Technical University of Denmark

Quantum mechanical operator equivalents used in the theory of magnetism

Danielsen, O.; Lindgård, Per-Anker

Publication date:
1972

Document Version
Publisher's PDF, also known as Version of record

Risö Report No. 259

Risö Report No. 259

Danish Atomic Energy Commission

Research Establishment Risö

Quantum Mechanical Operator Equivalents Used in the Theory of Magnetism

by O. Danielsen and P.-A. Lindgård

March 1972

Sales distributors: Jul. Gjellerup, 87, Sölvgade, DK-1307 Copenhagen K, Denmark

Available on exchange from: Library, Danish Atomic Energy Commission, Risö, DK-4000 Roskilde, Denmark

U. D. C.
530.145.6:538

March 1972 Risø Report No. 259

Quantum Mechanical Operator Equivalents Used in the Theory of Magnetism

by

O. Danielsen

and

P.-A. Lindgård

Danish Atomic Energy Commission
Research Establishment Risø
Physics Department

Abstract

Two sets of operator equivalents, the Racah operators and the Stevens operators are treated. Their definitions as angular momentum operators, the transformation properties under rotations, the approximate Bose operator expansions, and the mutual connection of the two sets of operators are treated in detail. The information is presented in a number of detailed tables to enable direct use in hand calculations.

ISBN 87 550 0135 1

CONTENTS

LIST OF TABLES

1. INTRODUCTION

1.1. Expansion in Spherical Harmonics

In physics it is in many cases convenient to perform an expansion of a function in spherical harmonics or linear combinations thereof. It is then possible to utilize the extensive work done on the transformation and combination properties of the spherical harmonics (angular momenta). We shall throughout use definitions and conventions as used by Edmonds[1].

The function to be expanded may be a potential energy function, a wave function, or any other function which in general is subject to some symmetry constraints. The spherical harmonics Y_{lm} form a natural ortho-normal set of basis functions for rotations and are therefore particularly useful in an expansion of a function with a number of rotation invariances. If, in addition, the function has inversion and reflection invariances, it is often more convenient to use the tesseral harmonics, being a ortho-normal set of functions defined in terms of the spherical harmonics as

$$C_{lm} = \frac{1}{\sqrt{2}}(Y_{l,-m} + (-1)^m Y_{l,m}) \qquad m \neq 0 \tag{1.1}$$

$$S_{lm} = \frac{i}{\sqrt{2}}(Y_{l,-m} - (-1)^m Y_{l,m}) \qquad m \neq 0$$

$$C_{lo} = Y_{lo}; \quad S_{lo} = 0 \qquad m = 0$$

Thus we may expand a function $V(\bar{r})$ as

$$V(\bar{r}) = v(\bar{r}) \sum_{l,m} B_{lm} \cdot Y_{lm} = v(\bar{r}) \sum_{l,m} (B^c_{lm} C_{lm} + B^s_{lm} S_{lm}) .$$

1.2. Angular Momentum Operator Equivalence

Angular momentum operators[x] operate in the following way on spherical harmonics[1]:

---

x) We use, as did Edmonds, J to denote a generalized angular momentum; L is restricted to denote an orbital momentum.

$$J_z Y_{lm} = m\, Y_{lm}$$

$$J^2 Y_{lm} = l(l+1)\, Y_{lm}$$

$$J^+ Y_{lm} = (J_x + iJ_y)\, Y_{lm} = \sqrt{l(l+1) - m(m+1)}\; Y_{l,m+1} \tag{1.2}$$

$$J^- Y_{lm} = (J_x - iJ_y)\, Y_{lm} = \sqrt{l(l+1) - m(m-1)}\; Y_{l,m-1}$$

With these properties it is possible to transform a function into a function of angular momentum operators, the so-called operator equivalents. The operator equivalents transforming as $\sqrt{\frac{4\pi}{2l+1}}\, Y_{l,m}$ are called Racah operators and denoted $\tilde{O}_{l,m}$. $\tilde{O}_{l,m}$ is a function of angular momenta $\tilde{O}_{l,m} = \tilde{O}_{l,m}(l_x, l_y, l_z)$. Functions transforming as $\sqrt{\frac{4\pi}{2l+1}}\, C_{l,m}$ and $\sqrt{\frac{4}{2l+1}}\, S_{l,m}$ are defined as

$$\tilde{O}^c_{l,m} = \frac{1}{\sqrt{2}} (\tilde{O}_{l,-m} + (-1)^m \tilde{O}_{l,m})$$

$$\tilde{O}^s_{l,m} = \frac{i}{\sqrt{2}} (\tilde{O}_{l,-m} - (-1)^m \tilde{O}_{l,m})\,. \tag{1.3}$$

Stevens[2)] was the first to invent the operator equivalence method in crystal field calculations. Stevens introduced a different set of operator equivalents which have later been widely used in the literature. These "Stevens operators" denoted by $\tilde{O}^m_l$ have the disadvantage of not having the same systematic transformation properties as the Racah operators, however, they are convenient in "hand calculations" because they are defined such that a number of square-root factors disappear. They are therefore included in this report. Thus we can write the exact operator equivalent of $V(\bar{r})$ considered as an operator as

$$V(\bar{r}) = v(\bar{r}) \sum_{l,m} \tilde{B}_{l,m}\, \tilde{O}_{l,m}(l_x, l_y, l_z)$$

$$= v(\bar{r}) \sum_{l,m} (B^c_{l,m} O^c_{l,m} + B^s_{l,m} O^s_{l,m}) \tag{1.4}$$

$$= v(\bar{r}) \sum_{l,m} B^m_l\, O^m_l\,.$$

1.3. Bose Operator Equivalence

The commutator relation for angular momenta is

$$[J_x, J_y] = iJ_z \quad \text{and cyclic permutations.} \tag{1.5}$$

It is inconvenient that the commutator is a new operator. In approximate calculations it is often advantageous to use Bose operators having the following commutator relation:

$$[a, a^+] = 1 . \tag{1.6}$$

It is then possible to utilize the extensive theory in the literature for non-interacting and interacting Bose systems. In spinwave calculations this was first utilized by Holstein and Primakoff[3] who found the Bose operator expansion of single angular momentum operators. Here we shall go further and include all the $\tilde{O}_{l,m}$ operators. In crystal field calculations a transformation to approximate Bose operators was first done by Trammel and Grover[4]. For this purpose it is necessary to calculate matrix elements of the Racah operators. These have been tabulated by B. Birgeneau[5].

2. RACAH OPERATOR EQUIVALENTS $\tilde{O}_{l,m}$

A set of angular momentum operators $\tilde{O}_{l,m}$, which transform under rotations of the frame of coordinates in the same way as $\sqrt{\frac{4\pi}{2l+1}}$ times the spherical harmonics[x],

$$\tilde{O}_{l,m} \text{ transform as } \sqrt{\frac{4\pi}{2l+1}}\, Y_{l,m} \, , \tag{2.1}$$

has been tabulated in terms of angular momentum operators J_x, J_y, J_z by Buckmaster[6]. The angular momentum operators can be obtained by the Stevens operator equivalence method in which the spherical harmonics $Y_{l,m}$ expressed in $\frac{x}{r}, \frac{y}{r}, \frac{z}{r}$ are translated into functions $\tilde{O}_{l,m}$ of J_x, J_y, J_z with due respect to the non-commutativity of the angular momentum components. It is, however, convenient to use this method only for the simplest case, $Y_{1,1} \rightarrow \tilde{O}_{1,1}$, where no commutation problems occur, and generate $\tilde{O}_{l,m}$ by successive commutations with J^-. This is essentially the method described by Racah[7].

---

[x] Operators with these transformation properties are per definition tensor operators.

The commutator between $J^- = J_x - iJ_y$ and a Racah operator $\tilde{O}_{l,m}$ is according to Racah

$$[J^-, O_{l,m}] = \sqrt{l(l+1) - m(m-1)}\ \tilde{O}_{l,m-1} \ . \tag{2.2}$$

As a starting point the operator with the maximum m-value, namely $m = l$, is calculated. The operators with lower m-values then are generated by a straight-forward, but lengthy calculation using (2.2). According to the definition of the spherical harmonics (7.1) we have

$$Y_{ll}(\theta, \varphi) = (-1)^l \sqrt{\frac{2l+1}{4\pi (2l)!}}\ P_l^l(\cos\theta) e^{il\varphi}$$

and from the associated Legendre functions $P_l^m(\cos\theta)$ we have according to Jahnke and Emde[8]

$$P_l^l(\cos\theta) = \frac{(2l)!}{2^l l!} (\sin\theta)^l$$

When we introduce Cartesian coordinates, we find from these two relations

$$Y_{ll}(\theta, \varphi) = \frac{(-1)^l}{2^l l!} \sqrt{\frac{(2l+1)!}{4\pi}}\ (\sin\theta)^l e^{il\varphi} = \frac{(-1)^l}{2^l l!} \sqrt{\frac{(2l+1)!}{4\pi}} \left(\frac{x+iy}{r}\right)^l$$

It is clearly convenient to multiply by $\sqrt{\frac{4\pi}{2l+1}}$ and by the operator equivalence method, whereby $\frac{x+iy}{r}$ is replaced by $J_x + iJ_y = J^+$; we have then in accordance with (2.1)

$$\tilde{O}_{ll} = \sqrt{\frac{4\pi}{2l+1}}\ \ \frac{(-1)^l}{2^l l!} \sqrt{\frac{(2l+1)!}{4\pi}}\ (J^+)^l$$

or

$$\tilde{O}_{ll} = \frac{(-1)^l}{2^l l!} \sqrt{(2l)!}\ \ (J^+)^l \tag{2.3}$$

The operators $\tilde{O}_{l,-m}$ are obtained by means of the relation

$$\tilde{O}_{l,-m} = (-1)^m\ \tilde{O}^+_{l,m} \tag{2.4}$$

In table 1 the Racah operator equivalents for all l up to $l = 8$ are tabulated.

As an example we shall explicitly perform the calculations of the Racah operator equivalents for $l = 2$. From (2.3) we find

$$\underline{\tilde{O}_{2,2} = \sqrt{\frac{3}{8}}\,(J^{+})^{2}}$$

and by Hermitian conjugation we obtain from (2.4)

$$\underline{\tilde{O}_{2,-2} = \sqrt{\frac{3}{8}}\,(J^{-})^{2}} \quad .$$

Using the commutator relation (2.2) we find the following operators

$$[J^{-},\ \tilde{O}_{2,2}] = \sqrt{2\cdot 3 - 2\cdot 1}\ \tilde{O}_{2,1}$$

$$\tilde{O}_{2,1} = \frac{1}{2}\sqrt{\frac{3}{8}}\ [J^{-},\ (J^{+})^{2}]$$

$$\underline{\tilde{O}_{2,1} = -\sqrt{\frac{3}{2}}\,\frac{1}{2}(J^{+}J_{z} + J_{z}J^{+})}$$

and with (2.4)

$$\underline{\tilde{O}_{2,-1} = \sqrt{\frac{3}{2}}\,\frac{1}{2}(J^{-}J_{z} + J_{z}J^{-})} \quad .$$

With one additional commutation we obtain $\tilde{O}_{2,0}$

$$[J^{-},\ \tilde{O}_{2,1}] = \sqrt{2\cdot 3}\ \tilde{O}_{2,0}$$

$$\tilde{O}_{2,0} = \frac{1}{\sqrt{6}}\,\frac{1}{2}\sqrt{\frac{3}{8}}\,[J^{-}, [J^{-},\ (J^{+})^{2}]]$$

$$\underline{\tilde{O}_{2,0} = \frac{1}{2}(3J_{z}^{2} - J(J+1))}$$

3. RACAH OPERATOR EQUIVALENTS EXPANDED IN BOSE OPERATORS

In approximate calculations it is useful to expand the operator equivalents $\tilde{O}_{l,m}$ in Bose operators.

3.1. Crystal Field Calculations

In crystal field calculations it is possible to express approximately an excitation operator between the ground state and an excited state as a matrix element of the operator between the states times a Bose operator. In this theory it is necessary to know the matrix element of the $\tilde{O}_{l,m}$ operators between various crystal field states. These have been calculated by Birgeneau[5], and we shall refer to that article for the numerical values.

3.2. Spin Wave Calculations

Suppose a system with total angular momentum J has a sequence of states beginning with the ground state as follows:

$$|J\rangle,\ |J-1\rangle,\ \ldots,\ |J-n\rangle,\ \ldots\ |-J\rangle \qquad \text{angular momentum states}$$

$$|0\rangle,\ |1\rangle,\ \ldots,\ |n\rangle,\ \ldots\ |2J+1\rangle \qquad \text{deviation states} \tag{3.1}$$

$$E_o < E_1 < \ldots < E_n < \ldots < E_{2J+1} \qquad \text{energies}$$

Let us introduce Bose operators a, a^+ as the spin deviation operators acting on a state $|p\rangle$ with p deviations: annihilation operator, $a\,|p\rangle = \sqrt{p}\,|p-1\rangle$ and creationoperator, $a^+|p\rangle = \sqrt{p+1}\,|p+1\rangle$. This is to be compared with the action of the operators $\tilde{O}_{l,m}$ on the state $|J-p\rangle$.

The idea of expanding an angular momentum operator in a power series of Bose operators was first used by Holstein and Primakoff[3] for a single angular momentum operator. This method was used by Goodings et al.[9] in finding the Bose operator expansion for a number of $\tilde{O}_{l,m}$ operators. Each angular momentum in the expression for $\tilde{O}_{l,m}$ (table 1) was replaced by its, Holstein-Primakoff-expansion, and finally all a^+ operators were commuted to the left of the expression (well ordering). This is a very cumbersome method, and only a few $\tilde{O}_{l,m}$ operators were expanded. We shall use a different method which is physically cleaer and easier to perform. We expand the $\tilde{O}_{l,m}$ formally in a well-ordered series of Bose operators and require the matrix elements between corresponding states to be identical. If we only require correct matrix elements between the ground and the first excited state, it can be shown that the result of the expansion is identical to the well-ordered Holstein Primakoff transformation as done by Goodings et al.[9].

The well-ordered expansion of $\tilde{O}_{l,m}$ is

$$\tilde{O}_{l,m} = (A^l_{m,o} + A^l_{m,1}a^+a + A^l_{m,2}a^+a^+aa + \ldots)a^m \tag{3.2}$$

We find the coefficients of $\tilde{O}_{l,-m}$ using (2.4).

The coefficients are real and determined by matching the matrix elements in the following way:

$$\langle J-p|\tilde{O}_{l,m}|J-(p+m)\rangle = \langle p|(A^{l}_{m,o}+A^{l}_{m,1}a^{+}a + A^{l}_{m,2}a^{+}a^{+}aa)a^{m}|p+m\rangle \tag{3.3}$$

hence according to (7.9)

$$(-1)^{J-p}\begin{pmatrix} J & l & J \\ -J+p & m & J-p-m \end{pmatrix}\langle J||\tilde{O}_l||J\rangle = \sqrt{\frac{(p+m)!}{m!}}(A^{l}_{m,o}+pA^{l}_{m,1}+p(p-1)A^{l}_{m,2}+\ldots)$$

where we have used the standard formula for the matrix elements of the angular momentum and Bose operators. From (3.3) we find the coefficients.

$$A^{l}_{m,o} = (-1)^{J}\langle J||\tilde{O}_l||J\rangle\begin{pmatrix} J & l & J \\ -J & m & J-m \end{pmatrix}$$

$$A^{l}_{m,1} = (-1)^{J-1}\langle J||\tilde{O}_l||J\rangle\left\{\begin{pmatrix} J & l & J \\ -J & m & J-m \end{pmatrix} + \frac{1}{\sqrt{m+1}}\begin{pmatrix} J & l & J \\ -J+1 & m & J-1-m \end{pmatrix}\right\}$$

$$A^{l}_{m,2} = \frac{(-1)^{J}}{2}\langle J||\tilde{O}_l||J\rangle\left\{\begin{pmatrix} J & l & J \\ -J & m & J-m \end{pmatrix} + \frac{2}{\sqrt{m+1}}\begin{pmatrix} J & l & J \\ -J+1 & m & J-1-m \end{pmatrix} + \frac{1}{\sqrt{(m+1)(m+2)}}\begin{pmatrix} J & l & J \\ -J+2 & m & J-2-m \end{pmatrix}\right\}$$

(3.4)

The reduced matrix element is

$$\langle J||\tilde{O}_l||J\rangle = \frac{1}{2^l}\sqrt{\frac{(2J+l+1)!}{(2J-l)}} \tag{3.5}$$

and the 3j symbols can be found either in tables or by means of recursive formulae. In the latter way we find the first coefficients in (3.2)

$$A^{l}_{m,o} = (-1)^{m} \frac{1}{m!} \sqrt{\frac{(l+m)!}{2^{m}(l-m)!}} \frac{S_l}{\sqrt{S_m}}$$

$$A^{l}_{m,1} = -A^{l}_{m,o} \sqrt{\frac{S_m}{S_1 S_{m+1}}} \left[\frac{(l-m)(l+m+1)}{2(m+1)} + \sqrt{\frac{S_1 S_{m+1}}{S_m}} - \frac{S_{m+1}}{S_m} \right] \qquad (3.6)$$

$$A^{l}_{l,2} = -A^{l}_{l,o} \left[\frac{(l-1)(l+2)}{4} \left\{ \frac{(l-2)(l+3)}{12} \frac{S_1}{S_2} + \frac{1}{\sqrt{S_2}} \left(1 - \frac{\sqrt{S_1 S_3}}{S_2} \right) \right\} + \frac{1}{2} \left(1 + \sqrt{\frac{S_3}{S_1 S_2}} \right) - \frac{\sqrt{S_2}}{S_1}$$

where $S_l = \frac{1}{2^l} \frac{(2J)!}{(2J-l)!} = J(J-\frac{1}{2})(J-1)(J-\frac{3}{2}) \ldots (J-\frac{l-1}{2})$

By means of these formulae the Racah operator equivalents are tabulated for odd l as well as even l up to l = 8 - table 2.

It should be added that the operators $\tilde{O}_{10}$, $\tilde{O}_{20}$, ... $\tilde{O}_{80}$ are finite expansions, whereas all other operators are infinite expansions in Bose operators. In all operators only terms with up to five Bose operators are written out because of the limited validity of the spin deviation representation.

4. TRANSFORMATIONS UNDER ROTATIONS OF SPHERICAL HARMONICS AND RACAH OPERATORS

We shall in this section summarize the transformation properties of the spherical harmonics and of the Racah operators under rotations. Although this transformation theory has been well developed (Edmonds[1], Rose[10], Tinkham[11], Judd[12], and Rothenberg[13]), it often causes confusion that the term rotation in the literature sometimes refers to the physical body, sometimes to the axes of the coordinate system. This point will therefore be discussed in some detail.

4.1. Description of the Result of a Rotation

An arbitrary rotation of the frame of coordinates is most conveniently expressed by the three Euler angles α, β, γ. The definitions of the Euler angles are shown in fig. 4.1.

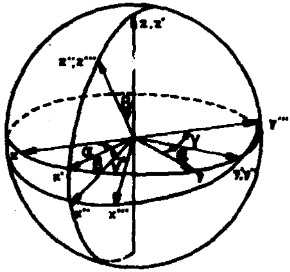


Fig. 4.1.
The Euler angles

1. A rotation $\alpha (0 \leq \alpha \leq 2\pi)$ about the z-axis, bringing the frame of axes from the initial position S into the position S'. The axis of this rotation is commonly called the vertical axis.
2. A rotation $\beta (0 \leq \beta \leq \pi)$ about the y-axis of the frame S', called the line of nodes. Note that its position is in general different from the initial position of the y-axis of the frame S. The resulting position of the frame of axes is symbolized by "S".
3. A rotation $\gamma (0 \leq \gamma \leq 2\pi)$ about the z-axis of the frame of axes S", called the figure axis; the position of this axis depends on the previous rotations α and β. The final position of the frame is symbolized by S'''.

It should be noted that the polar coordinates φ and θ with respect to the original frame S of the z-axis in its final position are identical with the Euler angles α and β respectively.

4.1.1. Transformation of Cartesian Coordinates

Before dealing with the general transformation theory we shall set up the much simpler formalism of transformation of Cartesian coordinates which may serve as a guideline for the more complex general transformations.

If the coordinate system is rotated through the Euler angles α, β, γ, the Cartesian coordinates x, y, z of a point P which is stationary under the rotation will be changed to x', y', z'. Let the two sets of Cartesian coordinates be written as column vectors $\underline{r}$ and $\underline{r}'$

$$\underline{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad ; \qquad \underline{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

and let the matrix $\underline{\underline{P}}(\gamma, \beta, \alpha)$ be the 3 x 3 matrix which connects them, then

$$\underline{r}' = \underline{\underline{P}} \cdot \underline{r} \tag{4.1}$$

and

$$\underline{\underline{P}}(\gamma, \beta, \alpha) = \underline{\underline{P}}_{z''}(\gamma) \cdot \underline{\underline{P}}_{y'}(\beta) \cdot \underline{\underline{P}}_{z}(\alpha)$$

where first $\underline{\underline{P}}_z(\alpha)$ performs a rotation about the original z-axis, then $\underline{\underline{P}}_{y'}(\beta)$ a rotation about the y-axis in S':y', and finally $\underline{\underline{P}}_{z''}(\gamma)$ a rotation about the z-axis in S'':z''. The three rotation matrices are given by

$$\underline{\underline{P}}_z(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{P}}_{y'}(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \tag{4.2}$$

$$\underline{\underline{P}}_{z''}(\gamma) = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The resulting transformation matrix is therefore

$$\underline{\underline{P}}(\gamma, \beta, \alpha) = \begin{bmatrix} (\cos\alpha \cos\beta \cos\gamma - \sin\alpha \sin\gamma) & (\sin\alpha \cos\beta \cos\gamma + \cos\alpha \sin\gamma) & -\sin\beta\cos \\ (-\cos\alpha \cos\beta \sin\gamma - \sin\alpha \cos\gamma) & (-\sin\alpha \cos\beta \sin\gamma + \cos\alpha \cos\gamma) & \sin\beta\sin \\ \cos\alpha \sin\beta & \sin\alpha \sin\beta & \cos\beta \end{bmatrix} \tag{4.3}$$

If one wants to rotate the physical body (the point P) instead of rotating the frame of coordinates, this is simply done by using the inverse rotation matrix, constructed in the following way

$$\begin{aligned} \underline{\underline{P}}(\gamma, \beta, \alpha)^{-1} &= (\underline{\underline{P}}_{z''}(\gamma)\, \underline{\underline{P}}_{y'}(\beta)\, \underline{\underline{P}}_z(\alpha))^{-1} = \underline{\underline{P}}_z(\alpha)^{-1}\, \underline{\underline{P}}_{y'}(\beta)^{-1}\, \underline{\underline{P}}_{z''}(\gamma)^{-1} \\ &= \underline{\underline{P}}_z(-\alpha)\, \underline{\underline{P}}_{y'}(-\beta)\, \underline{\underline{P}}_{z''}(-\gamma) = \underline{\underline{P}}(-\alpha, -\beta, -\gamma) \end{aligned} \tag{4.4}$$

$\underline{\underline{P}}(\gamma, \beta, \alpha)^{-1}$ is the transposed of $\underline{\underline{P}}(\gamma, \beta, \alpha)$.

4.1.2. General Transformations

Conventionally, there are four equivalent ways of describing the result of a rotation depending on the choice of the object to be rotated and on the choice of the frame of reference.

Either (1) the function (the body) or (2) the axes of frame are rotated. The rotations may be performed either (a) with respect to a rotated coordinate system (local system) or (b) with respect to a fixed coordinate system (global system).

It should be mentioned that the rotated functions are considered to be represented in terms of basis functions set up with respect to the global system irrespective of how the rotations are performed (cases 1a and 1b). The various cases are described in the following, and a summary presented in fig. 4.2.

(a) Rotated Coordinate System

In fig. 4.1 the successive rotations are made about the rotated axes (local system). Starting with the x, y, z coordinate system (S) we rotate by α about the z-axis, and denote the new axes x', y', z' (S'). Then we rotate by β about the y'-axis, and denote the resulting axes x'', y'', z'' (S''). Finally we rotate by γ about the z''-axis, and label the final axes x''', y''', z''' (S'''). In operator form we express these rotations as

$$P_{z''}(\gamma)\, P_{y'}(\beta)\, P_z(\alpha)\,.$$

(b) Fixed Coordinate System

The rotations can also be described with respect to a permanent set of space-fixed axes (global system). It can be shown, Tinkham[11], that the same final axes x''', y''', z''' might have been obtained by carrying out the rotations in reverse order about fixed axes. That is, we might have rotated by γ about z, then by β about y, and finally by α about z again. In operator form we have for these rotations which define the combined rotation operator $D(\alpha, \beta, \gamma)$

$$D(\alpha, \beta, \gamma) \equiv P_z(\alpha)\, P_y(\beta)\, P_z(\gamma)\,. \qquad (4.5)$$

(1) Rotation of a Function with Respect to a Fixed (Global) Coordinate System

A finite rotation of a function with respect to a fixed coordinate system can be looked upon as a succession of infinitesimal rotations.

A rotation through an angle φ about a fixed z-axis of a one-particle wave function $\Psi(x, y, z)$ can be described by

$$P_z(\varphi)\Psi(x, y, z) = \Psi(x\cos\varphi + y\sin\varphi, \ -x\sin\varphi + y\cos\varphi, \ z).$$

In order to find the operator $P_z(\varphi)$ we first consider an infinitesimal rotation $d\varphi$

$$P_z(d\varphi)\Psi(x, y, z) \simeq \Psi(x+yd\varphi, \ y-xd\varphi, \ z)$$

$$\simeq \Psi(x, y, z) + d\varphi\left(y\frac{\partial\Psi(x, y, z)}{\partial x} - x\frac{\partial\Psi(x, y, z)}{\partial y}\right)$$

$$= \left(1 - d\varphi\frac{i}{\hbar}l_z\right)\Psi(x, y, z)$$

hence the infinitesimal rotation operator is

$$P_z(d\varphi) = \left(1 - d\varphi\frac{i}{\hbar}l_z\right)$$

When rotating about a direction specified by $\underline{u}$, the infinitesimal rotation operator is

$$P_{\underline{u}}(d\varphi) = \left(1 - d\varphi\frac{i}{\hbar}\underline{l}\cdot\underline{u}\right)$$

If $\underline{J}$ is the total angular momentum of a many-body system, then its component along any axis $\underline{u}$ is related to the operator of infinitesimal rotation about the axis by the relation, Messiah[14)]

$$P_{\underline{u}}(d\varphi) = 1 - d\varphi\frac{i}{\hbar}\underline{J}\cdot\underline{u} \tag{4.6}$$

Considering a finite rotation $P_{\underline{u}}(\varphi)$ about the axis in the $\underline{u}$-direction, and putting $\underline{J}\cdot\underline{u} = J_u$

$$P_{\underline{u}}(\varphi + d\varphi) = P_{\underline{u}}(d\varphi)\cdot P_{\underline{u}}(\varphi) = \left(1 - d\varphi\frac{i}{\hbar}J_u\right)P_{\underline{u}}(\varphi)$$

which is equivalent with the equation

$$\frac{dP_{\underline{u}}(\varphi)}{d\varphi} = -\frac{i}{\hbar}J_uP_{\underline{u}}(\varphi).$$

This equation has the following solution under the condition $P_{\underline{u}}(o) = 1$

$$P_{\underline{u}}(\varphi) = e^{-i\varphi \frac{J_u}{\hbar}} \tag{4.7}$$

This is the rotation operator for the rotation around the axes $\underline{u}$.
From (4.7) we immediately find for the combined rotation operator $D(\alpha, \beta, \gamma)$

$$D(\alpha, \beta, \gamma) = P_z(\alpha) P_y(\beta) P_z(\gamma) = e^{-i\alpha \frac{J_z}{\hbar}} e^{-i\beta \frac{J_y}{\hbar}} e^{-i\gamma \frac{J_z}{\hbar}} \tag{4.8}$$

Now we want to find the matrices $\underline{\underline{D}}^l(\alpha, \beta, \gamma)$ which describe how the angular momentum eigenfunctions $Y_{lm}(\theta, \varphi)$ transform under the general rotation operator $D(\alpha, \beta, \gamma)$. Because of the choice of the z-axis as quantization axis, J_z is a diagonal operator and the effect of the rotations by α and γ is easily written by using the exponential form of the rotation operator. The rotation $P_y(\beta)$ will have a non-diagonal representation, which is denoted $\underline{\underline{d}}^l(\beta)$.

The transformed spherical harmonic function $Y_{lm}(\theta', \varphi')$ (analogue to the vector $\underline{r}'$ in (4.1)) is then expressed by

$$Y_{lm}(\theta', \varphi') = D(\alpha, \beta, \gamma)\, Y_{lm}(\theta, \varphi) = \sum_{m'=-l}^{l} Y_{lm'}(\theta, \varphi)\, D^{(l)}_{m'm}(\alpha, \beta, \gamma) \tag{4.9}$$

The matrix elements of the rotation operator $D(\alpha, \beta, \gamma)$ is

$$D^{(l)}_{m'm}(\alpha, \beta, \gamma) = \langle lm' | D(\alpha, \beta, \gamma) | lm \rangle = e^{-im'\alpha}\, d^{(l)}_{m'm}(-\beta)\, e^{-im\gamma} \tag{4.10}$$

where

$$d^{(l)}_{m'm}(\beta) = \left[\frac{(l+m')!\,(l-m')!}{(l+m)!\,(l-m)!} \right]^{1/2} \sum_{\sigma} \binom{l+m}{l-m'-\sigma} \binom{l-m}{\sigma} (-1)^{l-m'-\sigma} \left(\cos\frac{\beta}{2}\right)^{2\sigma+m'+m} \times$$

$$\left(\sin\frac{\beta}{2}\right)^{2l-2\sigma} \tag{4.11}$$

(the summation is over all positive σ such that the factorial terms are non-negative).

