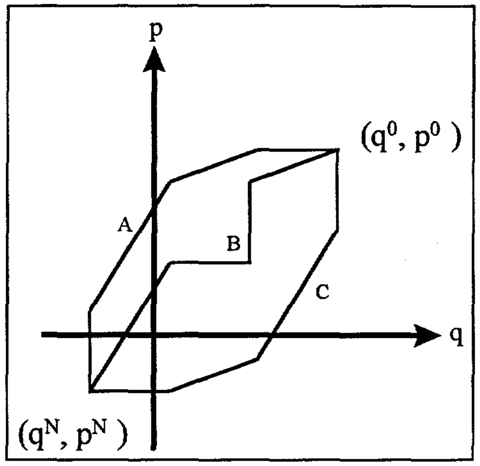

Figure 7 : Expansion lines for four units.
A : Increasing c^m - values.
B : Arbitrary sequence.
C : Decreasing c^m - values.

As explained, the points on F_N corresponding to a given outage pattern are points that may be reached by applying the units with full capacities $\left(c_i^q, c_i^p\right)$. However, the production of a unit may be reduced to $\left(s_i c_i^q, s_i c_i^p\right)$ with the capacity factor s_i satisfying $0 \le s_i \le 1$. By appropriate selection of s_i not only may the individual points described above be reached, but also any point on the line which connects two consecutive units. We call this piecewise linear line, from first to last point, the *expansion line*. Observe that for any given CHP system, the expansion line depends on the loading order of the units and on the outage pattern.

In Figure 7 we illustrate the three corresponding expansion lines. Observe that the three expansion lines start at the same point, $\left(q^0, p^0\right)$, and end at the same point, $\left(q^N, p^N\right)$. Further, it is seen that the expansion line *A* is the upper line, expansion line *C* is the lower line, with expansion line *B* in between. This is not incidental as expressed in the result below. Moreover, by suitable selection of capacity factors for the units it will be possible to end up at any point *(q, p)* located between expansion lines *A* and *C*. Such selection of capacity factors will be called a *dispatch*.

Lemma 1: Consider for a given outage pattern the expansion lines *A*, *B* and *C* corresponding to loading orders with increasing, arbitrary and decreasing c^m-values, respectively. Then the following holds:

A. The expansion lines start and end at the same points, viz., $\left(q^0, p^0\right)$ and $\left(q^N, p^N\right)$, respectively.

B. For any $q, q^N \le q \le q^0$, and any points $\left(q, p^A\right)$, $\left(q, p^B\right)$, $\left(q, p^C\right)$, located at expansion lines *A*, *B* and *C*, respectively, there holds $p^C \le p^B \le p^A$.

C. For any $p, p^N \le p \le p^0$, and any points $\left(q^A, p\right)$, $\left(q^B, p\right)$, $\left(q^C, p\right)$ located at expansion lines *A*, *B* and *C*, respectively, there holds $q^A \le q^B \le q^C$.

D. For any *(q, p)* located between expansion lines *A* and *C* there exists a dispatch that ends up in this point.

Proof:

A. Considering the capacities $\left(c_i^q, c_i^p\right)$ as vectors in R^2 we see that the addition of all capacities in the outage pattern brings us from point $\left(q^0, p^0\right)$ to point $\left(q^N, p^N\right)$. For vectors this result is independent of the sequence in which they are added.

B. Consider a given $q, q^N \leq q \leq q^0$. We want to find the maximum value of p corresponding to this. We therefore formulate the following optimization problem:

$$\max\left[p^0 - \sum_{i=1}^{N} p_i\right]$$

$$\sum_{i=1}^{N} q_i = q$$

$$p_i = c_i^m q_i \text{ for back-pressure units}$$

$$0 \leq p_i \leq c_i^p,\ i = 1,\ldots,N$$

$$0 \leq q_i \leq c_i^q,\ i = 1,\ldots,N$$

Here units corresponding to a 0 in the outage pattern have been assigned $c_i^p = c_i^q = 0$. The variables in the optimization problem are $p_i, i = 1,\ldots N$ and $q_i, i = 1,\ldots,N$. Thus, all units are formally assumed to be able to produce both heat and power, but a heat unit has $c_i^p = 0$ and a condensing unit has $c_i^q = 0$.

Obviously condensing units shall not be used, as they do not really enter the additive constraint $\sum_{i=1}^{N} q_i = q$, i.e., $p_i = 0, i \in c$, where c is the set of condensing units. The criterion may therefore be rewritten,

$$\max\left[p^0 - \sum_{i \in NC} p_i\right]$$

where NC is the set of non-condensing units. Finally using $p_i = c_i^m q_i$ these constraints may be omitted and the criterion may be rewritten,

$$\max\left[p^0 - \sum_{i \in NC} c_i^m q_i\right].$$

The problem is a linear programming problem. It is well-known that the optimal solution may be found by a sorting procedure, taking the units in a sequence corresponding to increasing c^m-values. Alternatively, a complete proof is given in relation to Lemma 2 below.

This loading order therefore gives the maximum p-value to the given q-value, and it follows that $p^B \leq p^A$.

Changing the criterion from maximization to minimization we will by similar argumentation obtain that $p^C \leq p^B$. Combining the results yields the desired relation $p^C \leq p^B \leq p^A$.

C. We may obtain the third result in a similar way by reversing the roles of p and q.

D. See the proof of Lemma 2 below. □

The importance of the Lemma is the following: First we have found absolute limits for the intervals in which we can end up. Second, we have established that any point within the border given by expansion lines A and C may be reached by a suitable dispatch. (Observe that to reach this result we do not actually need to find the dispatch.)

As we shall see in Section 3.4, the consequence of this is that we will be able to determine the values of *EOE* and *EUE*. This will be done by performing two sets of convolutions, viz., one in which we follow expansion line A, and one in which we follow expansion line C.

The exact way of doing this, in particular knowing when to stop applying more units, will depend on the criterion for balancing overflow energy against unserved energy for power and heat. Thus, the above illustration is fairly general and needs to be specified for full implementation.

In the sequel we limit ourselves to the criterion of heat priority. First we will show how to include extraction units in this analysis (Subsection 3.3), and then we show how to define and derive *EOE* and *EUE* for this criterion (Subsection 3.4).

3.3 Extraction units with heat priority criterion

Now we introduce extraction units. These are more complicated because there is freedom of choice of production combinations of p and q. The analysis leaves slightly less general results. Therefore, we consider only the case of heat priority.

The loading possibilities of an extraction unit could also be described by a capacity factor (two-dimensional in this case), cf. Subsection 3.2, but we shall keep the description verbal until the end of Subsection 3.4. This is sufficient, because, as will be seen, we need only consider loading on the line AB and on the line OC, cf. Figure 8.

We may therefore consider the following main types of loading of an extraction unit,

- As a fully loaded back-pressure unit, point C in Figure 8.
- As a partially loaded back-pressure unit, a point on the line OC in Figure 8.
- Maximum q load and, relative to this, also maximum p load, point B in Figure 8.
- Partially loaded on the c^v-line, i.e., a point on the line AB in Figure 8.

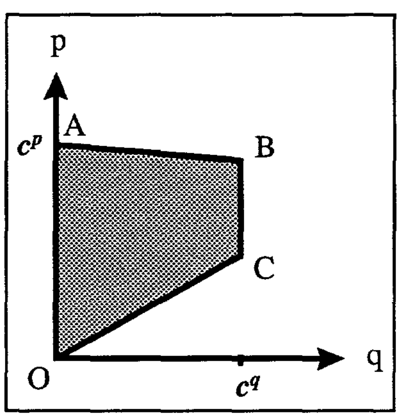

Figure 8 : Loading of an extraction unit.

As above, we consider two specific loading orders for any given outage pattern.

A. Treat all extraction units as back pressure units. Then apply all units sorted according to increasing c^m - values.

B. First apply all non-extraction units sorted according to decreasing c^m-values. Then apply all extraction units sorted according to increasing c^v-values.

In both cases the dispatch is adjusted according to the heat priority criterion. As before, heat units are formally defined with $c^m = 0$, while condensing units are defined with $c^m = \infty$.

In case "1", this implies that as long as no q-overflow is generated the units are used with full capacities (extraction units are treated as back-pressure units , i.e., point C in Figure 8 is used). Then capacity is reduced so that no q-overflow is generated (extraction units produce at a point on the line OC in Figure 8), or units do not produce at all. Case "1" may be characterized as minimum p-production to any given q-production.

In case "2", condensing units are applied first, at full capacities (p-overflow need not be avoided). Then back-pressure units are applied, followed by heat units: both types with suitable adjustments of production, according to the heat priority criterion. And finally extraction units are loaded as follows: As long as no q-overflow is generated, they are loaded to point B in Figure 8. If the point B in Figure 8 will produce q-overflow, then production is reduced to a point on the line AB so that no q-overflow is attained and (possibly) no unserved q-energy remains. Observe that extraction units must be applied even if no unserved q-energy exists (in which case they are loaded to point A). Case "2" may be characterized as maximum p-production to any given q-production.

Observe that loading order "2" may be described as taking all the units in sequence according to decreasing slope of the "upper" limit of the working areas, this being c_i^m for non-extraction units and $-c_i^v$ for extraction units. Recall that the c^v -value for a given unit i is defined as the negative of the slope, i.e., $c^v{}_i$ is positive. Observe also that if there are no extraction units then for $q>0$ the two loading orders correspond to those of expansion lines A and C of Lemma 1.

Figure 9 illustrates the two loading orders and possible loading points.

We call the resulting end points (q^1 ,p^1) and (q^2, p^2).

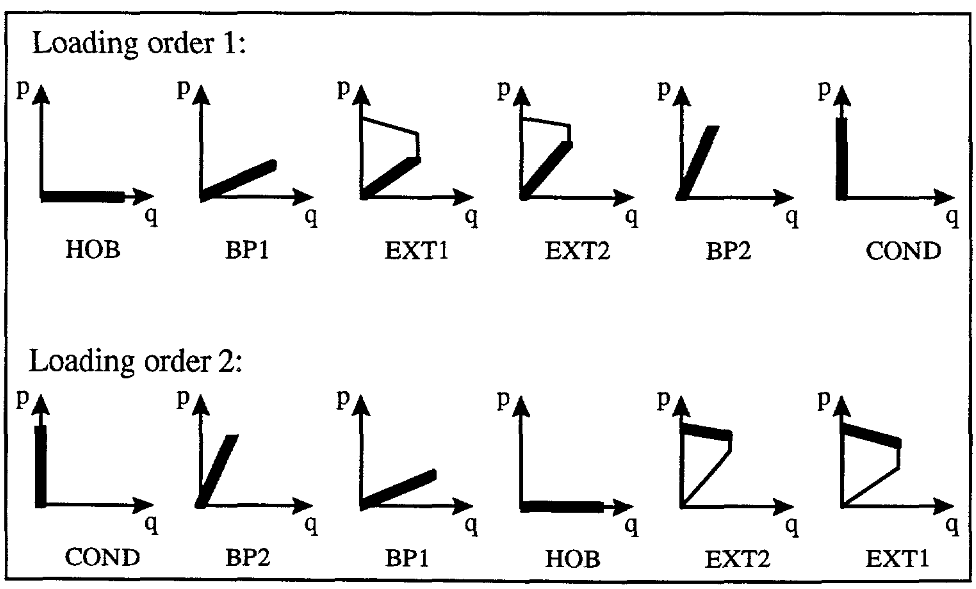

Figure 9 : The two loading orders and possible loading points.

The following result combines the ideas of Lemma 1 with extraction units and specialization to the case of heat priority.

Lemma 2: For any outage pattern and demand point we have the following, using dispatches according to the heat priority criterion

- $q^1 = q^2$ and $p^2 \leq p^1$.
- All other sequences or dispatches give end points (q,p) satisfying $q = q^1 = q^2$ and $p^2 \leq p \leq p^1$.
- For any $p^2 \leq p \leq p^1$ there exists a dispatch that gives the end point (q^1, p).

Proof: The idea of the proof of this Lemma follows closely that of Lemma 1. The first optimization problem in the proof may be indicated as follows:

$$\max\left[p^0 - \sum_{i=1}^{N} p_i\right]$$

$$\sum_{i=1}^{N} q_i = q^1$$

$$p_i = c_i^m q_i \quad \text{for back-pressure units}$$
$$p_i = 0 \quad \text{for heat only units}$$
$$q_i = 0 \quad \text{for condensing units}$$
$$c_i^m q_i \leq p_i \leq c_i^p - c_i^v q_i \quad \text{for extraction units}$$
$$0 \leq p_i \leq c_i^p \quad , i = 1,\ldots,N$$
$$0 \leq q_i \leq c_i^q \quad , i = 1,\ldots,N$$

We now give a detailed proof.

The production adjustment (dispatch) strategies in the two cases are identical with respect to q–production, viz., maximal q–production as long as there is unserved q–demand, and then adjustment of q–production such that no overflow is attained. Therefore $q^1 = q^2$.

We define the sets B, C, E and H of unit indexes $i = 1,\ldots,N$ representing the sets of back-pressure, condensing, extraction and heat units, respectively. The following optimization problem may then be formulated:

$$\max\left[p^0 - \sum_{i\in B} p_i - \sum_{i\in C} p_i - \sum_{i\in E} p_i\right]$$

$$\begin{array}{ll}
\sum_{i=1}^{N} q_i = q^0 - q^1 & \\
p_i = c_i^m q_i & i \in B \\
c_i^m q_i \leq p_i \leq c_i^p - c_i^v q_i & i \in E \\
0 \leq p_i \leq c_i^p & \forall i \\
0 \leq q_i \leq c_i^q & \forall i
\end{array}$$

In this, p^0 is the initial p–demand and q^0 is the initial q–demand. The optimization problem may be interpreted as one of finding the maximal remaining p–demand or minimal

p–production, given the total q–production, $q^0 - q^1$. Optimization is with respect to variables q_i and p_i, $i = 1, \ldots, N$. Thus, all units are formally assumed to have both power and heat production, but heat unit i has $c_i^p = 0$ and condensing unit i has $c_i^q = 0$.

We shall now show that the production on the units as given by loading order and dispatch "1" gives an optimal solution to this optimization problem.

