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Ultimate Loading of Wind Turbines

Gunner Chr. Larsen, Knut Ronold, Hans E. Jørgensen, Kimon Argyriadis and Jaap de Boer

Abstract

An extreme loading study has been conducted comprising a general wind climate analysis as well as a wind turbine reliability study.

In the wind climate analysis, the distribution of the (horizontal) turbulence standard deviation, conditioned on the mean wind speed, has been approximated by fitting a three parameter Weibull distribution to the measured on-shore and off-shore data for wind speed variations. Specific recommendations on off-shore design turbulence intensities are lacking in the present IEC-code. Based on the present analysis of the off-shore wind climate on two shallow water sites, a *design turbulence intensity* for off-shore application is proposed which, in the IEC code framework, is applicable for extreme as well as for fatigue load determination.

In order to establish a rational method to analyse wind turbine components with respect to failure in ultimate loading, and in addition to establish partial safety factors for design of such components against this failure mode, *structural reliability methods* must be applied. This type of analysis accounts for the variability of the external (wind) loading (as addressed in the analyses of the general wind climate) - and thereby the induced variability in the component stress response - as well as variability in material resistance. The present study comprises the development of a procedure suitable for dealing with this type of analyses. The main effort has been put on the methodology. Application of the procedure is illustrated by application to the event of failure in ultimate loading in flapwise bending in the normal operating condition of a site-specific turbine.

The present report has passed an internal review at the Wind Energy and Atmospheric Physics Department, Risø, performed by:

Carl Jørgen Christensen   Morten Thøgersen

Contents

1. Introduction

The verification of the structural integrity of a wind turbine structure involves analyses of fatigue loading as well as extreme loading arising from the environmental wind climate. With the trend of persistently growing turbines, the extreme loading seems to become relatively more important.

The extreme loading to be assessed in an ultimate limit state analyses may result from a number of extreme load events including transient operation (start/stop sequences), faults, and extreme wind events. Examples of extreme wind events are extreme mean wind speeds with a recurrence period of 50 years, extreme wind shear, extreme wind speed gusts and extreme wind direction gusts. The present study addresses extreme wind turbine loading arising only from (a particular class of) extreme wind events.

The extreme wind events explicitly accentuated above are included in the currently available draft of the IEC-standard (IEC 61400-1, 1998) as extreme load conditions that must be considered as ultimate load cases when designing a wind turbine. Within the framework of the IEC-standard, these load situations are defined in terms of two independent site variables - a reference mean wind speed and a characteristic turbulence intensity.

The available experimental data material relates to the mean wind speed regime between 5m/s and 25m/s, and the present study is consequently limited to extreme wind conditions occurring during normal operation of the wind turbine. These are in the code described exclusively in terms of the turbulence intensity. In addition to the code, which is somewhat empirically based, theoretical models, based on probabilistic analysis of multi-variate random processes, exist that predict probability density functions of gust events (Chaviaropoulos, 1997). Also these models rely heavily on the site turbulence intensity. Thus the turbulence intensity - defined as the standard deviation of the wind speed divided by the mean wind speed - is the crucial parameter concerning modelling of this class of extreme wind conditions.

In order to gain a deeper insight into the modelling of the turbulence intensity, an investigation of the *general wind climate* related to two important terrain categories - a flat and homogeneous terrain and a shallow water off-shore site - is conducted.

The wind speed standard deviation, determined from a finite sampling time, is not a constant for a given site and a given mean wind speed, but exhibits a statistical distribution around a mean standard deviation. Among the reasons for this variability are varying atmospheric stability conditions, varying roughness lengths for varying wind directions or time of the year and random errors. The wind climate analysis comprises investigations of the mean turbulence standard deviation, as well as variability of the turbulence standard deviation.

Expressions for the mean standard deviation, conditioned on the mean wind speed, as well as for the probability density function of the standard deviation, conditioned on the mean wind speed, are derived by fitting a huge amount measurements to suitable mathematical expressions. For the on-shore situation, the results are compared with the turbulence specifications given in the IEC-code. For the off-shore situation, the existing draft IEC-code contain no specific information. Based on the present analysis of the off-shore wind climate on two shallow water sites, a *design turbulence intensity* for off-shore application is proposed.

In order to establish a rational method to analyse wind turbine components with respect to failure in ultimate loading, and in addition to establish partial safety factors for design of such components against this failure mode, *structural reliability methods* must be applied. This type of analysis accounts for the variability of the external (wind) loading (as addressed in the analyses of the general wind climate) - and thereby the induced variability in the component stress response - as well variability in material resistance.

The present study comprises the development of a procedure suitable for dealing with this type of analyses. The main effort has been put on the methodology. Application of the procedure is illustrated by applying it to the event of failure in ultimate loading in flapwise bending on a site-specific turbine. Probabilistic models for the wind loading and its transfer to bending stresses are established together with a stochastic representation of the material resistance. Contributions to the failure probability from all wind climates, occurring during operation of the wind turbine over a 20-year design life, are integrated, and partial safety factors for load and material resistance determined.

2. General Wind Climate

In order to establish a simple statistical description of the standard deviation of the horizontal turbulence component, a parameterization is required. The first step in this process is to determine a relation between the mean of the standard deviation of the horizontal turbulence component and the mean wind speed under suitable simplifying assumptions - or, in other words, to establish the mean value of the standard deviation distribution conditioned on the mean wind speed. Having estimated the mean of the distribution, the second and final step is to quantify the variability around the mean in terms of a probability density function.

2.1 Theoretical considerations

At a given site and in a given height above the ground (or water surface), the standard deviation, σ_u, of the arbitrary wind speed, conditioned on the mean wind speed, will follow some probability distribution in the long term. This reflects a natural variability over time owing to, e.g., varying atmospheric stability conditions, varying wind directions (and thus varying roughness conditions) etc.. It is appropriate to consider the standard deviation, $\sigma_{u,T}$, of the arbitrary wind speed in the short term - i.e. over some limited time span T. During this length of time the mean wind speed is U_T, and, for *on-shore* conditions, the mean value of the wind speed standard deviation can be expressed by a linear relationship (Larsen, 1999)

$$\left\langle \sigma_{u,T} \right\rangle = \alpha + \beta U_T \quad , \tag{2.1}$$

where α and β are constants to be determined by fitting to experimental data. In the *off-shore* situation, the analogous relationship takes the form of a power law (Larsen, 1999)

$$\left\langle \sigma_{u,T} \right\rangle = \alpha U_T^{\beta} + \delta \quad , \tag{2.2}$$

where the constants α, β and δ are to be determined by fitting the expression to measured data.

The variability of the turbulence standard deviation around the mean standard deviation, as formulated above (or alternatively around a theoretical prediction of the above (Larsen, 1999)), can be treated theoretically under certain simplifying assumptions. More explicitly, it can be shown that the standard deviation of the standard deviation estimate, arising from random errors introduced due to the finite time sampling only, follows a certain probability density function described in terms of the mean standard deviation and an "efficient" number of statistical degrees of freedom (Larsen, 1999).

However, in the succeeding data analyses it is intended to include also the turbulence variability contributions arising from varying atmospheric stability conditions and varying roughness conditions caused by varying mean wind direction. Therefore a more empirically oriented approach is selected for a parameterization of the experimental data.

It is well known that the three parameter Weibull probability density function

$$f\left(x;k,\alpha,\beta\right) = \frac{k}{\beta}\left(\frac{x-\alpha}{\beta}\right)^{k-1} \exp\left[-\left(\frac{x-\alpha}{\beta}\right)^{k}\right]; \ x \geq \alpha \ , \tag{2.3}$$

where k is a shape parameter, α a position parameter and β a scaling parameter (k and β are required positive) is a very "flexible" distribution type, and consequently it was decided to base the parameterization of the data material on this generic distribution form. Subsequent numerical testing has shown that it is suitable for representation of the body of the distribution, however, it has a tendency of underestimating the upper tail. Recent investigations (Ronold, to appear) indicate that a lognormal distribution, at least for on-shore sites, might be a more adequate choice.

2.2 Data Analyses

The analysis is based on statistics from a huge amount of measured wind speeds originating from 3 different measuring campaigns in Denmark. The 3 sites - Vindeby, Gedser and Lammefjord - represent on-shore as well as off-shore conditions. All the available statistics on the data material have been transformed to 10-minute statistics.

At the Vindeby site the investigated wind data originate from a meteorological mast erected very close to the coast line. The data are recorded at level 37.5m and, depending on the wind direction a sea fetch, a land fetch and a mixed fetch are represented at this site. In the present investigation only the pure sea fetch and the pure land fetch are considered. The sea fetch is characterised by having more than 15 km of sea upstream (Barthelmie, 1994). The land fetch is characterised by being a flat and homogeneous terrain with roughness length of the order of magnitude 0.1m (Barthelmie, 1994). The data are selected in order to avoid mast and boom effects. With the defined selection criteria, the available data material constitutes 3487 10-minute time series for the land fetch and 5566 10-minute time series for the sea fetch. The overall mean wind speeds for the land- and sea fetch were 6.87m/s and 7.92m/s, respectively, reflecting the larger off-shore wind potential.

The meteorological mast at the Gedser site is also erected close to the coast line providing the possibility of analysing both land- and sea fetches. The present investigation relates to wind observations at level 30.0m. Also here the selected land fetch corresponds to flat and homogeneous terrain conditions. 10078 10-minute time series are available for the land fetch yielding an overall mean wind speed of 5.99m/s. The overall mean wind speed associated with the sea fetch is 7.87m/s, and the value is based on 21622 10-minute time series.

Lammefjord is an inland site situated in a large dried-up fjord with very flat and homogeneous terrain. The meteorological instrumentation includes three mast with cup anemometers in three different heights - for the present study recordings associated with the instrument positioned at the 30.0 m level were selected. The available data material is constituted by 28864 samples of 10-minute statistics with the overall mean wind speed equal to 6.53m/s.

The initial step in the present data analysis is to determine a suitable expression for the mean standard deviation by fitting the available data to the generic forms presented in Section 2.1. Having determined the mean standard deviation, conditioned on the mean wind speed, a zero mean process of the turbulence standard deviation can be identified. Applying a suitable binning matrix in the mean wind speed and in the standard deviation, the conditional probability density function of the standard deviation given the mean wind speed, expressed by

$$f_\sigma\left(x|U_T\right) = \frac{P\left(x \le \sigma_{u,T} \le x + dx|U_T\right)}{dx} \tag{2.4}$$

can be estimated in terms of histograms provided a sufficiently large number of bins are used as a basis for the histograms. In equation (2.4), P denotes probability.

The selection of bin intervals is a compromise between sufficient resolution and a reasonable number of data within each bin - in the present analysis the mean wind speed has been binned using a bin interval size equal to 2 m/s. As for the binning in the standard deviation, a total of 20 bin intervals have been selected which is somewhat more than the number of bin intervals proposed in (Conradsen, 1976).

2.3 Results

The on-shore situation and the off-shore situation are treated separately due to fundamental physical differences in the behaviour of the turbulence standard deviation. The difference is primary a result of the off-shore roughness increasing with the mean wind speed whereas the on-shore terrain roughness is a constant associated with a particular invariable topography (and wind direction). Further details can be found in (Larsen, 1999).

2.3.1 On-shore

The on-shore situation is represented both in the Vindeby, the Gedser and the Lammefjord measuring data. The three experiments are treated analogously but separately in the analysis, and as an example the analysis of the Gedser data is presented below.