For $d^{(l)}_{m'm}(\beta)$ we have the following symmetry relations

$$d^{(l)}_{m'm}(-\beta) = d^{(l)}_{mm'}(\beta) \tag{4.12}$$

$$d^{(l)}_{m'm}(\beta) = (-1)^{m'-m}\, d^{(l)}_{mm'}(\beta) \tag{4.13}$$

$$d^{(l)}_{m'm}(\beta) = (-1)^{m'-m}\, d^{(l)}_{-m',-m}(\beta) \tag{4.14}$$

The matrix elements $D^{(l)}_{m'm}(\alpha, \beta, \gamma)$ are therefore

$$D^{(l)}_{m'm}(\alpha, \beta, \gamma) = e^{-im'\alpha}\, d^{(l)}_{m'm}(\beta)\, e^{-im\gamma} \tag{4.15}$$

(2) Permanent Function and Rotated Axes

Consider the case where the function is held fixed in space, and the coordinate frame is rotated. We note that if the same rotation was applied to both the axes and the function, the relative position would be unchanged, which can be expressed by the relation

$$\underline{\underline{D}}^{l}_{axes}(\alpha, \beta, \gamma)\ \underline{\underline{D}}^{l}_{function}(\alpha, \beta, \gamma) = \underline{\underline{E}} \tag{4.16}$$

hence

$$\underline{\underline{D}}^{l}_{function}(\alpha, \beta, \gamma) = \underline{\underline{D}}^{l}_{axes}(\alpha, \beta, \gamma)^{-1} = \underline{\underline{D}}^{l}_{axes}(-\gamma, -\beta, -\alpha)$$

That is, we can use the same $\underline{\underline{D}}(\alpha, \beta, \gamma)$ to represent rotation of axes by $-\alpha$, $-\beta$, $-\gamma$ in the reverse order to that used if the functions were being rotated.

The discussion of this section can be summarized in the following figure 4.2.

Case	Rotated	With respect to	Operator
1a	function	rotated axes	$P_{z''}(\gamma)\,P_{y'}(\beta)\,P_z(\alpha)$
1b	function	fixed axes	$P_z(\alpha)\,P_y(\beta)\,P_z(\gamma)$
2a	axes	rotated axes	$P_{z''}(-\alpha)\,P_{y'}(-\beta)\,P_z(-\gamma)$
2b	axes	fixed axes	$P_z(-\gamma)\,P_y(-\beta)\,P_z(-\alpha)$

Fig. 4.2.

The alternative possibilities in transformation theory.

4.1.3. Rotation of Racah Operators

The Racah operators transform per definition under the unitary transformation comprising the rotation with respect to a fixed frame in the same way as the spherical harmonics, and thus according to fig. 4.2: case 1b we have

$$\tilde{O}_{lm}(J'_x, J'_y, J'_z) = D(\alpha,\beta,\gamma)\,\tilde{O}_{lm}(J_x, J_y, J_z)$$
$$= \sum_{m'=-l}^{l} \tilde{O}_{lm'}(J_x, J_y, J_z)\,\langle lm' \,|D(\alpha,\beta,\gamma)|\, lm\rangle \qquad (4.17)$$

The same relation can in a short hand matrix notation be written as

$$\underline{\tilde{O}}_l = \underline{\tilde{O}}'_l \cdot \underline{\underline{D}}^l(\alpha,\beta,\gamma) \qquad (4.18)$$

(The primed operators are angular momentum components in the triple-primed coordinate system, fig. 4.1).

4.2. Calculations of the Rotation Matrices $\underline{\underline{d}}^l(\beta)$

In table 3 the $\underline{\underline{d}}^l(\beta)$ matrices for all values of $l = 1, 2, \dots 8$ are given in the form

$$d^{(l)}_{m'm}(\beta) = C \cdot (\sin\tfrac{\beta}{2})^{2l}\left\{A_0 + A_1\cot\tfrac{\beta}{2} + A_2(\cot\tfrac{\beta}{2})^2 + \dots + A_{2l}(\cot\tfrac{\beta}{2})^{2l}\right\}_{m'm} \qquad (4.19)$$

where C and A_n are numerical constants.

As an example of the use of table 3 the $\underline{d}^1(\beta)$ - matrix is constructed

$$
\begin{aligned}
d^1_{11}(\beta) &= 1 \cdot (\sin\tfrac{\beta}{2})^2 (\cot\tfrac{\beta}{2})^2 &&= \tfrac{1}{2}(1+\cos\beta) \\
d^1_{10}(\beta) &= \sqrt{2}(\sin\tfrac{\beta}{2})^2 \cot\tfrac{\beta}{2} &&= \tfrac{1}{\sqrt{2}}\sin\beta \\
d^1_{1-1}(\beta) &= 1 \cdot (\sin\tfrac{\beta}{2})^2 \cdot 1 &&= \tfrac{1}{2}(1-\cos\beta) \\
d^1_{oo}(\beta) &= 1 \cdot (\sin\tfrac{\beta}{2})^2 \;[-1+(\cot\tfrac{\beta}{2})^2] &&= \cos\beta
\end{aligned}
\qquad (4.20)
$$

The rest of the matrix is obtained using the symmetry relations (4.13) and (4.14)

$$
\underline{\underline{d}}^1(\beta):\quad
\begin{array}{c|c|c|c|}
m' \backslash m & 1 & 0 & -1 \\
\hline
1 & \frac{1}{2}(1+\cos\beta) & \frac{\sqrt{2}}{2}\sin\beta & \frac{1}{2}(1-\cos\beta) \\
\hline
0 & -\frac{\sqrt{2}}{2}\sin\beta & \cos\beta & \frac{\sqrt{2}}{2}\sin\beta \\
\hline
-1 & \frac{1}{2}(1-\cos\beta) & -\frac{\sqrt{2}}{2}\sin\beta & \frac{1}{2}(1+\cos\beta) \\
\hline
\end{array}
\qquad (4.21)
$$

4.3. Rotation of the Angular Momentum Vectors

As an example of the transformation of the Racah operators we shall express the relations between $\underline{J}' = (J_x, J_y, J_z)$ in the rotated coordinate system - the local coordinate system - and $\underline{J} = (J_\xi, J_\eta, J_\zeta)$ in the stationary or global coordinate system. This transformation gives a check on the formalism as it can easily be calculated directly.

From table 1 we have

$$
[\tilde{O}_{11}, \tilde{O}_{10}, \tilde{O}_{1-1}] = [J_x, J_y, J_z] \cdot \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix} = [J_x, J_y, J_z] \cdot \underline{\underline{U}}
$$

or in shorthand notation

$$
\underline{\tilde{O}}_1 = \underline{J} \cdot \underline{\underline{U}} \qquad (4.22)
$$

and the opposite

$$\underline{J} = \underline{\tilde{O}_1} \cdot \underline{\underline{U}}^{-1} = \underline{\tilde{O}_1} \cdot \underline{\underline{U}}^{+} \tag{4.23}$$

The corresponding relations of course hold for $\underline{J}'$ and $\underline{\tilde{O}'_1}$.

The relation between $\underline{\tilde{O}'_1}$ and $\underline{\tilde{O}_1}$ is for a rotation through the Euler angles α and β:

$$\underline{\tilde{O}'_1} = \underline{\tilde{O}_1} \cdot \underline{\underline{D}}^1(\alpha, \beta, 0)$$

By use of (4.22) we find the relation between $\underline{J}'$ and $\underline{J}$

$$\underline{J}' \cdot \underline{\underline{U}} = \underline{J} \cdot \underline{\underline{U}} \cdot \underline{\underline{D}}^1(\alpha, \beta, 0) \qquad \text{or}$$

$$\underline{J}' = \underline{J} \cdot \underline{\underline{U}} \cdot \underline{\underline{D}}^1(\alpha, \beta, 0) \cdot \underline{\underline{U}}^{-1}$$

From (4.15) and (4.21) we find for $\underline{\underline{D}}^1(\alpha, \beta, 0)$:

$$\underline{\underline{D}}^1(\alpha, \beta, 0) = \begin{bmatrix} e^{-i\alpha} \frac{1+\cos\beta}{2} & -e^{-i\alpha} \frac{\sin\beta}{\sqrt{2}} & e^{-i\alpha} \frac{1-\cos\beta}{2} \\ \frac{\sin\beta}{\sqrt{2}} & \cos\beta & -\frac{\sin\beta}{\sqrt{2}} \\ e^{i\alpha} \frac{1-\cos\beta}{2} & e^{i\alpha} \frac{\sin\beta}{\sqrt{2}} & e^{i\alpha} \frac{1+\cos\beta}{2} \end{bmatrix} \tag{4.24}$$

so the final relation between $\underline{J}$ and $\underline{J}'$ is

$$[J_x, J_y, J_z] = [J_\xi, J_\eta, J_\zeta] \cdot \begin{bmatrix} \cos\alpha \cos\beta & -\sin\alpha & \cos\alpha \sin\beta \\ \sin\alpha \cos\beta & \cos\alpha & \sin\alpha \sin\beta \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \tag{4.25}$$

which is in accordance with the direct calculation using $\underline{\underline{P}}(-\alpha, -\beta, 0)$ in equation (4.4) and the result (4.16).

4.4. $\pi/2$ - Transformation of Racah Operators

In practice it is often convenient to *transform* a set of operators given in an axially symmetric system with the z-axis along the symmetry axis into a coordinate system with the z'-axis in the perpendicular plane xy so that z' makes an angle with x, and y' is parallel to z.

The transformation relating the operators in the new (rotated) frame of reference is

$$\underline{\tilde{O}}_l = \underline{\tilde{O}}'_l \cdot \underline{\underline{D}}^l(\alpha, \tfrac{\pi}{2}, \tfrac{\pi}{2})^{-1} \tag{4.26}$$

where

$$\underline{\tilde{O}}_l = (\tilde{O}_{l,l}, \tilde{O}_{l,l-1}, \ldots, \tilde{O}_{l,-l})$$

For $l = 1$ the expression gives

$$\underset{\text{(old)}}{[\tilde{O}_{11}, \tilde{O}_{10}, \tilde{O}_{1,-1}]} = \underset{\text{(new)}}{[\tilde{O}'_{1,1}, \tilde{O}'_{1,0}, \tilde{O}'_{1,-1}]} \begin{bmatrix} \frac{i}{2}e^{i\alpha} & \frac{i}{\sqrt{2}} & \frac{i}{2}e^{-i\alpha} \\ -\frac{1}{\sqrt{2}}e^{i\alpha} & 0 & \frac{1}{\sqrt{2}}e^{-i\alpha} \\ -\frac{i}{2}e^{i\alpha} & \frac{i}{\sqrt{2}} & -\frac{i}{2}e^{-i\alpha} \end{bmatrix} \tag{4.27}$$

In table 4 the matrices $\underline{\underline{D}}^l(\alpha, \frac{\pi}{2}, \frac{\pi}{2})^{-1}$ are calculated for $l = 1, 2, 3 \ldots, 8$.

5. STEVENS OPERATOR EQUIVALENTS, O_l^m

The operator *equivalents defined* by Stevens[2] are closely related to the *tensor operators* defined by Racah[7]. The Stevens operators do not have the systematic transformation properties of the tensor operators; however, they are convenient in hand calculations as a number of numerical factors are included in the definition.

The Stevens operators are in terms of the Racah operators defined as follows

$$O_l^m(c) = \frac{1}{K_l^m}\sqrt{\frac{2l+1}{4\pi}}\ \frac{1}{\sqrt{2}}\left(\tilde{O}_{l,-m} + (-1)^m\,\tilde{O}_{l,m}\right)$$

$$O_l^m(s) = \frac{1}{K_l^m}\sqrt{\frac{2l+1}{4\pi}}\ \frac{i}{\sqrt{2}}\left(\tilde{O}_{l,-m} - (-1)^m\,\tilde{O}_{l,m}\right) \qquad (5.1)$$

$$O_l^o(c) = \frac{1}{K_l^o}\sqrt{\frac{2l+1}{4\pi}}\ \tilde{O}_{l,o}$$

$$O_l^o(s) = 0$$

where K_l^m are the normalizing coefficients in the tesseral harmonics. These coefficients and $\frac{1}{K_l^m}\sqrt{\frac{2l+1}{4\pi}}$ are given in table 5 for l up to 8. In table 6 the Stevens operators for all even values of l up to 8 are given explicitly in terms of angular momentum operators. In table 7 the same Stevens operators are expanded in Bose operators.

We shall give an example of the method of finding the Stevens operators in terms of the angular-momentum operators using tables 5 and 6:

$$O_2^o(c) = \frac{1}{K_2^o}\sqrt{\frac{5}{4\pi}}\ \tilde{O}_{2,o} = 2\,\tilde{O}_{2,o} \doteq 3\,J_z^2 - J(J+1) \qquad (5.2)$$

$$O_2^2(c) = \frac{1}{K_2^2}\sqrt{\frac{5}{4\pi}}\,\frac{1}{\sqrt{2}}\left(\tilde{O}_{2,-2}+\tilde{O}_{2,2}\right) = \frac{2}{\sqrt{3}}\,\frac{1}{\sqrt{2}}\left(\tilde{O}_{2,-2}+\tilde{O}_{2,2}\right) = \frac{1}{2}\left((J^+)^2+(J^-)^2\right)$$

$$O_2^2(s) = \frac{1}{K_2^2}\sqrt{\frac{5}{4\pi}}\ \frac{i}{\sqrt{2}}\left(\tilde{O}_{2,-2}-\tilde{O}_{2,2}\right) = \frac{2}{\sqrt{3}}\,\frac{i}{\sqrt{2}}\left(\tilde{O}_{2,-2}-\tilde{O}_{2,2}\right) = -\frac{i}{2}\left((J^+)^2-(J^-)^2\right)$$

5.1. Transformation of the Stevens Operators under Rotation of the Frame of Coordinates

The transformation properties of the Stevens operators are found by using their relations to the Racah operators. Table 8 gives the transformations of the Stevens operators under the frequently occurring rotation of frame where the new z-axis is perpendicular to the old one and makes an angle α with the old x-axis. This transformation is generated by $D(\alpha, \frac{\pi}{2}, \frac{\pi}{2})^{-1}$ as described in section 4.

We shall give a few examples using tables 4 and 5

$O_2^o(c)$ - operator

$$\tilde{O}_{2,o} \rightarrow -\frac{1}{2}\tilde{O}_{2,o} - \frac{\sqrt{6}}{4}(\tilde{O}_{2,-2} + \tilde{O}_{2,2})$$

$$\frac{1}{2}O_2^o(c) \rightarrow -\frac{1}{2}\left(\frac{1}{2}O_2^o(c)\right) - \frac{\sqrt{6}}{4}\frac{\sqrt{6}}{2}O_2^2(c) \quad \text{or} \tag{5.3}$$

$$O_2^o(c) \rightarrow -\frac{1}{2}O_2^o(c) - \frac{3}{2}O_2^2(c)$$

$O_2^2(c)$ - operator

$$(\tilde{O}_{2,-2}+\tilde{O}_{2,2}) \rightarrow \left(\frac{\sqrt{6}}{4}O_{2,o} - \frac{1}{4}(\tilde{O}_{2,-2}+\tilde{O}_{2,2})\right)2\cos 2\alpha + \frac{1}{2}(\tilde{O}_{2,-1}-\tilde{O}_{2,1})2i\sin 2\alpha$$

$$\frac{\sqrt{6}}{2}O_2^2(c) \rightarrow \left(\frac{\sqrt{6}}{4}\frac{1}{2}O_2^o(c) - \frac{1}{4}\frac{\sqrt{6}}{4}O_2^2(c)\right)2\cos 2\alpha - \frac{\sqrt{3}\sqrt{2}}{2}O_2^1(c)2\sin 2\alpha$$

$$O_2^2(c) \rightarrow \left(\frac{1}{2}O_2^o(c) - \frac{1}{2}O_2^2(c)\right)\cos 2\alpha - 2\,O_2^1(c)\sin 2\alpha$$

6. CRYSTAL POTENTIAL ENERGY IN CUBIC AND HEXAGONAL SYMMETRY

An electron in a crystal is under the influence of the surrounding electric charges constituting the so-called crystal field. The crystal field is restricted by the crystal symmetry and is commonly expressed with respect to some principal or high-symmetry direction in the crystal. Using the operator equivalence method we find for the crystal potential energy from the crystal field potential either by means of Stevens operators

$$H_{cf} = \sum_{l,m} B_l^m O_l^m \tag{6.1}$$

or Racah operators

$$H_{cf} = \sum_{l,m} \tilde{B}_l^m \tilde{O}_{l,m} \tag{6.2}$$

The potential functions in (6.1) and (6.2) are phenomenological and quite general. The actual calculation of the parameters is very difficult, and one is forced to use simplifying models such as the effective point charge model or ligand field theory. In this section we shall tabulate the crystal potential energy for the cubic and the hexagonal symmetries in the principal directions in terms of both Stevens and Racah operators, using the tables for their rotation properties.

Cubic Symmetry

In fig. 6.1 the principal directions in the cubic symmetry are shown, namely the (001)-direction which is a 4-fold axis, the (110)-direction which is a 2-fold axis, and the (111)-direction which is a 3-fold axis. The crystal potential energy expressed with respect to the 4-fold axis is given by Hutchings[15] and by means of table 8 the expressions for the energy with respect to the (110)- and (111)-directions are calculated. The expressions are given in tables 9 and 10.

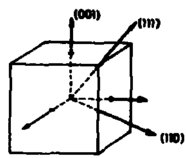


Fig. 6.1.
The principal directions of the cubic lattice

Hexagonal Symmetry

The crystal potential energy is in the hexagonal symmetry expressed with respect to the principal directions (0001), (1000), and (1200), see fig. 6.2. The (0001)-direction is a 6-fold axis, and the (1000)- and (1200)-directions are 2-fold axis. The expressions are given in tables 9 and 10.

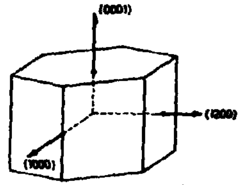


Fig. 5.2.
The principal directions of the hexagonal lattice.

7. DEFINITIONS AND RELATIONS FOR SPHERICAL HARMONIC FUNCTIONS AND RACAH OPERATORS

In this section we shall summarize the definitions and conventions for the spherical harmonics and the Racah operators.

7.1. Spherical Harmonics

The spherical harmonics are defined by Edmonds[1])

$$Y_{lm}(\theta,\varphi) = \frac{(-1)^{l+m}}{2^l\, l!}\left[\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}\right]^{1/2} (\sin\theta)^m \left[\frac{\partial}{\partial(\cos\theta)}\right]^{l+m} (\sin\theta)^{2l}\, e^{im\varphi} \tag{7.1}$$

$$= (-1)^m \left[\frac{(2l+1)}{4\pi}\,\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m\,(\cos\theta)e^{im\varphi}$$

where $P_l^m(\cos\theta)$ are the generalized (including negative-values of m) associated Legendre functions of the first kind. The spherical harmonics satisfy the ortho-normality relation

$$\int_0^{2} d\varphi \int_0^{\pi} d\theta\ [Y^*_{lm}(\theta,\varphi)\, Y_{l'm'}(\theta,\varphi)\,\sin\theta\,] = \delta_{ll'}\,\delta_{mm'} \tag{7.2}$$

and furthermore

$$Y_{l,-m}(\theta,\varphi) = (-1)^m \, Y^*_{l,m}(\theta,\varphi) \tag{7.3}$$

The product of two spherical harmonics which have the same arguments is given in terms of the 3j-symbols by

$$Y_{l_1 m_1}(\theta,\varphi)\, Y_{l_2 m_2}(\theta,\varphi) = \sum_{l,m} \left[\frac{(2l_1+1)(2l_2+1)(2l+1)}{4\pi}\right]^{1/2} \begin{pmatrix} l_1 & l_2 & l \\ m_1 & m_2 & m \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l \\ 0 & 0 & 0 \end{pmatrix} Y^*_{lm}(\theta,\varphi) \tag{7.4}$$

By use of (7.4) and the orthogonality properties of the 3j-symbols we obtain the reverse formula

$$Y^*_{lm}(\theta,\varphi) = (2l+1)^2 \sqrt{\frac{4\pi}{(2l_1+1)(2l_2+1)(2l+1)}} \sum_{m_1 m_2} \begin{pmatrix} l_1 & l_2 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l \\ m_1 & m_2 & m \end{pmatrix} Y_{l_1 m_1}(\theta,\varphi) Y_{l_2 m_2}(\theta,\varphi) \tag{7.5}$$

7.2. Racah Operators

Definition of the Operators

The Racah operators are irreducible tensor operators, which means that the set of $2l+1$ operators $\tilde{O}_{lm}$ ($m = l, l-1, l-2, \ldots, -l$) transforms under rotations of the frame of coordinates as $\sqrt{\frac{4\pi}{2l+1}}\, Y_{lm}(\theta,\varphi)$, namely

$$D(\alpha,\beta,\gamma)\, \tilde{O}_{l,m}\, D(\alpha,\beta,\gamma)^{-1} = \sum_{m'=-l}^{l} \tilde{O}_{lm'}\, D^{(l)}_{m'm}(\alpha,\beta,\gamma) \tag{7.6}$$

An Equivalent Definition

Since the operators of total angular momentum of the system are multiples of the infinitesimal rotation operators, we may replace the unitary transformation on the left by a commutator, giving for any component of angular momentum J_α

$$[J_\alpha, \tilde{O}_{lm}] = \sum_{m'=-l}^{l} \tilde{O}_{lm'} \langle lm' | J_\alpha | lm \rangle \tag{7.7}$$

In that way we find the equivalent definition of the irreducible tensor operators given by Racah[7], namely the commutator relations

$$[J^{\pm}, \tilde{O}_{lm}] = \sqrt{l(l+1) - m(m \pm 1)}\ \tilde{O}_{l,m\pm 1} \quad \text{and}$$

$$[J_z, \tilde{O}_{lm}] = m\, \tilde{O}_{lm} \tag{7.8}$$

The matrix element of a Racah operator is given by

$$\langle Jm | \tilde{O}_{k,q} | J'm' \rangle = (-1)^{J-m} \begin{pmatrix} J & k & J' \\ -m & q & m' \end{pmatrix} \langle J || \tilde{O}_k || J' \rangle \tag{7.9}$$

It should be noted that a tensor operator is characterized by its reduced matrix element, here $\langle J || \tilde{O}_k || J' \rangle$ for the Racah operators.

When dealing with the Racah operators one has to distinguish between operators that commute and operators that do not commute. If the operators are acting on different parts of the system, say spin and orbit, they commute. This can also be described by saying that the operators act on part i and j in the system. If the operators are acting on the same dynamical variable, part i in the system, the commutator relation is not in general zero.

In magnetic systems the interactions may be expressed as tensor products of Racah operators. General expressions for tensor products of Racah operators and matrix elements of tensor products of both commuting- and non-commuting Racah operators are therefore given.

7.2.1. Non-commuting Racah Operators

The product of two non-commuting Racah operators $\tilde{O}_{k_1q_1}(i)$ and $\tilde{O}_{k_2q_2}(i)$ is given by Judd[12] for tensor operators of order zero. This result is here generalized to tensor operators of order k.

$$\tilde{O}_{k_1q_1}(i)\,\tilde{O}_{k_2q_2}(i) =$$

$$\sum_{q_3=-k_3}^{k_3}\ \sum_{k_3=|k_1-k_2|}^{k_1+k_2} (-1)^{k_1+k_2+k_3}\,(2k_3+1) \begin{Bmatrix} k_1 & k_2 & k_3 \\ J & J & J \end{Bmatrix} \begin{pmatrix} k_1 & k_2 & k_3 \\ q_1 & q_2 & q_3 \end{pmatrix} \times$$

$$\frac{\langle J\,||\,\tilde{O}_{k_1}(i)\,||\,J\rangle\,\langle J\,||\,\tilde{O}_{k_2}(i)\,||\,J\rangle}{\langle J\,||\,\tilde{O}_{k_3}\,||\,J\rangle}\ \tilde{O}^{+}_{k_3q_3}(i) \qquad (7.10)$$

Using the symmetry relation for 3j-symbols, namely

$$\begin{pmatrix} k_1 & k_2 & k_3 \\ q_1 & q_2 & q_3 \end{pmatrix} = (-1)^{k_1+k_2+k_3} \begin{pmatrix} k_2 & k_1 & k_3 \\ q_2 & q_1 & q_3 \end{pmatrix}$$

we find the commutator relation

$$[\tilde{O}_{k_1q_1}(i),\ \tilde{O}_{k_2q_2}(i)] =$$

$$\sum_{q_3=-k_3}^{k_3}\ \sum_{k_3=|k_1-k_2|}^{k_1+k_2} \{(-1)^{k_1+k_2+k_3}-1\}\,(2k_3+1) \begin{Bmatrix} k_1 & k_2 & k_3 \\ J & J & J \end{Bmatrix} \begin{pmatrix} k_1 & k_2 & k_3 \\ q_1 & q_2 & q_3 \end{pmatrix} \times$$

$$\frac{\langle J\,||\,\tilde{O}_{k_1}(i)\,||\,J\rangle\,\langle J\,||\,\tilde{O}_{k_2}(i)\,||\,J\rangle}{\langle J\,||\,\tilde{O}_{k_3}(i)\,||\,J\rangle}\ \tilde{O}^{+}_{k_3q_3}(i) \qquad (7.11)$$

The reduced matrix element is

$$\langle J || \tilde{O}_k || J \rangle = \frac{1}{2^k} \sqrt{\frac{(2J+k+1)!}{(2J-k)!}} \tag{7.12}$$

It should be pointed out that the commutator relation does not depend on the magnitude of J. This can be seen directly by using the angular momentum expressions for the $\tilde{O}_{lm}$ operators (table 1) and the commutator relations for J_x, J_y, J_z.

The tensor product of two non-commuting Racah operators is defined by

$$\left(\tilde{O}^{(k_1)} \tilde{O}^{(k_2)}\right)^{(K)}_Q =$$

$$\sum_{q_1=-k_1}^{k_1} \sum_{q_2=-k_2}^{k_2} (1)^{-k_1+k_2-Q} \sqrt{2K+1} \begin{pmatrix} k_1 & k_2 & K \\ q_1 & q_2 & -Q \end{pmatrix} \tilde{O}_{k_1 q_1}(i)\, \tilde{O}_{k_2 q_2}(i) \tag{7.13}$$

and for the scalar product of two non-commuting Racah operators we have

$$\left(\tilde{O}_i^{(K)} \cdot \tilde{O}_i^{(K)}\right) = (-1)^K \sqrt{2K+1} \left(\tilde{O}^{(K)} \tilde{O}^{(K)}\right)^{(0)}_0 \tag{7.14}$$

The matrix element of the tensor product of two non-commuting Racah operators is

$$\langle Jm | \left(\tilde{O}^{(k_1)} \tilde{O}^{(k_2)}\right)^{(K)}_Q | Jm' \rangle$$

$$= (-1)^{J-m} \begin{pmatrix} J & K & J \\ -m & Q & m' \end{pmatrix} (-1)^K \sqrt{2K+1} \begin{Bmatrix} k_1 & k_2 & K \\ J & J & J \end{Bmatrix} \langle J || \tilde{O}_{k_1}(i) || J \rangle \langle J || \tilde{O}_{k_2}(i) || J \rangle$$

$$= (-1)^{J-m} \begin{pmatrix} J & K & J \\ -m & Q & m' \end{pmatrix} \langle J || \left(\tilde{O}^{(k_1)} \tilde{O}^{(k_2)}\right)^{(K)}_Q || J \rangle \tag{7.15}$$

from which we have for the reduced matrix element

$$\langle J \| \left(\tilde{O}^{(k_1)}\tilde{O}^{(k_2)}\right)^{(K)}_{Q} \| J \rangle = (-1)^{K}\sqrt{2K+1} \begin{Bmatrix} k_1 & k_2 & K \\ J & J & J \end{Bmatrix} \langle J \| \tilde{O}_{k_1}(i) \| J \rangle \langle J \| \tilde{O}_{k_2}(i) \| J \rangle \tag{7.16}$$

7.2.2. Commuting Racah Operators

As the two operators $\tilde{O}_{k_1 q_1}(i)$ and $\tilde{O}_{k_2 q_2}(j)$ commute, we immediately have

$$[\tilde{O}_{k_1 q_1}(i), \tilde{O}_{k_2 q_2}(j)] = 0 \tag{7.17}$$

The tensor product of two commuting Racah operators is defined by

$$\left\{\tilde{O}^{(k_1)}\tilde{O}^{(k_2)}\right\}^{(K)}_{Q} = \sum_{q_1=k_1}^{k_1} \sum_{q_2=-k_2}^{k_2} (-1)^{-k_1+k_2-Q}\sqrt{2K+1} \begin{pmatrix} k_1 & k_2 & K \\ q_1 & q_2 & -Q \end{pmatrix} \tilde{O}_{k_1 q_1}(i)\tilde{O}_{k_2 q_2}(j) \tag{7.18}$$

and for the scalar product of two commuting Racah operators we have

$$\left(\tilde{O}_i^{(K)} \cdot \tilde{O}_j^{(K)}\right) = (-1)^{K}\sqrt{2K+1} \left\{\tilde{O}^{(K)}\tilde{O}^{(K)}\right\}^{(0)}_{0} \tag{7.19}$$

The matrix element of the tensor product of two commuting Racah operators is

$$\langle j_1 j_2 J m | \left\{\tilde{O}^{(k_1)}\tilde{O}^{(k_2)}\right\}^{(K)}_{Q} | j_1' j_2' J' m' \rangle$$

$$= (-1)^{J-m} \begin{pmatrix} J & K & J' \\ -m & Q & m' \end{pmatrix} \sqrt{(2J+1)(2J'+1)(2K+1)} \begin{Bmatrix} j_1 & j_2 & J \\ j_1' & j_2' & J' \\ k_1 & k_2 & K \end{Bmatrix} \langle j_1 \| \tilde{O}_{k_1}(i) \| j_1' \rangle \langle j_2 \| \tilde{O}_{k_2}(j) \| j_2' \rangle$$

$$= (-1)^{J-m} \begin{pmatrix} J & K & J' \\ -m & Q & m' \end{pmatrix} \langle j_1 j_2 J \| \left\{\tilde{O}^{(k_1)}\tilde{O}^{(k_2)}\right\}^{(K)}_{Q} \| j_1' j_2' J' \rangle \tag{7.20}$$

So the reduced matrix element is

$$\langle j_1 j_2 J \| \left\{\tilde{O}^{(k_1)}\,\tilde{O}^{(k_2)}\right\}^{(K)}_{Q} \| j_1' j_2' J' \rangle \quad =$$

(7.21)

$$= \sqrt{(2J+1)(2J'+1)(2K+1)} \begin{Bmatrix} j_1 & j_2 & J \\ j_1' & j_2' & J' \\ k_1 & k_2 & K \end{Bmatrix} \langle j_1 \| \tilde{O}_{k_1}(i) \| j_1' \rangle \langle j_2 \| \tilde{O}_{k_2}(j) \| j_2' \rangle$$

7.3. 3j- and 6j-Symbols

The 3j- and 6j-symbols occur in many of the preceding expressions. Their definitions and symmetry properties are therefore given following Rothenberg et al.[13], whose definition is equivalent to that of Edmonds[1].