We may eliminate the variables p_i, $i \in B$, from the criterion, using the relation $p_i = c_i^m q_i$. The constraints $p_i = c_i^m q_i$, $i \in B$, are then eliminated. The criterion function is then reformulated as follows:

$$\max \left[p^0 - \sum_{i \in B} c_i^m q_i - \sum_{i \in C} p_i - \sum_{i \in E} p_i \right]$$

The problem is a linear programming problem (see e.g. Luenberger (1984)). There is a feasible solution because of the choice of right hand side $q^0 - q^1$, and it is specified by the merit order and production adjustment. We show that the specified feasible solution is optimal in the optimization problem by using necessary and sufficient optimality conditions expressed by the Kuhn–Tucker conditions (Luenberger (1984)). We introduce the multiplier $\mu \in R$ relative to the constraint $\sum_{i=1}^N q_i = q^0 - q^1$. We introduce the non-negative multipliers $\underline{\lambda}_i$, $i \in E$, relative to $c_i^m q_i - p_i \leq 0$, and the non-negative multipliers $\overline{\lambda}_i$, $i \in E$, relative to $p_i - c_i^p + c_i^v q_i \leq 0$ (these two sets of constraints are equivalent to $c_i^m q_i \leq p \leq c_i^p - c_i^v q_i$). We introduce multipliers δ_i^p relative to lower and upper bounds on all p_i–values, and δ_i^q relative to lower and upper bounds on all q_i–values.

The Kuhn–Tucker complementary slackness conditions in relation to the multipliers (except μ) and inequality constraints are for all i:

$\underline{\lambda}_i \geq 0$, and $\underline{\lambda}_i = 0$	if $c_i^m q_i - p_i < 0$ (i.e., not at the c^m–line)
$\overline{\lambda}_i \geq 0$, and $\overline{\lambda}_i = 0$	if $p_i - c_i^p + c_i^v q_i < 0$ (i.e., not at the c^v–line)
$\delta_i^p \leq 0$	if $p_i = 0$ (i.e., at the lower bound)
$\delta_i^p \geq 0$	if $p_i = c_i^p$ (i.e., at the upper bound)
$\delta_i^p = 0$	if $0 < p_i < c_i^p$
$\delta_i^q \leq 0$	if $q_i = 0$ (i.e., at the lower bound)
$\delta_i^q \geq 0$	if $q_i = c_i^q$ (i.e., at the upper bound)
$\delta_i^q = 0$	if $0 < q_i < c_i^q$

Combine now $\underline{\lambda}_i$, $i \in E$, into the vector $\underline{\lambda}$, $\overline{\lambda}_i$, $i \in E$, into the vector $\overline{\lambda}$, δ_i^p, $i \in E$ into δ_E^p, etc., and c_i^m into $\mathbf{c}^m$. Let $\underline{\lambda}\mathbf{c}^m$ denote the vector with elements $\underline{\lambda}_i c_i^m$, $i \in E$, and let $\overline{\lambda}\mathbf{c}^v$ denote the vector with elements $\overline{\lambda}_i c_i^v$, $i \in E$. Further, $\mathbf{-1}$ is a vector with all entries equal to -1, and $\mathbf{0}$ is the zero vector.

The Kuhn–Tucker stationary conditions with respect to variables p_i may now be specified as,

$$\begin{aligned}
\mathbf{0} &= \mu\mathbf{0} + \delta_B^p && (i \in B) \\
\mathbf{-1} &= \mu\mathbf{0} + \delta_C^p && (i \in C) \\
\mathbf{-1} &= \mu\mathbf{0} - \underline{\lambda} + \overline{\lambda} + \delta_E^p && (i \in E) \\
\mathbf{0} &= \mu\mathbf{0} + \delta_H^p && (i \in H)
\end{aligned}$$

We now let $\underline{\lambda}_i = 1$ and $\overline{\lambda}_i = 0$, $i \in E$ (indicating that the solution specified by the loading order and dispatch is at the c^m–line, which is true). We let $\delta_i^p = -1$, $i \in C$ (indicating that the solution is at the lower bound, which is true). We let $\delta_i^p = 0$, $i \in B$, $i \in E$, $i \in H$ (no indication implied).

It is seen that this satisfies the above stationary conditions for arbitrary choice of μ, and it also satisfies the complementary slackness conditions.

Finally we may formulate the Kuhn–Tucker stationarity condition with respect to variable q_i as follows,

$$\begin{aligned} -\mathbf{c}^m &= \mu\mathbf{1} + \delta_B^q & (i \in B) \\ \mathbf{0} &= \mu\mathbf{1} + \delta_C^q & (i \in C) \\ \mathbf{0} &= \mu\mathbf{1} + \underline{\lambda}\mathbf{c}^m + \overline{\lambda}\mathbf{c}^v + \delta_E^q & (i \in E) \\ \mathbf{0} &= \mu\mathbf{1} + \delta_H^q & (i \in H) \end{aligned}$$

Now let i^* denote the index of the last unit in the merit order which has a positive q–production. Take $\mu = -c_{i^*}^m$. We observe $\mu \leq 0$. Due to the sorting with respect to increasing c^m–values this implies $-c_i^m \geq \mu$, $i < i^*$, and $-c_i^m \leq \mu$, $i^* < i$.

Let $\delta_i^q = -c_i^m - \mu$, $i \in B$, $i \in E$. This is non-negative if $i < i^*$ (indicating that the solution is at the upper bound, which is true), non-positive if $i^* < i$ (indicating that the solution is at the lower bound, which is true). Let $\delta_i^q = -\mu$, $i \in C$, $i \in H$ (non-negative, indicating upper bound, which is true).

It is seen that this satisfies the above stationarity conditions and the complementary slackness conditions. We have therefore shown that the specified solution with the specified choice of multiplier values satisfies the Kuhn–Tucker conditions, and the solution is therefore optimal.

It follows that the optimal criterion value is p^1.

We now specify another optimization problem, identical to the first one except that maximization is replaced by minimization. This may be interpreted to mean that the maximal p–production or the minimum remaining p–demand should be found, relative to the given total q–production $q^0 - q^1$.

In a way similar to the above it may be shown that the solution specified by loading order and dispatch "2" is feasible and optimal in this problem, and that the optimal criterion value is p^2. We omit this.

As p^1 is obtained by maximization and p^2 is obtained by minimization it follows that $p^2 \leq p^1$.

As all other sequences and production adjustments are identical with respect to total q–production, it follows that any end point (q,p) satisfies $q = q^1 = q^2$ and $p^2 \leq p \leq p^1$.

Finally we show that for any $p^2 \leq p \leq p^1$ there exists a dispatch that gives the end point (q^1,p). Let the optimal solutions to loading order "1" be denoted (q^{1*},p^{1*}) and (q^{2*},p^{2*}), respectively. It follows that,

$$\begin{aligned} (q^1,p^1) &= \left(\sum_{i=1}^{N} q_i^{1*}, \sum_{i=1}^{N} p_i^{1*}\right) \\ (q^2,p^2) &= \left(\sum_{i=1}^{N} q_i^{2*}, \sum_{i=1}^{N} p_i^{2*}\right) \end{aligned}$$

For some $\alpha \in [0,1]$ there holds $(q^1,p) = \alpha(q^1,p^1) + (1-\alpha)(q^2,p^2)$ because $q^1 = q^2$ and $p^2 \leq p \leq p^1$. Therefore also,

$$(q^1,p) = \left(\sum_{i=1}^{N} \alpha q_i^{1*}, \sum_{i=1}^{N} \alpha p_i^{1*}\right) + \left(\sum_{i=1}^{N}(1-\alpha)q_i^{2*}, \sum_{i=1}^{N}(1-\alpha)p_i^{2*}\right)$$

The production $(\alpha q_i^{1*} + (1-\alpha)q_i^{2*}, \alpha p_i^{1*} + (1-\alpha)p_i^{2*})$ on unit i is feasible because (q_i^{1*}, p_i^{1*}) and (q_i^{2*}, p_i^{2*}) are feasible, $\alpha \in [0,1]$ and the working area for any unit is a convex set (cf. Luenberger (1984)). Therefore there exists a dispatch that gives the end point (q^1,p). □

The two Lemmas may be interpreted as follows in relation to the p-overflow criterion: For a given demand point with associated probability mass and for a given outage pattern we can calculate lower and upper limits on the p-overflow and unserved p-energy by performing two calculations, one for each of the two loading orders. Loading order "1" gives a minimum p-production to any given q-production, and therefore sets the lower limit on p-overflow. If in particular the end point (q^1, p^1) has $0 \leq p^1$ then p-overflow is zero, otherwise it is $-p^1$. Loading order "2" gives maximum p-production and therefore sets the lower limit on unserved p-energy. If in particular the end point (q^2, p^2) has $p^2 \leq 0$ then the unserved p-energy is zero, otherwise it is p^2.

Apart from identifying these limits the Lemmas also may be interpreted to state that any value between these limits may be attained by a suitable dispatch; this is relevant when $p^2 \leq 0 \leq p^1$. It is worth observing that the desirable dispatch has been shown to exist. However, it will not actually be calculated by the method.

The assumption underlying the derived result on the desirable dispatch is that in real time operation the control room operator knows the demand at the given time. He also knows which units are available, i.e., he knows which outage pattern is relevant for his dispatch. He is therefore able to perform the dispatch that will give the most desirable result.

The results may therefore be summarized as follows:

- If $0 \leq p^2$ then the minimum unserved p-energy is p^2 and there is no p-overflow.
- If $p^1 \leq 0$ then the minimum p-overflow is $-p^1$ and there is no unserved p-energy.
- If $p^2 \leq 0 \leq p^1$ then there exists a dispatch so that the end point (p,q) has p=0 implying that there is no p-overflow and no unserved p-energy.

These results refer to a specific demand point and outage pattern. The results suggest that the quantity EOE^p may be calculated from convolution "1" and the quantity EUE^p may be calculated from convolution "2", whereas EUE^q may be calculated from either (and EOE^q=0 by the definition of the heat priority criterion). Further, it is suggested that there is no contradiction between the objectives of getting small values of EOE^p and EUE^p, i.e. that a trade-off between these quantities is not necessary. We now extend the result in this direction for all demand points. For this purpose, we interpret a specific outage pattern as a system with fully reliable units.

Lemma 3: Assume that we have a system with fully reliable units (FOR=0) and consider the heat priority case. Perform the convolution "1" and calculate EOE^p and EUE^q. Then perform the convolution "2" and calculate EUE^p and EUE^q.

Then EOE^q=0; the two values of EUE^q are equal ; and no dispatches can give lower value of EOE^p, EUE^p or EUE^q.

Proof: The results of Lemma 2 were derived for one demand point. We first show that the results may be extended to hold for all demand points treated together.

The derivation of the limits involved in Lemma 2 calls for an optimized dispatch (minimization in case "1", maximization in case "2") for the given demand point and outage pattern. As shown in the proof of Lemma 2, this optimization is achieved by applying the units in a specific loading order and using specific loading rules. The decisive property of this is that the loading of a particular unit depends only on the convolution in question ("1" or "2") and on the demand point (q,p) to which it is applied; in particular, it does not depend on the position of the unit in the loading order nor on the initial point of demand to which the first unit in the loading order was applied. Therefore the optimal dispatch is also achieved if all demand points are treated together.

It is observed that the dispatches involved in Lemma 2, one for each demand point, need not actually be calculated; it is sufficient that they exist, which they do also if all points are treated together in the convolution. Therefore, it follows that the limits involved in Lemma 2 still hold, as well as the conclusions concerning the overflows and unserved energies.

Obviously, the analysis of all demand points for one outage pattern may be interpreted as a convolution where all the units which are on in that outage pattern are fully reliable. Therefore, the quantities *EOE* and *EUE*, defined in Subsection 2.5, may be calculated from the convolutions "1" and "2", respectively.

From the definition of the heat priority case it follows that EOE^q=0, and obviously this quantity cannot be reduced. The values EUE^q will be equal, as follows from Lemma 2, and it follows from the definition of the heat priority case that they are minimal. Finally, it follows from Lemma 2 that EOE^p and EUE^p may be calculated by convolutions "1" and "2", respectively, and that the values are minimal. □

In the next subsection we extend the result to the normal case of not fully reliable units. However, first we discuss a graphical illustration of the above results.

Graphical illustration :

We turn to a graphical illustration in line with that of Figure 7, where no extraction units were introduced. It turns out that it is not straightforward, which is why we take it here at the end.

There is no need for any special treatment in order to reach the upper expansion line.

In order to reach the lower expansion line, we have to split the extraction units into two parts. This particular split is possible for two reasons: First it exploits the partial independence between *p* and *q* production on a extraction unit. Second, under the assumptions taken here we know the particular outage pattern in question, and therefore no difficulties of a stochastic nature are present.

The first part of the split extraction unit *i* corresponds to a condensing unit with capacity c_i^p. The second part corresponds to a back-pressure unit with heat capacity c_i^q and back-pressure slope $-c_i^v$ (negative).

The condensing part is taken into the loading sequence together with the ordinary condensing units (i.e., first in convolution "2"). The back-pressure part is taken into the loading sequence together with the ordinary back-pressure units (where they will come last, because the back-pressure slope $-c^{v}$ is negative).

An illustration is given in Figure 10.

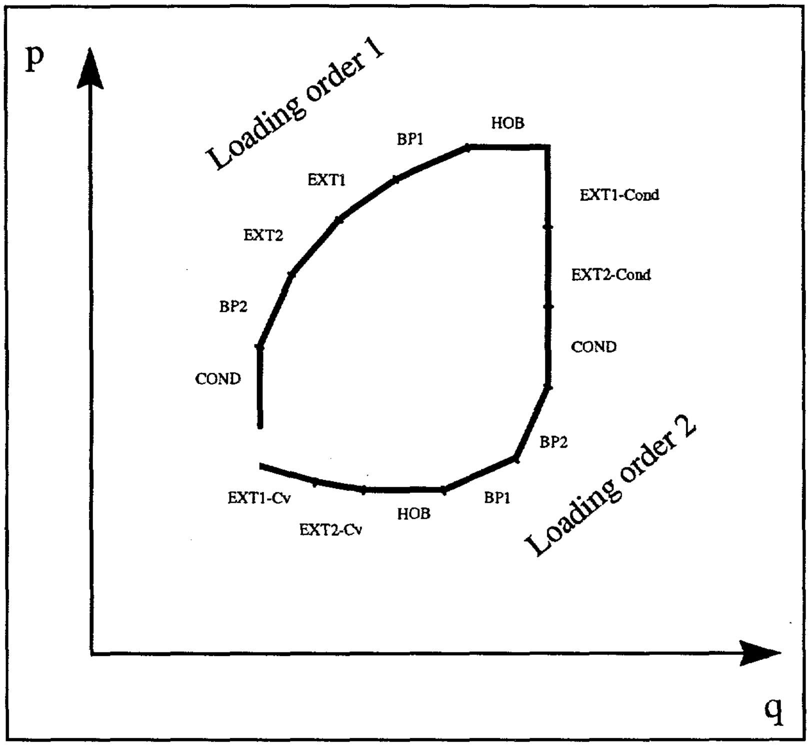

Figure 10 : Loading of the units for the two loading orders.