Gedser

The data material covers 10-minute mean wind speeds ranging from 2m/s to approximately 15m/s. Initially, the standard deviation of the wind speed is plotted as a function of the mean wind speed in Figure(2.3-1). A linear expression, as suggested in Section 2.1, has been fitted to the data material by a least square fit, and the fit seems to offer a satisfactory representation of the mean standard deviation. The resulting linear fit, as expressed in equation (2.1), is

$$\left\langle \sigma_{u,10} \right\rangle = 0.151\, U_{10} - 0.119 \quad . \tag{2.5}$$
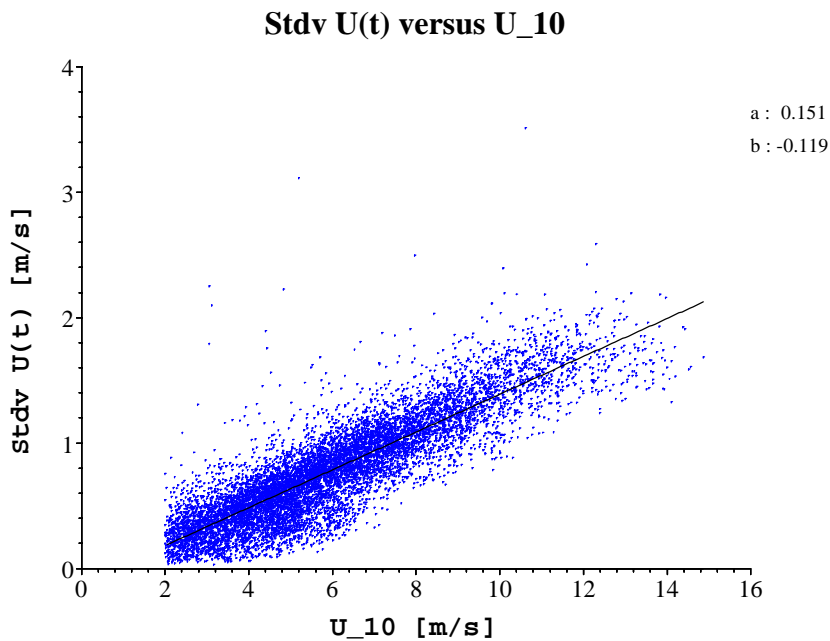
Stdv U(t) versus U_10

Figure 2.3-1 Standard deviation of wind speed as function of the mean wind speed with a linear expression fitted to the data points.

The *distribution* of the standard deviation is subsequently evaluated for each mean wind speed bin. Figure (2.3-2) illustrates the result for the mean wind speed bin interval extending from 8m/s to 10m/s. The (discrete) distribution is depicted both in terms of its probability density function (PDF) and its cumulative probability distribution function (CDF).
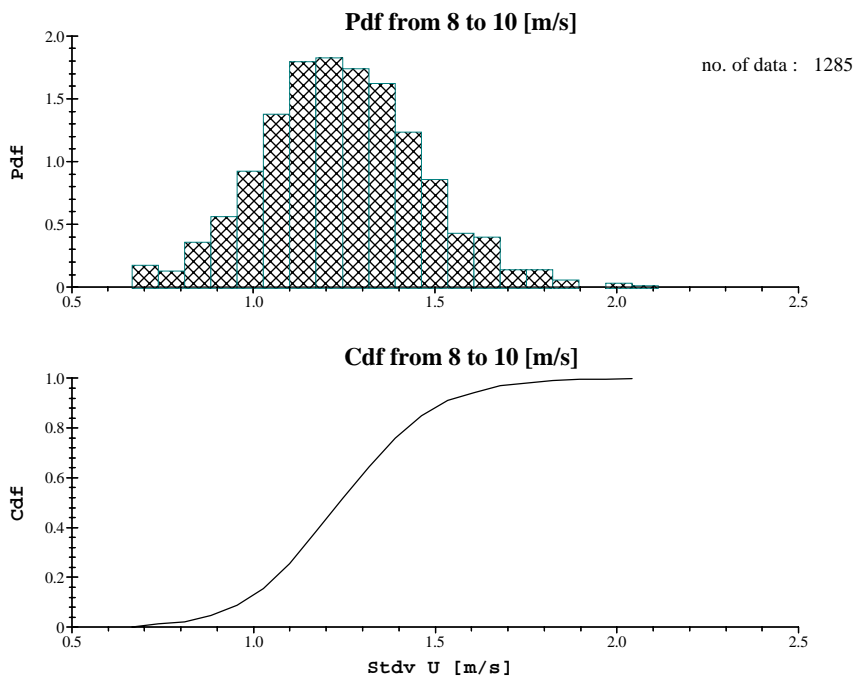

Figure 2.3-2 Discrete PDF and CDF of the wind speed standard deviation associated with the mean wind speed bin interval ranging from 8m/s to 10m/s.

Introducing a shift in the standard deviation co-ordinate equal to the difference between the mean standard deviation associated with a particular mean wind speed bin and the mean standard deviation associated with a reference bin (here chosen as the bin representing the lowest mean wind speed), the resulting PDF's for all mean wind speed bins are shown in Figure (2.3-3).
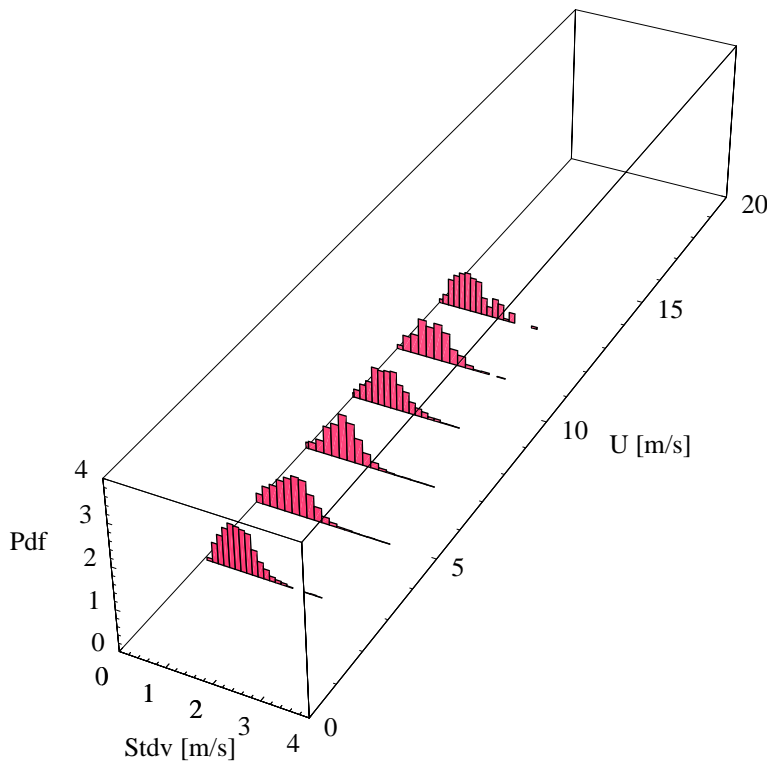

Figure 2.3-3 Discrete PDF of the wind speed standard deviation estimated for all mean wind speed bin intervals.

The discrete probability density functions presented in Figure (2.3-3) are subsequently parameterized by fitting with a three parameter Weibull distribution. The fitting is performed by means the non-linear Levenberg-Marquardt least-square regression (Press, 1987), and the results are given in Table (2.3-1), where U_c denotes the "centre" of the mean wind speed bin interval.

U_c (m/s)	k	β	α
3	2.50	0.45	0.00
5	2.27	0.50	0.24
7	2.70	0.51	0.56
9	2.25	0.48	0.87
11	2.31	0.51	1.19
13	1.97	0.45	1.33

Table 2.3-1 Weibull parameters obtained from the performed fitting procedure.

The performance of the Weibull fit can be visually inspected in Figure (2.3-4), where the measured discrete PDF is compared with the resulting Weibull approximation for the mean wind speed bin ranging from 8m/s to 10m/s.
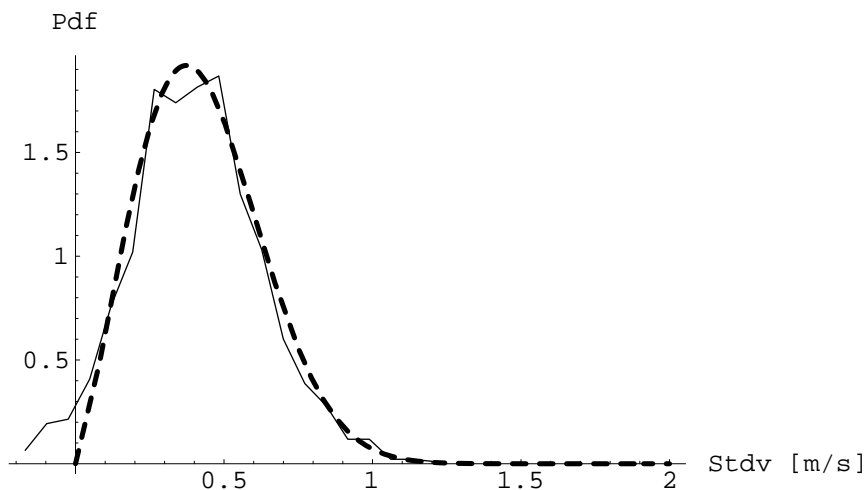
Figure 2.3-4 Probability density functions representing the mean wind speed bin interval extending from 8m/s to 10m/s. The abscissa values refer to shifted standard deviations.

2.3.2 Off-shore

The off-shore situation is represented only in the Vindeby and the Gedser measuring campaigns. As for the on-shore situation, the two experiments are treated analogously but separately in the analysis. The analysis of the Gedser off-shore data is presented below as an example.

Gedser

The data material covers 10-minute mean wind speeds ranging from 2m/s to approximately 22m/s. The standard deviation of the wind speed is plotted as function of the mean wind speed in Figure (2.3-5). The expression for the power law fit performed in Figure (2.3.5) is

$$\left\langle \sigma_{u,10} \right\rangle = 0.005 \, U_{10}^{1.937} + 0.279 \quad , \tag{2.6}$$

and, as seen, it seems to offer a good representation of the trend in the data material over the considered mean wind speed regime. It is evident that the trend in the data material presented in Figure (2.3-5) for the off-shore data is of a different character than the trend in the on-shore data shown in Figure (2.3-1).
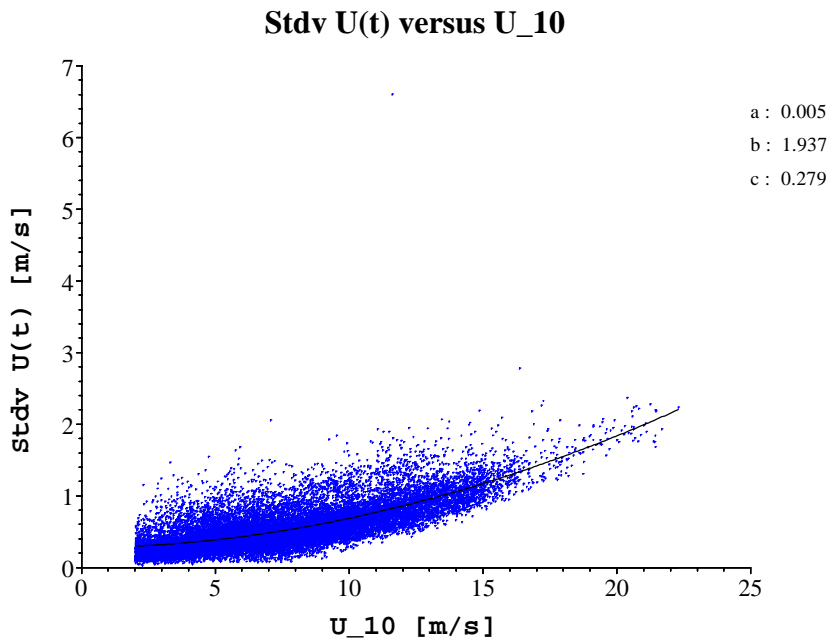
Stdv U(t) versus U_10

Figure 2.3-5 data: a : 0.005, b : 1.937, c : 0.279

Figure 2.3-5 Standard deviation of wind speed as function of the mean wind speed with a power law type expression fitted to the data points.