The 3j-symbol is defined as

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = (-1)^{j_1 - j_2 - m_3}$$

$$\sqrt{\frac{(j_1+j_2-j_3)!\,(j_1-j_2+j_3)!\,(-j_1+j_2+j_3)!\,(j_1+m_1)!\,(j_1-m_1)!\,(j_2+m_2)!\,(j_2-m_2)!\,(j_3+m_3)!\,(j_3-m_3)!}{(j_1+j_2+j_3+1)!}}$$

$$\times \sum_k \frac{(-1)^k}{k!\,(j_1+j_2-j_3-k)!\,(j_1-m_1-k)!\,(j_2+m_2-k)!\,(j_3-j_2+m_1+k)!\,(j_3-j_1-m_2+k)!}$$

(7.22)

Under an <u>even</u> permutation of the columns the symmetry properties of the 3j-symbols are given by

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = \begin{pmatrix} j_2 & j_3 & j_1 \\ m_2 & m_3 & m_1 \end{pmatrix} = \begin{pmatrix} j_3 & j_1 & j_2 \\ m_3 & m_1 & m_2 \end{pmatrix} \tag{7.23}$$

and under an <u>odd</u> permutation by

$$(-1)^{j_1+j_2+j_3}\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = \begin{pmatrix} j_2 & j_1 & j_3 \\ m_2 & m_1 & m_3 \end{pmatrix} = \begin{pmatrix} j_1 & j_3 & j_2 \\ m_1 & m_3 & m_2 \end{pmatrix} = \begin{pmatrix} j_3 & j_2 & j_1 \\ m_3 & m_2 & m_1 \end{pmatrix} \tag{7.24}$$

When changing the signs of the m's we have

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ -m_1 & -m_2 & -m_3 \end{pmatrix} = (-1)^{j_1+j_2+j_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \tag{7.25}$$

The 3j-symbol automatically equals zero unless both $m_1 + m_2 + m_3 = 0$ and j_1, j_2, and j_3 satisfy the triangle conditions

$$j_1 + j_2 - j_3 \gtreqless 0; \quad j_1 - j_2 + j_3 \gtreqless 0; \quad -j_1 + j_2 + j_3 \gtreqless 0 \tag{7.26}$$

Besides the sum $j_1 + j_2 + j_3$ must be an integer.

The 6j-symbol is defined as

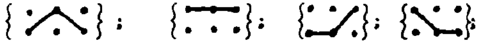
$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ l_1 & l_2 & l_3 \end{Bmatrix} = (-1)^{j_1+j_2+l_1+l_2} \Delta(j_1 j_2 j_3)\Delta(l_1 l_2 j_3)\Delta(l_1 j_2 l_3)\Delta(j_1 l_2 l_3) \times$$

$$\sum_k \frac{(-1)^k (j_1+j_2+l_1+l_2+1-k)!}{k!(j_1+j_2-j_3-k)!(l_1+l_2-j_3-k)!(j_1+l_2-l_3-k)!(l_1+j_2-l_3-k)!(-j_1-l_1+l_3+k)!(-j_2-l_2+j_3+l_3+k)!}$$

where

$$\Delta(a\,b\,c) = \left[\frac{(a+b-c)!\ (a-b+c)!\ (-a+b+c)!}{(a+b+c+1)!} \right]^{1/2} \tag{7.27}$$

The 6j-symbol stands invariant under interchange of columns. It is also invariant at interchange of any two numbers in the bottom row with the corresponding two numbers in the top row. The 6j-symbol is automatically zero unless each of the four triads (j_1, j_2, j_3), (l_1, l_2, j_3), (j_1, l_2, l_3), and (l_1, j_2, l_3) satisfy the triangle conditions (7.26). The elements of each triad also must sum to an integer. The four triangle conditions for the non-vanishing of the 6j-symbol might be given in a diagram form easier to remember



(7.28)

In tables 11 and 12 the 3j- and 6j-symbols are calculated by means of the Borroughs 6700 computer for all integers up to 6. A more extensive table including all integers and half-integers up to 8 has been given by Rothenberg et al.[13].

REFERENCES

1) A. R. Edmonds, Angular Momentum in Quantum Mechanics (Princeton University Press, Princeton, 1957). 146 pp.

2) K. W. H. Stevens, Proc. Phys. Soc. A65 (1952) 209-215.

3) T. Holstein and H. Primakoff, Phys. Rev. 58 (1940) 1098-1113.

4) G. T. Trammell, J. Appl. Phys. 31 (1960) 362S-363S.
G. T. Trammell, Phys. Rev. 131 (1963) 932-948.
B. Grover, Phys. Rev. A140 (1965) 1944-1951.

5) R. J. Birgeneau, Can. J. Phys. 45 (1967) 3761-3771.

6) H. A. Buckmaster, Can. J. Phys. 40 (1962) 1670-1676.

7) G. Racah, Phys. Rev. 62 (1942) 438-462.

8) E. Jahnke and F. Emde, Tables of Functions with Formulae and Curves (Dover, New York, 1945). 380 pp.

9) D. A. Goodings and B. W. Southern, Can. J. Phys. 49 (1971) 1137-1161.

10) M. E. Rose, Elementary Theory of Angular Momentum (Wiley, New York, 1957). 248 pp.

11) M. Tinkham, Group Theory and Quantum Mechanics (Mc-Graw-Hill, New York, 1964). 340 pp.

12) B. R. Judd, Operator Techniques in Atomic Spectroscopy (Mc-Graw-Hill, New York, 1963). 242 pp.

13) M. Rothenberg, R. Bivins, N. Metropolis and J. K. Wooten, The 3-j and 6-j-symbols (Massachusetts Institute of Technology, Cambridge, Mass., 1959). 498 pp.

14) A. Messiah, Quantum Mechanics 2 (North Holland Publishing Co. Amsterdam, 1962).

15) M. T. Hutchings, Sol. State Phys. 16 (1964) 227-273.

Table 1

Racah operator equivalents

$X = J(J+1)$

$$\tilde{O}_{0,0} = 1$$

$$\tilde{O}_{1,0} = J_z$$

$$\tilde{O}_{1,\pm 1} = \mp\sqrt{\tfrac{1}{2}}\,J^{\pm}$$

$$\tilde{O}_{2,0} = \tfrac{1}{2}\left[3J_z^2 - X\right]$$

$$\tilde{O}_{2,\pm 1} = \mp\sqrt{\tfrac{3}{2}}\,\tfrac{1}{2}\left[J_z J^{\pm} + J^{\pm} J_z\right]$$

$$\tilde{O}_{2,\pm 2} = \sqrt{\tfrac{3}{8}}\,(J^{\pm})^2$$

$$\tilde{O}_{3,0} = \tfrac{1}{2}\left[5J_z^3 - \{3X - 1\}J_z\right]$$

$$\tilde{O}_{3,\pm 1} = \mp\sqrt{\tfrac{3}{16}}\,\tfrac{1}{2}\left[\{5J_z^2 - X - \tfrac{1}{2}\}J^{\pm} + J^{\pm}\{\cdots\}\right]$$

$$\tilde{O}_{3,\pm 2} = \sqrt{\tfrac{15}{8}}\,\tfrac{1}{2}\left[J_z(J^{\pm})^2 + (J^{\pm})^2 J_z\right]$$

$$\tilde{O}_{3,\pm 3} = \mp\sqrt{\tfrac{5}{16}}\,(J^{\pm})^3$$

$$\tilde{O}_{4,0} = \tfrac{1}{8}\left[35J_z^4 - \{30X - 25\}J_z^2 + 3X^2 - 6X\right]$$

$$\tilde{O}_{4,\pm 1} = \mp\sqrt{\tfrac{5}{16}}\,\tfrac{1}{2}\left[\{7J_z^3 - (3X+1)J_z\}J^{\pm} + J^{\pm}\{\cdots\}\right]$$

$$\tilde{O}_{4,\pm 2} = \sqrt{\tfrac{5}{32}}\,\tfrac{1}{2}\left[\{7J_z^2 - X - 5\}(J^{\pm})^2 + (J^{\pm})^2\{\cdots\}\right]$$

$$\tilde{O}_{4,\pm 3} = \mp\sqrt{\tfrac{35}{16}}\,\tfrac{1}{2}\left[J_z(J^{\pm})^3 + (J^{\pm})^3 J_z\right]$$

$$\tilde{O}_{4,\pm 4} = \sqrt{\tfrac{35}{128}}\,(J^{\pm})^4$$

$$\tilde{O}_{5,0} = \tfrac{1}{8}\left[63J_z^5 - \{70X - 105\}J_z^3 + \{15X^2 - 50X + 12\}J_z\right]$$

$$\tilde{O}_{5,\pm 1} = \mp\sqrt{\tfrac{15}{128}}\,\tfrac{1}{2}\left[\{21J_z^4 - 14XJ_z^2 + X^2 - X + \tfrac{3}{2}\}J^{\pm} + J^{\pm}\{\cdots\}\right]$$

$$\tilde{O}_{5,\pm 2} = \sqrt{\tfrac{105}{32}}\,\tfrac{1}{2}\left[\{3J_z^3 - (X+6)J_z\}(J^{\pm})^2 + (J^{\pm})^2\{\cdots\}\right]$$

$$\tilde{O}_{5,\pm 3} = \mp\sqrt{\tfrac{35}{256}}\,\tfrac{1}{2}\left[\{9J_z^2 - X - \tfrac{33}{2}\}(J^{\pm})^3 + (J^{\pm})^3\{\cdots\}\right]$$

$$\tilde{O}_{5,\pm 4} = \sqrt{\tfrac{315}{128}}\,\tfrac{1}{2}\left[J_z(J^{\pm})^4 + (J^{\pm})^4 J_z\right]$$

$$\tilde{O}_{5,\pm 5} = \mp\sqrt{\tfrac{63}{256}}\,(J^{\pm})^5$$

$$\tilde{O}_{6,0} = \frac{1}{16}\left[231\, J_z^6 - \{315\, X - 735\}J_z^4 + \{105\, X^2 - 525\, X + 294\}J_z^2 - 5\, X^3 + 40\, X^2 - 60\, X\right]$$

$$\tilde{O}_{6,\pm 1} = \mp\sqrt{\frac{21}{128}}\,\frac{1}{2}\left[\{33\, J_z^5 - (30\, X - 15)J_z^3 + (5\, X^2 - 10\, X + 12)J_z\}J^{\pm} + J^{\pm}\{\cdots\}\right]$$

$$\tilde{O}_{6,\pm 2} = \sqrt{\frac{105}{1024}}\,\frac{1}{2}\left[\{33\, J_z^4 - (18\, X + 123)J_z^2 + X^2 + 10\, X + 102\}(J^{\pm})^2 + (J^{\pm})^2\{\cdots\}\right]$$

$$\tilde{O}_{6,\pm 3} = \mp\sqrt{\frac{105}{256}}\,\frac{1}{2}\left[\{11\, J_z^3 - (3\, X + 59)J_z\}(J^{\pm})^3 + (J^{\pm})^3\{\cdots\}\right]$$

$$\tilde{O}_{6,\pm 4} = \sqrt{\frac{63}{512}}\,\frac{1}{2}\left[\{11\, J_z^2 - X - 38\}(J^{\pm})^4 + (J^{\pm})^4\{\cdots\}\right]$$

$$\tilde{O}_{6,\pm 5} = \mp\sqrt{\frac{693}{256}}\,\frac{1}{2}\left[J_z(J^{\pm})^5 + (J^{\pm})^5 J_z\right]$$

$$\tilde{O}_{6,\pm 6} = \sqrt{\frac{231}{1024}}\,(J^{\pm})^6$$

$$\tilde{O}_{7,0} = \frac{1}{16}\left[429\, J_z^7 - \{693\, X - 2310\}J_z^5 + \{315\, X^2 - 2205\, X - 2121\}J_z^3 - \{35\, X^3 - 385\, X^2 + 882\, X - 180\}J_z\right]$$

$$\tilde{O}_{7,\pm 1} = \mp\sqrt{\frac{7}{32\,768}}\,\frac{1}{2}\left[\{1716\, J_z^6 - (1980\, X - 2310)J_z^4 + (540\, X^2 - 1800\, X + 2184)J_z^2 - (20\, X^3 - 130\, X^2 + 270\, X + 225)\}J^{\pm} + J^{\pm}\{\cdots\right.$$

$$\tilde{O}_{7,\pm 2} = \sqrt{\frac{21}{1024}}\,\frac{1}{2}\left[\{143\, J_z^5 - (110\, X + 825)J_z^3 + (15\, X^2 + 170\, X + 2092)J_z\}(J^{\pm})^2 + (J^{\pm})^2\{\cdots\}\right]$$

$$\tilde{O}_{7,\pm 3} = \mp\sqrt{\frac{21}{8192}}\,\frac{1}{2}\left[\{286\, J_z^4 - (132\, X + 2992)J_z^2 + 6\, X^2 + 222\, X + 5481\}(J^{\pm})^3 + (J^{\pm})^3\{\cdots\}\right]$$

$$\tilde{O}_{7,\pm 4} = \sqrt{\frac{231}{512}}\,\frac{1}{2}\left[\{13\, J_z^3 - (3\, X + 133)J_z\}(J^{\pm})^4 + (J^{\pm})^4\{\cdots\}\right]$$

$$\tilde{O}_{7,\pm 5} = \mp\sqrt{\frac{231}{2048}}\,\frac{1}{2}\left[\{13\, J_z^2 - X - \frac{145}{2}\}(J^{\pm})^5 + (J^{\pm})^5\{\cdots\}\right]$$

$$\tilde{O}_{7,\pm 6} = \sqrt{\frac{9003}{1024}}\,\frac{1}{2}\left[J_z(J^{\pm})^6 + (J^{\pm})^6 J_z\right]$$

$$\tilde{O}_{7,\pm 7} = \mp\sqrt{\frac{429}{2048}}\,(J^{\pm})^7$$

$$\tilde{O}_{8,0} = \frac{1}{128}\left[6435\, J_z^8 - \{12012\, X - 54054\}J_z^6 + \{6930\, X^2 - 64680\, X + 93555\}J_z^4 - \right.$$
$$\left.\{1260\, X^3 - 18270\, X^2 + 59388\, X - 21390\}J_z^2 + 35\, X^4 - 700\, X^3 + 3780\, X^2 - 5040\, X\right]$$

$$\tilde{O}_{8,\pm 1} = \mp\sqrt{\frac{9}{2048}}\,\frac{1}{2}\left[\{715\, J_z^7 - (1001\, X - 2002)J_z^5 + (385\, X^2 - 1925\, X + 2695)J_z^3 - (35\, X^3 - 315\, X^2 + 854\, X + 372)J_z\}(J^{\pm}) + (J^{\pm})\{\cdots\}\right]$$

$$\tilde{O}_{8,\pm 2} = \sqrt{\frac{315}{4096}}\,\frac{1}{2}\left[\{143\, J_z^6 - (143\, X + 1144)J_z^4 + (33\, X^2 + 407\, X + 5951)J_z^2 - (X^3 + 13\, X^2 + 372\, X + 4806)\}(J^{\pm})^2 + (J^{\pm})^2\{\cdots\}\right]$$

$$\tilde{O}_{8,\pm 3} = \mp\sqrt{\frac{231}{2048}}\,\frac{1}{2}\left[\{39\, J_z^5 - (26\, X + 663)J_z^3 + (3\, X^2 + 128\, X + 3648)J_z\}(J^{\pm})^3 + (J^{\pm})^3\{\cdots\}\right]$$

$$\tilde{O}_{8,\pm 4} = \sqrt{\frac{693}{8192}}\,\frac{1}{2}\left[\{65\, J_z^4 - (26\, X + 1313)J_z^2 + X^2 + 86\, X + 4284\}(J^{\pm})^4 + (J^{\pm})^4\{\cdots\}\right]$$

$$\tilde{O}_{8,\pm 5} = \mp\sqrt{\frac{9009}{2048}}\,\frac{1}{2}\left[\{5\, J_z^3 - (X + 63)J_z\}(J^{\pm})^5 + (J^{\pm})^5\{\cdots\}\right]$$

$$\tilde{O}_{8,\pm 6} = \sqrt{\frac{429}{4096}}\,\frac{1}{2}\left[\{15\, J_z^2 - X - 123\}(J^{\pm})^6 + (J^{\pm})^6\{\cdots\}\right]$$

$$\tilde{O}_{8,\pm 7} = \mp\sqrt{\frac{6435}{2048}}\,\frac{1}{2}\left[J_z(J^{\pm})^7 + (J^{\pm})^7 J_z\right]$$

$$\tilde{O}_{8,\pm 8} = \sqrt{\frac{6435}{32\,768}}\,(J^{\pm})^8$$

Table 2

Racah Operator Equivalents expanded in Bose operators

$$S_n = J(J-\tfrac{1}{2})(J-1)\cdots(J-\tfrac{n-1}{2})$$

$$\tilde{O}_{00} = 1$$

$$\tilde{O}_{10} = S_1 \Big[1 - \frac{1}{S_1}\, a^+a \Big]$$

$$\tilde{O}_{11} = -\sqrt{S_1}\, \Big[a - \frac{1}{\sqrt{S_2}} \Big[\sqrt{S_2} - \frac{S_2}{S_1} \Big] a^+aa + \Big[\frac{1}{2}\, (1 + \sqrt{\frac{S_3}{S_1 S_2}}) - \frac{\sqrt{S_2}}{S_1} \Big] a^+a^+aaa + \cdots \Big]$$

$$\tilde{O}_{20} = S_2 \Big[1 - \frac{3}{S_1}\, a^+a + \frac{3}{2S_2}\, a^+a^+aa \Big]$$

$$\tilde{O}_{21} = -\sqrt{3}\, \frac{S_2}{\sqrt{S_1}} \Big[a - \frac{1}{\sqrt{S_2}} \Big[1 + \sqrt{S_2} - \frac{S_2}{S_1} \Big] a^+aa + \Big[\frac{1}{\sqrt{S_2}}\, (1 - \frac{\sqrt{S_1 S_3}}{S_2}) + \frac{1}{2}\, (1 + \sqrt{\frac{S_3}{S_1 S_2}}) - \frac{\sqrt{S_2}}{S_1} \Big] a^+a^+aaa + \cdots \Big]$$

$$\tilde{O}_{22} = \frac{\sqrt{6}}{2}\, \sqrt{S_2} \Big[aa - \sqrt{\frac{S_2}{S_1 S_3}}\, \Big[\sqrt{\frac{S_1 S_3}{S_2}} - \frac{S_3}{S_2} \Big] a^+aaa + \cdots \Big]$$

$$\tilde{O}_{30} = S_3 \Big[1 - \frac{6}{S_1}\, a^+a + \frac{15}{2S_2}\, a^+a^+aa + \cdots \Big]$$

$$\tilde{O}_{31} = -\sqrt{6}\, \frac{S_3}{\sqrt{S_1}} \Big[a - \frac{1}{\sqrt{S_2}} \Big[\frac{5}{2} + \sqrt{S_2} - \frac{S_2}{S_1} \Big] a^+aa + \Big[\frac{5}{2} \Big[\frac{1}{2}\, (\frac{S_1}{S_2}) + \frac{1}{\sqrt{S_2}}\, (1 - \frac{\sqrt{S_1 S_3}}{S_2}) \Big] + \frac{1}{2}\, (1 + \sqrt{\frac{S_3}{S_1 S_2}}) - \frac{\sqrt{S_2}}{S_1} \Big]\, a^+a^+aaa + \cdots \Big]$$

$$\tilde{O}_{32} = \frac{\sqrt{30}}{2} \frac{S_3}{\sqrt{S_2}} \Big[aa - \sqrt{\frac{S_2}{S_1 S_3}}\, \Big[1 + \sqrt{\frac{S_1 S_3}{S_2}} - \frac{S_3}{S_2} \Big] a^+aaa + \cdots \Big]$$

$$\tilde{O}_{33} = -\frac{\sqrt{10}}{2}\, \sqrt{S_3}\, \Big[aaa - \sqrt{\frac{S_3}{S_1 S_4}}\, \Big[\sqrt{\frac{S_1 S_4}{S_3}} - \frac{S_4}{S_3} \Big] a^+aaaa + \cdots \Big]$$

$$\tilde{O}_{40} = S_4 \Big[1 - \frac{10}{S_1}\, a^+a + \frac{45}{2S_2}\, a^+a^+aa + \cdots \Big]$$

$$\tilde{O}_{41} = -\sqrt{10}\, \frac{S_4}{\sqrt{S_1}} \Big[a - \frac{1}{\sqrt{S_2}} \Big[\frac{9}{2} + \sqrt{S_2} - \frac{S_2}{S_1} \Big] a^+aa + \Big[\frac{9}{2} \Big[\frac{7}{6} \cdot (\frac{S_1}{S_2}) + \frac{1}{\sqrt{S_2}}\, (1 - \frac{\sqrt{S_1 S_3}}{S_2}) \Big] + \frac{1}{2}\, (1 + \sqrt{\frac{S_3}{S_1 S_2}}) - \frac{\sqrt{S_2}}{S_1} \Big]\, a^+a^+aaa + \cdots)$$

$$\tilde{O}_{42} = \frac{3\sqrt{10}}{2}\, \frac{S_4}{\sqrt{S_2}} \Big[aa - \sqrt{\frac{S_2}{S_1 S_3}}\, \Big[\frac{7}{3} + \sqrt{\frac{S_1 S_3}{S_2}} - \frac{S_3}{S_2} \Big] a^+aaa + \cdots \Big]$$

$$\tilde{O}_{43} = -\frac{\sqrt{70}}{2}\, \frac{S_4}{\sqrt{S_3}} \Big[aaa - \sqrt{\frac{S_3}{S_1 S_4}}\, \Big[1 + \sqrt{\frac{S_1 S_4}{S_3}} - \frac{S_4}{S_3} \Big]\, a^+aaaa + \cdots \Big]$$

$$\tilde{O}_{44} = \frac{\sqrt{70}}{4}\, \sqrt{S_4}\, aaaa + \cdots\cdot$$

$$\tilde{O}_{50} = S_5 \left[1 - \frac{15}{S_1} a^+a + \frac{105}{2S_2} a^+a^+aa + \cdots \right]$$

$$\tilde{O}_{51} = -\sqrt{15}\, \frac{S_5}{\sqrt{S_1}} \left[a - \frac{1}{\sqrt{S_2}} \left[7 + \sqrt{S_2} - \frac{S_2}{S_1} \right] a^+aa + \left[7 \left[2 \cdot \left(\frac{S_1}{S_2}\right) + \frac{1}{\sqrt{S_2}} \left(1 - \frac{\sqrt{S_1S_3}}{S_2}\right) \right] + \frac{1}{2} \left(1 + \sqrt{\frac{S_3}{S_1S_2}}\right) - \frac{\sqrt{S_2}}{S_1} \right] a^+a^+aaa + \cdots \right]$$

$$\tilde{O}_{52} = \frac{\sqrt{210}}{2} \frac{S_5}{\sqrt{S_2}} \left[aa - \sqrt{\frac{S_2}{S_1S_3}} \left[4 + \sqrt{\frac{S_1S_3}{S_2}} - \frac{S_3}{S_2} \right] a^+aaa + \cdots \right]$$

$$\tilde{O}_{53} = -\sqrt{70}\, \frac{S_5}{\sqrt{S_3}} \left[aaa - \sqrt{\frac{S_3}{S_1S_4}} \left[\frac{9}{4} + \sqrt{\frac{S_1S_4}{S_3}} - \frac{S_4}{S_3} \right] a^+aaaa + \cdots \right]$$

$$\tilde{O}_{54} = \frac{3}{4} \sqrt{70}\, \frac{S_5}{\sqrt{S_4}}\, aaaa + \cdots$$

$$\tilde{O}_{55} = -\frac{3}{4} \sqrt{14}\, \sqrt{S_5}\, aaaaa + \cdots$$

$$\tilde{O}_{60} = S_6 \left[1 - \frac{21}{S_1} a^+a + \frac{105}{S_2} a^+a^+aa + \cdots \right]$$

$$\tilde{O}_{61} = -\sqrt{21}\, \frac{S_6}{\sqrt{S_1}} \left[a - \frac{1}{\sqrt{S_2}} \left[10 + \sqrt{S_2} - \frac{S_2}{S_1} \right] a^+aa + \left[10 \left[3 \cdot \left(\frac{S_1}{S_2}\right) + \frac{1}{\sqrt{S_2}} \left(1 - \frac{\sqrt{S_1S_3}}{S_2}\right) \right] + \frac{1}{2} \left(1 + \sqrt{\frac{S_3}{S_1S_2}}\right) - \frac{\sqrt{S_2}}{S_1} \right] a^+a^+aaa + \cdots \right]$$

$$\tilde{O}_{62} = \sqrt{105}\, \frac{S_6}{\sqrt{S_2}} \left[aa - \sqrt{\frac{S_2}{S_1S_3}} \left[6 + \sqrt{\frac{S_1S_3}{S_2}} - \frac{S_3}{S_2} \right] a^+aaa + \cdots \right]$$

$$\tilde{O}_{63} = -\sqrt{210}\, \frac{S_6}{\sqrt{S_3}} \left[aaa - \sqrt{\frac{S_3}{S_1S_4}} \left[\frac{15}{4} + \sqrt{\frac{S_1S_4}{S_3}} - \frac{S_4}{S_3} \right] a^+aaaa + \cdots \right]$$

$$\tilde{O}_{64} = \frac{15}{4} \sqrt{14}\, \frac{S_6}{\sqrt{S_4}}\, aaaa + \cdots$$

$$\tilde{O}_{65} = -\frac{3}{4} \sqrt{154}\, \frac{S_6}{\sqrt{S_5}}\, aaaaa + \cdots$$

$$\tilde{O}_{66} = \cdots$$

$$\tilde{O}_{70} = S_7 \left[1 - \frac{28}{S_1} a^+a + \frac{189}{S_2} a^+a^+aa + \cdots \right]$$

$$\tilde{O}_{71} = -\sqrt{28}\, \frac{S_7}{\sqrt{S_1}} \left[a - \frac{1}{\sqrt{S_2}} \left[\frac{27}{2} + \sqrt{S_2} - \frac{S_2}{S_1} \right] a^+aa + \left[\frac{27}{2} \left[\frac{25}{6} \left(\frac{S_1}{S_2}\right) + \frac{1}{\sqrt{S_2}} \left(1 - \frac{\sqrt{S_1S_3}}{S_2}\right) \right] + \frac{1}{2} \left(1 + \sqrt{\frac{S_3}{S_1S_2}}\right) - \frac{\sqrt{S_2}}{S_1} \right] a^+a^+aaa + \cdots \right]$$

$$\tilde{O}_{72} = 3\sqrt{21}\, \frac{S_7}{\sqrt{S_2}} \left[aa - \sqrt{\frac{S_2}{S_1S_3}} \left[\frac{25}{3} + \sqrt{\frac{S_1S_3}{S_2}} - \frac{S_3}{S_2} \right] a^+aaa + \cdots \right]$$

$$\tilde{O}_{73} = -5\sqrt{21}\, \frac{S_7}{\sqrt{S_3}} \left[aaa - \sqrt{\frac{S_3}{S_1S_4}} \left[\frac{11}{2} + \sqrt{\frac{S_1S_4}{S_3}} - \frac{S_4}{S_3} \right] a^+aaaa + \cdots \right]$$

$$\tilde{O}_{74} = \frac{5}{4} \sqrt{462}\, \frac{S_7}{\sqrt{S_4}}\, aaaa + \cdots$$

$$\tilde{O}_{75} = -\frac{3}{2} \sqrt{231}\, \frac{S_7}{\sqrt{S_5}}\, aaaaa + \cdots$$

$$\tilde{O}_{76} = \cdots$$

$$\tilde{O}_{77} = \cdots$$

$$\tilde{O}_{80} = S_8 \left[1 - \frac{36}{S_1} a^+ a + \frac{315}{S_2} a^+ a^+ aa + \cdots \right]$$

$$\tilde{O}_{81} = - \sqrt{36}\ \frac{S_8}{\sqrt{S_1}} \left[a - \left[\frac{35}{2} + \sqrt{S_2} - \frac{S_2}{S_1} \right] a^+ aa + \left| \frac{35}{2} \left[\frac{11}{2} \left(\frac{S_1}{S_2}\right) + \frac{1}{\sqrt{S_2}} \left(1 - \frac{\sqrt{S_1 S_3}}{S_2}\right) \right] + \frac{1}{2} \left(1 + \sqrt{\frac{S_3}{S_1 S_2}}\right) - \frac{\sqrt{S_2}}{S_1} \right| a^+ a^+ aaa + \cdots \right].$$

$$\tilde{O}_{82} = 3\sqrt{35}\, \frac{S_8}{\sqrt{S_2}} \left[aa - \sqrt{\frac{S_2}{S_1 S_3}} \left[11 + \sqrt{\frac{S_1 S_3}{S_2}} - \frac{S_3}{S_2} \right] a^+ aaa + \cdots \right]$$

$$\tilde{O}_{83} = - \sqrt{1155}\ \frac{S_8}{\sqrt{S_3}} \left[aaa - \sqrt{\frac{S_3}{S_1 S_4}} \left[\frac{15}{2} + \sqrt{\frac{S_1 S_4}{S_3}} - \frac{S_4}{S_3} \right] a^+ aaaa + \cdots \right]$$

$$\tilde{O}_{84} = \frac{15}{4} \sqrt{154}\ \frac{S_8}{\sqrt{S_4}} aaaa + \cdots$$

$$\tilde{O}_{85} = - \frac{3}{2} \sqrt{1001}\ \frac{S_8}{\sqrt{S_5}} aaaaa + \cdots$$

$$\tilde{O}_{86} = \cdots$$

$$\tilde{O}_{87} = \cdots$$

$$\tilde{O}_{88} = \cdots$$

Table 3 The Rotation Matrices $d^{\ell}(\beta)$

L= 1					
M1	M	P	M[0]	M[1]	M[2]
1	1	1.0000	0	0	1
1	0	1.4142	0	1	0
1	-1	1.0000	1	0	0
0	0	1.0000	-1	0	1

L= 2							
M1	M	P	M[0]	M[1]	M[2]	M[3]	M[4]
2	2	1.0000	0	0	0	0	1
2	1	2.0000	0	0	0	1	0
2	0	2.4495	0	0	1	0	0
2	-1	2.0000	0	1	0	0	0
2	-2	1.0000	1	0	0	0	0
1	1	1.0000	0	0	-3	0	1
1	0	1.2247	0	-2	0	2	0
1	-1	1.0000	-1	0	3	0	0
0	0	1.0000	1	0	-4	0	1