This way of splitting units will give the same result as described in Lemma 2, as may be verified by adapting the proof of the Lemma to include split units.

We close with an important observation concerning the loading orders. This observation is that a condensing unit may be taken at any position in the loading order, without influencing the result relative to the p^2-value of Lemma 2. This also holds if the extraction units are split into two units as described in relation to the graphical analysis.

In particular, this means that an artificial condensing unit may or may not be placed next to the artificial back-pressure unit that resulted from a split of an extraction unit. In other words, this confirms that there is freedom to choose between the originally described loading order "2", and the split loading order described in relation to the graphical analysis, cf. Figure 10. Why then present the different loading orders?

The advantage of the split loading order is that it leads to a neat graphical analysis. However, this analysis is performed in relation to a given outage pattern, i.e. stochasticity is not present.

The advantage of the loading order "2" is precisely that it takes an extraction unit as one unit, not two independent subunits. When stochasticity is to be treated this implies that difficulties concerning dependence between outages of the two subunits do not arise. Therefore, this loading order is better suited for the subsequent analysis.

3.4 Results for the heat priority criterion

We are now in a position to see how the quantities *EOE* and *EUE* may be derived for the heat priority criterion. This is done by performing two convolutions. In the first one, "1", units are loaded according to increasing c^m-values. This convolution, characterized as minimum p-production for given q-production, is used to determine EOE^p and EUE^q. In the second one, "2", the units are loaded according to decreasing c^m-values (non-extraction units first) and then increasing c^v-values (extraction units). This convolution, characterized as maximum p-production for given q-production, is used to determine EUE^p and EUE^q. Observe that by definition EOE^q=0.

Now we extend the above results in two ways as follows:

First, we show that the heat priority criterion yields well-defined minimum values of EUE^q, EOE^p and EUE^p (recall that EOE^q=0). This in particular means that there is no trade-off between these values.

Second, we show that these quantities may be calculated by performing two convolutions and we specify the loading orders and production adjustment strategies to be used.

Proposition:

Consider the heat priority criterion.

Perform two sets of convolutions with loading orders and dispatches as described above, "1" and "2". For "1" calculate EOE^p and EUE^q and denote them EOE^{p1} and EUE^{q1}, respectively. For "2" calculate EUE^p and EUE^q and denote them EUE^{p2} and EUE^{q2}, respectively.

Then $EOE^q = 0$ and $EUE^{q1} = EUE^{q2}$.

Define $\mathrm{EOE}^p = \mathrm{EOE}^{p1}$, $\mathrm{EUE}^p = \mathrm{EUE}^{p2}$, and $\mathrm{EUE}^q = \mathrm{EUE}^{q1} = \mathrm{EUE}^{q2}$.

Then these quantities are well defined in the sense that no loading order or production adjustment can give lower values of any of them.

Proof: As shown in Lemma 3 for one outage pattern, interpreted as a system with fully reliable units, we can calculate the desired quantities *EOE* and *EUE,* and these are well-defined, minimal values. Obviously, $EOE^q = 0$. For all possible outage patterns these quantities may be calculated by taking the sum over all the 2^N outage patterns of the quantities, weighted by the respective probabilities of the individual outage patterns. We shall show that by making a traditional convolution we get the same result.

The derivation of the limits involved in Lemma 2 and applied in Lemma 3 calls for an optimized dispatch (maximization in case "1" , minimization in case "2") for the given demand point and outage pattern. As shown in Lemma 2, this optimization is achieved by applying the units in a specific loading order and using specific loading rules (dispatches). The decisive property of this is that the loading of a particular unit depends only on the convolution in question ("1" or "2") and on the demand point (q,p) to which it is applied; in particular, it does not depend on the outage. Therefore, the optimal dispatch is also achieved if the traditional convolution is applied.

It follows that making a traditional convolution corresponds to adding the 2^N outage patterns, weighted by the respective probabilities of the individual outage patterns. Therefore, the calculation for all outage patterns may be performed simultaneously in the form of a traditional convolution. □

Observe that slight savings in computation may be obtained because in the convolution "1" the condensing units need not be applied in order to calculate $EOE^p = EOE^{p1}$ and $EUE^q = EUE^{q1}$. Further, in the convolution "2" the condensing units may be placed anywhere in the loading order. Thus, by placing them last only that part of the probability space with $p>0$ need be considered because the purpose of the convolution "2" is the calculation of $EUE^p = EUE^{p2}$ and $EUE^q = EUE^{q2}$. And finally, since $EUE^{q1} = EUE^{q2}$, only one of these quantities need be calculated.

The application of the proposition is illustrated in Figure 11. The plants of the CHP system used are described in Table 1.

Unit	Type	FOR	Capacity		c^m	c^v
			Heat MJ/s	Power MW	MW/(MJ/s)	MW/(MJ/s)
HOB	Heat unit	0.1	1000			
COND	Condensing unit	0.1		300		
BP1	Back pressure unit	0.1	250	100	0.40	
BP2	Back pressure unit	0.1	200	150	0.75	
EXT	Extraction unit	0.1	500	575	0.80	0.15

Table 1 : Plant data.

Figure 11 shows the two frequency functions after convolution with minimum power production relative to heat production (convolution "1") and maximum power production relative to heat production (convolution "2"). In convolution "1" the units are applied in the loading order HOB, BP1, BP2, EXT, and COND, while in convolution "2" the loading order is COND, BP2, BP1, HOB, and EXT.

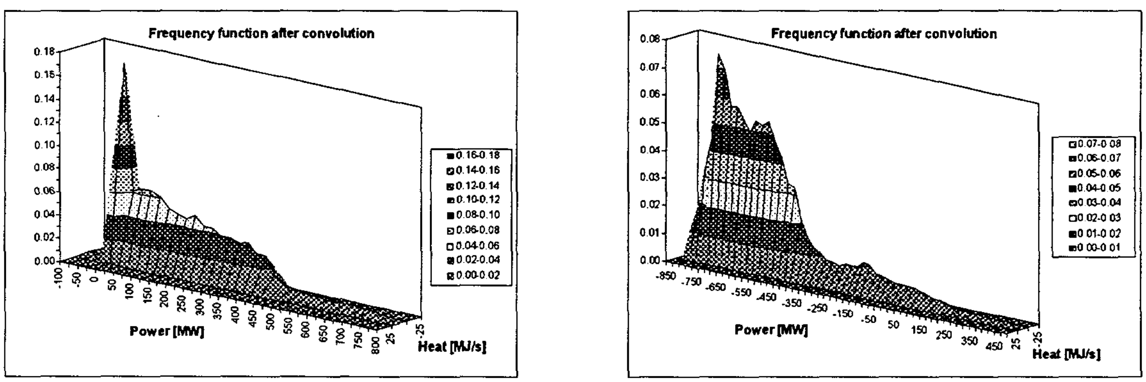

Figure 11 : Frequency functions after convolution.
Left : Minimum power production relative to heat production.
Right : Maximum power production relative to heat production.

Figure 12 shows the corresponding marginal frequency functions for heat and power.

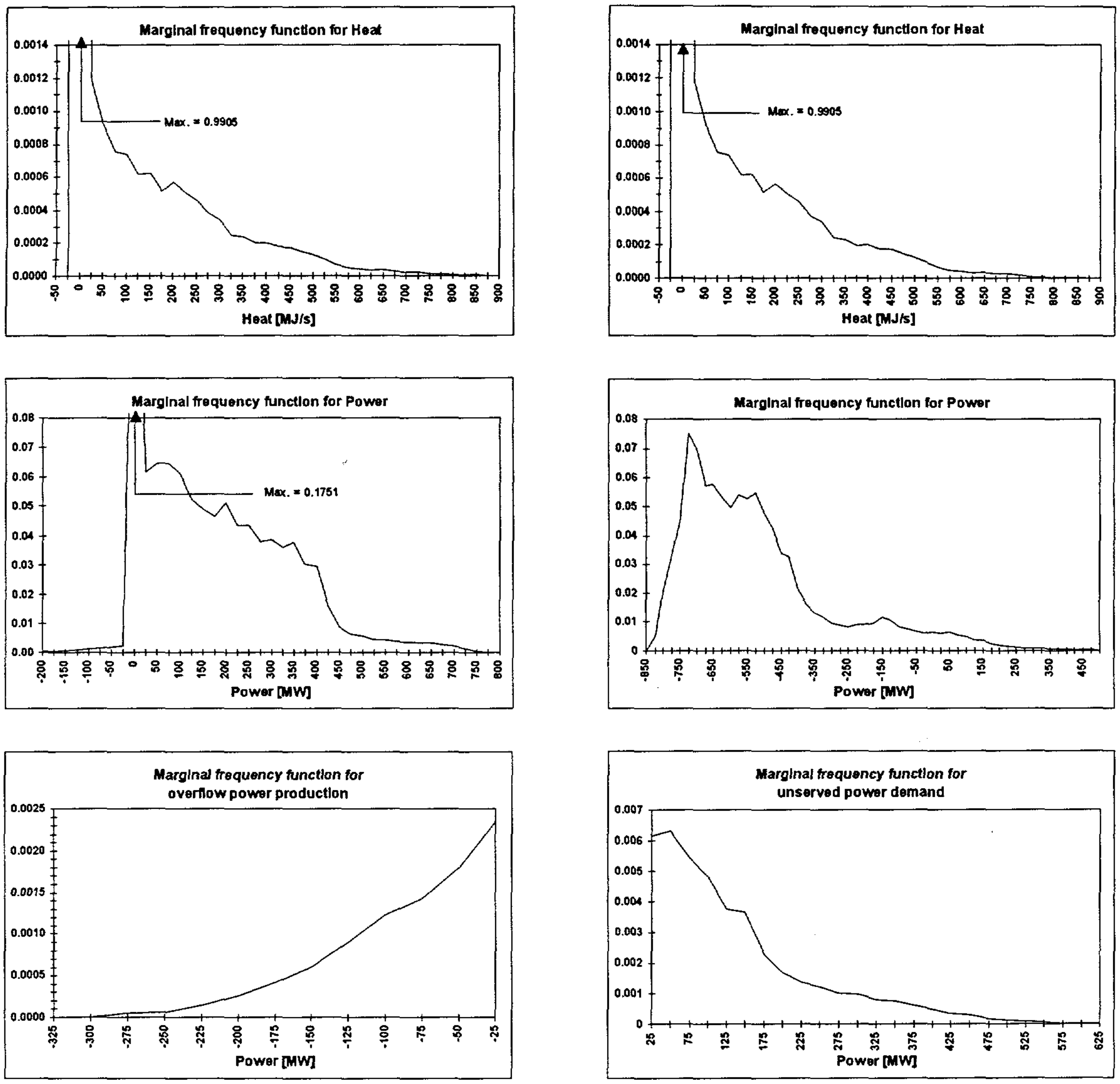

Figure 12 : Marginal frequency functions after convolution.
Left : Minimum power production relative to heat production.
Right : Maximum power production relative to heat production.

Observe that the two marginal frequency functions for heat (top graphs) are identical. The two bottom graphs in Figure 12, indicating the overflow power production and the unserved power demand, are enlargements of the left and right parts of the middle graphs in Figure 12.

By taking the two bottom graphs of Figure 12 together, as shown in Figure 13, we get a marginal frequency function for the unbalances between power demand and production, i.e. the power overflow and the unserved power demand. Most of the power demand is satisfied exactly. This gives a large probability value at $p = 0$ (not shown in the figure).

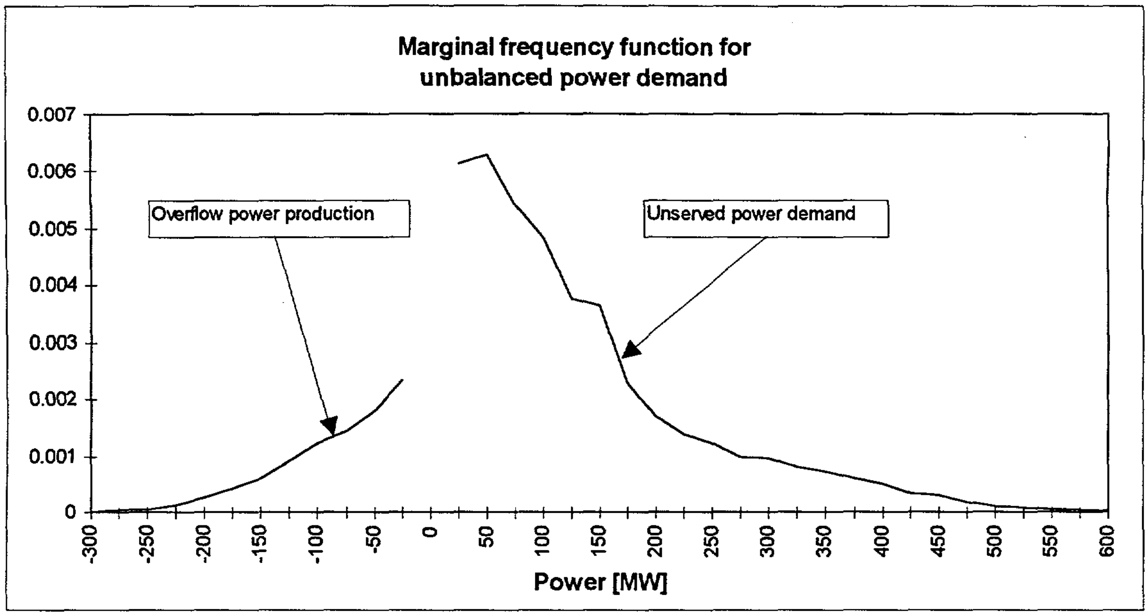

Figure 13 : Marginal frequency function for the unbalances between power demand and production.

We conclude by specifying how the production levels during convolutions may be reduced in order to avoid q-overflow. This subject was postponed in Subsection 2.3, because we needed the above results.

Consider a point which before convolution of unit n has coordinates $(\overline{q},\overline{p})$, with $0 \le \overline{q} < c_n^q$. For this point the production level must be reduced.

Consider first a unit loaded on the back-pressure line with slope c_n^m (a heat unit, a back-pressure unit, or an extraction unit in the convolution "1"). The production of this unit should be (sc_n^q, sc_n^p), where the scalar s has the value $s = \overline{q} \ / \ c_n^q$. The resulting remaining load from the point $(\overline{q},\overline{p})$ will then be located at the position $(\overline{q},\overline{p})$-$(sc_n^q, sc_n^p) = (0, \overline{p} - sc_n^p)$.