The distribution of the standard deviation is now evaluated for each mean wind speed bin. Figure (2.3-6) illustrates the result for the mean wind speed bin interval extending from 8m/s to 10m/s. The (discrete) distribution is depicted both in terms of its probability density function and its cumulative probability distribution function.
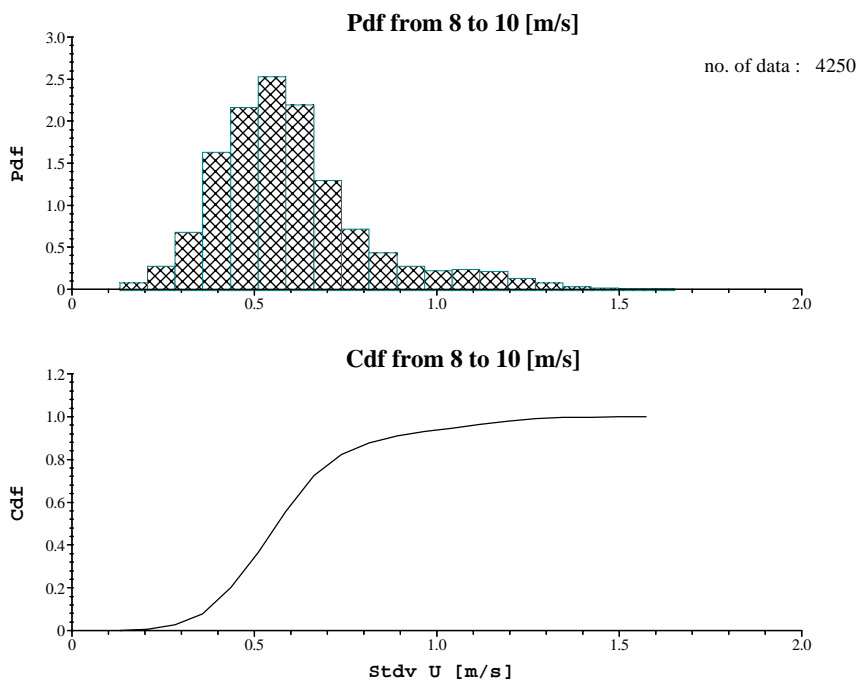

Pdf from 8 to 10 [m/s]

no. of data : 4250

Cdf from 8 to 10 [m/s]

Figure 2.3-6 Discrete PDF and CDF of the wind speed standard deviation associated with the mean wind speed bin interval ranging from 8m/s to 10m/s.

Introducing the usual shift in the standard deviation co-ordinate the resulting PDF's for all mean wind speed bins are shown in Figure (2.3-7).
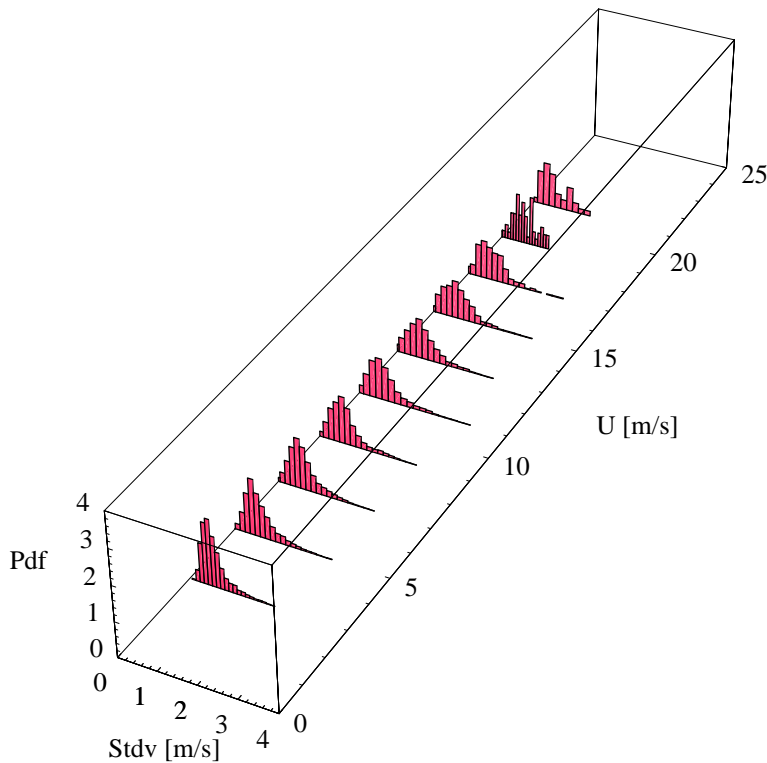

Figure 2.3-7 Discrete PDF of the wind speed standard deviation estimated for all mean wind speed bin intervals.

As for the on-shore situation, the above discrete distributions are consequently parameterized by fitting with a three parameter Weibull distribution. The results, corresponding to the present data, are given in Table (2.3-2). U_c denotes the "centre" of the bin interval.

U_c (m/s)	k	β	α
3	2.82	0.31	0.00
5	2.12	0.33	0.11
7	2.14	0.35	0.18
9	2.11	0.36	0.30
11	1.75	0.37	0.47
13	1.83	0.41	0.66
15	1.81	0.39	0.84
17	1.62	0.37	1.12
19	1.90	0.44	1.42
21	1.55	0.40	1.68

Table 2.3-2 Weibull parameters obtained from the performed fitting procedure.

In analogy with the investigation of the on-shore situation, the performance of the Weibull fit can be visually inspected in Figure (2.3-8), where the measured discrete PDF is

compared with the resulting Weibull approximation for the mean wind speed bin ranging from 8m/s to 10m/s.
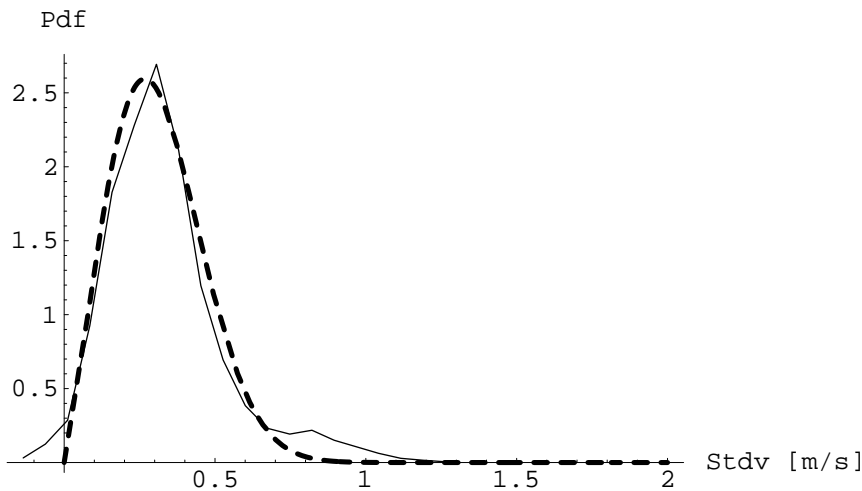

Figure 2.3-8 Probability density functions representing the mean wind speed bin interval extending from 8m/s to 10m/s. The abscissa values refer to shifted standard deviations.

2.4 Relation to IEC 61400-1

The standard deviation of the horizontal turbulence component plays a prominent role in the IEC 61400-1 formulation of the fatigue and extreme loading of wind turbines. It is therefore of interest to compare the turbulence intensity specifications in the standard with the probability density functions of the horizontal turbulence component evaluated in the present analysis.

The values for the class B turbulence intensities in the code claim to represent an 80% quantile level for a data material including measured turbulence characteristics covering "all wind turbine relevant (*on-shore*) sites". The turbulence intensity specification in the code is thus intended to represent the turbulence characteristics of many different sites rather than to give precise information of one particular site. The available data material in the present project represents three on-shore sites in *flat and homogeneous terrain*.

The IEC 61400-1 does not provide any specific information on turbulence intensities suitable for *off-shore* sites. Although differences in wind climates for off-shore sites definitely exist, these differences seem smaller than the mutual differences between wind climates related to on-shore sites in general, where large variations in terrain forms inevitably are represented. It is consequently expected that data from relative few off-shore sites can provide sufficient information to give a general idea of the off-shore wind climate. Based on the present analysis of two off-shore sites and a simple (heuristic) fatigue load assumption, a proposal will thus be formulated for a suitable off-shore *design turbulence intensity*.

2.4.1 Comparison with on-shore predictions

In the Figures (2.4-1) to (2.4-3), the 80% quantile predictions from the IEC 61400-1 code are compared to the 80% quantile predictions originating from the present three on-shore sites. The evaluation of the 80% quantiles from the present data analysis are based on the Weibull fitted distributions of the turbulence standard deviation.
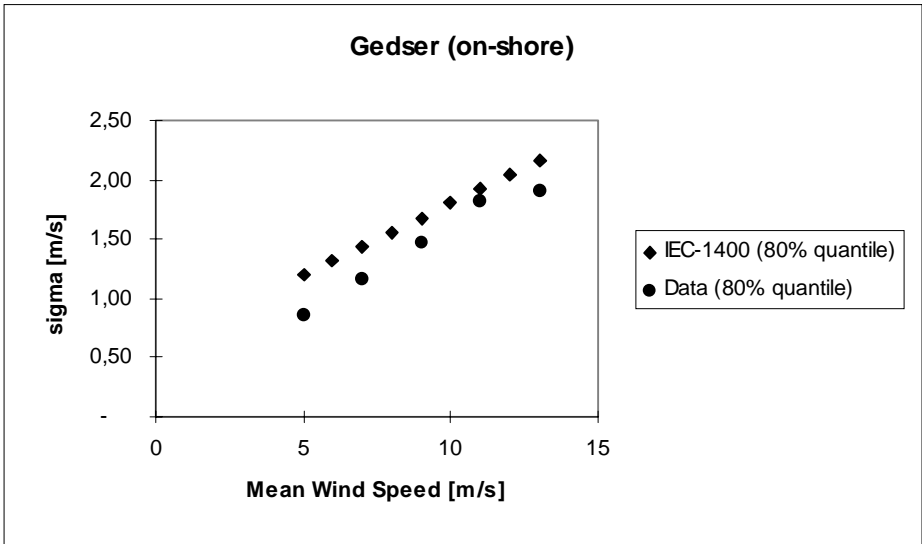
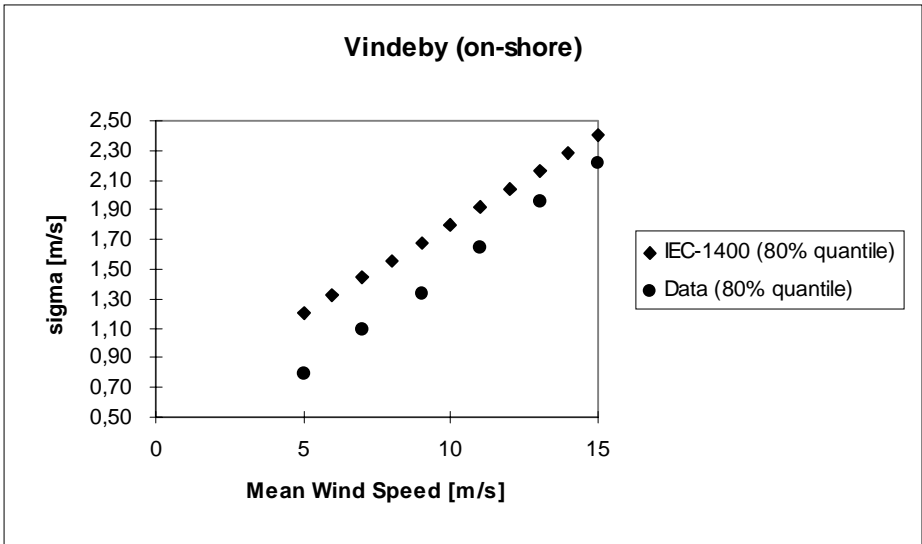**Figure 2.4-1 Standard deviation of horizontal wind turbulence component.**

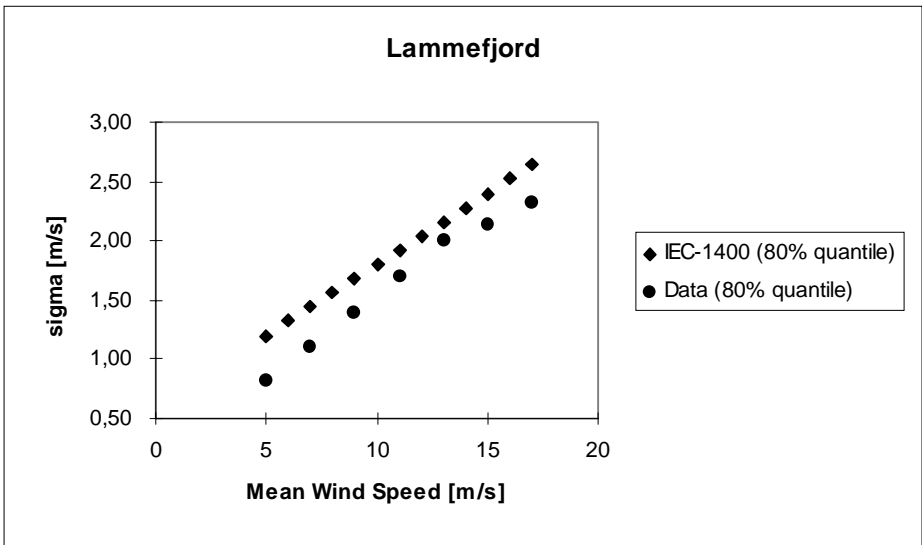**Figure 2.4-2 Standard deviation of horizontal wind turbulence component.**

Figure 2.4-3 Standard deviation of horizontal wind turbulence component.