L= 3									
M1	M	P	M[0]	M[1]	M[2]	M[3]	M[4]	M[5]	M[6]
3	3	1.0000	0	0	0	0	0	0	1
3	2	2.4495	0	0	0	0	0	1	0
3	1	3.8730	0	0	0	0	1	0	0
3	0	4.4721	0	0	0	1	0	0	0
3	-1	3.8730	0	0	1	0	0	0	0
3	-2	2.4495	0	1	0	0	0	0	0
3	-3	1.0000	1	0	0	0	0	0	0
2	2	1.0000	0	0	0	0	-5	0	1
2	1	1.5811	0	0	0	-4	0	2	0
2	0	1.8257	0	0	-3	0	3	0	0
2	-1	1.5811	0	-2	0	4	0	0	0
2	-2	1.0000	-1	0	5	0	0	0	0
1	1	1.0000	0	0	6	0	-8	0	1
1	0	1.1547	0	3	0	-9	0	3	0
1	-1	1.0000	1	0	-8	0	6	0	0
0	0	1.0000	-1	0	9	0	-9	0	1

L= 4											
M1	M	P	M[0]	M[1]	M[2]	M[3]	M[4]	M[5]	M[6]	M[7]	M[8]
4	4	1.0000	0	0	0	0	0	0	0	0	1
4	3	2.8284	0	0	0	0	0	0	0	1	0
4	2	5.2915	0	0	0	0	0	0	1	0	0
4	1	7.4833	0	0	0	0	0	1	0	0	0
4	0	8.3666	0	0	0	0	1	0	0	0	0
4	-1	7.4833	0	0	0	1	0	0	0	0	0
4	-2	5.2915	0	0	1	0	0	0	0	0	0
4	-3	2.8284	0	1	0	0	0	0	0	0	0
4	-4	1.0000	1	0	0	0	0	0	0	0	0
3	3	1.0000	0	0	0	0	0	0	-7	0	1
3	2	1.8708	0	0	0	0	0	-6	0	2	0
3	1	2.6458	0	0	0	0	-5	0	3	0	0
3	0	2.9580	0	0	0	-4	0	4	0	0	0
3	-1	2.6458	0	0	-3	0	5	0	0	0	0
3	-2	1.8708	0	-2	0	0	0	0	0	0	0
3	-3	1.0000	-1	0	7	0	0	0	0	0	0
2	2	1.0000	0	0	0	0	15	0	-12	0	1
2	1	1.4142	0	0	0	10	0	-15	0	3	0
2	0	1.9811	0	0	6	0	-16	0	6	0	0
2	-1	1.4142	0	3	0	-15	0	10	0	0	0
2	-2	1.0000	1	0	-12	0	15	0	0	0	0
1	1	1.0000	0	0	-10	0	30	0	-15	0	1
1	0	1.1180	0	-4	0	24	0	-24	0	0	0
1	-1	1.0000	-1	0	15	0	-30	0	10	0	0
0	0	1.0000	1	0	-16	0	36	0	-16	0	1

L= 5

NI	M	P	H(0)	H(1)	H(2)	H(3)	H(4)	H(5)	H(6)	H(7)	H(8)	H(9)	H(10)
5	5	1.0000	0	0	0	0	0	0	0	0	0	0	1
5	4	3.1623	0	0	0	0	0	0	0	0	0	1	0
5	3	6.7082	0	0	0	0	0	0	0	0	1	0	0
5	2	10.9545	0	0	0	0	0	0	0	1	0	0	0
5	1	14.4914	0	0	0	0	0	0	1	0	0	0	0
5	0	15.8745	0	0	0	0	0	1	0	0	0	0	0
5	-1	14.4914	0	0	0	0	1	0	0	0	0	0	0
5	-2	10.9545	0	0	0	1	0	0	0	0	0	0	0
5	-3	6.7082	0	0	1	0	0	0	0	0	0	0	0
5	-4	3.1623	0	1	0	0	0	0	0	0	0	0	0
5	-5	1.0000	1	0	0	0	0	0	0	0	0	0	0
4	4	1.0000	0	0	0	0	0	0	0	0	-9	0	1
4	3	2.1213	0	0	0	0	0	0	0	-8	0	2	0
4	2	3.4641	0	0	0	0	0	0	-7	0	3	0	0
4	1	4.5826	0	0	0	0	0	-6	0	4	0	0	0
4	0	5.2915	0	0	0	0	-5	0	5	0	0	0	0
4	-1	4.5826	0	0	0	-4	0	6	0	0	0	0	0
4	-2	3.4641	0	0	-3	0	7	0	0	0	0	0	0
4	-3	2.1213	0	-2	0	8	0	0	0	0	0	0	0
4	-4	1.0000	-1	0	9	0	0	0	0	0	0	0	0
3	3	1.0000	0	0	0	0	0	0	28	0	-16	0	1
3	2	1.6330	0	0	0	0	0	21	0	-21	0	3	0
3	1	2.1602	0	0	0	0	15	0	-24	0	6	0	0
3	0	2.3664	0	0	0	10	0	-25	0	10	0	0	0
3	-1	2.1602	0	0	6	0	-24	0	15	0	0	0	0
3	-2	1.6330	0	3	0	-21	0	21	0	0	0	0	0
3	-3	1.0000	1	0	-16	0	28	0	0	0	0	0	0
2	2	1.0000	0	0	0	0	-35	0	63	0	-21	0	1
2	1	1.3229	0	0	0	-28	0	60	0	-36	0	4	0
2	0	1.4491	0	0	-10	0	50	0	-50	0	10	0	0
2	-1	1.3229	0	-4	0	36	0	-60	0	28	0	0	0
2	-2	1.0000	-1	0	21	0	-63	0	35	0	0	0	0
1	1	1.0000	0	0	15	0	-80	0	90	0	-24	0	1
1	0	1.0954	0	5	0	-50	0	100	0	-50	0	5	0
1	-1	1.0000	1	0	-24	0	90	0	-80	0	15	0	0
0	0	1.0000	-1	0	25	0	-100	0	100	0	-25	0	1

L= 6

NI	M	P	H(0)	H(1)	H(2)	H(3)	H(4)	H(5)	H(6)	H(7)	H(8)	H(9)	H(10)	H(11)	H(12)
6	6	1.0000	0	0	0	0	0	0	0	0	0	0	0	0	1
6	5	3.4641	0	0	0	0	0	0	0	0	0	0	0	1	0
6	4	8.1240	0	0	0	0	0	0	0	0	0	0	1	0	0
6	3	14.8324	0	0	0	0	0	0	0	0	0	1	0	0	0
6	2	22.2486	0	0	0	0	0	0	0	0	1	0	0	0	0
6	1	28.1425	0	0	0	0	0	0	0	1	0	0	0	0	0
6	0	30.3974	0	0	0	0	0	0	1	0	0	0	0	0	0
6	-1	28.1425	0	0	0	0	0	1	0	0	0	0	0	0	0
6	-2	22.2486	0	0	0	0	1	0	0	0	0	0	0	0	0
6	-3	14.8324	0	0	0	1	0	0	0	0	0	0	0	0	0
6	-4	8.1240	0	0	1	0	0	0	0	0	0	0	0	0	0
6	-5	3.4641	0	1	0	0	0	0	0	0	0	0	0	0	0
6	-6	1.0000	1	0	0	0	0	0	0	0	0	0	0	0	0
5	5	1.0000	0	0	0	0	0	0	0	0	0	0	-11	0	1
5	4	2.3452	0	0	0	0	0	0	0	0	0	-10	0	2	0
5	3	4.2817	0	0	0	0	0	0	0	0	-9	0	3	0	0
5	2	6.4226	0	0	0	0	0	0	0	-8	0	4	0	0	0
5	1	8.1240	0	0	0	0	0	0	-7	0	5	0	0	0	0
5	0	8.7750	0	0	0	0	0	-6	0	6	0	0	0	0	0
5	-1	8.1240	0	0	0	0	-5	0	7	0	0	0	0	0	0
5	-2	6.4226	0	0	0	-4	0	8	0	0	0	0	0	0	0
5	-3	4.2817	0	0	-3	0	9	0	0	0	0	0	0	0	0
5	-4	2.3452	0	-2	0	10	0	0	0	0	0	0	0	0	0
5	-5	1.0000	-1	0	11	0	0	0	0	0	0	0	0	0	0
4	4	1.0000	0	0	0	0	0	0	0	0	45	0	-20	0	1
4	3	1.8257	0	0	0	0	0	0	0	36	0	-27	0	3	0
4	2	2.7386	0	0	0	0	0	0	28	0	-32	0	6	0	0
4	1	3.4641	0	0	0	0	0	21	0	-35	0	10	0	0	0
4	0	3.7417	0	0	0	0	15	0	-36	0	15	0	0	0	0
4	-1	3.4641	0	0	0	10	0	-35	0	21	0	0	0	0	0
4	-2	2.7386	0	0	6	0	-32	0	28	0	0	0	0	0	0
4	-3	1.8257	0	3	0	-27	0	36	0	0	0	0	0	0	0
4	-4	1.0000	1	0	-20	0	45	0	0	0	0	0	0	0	0
3	3	1.0000	0	0	0	0	0	0	-84	0	108	0	-27	0	1
3	2	1.5000	0	0	0	0	0	-56	0	112	0	-48	0	4	0
3	1	1.8974	0	0	0	0	-35	0	105	0	-70	0	10	0	0
3	0	2.0494	0	0	0	-20	0	90	0	-90	0	20	0	0	0
3	-1	1.8974	0	0	-10	0	70	0	-105	0	35	0	0	0	0
3	-2	1.5000	0	-4	0	48	0	-112	0	56	0	0	0	0	0
3	-3	1.0000	-1	0	27	0	-108	0	84	0	0	0	0	0	0
2	2	1.0000	0	0	0	0	70	0	-224	0	168	0	-32	0	1
2	1	1.2649	0	0	0	35	0	-175	0	210	0	-70	0	5	0
2	0	1.3663	0	0	15	0	-120	0	225	0	-120	0	15	0	0
2	-1	1.2649	0	5	0	-70	0	210	0	-175	0	35	0	0	0
2	-2	1.0000	1	0	-32	0	168	0	-224	0	70	0	0	0	0
1	1	1.0000	0	0	-21	0	175	0	-350	0	210	0	-35	0	1
1	0	1.0801	0	-6	0	90	0	-300	0	300	0	-90	0	6	0
1	-1	1.0000	-1	0	35	0	-210	0	350	0	-175	0	21	0	0
0	0	1.0000	1	0	-36	0	225	0	-400	0	225	0	-36	0	1

L= 7

M1	M	P	M[0]	M[1]	M[2]	M[3]	M[4]	M[5]	M[6]	M[7]	M[8]	M[9]	M[10]	M[11]	M[12]	M[13]	M[14]
7	7	1.0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
7	6	3.7417	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
7	5	9.5394	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
7	4	19.0786	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
7	3	31.6386	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
7	2	44.7437	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
7	1	54.7996	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
7	0	58.5833	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
7	-1	54.7996	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
7	-2	44.7437	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
7	-3	31.6386	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
7	-4	19.0786	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
7	-5	9.5394	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
7	-6	3.7417	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
7	-7	1.0000	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	6	1.0000	0	0	0	0	0	0	0	0	0	0	0	0	-13	0	1
6	5	2.5495	0	0	0	0	0	0	0	0	0	0	0	-12	0	2	0
6	4	5.0990	0	0	0	0	0	0	0	0	0	0	-11	0	3	0	0
6	3	8.4558	0	0	0	0	0	0	0	0	0	-10	0	4	0	0	0
6	2	11.9583	0	0	0	0	0	0	0	0	-9	0	5	0	0	0	0
6	1	14.6458	0	0	0	0	0	0	0	-8	0	6	0	0	0	0	0
6	0	15.6570	0	0	0	0	0	0	-7	0	7	0	0	0	0	0	0
6	-1	14.6458	0	0	0	0	0	-6	0	8	0	0	0	0	0	0	0
6	-2	11.9583	0	0	0	0	-5	0	9	0	0	0	0	0	0	0	0
6	-3	8.4558	0	0	0	-4	0	10	0	0	0	0	0	0	0	0	0
6	-4	5.0990	0	0	-3	0	11	0	0	0	0	0	0	0	0	0	0
6	-5	2.5495	0	-2	0	12	0	0	0	0	0	0	0	0	0	0	0
6	-6	1.0000	-1	0	13	0	0	0	0	0	0	0	0	0	0	0	0
5	5	1.0000	0	0	0	0	0	0	0	0	0	0	66	0	-24	0	1
5	4	2.0000	0	0	0	0	0	0	0	0	0	55	0	-33	0	3	0
5	3	3.3166	0	0	0	0	0	0	0	0	45	0	-40	0	6	0	0
5	2	4.6904	0	0	0	0	0	0	0	36	0	-45	0	10	0	0	0
5	1	5.7446	0	0	0	0	0	0	28	0	-48	0	15	0	0	0	0
5	0	6.1412	0	0	0	0	0	21	0	-49	0	21	0	0	0	0	0
5	-1	5.7446	0	0	0	0	15	0	-48	0	28	0	0	0	0	0	0
5	-2	4.6904	0	0	0	10	0	-45	0	36	0	0	0	0	0	0	0
5	-3	3.3166	0	0	6	0	-40	0	45	0	0	0	0	0	0	0	0
5	-4	2.0000	0	3	0	-33	0	55	0	0	0	0	0	0	0	0	0
5	-5	1.0000	1	0	-24	0	66	0	0	0	0	0	0	0	0	0	0
4	4	1.0000	0	0	0	0	0	0	0	0	-165	0	165	0	-33	0	1
4	3	1.6583	0	0	0	0	0	0	0	-120	0	180	0	-60	0	4	0
4	2	2.3452	0	0	0	0	0	0	-84	0	180	0	-90	0	10	0	0
4	1	2.8723	0	0	0	0	0	-56	0	168	0	-120	0	20	0	0	0
4	0	3.0706	0	0	0	0	-35	0	147	0	-147	0	35	0	0	0	0
4	-1	2.8723	0	0	0	-20	0	120	0	-168	0	56	0	0	0	0	0
4	-2	2.3452	0	0	-10	0	90	0	-180	0	84	0	0	0	0	0	0
4	-3	1.6583	0	-4	0	44	0	-180	0	120	0	0	0	0	0	0	0
4	-4	1.0000	-1	0	33	0	-165	0	165	0	0	0	0	0	0	0	0
3	3	1.0000	0	0	0	0	0	0	210	0	-480	0	270	0	-40	0	1
3	2	1.4142	0	0	0	0	0	126	0	-420	0	360	0	-90	0	5	0
3	1	1.7321	0	0	0	0	70	0	-336	0	420	0	-160	0	15	0	0
3	0	1.8516	0	0	0	35	0	-245	0	441	0	-24.	0	35	0	0	0
3	-1	1.7321	0	0	15	0	-160	0	420	0	-336	0	70	0	0	0	0
3	-2	1.4142	0	5	0	-90	0	360	0	-420	0	126	0	0	0	0	0
3	-3	1.0000	1	0	-40	0	270	0	-480	0	210	0	0	0	0	0	0
2	2	1.0000	0	0	0	0	-126	0	630	0	-840	0	360	0	-45	0	1
2	1	1.2247	0	0	0	-56	0	420	0	-840	0	560	0	-120	0	6	0
2	0	1.3093	0	0	-21	0	245	0	-735	0	735	0	-245	0	21	0	0
2	-1	1.2247	0	-6	0	120	0	-560	0	840	0	-420	0	56	0	0	0
2	-2	1.0000	-1	0	45	0	-360	0	840	0	-630	0	126	0	0	0	0
1	1	1.0000	0	0	28	0	-336	0	1050	0	-1120	0	420	0	-48	0	1
1	0	1.0690	0	7	0	-147	0	735	0	-1225	0	735	0	-147	0	7	0
1	-1	1.0000	1	0	-48	0	420	0	-1120	0	1050	0	-336	0	28	0	0
0	0	1.0000	-1	0	49	0	-441	0	1225	0	-1225	0	441	0	-49	0	1

L= 8 M1	M	P	H[0]	H[1]	H[2]	H[3]	H[4]	H[5]	H[6]	H[7]	H[8]	H[9]	H[10]	H[11]	H[12]	H[13]	H[14]	H[15]	H[16]
8	8	1.0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
8	7	4.0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
8	6	10.9545	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
8	5	23.6643	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
8	4	42.6615	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
8	3	66.0908	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
8	2	89.4874	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
8	1	106.9579	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
8	0	113.4460	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
8	-1	106.9579	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
8	-2	89.4874	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
8	-3	66.0908	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
8	-4	42.6615	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
8	-5	23.6643	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
8	-6	10.9545	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	-7	4.0000	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	-8	1.0000	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	7	1.0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-15	0	1
7	6	2.7386	0	0	0	0	0	0	0	0	0	0	0	0	0	-14	0	2	0
7	5	5.9161	0	0	0	0	0	0	0	0	0	0	0	0	-13	0	3	0	0
7	4	10.6654	0	0	0	0	0	0	0	0	0	0	0	-12	0	4	0	0	0
7	3	16.5227	0	0	0	0	0	0	0	0	0	0	-11	0	5	0	0	0	0
7	2	22.3719	0	0	0	0	0	0	0	0	0	-10	0	6	0	0	0	0	0
7	1	26.7395	0	0	0	0	0	0	0	0	-9	0	7	0	0	0	0	0	0
7	0	28.3615	0	0	0	0	0	0	0	-8	0	8	0	0	0	0	0	0	0
7	-1	26.7395	0	0	0	0	0	0	-7	0	9	0	0	0	0	0	0	0	0
7	-2	22.3719	0	0	0	0	0	-6	0	10	0	0	0	0	0	0	0	0	0
7	-3	16.5227	0	0	0	0	-5	0	11	0	0	0	0	0	0	0	0	0	0
7	-4	10.6654	0	0	0	-4	0	12	0	0	0	0	0	0	0	0	0	0	0
7	-5	5.9161	0	0	-3	0	13	0	0	0	0	0	0	0	0	0	0	0	0
7	-6	2.7386	0	-2	0	14	0	0	0	0	0	0	0	0	0	0	0	0	0
7	-7	1.0000	-1	0	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	6	1.0000	0	0	0	0	0	0	0	0	0	0	0	0	91	0	-28	0	1
6	5	2.1602	0	0	0	0	0	0	0	0	0	0	0	78	0	-39	0	3	0
6	4	3.8944	0	0	0	0	0	0	0	0	0	0	66	0	-48	0	6	0	0
6	3	6.0332	0	0	0	0	0	0	0	0	0	55	0	-55	0	10	0	0	0
6	2	8.1690	0	0	0	0	0	0	0	0	45	0	-60	0	15	0	0	0	0
6	1	9.7639	0	0	0	0	0	0	0	36	0	-63	0	21	0	0	0	0	0
6	0	10.3562	0	0	0	0	0	0	25	0	-64	0	28	0	0	0	0	0	0
6	-1	9.7639	0	0	0	0	0	21	0	-63	0	36	0	0	0	0	0	0	0
6	-2	8.1690	0	0	0	0	15	0	-60	0	45	0	0	0	0	0	0	0	0
6	-3	6.0332	0	0	0	10	0	-55	0	55	0	0	0	0	0	0	0	0	0
6	-4	3.8944	0	0	6	0	-48	0	66	0	0	0	0	0	0	0	0	0	0
6	-5	2.1602	0	3	0	-39	0	78	0	0	0	0	0	0	0	0	0	0	0
6	-6	1.0000	1	0	-28	0	91	0	0	0	0	0	0	0	0	0	0	0	0
5	5	1.0000	0	0	0	0	0	0	0	0	0	0	-286	0	234	0	-39	0	1
5	4	1.8028	0	0	0	0	0	0	0	0	0	-220	0	264	0	-72	0	4	0
5	3	2.7928	0	0	0	0	0	0	0	0	-165	0	275	0	-110	0	10	0	0
5	2	3.7815	0	0	0	0	0	0	0	-120	0	270	0	-150	0	20	0	0	0
5	1	4.5198	0	0	0	0	0	0	-84	0	252	0	-189	0	35	0	0	0	0
5	0	4.7980	0	0	0	0	0	-56	0	224	0	-224	0	56	0	0	0	0	0
5	-1	4.5198	0	0	0	0	-35	0	189	0	-252	0	84	0	0	0	0	0	0
5	-2	3.7815	0	0	0	-20	0	150	0	-270	0	120	0	0	0	0	0	0	0
5	-3	2.7928	0	0	-10	0	110	0	-275	0	165	0	0	0	0	0	0	0	0
5	-4	1.8028	0	-4	0	72	0	-264	0	220	0	0	0	0	0	0	0	0	0
5	-5	1.0000	-1	0	39	0	-234	0	286	0	0	0	0	0	0	0	0	0	0
4	4	1.0000	0	0	0	0	0	0	0	0	495	0	-880	0	396	0	-48	0	1
4	3	1.5492	0	0	0	0	0	0	0	330	0	-825	0	550	0	-110	0	5	0
4	2	2.0976	0	0	0	0	0	0	210	0	-720	0	675	0	-200	0	15	0	0
4	1	2.5071	0	0	0	0	0	126	0	-588	0	756	0	-315	0	35	0	0	0
4	0	2.6592	0	0	0	0	70	0	-448	0	784	0	-448	0	70	0	0	0	0
4	-1	2.5071	0	0	0	35	0	-315	0	756	0	-588	0	126	0	0	0	0	0
4	-2	2.0976	0	0	15	0	-200	0	675	0	-720	0	210	0	0	0	0	0	0
4	-3	1.5492	0	5	0	-110	0	550	0	-825	0	330	0	0	0	0	0	0	0
4	-4	1.0000	1	0	-48	0	396	0	-880	0	495	0	0	0	0	0	0	0	0
3	3	1.0000	0	0	0	0	0	0	-462	0	1650	0	-1650	0	550	0	-55	0	1
3	2	1.3540	0	0	0	0	0	-252	0	1260	0	-1800	0	900	0	-150	0	6	0
3	1	1.4183	0	0	0	0	-126	0	882	0	-1764	0	1260	0	-315	0	21	0	0
3	0	1.7165	0	0	0	-56	0	560	0	-1568	0	1568	0	-560	0	56	0	0	0
3	-1	1.6183	0	0	-21	0	315	0	-1260	0	1764	0	-882	0	126	0	0	0	0
3	-2	1.3540	0	-6	0	150	0	-900	0	1800	0	-1260	0	252	0	0	0	0	0
3	-3	1.0000	-1	0	55	0	-550	0	1650	0	-1650	0	462	0	0	0	0	0	0
2	2	1.0000	0	0	0	0	210	0	-1512	0	3150	0	-2400	0	675	0	-60	0	1
2	1	1.1952	0	0	0	84	0	-882	0	2646	0	-2940	0	1260	0	-189	0	7	0
2	0	1.2677	0	0	28	0	-448	0	1960	0	-3136	0	1960	0	-448	0	28	0	0
2	-1	1.1952	0	7	0	-189	0	1260	0	-2940	0	2646	0	-882	0	84	0	0	0
2	-2	1.0000	1	0	-60	0	675	0	-2400	0	3150	0	-1512	0	210	0	0	0	0
1	1	1.0000	0	0	-36	0	588	0	-2646	0	4410	0	-2940	0	756	0	-63	0	1
1	0	1.0607	0	-6	0	224	0	-1568	0	3920	0	-3920	0	1568	0	-224	0	8	0
1	-1	1.0000	-1	0	63	0	-756	0	2940	0	-4410	0	2646	0	-588	0	36	0	0
0	0	1.0000	1	0	-64	0	784	0	-3136	0	4900	0	-3136	0	784	0	-64	0	1

Table 4.