It follows that the convolution formula (5) for values on the p-axis should be substituted by

$$f_n(0,p) = (1-r_n)\int_0^1 f_{n-1}(sc_n^q, sc_n^p)ds + r_n f_{n-1}(0,p) \tag{14}$$

Consider now the convolution "2". Here, an extraction unit is loaded along the c^v-line. It follows that for this unit the formula (5) should be substituted by,

$$f_n(0,p) = (1-r_n)\int_0^1 f_{n-1}(sc_n^q, c_n^p - sc_n^q c_n^v)ds + r_n f_{n-1}(0,p) \tag{15}$$

For heat unit and back-pressure units we still use (14).

For condensing units (5) still has validity in both convolutions, "1" and "2", as this unit will not generate q-overflow.

This section concludes the basic development of the method since it shows how to find the quantities *EOE* and *EUE* for the heat priority case.

The method, as presented here, deals with one heat area only. We expect that it will be straightforward to extend the method to deal with more than one heat area for the heat priority case. However, we omit further discussion on this subject.

Therefore, we now turn to questions of implementation.

4. DISCRETIZATION AND ANALYTICAL REPRESENTATIONS

The above analysis indicates that certain functions have to be identified and certain calculations have to be performed in order to derive quantitative results. This may be done in various ways, see e.g. Lin et al. (1989) or Caramanis et al. (1983).

A discretization of the probability space may be used, as we shall discuss in Subsection 4.1. Also a continuous representation of the probability space may be adopted, using approximations by analytical functions. This will be discussed in Subsection 4.2, and specialized to the Fourier series in Subsection 4.3.

In general there are the following three essential steps in applying the method:

1. Identification of the initial probability function $\left(f_0 \text{ or } F_0\right)$.
2. Convolution, i.e. generation of f_{n+1} or F_{n+1}, from f_n or F_n and the characteristics of unit $n+1$.
3. Calculation of desired quantities EUE , etc.

We discuss numerical methods in relation to these steps.

4.1 Discretization

The idea of this method is first to discretize the state-space for $(x^q,\ x^p)$ into squares and then treat all probabilities as discrete probabilities.

For specificity, assume that a grid with 25 $[\mathrm{MJ/s}] \times 25\ [\mathrm{MW}]$ squares is used. This is interpreted to mean that all probability mass is located at points $(25i,\ 25j)$, where i and j are integers. The probability mass in a specific point $\left(x^q_{i*},\ x^p_{j*}\right) = \left(25i^*, 25j^*\right)$ is assumed to represent all the probability mass distributed over the square with x^q-co-ordinate satisfying $25i^* - 12.5 \le x^q < 25i^* + 12.5$ and x^p -co-ordinate satisfying $25j^* - 12.5 \le x^p < 25j^* + 12.5$.

The advantage of the discretization procedure is that it is relatively straightforward to conceive and implement. The disadvantage is that some errors are introduced, as we now explain in relation to the three steps of the introduction to Section 4.

When we identify the initial probability function (Step 1) an error may or may not be introduced, depending on how the initial function is given. If it is given as a discretized function with the same grid-size then actually no error is introduced in defining f_0 and / or F_0. Otherwise (i.e. the initial function is given as a discretized function with different grid-size or it is given as a continuous function), an error will almost certainly be introduced.

In the second step (convolution) errors will most probably be introduced, due to inconsistencies between the grid-sizes and the magnitudes c^q, c^p and c^p / c^q of the individual units. In one-dimensional analysis this may be avoided by applying grid-sizes that are consistent with the capacities (see e.g. Lin, Breipol and Lee (1989)). But with two or more dimensions this is most unlikely to be avoidable. Thus, ad hoc approximation and interpolation must be developed (see e.g. Søndergren (1994) for ideas).

Finally, in Step 3 no additional error will be introduced.

The computational complexity of the discretization method may be indicated as follows: Assume grid-sizes so that a total of D^2 grid squares cover the relevant *qp*-area. Then convolution of one unit requires arithmetical operations proportional to D^2, and one set of calculations for N units will require arithmetical operations roughly proportional to D^2 N. Therefore, e.g. a halving of the grid-length in each of the two dimensions is paid for by a four times longer computational time.

Alternative implementations

The above description was based on the idea of an adjustment of the production of unit *n*, if full production would give overflow that was undesirable according to the criterion applied (in our basic case: heat priority). This results in an accumulation of probability mass on the *p*-axis.

Now we describe an alternative idea: Rather than collect probability mass at the *p*-axis, we shall save this probability mass in a different function called $\tilde{f}$. We exploit the insight gained in Subsection 3.4.

The computational complexity of this alternative method is the same as that above, i.e. D^2 N. The advantage is that it points to ideas that are more conveniently handled in connection with continuous functions, cf. the next subsection.

Following the convolution of unit *n*, and prior to the convolution of unit *n+1*, a corrective calculation is performed. The aim is to find out what the result would have been if the load had been adjusted according to the heat priority criterion.

Consider first a unit loaded on the back-pressure line with slope c^m (a back-pressure unit, or an extraction unit in the convolution "1", yielding minimum *p*-production to any *q*-production, cf. Subsection 3.3).

We define the function $\tilde{f}_n : R \to R$ as that which holds the probability mass at the *p*-axis. We define $\tilde{f}_0(p) \equiv 0$. After convolution of unit *n* we define,

$$\tilde{f}_n(p) = \tilde{f}_{n-1}(p) + (1 - r_n)\int_0^c f_n(q, p + c_n^m q)\, dq \tag{16}$$

where $c = c_n^q$.

For $n=N$ this function therefore holds the probability that the *q*-load has been exactly fulfilled, while the resulting *p*-load has the magnitude *p*. The *p*-overflow may be calculated (c.f. also (13), and Subsection 3.4) as

$$EOE^p = -\tau\int_{-\infty}^0 p\tilde{f}_N(p)dp - \tau\int_{-\infty}^0 p\int_0^\infty f_N(q,p)dqdp \tag{17}$$

Now consider a unit loaded according to the convolution "2" of Subsection 3.3. Again a back-pressure unit is loaded along the c^m-line, while an extraction unit is loaded along the c^v-line. The argumentation proceeds as above. We reach the same expression as in (16) for the contribution of the back-pressure unit, while the similar expression for the extraction unit is,

$$(1 - r_n)\int_0^c f_n(q, p - c_n^v q)\, dq \tag{18}$$

where $c = c_n^q$.

The EUE^p formula corresponding to (17) will be,

$$EUE^p = \tau\int_0^\infty p\tilde{f}_N(p)dp + \tau\int_0^\infty p\int_0^\infty f_N(q,p)dqdp \qquad (19)$$

The main conceptual difference between the two methods is that a certain part of the probability mass in the first method is contained in a specific segment of f_n (viz., along the p-axis) while in the alternative method this part of the probability mass is contained in the specific function $\tilde{f}_n$.

The implementation of this idea in the discretized method is straightforward. The results are, of course, the same. And the complexities of the calculations are also the same.

The purpose of introducing the alternative version here is therefore to be able to have a parallel, in the discretized version, to what will be applied in the next subsection on analytical representations.

The difficulty with the version presented first, in relation to the analytical representations, is that the functions f_n for $n \geq 1$ will display a discontinuity along the p-axis. By storing a certain part of the probability mass in the function $\tilde{f}_n$, this discontinuity is avoided, facilitating the analytical representation.

4.2 Analytical representations

In the above we have been working with a discrete representation of the distribution and/or frequency functions. This means that in the one-dimensional case the functions F_n and/or f_n are defined in discrete points $x_j, j = 1,\ldots,J,$ through the values $F_n(x_j)$ and/or $f_n(x_j)$.

The advantage of this approach is that it is relatively straightforward to explain and also relatively straightforward to implement in computer calculations. Moreover, f_0 or F_0 will often be given as such discrete functions, based on underlying data material. However, the calculation may be time consuming. The key part of the calculations is to perform the convolution. It is therefore desirable to find other ways.

A number of techniques have been proposed that exploit an approximations of the functions involved by analytical functions. This permits a derivation of convenient relations between F_{n+1} , and F_n (and between f_{n+1} and f_n). In particular, relations between the statistical aspects, cumulants and moments (see Rau et al. (1980), Stremel et al. (1980)) of the functions are exploited.

In applying the convolutions there are essentially the following three steps as defined earlier:

1. Identification of f_0 or F_0.
2. Convolution, i.e. identification of f_{n+1} (or F_{n+1}) from f_n (or F_n) and from characteristics of unit $n+1$ (the *FOR* r_{n+1} and capacity (working area)).
3. Calculation of *EUE*, etc., to be performed as an integration of f_N or calculation of values of F_N.

Analytical functions will typically be given as a finite series, i.e., of the form

$$F_n(x)=\sum_{k=0}^{K}c_n^k\varphi^k(x) \tag{20}$$

where c_n^k is a coefficient and φ^k is a given function; a similar expression is used for f_n.

The above three steps may therefore be reformulated as,

1. Identification of c_0^k, $k=0,\dots,K$
2. Identification of c_{n+1}^k from c_n^k, $k=0,\dots,K$ and characteristics of unit $n+1$.
3. Calculation of values or integrals of $\varphi^k, k=0,\dots,K$.

As a simple example, assume that F_n is one-dimensional and given as a polynomial of degree *K*, i.e. $\varphi^k(x)=(x)^k$:

$$F_n(x)=\sum_{k=0}^{K}c_n^k(x)^k \tag{21}$$

The first of the above steps may be performed by determining $c_0^k, k=0,\dots,K$, e.g. by minimizing the sum of square of deviation between defined in (21), and the given function $\widetilde{F}_0$, say . In other words, the c_0^k are determined by solving the problem,

$$\min\left[\sum_{j=1}^{J}\left(\left(\sum_{k=0}^{k}c_0^k\left(x_j\right)^k\right)-\widetilde{F}_0\left(x_j\right)\right)^2\right] \tag{22}$$

with respect to the variables $c_0^k, k=0,\dots,K$. The distribution function given originally $\widetilde{F}_0$ is here assumed to be defined through the J values $F_0\left(x_j\right), j=1,\dots,J$.

The convolution is defined as,

$$\begin{aligned}F_{n+1}(x)&=r_{n+1}F_n(x)+\left(1-r_{n+1}\right)F_n\left(x+\bar{x}_{n+1}\right)\\&=r_{n+1}\left(\sum_{k=0}^{K}c_n^k(x)^{\,k}\right)+\left(1-r_{n+1}\right)\left(\sum_{k=0}^{K}c_n^k\left(x+\bar{x}_{n+1}\right)^{\,k}\right)\end{aligned} \tag{23}$$

Obviously the expression is again a polynomial of degree *K*, and the values of the coefficients c_{n+1}^k may be determined analytically, so that F_{n+1} may also be written in the form (21). Finally, a calculation of values F_N may be performed analytically term by term.

The polynomial approximation therefore permits the three steps to be performed.

However, as is well-known, it is not desirable to us the polynomial (21) as approximation. This is to say that this approximation is weak in relation to step 1 above. Rather, orthogonal functions should be used in order to have better approximation performance.

Therefore, the task is to find series that combine good approximation properties (in relation to step 1) with convenient formulas for the two other steps.

In the one-dimensional analysis the following functions have been used : Gram-Charlier (Rau, Toy and Schenk (1980)), Edgeworth (Levy and Kahn (1982)), Legendre (Jørgensen (1990)), Mixture of Normals (Gross, Garapic and McNutt (1988)) and the Fourier series presented below.

4.3 The Fourier Series

In the sequel we analyze the application of the Fourier series in the two-dimensional analysis. Apparently this series has been used only in the one-dimensional case in Lin, Breipol and Lee (1989), Rau and Schenk (1980) and in Jenkins and Vorce (1977). However, only the first one of these works has been available to us.

The two-dimensional analysis below includes the one-dimensional analysis as a special case, and as such it contributes to the one-dimensional analysis as well.

For an introduction to multidimensional Fourier series see Canuto et al. (1986), and see Press et al. (1992) for computational aspects.

In relation to the double Fourier series f_n (or F_n) may be defined as follows, with I and J indicating the number of terms in each dimension

$$\begin{aligned} f_n(x,y) = & \sum_{i=0}^{I} \sum_{j=0}^{J} \left(c_j^{ss}\right)_n \sin(ix)\sin(jy) \\ & + \sum_{i=0}^{I} \sum_{j=0}^{J} \left(c_{ij}^{cc}\right)_n \cos(ix)\cos(jy) \\ & + \sum_{i=0}^{I} \sum_{j=0}^{J} \left(c_{ij}^{sc}\right)_n \sin(ix)\cos(jy) \\ & + \sum_{i=0}^{I} \sum_{j=0}^{J} \left(c_{ij}^{cs}\right)_n \cos(ix)\sin(jy) \end{aligned} \tag{24}$$

Figures 14 and 15 show the ability of the Fourier series of representing the marginal frequency functions for heat and power demand. Figure 17 gives the same information for the two-dimensional frequency function.

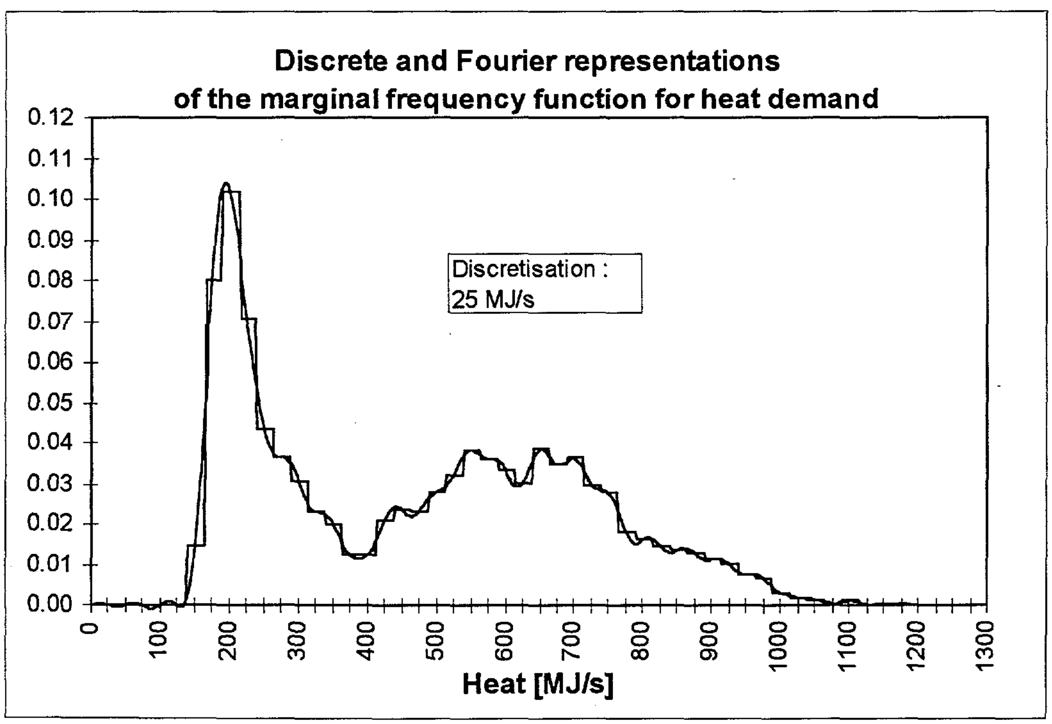

Figure 14 : Comparison of discrete and Fourier representations of the marginal frequency function for heat demand.