The present on-shore sites are characterised by providing a less severe turbulence loading than specified by the IEC 61400-1 code. This is an expected outcome as the present sites only represent flat and homogeneous terrain forms, whereas the specifications in the code also are intended to cover more complex topographies. The linear trend expressed in the code is also approximately found in the predictions from the present data material, reflecting that the change in distribution type is modest with the mean wind speed. Note, that the mean of the distribution is known to be linear with the mean wind speed.

2.4.2 Proposal for off-shore load condition

The safety margin provided by the code specification compared to the results from the data analysis associated with the off-shore sites is dramatically increased compared to the on-shore situation. This is illustrated in the Figures (2.4-4) and (2.4-5), where the 80% quantile predictions from the IEC 61400-1 code are compared to the 80% quantile predictions originating from the present two off-shore sites
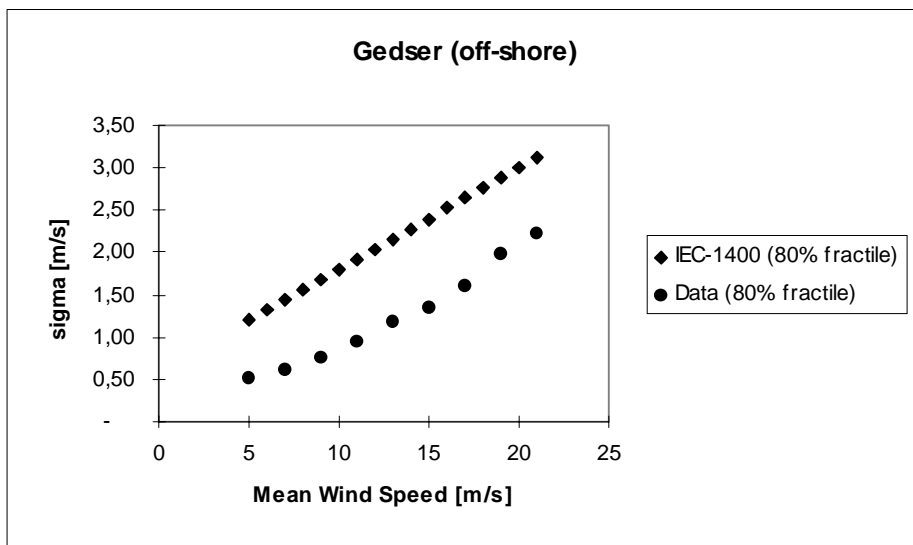

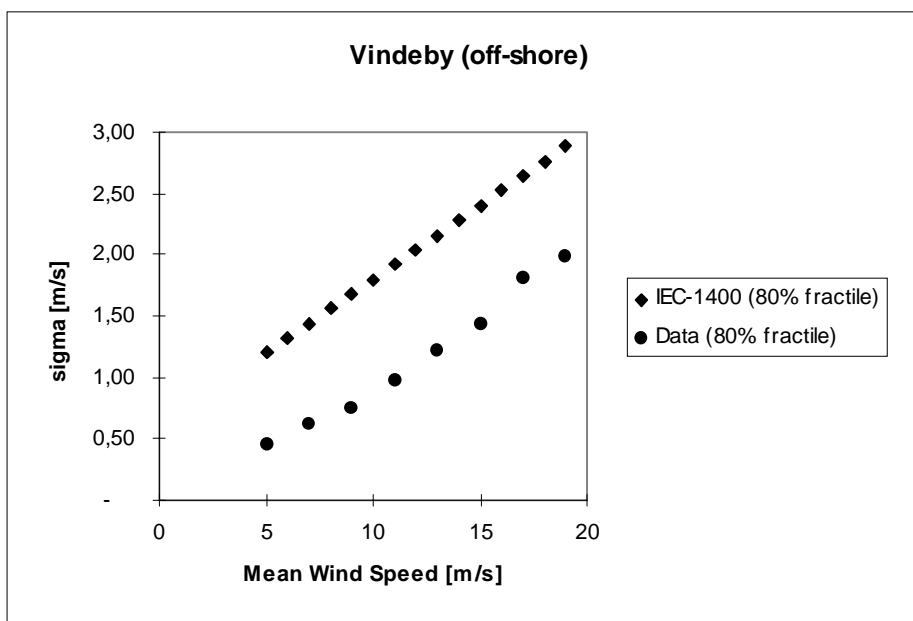Figure 2.4-4 Standard deviation of horizontal wind turbulence component.

Figure 2.4-5 Standard deviation of horizontal wind turbulence component.

Note that the increased safety margin is of same order of magnitude for the two sites. Assuming off-shore sites in general to be mutually relatively more homogeneous than on-shore sites, the above observations provide good reasons to suggest a reduction of the design turbulence specifications in the codes when applied to turbines dedicated for off-shore siting.

The proposal will be based on the conditional distributions of the standard deviation of the horizontal wind fluctuations as obtained from the present analysis of only two off-shore sites in shallow water regions. The application of the proposal will, until further analyses of other water regions are available, be limited to *shallow water regions*. However, at present most off-shore turbines are erected at such sites.

The basic presumption is that the fatigue load spectrum, in a first order approximation, is proportional to the standard deviation of the horizontal wind fluctuations. The physics behind the assumption is the following: for a given mean wind speed (expansion point) the dynamic wind loading of a turbine can be approximated by a "gradient" multiplied with the (horizontal) turbulence fluctuations. It is here further assumed that this "gradient" is independent of the size of the fluctuations.

For a given mean wind speed, U_T, the *mean fatigue loading*, $L_f(U_T)$, of the turbine is then symbolically determined from the expression

$$L_f\left(U_T\right) = C \int_0^\infty f_\sigma\left(\sigma_{u,T} \big| U_T\right) \sigma_{u,T}^m \, d\sigma_{u,T} \quad , \tag{2.7}$$

where C is a characteristic constant for the particular load on the particular wind turbine, and m is the Wöhler exponent for the particular material. In the above formulation it is implicitly assumed that the fatigue loading is caused exclusively by the stochastic part of the wind field.

Defining the *design standard deviation* as the particular standard deviation, $\sigma_{d,T}(U_T)$, giving rise to the above mean fatigue loading we find

$$L_f\left(U_T\right) = C\,\sigma_{d,T}\left(U_T\right)^m = C \int_0^\infty f_\sigma\left(\sigma_{u,T} \big| U_T\right) \sigma_{u,T}^m \, d\sigma_{u,T} \quad , \tag{2.8}$$

whereby the design standard deviation is expressed only in terms of the conditional distributions of the standard deviation of the horizontal turbulence component as

$$\sigma_{d,T}\left(U_T\right) = \sqrt[m]{\int_0^\infty f_\sigma\left(\sigma_{u,T} \big| U_T\right) \sigma_{u,T}^m \, d\sigma_{u,T}} \quad . \tag{2.9}$$

In case the fatigue loading, in addition to the turbulence contribution, originates from a periodic deterministic load component, it can be shown that the design standard deviation defined according to equation (2.9) will be conservative.

Due to the non-linear weighting resulting from the equation above, the design standard deviation do not correspond to a 50% quantile (or a mean value) in the associated empirical distributions conditioned on the mean wind speed. The quantile level depends on the value of the Wöhler exponents. For the present sites, the quantile values are shown in Figures (2.4-6) and (2.4-7) for a conservative choice of the Wöhler exponent (m=12).
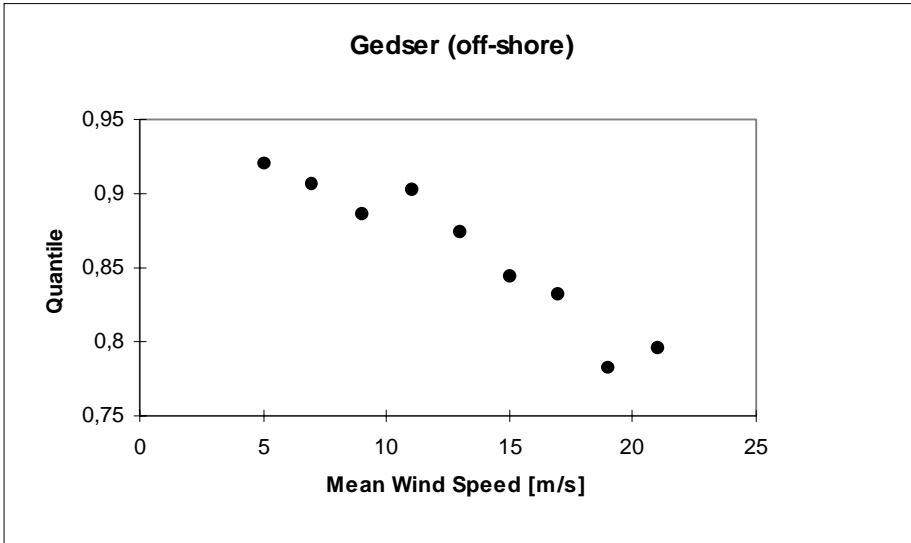
Figure 2.4-6 Quantiles in the estimated (off-shore) Gedser distributions corresponding to the design standard deviation determined for m=12.
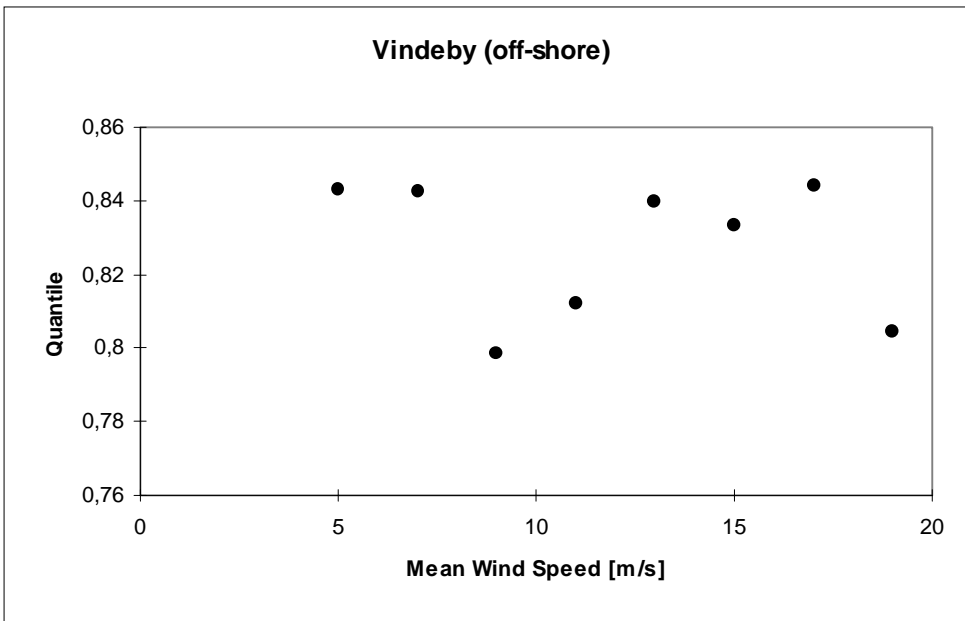
Figure 2.4-7 Quantiles in the estimated (off-shore) Vindeby distributions corresponding to the design standard deviation determined for m=12.

It is observed that the quantile levels vary moderately with the mean wind speed, and that the results, originating from the two independent analyses of the Gedser and Vindeby data, are of approximately the same size. Moreover, the quantile levels are of the same order of magnitude as the code specifications for on-shore siting, where the values for class B turbulence intensities claim to represent a 80% quantile level for relevant data.