The rotation-matrices $\mathcal{D}^j(\alpha,\frac{\pi}{2},\frac{\pi}{2})^{-1}$

$\mathcal{D}^1(\alpha,\frac{\pi}{2},\frac{\pi}{2})^{-1}$

$$\begin{bmatrix} \frac{i}{2}e^{i\alpha} & \frac{i}{\sqrt{2}} & \frac{i}{2}e^{-i\alpha} \\ -\frac{1}{\sqrt{2}}e^{i\alpha} & 0 & \frac{1}{\sqrt{2}}e^{-i\alpha} \\ -\frac{i}{2}e^{i\alpha} & \frac{i}{\sqrt{2}} & -\frac{i}{2}e^{-i\alpha} \end{bmatrix}$$

$\mathcal{D}^2(\alpha,\frac{\pi}{2},\frac{\pi}{2})^{-1}$

$$\begin{bmatrix} -\frac{1}{4}e^{i2\alpha} & -\frac{1}{2}e^{i\alpha} & -\frac{\sqrt{6}}{4} & -\frac{1}{2}e^{-i\alpha} & -\frac{1}{4}e^{-i2\alpha} \\ -\frac{i}{2}e^{i2\alpha} & -\frac{i}{2}e^{i\alpha} & 0 & \frac{i}{2}e^{-i\alpha} & \frac{i}{2}e^{-i2\alpha} \\ \frac{\sqrt{6}}{4}e^{i2\alpha} & 0 & -\frac{1}{2} & 0 & \frac{\sqrt{6}}{4}e^{-i2\alpha} \\ \frac{i}{2}e^{i2\alpha} & -\frac{i}{2}e^{i\alpha} & 0 & \frac{i}{2}e^{-i\alpha} & -\frac{i}{2}e^{-i2\alpha} \\ -\frac{1}{4}e^{i2\alpha} & \frac{1}{2}e^{i\alpha} & -\frac{\sqrt{6}}{4} & \frac{1}{2}e^{-i\alpha} & -\frac{1}{4}e^{-i2\alpha} \end{bmatrix}$$

$\mathcal{D}^3(\alpha,\frac{\pi}{2},\frac{\pi}{2})^{-1}$

$$\begin{bmatrix} -\frac{i}{8}e^{i3\alpha} & -i\frac{\sqrt{6}}{8}e^{i2\alpha} & -i\frac{\sqrt{15}}{8}e^{i\alpha} & -i\frac{\sqrt{5}}{4} & -i\frac{\sqrt{15}}{8}e^{-i\alpha} & -i\frac{\sqrt{6}}{8}e^{-i2\alpha} & -\frac{i}{8}e^{-i3\alpha} \\ \frac{\sqrt{6}}{8}e^{i3\alpha} & \frac{1}{2}e^{i2\alpha} & \frac{\sqrt{10}}{8}e^{i\alpha} & 0 & -\frac{\sqrt{10}}{8}e^{-i\alpha} & -\frac{1}{2}e^{-i2\alpha} & -\frac{\sqrt{6}}{8}e^{-i3\alpha} \\ i\frac{\sqrt{15}}{8}e^{i3\alpha} & i\frac{\sqrt{10}}{8}e^{i2\alpha} & -\frac{i}{8}e^{i\alpha} & -i\frac{\sqrt{3}}{4} & -\frac{i}{8}e^{-i\alpha} & i\frac{\sqrt{10}}{8}e^{-i2\alpha} & i\frac{\sqrt{15}}{8}e^{-i3\alpha} \\ -\frac{\sqrt{5}}{4}e^{i3\alpha} & 0 & \frac{\sqrt{3}}{4}e^{i\alpha} & 0 & -\frac{\sqrt{3}}{4}e^{-i\alpha} & 0 & \frac{\sqrt{5}}{4}e^{-i3\alpha} \\ -i\frac{\sqrt{15}}{8}e^{i3\alpha} & i\frac{\sqrt{10}}{8}e^{i2\alpha} & \frac{i}{8}e^{i\alpha} & -i\frac{\sqrt{3}}{4} & \frac{i}{8}e^{-i\alpha} & i\frac{\sqrt{10}}{8}e^{-i2\alpha} & -i\frac{\sqrt{15}}{8}e^{-i3\alpha} \\ \frac{\sqrt{6}}{8}e^{i3\alpha} & -\frac{1}{2}e^{i2\alpha} & \frac{\sqrt{10}}{8}e^{i\alpha} & 0 & -\frac{\sqrt{10}}{8}e^{-i\alpha} & \frac{1}{2}e^{-i2\alpha} & -\frac{\sqrt{6}}{8}e^{-i3\alpha} \\ \frac{i}{8}e^{i3\alpha} & -i\frac{\sqrt{6}}{8}e^{i2\alpha} & i\frac{\sqrt{15}}{8}e^{i\alpha} & -i\frac{\sqrt{5}}{4} & i\frac{\sqrt{15}}{8}e^{-i\alpha} & -i\frac{\sqrt{6}}{8}e^{-i2\alpha} & \frac{i}{8}e^{-i3\alpha} \end{bmatrix}$$

$\mathcal{D}^4(\alpha,\frac{\pi}{2},\frac{\pi}{2})^{-1}$

$$\begin{bmatrix} \frac{1}{16}e^{i4\alpha} & \frac{\sqrt{2}}{8}e^{i3\alpha} & \frac{\sqrt{7}}{8}e^{i2\alpha} & \frac{\sqrt{14}}{8}e^{i\alpha} & \frac{\sqrt{70}}{16} & \frac{\sqrt{14}}{8}e^{-i\alpha} & \frac{\sqrt{7}}{8}e^{-i2\alpha} & \frac{\sqrt{2}}{8}e^{-i3\alpha} & \frac{1}{16}e^{-i4\alpha} \\ i\frac{\sqrt{2}}{8}e^{i4\alpha} & i\frac{3}{8}e^{i3\alpha} & i\frac{\sqrt{14}}{8}e^{i2\alpha} & i\frac{\sqrt{7}}{8}e^{i\alpha} & 0 & -i\frac{\sqrt{7}}{8}e^{-i\alpha} & -i\frac{\sqrt{14}}{8}e^{-i2\alpha} & -i\frac{3}{8}e^{-i3\alpha} & -i\frac{\sqrt{2}}{8}e^{-i4\alpha} \\ -\frac{\sqrt{7}}{8}e^{i4\alpha} & -\frac{\sqrt{14}}{8}e^{i3\alpha} & -\frac{1}{4}e^{i2\alpha} & \frac{\sqrt{2}}{8}e^{i\alpha} & \frac{\sqrt{10}}{8} & \frac{\sqrt{2}}{8}e^{-i\alpha} & -\frac{1}{4}e^{-i2\alpha} & -\frac{\sqrt{14}}{8}e^{-i3\alpha} & -\frac{\sqrt{7}}{8}e^{-i4\alpha} \\ -i\frac{\sqrt{14}}{8}e^{i4\alpha} & -i\frac{\sqrt{7}}{8}e^{i3\alpha} & i\frac{\sqrt{2}}{8}e^{i2\alpha} & i\frac{1}{8}e^{i\alpha} & 0 & -i\frac{1}{8}e^{-i\alpha} & -i\frac{\sqrt{2}}{8}e^{-i2\alpha} & i\frac{\sqrt{7}}{8}e^{-i3\alpha} & i\frac{\sqrt{14}}{8}e^{-i4\alpha} \\ \frac{\sqrt{70}}{16}e^{i4\alpha} & 0 & -\frac{\sqrt{10}}{8}e^{i2\alpha} & 0 & \frac{3}{8} & 0 & -\frac{\sqrt{10}}{8}e^{-i2\alpha} & 0 & \frac{\sqrt{70}}{16}e^{-i4\alpha} \\ i\frac{\sqrt{14}}{8}e^{i4\alpha} & -i\frac{\sqrt{7}}{8}e^{i3\alpha} & -i\frac{\sqrt{2}}{8}e^{i2\alpha} & i\frac{1}{8}e^{i\alpha} & 0 & -i\frac{1}{8}e^{-i\alpha} & i\frac{\sqrt{2}}{8}e^{-i2\alpha} & i\frac{\sqrt{7}}{8}e^{-i3\alpha} & -i\frac{\sqrt{14}}{8}e^{-i4\alpha} \\ -\frac{\sqrt{7}}{8}e^{i4\alpha} & \frac{\sqrt{14}}{8}e^{i3\alpha} & -\frac{1}{4}e^{i2\alpha} & -\frac{\sqrt{2}}{8}e^{i\alpha} & \frac{\sqrt{10}}{8} & -\frac{\sqrt{2}}{8}e^{-i\alpha} & -\frac{1}{4}e^{-i2\alpha} & \frac{\sqrt{14}}{8}e^{-i3\alpha} & -\frac{\sqrt{7}}{8}e^{-i4\alpha} \\ -i\frac{\sqrt{2}}{8}e^{i4\alpha} & i\frac{3}{8}e^{i3\alpha} & -i\frac{\sqrt{14}}{8}e^{i2\alpha} & i\frac{\sqrt{7}}{8}e^{i\alpha} & 0 & -i\frac{\sqrt{7}}{8}e^{-i\alpha} & i\frac{\sqrt{14}}{8}e^{-i2\alpha} & -i\frac{3}{8}e^{-i3\alpha} & i\frac{\sqrt{2}}{8}e^{-i4\alpha} \\ \frac{1}{16}e^{i4\alpha} & -\frac{\sqrt{2}}{8}e^{i3\alpha} & \frac{\sqrt{7}}{8}e^{i2\alpha} & -\frac{\sqrt{14}}{8}e^{i\alpha} & \frac{\sqrt{70}}{16} & -\frac{\sqrt{14}}{8}e^{-i\alpha} & \frac{\sqrt{7}}{8}e^{-i2\alpha} & -\frac{\sqrt{2}}{8}e^{-i3\alpha} & \frac{1}{16}e^{-i4\alpha} \end{bmatrix}$$

$\underline{\mathfrak{D}}^{5}(\alpha,\frac{\pi}{2},\frac{\pi}{2})^{-1}$

$$
\begin{bmatrix}
i\frac{1}{32}e^{5i\alpha} & i\frac{\sqrt{10}}{32}e^{4i\alpha} & i\frac{3\sqrt{5}}{32}e^{3i\alpha} & i\frac{\sqrt{30}}{16}e^{2i\alpha} & i\frac{\sqrt{210}}{32}e^{i\alpha} & i\frac{3\sqrt{7}}{16} & i\frac{\sqrt{210}}{32}e^{-i\alpha} & i\frac{\sqrt{30}}{16}e^{-2i\alpha} & i\frac{3\sqrt{5}}{32}e^{-3i\alpha} & i\frac{\sqrt{10}}{32}e^{-4i\alpha} & i\frac{1}{32}e^{-5i\alpha} \\
-\frac{\sqrt{10}}{32}e^{5i\alpha} & -\frac{1}{4}e^{4i\alpha} & -\frac{9\sqrt{2}}{32}e^{3i\alpha} & -\frac{\sqrt{3}}{4}e^{2i\alpha} & -\frac{\sqrt{21}}{16}e^{i\alpha} & 0 & \frac{\sqrt{21}}{16}e^{-i\alpha} & \frac{\sqrt{3}}{4}e^{-2i\alpha} & \frac{9\sqrt{2}}{32}e^{-3i\alpha} & \frac{1}{4}e^{-4i\alpha} & \frac{\sqrt{10}}{32}e^{-5i\alpha} \\
-i\frac{3\sqrt{5}}{32}e^{5i\alpha} & -i\frac{9\sqrt{2}}{32}e^{4i\alpha} & -i\frac{13}{32}e^{3i\alpha} & -i\frac{\sqrt{6}}{16}e^{2i\alpha} & i\frac{\sqrt{42}}{32}e^{i\alpha} & i\frac{\sqrt{35}}{16} & i\frac{\sqrt{42}}{32}e^{-i\alpha} & -i\frac{\sqrt{6}}{16}e^{-2i\alpha} & -i\frac{13}{32}e^{-3i\alpha} & -i\frac{9\sqrt{2}}{32}e^{-4i\alpha} & -i\frac{3\sqrt{5}}{32}e^{-5i\alpha} \\
\frac{\sqrt{30}}{16}e^{5i\alpha} & \frac{\sqrt{3}}{4}e^{4i\alpha} & \frac{\sqrt{6}}{16}e^{3i\alpha} & -\frac{1}{4}e^{2i\alpha} & -\frac{\sqrt{7}}{8}e^{i\alpha} & 0 & \frac{\sqrt{7}}{8}e^{-i\alpha} & \frac{1}{4}e^{-2i\alpha} & -\frac{\sqrt{6}}{16}e^{-3i\alpha} & -\frac{\sqrt{3}}{4}e^{-4i\alpha} & -\frac{\sqrt{30}}{16}e^{-5i\alpha} \\
i\frac{\sqrt{210}}{32}e^{5i\alpha} & i\frac{\sqrt{21}}{16}e^{4i\alpha} & -i\frac{\sqrt{42}}{32}e^{3i\alpha} & -i\frac{\sqrt{7}}{8}e^{2i\alpha} & i\frac{1}{16}e^{i\alpha} & i\frac{\sqrt{30}}{16} & i\frac{1}{16}e^{-i\alpha} & -i\frac{\sqrt{7}}{8}e^{-2i\alpha} & -i\frac{\sqrt{42}}{32}e^{-3i\alpha} & i\frac{\sqrt{21}}{16}e^{-4i\alpha} & i\frac{\sqrt{210}}{32}e^{-5i\alpha} \\
-\frac{3\sqrt{7}}{16}e^{5i\alpha} & 0 & \frac{\sqrt{35}}{16}e^{3i\alpha} & 0 & -\frac{\sqrt{30}}{16}e^{i\alpha} & 0 & \frac{\sqrt{30}}{16}e^{-i\alpha} & 0 & -\frac{\sqrt{35}}{16}e^{-3i\alpha} & 0 & \frac{3\sqrt{7}}{16}e^{-5i\alpha} \\
-i\frac{\sqrt{210}}{32}e^{5i\alpha} & i\frac{\sqrt{21}}{16}e^{4i\alpha} & -i\frac{\sqrt{42}}{32}e^{3i\alpha} & -i\frac{\sqrt{7}}{8}e^{2i\alpha} & -i\frac{1}{16}e^{i\alpha} & i\frac{\sqrt{30}}{16} & -i\frac{1}{16}e^{-i\alpha} & -i\frac{\sqrt{7}}{8}e^{-2i\alpha} & i\frac{\sqrt{42}}{32}e^{-3i\alpha} & i\frac{\sqrt{21}}{16}e^{-4i\alpha} & -i\frac{\sqrt{210}}{32}e^{-5i\alpha} \\
\frac{\sqrt{30}}{16}e^{5i\alpha} & -\frac{\sqrt{3}}{4}e^{4i\alpha} & -\frac{\sqrt{6}}{16}e^{3i\alpha} & \frac{1}{4}e^{2i\alpha} & -\frac{\sqrt{7}}{8}e^{i\alpha} & 0 & \frac{\sqrt{7}}{8}e^{-i\alpha} & -\frac{1}{4}e^{-2i\alpha} & -\frac{\sqrt{6}}{16}e^{-3i\alpha} & \frac{\sqrt{3}}{4}e^{-4i\alpha} & -\frac{\sqrt{30}}{16}e^{-5i\alpha} \\
i\frac{3\sqrt{5}}{32}e^{5i\alpha} & -i\frac{9\sqrt{2}}{32}e^{4i\alpha} & -i\frac{13}{32}e^{3i\alpha} & -i\frac{\sqrt{6}}{16}e^{2i\alpha} & -i\frac{\sqrt{42}}{32}e^{i\alpha} & i\frac{\sqrt{35}}{16} & -i\frac{\sqrt{42}}{32}e^{-i\alpha} & -i\frac{\sqrt{6}}{16}e^{-2i\alpha} & i\frac{13}{32}e^{-3i\alpha} & -i\frac{9\sqrt{2}}{32}e^{-4i\alpha} & i\frac{3\sqrt{5}}{32}e^{-5i\alpha} \\
-\frac{\sqrt{10}}{32}e^{5i\alpha} & \frac{1}{4}e^{4i\alpha} & \frac{9\sqrt{2}}{32}e^{3i\alpha} & \frac{\sqrt{3}}{4}e^{2i\alpha} & -\frac{\sqrt{21}}{16}e^{i\alpha} & 0 & \frac{\sqrt{21}}{16}e^{-i\alpha} & -\frac{\sqrt{3}}{4}e^{-2i\alpha} & \frac{9\sqrt{2}}{32}e^{-3i\alpha} & -\frac{1}{4}e^{-4i\alpha} & \frac{\sqrt{10}}{32}e^{-5i\alpha} \\
-i\frac{1}{32}e^{5i\alpha} & i\frac{\sqrt{10}}{32}e^{4i\alpha} & -i\frac{3\sqrt{5}}{32}e^{3i\alpha} & i\frac{\sqrt{30}}{16}e^{2i\alpha} & -i\frac{\sqrt{210}}{32}e^{i\alpha} & i\frac{3\sqrt{7}}{16} & -i\frac{\sqrt{210}}{32}e^{-i\alpha} & i\frac{\sqrt{30}}{16}e^{-2i\alpha} & -i\frac{3\sqrt{5}}{32}e^{-3i\alpha} & i\frac{\sqrt{10}}{32}e^{-4i\alpha} & -i\frac{1}{32}e^{-5i\alpha}
\end{bmatrix}
$$

$\underline{\mathfrak{D}}^{6}(\alpha,\frac{\pi}{2},\frac{\pi}{2})^{-1}$

$$
\begin{bmatrix}
-\frac{1}{64}e^{6i\alpha} & -\frac{\sqrt{3}}{32}e^{5i\alpha} & -\frac{\sqrt{66}}{64}e^{4i\alpha} & -\frac{\sqrt{55}}{32}e^{3i\alpha} & -\frac{3\sqrt{55}}{64}e^{2i\alpha} & -\frac{3\sqrt{22}}{32}e^{i\alpha} & -\frac{\sqrt{231}}{32} & -\frac{3\sqrt{22}}{32}e^{-i\alpha} & -\frac{3\sqrt{55}}{64}e^{-2i\alpha} & -\frac{\sqrt{55}}{32}e^{-3i\alpha} & -\frac{\sqrt{66}}{64}e^{-4i\alpha} & -\frac{\sqrt{3}}{32}e^{-5i\alpha} & -\frac{1}{64}e^{-6i\alpha} \\
-i\frac{\sqrt{3}}{32}e^{6i\alpha} & -i\frac{5}{32}e^{5i\alpha} & -i\frac{\sqrt{22}}{16}e^{4i\alpha} & -i\frac{\sqrt{165}}{32}e^{3i\alpha} & -i\frac{\sqrt{165}}{32}e^{2i\alpha} & -i\frac{\sqrt{66}}{32}e^{i\alpha} & 0 & i\frac{\sqrt{66}}{32}e^{-i\alpha} & i\frac{\sqrt{165}}{32}e^{-2i\alpha} & i\frac{\sqrt{165}}{32}e^{-3i\alpha} & i\frac{\sqrt{22}}{16}e^{-4i\alpha} & i\frac{5}{32}e^{-5i\alpha} & i\frac{\sqrt{3}}{32}e^{-6i\alpha} \\
\frac{\sqrt{66}}{64}e^{6i\alpha} & \frac{\sqrt{22}}{16}e^{5i\alpha} & \frac{13}{32}e^{4i\alpha} & \frac{\sqrt{30}}{16}e^{3i\alpha} & \frac{\sqrt{30}}{64}e^{2i\alpha} & -\frac{\sqrt{3}}{8}e^{i\alpha} & -\frac{3\sqrt{14}}{32} & -\frac{\sqrt{3}}{8}e^{-i\alpha} & \frac{\sqrt{30}}{64}e^{-2i\alpha} & \frac{\sqrt{30}}{16}e^{-3i\alpha} & \frac{13}{32}e^{-4i\alpha} & \frac{\sqrt{22}}{16}e^{-5i\alpha} & \frac{\sqrt{66}}{64}e^{-6i\alpha} \\
i\frac{\sqrt{55}}{32}e^{6i\alpha} & i\frac{\sqrt{165}}{32}e^{5i\alpha} & i\frac{\sqrt{30}}{16}e^{4i\alpha} & i\frac{1}{32}e^{3i\alpha} & -i\frac{9}{32}e^{2i\alpha} & -i\frac{3\sqrt{10}}{32}e^{i\alpha} & 0 & i\frac{3\sqrt{10}}{32}e^{-i\alpha} & i\frac{9}{32}e^{-2i\alpha} & -i\frac{1}{32}e^{-3i\alpha} & -i\frac{\sqrt{30}}{16}e^{-4i\alpha} & -i\frac{\sqrt{165}}{32}e^{-5i\alpha} & -i\frac{\sqrt{55}}{32}e^{-6i\alpha} \\
-\frac{3\sqrt{55}}{64}e^{6i\alpha} & -\frac{\sqrt{165}}{32}e^{5i\alpha} & -\frac{\sqrt{30}}{64}e^{4i\alpha} & \frac{9}{32}e^{3i\alpha} & \frac{17}{64}e^{2i\alpha} & -\frac{\sqrt{10}}{32}e^{i\alpha} & -\frac{\sqrt{105}}{32} & -\frac{\sqrt{10}}{32}e^{-i\alpha} & \frac{17}{64}e^{-2i\alpha} & \frac{9}{32}e^{-3i\alpha} & -\frac{\sqrt{30}}{64}e^{-4i\alpha} & -\frac{\sqrt{165}}{32}e^{-5i\alpha} & -\frac{3\sqrt{55}}{64}e^{-6i\alpha} \\
-i\frac{3\sqrt{22}}{32}e^{6i\alpha} & -i\frac{\sqrt{66}}{32}e^{5i\alpha} & i\frac{\sqrt{3}}{8}e^{4i\alpha} & i\frac{3\sqrt{10}}{32}e^{3i\alpha} & -i\frac{\sqrt{10}}{32}e^{2i\alpha} & -i\frac{5}{16}e^{i\alpha} & 0 & i\frac{5}{16}e^{-i\alpha} & i\frac{\sqrt{10}}{32}e^{-2i\alpha} & -i\frac{3\sqrt{10}}{32}e^{-3i\alpha} & -i\frac{\sqrt{3}}{8}e^{-4i\alpha} & i\frac{\sqrt{66}}{32}e^{-5i\alpha} & i\frac{3\sqrt{22}}{32}e^{-6i\alpha} \\
\frac{\sqrt{231}}{32}e^{6i\alpha} & 0 & -\frac{3\sqrt{14}}{32}e^{4i\alpha} & 0 & \frac{\sqrt{105}}{32}e^{2i\alpha} & 0 & -\frac{5}{16} & 0 & \frac{\sqrt{105}}{32}e^{-2i\alpha} & 0 & -\frac{3\sqrt{14}}{32}e^{-4i\alpha} & 0 & \frac{\sqrt{231}}{32}e^{-6i\alpha} \\
i\frac{3\sqrt{22}}{32}e^{6i\alpha} & -i\frac{\sqrt{66}}{32}e^{5i\alpha} & -i\frac{\sqrt{3}}{8}e^{4i\alpha} & i\frac{3\sqrt{10}}{32}e^{3i\alpha} & i\frac{\sqrt{10}}{32}e^{2i\alpha} & -i\frac{5}{16}e^{i\alpha} & 0 & i\frac{5}{16}e^{-i\alpha} & -i\frac{\sqrt{10}}{32}e^{-2i\alpha} & -i\frac{3\sqrt{10}}{32}e^{-3i\alpha} & i\frac{\sqrt{3}}{8}e^{-4i\alpha} & i\frac{\sqrt{66}}{32}e^{-5i\alpha} & -i\frac{3\sqrt{22}}{32}e^{-6i\alpha} \\
-\frac{3\sqrt{55}}{64}e^{6i\alpha} & \frac{\sqrt{165}}{32}e^{5i\alpha} & -\frac{\sqrt{30}}{64}e^{4i\alpha} & -\frac{9}{32}e^{3i\alpha} & \frac{17}{64}e^{2i\alpha} & \frac{\sqrt{10}}{32}e^{i\alpha} & -\frac{\sqrt{105}}{32} & \frac{\sqrt{10}}{32}e^{-i\alpha} & \frac{17}{64}e^{-2i\alpha} & -\frac{9}{32}e^{-3i\alpha} & -\frac{\sqrt{30}}{64}e^{-4i\alpha} & \frac{\sqrt{165}}{32}e^{-5i\alpha} & -\frac{3\sqrt{55}}{64}e^{-6i\alpha} \\
-i\frac{\sqrt{55}}{32}e^{6i\alpha} & i\frac{\sqrt{165}}{32}e^{5i\alpha} & -i\frac{\sqrt{30}}{16}e^{4i\alpha} & i\frac{1}{32}e^{3i\alpha} & i\frac{9}{32}e^{2i\alpha} & -i\frac{3\sqrt{10}}{32}e^{i\alpha} & 0 & i\frac{3\sqrt{10}}{32}e^{-i\alpha} & -i\frac{9}{32}e^{-2i\alpha} & -i\frac{1}{32}e^{-3i\alpha} & i\frac{\sqrt{30}}{16}e^{-4i\alpha} & -i\frac{\sqrt{165}}{32}e^{-5i\alpha} & i\frac{\sqrt{55}}{32}e^{-6i\alpha} \\
\frac{\sqrt{66}}{64}e^{6i\alpha} & -\frac{\sqrt{22}}{16}e^{5i\alpha} & \frac{13}{32}e^{4i\alpha} & -\frac{\sqrt{30}}{16}e^{3i\alpha} & \frac{\sqrt{30}}{64}e^{2i\alpha} & \frac{\sqrt{3}}{8}e^{i\alpha} & -\frac{3\sqrt{14}}{32} & \frac{\sqrt{3}}{8}e^{-i\alpha} & \frac{\sqrt{30}}{64}e^{-2i\alpha} & -\frac{\sqrt{30}}{16}e^{-3i\alpha} & \frac{13}{32}e^{-4i\alpha} & -\frac{\sqrt{22}}{16}e^{-5i\alpha} & \frac{\sqrt{66}}{64}e^{-6i\alpha} \\
i\frac{\sqrt{3}}{32}e^{6i\alpha} & -i\frac{5}{32}e^{5i\alpha} & i\frac{\sqrt{22}}{16}e^{4i\alpha} & -i\frac{\sqrt{165}}{32}e^{3i\alpha} & i\frac{\sqrt{165}}{32}e^{2i\alpha} & -i\frac{\sqrt{66}}{32}e^{i\alpha} & 0 & i\frac{\sqrt{66}}{32}e^{-i\alpha} & -i\frac{\sqrt{165}}{32}e^{-2i\alpha} & i\frac{\sqrt{165}}{32}e^{-3i\alpha} & -i\frac{\sqrt{22}}{16}e^{-4i\alpha} & i\frac{5}{32}e^{-5i\alpha} & -i\frac{\sqrt{3}}{32}e^{-6i\alpha} \\
-\frac{1}{64}e^{6i\alpha} & \frac{\sqrt{3}}{32}e^{5i\alpha} & -\frac{\sqrt{66}}{64}e^{4i\alpha} & \frac{\sqrt{55}}{32}e^{3i\alpha} & -\frac{3\sqrt{55}}{64}e^{2i\alpha} & \frac{3\sqrt{22}}{32}e^{i\alpha} & -\frac{\sqrt{231}}{32} & \frac{3\sqrt{22}}{32}e^{-i\alpha} & -\frac{3\sqrt{55}}{64}e^{-2i\alpha} & \frac{\sqrt{55}}{32}e^{-3i\alpha} & -\frac{\sqrt{66}}{64}e^{-4i\alpha} & \frac{\sqrt{3}}{32}e^{-5i\alpha} & -\frac{1}{64}e^{-6i\alpha}
\end{bmatrix}
$$