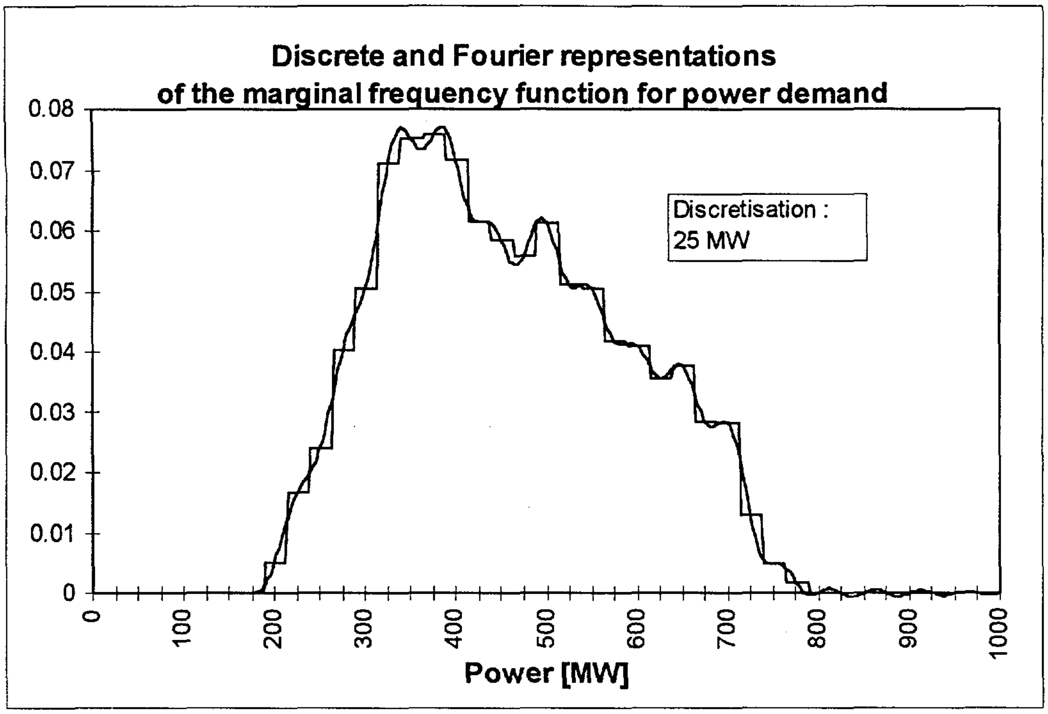

Figure 15 : Comparison of discrete and Fourier representations of the marginal frequency function for power demand.

Now we discuss the application of the Fourier series in relation to the three steps:

Step 1

The first step is to identify f_0 and/or F_0, i.e., to determine the coefficients $\left(c_{ij}\right)_0$. We discuss it in relation to f_0 only, as F_0 may be treated similarly.

The data representing the two-dimensional demand function, conceived as a joint probability frequency function, will typically be given as values in discrete points. Thus, for each index pairs *(a,b)*, $a=1,\dots,A$, $b=1,\dots,B$, we have values $\hat{f}_0\left(q_a, p_b\right)$, representing the probability of having demand q_a, p_b, corresponding to the index pair *(a,b)*.

During the convolution of the units the function f_n will be shifted in the downwards and leftwards directions in the *(q,p)*-plane. Therefore, the function f_0 must be defined in order to accommodate this.

Thus, let $q_d^{\max}$ be the largest value of heat demand which will be assumed to occur. It is assumed that $q_d^{\max}$ is a finite value. Similarly the finite value $p_d^{\max}$ is the largest power demand which will occur. Then the function $\hat{f}_0$ must be defined with *q*-arguments up to

$$q^{\max} = q_d^{\max} + \sum_{n=1}^{N} c_n^q \tag{25}$$

and with *p*-argument up to

$$p^{\max} = p_d^{\max} + \sum_{n=1}^{N} c_n^p \tag{26}$$

In this, heat units have $c^p = 0$ and condensing units have $c^q = 0$.

All the values (*q,p*) within the rectangle $0 \le q \le q^{\max}$, $0 \le p \le p^{\max}$, for which no demand is expected, will be defined with $\hat{f}_0(q,p) = 0$. By definition this holds for $q_d^{\max} \le q \le q^{\max}$ and $p_d^{\max} \le p \le p^{\max}$.

The Fourier series is periodic with period 2π. In order to apply it to the given problem we consider only one period. Further, we scale the variables so that the *q*-interval $[0, q^{\max}]$ is mapped onto $[0,2\pi]$, and the *p*-interval $[0, p^{\max}]$ is mapped onto $[0,2\pi]$.

This is accomplished as follows: Define

$$q_a = \frac{a}{A} \cdot q^{\max} \qquad , a = 0,\ldots,\mathrm{A} \tag{27}$$

$$p_b = \frac{b}{B} \cdot p^{\max} \qquad , b = 0,\ldots,B \tag{28}$$

Then the coefficients are given by expressions similar to the following one :

$$\left(c_{ij}^{ss}\right)_0 = \frac{4}{q^{\max} p^{\max}} \sum_{a=0}^{A-1} \sum_{b=0}^{B-1} \hat{f}_0(q_a, q_b) \sin\left(\frac{i2\pi q_a}{q^{\max}}\right) \sin\left(\frac{j2\pi p_b}{p^{\max}}\right)$$

However, observe that the formulas depend on the specific definition (24) given, and in particular also that advantage may be taken of a reformulation using complex numbers, see Press et al. (1992).

<u>Step 2</u>

We now specify how to perform step 2, the convolutions. For this we have the following result:

Proposition (Convolution Formula for Double Fourier Series):

Assume,

$$\begin{aligned} f_n(x,y) = & \sum_{i=0}^{I} \sum_{j=0}^{J} \left(c_{ij}^{ss}\right)_n \sin(ix) \sin(jy) \\ & + \sum_{i=0}^{I} \sum_{j=0}^{J} \left(c_{ij}^{cc}\right)_n \cos(ix) \cos(jy) \\ & + \sum_{i=0}^{I} \sum_{j=0}^{J} \left(c_{ij}^{sc}\right)_n \sin(ix) \cos(jy) \\ & + \sum_{i=0}^{I} \sum_{j=0}^{J} \left(c_{ij}^{cs}\right)_n \cos(ix) \sin(jy) \end{aligned}$$

Define,

$$f_{n+1}(x,y) = p_{n+1} f_n(x,y) + (1 - p_{n+1}) f_n(x + d_x, y + d_y)$$

Then f_{n+1} may be written in the same form as f_n, and the coefficients are given as,

$$\left(c_{ij}^{ss}\right)_{n+1} = p_{n+1}\left(c_{ij}^{ss}\right)_n + \left(1 - p_{n+1}\right)\Big[\left(c_{ij}^{ss}\right)_n \cos\left(id_x\right)\cos\left(jd_y\right)$$
$$+\left(c_{ij}^{cc}\right)_n \sin\left(id_x\right)\sin\left(jd_y\right)$$
$$-\left(c_{ij}^{sc}\right)_n \cos\left(id_x\right)\sin\left(jd_y\right) - \left(c_{ij}^{cs}\right)_n \sin\left(id_x\right)\cos\left(jd_y\right)\Big]$$

$$\left(c_{ij}^{cc}\right)_{n+1} = p_{n+1}\left(c_{ij}^{cc}\right)_n + \left(1 - p_{n+1}\right)\Big[\left(c_{ij}^{ss}\right)_n \sin\left(id_x\right)\sin\left(jd_y\right)$$
$$+\left(c_{ij}^{cc}\right)_n \cos\left(id_x\right)\cos\left(jd_y\right)$$
$$+\left(c_{ij}^{sc}\right)_n \sin\left(id_x\right)\cos\left(jd_y\right) + \left(c_{ij}^{cs}\right)_n \cos\left(id_x\right)\sin\left(jd_y\right)\Big]$$

$$\left(c_{ij}^{sc}\right)_{n+1} = p_{n+1}\left(c_{ij}^{sc}\right)_n + \left(1 - p_{n+1}\right)\Big[\left(c_{ij}^{ss}\right)_n \cos\left(id_x\right)\sin\left(jd_y\right)$$
$$-\left(c_{ij}^{cc}\right)_n \sin\left(id_x\right)\cos\left(jd_y\right)$$
$$+\left(c_{ij}^{sc}\right)_n \cos\left(id_x\right)\cos\left(jd_y\right) - \left(c_{ij}^{cs}\right)_n \sin\left(id_x\right)\sin\left(jd_y\right)\Big]$$

$$\left(c_{ij}^{cs}\right)_{n+1} = p_{n+1}\left(c_{ij}^{cs}\right)_n + \left(1 - p_{n+1}\right)\Big[\left(c_{ij}^{ss}\right)_n \sin\left(id_x\right)\cos\left(jd_y\right)$$
$$-\left(c_{ij}^{cc}\right)_n \cos\left(id_x\right)\sin\left(jd_y\right)$$
$$-\left(c_{ij}^{sc}\right)_n \sin\left(id_x\right)\sin\left(jd_y\right) + \left(c_{ij}^{cs}\right)_n \cos\left(id_x\right)\cos\left(jd_y\right)\Big]$$

Proof:

We use the formulas,

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

to rewrite the series expression of $f_n\,(x+d_x, y+d_y)$:

$$f_n\,(x+d_x, y+d_y) =$$

$$\sum_{i=0}^{I}\sum_{j=0}^{J}\left(c_{ij}^{ss}\right)_n \left(\sin(ix)\cos\left(id_x\right) + \cos\left(id_x\right)\sin\left(id_x\right)\right)$$
$$\left(\sin\left(jy\right)\cos\left(jd_y\right) + \cos\left(jy\right)\sin\left(jd_y\right)\right)$$

$$+\sum_{i=0}^{I}\sum_{j=0}^{J}\left(c_{ij}^{cc}\right)_n \left(\cos(ix)\cos\left(id_x\right) - \sin(ix)\sin\left(id_x\right)\right)$$
$$\left(\cos\left(jy\right)\cos\left(jd_y\right) - \sin\left(jy\right)\sin\left(jdy\right)\right)$$

$$+\sum_{i=0}^{I}\sum_{j=0}^{J}\left(c_{ij}^{sc}\right)_n \left(\sin(ix)\cos\left(id_x\right)+\cos(ix)\sin\left(id_x\right)\right)$$

$$\left(\cos\left(jy\right)\cos\left(jd_y\right)-\sin\left(jy\right)\sin\left(jd_y\right)\right)$$

$$+\sum_{i=0}^{I}\sum_{j=0}^{J}\left(c_{ij}^{cs}\right)_n \left(\cos(ix)\cos\left(id_x\right)-\sin(ix)\sin\left(id_x\right)\right)$$

$$\left(\sin\left(jy\right)\cos\left(jd_y\right)+\cos\left(jy\right)\sin\left(jd_y\right)\right)$$

Substituting this into the definition of f_{n+1} , using the definition of f_n and rearranging terms we get the desired result. □

Again advantage may be taken of complex notation.

Step 3

In Step 3 we calculate the quantities *EOE* and *EUE*.

Using the frequency function f_n and using Subsection 2.5 and the alternative implementation defined in Subsection 4.1, the required calculations may be performed term by term.

In conclusion we see that it is possible to perform the operations required in the three steps when using the double Fourier series.

We finish this section by an example.

The data in this case is sampled in 25MJ/s intervals for heat and 25MW for power. Figure 16 shows the sampled two dimensional histogram for heat and power. Actually, Figure 3 and 16 show the same data.

In order to define a big enough envelope for the data set, to be able to represent it by the periodic Fourier series, we must know the maximum load and the capacities of the plants. The capacities shown in Table 1 indicate a minimum envelope of 3025MJ/s for heat and 1600MW for power. In terms of 25MJ/s × 25MW intervals, this leads to 121×64 points, which are rounded to 128×64 to improve the speed of the FFT routine (see Press et. al. (1992)).

Figure 17 shows the interpolated frequency function. The function is shown with point in between the 25MJ/s × 25MW intervals, as well as the original points. This is done to give indication of the oscillatory nature of the Fourier interpolation. Some small oscillations can be observed in the figure, but it seems to fit well to Figure 16.

The next two figures show the functions f and $\tilde{f}$ with the first plant loaded. Note that the two dimensional mesh is periodic so the data appears at the corners of the mesh.

The following figures show the two functions as they are generated by loading the remaining units.