In order to investigate the potential for evaluating a "general" shallow water expression for the design turbulence intensity, the results originating from the two independent off-shore analyses have been visually compared in Figure (2.4-8).
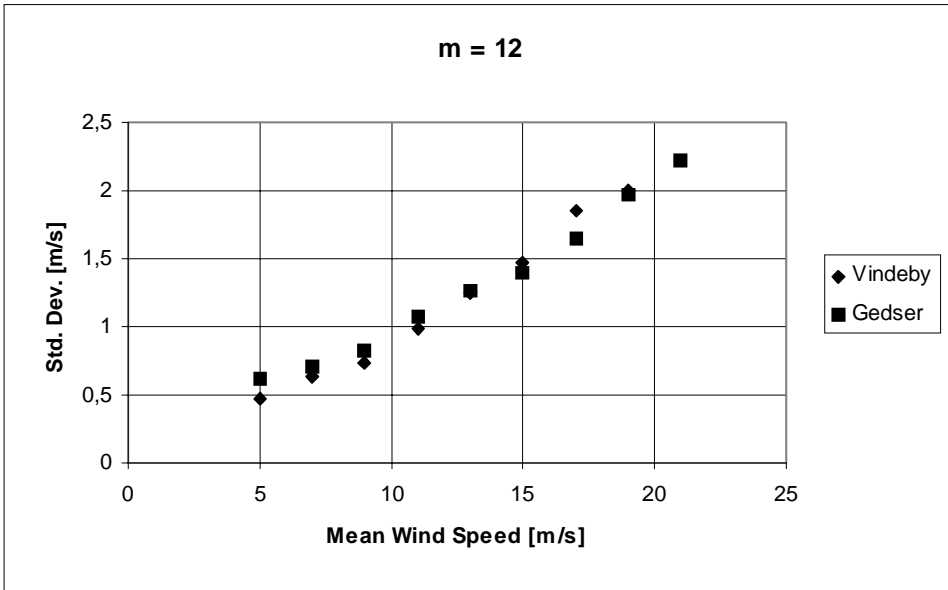
Figure 2.4-8 Design standard deviation as function of mean wind speed based on the Wöhler exponent m = 12.

The obtained results for the two independent analyses are in good agreement, and it consequently makes sense to determine a "general" expression for the design standard deviation describing shallow water off-shore site conditions.

To determine a simple expression for the design turbulence intensity, the design standard deviation is plotted as function of the mean wind speed, and it turns out that there exists an approximate second order polynomial relationship between these quantities. Hence, a second order polynomial least square fit is subsequently fitted to the data, and the result is presented in Figure (2.4-9) and Figure (2.4-10).

Figure 2.4-9 Second order polynomial least square fit of the design standard deviation as function of mean wind speed for the Gedser data based on m=12.

Figure 2.4-10 Second order polynomial least square fit of design standard deviation as function of mean wind speed for the Vindeby data based on m=12.

The analysis thus suggests the following relationships between design turbulence intensity and mean wind speed

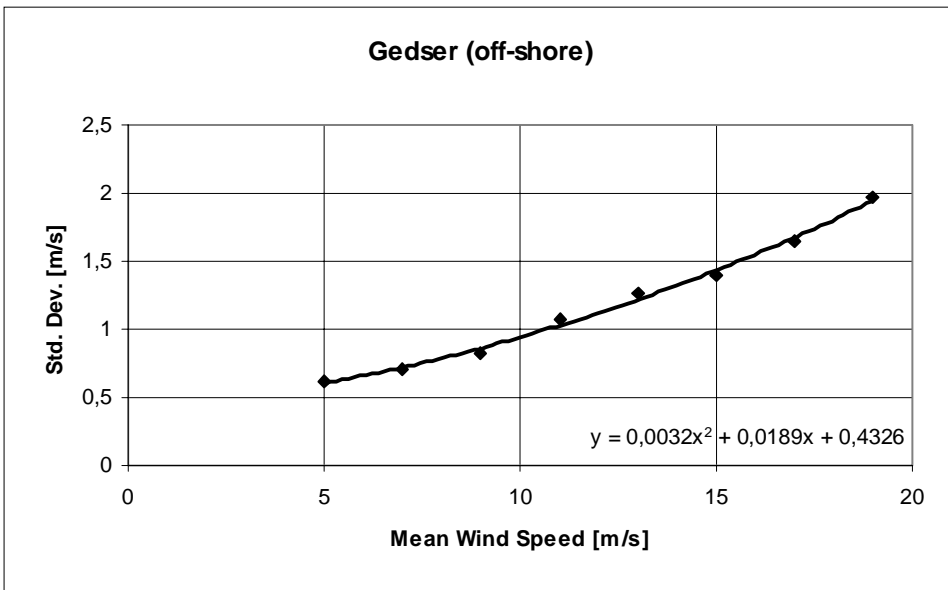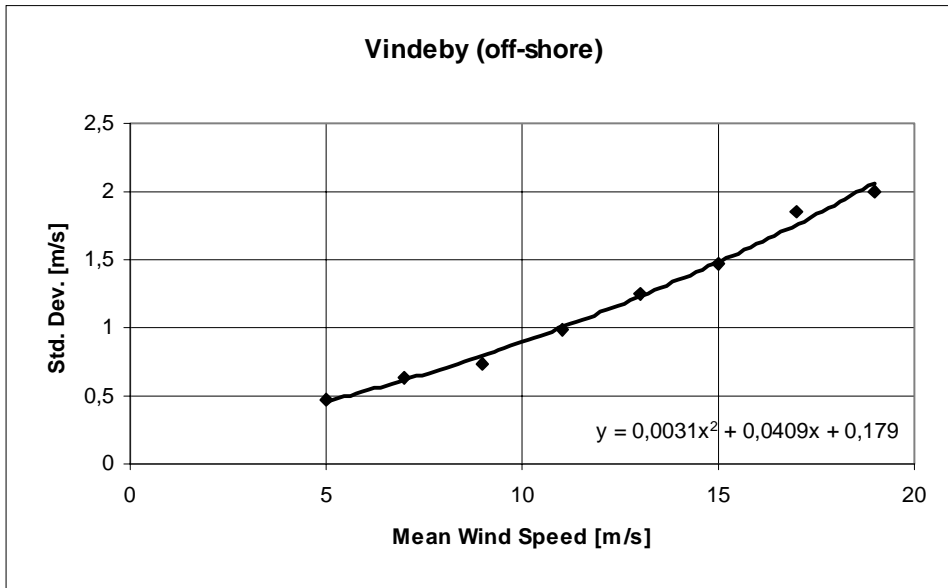$$TI_G = 0.0032\ U_{10} + 0.0189 + \frac{0.4326}{U_{10}} \quad,$$

$$TI_V = 0.0031\ U_{10} + 0.0409 + \frac{0.1790}{U_{10}} \quad,$$

where TI_G and TI_V denote design turbulence intensities related to the Gedser results and to the Vindeby results, respectively. U_{10} denotes mean wind speed based on a 10-minute averaging period.

The proposal for an off-shore design turbulence intensity, TI_D, is based on a weighted mean of the results arising from the two investigated sites. The weighting factors are selected as the relative number of the total number of available data series associated with each site (5566/27188 for the Vindeby experiment and 21622/27188 for the Gedser experiment). The resulting expression for the design turbulence intensity is

$$TI_D = 0.0032\ U_{10} + 0.0234 + \frac{0.3807}{U_{10}} \quad. \tag{2.10}$$

The above expression applies to extreme design load as well as to fatigue design provided that the guidelines in IEC 64100-1 are adopted.

Note that the present investigation only includes mean wind speed values up to approximately 22m/s, and that the above specification of design turbulence intensity consequently should be used with care in wind regimes outside this range. However, compared to the conventional practice where off-shore conditions, according to the IEC 61400-1 standard must be considered as an "S" class type situation (where the required parameters must be entirely supplied (and documented) by the designer), the present proposal gives considerable guidance.

3. Structural reliability

The present section describes a structural reliability method that can be applied to deal with wind turbine components in ultimate loading. The procedure is exemplified by application to the prediction of the probability of structural failure of a wind turbine blade in flapwise loading. A particular wind turbine at a particular site is considered. The result from the analyses is used in the assessment of partial safety factors associated with the considered failure mode.

3.1 Load modelling theory

The wind climate that governs the loading of a wind turbine and its rotor blades is commonly described by the 10-minute mean wind speed U_{10} at the site in conjunction with the standard deviation σ_u of the wind speed. The long-term distribution of the 10-minute mean wind speed can be taken as a Weibull distribution

$$F_{U_{10}}(u) = 1 - \exp(-(\frac{u}{A})^k) \qquad (3.1)$$

in which k and A are site- and height-dependent coefficients. Only normal operation of the wind turbine is considered. The turbine will stop whenever the cut-out wind speed u_C is exceeded. For analysis of failure in ultimate loading in the normal operating condition, it is therefore of interest to represent the 10-minute mean wind speed by a Weibull distribution, which is truncated at the cut-out speed,

$$F_{U_{10}}(u) = \frac{1 - \exp(-(\frac{u}{A})^k)}{1 - \exp(-(\frac{u_C}{A})^k)} \;; \qquad 0 < u < u_C \qquad (3.2)$$

This is the cumulative distribution function of the 10-minute mean wind speed in an arbitrary 10-minute period whose mean wind speed does not exceed the threshold u_C. The distribution of the 10-minute mean wind speed in the most severe 10-minute wind climate in N 10-minute periods whose mean wind speed does not exceed the threshold u_C can be expressed as

$$F_{U_{10,\max}}(u) = (F_{U_{10}}(u))^N$$

under an assumption of independence between successive 10-minute periods. This will be applied in the following where N will refer to the normal operating condition during the design life of a wind turbine.

The standard deviation σ_u of the wind speed depends to some extent on the 10-minute mean wind speed U_{10} as addressed in Chapter 2. For analysis of failure in ultimate loading, the upper tail of the distribution of σ_u conditioned on U_{10} is of most interest. The upper tail of this distribution can reasonably well be represented by a Weibull distribution

$$F_{\sigma_U}(\sigma) = 1 - \exp(-(\exp b_0) \cdot \sigma^{b_1}) \qquad (3.3)$$

in which the coefficients b_0 and b_1 are functions of U_{10} as follows

$$b_0 = c_0 + c_1 U_{10}$$

$$b_1 = d_0 + d_1 U_{10}$$

(3.4)

One rotor blade is considered in the following. Let X denote the bending moment at the blade root in flapwise bending. The bending moment process X is characterised by its mean value μ, standard deviation σ, skewness α_3, kurtosis α_4, and regularity factor α. Based on the first four moments, the distribution of X can be represented by a Hermite moment transformation (fourth-moment Hermite polynomial expansion) of a standard Gaussian variable U. Reference is made to (Winterstein, 1988). For kurtosis $\alpha_4 > 3$, the transformation reads

$$X = \mu + \kappa\sigma(U + c_3(U^2 - 1) + c_4(U^3 - 3U))$$

$$c_4 = \frac{\sqrt{1 + 36h_4} - 1}{18}; \quad h_4 = \frac{\alpha_4 - 3}{24}$$

(3.5)

$$c_3 = \frac{h_3}{1 + 6c_4}; \quad h_3 = \frac{\alpha_3}{6}$$

$$\kappa = \frac{1}{\sqrt{1 + 2c_3^2 + 6c_4^2}}$$

For kurtosis $\alpha_4 < 3$, the transformation reads

$$X = \mu + \sigma((\sqrt{c^2 + k} + c)^{1/3} - (\sqrt{c^2 + k} - c)^{1/3} - a)$$

$$c = 1.5b(a + U) - a^3$$

(3.6)

$$b = -\frac{1}{3h_4}; \quad h_3 = \frac{\alpha_3}{6}$$

$$a = \frac{h_3}{3h_4}; \quad h_4 = \frac{\alpha_4 - 3}{24}$$

$$k = (b - 1 - a^2)^3$$

In either case, the transformation is monotonous, which in particular implies that local maxima of the underlying Gaussian process U are transformed to local maxima of X. Consider now an arbitrary 10-minute period during which the 10-minute mean wind speed can be considered constant equal to a realisation of the distribution in equation (3.2). The 10-minute period is assumed to be short enough that other local maxima than the largest maximum X_{max} of X in the 10 minutes will not contribute significantly to the probability of failure in ultimate loading. The validity of this assumption is discussed later. The largest maximum X_{max} in an arbitrary 10-minute period is therefore dealt with in the following. Consider now the corresponding maximum U_{max} of the underlying standard Gaussian process U. The distribution of local maxima of U is a Rice distribution. This implies that the distribution of U_{max} is an extreme-value distribution, which can be approximated by

$$F_{U_{max}}(u) \approx \exp(-\alpha N_{max} \exp(-\frac{u^2}{2}))$$

(3.7)

where α is the regularity factor and N_{max} is the number of local maxima in 10 minutes. Reference is made to (Ronold, 1993). Once U_{max} has been determined from this distribution, X_{max} can be found from U_{max} through the transformations in equations (3.5-6). The corresponding bending stress is defined as $S_{max} = X_{max}/W$, where W is the section modulus of the rotor blade at the blade root.