$\mathfrak{D}^7(\alpha,\frac{\pi}{2},\frac{\pi}{2})^{-1}$

$$\left[\begin{array}{ccccccccccccccc}
-i\frac{1}{128}e^{7i\alpha} & -i\frac{\sqrt{14}}{128}e^{6i\alpha} & -i\frac{\sqrt{91}}{128}e^{5i\alpha} & -i\frac{\sqrt{91}}{64}e^{4i\alpha} & -i\frac{\sqrt{1001}}{128}e^{3i\alpha} & -i\frac{\sqrt{2002}}{128}e^{2i\alpha} & -i\frac{\sqrt{3003}}{128}e^{i\alpha} & -i\frac{\sqrt{858}}{64} & -i\frac{\sqrt{3003}}{128}e^{-i\alpha} & -i\frac{\sqrt{2002}}{128}e^{-2i\alpha} & -i\frac{\sqrt{1001}}{128}e^{-3i\alpha} & -i\frac{\sqrt{91}}{64}e^{-4i\alpha} & -i\frac{\sqrt{91}}{128}e^{-5i\alpha} & -i\frac{\sqrt{14}}{128}e^{-6i\alpha} & -i\frac{1}{128}e^{-7i\alpha} \\
\frac{\sqrt{14}}{128}e^{7i\alpha} & \frac{3}{32}e^{6i\alpha} & \frac{5\sqrt{26}}{128}e^{5i\alpha} & \frac{\sqrt{26}}{16}e^{4i\alpha} & \frac{3\sqrt{286}}{128}e^{3i\alpha} & \frac{\sqrt{143}}{32}e^{2i\alpha} & \frac{\sqrt{858}}{128}e^{i\alpha} & 0 & -\frac{\sqrt{858}}{128}e^{-i\alpha} & -\frac{\sqrt{143}}{32}e^{-2i\alpha} & -\frac{3\sqrt{286}}{128}e^{-3i\alpha} & -\frac{\sqrt{26}}{16}e^{-4i\alpha} & -\frac{5\sqrt{26}}{128}e^{-5i\alpha} & -\frac{3}{32}e^{-6i\alpha} & -\frac{\sqrt{14}}{128}e^{-7i\alpha} \\
i\frac{\sqrt{91}}{128}e^{7i\alpha} & i\frac{5\sqrt{26}}{128}e^{6i\alpha} & i\frac{43}{128}e^{5i\alpha} & i\frac{25}{64}e^{4i\alpha} & i\frac{11\sqrt{11}}{128}e^{3i\alpha} & i\frac{\sqrt{22}}{128}e^{2i\alpha} & -i\frac{5\sqrt{33}}{128}e^{i\alpha} & -i\frac{\sqrt{462}}{64} & -i\frac{5\sqrt{33}}{128}e^{-i\alpha} & i\frac{\sqrt{22}}{128}e^{-2i\alpha} & i\frac{11\sqrt{11}}{128}e^{-3i\alpha} & i\frac{25}{64}e^{-4i\alpha} & i\frac{43}{128}e^{-5i\alpha} & i\frac{5\sqrt{26}}{128}e^{-6i\alpha} & i\frac{\sqrt{91}}{128}e^{-7i\alpha} \\
-\frac{\sqrt{91}}{64}e^{7i\alpha} & -\frac{\sqrt{26}}{16}e^{6i\alpha} & -\frac{25}{64}e^{5i\alpha} & -\frac{1}{4}e^{4i\alpha} & \frac{\sqrt{11}}{64}e^{3i\alpha} & \frac{\sqrt{22}}{16}e^{2i\alpha} & \frac{3\sqrt{33}}{64}e^{i\alpha} & 0 & -\frac{3\sqrt{33}}{64}e^{-i\alpha} & -\frac{\sqrt{22}}{16}e^{-2i\alpha} & -\frac{\sqrt{11}}{64}e^{-3i\alpha} & \frac{1}{4}e^{-4i\alpha} & \frac{25}{64}e^{-5i\alpha} & \frac{\sqrt{26}}{16}e^{-6i\alpha} & \frac{\sqrt{91}}{64}e^{-7i\alpha} \\
-i\frac{\sqrt{1001}}{128}e^{7i\alpha} & -i\frac{3\sqrt{286}}{128}e^{6i\alpha} & -i\frac{11\sqrt{11}}{128}e^{5i\alpha} & i\frac{\sqrt{11}}{64}e^{4i\alpha} & i\frac{39}{128}e^{3i\alpha} & i\frac{19\sqrt{2}}{128}e^{2i\alpha} & -i\frac{9\sqrt{3}}{128}e^{i\alpha} & -i\frac{3\sqrt{42}}{64} & -i\frac{9\sqrt{3}}{128}e^{-i\alpha} & i\frac{19\sqrt{2}}{128}e^{-2i\alpha} & i\frac{39}{128}e^{-3i\alpha} & i\frac{\sqrt{11}}{64}e^{-4i\alpha} & -i\frac{11\sqrt{11}}{128}e^{-5i\alpha} & -i\frac{3\sqrt{286}}{128}e^{-6i\alpha} & -i\frac{\sqrt{1001}}{128}e^{-7i\alpha} \\
\frac{\sqrt{2002}}{128}e^{7i\alpha} & \frac{\sqrt{143}}{32}e^{6i\alpha} & \frac{\sqrt{22}}{128}e^{5i\alpha} & -\frac{\sqrt{22}}{16}e^{4i\alpha} & -\frac{19\sqrt{2}}{128}e^{3i\alpha} & \frac{5}{32}e^{2i\alpha} & \frac{15\sqrt{6}}{128}e^{i\alpha} & 0 & -\frac{15\sqrt{6}}{128}e^{-i\alpha} & -\frac{5}{32}e^{-2i\alpha} & \frac{19\sqrt{2}}{128}e^{-3i\alpha} & \frac{\sqrt{22}}{16}e^{-4i\alpha} & -\frac{\sqrt{22}}{128}e^{-5i\alpha} & -\frac{\sqrt{143}}{32}e^{-6i\alpha} & -\frac{\sqrt{2002}}{128}e^{-7i\alpha} \\
i\frac{\sqrt{3003}}{128}e^{7i\alpha} & i\frac{\sqrt{858}}{128}e^{6i\alpha} & -i\frac{5\sqrt{33}}{128}e^{5i\alpha} & -i\frac{3\sqrt{33}}{64}e^{4i\alpha} & i\frac{9\sqrt{3}}{128}e^{3i\alpha} & i\frac{15\sqrt{6}}{128}e^{2i\alpha} & -i\frac{5}{128}e^{i\alpha} & -i\frac{5\sqrt{14}}{64} & -i\frac{5}{128}e^{-i\alpha} & i\frac{15\sqrt{6}}{128}e^{-2i\alpha} & i\frac{9\sqrt{3}}{128}e^{-3i\alpha} & -i\frac{3\sqrt{33}}{64}e^{-4i\alpha} & -i\frac{5\sqrt{33}}{128}e^{-5i\alpha} & i\frac{\sqrt{858}}{128}e^{-6i\alpha} & i\frac{\sqrt{3003}}{128}e^{-7i\alpha} \\
-\frac{\sqrt{858}}{64}e^{7i\alpha} & 0 & \frac{\sqrt{462}}{64}e^{5i\alpha} & 0 & -\frac{3\sqrt{42}}{64}e^{3i\alpha} & 0 & \frac{5\sqrt{14}}{64}e^{i\alpha} & 0 & -\frac{5\sqrt{14}}{64}e^{-i\alpha} & 0 & \frac{3\sqrt{42}}{64}e^{-3i\alpha} & 0 & -\frac{\sqrt{462}}{64}e^{-5i\alpha} & 0 & \frac{\sqrt{858}}{64}e^{-7i\alpha} \\
-i\frac{\sqrt{3003}}{128}e^{7i\alpha} & i\frac{\sqrt{858}}{128}e^{6i\alpha} & i\frac{5\sqrt{33}}{128}e^{5i\alpha} & -i\frac{3\sqrt{33}}{64}e^{4i\alpha} & -i\frac{9\sqrt{3}}{128}e^{3i\alpha} & i\frac{15\sqrt{6}}{128}e^{2i\alpha} & i\frac{5}{128}e^{i\alpha} & -i\frac{5\sqrt{14}}{64} & i\frac{5}{128}e^{-i\alpha} & i\frac{15\sqrt{6}}{128}e^{-2i\alpha} & -i\frac{9\sqrt{3}}{128}e^{-3i\alpha} & -i\frac{3\sqrt{33}}{64}e^{-4i\alpha} & i\frac{5\sqrt{33}}{128}e^{-5i\alpha} & i\frac{\sqrt{858}}{128}e^{-6i\alpha} & -i\frac{\sqrt{3003}}{128}e^{-7i\alpha} \\
\frac{\sqrt{2002}}{128}e^{7i\alpha} & -\frac{\sqrt{143}}{32}e^{6i\alpha} & \frac{\sqrt{22}}{128}e^{5i\alpha} & \frac{\sqrt{22}}{16}e^{4i\alpha} & -\frac{19\sqrt{2}}{128}e^{3i\alpha} & -\frac{5}{32}e^{2i\alpha} & \frac{15\sqrt{6}}{128}e^{i\alpha} & 0 & -\frac{15\sqrt{6}}{128}e^{-i\alpha} & \frac{5}{32}e^{-2i\alpha} & \frac{19\sqrt{2}}{128}e^{-3i\alpha} & -\frac{\sqrt{22}}{16}e^{-4i\alpha} & -\frac{\sqrt{22}}{128}e^{-5i\alpha} & \frac{\sqrt{143}}{32}e^{-6i\alpha} & -\frac{\sqrt{2002}}{128}e^{-7i\alpha} \\
i\frac{\sqrt{1001}}{128}e^{7i\alpha} & -i\frac{3\sqrt{286}}{128}e^{6i\alpha} & i\frac{11\sqrt{11}}{128}e^{5i\alpha} & i\frac{\sqrt{11}}{64}e^{4i\alpha} & -i\frac{39}{128}e^{3i\alpha} & i\frac{19\sqrt{2}}{128}e^{2i\alpha} & i\frac{9\sqrt{3}}{128}e^{i\alpha} & -i\frac{3\sqrt{42}}{64} & i\frac{9\sqrt{3}}{128}e^{-i\alpha} & i\frac{19\sqrt{2}}{128}e^{-2i\alpha} & -i\frac{39}{128}e^{-3i\alpha} & i\frac{\sqrt{11}}{64}e^{-4i\alpha} & i\frac{11\sqrt{11}}{128}e^{-5i\alpha} & -i\frac{3\sqrt{286}}{128}e^{-6i\alpha} & i\frac{\sqrt{1001}}{128}e^{-7i\alpha} \\
-\frac{\sqrt{91}}{64}e^{7i\alpha} & \frac{\sqrt{26}}{16}e^{6i\alpha} & -\frac{25}{64}e^{5i\alpha} & \frac{1}{4}e^{4i\alpha} & \frac{\sqrt{11}}{64}e^{3i\alpha} & -\frac{\sqrt{22}}{16}e^{2i\alpha} & \frac{3\sqrt{33}}{64}e^{i\alpha} & 0 & -\frac{3\sqrt{33}}{64}e^{-i\alpha} & \frac{\sqrt{22}}{16}e^{-2i\alpha} & -\frac{\sqrt{11}}{64}e^{-3i\alpha} & -\frac{1}{4}e^{-4i\alpha} & \frac{25}{64}e^{-5i\alpha} & -\frac{\sqrt{26}}{16}e^{-6i\alpha} & \frac{\sqrt{91}}{64}e^{-7i\alpha} \\
-i\frac{\sqrt{91}}{128}e^{7i\alpha} & i\frac{5\sqrt{26}}{128}e^{6i\alpha} & -i\frac{43}{128}e^{5i\alpha} & i\frac{25}{64}e^{4i\alpha} & -i\frac{11\sqrt{11}}{128}e^{3i\alpha} & i\frac{\sqrt{22}}{128}e^{2i\alpha} & i\frac{5\sqrt{33}}{128}e^{i\alpha} & -i\frac{\sqrt{462}}{64} & i\frac{5\sqrt{33}}{128}e^{-i\alpha} & i\frac{\sqrt{22}}{128}e^{-2i\alpha} & -i\frac{11\sqrt{11}}{128}e^{-3i\alpha} & i\frac{25}{64}e^{-4i\alpha} & -i\frac{43}{128}e^{-5i\alpha} & i\frac{5\sqrt{26}}{128}e^{-6i\alpha} & -i\frac{\sqrt{91}}{128}e^{-7i\alpha} \\
\frac{\sqrt{14}}{128}e^{7i\alpha} & -\frac{3}{32}e^{6i\alpha} & \frac{5\sqrt{26}}{128}e^{5i\alpha} & -\frac{\sqrt{26}}{16}e^{4i\alpha} & \frac{3\sqrt{286}}{128}e^{3i\alpha} & -\frac{\sqrt{143}}{32}e^{2i\alpha} & \frac{\sqrt{858}}{128}e^{i\alpha} & 0 & -\frac{\sqrt{858}}{128}e^{-i\alpha} & \frac{\sqrt{143}}{32}e^{-2i\alpha} & -\frac{3\sqrt{286}}{128}e^{-3i\alpha} & \frac{\sqrt{26}}{16}e^{-4i\alpha} & -\frac{5\sqrt{26}}{128}e^{-5i\alpha} & \frac{3}{32}e^{-6i\alpha} & -\frac{\sqrt{14}}{128}e^{-7i\alpha} \\
i\frac{1}{128}e^{7i\alpha} & -i\frac{\sqrt{14}}{128}e^{6i\alpha} & i\frac{\sqrt{91}}{128}e^{5i\alpha} & -i\frac{\sqrt{91}}{64}e^{4i\alpha} & i\frac{\sqrt{1001}}{128}e^{3i\alpha} & -i\frac{\sqrt{2002}}{128}e^{2i\alpha} & i\frac{\sqrt{3003}}{128}e^{i\alpha} & -i\frac{\sqrt{858}}{64} & i\frac{\sqrt{3003}}{128}e^{-i\alpha} & -i\frac{\sqrt{2002}}{128}e^{-2i\alpha} & i\frac{\sqrt{1001}}{128}e^{-3i\alpha} & -i\frac{\sqrt{91}}{64}e^{-4i\alpha} & i\frac{\sqrt{91}}{128}e^{-5i\alpha} & -i\frac{\sqrt{14}}{128}e^{-6i\alpha} & i\frac{1}{128}e^{-7i\alpha}
\end{array}\right]$$

$$\mathfrak{D}^{8}\left(\alpha,\tfrac{\pi}{2},\tfrac{\pi}{2}\right)^{-1}$$

$$\left[\begin{array}{ccccccccccccccccc}
\frac{1}{256}e^{i8\alpha} & \frac{1}{64}e^{i7\alpha} & \frac{\sqrt{30}}{128}e^{i6\alpha} & \frac{\sqrt{35}}{64}e^{i5\alpha} & \frac{\sqrt{455}}{128}e^{i4\alpha} & \frac{\sqrt{273}}{64}e^{i3\alpha} & \frac{\sqrt{2002}}{128}e^{i2\alpha} & \frac{\sqrt{715}}{64}e^{i\alpha} & \frac{3\sqrt{1430}}{256} & \frac{\sqrt{715}}{64}e^{-i\alpha} & \frac{\sqrt{2002}}{128}e^{-i2\alpha} & \frac{\sqrt{273}}{64}e^{-i3\alpha} & \frac{\sqrt{455}}{128}e^{-i4\alpha} & \frac{\sqrt{35}}{64}e^{-i5\alpha} & \frac{\sqrt{30}}{128}e^{-i6\alpha} & \frac{1}{64}e^{-i7\alpha} & \frac{1}{256}e^{-i8\alpha} \\
i\frac{1}{64}e^{i8\alpha} & i\frac{7}{128}e^{i7\alpha} & i\frac{3\sqrt{30}}{128}e^{i6\alpha} & i\frac{5\sqrt{35}}{128}e^{i5\alpha} & i\frac{\sqrt{455}}{64}e^{i4\alpha} & i\frac{3\sqrt{273}}{128}e^{i3\alpha} & i\frac{\sqrt{2002}}{128}e^{i2\alpha} & i\frac{\sqrt{715}}{128}e^{i\alpha} & 0 & -i\frac{\sqrt{715}}{128}e^{-i\alpha} & -i\frac{\sqrt{2002}}{128}e^{-i2\alpha} & -i\frac{3\sqrt{273}}{128}e^{-i3\alpha} & -i\frac{\sqrt{455}}{64}e^{-i4\alpha} & -i\frac{5\sqrt{35}}{128}e^{-i5\alpha} & -i\frac{3\sqrt{30}}{128}e^{-i6\alpha} & -i\frac{7}{128}e^{-i7\alpha} & -i\frac{1}{64}e^{-i8\alpha} \\
-\frac{\sqrt{30}}{128}e^{i8\alpha} & -\frac{3\sqrt{30}}{128}e^{i7\alpha} & -\frac{1}{4}e^{i6\alpha} & -\frac{7\sqrt{42}}{128}e^{i5\alpha} & -\frac{\sqrt{546}}{64}e^{i4\alpha} & -\frac{\sqrt{910}}{128}e^{i3\alpha} & 0 & \frac{\sqrt{858}}{128}e^{i\alpha} & \frac{\sqrt{429}}{64} & \frac{\sqrt{858}}{128}e^{-i\alpha} & 0 & -\frac{\sqrt{910}}{128}e^{-i3\alpha} & -\frac{\sqrt{546}}{64}e^{-i4\alpha} & -\frac{7\sqrt{42}}{128}e^{-i5\alpha} & -\frac{1}{4}e^{-i6\alpha} & -\frac{3\sqrt{30}}{128}e^{-i7\alpha} & -\frac{\sqrt{30}}{128}e^{-i8\alpha} \\
-i\frac{\sqrt{35}}{64}e^{i8\alpha} & -i\frac{5\sqrt{35}}{128}e^{i7\alpha} & -i\frac{7\sqrt{42}}{128}e^{i6\alpha} & -i\frac{45}{128}e^{i5\alpha} & -i\frac{3\sqrt{13}}{64}e^{i4\alpha} & i\frac{\sqrt{195}}{128}e^{i3\alpha} & i\frac{\sqrt{1430}}{128}e^{i2\alpha} & i\frac{\sqrt{1001}}{128}e^{i\alpha} & 0 & -i\frac{\sqrt{1001}}{128}e^{-i\alpha} & -i\frac{\sqrt{1430}}{128}e^{-i2\alpha} & -i\frac{\sqrt{195}}{128}e^{-i3\alpha} & i\frac{3\sqrt{13}}{64}e^{-i4\alpha} & i\frac{45}{128}e^{-i5\alpha} & i\frac{7\sqrt{42}}{128}e^{-i6\alpha} & i\frac{5\sqrt{35}}{128}e^{-i7\alpha} & i\frac{\sqrt{35}}{64}e^{-i8\alpha} \\
\frac{\sqrt{455}}{128}e^{i8\alpha} & \frac{\sqrt{455}}{64}e^{i7\alpha} & \frac{\sqrt{546}}{64}e^{i6\alpha} & \frac{3\sqrt{13}}{64}e^{i5\alpha} & -\frac{9}{64}e^{i4\alpha} & -\frac{5\sqrt{15}}{64}e^{i3\alpha} & -\frac{\sqrt{110}}{64}e^{i2\alpha} & \frac{\sqrt{308}}{128}e^{i\alpha} & \frac{\sqrt{1386}}{128} & \frac{\sqrt{308}}{128}e^{-i\alpha} & -\frac{\sqrt{110}}{64}e^{-i2\alpha} & -\frac{5\sqrt{15}}{64}e^{-i3\alpha} & -\frac{9}{64}e^{-i4\alpha} & \frac{3\sqrt{13}}{64}e^{-i5\alpha} & \frac{\sqrt{546}}{64}e^{-i6\alpha} & \frac{\sqrt{455}}{64}e^{-i7\alpha} & \frac{\sqrt{455}}{128}e^{-i8\alpha} \\
i\frac{\sqrt{273}}{64}e^{i8\alpha} & i\frac{3\sqrt{273}}{128}e^{i7\alpha} & i\frac{\sqrt{910}}{128}e^{i6\alpha} & -i\frac{\sqrt{195}}{128}e^{i5\alpha} & -i\frac{5\sqrt{15}}{64}e^{i4\alpha} & -i\frac{17}{128}e^{i3\alpha} & i\frac{\sqrt{594}}{128}e^{i2\alpha} & i\frac{\sqrt{1155}}{128}e^{i\alpha} & 0 & -i\frac{\sqrt{1155}}{128}e^{-i\alpha} & -i\frac{\sqrt{594}}{128}e^{-i2\alpha} & i\frac{17}{128}e^{-i3\alpha} & i\frac{5\sqrt{15}}{64}e^{-i4\alpha} & i\frac{\sqrt{195}}{128}e^{-i5\alpha} & -i\frac{\sqrt{910}}{128}e^{-i6\alpha} & -i\frac{3\sqrt{273}}{128}e^{-i7\alpha} & -i\frac{\sqrt{273}}{64}e^{-i8\alpha} \\
-\frac{\sqrt{2002}}{128}e^{i8\alpha} & -\frac{\sqrt{2002}}{128}e^{i7\alpha} & 0 & \frac{\sqrt{1430}}{128}e^{i5\alpha} & \frac{\sqrt{110}}{64}e^{i4\alpha} & -\frac{\sqrt{594}}{128}e^{i3\alpha} & -\frac{1}{4}e^{i2\alpha} & \frac{\sqrt{70}}{128}e^{i\alpha} & \frac{\sqrt{315}}{64} & \frac{\sqrt{70}}{128}e^{-i\alpha} & -\frac{1}{4}e^{-i2\alpha} & -\frac{\sqrt{594}}{128}e^{-i3\alpha} & \frac{\sqrt{110}}{64}e^{-i4\alpha} & \frac{\sqrt{1430}}{128}e^{-i5\alpha} & 0 & -\frac{\sqrt{2002}}{128}e^{-i7\alpha} & -\frac{\sqrt{2002}}{128}e^{-i8\alpha} \\
-i\frac{\sqrt{715}}{64}e^{i8\alpha} & -i\frac{\sqrt{715}}{128}e^{i7\alpha} & i\frac{\sqrt{858}}{128}e^{i6\alpha} & i\frac{\sqrt{1001}}{128}e^{i5\alpha} & -i\frac{\sqrt{308}}{128}e^{i4\alpha} & -i\frac{\sqrt{1155}}{128}e^{i3\alpha} & i\frac{\sqrt{70}}{128}e^{i2\alpha} & i\frac{35}{128}e^{i\alpha} & 0 & -i\frac{35}{128}e^{-i\alpha} & -i\frac{\sqrt{70}}{128}e^{-i2\alpha} & i\frac{\sqrt{1155}}{128}e^{-i3\alpha} & i\frac{\sqrt{308}}{128}e^{-i4\alpha} & -i\frac{\sqrt{1001}}{128}e^{-i5\alpha} & -i\frac{\sqrt{858}}{128}e^{-i6\alpha} & i\frac{\sqrt{715}}{128}e^{-i7\alpha} & i\frac{\sqrt{715}}{64}e^{-i8\alpha} \\
\frac{3\sqrt{1430}}{256}e^{i8\alpha} & 0 & -\frac{\sqrt{429}}{64}e^{i6\alpha} & 0 & \frac{\sqrt{1386}}{128}e^{i4\alpha} & 0 & -\frac{\sqrt{315}}{64}e^{i2\alpha} & 0 & \frac{35}{128} & 0 & -\frac{\sqrt{315}}{64}e^{-i2\alpha} & 0 & \frac{\sqrt{1386}}{128}e^{-i4\alpha} & 0 & -\frac{\sqrt{429}}{64}e^{-i6\alpha} & 0 & \frac{3\sqrt{1430}}{256}e^{-i8\alpha} \\
i\frac{\sqrt{715}}{64}e^{i8\alpha} & -i\frac{\sqrt{715}}{128}e^{i7\alpha} & -i\frac{\sqrt{858}}{128}e^{i6\alpha} & i\frac{\sqrt{1001}}{128}e^{i5\alpha} & i\frac{\sqrt{308}}{128}e^{i4\alpha} & -i\frac{\sqrt{1155}}{128}e^{i3\alpha} & -i\frac{\sqrt{70}}{128}e^{i2\alpha} & i\frac{35}{128}e^{i\alpha} & 0 & -i\frac{35}{128}e^{-i\alpha} & i\frac{\sqrt{70}}{128}e^{-i2\alpha} & i\frac{\sqrt{1155}}{128}e^{-i3\alpha} & -i\frac{\sqrt{308}}{128}e^{-i4\alpha} & -i\frac{\sqrt{1001}}{128}e^{-i5\alpha} & i\frac{\sqrt{858}}{128}e^{-i6\alpha} & i\frac{\sqrt{715}}{128}e^{-i7\alpha} & -i\frac{\sqrt{715}}{64}e^{-i8\alpha} \\
-\frac{\sqrt{2002}}{128}e^{i8\alpha} & \frac{\sqrt{2002}}{128}e^{i7\alpha} & 0 & -\frac{\sqrt{1430}}{128}e^{i5\alpha} & \frac{\sqrt{110}}{64}e^{i4\alpha} & \frac{\sqrt{594}}{128}e^{i3\alpha} & -\frac{1}{4}e^{i2\alpha} & -\frac{\sqrt{70}}{128}e^{i\alpha} & \frac{\sqrt{315}}{64} & -\frac{\sqrt{70}}{128}e^{-i\alpha} & -\frac{1}{4}e^{-i2\alpha} & \frac{\sqrt{594}}{128}e^{-i3\alpha} & \frac{\sqrt{110}}{64}e^{-i4\alpha} & -\frac{\sqrt{1430}}{128}e^{-i5\alpha} & 0 & \frac{\sqrt{2002}}{128}e^{-i7\alpha} & -\frac{\sqrt{2002}}{128}e^{-i8\alpha} \\
-i\frac{\sqrt{273}}{64}e^{i8\alpha} & i\frac{3\sqrt{273}}{128}e^{i7\alpha} & -i\frac{\sqrt{910}}{128}e^{i6\alpha} & -i\frac{\sqrt{195}}{128}e^{i5\alpha} & i\frac{5\sqrt{15}}{64}e^{i4\alpha} & -i\frac{17}{128}e^{i3\alpha} & -i\frac{\sqrt{594}}{128}e^{i2\alpha} & i\frac{\sqrt{1155}}{128}e^{i\alpha} & 0 & -i\frac{\sqrt{1155}}{128}e^{-i\alpha} & i\frac{\sqrt{594}}{128}e^{-i2\alpha} & i\frac{17}{128}e^{-i3\alpha} & -i\frac{5\sqrt{15}}{64}e^{-i4\alpha} & i\frac{\sqrt{195}}{128}e^{-i5\alpha} & i\frac{\sqrt{910}}{128}e^{-i6\alpha} & -i\frac{3\sqrt{273}}{128}e^{-i7\alpha} & i\frac{\sqrt{273}}{64}e^{-i8\alpha} \\
\frac{\sqrt{455}}{128}e^{i8\alpha} & -\frac{\sqrt{455}}{64}e^{i7\alpha} & \frac{\sqrt{546}}{64}e^{i6\alpha} & -\frac{3\sqrt{13}}{64}e^{i5\alpha} & -\frac{9}{64}e^{i4\alpha} & \frac{5\sqrt{15}}{64}e^{i3\alpha} & -\frac{\sqrt{110}}{64}e^{i2\alpha} & -\frac{\sqrt{308}}{128}e^{i\alpha} & \frac{\sqrt{1386}}{128} & -\frac{\sqrt{308}}{128}e^{-i\alpha} & -\frac{\sqrt{110}}{64}e^{-i2\alpha} & \frac{5\sqrt{15}}{64}e^{-i3\alpha} & -\frac{9}{64}e^{-i4\alpha} & -\frac{3\sqrt{13}}{64}e^{-i5\alpha} & \frac{\sqrt{546}}{64}e^{-i6\alpha} & -\frac{\sqrt{455}}{64}e^{-i7\alpha} & \frac{\sqrt{455}}{128}e^{-i8\alpha} \\
i\frac{\sqrt{35}}{64}e^{i8\alpha} & -i\frac{5\sqrt{35}}{128}e^{i7\alpha} & i\frac{7\sqrt{42}}{128}e^{i6\alpha} & -i\frac{45}{128}e^{i5\alpha} & i\frac{3\sqrt{13}}{64}e^{i4\alpha} & i\frac{\sqrt{195}}{128}e^{i3\alpha} & -i\frac{\sqrt{1430}}{128}e^{i2\alpha} & i\frac{\sqrt{1001}}{128}e^{i\alpha} & 0 & -i\frac{\sqrt{1001}}{128}e^{-i\alpha} & i\frac{\sqrt{1430}}{128}e^{-i2\alpha} & -i\frac{\sqrt{195}}{128}e^{-i3\alpha} & -i\frac{3\sqrt{13}}{64}e^{-i4\alpha} & i\frac{45}{128}e^{-i5\alpha} & -i\frac{7\sqrt{42}}{128}e^{-i6\alpha} & i\frac{5\sqrt{35}}{128}e^{-i7\alpha} & -i\frac{\sqrt{35}}{64}e^{-i8\alpha} \\
-\frac{\sqrt{30}}{128}e^{i8\alpha} & \frac{3\sqrt{30}}{128}e^{i7\alpha} & -\frac{1}{4}e^{i6\alpha} & \frac{7\sqrt{42}}{128}e^{i5\alpha} & -\frac{\sqrt{546}}{64}e^{i4\alpha} & \frac{\sqrt{910}}{128}e^{i3\alpha} & 0 & -\frac{\sqrt{858}}{128}e^{i\alpha} & \frac{\sqrt{429}}{64} & -\frac{\sqrt{858}}{128}e^{-i\alpha} & 0 & \frac{\sqrt{910}}{128}e^{-i3\alpha} & -\frac{\sqrt{546}}{64}e^{-i4\alpha} & \frac{7\sqrt{42}}{128}e^{-i5\alpha} & -\frac{1}{4}e^{-i6\alpha} & \frac{3\sqrt{30}}{128}e^{-i7\alpha} & -\frac{\sqrt{30}}{128}e^{-i8\alpha} \\
-i\frac{1}{64}e^{i8\alpha} & i\frac{7}{128}e^{i7\alpha} & -i\frac{3\sqrt{30}}{128}e^{i6\alpha} & i\frac{5\sqrt{35}}{128}e^{i5\alpha} & -i\frac{\sqrt{455}}{64}e^{i4\alpha} & i\frac{3\sqrt{273}}{128}e^{i3\alpha} & -i\frac{\sqrt{2002}}{128}e^{i2\alpha} & i\frac{\sqrt{715}}{128}e^{i\alpha} & 0 & -i\frac{\sqrt{715}}{128}e^{-i\alpha} & i\frac{\sqrt{2002}}{128}e^{-i2\alpha} & -i\frac{3\sqrt{273}}{128}e^{-i3\alpha} & i\frac{\sqrt{455}}{64}e^{-i4\alpha} & -i\frac{5\sqrt{35}}{128}e^{-i5\alpha} & i\frac{3\sqrt{30}}{128}e^{-i6\alpha} & -i\frac{7}{128}e^{-i7\alpha} & i\frac{1}{64}e^{-i8\alpha} \\
\frac{1}{256}e^{i8\alpha} & -\frac{1}{64}e^{i7\alpha} & \frac{\sqrt{30}}{128}e^{i6\alpha} & -\frac{\sqrt{35}}{64}e^{i5\alpha} & \frac{\sqrt{455}}{128}e^{i4\alpha} & -\frac{\sqrt{273}}{64}e^{i3\alpha} & \frac{\sqrt{2002}}{128}e^{i2\alpha} & -\frac{\sqrt{715}}{64}e^{i\alpha} & \frac{3\sqrt{1430}}{256} & -\frac{\sqrt{715}}{64}e^{-i\alpha} & \frac{\sqrt{2002}}{128}e^{-i2\alpha} & -\frac{\sqrt{273}}{64}e^{-i3\alpha} & \frac{\sqrt{455}}{128}e^{-i4\alpha} & -\frac{\sqrt{35}}{64}e^{-i5\alpha} & \frac{\sqrt{30}}{128}e^{-i6\alpha} & -\frac{1}{64}e^{-i7\alpha} & \frac{1}{256}e^{-i8\alpha}
\end{array}\right]$$