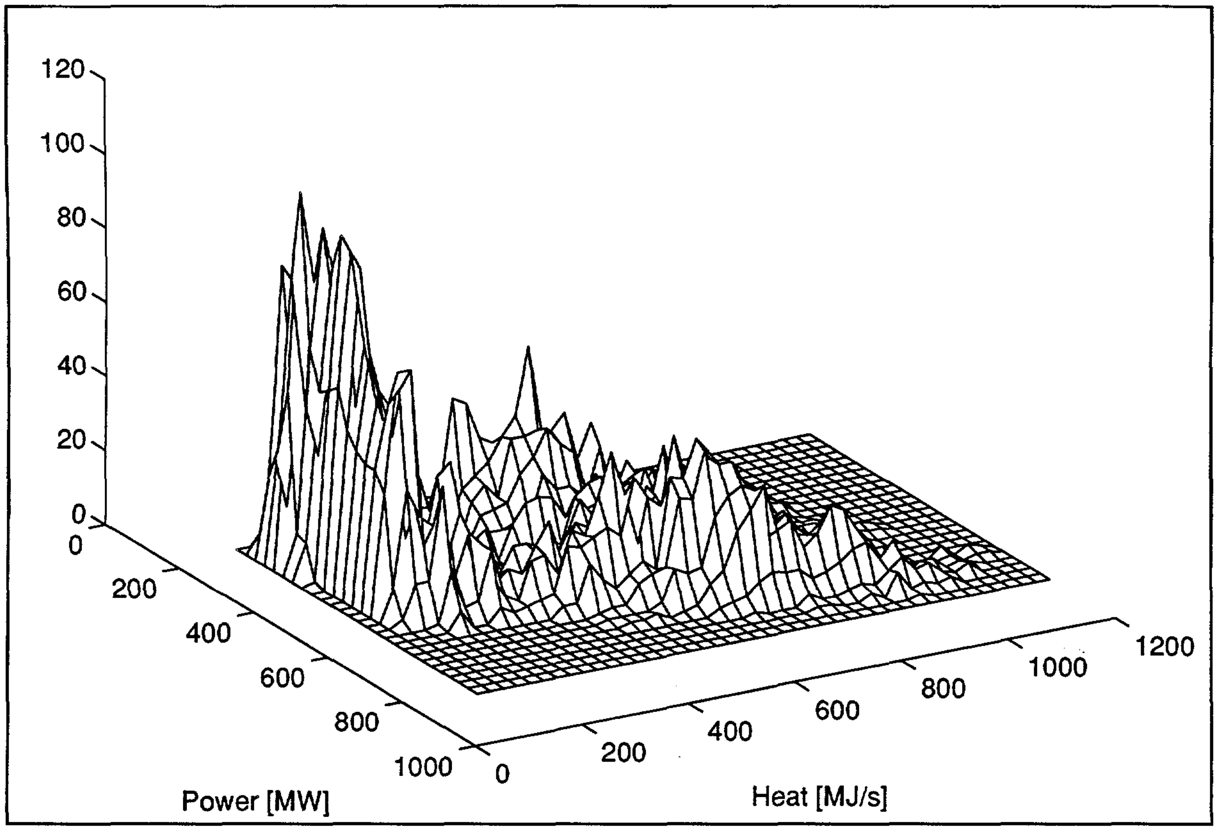

Figure 16 : Sampled histogram with 25MJ/s × 25MW sample intervals.

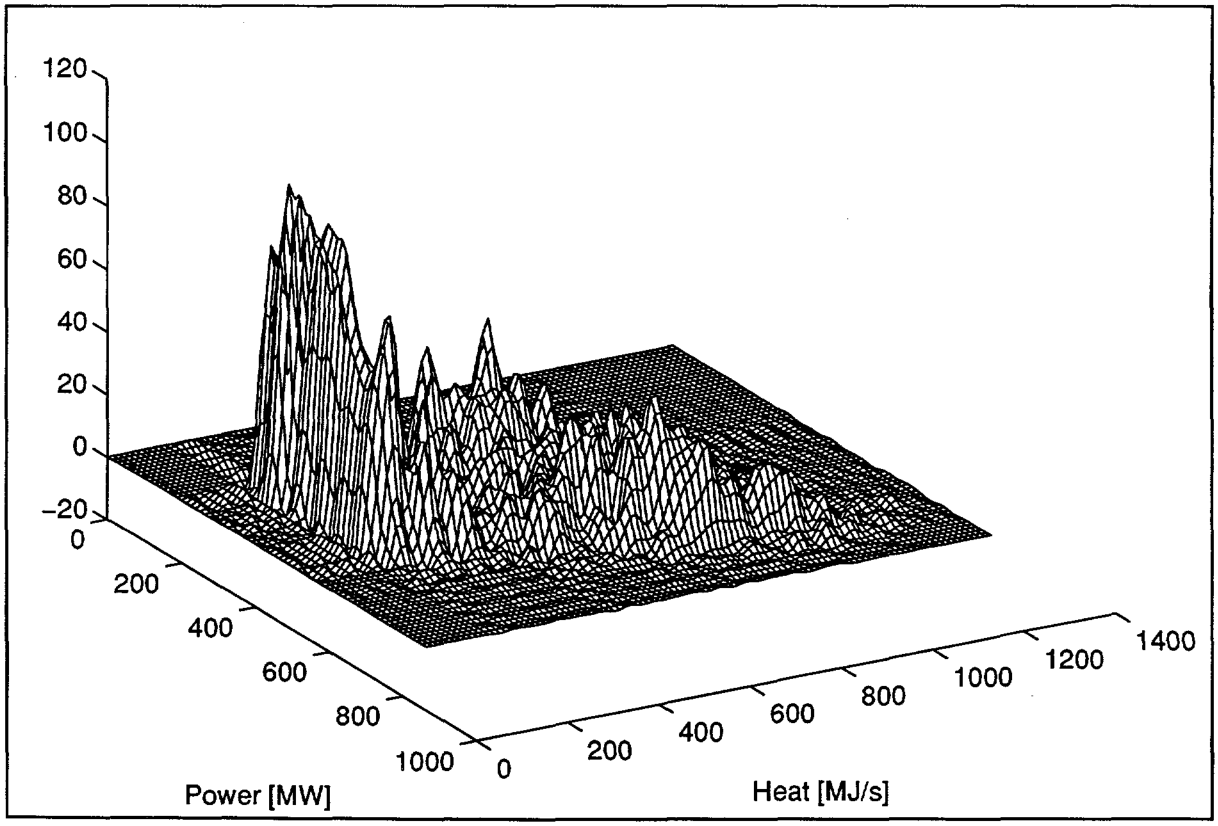

Figure 17 : Frequency function with 12.5MJ/s × 12.5MW intervals.

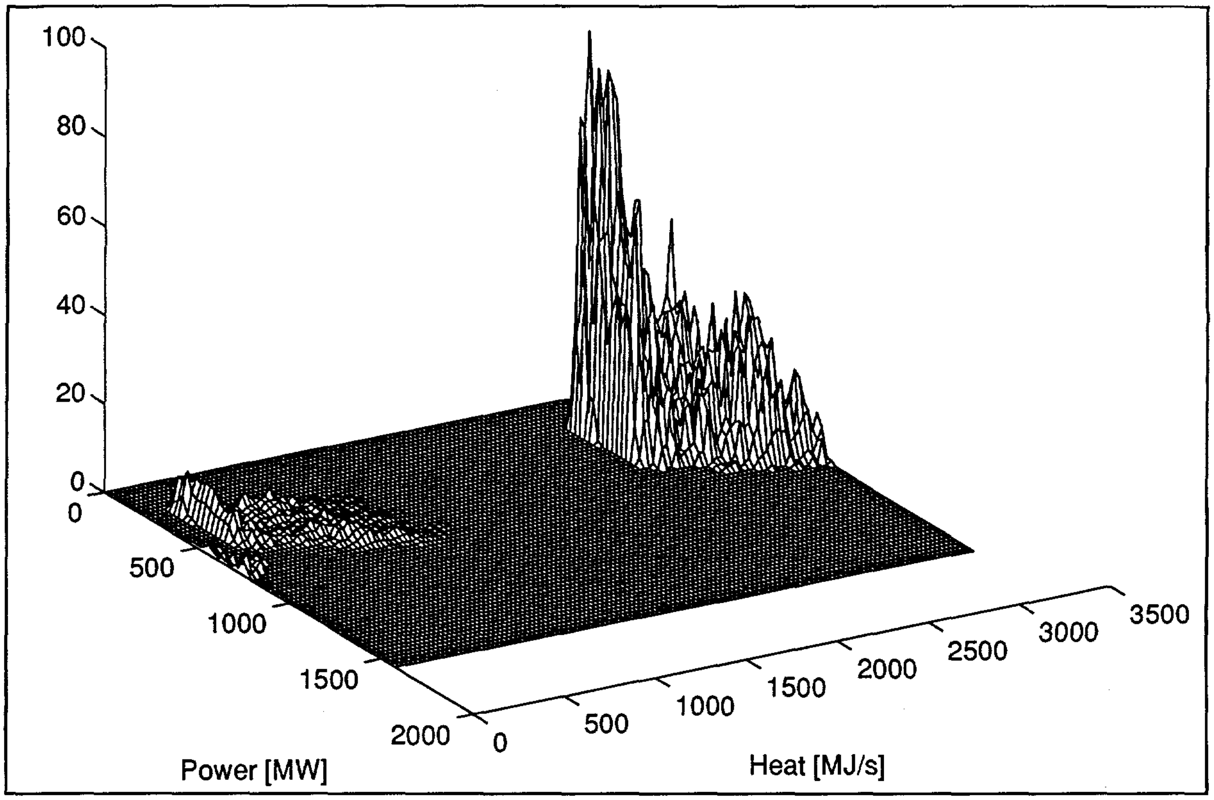

Figure 18 : Frequency function with first plant loaded.

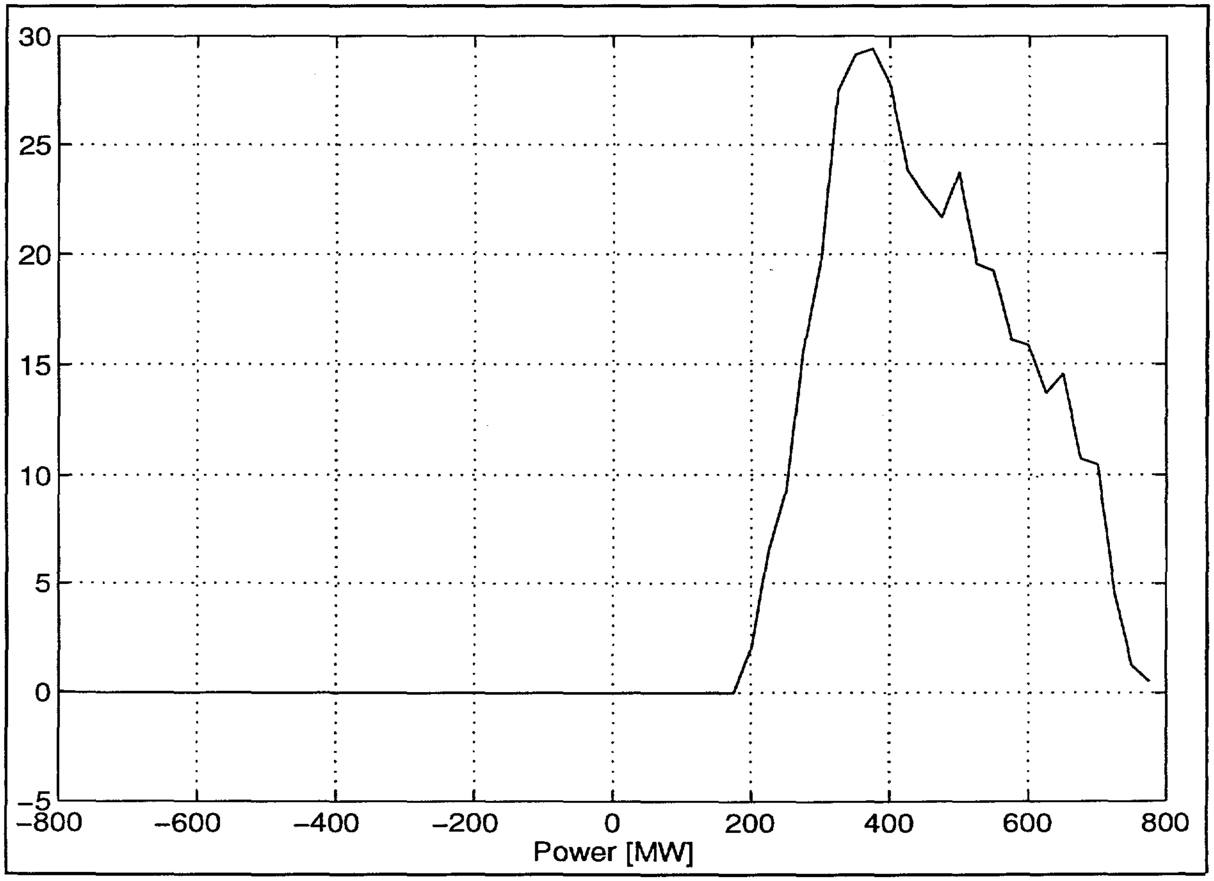

Figure 19 : The auxiliary function $\tilde{f}$ with first plant loaded.

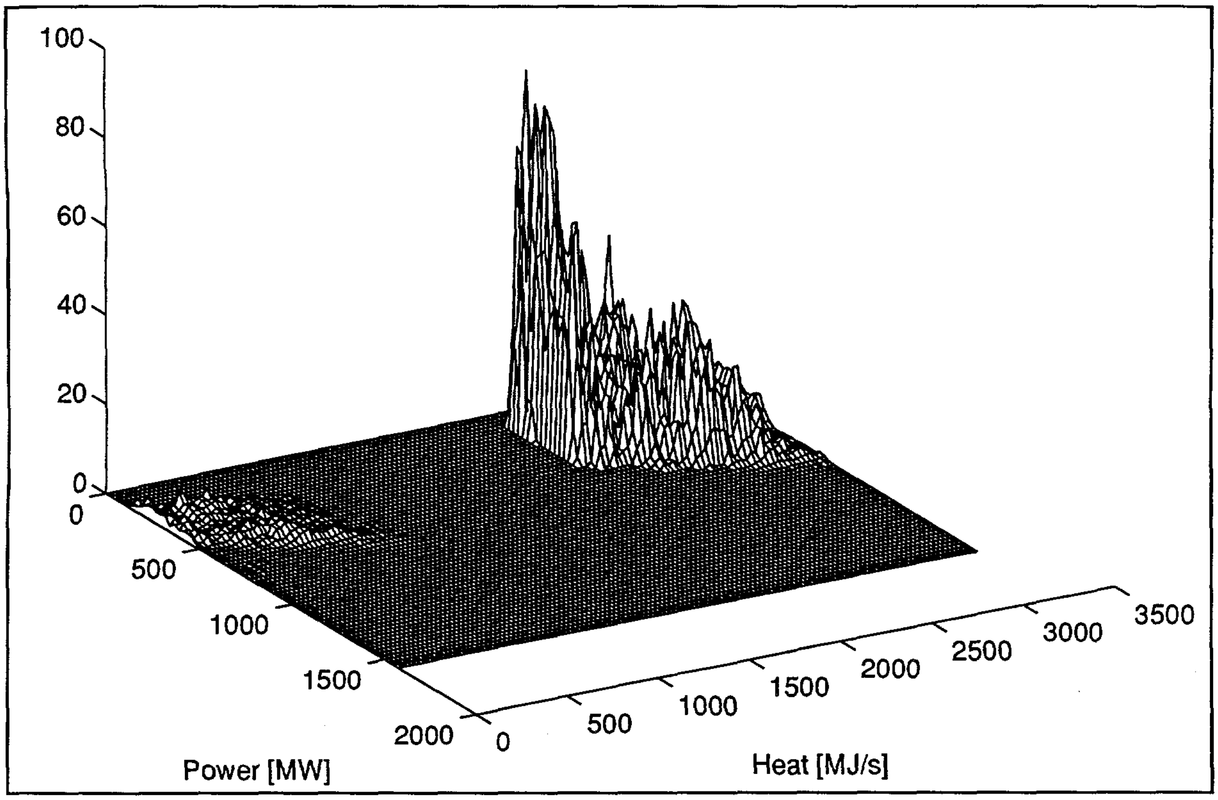

Figure 20 : Frequency function with second plant loaded.