3.2 Load model case study

The wind turbine in the present case study is considered for a location whose wind loading regime is characterised by a scale parameter $A=9.1$ m/sec, a slope parameter $k=1.9$, and a terrain roughness $z_0=0.05$ m. The cut-out wind speed for the turbine is $u_C=25$ m/sec. The design life is taken as $T_L=20$ years. There are 1051200 10-minute periods during this time span, and the turbine will be in its operating condition during $N=1050055$ of these 10-minute periods as determined from the frequency of wind speeds beyond the cut-out speed.

As described in Section 3.1, the upper tail of the distribution of the standard deviation σ_U of the wind speed, conditioned on the 10-minute mean wind speed U_{10}, can be represented by a Weibull distribution

$$F_{\sigma_U}(\sigma) = 1 - \exp(-(\exp b_0) \cdot \sigma^{b_1}) \tag{3.8}$$

Based on available wind climate data from the considered location, the coefficients b_0 and b_1 are represented as linear functions of U_{10} as follows

$b_0=3.2358-0.2174U_{10}$

$$\tag{3.9}$$

$b_1=-1.7563+0.2426U_{10}$

A total of 642 10-minute records of the flapwise bending moment response process X for various realisations of the 10-minute mean wind speed U_{10} are available. Based on these data, the mean value μ and the standard deviation σ of the process conditional on U_{10} are represented as functions of U_{10} and σ_U,

$$\mu = -156.77 + 213.45\sqrt{U_{10} - 2.1796} - 23.488U_{10} \tag{3.10}$$

$$\sigma = 17.545 - 1.9038U_{10} + 0.0939U_{10}^2 + 76.478\frac{\sigma_U}{U_{10}} + 335.87(\frac{\sigma_U}{U_{10}})^2 \tag{3.11}$$

For 10-minute mean wind speeds in excess of 15 m/sec, which is of practical interest here, the data indicates very stable values of the skewness α_3 and the kurtosis α_4 of the bending moment response process X, and they can both be modelled as constants

$\alpha_3=-0.0066$ $\tag{3.12}$

$\alpha_4=2.8174$ $\tag{3.13}$

Note that these values for skewness and kurtosis indicate a response process whose marginal distribution is very close to a Gaussian distribution in the high wind speed range. This may be reasonable for flapwise bending, which is considered here. For edgewise bending, where the response has a significant sinusoidal term owing to the effects of gravity, the marginal distribution of the response process will be far from any Gaussian distribution, in particular because its kurtosis which will be much smaller than 3.0. Based

on available data, the regularity factor of the bending moment response process is empirically represented as a function of U_{10} as follows,

$$\alpha = 0.02954\arctan(1.1541(U_{10} - 11.701)) + 0.16636 \qquad (3.14)$$

and the number of local maxima of this process in 10 minutes is

$$N_{\max} = 336.86\arctan(0.4857(U_{10} - 11.609)) + 2016.0 \qquad (3.15)$$

The above suffices to produce the distribution of the maximum bending moment response in 10 minutes, cfr. equations (3.5), (3.6), and (3.7).

3.3 Structural resistance

The rotor blade is made up of a fiber-reinforced polyester laminate. The strength that provides the capacity at the blade root in flapwise bending is the tensile strength σ_F in the direction of the fibers. Failure in ultimate loading is defined to occur whenever the tensile bending stress $S=X/W$ exceeds the tensile strength σ_F of the rotor blade. As significant contributions to the probability of failure in ultimate loading are produced only by the 10-minute maximum tensile bending stress $S_{\max}= X_{\max}/W$, it suffices to consider failure as any situation in which the 10-minute maximum tensile bending stress $S_{\max}$ exceeds the tensile strength σ_F.

This strength is characterised by a natural variability and is thus represented by its probability distribution. For the case study, the tensile strength of the composite laminate in the direction of the fibers is represented by a normal distribution with mean value

$$E[\sigma_F]=518000 \text{ kPa} \qquad (3.16)$$

and a coefficient of variation of 10%. Reference is made to (Ronold, 1997). The section modulus of the rotor blade at the blade root in flapwise bending is taken as $W=0.0013$ m^3.

3.4 Reliability Analyses

The reliability against failure of the considered rotor blade in ultimate loading in flapwise bending is analysed for the cyclic loading caused by wind over the design life. For this purpose, a limit state function is defined

$$g(\mathbf{X}) = \sigma_F - \frac{X_{\max}}{W} \qquad (3.17)$$

in which $\mathbf{X}$ denotes the vector of stochastic variables which include the load variables (U_{10}, σ_U, $X_{\max}$) and the strength variable σ_F.

The reliability is the complement of the failure probability

$$P_F = P[g(\mathbf{X}) \le 0] \qquad (3.18)$$

and may be expressed in terms of the reliability index $\beta=-\Phi^{-1}(P_F)$. The reliability is computed by means of a first-order reliability method as described in (Madsen, 1986). The probabilistic analysis program PROBAN, see (Tvedt , 1989), is used for this purpose. It is a standard approach to assume that the long-term reliability can be estimated by calculating the failure probability for the largest bending moment response in the most

severe 10-minute wind climate in the normal operating condition during the design life of the wind turbine. This can be done directly by a conventional first-order reliability analysis, where the stochastic variables denoted as **X** are input. The results of such an analysis is shown in Table (3.4-1).

Table 3.4-1 *Results of Conventional long-term Reliability Analysis*
Failure in Ultimate Loading for largest Bending Moment Response in most severe 10-minute Wind Climate in Normal Operating Condition

Rotor Blade, $W=0.0013$ m^3

Probability of Failure $P_F=0.21 \cdot 10^{-4}$
Reliability Index $\beta=4.09$

Variable	Distribution	Design point $\mathbf{x}^*$	Importance factor α^2
$U_{10,max}$	Extreme of truncated Weibull	25.000	0.000
$\sigma_U \mid U_{10,max}$	Weibull	1.694	0.010
$X_{max} \mid (U_{10,max}, \sigma_U)$	Transformed extreme-value	402.46	0.023
σ_F	Normal	309577.5	0.967

It is, however, reasonable to expect that also other 10-minute wind climates than the most severe one in the normal operating condition will contribute to the probability of failure. Integration of failure probability contributions from all N 10-minute wind climates in the normal operating condition during the design life requires solution of a series system of N failure events. This can practically be solved by a procedure, which involves nested applications of first-order reliability analyses in an iterative procedure, see (Bjerager, 1988) and (Wen, 1987).

The stochastic variables **X** are divided in two groups, **Y** and **Z**. **Z** covers the system variables, i.e., in this case only the strength variable σ_F, which is the same during all 10-minute wind climates. **Y** covers the wind climate and load response variables, here U_{10}, σ_U, and X_{max}, and they are assumed independent from one wind climate to another.

A given outcome **z** of **Z** produces a conditional failure probability for the rotor blade in an arbitrary 10-minute wind climate

$$P_{F_Z}(\mathbf{z}) = P\left[g(\mathbf{Y},\mathbf{Z}) \leq 0 \mid \mathbf{Z} = \mathbf{z}\right] \tag{3.19}$$

A conditional short-term reliability index corresponds to this probability and is found by a reliability analysis in which **Y** is modelled as stochastic variables and **Z**=**z** is fixed,

$$\beta_S = -\Phi^{-1}(P_{F_Z}(\mathbf{z})) \tag{3.20}$$

The N 10-minute wind climates are assumed to be independent, and when the probability is conditioned on an outcome **z** of the system variables **Z**, then the corresponding conditional safety margins for rotor blade failure will be independent. The conditional probability of failure during the N wind climates in normal operation during the design life can hence be calculated as

$$P_{F_{Z,N}}(\mathbf{z}) = 1 - (1 - P_{F_Z}(\mathbf{z}))^N \tag{3.21}$$

The total probability of failure during the N wind climates is found by integration over all possible outcomes **z** of **Z**

$$P_F = \int_{\mathbf{z}} P_{F_{Z,N}}(\mathbf{z}) f_{\mathbf{Z}}(\mathbf{z}) d\mathbf{z} \tag{3.22}$$

By introducing an auxiliary variable U_{aux}, which is standard normally distributed, this probability can be rewritten as

$$P_F = \int_{\mathbf{z}} P\left[U_{aux} \leq \Phi^{-1}(P_{F_{Z,N}}(\mathbf{z}))\right] f_{\mathbf{Z}}(\mathbf{z})d\mathbf{z}$$

$$= \int_{\mathbf{z}} P\left[U_{aux} - \Phi^{-1}(P_{F_{Z,N}}(\mathbf{Z})) \leq 0 \mid \mathbf{Z} = \mathbf{z}\right] f_{\mathbf{Z}}(\mathbf{z})d\mathbf{z} = P\left[U_{aux} - \Phi^{-1}(P_{F_{Z,N}}(\mathbf{Z})) \leq 0\right]$$

(3.23)

see (Bjerager, 1988). Substitution of the expressions from equations (3.20) and (3.21) into equation (3.23) yields

$$P_F = P\left[U_{aux} + \Phi^{-1}(\Phi(\beta_S(\mathbf{Z}))^N) \leq 0\right] \tag{3.24}$$

The sought-after failure probability P_F is then solved by a first-order reliability analysis under application of a limit state function

$$h = U_{aux} + \Phi^{-1}(\Phi(\beta_S(\mathbf{Z}))^N) \tag{3.25}$$

such that

$$P_F = P\left[h(U_{aux}, \mathbf{Z}) \leq 0\right] \tag{3.26}$$

and the corresponding unconditional long-term reliability index is $\beta_L = -\Phi^{-1}(P_F)$.

U_{aux} and $\mathbf{Z}$ are represented as stochastic variables, and β_L can be solved provided the partial derivatives of β_S with respect to $\mathbf{Z}$ can be computed. These derivatives are equal to the parametric sensitivity factors, which can be obtained as byproducts of the first-order computation of β_S as described above, see (Madsen, 1986). The procedure for solution of P_F and β_L is hence a nested application of first-order reliability analyses. This procedure is iterative in the sense that it has to be repeated until the conditional short-term reliability index β_S is calculated for a fixed set of the system variables $\mathbf{Z}=\mathbf{z}$ equal to the design point $\mathbf{Z}=\mathbf{z}^*$ pertaining to β_L. The results of the nested long-term reliability analysis are shown in Table (3.4-2).

Table 3.4-2 *Results of Nested long-term Reliability Analysis*			
Failure in Ultimate Loading for largest Bending Moment Response in each of all 10-minute Wind Climates in Normal Operating Condition			
Rotor Blade**,** $W=0.0013$ m^3			
Probability of Failure $P_F=0.27\cdot10^{-3}$			
Reliability Index $\beta_L=3.46$			
	Distribution		
Variable	Conditional short-term analysis	Unconditional long-term analysis	Design point $\mathbf{x}^*$
U_{10}	Truncated Weibull	Not included	24.692
$\sigma_U \mid U_{10}$	Weibull	Not included	2.114
$X_{max} \mid (U_{10}, \sigma_U)$	Transformed extreme-value	Not included	441.0
σ_F	Fixed	Normal	339247.8
U_{aux}	Not included	Normal	−0.287

By comparison of the results in Table (3.4-1) and Table (3.4-2), it appears that the nested reliability analysis produces a failure probability, which is more than ten times the failure probability produced by the conventional reliability analysis. This demonstrates the

importance of including contributions to the failure probability from all 10-minute wind climates that occur during the normal operating condition of the wind turbine. In other words, the failure probability will be significantly underestimated if it does not cover other probability contributions than that from the most severe 10-minute wind climate. This implies that the conventional reliability analysis is insufficient for estimation of the failure probability associated with the present failure problem, and the presented nested reliability analysis comes in handy as a more suitable approach.