Table 5

Coefficients relating Stevens operators to Racah operators

l	m	K_l^m	$\sqrt{\frac{2l+1}{4\pi}} \Big/ K_l^m$
1	0	$\frac{1}{2}\sqrt{\frac{3}{\pi}}$	1
1	1	$\frac{1}{2}\sqrt{\frac{3}{\pi}}$	1
2	0	$\frac{1}{4}\sqrt{\frac{5}{\pi}}$	2
2	1	$\frac{1}{2}\sqrt{\frac{15}{\pi}}$	$\frac{1}{\sqrt{3}}$
2	2	$\frac{1}{4}\sqrt{\frac{15}{\pi}}$	$\frac{2}{\sqrt{3}}$
3	0	$\frac{1}{4}\sqrt{\frac{7}{\pi}}$	2
3	1	$\frac{1}{8}\sqrt{\frac{42}{\pi}}$	$\frac{4}{\sqrt{6}}$
3	2	$\frac{1}{4}\sqrt{\frac{105}{\pi}}$	$\frac{2}{\sqrt{15}}$
3	3	$\frac{1}{8}\sqrt{\frac{70}{\pi}}$	$\frac{4}{\sqrt{10}}$
4	0	$\frac{1}{16}\sqrt{\frac{9}{\pi}}$	8
4	1	$\frac{3}{8}\sqrt{\frac{10}{\pi}}$	$\frac{4}{\sqrt{10}}$
4	2	$\frac{3}{8}\sqrt{\frac{5}{\pi}}$	$\frac{4}{\sqrt{5}}$
4	3	$\frac{3}{8}\sqrt{\frac{70}{\pi}}$	$\frac{4}{\sqrt{70}}$
4	4	$\frac{3}{16}\sqrt{\frac{35}{\pi}}$	$\frac{8}{\sqrt{35}}$
5	0	$\frac{1}{16}\sqrt{\frac{11}{\pi}}$	8
5	1	$\frac{1}{16}\sqrt{\frac{165}{\pi}}$	$\frac{8}{\sqrt{15}}$
5	2	$\frac{1}{8}\sqrt{\frac{1155}{\pi}}$	$\frac{4}{\sqrt{105}}$
5	3	$\frac{1}{32}\sqrt{\frac{770}{\pi}}$	$\frac{16}{\sqrt{70}}$
5	4	$\frac{3}{16}\sqrt{\frac{385}{\pi}}$	$\frac{8}{3\sqrt{35}}$
5	5	$\frac{3}{32}\sqrt{\frac{154}{\pi}}$	$\frac{16}{3\sqrt{14}}$
6	0	$\frac{1}{32}\sqrt{\frac{13}{\pi}}$	16
6	1	$\frac{1}{16}\sqrt{\frac{273}{\pi}}$	$\frac{8}{\sqrt{21}}$
6	2	$\frac{1}{64}\sqrt{\frac{2730}{\pi}}$	$\frac{32}{\sqrt{210}}$
6	3	$\frac{1}{32}\sqrt{\frac{2730}{\pi}}$	$\frac{16}{\sqrt{210}}$
6	4	$\frac{3}{32}\sqrt{\frac{91}{\pi}}$	$\frac{16}{3\sqrt{7}}$
6	5	$\frac{3}{32}\sqrt{\frac{2002}{\pi}}$	$\frac{16}{3\sqrt{154}}$
6	6	$\frac{1}{64}\sqrt{\frac{6006}{\pi}}$	$\frac{32}{\sqrt{462}}$
7	0	$\frac{1}{32}\sqrt{\frac{15}{\pi}}$	16
7	1	$\frac{1}{256}\sqrt{\frac{105}{\pi}}$	$\frac{128}{\sqrt{7}}$
7	2	$\frac{3}{64}\sqrt{\frac{70}{\pi}}$	$\frac{32}{\sqrt{42}}$
7	3	$\frac{3}{128}\sqrt{\frac{95}{\pi}}$	$\frac{64}{\sqrt{21}}$
7	4	$\frac{3}{256}\sqrt{\frac{35}{\pi}}$	$\frac{128}{\sqrt{21}}$
7	5	$\frac{3}{128}\sqrt{\frac{770}{\pi}}$	$\frac{64}{\sqrt{462}}$
7	6	$\frac{3}{64}\sqrt{\frac{10.010}{\pi}}$	$\frac{32}{\sqrt{6006}}$
7	7	$\frac{21}{64}\sqrt{\frac{715}{\pi}}$	$\frac{32}{7\sqrt{429}}$
8	0	$\frac{1}{256}\sqrt{\frac{17}{\pi}}$	128
8	1	$\frac{3}{64}\sqrt{\frac{17}{\pi}}$	$\frac{32}{3}$
8	2	$\frac{3}{128}\sqrt{\frac{1190}{\pi}}$	$\frac{64}{3\sqrt{70}}$
8	3	$\frac{1}{64}\sqrt{\frac{19.635}{\pi}}$	$\frac{32}{\sqrt{1155}}$
8	4	$\frac{3}{128}\sqrt{\frac{1309}{\pi}}$	$\frac{64}{3\sqrt{77}}$
8	5	$\frac{3}{64}\sqrt{\frac{17.017}{\pi}}$	$\frac{32}{3\sqrt{1001}}$
8	6	$\frac{1}{128}\sqrt{\frac{14.586}{\pi}}$	$\frac{64}{\sqrt{858}}$
8	7	$\frac{3}{64}\sqrt{\frac{12.155}{\pi}}$	$\frac{32}{3\sqrt{715}}$
8	8	$\frac{3}{256}\sqrt{\frac{12.155}{\pi}}$	$\frac{128}{3\sqrt{715}}$

Table 6

Stevens operator equivalents

$X = J(J+1)$

$$O_2^0(c) = 2 \quad \tilde{O}_{20} \qquad = 3J_z^2 - X$$

$$O_2^2(c) = \frac{2}{\sqrt{3}}\frac{1}{\sqrt{2}}(\tilde{O}_{2,-2} + \tilde{O}_{2,2}) \qquad = \frac{1}{2}\left[(J^+)^2 + (J^-)^2\right]$$

$$O_4^0(c) = 8 \quad \tilde{O}_{40} \qquad = 35J_z^4 - \{30X - 25\}J_z^2 + 3X^2 - 6X$$

$$O_4^2(c) = \frac{4}{\sqrt{5}}\frac{1}{\sqrt{2}}(\tilde{O}_{4,-2} + \tilde{O}_{4,2}) \qquad = \frac{1}{4}\left[\{7J_z^2 - X - 5\}\left[(J^+)^2 + (J^-)^2\right] + \left[(J^+)^2 + (J^-)^2\right]\{\cdots\}\right]$$

$$O_4^4(c) = \frac{8}{\sqrt{35}}\frac{1}{\sqrt{2}}(\tilde{O}_{4,-4} + \tilde{O}_{4,4}) \qquad = \frac{1}{2}\left[(J^+)^4 + (J^-)^4\right]$$

$$O_6^0(c) = 16 \quad \tilde{O}_{60} \qquad = 231\,J_z^6 - \{315X - 735\}J_z^4 + \{105X^2 - 525X + 294\}J_z^2 - 5X^3 + 40X^2 - 60X$$

$$O_6^2(c) = \frac{32}{\sqrt{210}}\frac{1}{\sqrt{2}}(\tilde{O}_{6,-2} + \tilde{O}_{6,2}) \qquad = \frac{1}{4}\left[\{33J_z^4 - (18X + 123)J_z^2 + X^2 + 10X + 102\}\left[(J^+)^2 + (J^-)^2\right] + \left[(J^+)^2 + (J^-)^2\right]\{\cdots\}\right]$$

$$O_6^4(c) = \frac{16}{3\sqrt{7}\sqrt{2}}(\tilde{O}_{6,-4} + \tilde{O}_{6,4}) \qquad = \frac{1}{4}\left[\{11J_z^2 - X - 38\}\left[(J^+)^4 + (J^-)^4\right] + \left[(J^+)^4 + (J^-)^4\right]\{\cdots\}\right]$$

$$O_6^6(c) = \frac{32}{\sqrt{462}}\frac{1}{\sqrt{2}}(\tilde{O}_{6,-6} + \tilde{O}_{6,6}) \qquad = \frac{1}{2}\left[(J^+)^6 + (J^-)^6\right]$$

$$\begin{aligned} O_8^0(c) = 128 \quad \tilde{O}_{80} \qquad &= 6435J_z^8 - \{12012X - 54054\}J_z^6 + \{6930X^2 - 64680X + 93\,555\}J_z^4 \\ &\quad + \{-1260X^3 + 18270X^2 - 59388X + 21\,390\}J_z^2 \\ &\quad + 35X^4 - 700X^3 + 3780X^2 - 5040X \end{aligned}$$

$$\begin{aligned} O_8^2(c) = \frac{64}{3\sqrt{70}}\frac{1}{\sqrt{2}}(\tilde{O}_{8,-2} + \tilde{O}_{8,2}) \qquad &= \frac{1}{4}\Big[\{143J_z^6 - (143X + 1144)J_z^4 + (33X^2 + 407X + 5951)J_z^2 \\ &\quad - \{X^3 + 13X^2 + 372X + 4806\}\left[(J^+)^2 + (J^-)^2\right] + \left[(J^+)^2 - (J^-)^2\right]\{\cdots\}\Big] \end{aligned}$$

$$O_8^4(c) = \frac{64}{3\sqrt{77}}\frac{1}{\sqrt{2}}(\tilde{O}_{8,-4} + \tilde{O}_{8,4}) \qquad = \frac{1}{4}\left[\{65J_z^4 - (26X + 1313)J_z^2 + X^2 + 86X + 4284\}\left[(J^+)^4 + (J^-)^4\right] + \left[(J^+)^4 + (J^-)^4\right]\{\cdots\}\right]$$

$$O_8^6(c) = \frac{64}{\sqrt{858}}\frac{1}{\sqrt{2}}(\tilde{O}_{8,-6} + \tilde{O}_{8,6}) \qquad = \frac{1}{4}\left[\{15J_z^2 - X - 123\}\left[(J^+)^6 + (J^-)^6\right] + \left[(J^+)^6 + (J^-)^6\right]\{\cdots\}\right]$$

$$O_8^8(c) = \frac{128}{3\sqrt{715}}\frac{1}{\sqrt{2}}(\tilde{O}_{8,-8} + \tilde{O}_{8,8}) \qquad = \frac{1}{2}\left[(J^+)^8 + (J^-)^8\right]$$

Table 7

Stevens Operator equivalents expanded in Bose operators

$$S_n = J(J-\tfrac{1}{2})(J-1)\dots(J-\tfrac{n-1}{2})$$

$$O_2^0(c) = 2\,\tilde{O}_{20} = 2S_2\left\{1 - \frac{3}{S_1}a^+a + \frac{3}{2S_2}a^+a^+aa\right\}$$

$$O_2^2(c) = \frac{2}{\sqrt{5}}\frac{1}{\sqrt{2}}(\tilde{O}_{2,-2} + \tilde{O}_{2,2}) = \sqrt{S_2}\left\{(a^+a^+ + aa) - \sqrt{\frac{S_2}{S_1S_3}}\left[\sqrt{\frac{S_1S_3}{S_2}} - \frac{S_3}{S_2}\right](a^+a^+a^+a + a^+aaa) + \cdots\right\}$$

$$O_4^0(c) = 8\,\tilde{O}_{40} = 8S_4\left\{1 - \frac{10}{S_1}a^+a + \frac{45}{2S_2}a^+a^+aa + \cdots\right\}$$

$$O_4^2(c) = \frac{4}{\sqrt{5}}\frac{1}{\sqrt{2}}(\tilde{O}_{4,-2} + \tilde{O}_{4,2}) = 6\frac{S_4}{\sqrt{S_2}}\left\{(a^+a^+ + aa) - \sqrt{\frac{S_2}{S_1S_3}}\left[\frac{7}{3} + \sqrt{\frac{S_1S_3}{S_2}} - \frac{S_3}{S_2}\right](a^+a^+a^+a + a^+aaa) + \cdots\right\}$$

$$O_4^4(c) = \frac{8}{\sqrt{35}}\frac{1}{\sqrt{2}}(\tilde{O}_{4,-4} + \tilde{O}_{4,4}) = 2\sqrt{S_4}\,(a^+a^+a^+a^+ + aaaa) + \cdots$$

$$O_6^0(c) = 16\,\tilde{O}_{60} = 16S_6\left\{1 - \frac{21}{S_1}a^+a + \frac{105}{S_2}a^+a^+aa + \cdots\right\}$$

$$O_6^2(c) = \frac{32}{\sqrt{210}}\frac{1}{\sqrt{2}}(\tilde{O}_{6,-2} + \tilde{O}_{6,2}) = 16\frac{S_6}{\sqrt{S_2}}\left\{(a^+a^+ + aa) - \sqrt{\frac{S_2}{S_1S_3}}\left[6 + \sqrt{\frac{S_1S_3}{S_2}} - \frac{S_3}{S_2}\right](a^+a^+a^+a + a^+aaa) + \cdots\right\}$$

$$O_6^4(c) = \frac{16}{3\sqrt{7}}\frac{1}{\sqrt{2}}(\tilde{O}_{6,-4} + \tilde{O}_{6,4}) = 20\frac{S_6}{\sqrt{S_4}}(a^+a^+a^+a^+ + aaaa) + \cdots$$

$$O_6^6(c) = \cdots$$

$$O_8^0(c) = 128\,\tilde{O}_{80} = 128\,S_8\left\{1 - \frac{36}{S_1}a^+a + \frac{315}{S_2}a^+a^+aa + \cdots\right\}$$

$$O_8^2(c) = \frac{64}{3\sqrt{70}}\frac{1}{\sqrt{2}}(\tilde{O}_{8,-2} + \tilde{O}_{8,2}) = 32\frac{S_8}{\sqrt{S_2}}\left\{(a^+a^+ + aa) - \sqrt{\frac{S_2}{S_1S_3}}\left[11 + \sqrt{\frac{S_1S_3}{S_2}} - \frac{S_3}{S_2}\right](a^+a^+a^+a + a^+aaa) + \cdots\right\}$$

$$O_8^4(c) = \frac{64}{3\sqrt{77}}\frac{1}{\sqrt{2}}(\tilde{O}_{8,-4} + \tilde{O}_{8,4}) = 80\frac{S_8}{\sqrt{S_4}}(a^+a^+a^+a^+ + aaaa) + \cdots$$

$$O_8^6(c) = \cdots$$

$$O_8^8(c) = \cdots$$

Table 8

Transformation of Stevens operators on Rotation of the z-axis into the perpendicular plane xy so that z' makes an angle α with x, and y' coincides with z.

Old frame — New frame

$$O_2^0(c) \longrightarrow -\tfrac{1}{2}O_2^0(c) - \tfrac{3}{2}O_2^2(c)$$

$$O_2^2(c) \longrightarrow \left[\tfrac{1}{2}O_2^0(c) - \tfrac{1}{2}O_2^2(c)\right]\cos 2\alpha - 2\,O_2^1(c)\sin 2\alpha$$

$$O_4^0(c) \longrightarrow \tfrac{3}{8}O_4^0(c) + \tfrac{5}{2}O_4^2(c) + \tfrac{23}{8}O_4^4(c)$$

$$O_4^2(c) \longrightarrow \left[-\tfrac{1}{8}O_4^0(c) - \tfrac{1}{2}O_4^2(c) + \tfrac{7}{8}O_4^4(c)\right]\cos 2\alpha + \left[\tfrac{1}{2}O_4^1(c) + \tfrac{7}{2}O_4^3(c)\right]\sin 2\alpha$$

$$O_4^3(c) \longrightarrow \left[\tfrac{1}{8}O_4^4(S) - \tfrac{1}{4}O_4^2(S)\right]\sin 3\alpha + \left[-\tfrac{3}{4}O_4^3(S) + \tfrac{1}{4}O_4^1(S)\right]\cos 3\alpha$$

$$O_4^4(c) \longrightarrow \left[\tfrac{1}{8}O_4^0(c) - \tfrac{1}{2}O_4^2(c) + \tfrac{1}{8}O_4^4(c)\right]\cos 4\alpha - \left[O_4^1(c) - O_4^3(c)\right]\sin 4\alpha$$

$$O_6^0(c) \longrightarrow -\tfrac{5}{16}O_6^0(c) - \tfrac{105}{32}O_6^2(c) - \tfrac{63}{16}O_6^4(c) - \tfrac{231}{32}O_6^6(c)$$

$$O_6^2(c) \longrightarrow \left[\tfrac{1}{16}O_6^0(c) + \tfrac{17}{32}O_6^2(c) + \tfrac{3}{16}O_6^4(c) - \tfrac{33}{32}O_6^6(c)\right]\cos 2\alpha - \left[\tfrac{1}{4}O_6^1(c) + \tfrac{9}{8}O_6^3(c) + \tfrac{33}{8}O_6^5(c)\right]\sin 2\alpha$$

$$O_6^3(c) \longrightarrow \left[-\tfrac{11}{32}O_6^6(S) + \tfrac{3}{8}O_6^4(S) + \tfrac{9}{32}O_6^2(S)\right]\sin 3\alpha + \left[\tfrac{33}{16}O_6^5(S) - \tfrac{1}{16}O_6^3(S) - \tfrac{3}{8}O_6^1(S)\right]\cos 3\alpha$$

$$O_6^4(c) \longrightarrow \left[-\tfrac{1}{16}O_6^0(c) - \tfrac{5}{32}O_6^2(c) + \tfrac{13}{16}O_6^4(c) - \tfrac{11}{32}O_6^6(c)\right]\cos 4\alpha + \left[\tfrac{1}{2}O_6^1(c) + \tfrac{5}{4}O_6^3(c) - \tfrac{11}{4}O_6^5(c)\right]\sin 4\alpha$$

$$O_6^6(c) \longrightarrow \left[\tfrac{1}{16}O_6^0(c) - \tfrac{15}{32}O_6^2(c) + \tfrac{3}{16}O_6^4(c) - \tfrac{1}{32}O_6^6(c)\right]\cos 6\alpha - \left[\tfrac{3}{4}O_6^1(c) - \tfrac{3}{8}O_6^3(c) + \tfrac{3}{8}O_6^5(c)\right]\sin 6\alpha$$

$$O_8^0(c) \longrightarrow \tfrac{35}{128}O_8^0(c) + \tfrac{315}{16}O_8^2(c) + \tfrac{693}{32}O_8^4(c) + \tfrac{429}{16}O_8^6(c) + \tfrac{6435}{128}O_8^8(c)$$

$$O_8^2(c) \longrightarrow \left[-\tfrac{1}{128}O_8^0(c) - \tfrac{1}{2}O_8^2(c) - \tfrac{11}{32}O_8^4(c) + \tfrac{143}{128}O_8^8(c)\right]\cos 2\alpha + \left[\tfrac{1}{32}O_8^1(c) + \tfrac{33}{32}O_8^3(c) + \tfrac{143}{32}O_8^5(c) + \tfrac{143}{32}O_8^7(c)\right]\sin 2\alpha$$

$$O_8^4(c) \longrightarrow \left[\tfrac{1}{128}O_8^0(c) + \tfrac{10}{32}O_8^2(c) - \tfrac{9}{32}O_8^4(c) + \tfrac{13}{16}O_8^6(c) + \tfrac{65}{128}O_8^8(c)\right]\cos 4\alpha - \left[\tfrac{1}{16}O_8^1(c) + \tfrac{57}{16}O_8^3(c) + \tfrac{39}{16}O_8^5(c) - \tfrac{65}{16}O_8^7(c)\right]\sin 4\alpha$$

$$O_8^6(c) \longrightarrow \left[-\tfrac{1}{128}O_8^0(c) + \tfrac{21}{32}O_8^4(c) - \tfrac{1}{2}O_8^6(c) + \tfrac{15}{128}O_8^8(c)\right]\cos 6\alpha + \left[\tfrac{3}{32}O_8^1(c) + \tfrac{35}{32}O_8^3(c) - \tfrac{147}{32}O_8^5(c) + \tfrac{45}{32}O_8^7(c)\right]\sin 6\alpha$$

$$O_8^8(c) \longrightarrow \left[\tfrac{1}{128}O_8^0(c) - \tfrac{7}{16}O_8^2(c) + \tfrac{7}{32}O_8^4(c) - \tfrac{1}{16}O_8^6(c) + \tfrac{1}{128}O_8^8(c)\right]\cos 8\alpha - \left[\tfrac{1}{8}O_8^1(c) - \tfrac{7}{8}O_8^3(c) + \tfrac{7}{8}O_8^5(c) - \tfrac{1}{8}O_8^7(c)\right]\sin 8\alpha$$

Table 9 *Crystal potential energy expressed in Stevens operators*

Symmetry	Direction	Form of crystal potential energy
Cubic	(001)	$H_{cf,1} = B_4^0\left[O_4^0(c) + 5O_4^4(c)\right] + B_6^0\left[O_6^0(c) - 21\,O_6^4(c)\right]$
	(110)	$H_{cf,2} = -\frac{1}{4}B_4^0\left[O_4^0(c) - 20\,O_4^2(c) - 15\,O_4^4(c)\right] - \frac{13}{8}B_6^0\left[O_6^0(c) + \frac{105}{26}O_6^2(c) - \frac{105}{15}O_6^4(c) + \frac{231}{26}O_6^6(c)\right]$
	(111)	$H_{cf,3} = -\frac{2}{3}B_4^0\left[O_4^0(c) - 20\sqrt{2}\,O_4^3(c)\right] + \frac{16}{9}B_6^0\left[O_6^0(c) + \frac{35\sqrt{2}}{4}O_6^3(c) + \frac{77}{8}O_6^6(c)\right]$
Hexagonal	(0001)	$H_{cf,6} = B_2^0 O_2^0(c) + B_4^0 O_4^0(c) + B_6^0\left[O_6^0(c) + \frac{77}{8}O_6^6(c)\right]$
	(1000)	$H_{cf,2} = -\frac{1}{2}B_2^0\left[O_2^0(c) + 3\,O_2^2(c)\right] + \frac{3}{8}B_4^0\left[O_4^0(c) + \frac{20}{3}O_4^2(c) + \frac{35}{3}O_4^4(c)\right] + \frac{37}{128}B_6^0\left[O_6^0(c) - \frac{1995}{74}O_6^2(c) - \frac{273}{37}O_6^4(c) - \frac{1925}{74}O_6^6(c)\right]$
	(1200)	$H_{cf,2} = -\frac{1}{2}B_2^0\left[O_2^0(c) + 3\,O_2^2(c)\right] + \frac{3}{8}B_4^0\left[O_4^0(c) + \frac{20}{3}O_4^2(c) + \frac{35}{3}O_4^4(c)\right] - \frac{117}{128}B_6^0\left[O_6^0(c) - \frac{35}{26}O_6^2(c) + \frac{245}{39}O_6^4(c) - \frac{1771}{234}O_6^6(c)\right]$

Table 10 Crystal potential energy expressed in Racah operators

Symmetry	Direction	Form of crystal potential energy
Cubic	(001)	$H_{cf,4}=8B_4^0\left[\tilde{O}_{4,0}+\frac{\sqrt{70}}{14}(\tilde{O}_{4,-4}+\tilde{O}_{4,4})\right]+16B_6^0\left[\tilde{O}_{6,0}-\frac{\sqrt{14}}{2}(\tilde{O}_{6,-4}+\tilde{O}_{6,4})\right]$
	(110)	$H_{cf,2}=-2B_4^0\left[\tilde{O}_{4,0}-\sqrt{10}(\tilde{O}_{4,-2}+\tilde{O}_{4,2})-\frac{3\sqrt{70}}{14}(\tilde{O}_{4,-4}+\tilde{O}_{4,4})\right]-26B_6^0\left[\tilde{O}_{6,0}+\frac{\sqrt{105}}{26}(\tilde{O}_{6,-2}+\tilde{O}_{6,2})-\frac{5\sqrt{14}}{26}(\tilde{O}_{6,-4}+\tilde{O}_{6,4})+\frac{\sqrt{231}}{26}(\tilde{O}_{6,-6}+\tilde{O}_{6,6})\right]$
	(111)	$H_{cf,3}=-\frac{16}{3}B_4^0\left[\tilde{O}_{4,0}-\frac{\sqrt{70}}{7}(\tilde{O}_{4,-3}-\tilde{O}_{4,3})\right]+\frac{256}{9}B_6^0\left[\tilde{O}_{6,0}+\frac{\sqrt{210}}{24}(\tilde{O}_{6,-3}-\tilde{O}_{6,3})+\frac{\sqrt{231}}{24}(\tilde{O}_{6,-6}+\tilde{O}_{6,6})\right]$
Hexagonal	(0001)	$H_{cf,6}=2B_2^0\tilde{O}_{2,0}+8B_4^0\tilde{O}_{4,0}+16B_6^0\left[\tilde{O}_{6,0}+\frac{\sqrt{231}}{24}(\tilde{O}_{6,-6}+\tilde{O}_{6,6})\right]$
	(1000)	$H_{cf,2}=-B_2^0\left[\tilde{O}_{2,0}+\frac{\sqrt{6}}{2}(\tilde{O}_{2,-2}+\tilde{O}_{2,2})\right]+3B_4^0\left[\tilde{O}_{4,0}+\frac{\sqrt{10}}{3}(\tilde{O}_{4,-2}+\tilde{O}_{4,2})+\frac{\sqrt{70}}{6}(\tilde{O}_{4,-4}+\tilde{O}_{4,4})\right]$ $+\frac{37}{8}B_6^0\left[\tilde{O}_{6,0}-\frac{19\sqrt{105}}{74}(\tilde{O}_{6,-2}+\tilde{O}_{6,2})-\frac{13\sqrt{14}}{74}(\tilde{O}_{6,-4}+\tilde{O}_{6,4})-\frac{25\sqrt{231}}{222}(\tilde{O}_{6,-6}+\tilde{O}_{6,6})\right]$
	(1200)	$H_{cf,2}=-B_2^0\left[\tilde{O}_{2,0}+\frac{\sqrt{6}}{2}(\tilde{O}_{2,-2}+\tilde{O}_{2,2})\right]+3B_4^0\left[\tilde{O}_{4,0}+\frac{\sqrt{10}}{3}(\tilde{O}_{4,-2}+\tilde{O}_{4,2})+\frac{\sqrt{70}}{6}(\tilde{O}_{4,-4}+\tilde{O}_{4,4})\right]$ $-\frac{117}{8}B_6^0\left[\tilde{O}_{6,0}-\frac{\sqrt{105}}{78}(\tilde{O}_{6,-2}+\tilde{O}_{6,2})+\frac{35\sqrt{14}}{234}(\tilde{O}_{6,-4}+\tilde{O}_{6,4})+\frac{23\sqrt{231}}{702}(\tilde{O}_{6,-6}+\tilde{O}_{6,6})\right]$

Table 11. 3 j - symbols

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

j_1	j_2	j_3	m_1	m_2	m_3		
0	0	0	0	0	0	1.00000	
1	1	0	1	-1	0	0.57735	
1	1	0	0	0	0	-0.57735	
1	1	0	-1	1	0	0.57735	
1	1	1	1	-1	0	0.40825	*
1	1	1	1	0	-1	-0.40825	*
1	1	1	0	-1	1	-0.40825	*
1	1	1	0	1	-1	0.40825	*
1	1	1	-1	0	1	0.40825	*
1	1	1	-1	1	0	-0.40825	*
2	1	1	2	-1	-1	0.44721	
2	1	1	1	-1	0	-0.31623	
2	1	1	1	0	-1	-0.31623	
2	1	1	0	-1	1	0.18257	
2	1	1	0	0	0	0.36515	
2	1	1	0	1	-1	0.18257	
2	1	1	-1	0	1	-0.31623	
2	1	1	-1	1	0	-0.31623	
2	1	1	-2	1	1	0.44721	
2	2	0	2	-2	0	0.44721	
2	2	0	1	-1	0	-0.44721	
2	2	0	0	0	0	0.44721	
2	2	0	-1	1	0	-0.44721	
2	2	0	-2	2	0	0.44721	
2	2	1	2	-2	0	0.36515	*
2	2	1	2	-1	-1	-0.25820	*
2	2	1	1	-2	1	-0.25820	*
2	2	1	1	-1	0	-0.18257	*
2	2	1	1	0	-1	0.31623	*
2	2	1	0	-1	1	0.31623	*
2	2	1	0	1	-1	-0.31623	*
2	2	1	-1	0	1	-0.31623	*
2	2	1	-1	1	0	0.18257	*
2	2	1	-1	2	-1	0.25820	*
2	2	1	-2	1	1	0.25820	*
2	2	1	-2	2	0	-0.36515	*
2	2	2	2	-2	0	0.23905	
2	2	2	2	-1	-1	-0.29277	
2	2	2	2	0	-2	0.23905	
2	2	2	1	-2	1	-0.29277	
2	2	2	1	-1	0	0.11952	
2	2	2	1	0	-1	0.11952	
2	2	2	1	1	-2	-0.29277	
2	2	2	0	-2	2	0.23905	
2	2	2	0	-1	1	0.11952	
2	2	2	0	0	0	-0.23905	
2	2	2	0	1	-1	0.11952	
2	2	2	0	2	-2	0.23905	
2	2	2	-1	-1	2	-0.29277	
2	2	2	-1	0	1	0.11952	
2	2	2	-1	1	0	0.11952	
2	2	2	-1	2	-1	-0.29277	
2	2	2	-2	0	2	0.23905	
2	2	2	-2	1	1	-0.29277	
2	2	2	-2	2	0	0.23905	
3	2	1	3	-2	-1	0.37796	
3	2	1	2	-2	0	-0.21822	
3	2	1	2	-1	-1	-0.30861	
3	2	1	1	-2	1	0.09759	
3	2	1	1	-1	0	0.27603	
3	2	1	1	0	-1	0.23905	
3	2	1	0	-1	1	-0.16903	
3	2	1	0	0	0	-0.29277	
3	2	1	0	1	-1	-0.16903	
3	2	1	-1	0	1	0.23905	
3	2	1	-1	1	0	0.27603	
3	2	1	-1	2	-1	0.09759	
3	2	1	-2	1	1	-0.30861	
3	2	1	-2	2	0	-0.21822	
3	2	1	-3	2	1	0.37796	
3	2	2	3	-2	-1	0.26726	*
3	2	2	3	-1	-2	-0.26726	*
3	2	2	2	-2	0	-0.26726	*
3	2	2	2	-1	-1	-0.00000	*
3	2	2	2	0	-2	0.26726	*
3	2	2	1	-2	1	0.20702	*
3	2	2	1	-1	0	0.16903	*
3	2	2	1	0	-1	-0.16903	*

j_1	j_2	j_3	m_1	m_2	m_3		
3	2	2	1	1	-2	-0.20702	*
3	2	2	0	-2	2	-0.11952	*
3	2	2	0	-1	1	-0.23905	*
3	2	2	0	1	-1	0.23905	*
3	2	2	0	2	-2	0.11952	*
3	2	2	-1	-1	2	0.20702	*
3	2	2	-1	0	1	0.16903	*
3	2	2	-1	1	0	-0.16903	*
3	2	2	-1	2	-1	-0.20702	*
3	2	2	-2	0	2	-0.26726	*
3	2	2	-2	1	1	0.00000	*
3	2	2	-2	2	0	0.26726	*
3	2	2	-3	1	2	0.26726	*
3	2	2	-3	2	1	-0.26726	*
3	3	0	3	-3	0	0.37796	
3	3	0	2	-2	0	-0.37796	
3	3	0	1	-1	0	0.37796	
3	3	0	0	0	0	-0.37796	
3	3	0	-1	1	0	0.37796	
3	3	0	-2	2	0	-0.37796	
3	3	0	-3	3	0	0.37796	
3	3	1	3	-3	0	0.32733	*
3	3	1	3	-2	-1	-0.18898	*
3	3	1	2	-3	1	-0.18898	*
3	3	1	2	-2	0	-0.21822	*
3	3	1	2	-1	-1	0.24398	*
3	3	1	1	-2	1	0.24398	*
3	3	1	1	-1	0	0.10911	*
3	3	1	1	0	-1	-0.26726	*
3	3	1	0	-1	1	-0.26726	*
3	3	1	0	1	-1	0.26726	*
3	3	1	-1	0	1	0.26726	*
3	3	1	-1	1	0	-0.10911	*
3	3	1	-1	2	-1	-0.24398	*
3	3	1	-2	1	1	-0.24398	*
3	3	1	-2	2	0	0.21822	*
3	3	1	-2	3	-1	0.18898	*
3	3	1	-3	2	1	0.18898	*
3	3	1	-3	3	0	-0.32733	*
3	3	2	3	-3	0	0.24398	
3	3	2	3	-2	-1	-0.24398	
3	3	2	3	-1	-2	0.15430	
3	3	2	2	-3	1	-0.24398	
3	3	2	2	-2	0	0.00000	
3	3	2	2	-1	-1	0.18898	
3	3	2	2	0	-2	-0.21822	
3	3	2	1	-3	2	0.15430	
3	3	2	1	-2	1	0.18898	
3	3	2	1	-1	0	-0.14639	
3	3	2	1	0	-1	-0.06901	
3	3	2	1	1	-2	0.23905	
3	3	2	0	-2	2	-0.21822	
3	3	2	0	-1	1	-0.06901	
3	3	2	0	0	0	0.19518	
3	3	2	0	1	-1	-0.06901	
3	3	2	0	2	-2	-0.21822	
3	3	2	-1	-1	2	0.23905	
3	3	2	-1	0	1	-0.06901	
3	3	2	-1	1	0	-0.14639	
3	3	2	-1	2	-1	0.18898	
3	3	2	-1	3	-2	0.15430	
3	3	2	-2	0	2	-0.21822	
3	3	2	-2	1	1	0.18898	
3	3	2	-2	2	0	0.00000	
3	3	2	-2	3	-1	-0.24398	
3	3	2	-3	1	2	0.15430	
3	3	2	-3	2	1	-0.24398	
3	3	2	-3	3	0	0.24398	
3	3	3	3	-3	0	0.15430	*
3	3	3	3	-2	-1	-0.21822	*
3	3	3	3	-1	-2	0.21822	*
3	3	3	3	0	-3	-0.15430	*
3	3	3	2	-3	1	-0.21822	*
3	3	3	2	-2	0	0.15430	*
3	3	3	2	-1	-1	0.00000	*
3	3	3	2	0	-2	-0.15430	*
3	3	3	2	1	-3	0.21822	*
3	3	3	1	-3	2	0.21822	*
3	3	3	1	-2	1	0.00000	*
3	3	3	1	-1	0	-0.15430	*
3	3	3	1	0	-1	0.15430	*
3	3	3	1	1	-2	0.00000	*
3	3	3	1	2	-3	-0.21822	*
3	3	3	0	-3	3	-0.15430	*
3	3	3	0	-2	2	-0.15430	*
3	3	3	0	-1	1	0.15430	*
3	3	3	0	1	-1	-0.15430	*
3	3	3	0	2	-2	0.15430	*
3	3	3	0	3	-3	0.15430	*
3	3	3	-1	-2	3	0.21822	*
3	3	3	-1	-1	2	0.00000	*
3	3	3	-1	0	1	-0.15430	*
3	3	3	-1	1	0	0.15430	*
3	3	3	-1	2	-1	-0.00000	*
3	3	3	-1	3	-2	-0.21822	*
3	3	3	-2	-1	3	-0.21822	*
3	3	3	-2	0	2	0.15430	*
3	3	3	-2	1	1	-0.00000	*
3	3	3	-2	2	0	-0.15430	*
3	3	3	-2	3	-1	0.21822	*
3	3	3	-3	0	3	0.15430	*
3	3	3	-3	1	2	-0.21822	*
3	3	3	-3	2	1	0.21822	*
3	3	3	-3	3	0	-0.15430	*
4	2	2	4	-2	-2	0.33333	
4	2	2	3	-2	-1	-0.23570	
4	2	2	3	-1	-2	-0.23570	
4	2	2	2	-2	0	0.15430	
4	2	2	2	-1	-1	0.25198	
4	2	2	2	0	-2	0.15430	