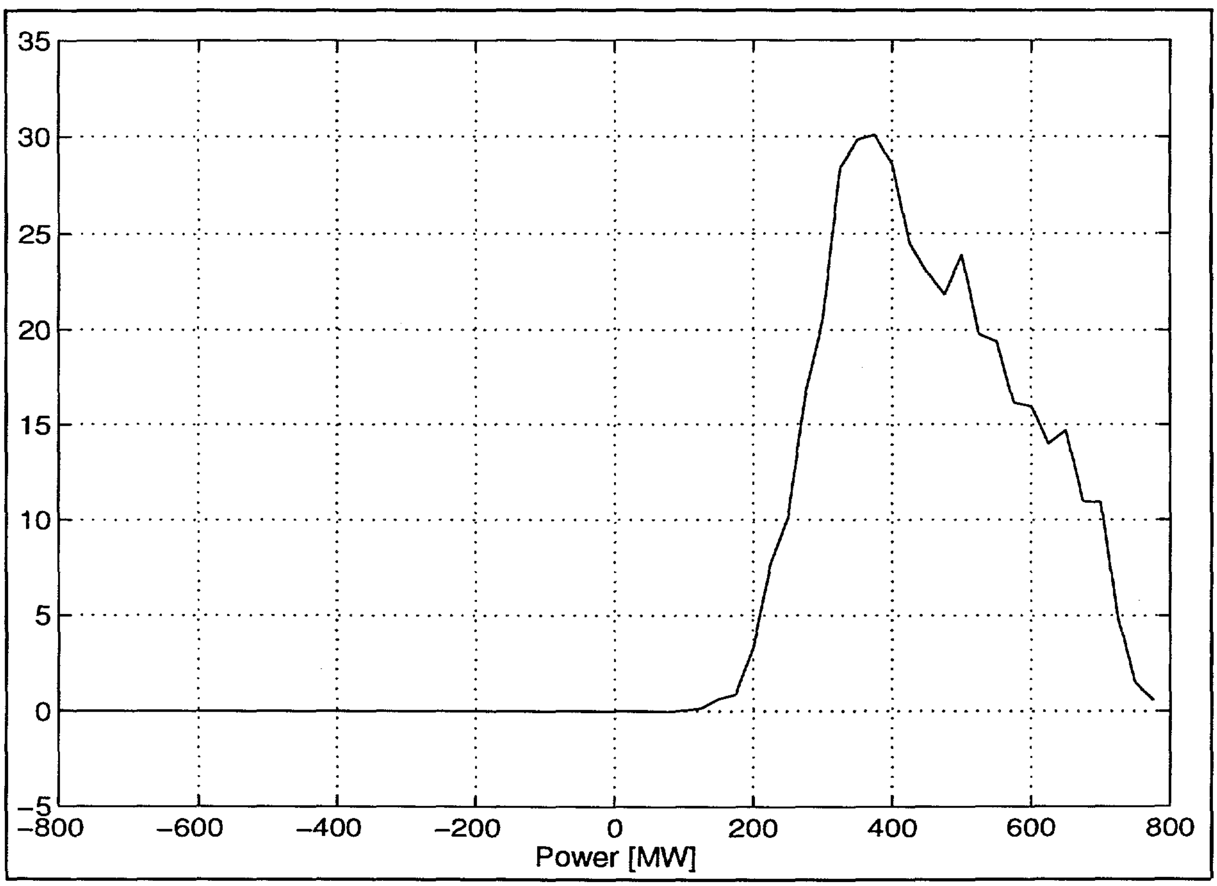

Figure 21 : The auxiliary function $\tilde{f}$ with second plant loaded.

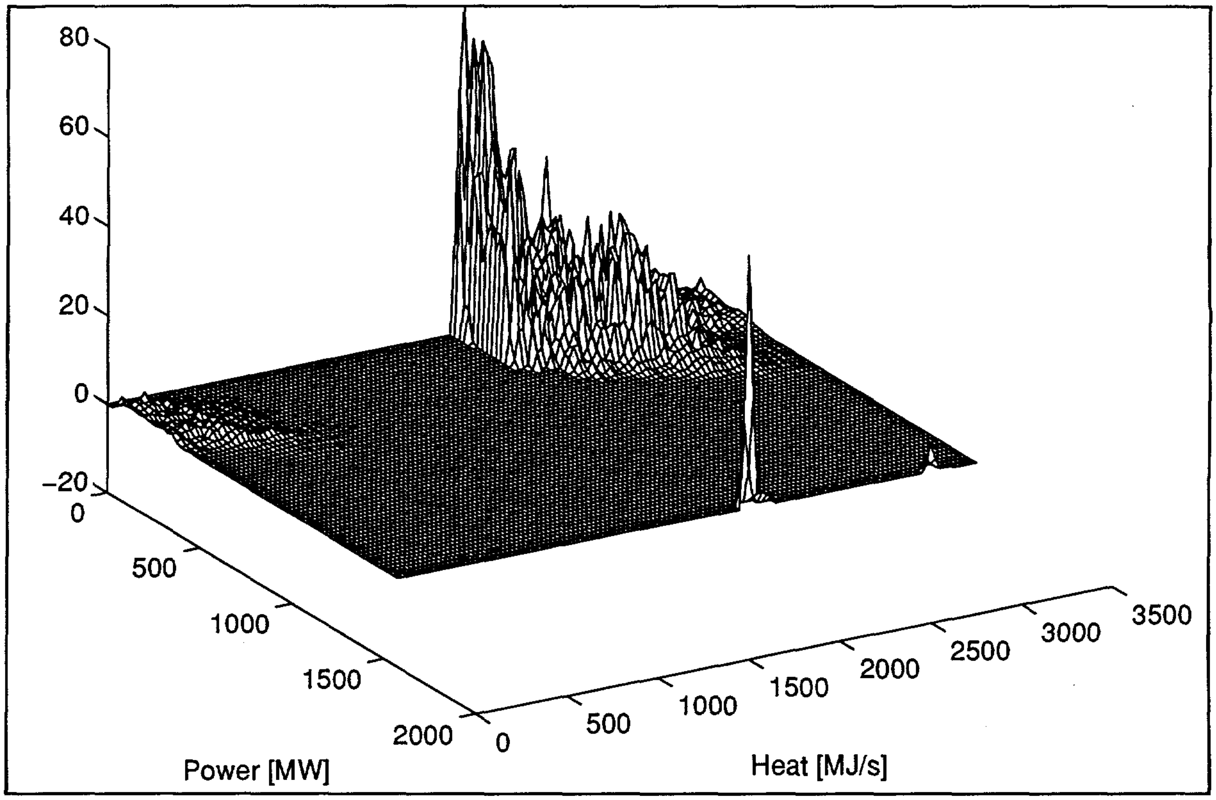

Figure 22 : Frequency function with third plant loaded.

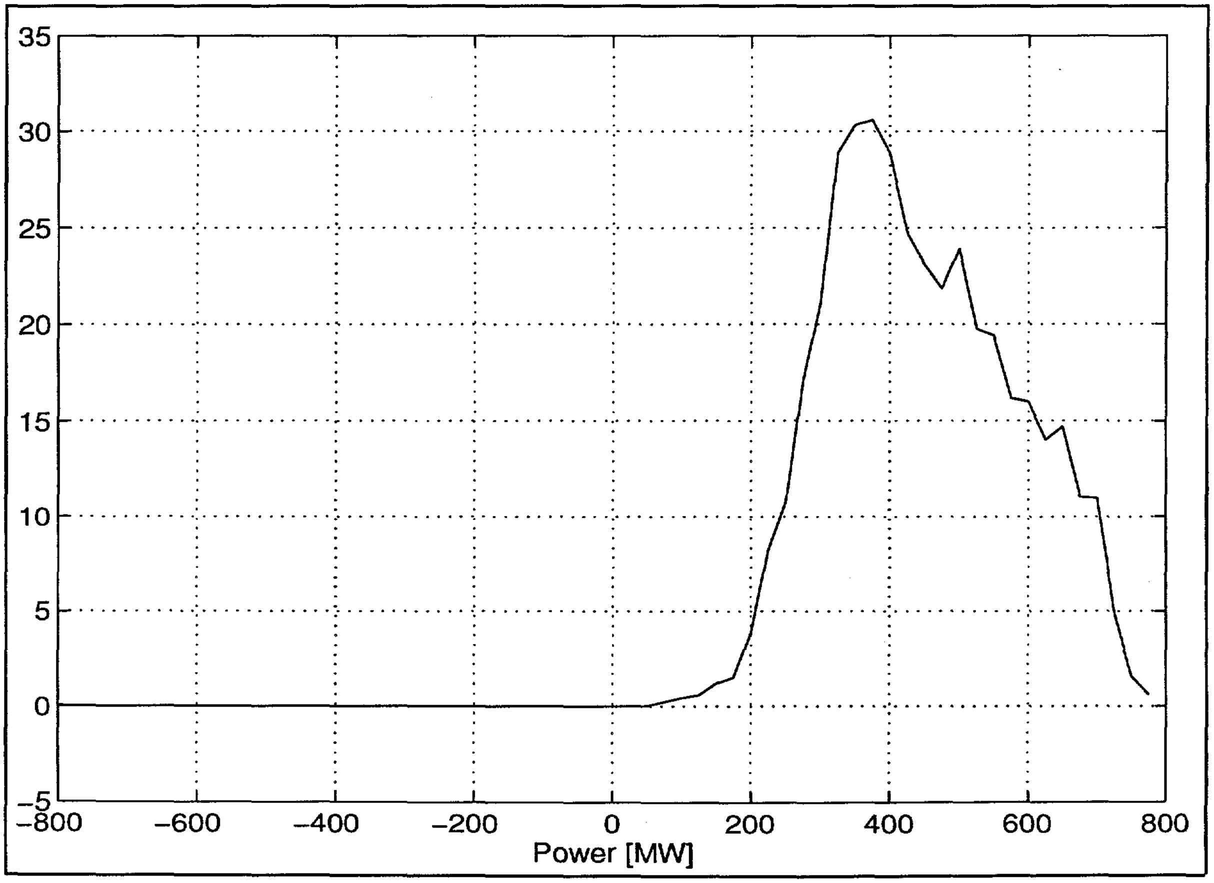

Figure 23 : The auxiliary function $\tilde{f}$ with third plant loaded.

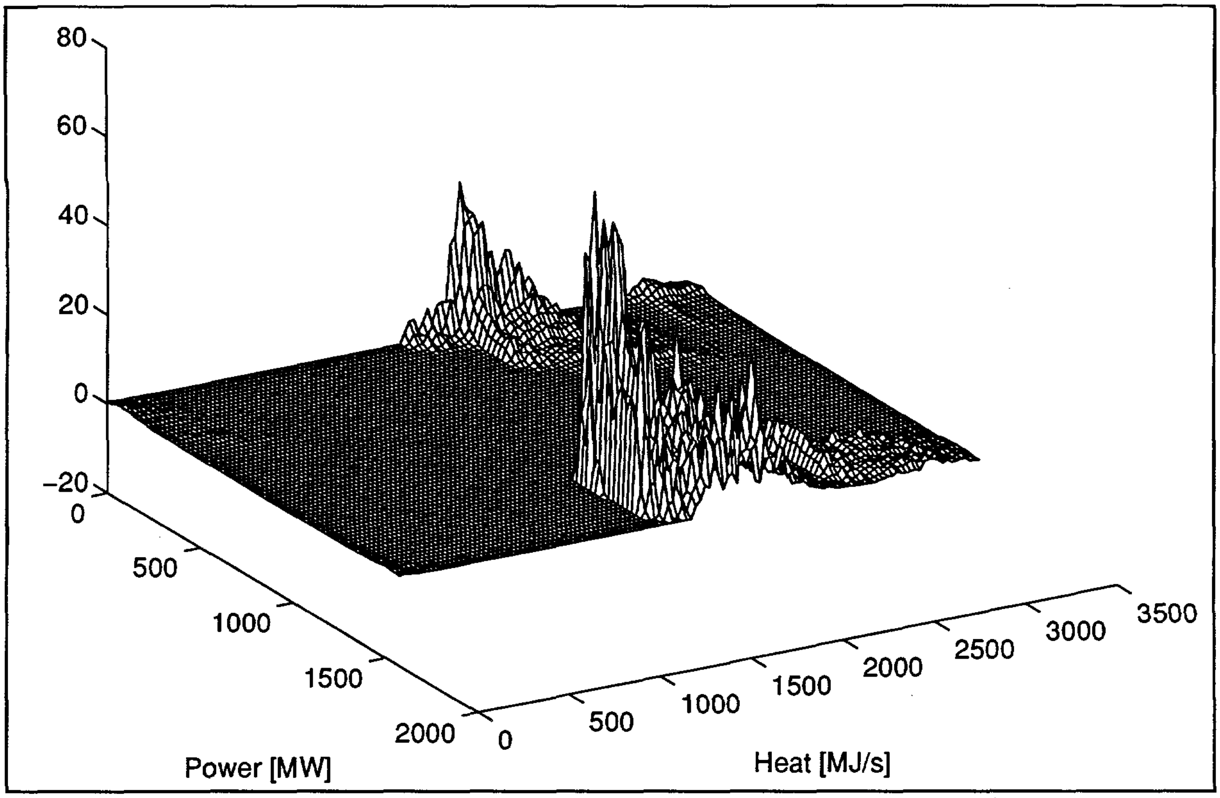

Figure 24 : Frequency function with fourth plant loaded.