The reliability analyses have been carried out under the assumption that within the duration of a 10-minute wind climate, only the largest bending moment response contributes to the failure probability. One may consider that also other local maxima of the bending moment response within such a 10-minute period may contribute to the failure probability. This can be investigated by carrying out a nested reliability analysis of failure in an arbitrary 10-minute period, integrating probability contributions from all local response maxima, and comparing with a conventional reliability analysis that deals only with failure for the largest response in the ten minutes. This has been done, and the comparison showed about 1% higher failure probability in the case where all local maxima were considered than in the case where only the largest response in the ten minutes was considered. This is a rather insignificant difference, and serves to support the assumption stated previously that, within a 10-minute period, it suffices to consider the probability contribution from the largest response only.

By studying the design point shown in the fourth column of Table (3.4-2), it appears that the variability in the tensile strength σ_F is by far the most important uncertainty source. The loading is confined and very much governed by the imposed cut-out wind speed, $u_C=25$ m/sec, which has been considered as fixed in the analyses. Without the cut-out speed included in the analyses, the variability in the mean wind speed U_{10} would have become a much more important uncertainty source with a design point value in the range 35-40 m/sec, which is rather unrealistic for operation of the wind turbine. This would have led to a significantly higher failure probability in ultimate loading than the one reported here. In order to maintain the reliability against failure of the rotor blade in ultimate loading in the normal operating condition, this demonstrates that it is of utmost importance to keep the cut-out wind speed at 25 m/sec as intended during the entire service life of the wind turbine.

3.5 Calibration of partial safety factors

It is of interest to demonstrate how reliability analysis results, obtained as outlined in the previous chapters, play a role in codified practice and design. With the first-order reliability method available, it is possible to determine sets of equivalent partial safety factors which result in rotor blade designs with a prescribed reliability. As a first step, a target reliability index β_t must be selected.

The choice for the target reliability index can be derived from a utility-based feasibility assessment in a decision analysis, or by requiring that the safety level as resulting from the design by a reliability analysis shall be the same as that resulting from current deterministic design practice. The latter approach is based on the assumption that current design practice is optimal with respect to safety and economy or, at least, leads to a safety level acceptable by society.

Referring to the present example, we consider a rotor blade design for high safety and less serious consequence, which seems to be a reasonable classification for a wind turbine design against fatigue where human life is at negligible risk. According to (Nordic Committee on Building Regulations, 1978), the requirement to the annual failure probability for design under such a classification is 10^{-5}. Under a Poissonian assumption

for a rare failure event, this implies that the acceptable failure probability in a 20-year lifetime is $2.0 \cdot 10^{-4}$, and the corresponding target reliability index is $\beta_t = 3.54$.

To proceed, characteristic values have to be selected for the governing load and resistance variables. For design in ultimate loading, the 98% quantile of the annual maximum load is traditionally used as the characteristic load value. The distribution of the maximum bending moment response, X_{max}, in an arbitrary 10-minute period can be determined on the basis of the probabilistic models described in Section 3.2.

Under an assumption of independence between the $N=52503$ 10-minute periods of $U_{10} < u_C = 25$ m/sec in one year, the distribution of the annual maximum of the bending moment response can be found as

$$F_{Xmax,1yr}(x) = F_{Xmax,10min}(x)^N \tag{3.27}$$

The 98% quantile can subsequently be determined to

$X_{max,c} = X_{max,1yr,98\%} = 457.6$ kNm.

For the tensile strength, it is a standard approach to select the 2% quantile as the characteristic value. Based on the normal distribution for σ_F quoted in Section 3.3, this leads to the characteristic strength

$\sigma_{F,c} = 411603$ kPa

Two partial safety factors are introduced. A load factor γ_f greater than 1.0 is applied as a factor on the characteristic bending moment $X_{max,c}$ and gives a design bending moment $X_{max,d}$

$$X_{max,d} = \gamma_f X_{max,c} \tag{3.28}$$

Correspondingly, γ_m is a material factor greater than 1.0. The characteristic strength $\sigma_{F,c}$ is divided by this number to give the design strength $\sigma_{F,d}$. Hence the design strength becomes

$\sigma_{F,d} = \sigma_{F,c} / \gamma_m$

In the design situation, it is required that the design load equals the design resistance. This gives the following deterministic design equation

$$\frac{\gamma_f X_{max,c}}{W} = \frac{\sigma_{F,c}}{\gamma_m} \tag{3.29}$$

This leads to a requirement to the product of the two partial safety factors, expressed as a function of the section modulus W

$$\gamma_f \gamma_m = \frac{\sigma_{F,c}}{X_{max,c}} W \tag{3.30}$$

For each value of the section modulus W that was considered for the reliability analyses, the requirement to the safety factor product can now be determined. The ultimate result of this is a required partial safety factor product $\gamma_f \gamma_m$ expressed as a function of the section modulus W, $\gamma_f \gamma_m = \gamma_f \gamma_m(W)$.

The reliability index as resulting from the reliability analysis depends on the geometrical quantities (or design parameters) of the wind turbine blade. For a rotor blade, the most

practicable parameter to adjust is the section modulus W, which is a function of the cross-sectional properties of the blade at the root. The reliability index is subsequently, by performing a series of reliability analyses, expressed as a function of the section modulus, $\beta=\beta\ (W)$.

Eliminating the section modulus from the safety factor product function and the reliability index function, the reliability index is finally expressed in terms of the calibrated partial safety factor product $\gamma_f\gamma_m$. For the present target reliability index, $\beta_t=3.54$, the required section modulus is determined from the reliability analyses to be $W=0.001316$ m^3, and equation (3.30) then gives a requirement to the product of the partial safety factors

$$\gamma_f\gamma_m = \frac{\sigma_{F,c}}{X_{max,c}} W = \frac{411603}{457.6} \cdot 0.001316 = 1.184 \ \ .$$

An infinite number of possible choices for the set of partial safety factors (γ_f, γ_m) exist for each β value, as the requirement is on their product. A robust choice of partial safety factors is usually a set which leads to design values of stresses and strengths as close as possible to the design point values resulting from the corresponding reliability analysis.

For the tensile strength, the characteristic value is $\sigma_{F,c}=411603$ kPa, and the design point value from the reliability analysis is $\sigma_F*=335936$ kPa. This gives the following robust choice for the material factor

$$\gamma_m = \frac{\sigma_{F,c}}{\sigma_F *} = 1.225 \tag{3.31}$$

This implies a corresponding value for the load factor

$$\gamma_f = \frac{\gamma_f \gamma_m}{\gamma_m} = \frac{1.184}{1.225} = 0.966 \tag{3.32}$$

It is of interest to notice that the requirement to the load factor becomes less than 1.0. This is due to the fact that in the reliability analysis the design point of the load happens to come out as a smaller value than the chosen characteristic value. In other words, the design point value of the load is in the particular case not a value particularly far out in the tail of the load distribution.

4. Conclusions

The extreme loading study has comprised a general wind climate analysis and a wind turbine reliability study.

In the wind climate analysis, the distribution of the turbulence standard deviation (around the mean turbulence standard deviation), conditioned on the mean wind speed, has been approximated by fitting a three parameter Weibull distribution to the measured data. The resulting Weibull parameters, associated with the on-shore data, are presented in Figures (4-1), (4-2) and (4-3).
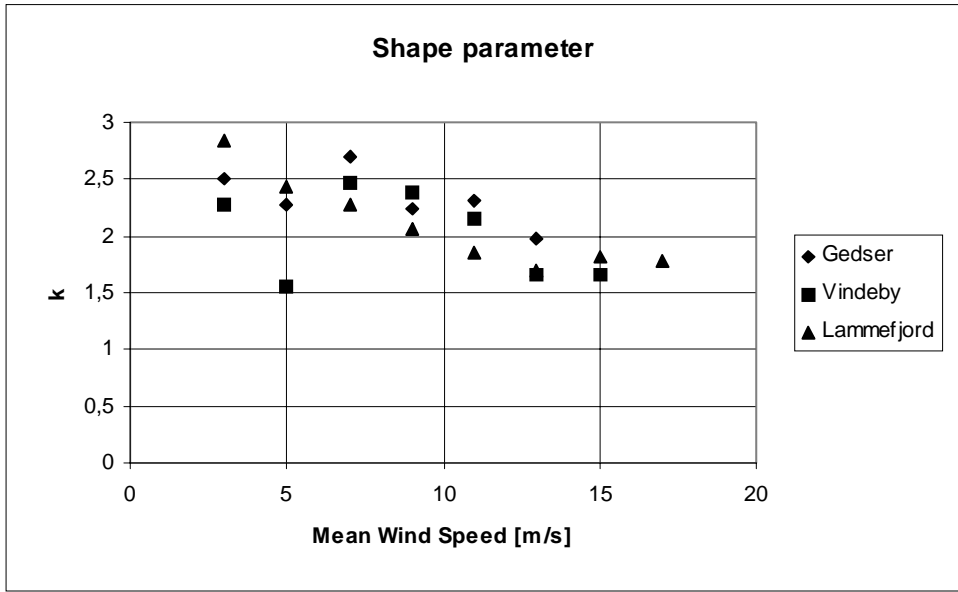
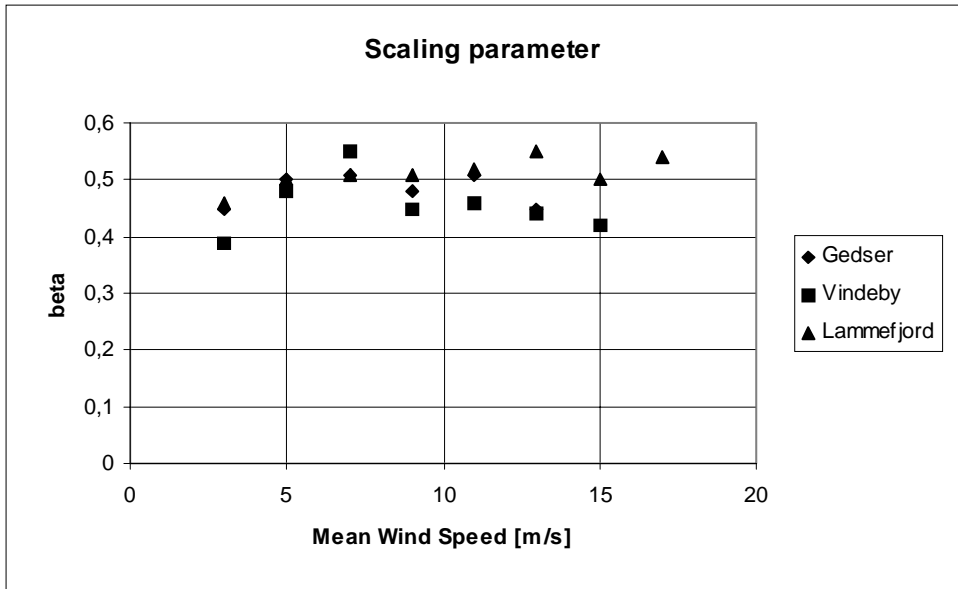**Figure 4-1 Shape parameter in the Weibull fit associated with on-shore data.**

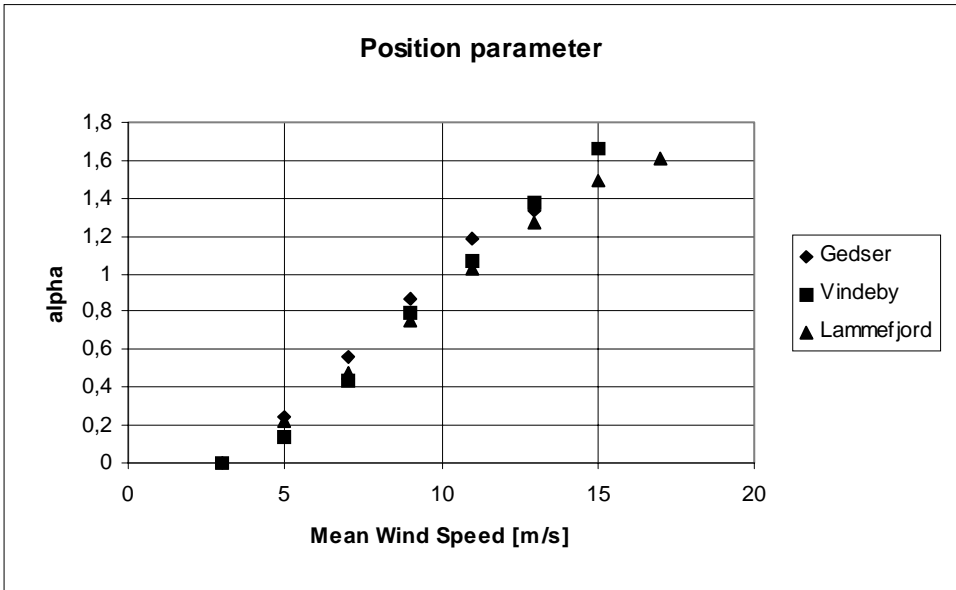**Figure 4-2 Scaling parameter in the Weibull fit associated with on-shore data.**

Figure 4-3 Position parameter in the Weibull fit associated with on-shore data.