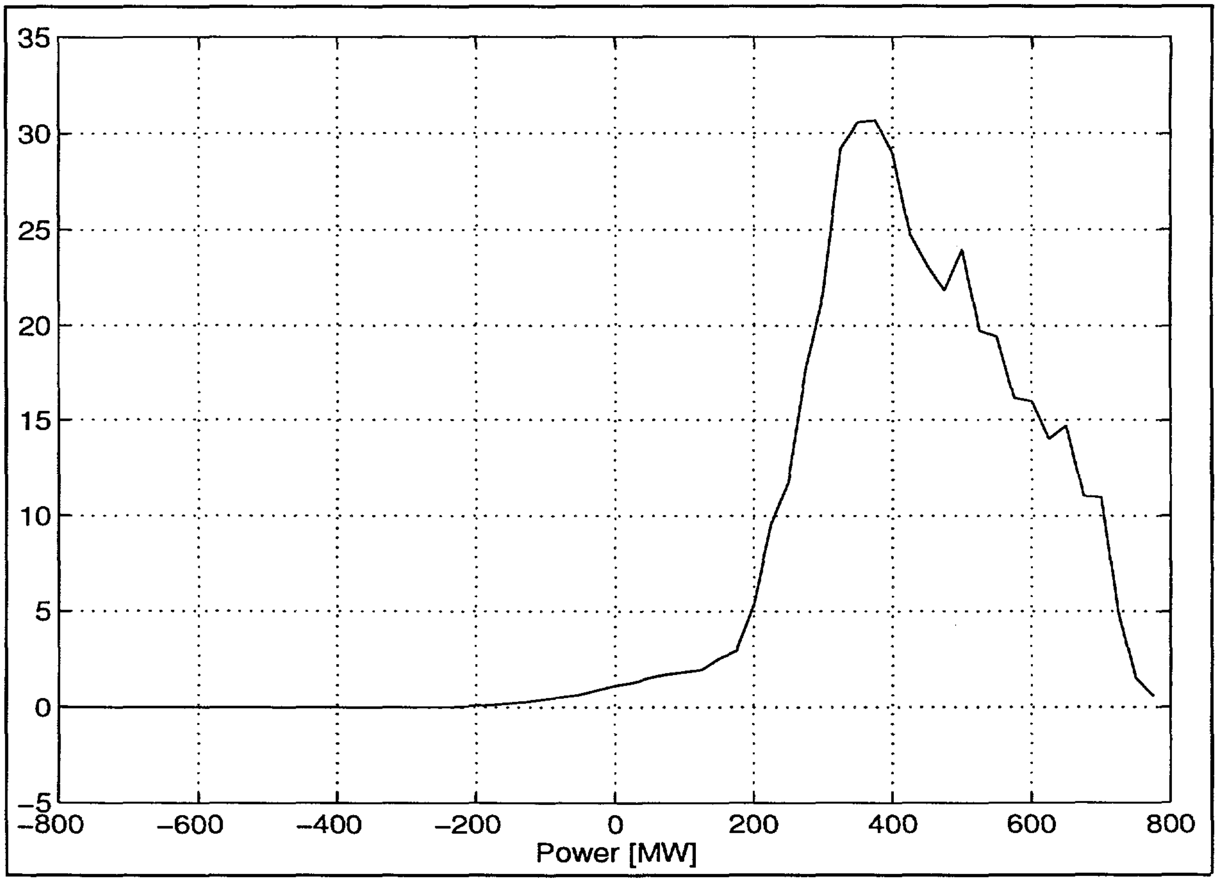

Figure 25 : The auxiliary function $\tilde{f}$ with fourth plant loaded.

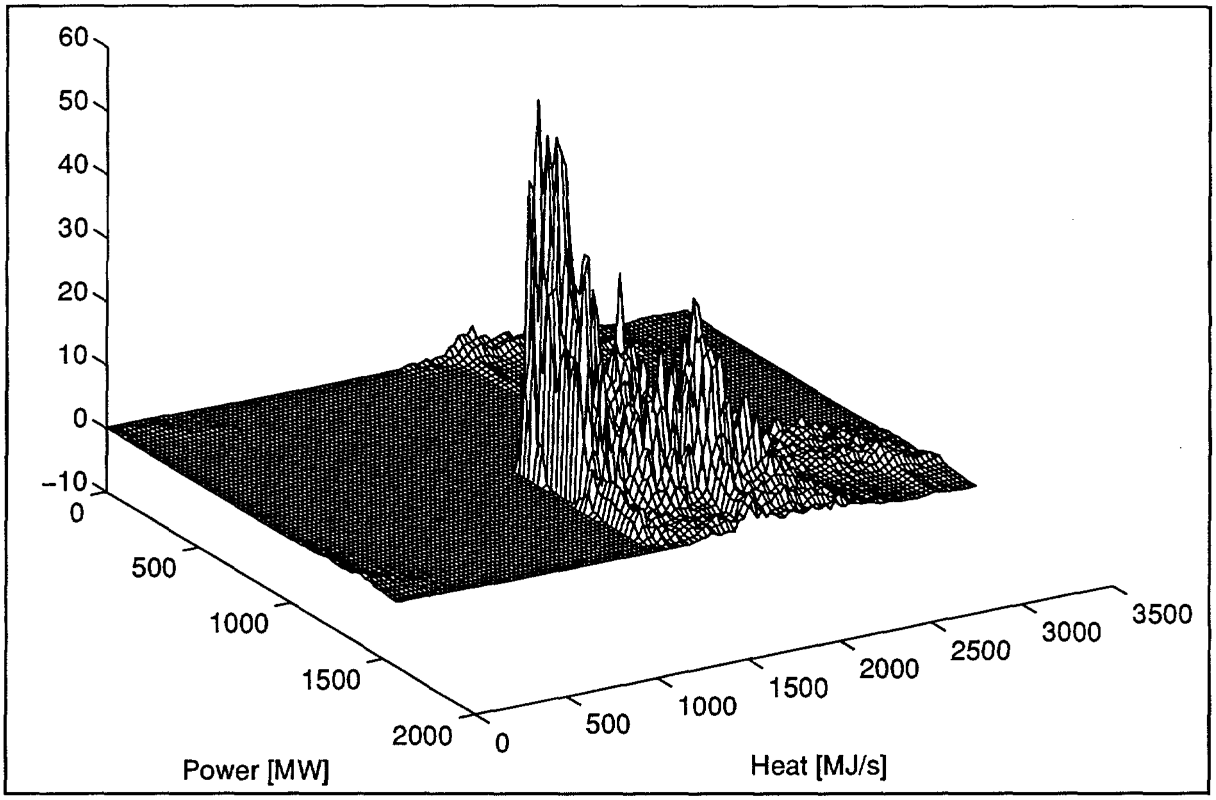

Figure 26 : Frequency function with fifth plant loaded.

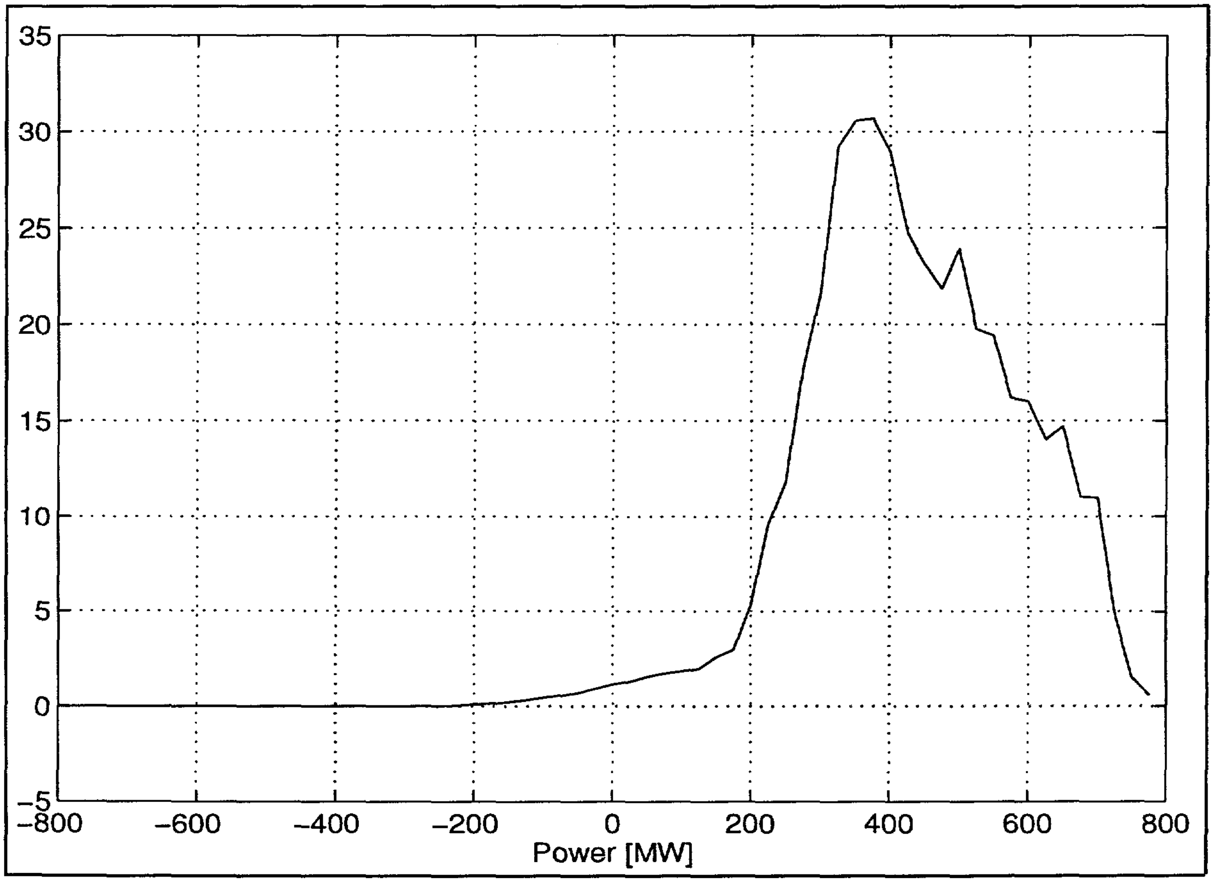

Figure 27 : The auxiliary function $\tilde{f}$ with fifth plant loaded.

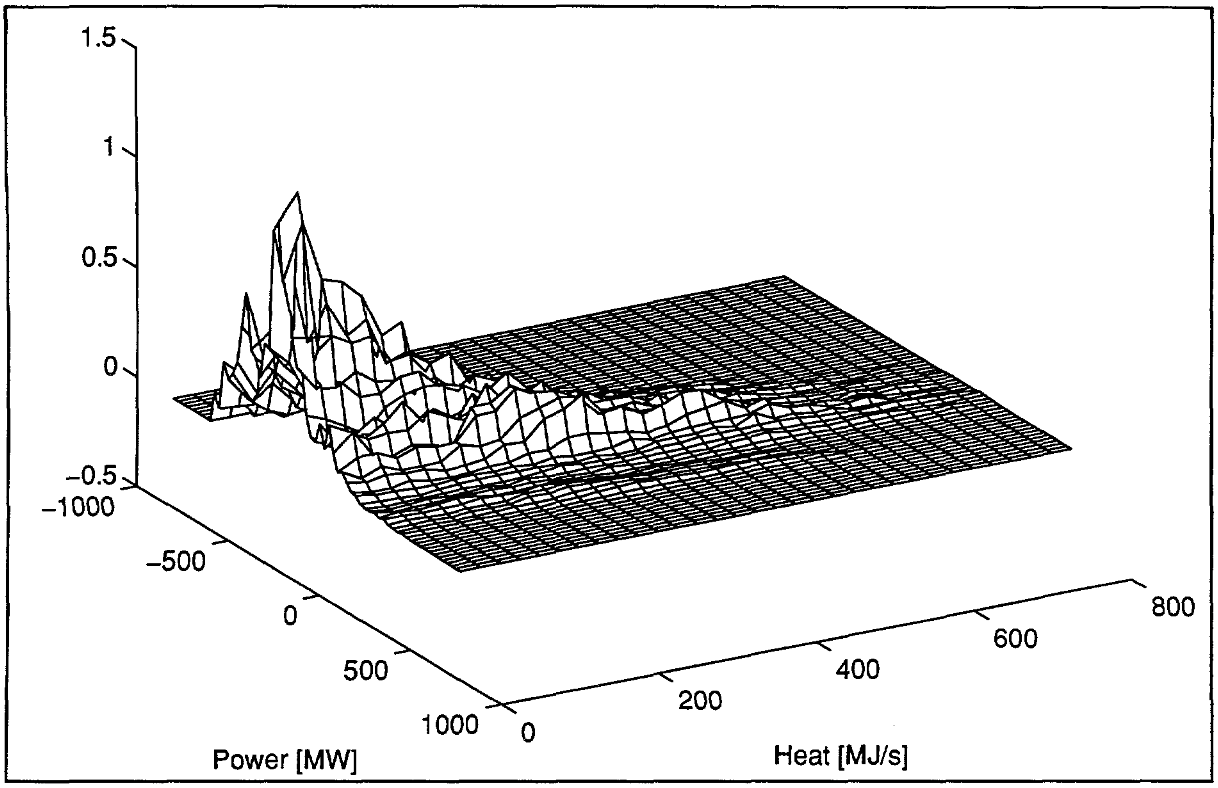

Figure 28 : Same as Figure 26, but with the power axis moved half a period to show the residual heat demand more clearly.

5. CONCLUSIONS

A probabilistic production simulation method has been presented for an energy system containing combined heat and power plants. The method permits incorporation of stochastic failures (forced outages) of the plants and is well suited for analysis of the dimensioning of the system, that is, for finding the appropriate types and capacities of production plants in relation to expansion planning.

The method is in the tradition of similar approaches for the analysis of power systems, based on the load duration curve. The present method extends on this by considering a two-dimensional load duration curve where the two dimensions represent heat and power.

The method permits the analysis of a combined heat and power system which includes all the basic relevant types of plants, viz., condensing plants, back-pressure plants, extraction plants and heat plants.

The focus of the method is on the situation where the heat side has priority. This implies that on the power side there may be imbalances between demand and production. The method permits quantification of the expected power overflow, the expected unserved power demand, and the expected unserved heat demand.

It is shown that a discretization method as well as the double Fourier series may be applied in algorithms based on the method.

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Abstract

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The method is in the tradition of similar approaches for the analysis of power systems, based on the load duration curve. The present method extends on this by considering a two-dimensional load duration curve where the two dimensions represent heat and power.

The method permits the analysis of a combined heat and power system which includes all the basic relevant types of plants, viz., condensing plants, back pressure plants, extraction plants and heat plants.

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It is shown that a discretization method as well as double Fourier series may be applied in algorithms based on the method.

Descriptors INIS/EDB

COGENERATION, COMPUTERIZED SIMULATION, FOURIER ANALYSIS, LOAD MANAGEMENT, OUTAGES, POWER DEMAND PLANNING, PROBABILISTIC ESTIMATION, SYSTEMS ANALYSIS

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