Even though the three investigated on-shore terrain forms is comparable, some mutual scatter on the distribution parameters are revealed, particularly concerning the shape parameter. Fitting appropriate average expressions to the distribution parameters the following approximations have been obtained

$$k = -0.002 \, U_{10}^2 - 0.022 \, U_{10} + 2.53 \quad ,$$
$$\beta = 0.023 \, \ln\left(U_{10}\right) + 0.244 \quad , \tag{4.1}$$
$$\alpha = 0.130 \, U_{10} - 0.400 \quad .$$

Note that the approximations are restricted to flat and homogeneous terrain for mean wind speeds in the range 3m/s to 17m/s.

In analogy with the on-shore situation, the resulting Weibull parameters associated with the off-shore data are presented in Figures (4-4), (4-5) and (4-6).
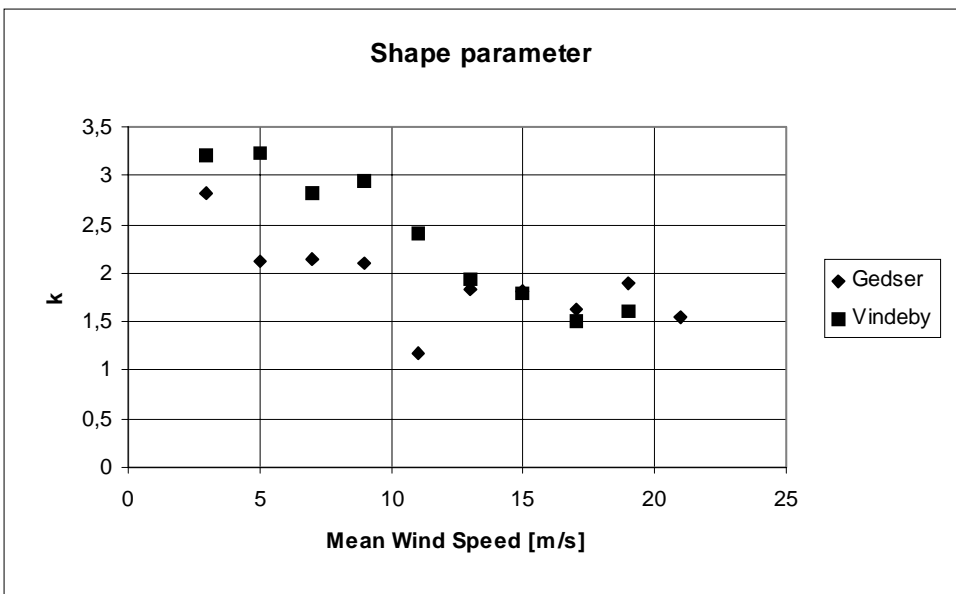

Figure 4-4 Shape parameter in the Weibull fit associated with off-shore data.
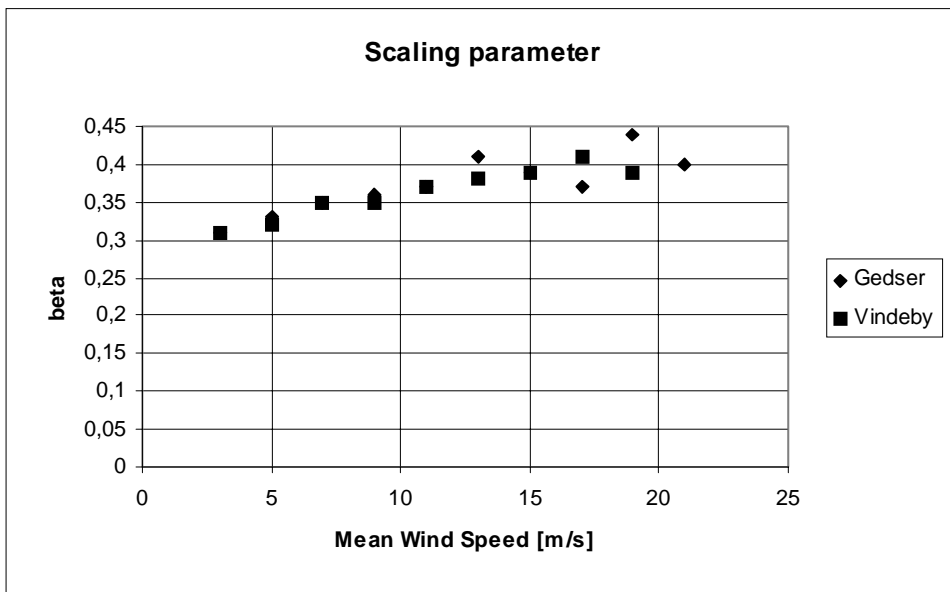
Risø-R-1111(EN)

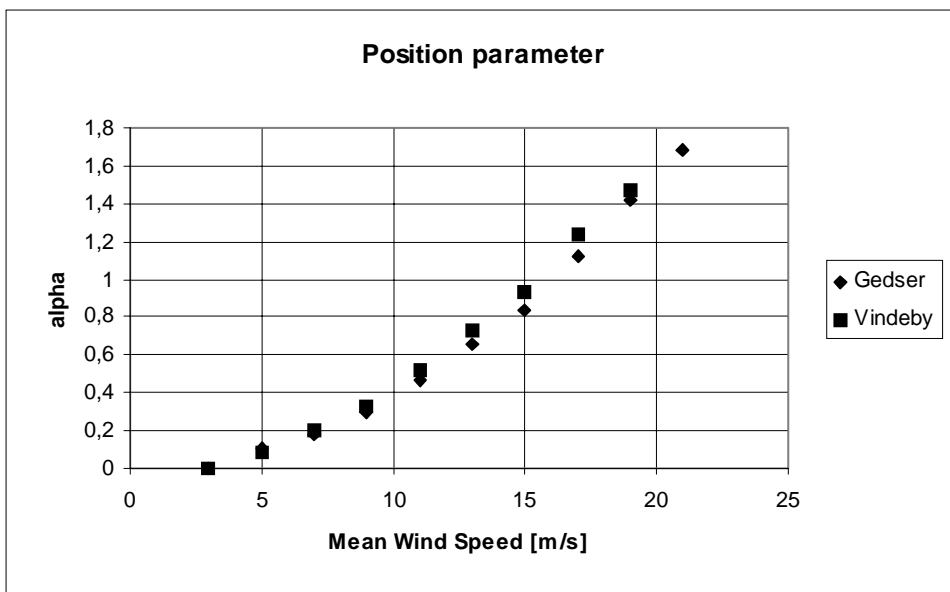**Figure 4-5 Scaling parameter in the Weibull fit associated with off-shore data.**

Figure 4-6 Position parameter in the Weibull fit associated with off-shore data.

Also for the off-shore situation some mutual scatter is revealed for the shape parameter. Fitting appropriate average expressions to the parameters, the following approximations are obtained

$$k = 0.0045\,U^2 - 0.1872\,U + 3.558 \quad,$$

$$\beta = 0.0541\ln(U) + 0.2441 \quad, \tag{4.2}$$

$$\alpha = 0.0034\,U^2\,0.0144\,U - 0.072 \quad.$$

Note that the available measurements are confined to shallow water off-shore situations in the wind range 3m/s to 22m/s, and the same limitation is consequently imposed on the obtained results.

Specific recommendations on off-shore design turbulence intensities are lacking in the present IEC-code, and the available on-shore specifications seem much too conservative for off-shore applications. Based on the present analysis of the off-shore wind climate on two shallow water sites, a design turbulence intensity for off-shore application is proposed, which in the IEC code framework is applicable for extreme load as for fatigue load determination. The proposed expression is

$$TI_D = 0.0032\, U_{10} + 0.0234 + \frac{0.3807}{U_{10}} \quad . \tag{4.3}$$

Note, however, that the extreme loading relates to atmospheric turbulence described in terms of a statistically stationary process. Gusts due to thunderstorms, tornados, down bursts etc. are not covered.

The performed reliability study have dealt with conventional as well as more advanced nested types of analyses. In the investigated case study, the nested reliability analysis produces a failure probability, which is more than ten times the failure probability produced by the conventional reliability analysis. This demonstrates the importance of including contributions to the failure probability from all 10-minute wind climates that occur during the normal operating condition of the wind turbine. This implies that the conventional reliability analysis is insufficient for estimation of the failure probability associated with the present failure problem, and the presented nested reliability analysis comes in handy as a more fulfilling approach. It has further been demonstrated that, within each of the investigated time series, it suffices to consider the probability contribution from the largest response only.

Without the cut-out speed included in the reliability analysis, the variability in the mean wind speed U_{10} would have become a much more important uncertainty source with a design point value in the range 35-40 m/sec, which is rather unrealistic for operation of the wind turbine. This would have led to a significantly higher failure probability in ultimate loading than the one reported here. In order to maintain the reliability against failure of the rotor blade in ultimate loading in the normal operating condition, this demonstrates that it is of utmost importance to keep the cut-out wind speed at 25 m/sec as intended during the entire service life of the wind turbine.

A target lifetime reliability, corresponding to an acceptable annual probability of failure of 10^{-5}, has been applied for the calibration of safety factors in a case study of a blade root flapwise loading situation. Based on the specific choice of characteristic values for load and resistance, a requirement to the product of load factor and material factor $\gamma_f \gamma_m = 1.184$ has come out. Based on the importance information of the underlying reliability analysis, a particular robust set of partial safety factors that fulfil this requirement has been determined, hence $\gamma_f = 0.966$ and $\gamma_m = 1.225$. This reflects that literally all uncertainty importance associated with failure in ultimate loading can be ascribed to the variability in the material properties. This is much an effect of the introduction of a cut-out wind speed that determines the particularly wind climates for which the wind turbine will not operate and from which there will be no contribution to the failure probability.

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Abstract (max. 2000 characters)

An extreme loading study has been conducted comprising a general wind climate analysis as well as a wind turbine reliability study.

In the wind climate analysis, the distribution of the turbulence standard deviation (around the mean turbulence standard deviation), conditioned on the mean wind speed has been approximated by fitting a three parameter Weibull distribution to the measured on-shore and off-shore data for wind speed variations. Specific recommendations on off-shore design turbulence intensities are lacking in the present IEC-code, and the available on-shore specifications seem much too conservative for off-shore applications. Based on the present analysis of the off-shore wind climate on two shallow water sites, a *design turbulence intensity* for off-shore application is proposed which, in the IEC code framework, is applicable for extreme as well as for fatigue load determination.

In order to establish a rational method to analyse wind turbine components with respect to failure in ultimate loading, and in addition to establish partial safety factors for design of such components against this failure mode, *structural reliability methods* must be applied. This type of analysis accounts for the variability of the external (wind) loading (as addressed in the analyses of the general wind climate) - and thereby the induced variability in the component stress response - as well as variability in material resistance. The present study comprises the development of a procedure suitable for dealing with this type of analyses. The main effort has been put on the methodology. Application of the procedure is illustrated by application to the event of failure in ultimate loading in flapwise bending in the normal operating condition of a site-specific turbine.