J_1	J_2	J_3	m_1	m_2	m_3		
4	2	2	1	-2	1	-0.08909	
4	2	2	1	-1	0	-0.21822	
4	2	2	1	0	-1	-0.21822	
4	2	2	1	1	-2	-0.08909	
4	2	2	0	-2	2	0.03984	
4	2	2	0	-1	1	0.15936	
4	2	2	0	0	0	0.23905	
4	2	2	0	1	-1	0.15936	
4	2	2	0	2	-2	0.03984	
4	2	2	-1	-1	2	-0.08909	
4	2	2	-1	0	1	-0.21822	
4	2	2	-1	1	0	-0.21822	
4	2	2	-1	2	-1	-0.08909	
4	2	2	-2	0	2	0.15430	
4	2	2	-2	1	1	0.25198	
4	2	2	-2	2	0	0.15430	
4	2	2	-3	1	2	-0.23570	
4	2	2	-3	2	1	-0.23570	
4	2	2	-4	2	2	0.33333	
4	3	1	4	-3	-1	0.33333	
4	3	1	3	-3	0	-0.16667	
4	3	1	3	-2	-1	-0.28868	
4	3	1	2	-3	1	0.06299	
4	3	1	2	-2	0	0.21822	
4	3	1	2	-1	-1	0.24398	
4	3	1	1	-2	1	-0.10911	
4	3	1	1	-1	0	-0.24398	
4	3	1	1	0	-1	-0.19920	
4	3	1	0	-1	1	0.15430	
4	3	1	0	0	0	0.25198	
4	3	1	0	1	-1	0.15430	
4	3	1	-1	0	1	-0.19920	
4	3	1	-1	1	0	-0.24398	
4	3	1	-1	2	-1	-0.10911	
4	3	1	-2	1	1	0.24398	
4	3	1	-2	2	0	0.21822	
4	3	1	-2	3	-1	0.06299	
4	3	1	-3	2	1	-0.28868	
4	3	1	-3	3	0	-0.16667	
4	3	1	-4	3	1	0.33333	
4	3	2	4	-3	-1	0.25820	*
4	3	2	4	-2	-2	-0.21082	*
4	3	2	3	-3	0	-0.22361	*
4	3	2	3	-2	-1	-0.07454	*
4	3	2	3	-1	-2	0.23570	*
4	3	2	2	-3	1	0.14639	*
4	3	2	2	-2	0	0.19518	*
4	3	2	2	-1	-1	-0.06299	*
4	3	2	2	0	-2	-0.21822	*
4	3	2	1	-3	2	-0.06901	*
4	3	2	1	-2	1	-0.19720	*
4	3	2	1	-1	0	-0.10911	*
4	3	2	1	0	-1	0.15430	*
4	3	2	1	1	-2	0.17817	*
4	3	2	0	-2	2	0.12599	*
4	3	2	0	-1	1	0.19920	*
4	3	2	0	1	-1	-0.19920	*
4	3	2	0	2	-2	-0.12599	*
4	3	2	-1	-1	2	-0.17817	*
4	3	2	-1	0	1	-0.15430	*
4	3	2	-1	1	0	0.10911	*
4	3	2	-1	2	-1	0.19720	*
4	3	2	-1	3	-2	0.06901	*
4	3	2	-2	0	2	0.21822	*
4	3	2	-2	1	1	0.06299	*
4	3	2	-2	2	0	-0.19518	*
4	3	2	-2	3	-1	-0.14639	*
4	3	2	-3	1	2	-0.23570	*
4	3	2	-3	2	1	0.07454	*
4	3	2	-3	3	0	0.22361	*
4	3	2	-4	2	2	0.21082	*
4	3	2	-4	3	1	-0.25820	*
4	3	3	4	-3	-1	0.17408	
4	3	3	4	-2	-2	-0.22473	
4	3	3	4	-1	-3	0.17408	
4	3	3	3	-3	0	-0.21320	
4	3	3	3	-2	-1	0.10050	
4	3	3	3	-1	-2	0.10050	
4	3	3	3	0	-3	-0.21320	
4	3	3	2	-3	1	0.19739	
4	3	3	2	-2	0	0.04652	
4	3	3	2	-1	-1	-0.16988	
4	3	3	2	0	-2	0.04652	
4	3	3	2	1	-3	0.19739	
4	3	3	1	-3	2	-0.14712	
4	3	3	1	-2	1	-0.15195	
4	3	3	1	-1	0	0.10403	
4	3	3	1	0	-1	0.10403	
4	3	3	1	1	-2	-0.15195	
4	3	3	1	2	-3	-0.14712	
4	3	3	0	-3	3	0.08058	
4	3	3	0	-2	2	0.18803	
4	3	3	0	-1	1	0.02686	
4	3	3	0	0	0	-0.16116	
4	3	3	0	1	-1	0.02686	
4	3	3	0	2	-2	0.18803	
4	3	3	0	3	-3	0.08058	
4	3	3	-1	-2	3	-0.14712	
4	3	3	-1	-1	2	-0.15195	
4	3	3	-1	0	1	0.10403	
4	3	3	-1	1	0	0.10403	
4	3	3	-1	2	-1	-0.15195	
4	3	3	-1	3	-2	-0.14712	
4	3	3	-2	-1	3	0.19739	
4	3	3	-2	0	2	0.04652	
4	3	3	-2	1	1	-0.16988	
4	3	3	-2	2	0	0.04652	
4	3	3	-2	3	-1	0.19739	
4	3	3	-3	0	3	-0.21320	
4	3	3	-3	1	2	0.10050	
4	3	3	-3	2	1	0.10050	
4	3	3	-3	3	0	-0.21320	
4	3	3	-4	1	3	0.17408	
4	3	3	-4	2	2	-0.22473	
4	3	3	-4	3	1	0.17408	

j_1	j_2	j_3	m_1	m_2	m_3		
4	4	0	4	-4	0	0.33333	
4	4	0	3	-3	0	-0.33333	
4	4	0	2	-2	0	0.33333	
4	4	0	1	-1	0	-0.33333	
4	4	0	0	0	0	0.33333	
4	4	0	-1	1	0	-0.33333	
4	4	0	-2	2	0	0.33333	
4	4	0	-3	3	0	-0.33333	
4	4	0	-4	4	0	0.33333	
4	4	1	4	-4	0	0.29814	*
4	4	1	4	-3	-1	-0.14907	*
4	4	1	3	-4	1	-0.14907	*
4	4	1	3	-3	0	-0.22361	*
4	4	1	3	-2	-1	0.19720	*
4	4	1	2	-3	1	0.19720	*
4	4	1	2	-2	0	0.14907	*
4	4	1	2	-1	-1	-0.22361	*
4	4	1	1	-2	1	-0.22361	*
4	4	1	1	-1	0	-0.07454	*
4	4	1	1	0	-1	0.23570	*
4	4	1	0	-1	1	0.23570	*
4	4	1	0	1	-1	-0.23570	*
4	4	1	-1	0	1	-0.23570	*
4	4	1	-1	1	0	0.07454	*
4	4	1	-1	2	-1	0.22361	*
4	4	1	-2	1	1	0.22361	*
4	4	1	-2	2	0	-0.14907	*
4	4	1	-2	3	-1	-0.19720	*
4	4	1	-3	2	1	-0.19720	*
4	4	1	-3	3	0	0.22361	*
4	4	1	-3	4	-1	0.14907	*
4	4	1	-4	3	1	0.14907	*
4	4	1	-4	4	0	-0.29814	*
4	4	2	4	-4	0	0.23784	
4	4	2	4	-3	-1	-0.20597	
4	4	2	4	-2	-2	0.11010	
4	4	2	3	-4	1	-0.20597	
4	4	2	3	-3	0	-0.05946	
4	4	2	3	-2	-1	0.19462	
4	4	2	3	-1	-2	-0.16514	
4	4	2	2	-4	2	0.11010	
4	4	2	2	-3	1	0.19462	
4	4	2	2	-2	0	-0.06795	
4	4	2	2	-1	-1	-0.13241	
4	4	2	2	0	-2	0.19739	
4	4	2	1	-3	2	-0.16514	
4	4	2	1	-2	1	-0.13241	
4	4	2	1	-1	0	0.14440	
4	4	2	1	0	-1	0.04652	
4	4	2	1	1	-2	-0.20806	
4	4	2	0	-2	2	0.19739	
4	4	2	0	-1	1	0.04652	
4	4	2	0	0	0	-0.16988	
4	4	2	0	1	-1	0.04652	
4	4	2	0	2	-2	0.19739	
4	4	2	-1	-1	2	-0.20806	
4	4	2	-1	0	1	0.04652	
4	4	2	-1	1	0	0.14440	
4	4	2	-1	2	-1	-0.13241	
4	4	2	-1	3	-2	-0.16514	
4	4	2	-2	0	2	0.19739	
4	4	2	-2	1	1	-0.13241	
4	4	2	-2	2	0	-0.06795	
4	4	2	-2	3	-1	0.19462	
4	4	2	-2	4	-2	0.11010	
4	4	2	-3	1	2	-0.16514	
4	4	2	-3	2	1	0.19462	
4	4	2	-3	3	0	-0.05946	
4	4	2	-3	4	-1	-0.20597	
4	4	2	-4	2	2	0.11010	
4	4	2	-4	3	1	-0.20597	
4	4	2	-4	4	0	0.23784	
4	4	3	4	-4	0	0.16818	*
4	4	3	4	-3	-1	-0.20597	*
4	4	3	4	-2	-2	0.17408	*
4	4	3	4	-1	-3	-0.10050	*
4	4	3	3	-4	1	-0.20597	*
4	4	3	3	-3	0	0.08409	*
4	4	3	3	-2	-1	0.07785	*
4	4	3	3	-1	-2	-0.17408	*
4	4	3	3	0	-3	0.15891	*
4	4	3	2	-4	2	0.17408	*
4	4	3	2	-3	1	0.07785	*
4	4	3	2	-2	0	-0.15616	*
4	4	3	2	-1	-1	0.05885	*
4	4	3	2	0	-2	0.10403	*
4	4	3	2	1	-3	-0.18993	*
4	4	3	1	-4	3	-0.10050	*
4	4	3	1	-3	2	-0.17408	*
4	4	3	1	-2	1	0.05885	*
4	4	3	1	-1	0	0.10811	*
4	4	3	1	0	-1	-0.13957	*
4	4	3	1	1	-2	0.00000	*
4	4	3	1	2	-3	0.18993	*
4	4	3	0	-3	3	0.15891	*
4	4	3	0	-2	2	0.10403	*
4	4	3	0	-1	1	-0.13957	*
4	4	3	0	1	-1	0.13957	*
4	4	3	0	2	-2	-0.10403	*
4	4	3	0	3	-3	-0.15891	*
4	4	3	-1	-2	3	-0.18993	*
4	4	3	-1	-1	2	0.00000	*
4	4	3	-1	0	1	0.13957	*
4	4	3	-1	1	0	-0.10811	*
4	4	3	-1	2	-1	-0.05885	*
4	4	3	-1	3	-2	0.17408	*
4	4	3	-1	4	-3	0.10050	*
4	4	3	-2	-1	3	0.18993	*
4	4	3	-2	0	2	-0.10403	*
4	4	3	-2	1	1	-0.05885	*
4	4	3	-2	2	0	0.15616	*
4	4	3	-2	3	-1	-0.07785	*
4	4	3	-2	4	-2	-0.17408	*

j_1	j_2	j_3	m_1	m_2	m_3		
4	4	3	-3	0	3	-0.15891	*
4	4	3	-3	1	2	0.17408	*
4	4	3	-3	2	1	-0.07785	*
4	4	3	-3	3	0	-0.08409	*
4	4	3	-3	4	-1	0.20597	*
4	4	3	-4	1	3	0.10050	*
4	4	3	-4	2	2	-0.17408	*
4	4	3	-4	3	1	0.20597	*
4	4	3	-4	4	0	-0.16818	*
4	4	4	4	-4	0	0.10430	
4	4	4	4	-3	-1	-0.16491	
4	4	4	4	-2	-2	0.18699	
4	4	4	4	-1	-3	-0.16491	
4	4	4	4	0	-4	0.10430	
4	4	4	3	-4	1	-0.16491	
4	4	4	3	-3	0	0.15645	
4	4	4	3	-2	-1	-0.06233	
4	4	4	3	-1	-2	-0.06233	
4	4	4	3	0	-3	0.15645	
4	4	4	3	1	-4	-0.16491	
4	4	4	2	-4	2	0.18699	
4	4	4	2	-3	1	-0.06233	
4	4	4	2	-2	0	-0.08195	
4	4	4	2	-1	-1	0.14135	
4	4	4	2	0	-2	-0.08195	
4	4	4	2	1	-3	-0.06233	
4	4	4	2	2	-4	0.18699	
4	4	4	1	-4	3	-0.16491	
4	4	4	1	-3	2	-0.06233	
4	4	4	1	-2	1	0.14135	
4	4	4	1	-1	0	-0.06705	
4	4	4	1	0	-1	-0.06705	
4	4	4	1	1	-2	0.14135	
4	4	4	1	2	-3	-0.06233	
4	4	4	1	3	-4	-0.16491	
4	4	4	0	-4	4	0.10430	
4	4	4	0	-3	3	0.15645	
4	4	4	0	-2	2	-0.08195	
4	4	4	0	-1	1	-0.06705	
4	4	4	0	0	0	0.13410	
4	4	4	0	1	-1	-0.06705	
4	4	4	0	2	-2	-0.08195	
4	4	4	0	3	-3	0.15645	
4	4	4	0	4	-4	0.10430	
4	4	4	-1	-3	4	-0.16491	
4	4	4	-1	-2	3	-0.06233	
4	4	4	-1	-1	2	0.14135	
4	4	4	-1	0	1	-0.06705	
4	4	4	-1	1	0	-0.06705	
4	4	4	-1	2	-1	0.14135	
4	4	4	-1	3	-2	-0.06233	
4	4	4	-1	4	-3	-0.16491	
4	4	4	-2	-2	4	0.18699	
4	4	4	-2	-1	3	-0.06233	
4	4	4	-2	0	2	-0.08195	
4	4	4	-2	1	1	0.14135	
4	4	4	-2	2	0	-0.08195	
4	4	4	-2	3	-1	-0.06233	
4	4	4	-2	4	-2	0.18699	
4	4	4	-3	-1	4	-0.16491	
4	4	4	-3	0	3	0.15645	
4	4	4	-3	1	2	-0.06233	
4	4	4	-3	2	1	-0.06233	
4	4	4	-3	3	0	0.15645	
4	4	4	-3	4	-1	-0.16491	
4	4	4	-4	0	4	0.10430	
4	4	4	-4	1	3	-0.16491	
4	4	4	-4	2	2	0.18699	
4	4	4	-4	3	1	-0.16491	
4	4	4	-4	4	0	0.10430	
5	3	2	5	-3	-2	0.30151	
5	3	2	4	-3	-1	-0.19069	
5	3	2	4	-2	-2	-0.23355	
5	3	2	3	-3	0	0.11010	
5	3	2	3	-2	-1	0.22019	
5	3	2	3	-1	-2	0.17408	
5	3	2	2	-3	1	-0.05505	
5	3	2	2	-2	0	-0.16514	
5	3	2	2	-1	-1	-0.21320	
5	3	2	2	0	-2	-0.12309	
5	3	2	1	-3	2	0.02081	
5	3	2	1	-2	1	0.10193	
5	3	2	1	-1	0	0.19739	
5	3	2	1	0	-1	0.18610	
5	3	2	1	1	-2	0.08058	
5	3	2	0	-2	2	-0.04652	
5	3	2	0	-1	1	-0.14712	
5	3	2	0	0	0	-0.20806	
5	3	2	0	1	-1	-0.14712	
5	3	2	0	2	-2	-0.04652	
5	3	2	-1	-1	2	0.08058	
5	3	2	-1	0	1	0.18610	
5	3	2	-1	1	0	0.19739	
5	3	2	-1	2	-1	0.10193	
5	3	2	-1	3	-2	0.02081	
5	3	2	-2	0	2	-0.12309	
5	3	2	-2	1	1	-0.21320	
5	3	2	-2	2	0	-0.16514	
5	3	2	-2	3	-1	-0.05505	
5	3	2	-3	1	2	0.17408	
5	3	2	-3	2	1	0.22019	
5	3	2	-3	3	0	0.11010	
5	3	2	-4	2	2	-0.23355	
5	3	2	-4	3	1	-0.19069	
5	3	2	-5	3	2	0.30151	
5	3	3	5	-3	-2	0.21320	*
5	3	3	5	-2	-3	-0.21320	*
5	3	3	4	-3	-1	-0.21320	*
5	3	3	4	-2	-2	-0.00000	*
5	3	3	4	-1	-3	0.21320	*
5	3	3	3	-3	0	0.17408	*
5	3	3	3	-2	-1	0.12309	*

j_1	j_2	j_3	m_1	m_2	m_3		
5	3	3	3	-1	-2	-0.12309	*
5	3	3	3	0	-3	-0.17408	*
5	3	3	2	-3	1	-0.12309	*
5	3	3	2	-2	0	-0.17408	*
5	3	3	2	-1	-1	0.00000	*
5	3	3	2	0	-2	0.17408	*
5	3	3	2	1	-3	0.12309	*
5	3	3	1	-3	2	0.07356	*
5	3	3	1	-2	1	0.17094	*
5	3	3	1	-1	0	0.10403	*
5	3	3	1	0	-1	-0.10403	*
5	3	3	1	1	-2	-0.17094	*
5	3	3	1	2	-3	-0.07356	*
5	3	3	0	-3	3	-0.03290	*
5	3	3	0	-2	2	-0.13159	*
5	3	3	0	-1	1	-0.16449	*
5	3	3	0	1	-1	0.16449	*
5	3	3	0	2	-2	0.13159	*
5	3	3	0	3	-3	0.03290	*
5	3	3	-1	-2	3	0.07356	*
5	3	3	-1	-1	2	0.17094	*
5	3	3	-1	0	1	0.10403	*
5	3	3	-1	1	0	-0.10403	*
5	3	3	-1	2	-1	-0.17094	*
5	3	3	-1	3	-2	-0.07356	*
5	3	3	-2	-1	3	-0.12309	*
5	3	3	-2	0	2	-0.17408	*
5	3	3	-2	1	1	-0.00000	*
5	3	3	-2	2	0	0.17408	*
5	3	3	-2	3	-1	0.12309	*
5	3	3	-3	0	3	0.17408	*
5	3	3	-3	1	2	0.12309	*
5	3	3	-3	2	1	-0.12309	*
5	3	3	-3	3	0	-0.17408	*
5	3	3	-4	1	3	-0.21320	*
5	3	3	-4	2	2	0.00000	*
5	3	3	-4	3	1	0.21320	*
5	3	3	-5	2	3	0.21320	*
5	3	3	-5	3	2	-0.21320	*
5	4	1	5	-4	-1	0.30151	
5	4	1	4	-4	0	-0.13484	
5	4	1	4	-3	-1	-0.26968	
5	4	1	3	-4	1	0.04495	
5	4	1	3	-3	0	0.17979	
5	4	1	3	-2	-1	0.23784	
5	4	1	2	-3	1	-0.07785	
5	4	1	2	-2	0	-0.20597	
5	4	1	2	-1	-1	-0.20597	
5	4	1	1	-2	1	0.11010	
5	4	1	1	-1	0	0.22019	
5	4	1	1	0	-1	0.17408	
5	4	1	0	-1	1	-0.14213	
5	4	1	0	0	0	-0.22473	
5	4	1	0	1	-1	-0.14213	
5	4	1	-1	0	1	0.17408	
5	4	1	-1	1	0	0.22019	
5	4	1	-1	2	-1	0.11010	
5	4	1	-2	1	1	-0.20597	
5	4	1	-2	2	0	-0.20597	
5	4	1	-2	3	-1	-0.07785	
5	4	1	-3	2	1	0.23784	
5	4	1	-3	3	0	0.17979	
5	4	1	-3	4	-1	0.04495	
5	4	1	-4	3	1	-0.26968	
5	4	1	-4	4	0	-0.13484	
5	4	1	-5	4	1	0.30151	
5	4	2	5	-4	-1	0.24618	*
5	4	2	5	-3	-2	-0.17408	*
5	4	2	4	-4	0	-0.19069	*
5	4	2	4	-3	-1	-0.11010	*
5	4	2	4	-2	-2	0.20597	*
5	4	2	3	-4	1	0.11010	*
5	4	2	3	-3	0	0.19069	*
5	4	2	3	-2	-1	0.00000	*
5	4	2	3	-1	-2	-0.20597	*
5	4	2	2	-4	2	-0.04495	*
5	4	2	2	-3	1	-0.15891	*
5	4	2	2	-2	0	-0.14564	*
5	4	2	2	-1	-1	0.08409	*
5	4	2	2	0	-2	0.18803	*
5	4	2	1	-3	2	0.08409	*
5	4	2	1	-2	1	0.17979	*
5	4	2	1	-1	0	0.07785	*
5	4	2	1	0	-1	-0.14213	*
5	4	2	1	1	-2	-0.15891	*
5	4	2	0	-2	2	-0.12309	*
5	4	2	0	-1	1	-0.17408	*
5	4	2	0	1	-1	0.17408	*
5	4	2	0	2	-2	0.12309	*
5	4	2	-1	-1	2	0.15891	*
5	4	2	-1	0	1	0.14213	*
5	4	2	-1	1	0	-0.07785	*
5	4	2	-1	2	-1	-0.17979	*
5	4	2	-1	3	-2	-0.08409	*
5	4	2	-2	0	2	-0.18803	*
5	4	2	-2	1	1	-0.08409	*
5	4	2	-2	2	0	0.14564	*
5	4	2	-2	3	-1	0.15891	*
5	4	2	-2	4	-2	0.04495	*
5	4	2	-3	1	2	0.20597	*
5	4	2	-3	2	1	0.00000	*
5	4	2	-3	3	0	-0.19069	*
5	4	2	-3	4	-1	-0.11010	*
5	4	2	-4	2	2	-0.20597	*
5	4	2	-4	3	1	0.11010	*
5	4	2	-4	4	0	0.19069	*
5	4	2	-5	3	2	0.17408	*
5	4	2	-5	4	1	-0.24618	*
5	4	3	5	-4	-1	0.18065	
5	4	3	5	-3	-2	-0.20197	
5	4	3	5	-2	-3	0.13222	
5	4	3	4	-4	0	-0.19789	
5	4	3	4	-3	-1	0.04039	

j_1	j_2	j_3	m_1	m_2	m_3	
5	4	3	4	-2	-2	0.13656
5	4	3	4	-1	-3	-0.17739
5	4	3	3	-4	1	0.16158
5	4	3	3	-3	0	0.09895
5	4	3	3	-2	-1	-0.13741
5	4	3	3	-1	-2	-0.03414
5	4	3	3	0	-3	0.18699
5	4	3	2	-4	2	-0.10430
5	4	3	2	-3	1	-0.16325
5	4	3	2	-2	0	0.02159
5	4	3	2	-1	-1	0.14104
5	4	3	2	0	-2	-0.06233
5	4	3	2	1	-3	-0.17070
5	4	3	1	-4	3	0.04828
5	4	3	1	-3	2	0.15331
5	4	3	1	-2	1	0.10130
5	4	3	1	-1	0	-0.10964
5	4	3	1	0	-1	-0.08195
5	4	3	1	1	-2	0.13170
5	4	3	1	2	-3	0.13686
5	4	3	0	-3	3	-0.09349
5	4	3	0	-2	2	-0.16322
5	4	3	0	-1	1	-0.00912
5	4	3	0	0	0	0.14135
5	4	3	0	1	-1	-0.00912
5	4	3	0	2	-2	-0.16322
5	4	3	0	3	-3	-0.09349
5	4	3	-1	-2	3	0.13686
5	4	3	-1	-1	2	0.13170
5	4	3	-1	0	1	-0.08195
5	4	3	-1	1	0	-0.10964
5	4	3	-1	2	-1	0.10130
5	4	3	-1	3	-2	0.15331
5	4	3	-1	4	-3	0.04828
5	4	3	-2	-1	3	-0.17070
5	4	3	-2	0	2	-0.06233
5	4	3	-2	1	1	0.14104
5	4	3	-2	2	0	0.02159
5	4	3	-2	3	-1	-0.16325
5	4	3	-2	4	-2	-0.10430
5	4	3	-3	0	3	0.18699
5	4	3	-3	1	2	-0.03414
5	4	3	-3	2	1	-0.13741
5	4	3	-3	3	0	0.09895
5	4	3	-3	4	-1	0.16158
5	4	3	-4	1	3	-0.17739
5	4	3	-4	2	2	0.13656
5	4	3	-4	3	1	0.04039
5	4	3	-4	4	0	-0.19789
5	4	3	-5	2	3	0.13222
5	4	3	-5	3	2	-0.20197
5	4	3	-5	4	1	0.18065
5	4	4	5	-4	-1	0.11826 *
5	4	4	5	-3	-2	-0.17739 *
5	4	4	5	-2	-3	0.17739 *
5	4	4	5	-1	-4	-0.11826 *
5	4	4	4	-4	0	-0.16725 *
5	4	4	4	-3	-1	0.13222 *

j_1	j_2	j_3	m_1	m_2	m_3	
5	4	4	4	-2	-2	0.00000 *
5	4	4	4	-1	-3	-0.13222 *
5	4	4	4	0	-4	0.16725 *
5	4	4	3	-4	1	0.17630 *
5	4	4	3	-3	0	-0.02787 *
5	4	4	3	-2	-1	-0.11661 *
5	4	4	3	-1	-2	0.11661 *
5	4	4	3	0	-3	0.02787 *
5	4	4	3	1	-4	-0.17630 *
5	4	4	2	-4	2	-0.15268 *
5	4	4	2	-3	1	-0.07634 *
5	4	4	2	-2	0	0.12774 *
5	4	4	2	-1	-1	-0.00000 *
5	4	4	2	0	-2	-0.12774 *
5	4	4	2	1	-3	0.07634 *
5	4	4	2	2	-4	0.15268 *
5	4	4	1	-4	3	0.10796 *
5	4	4	1	-3	2	0.14282 *
5	4	4	1	-2	1	-0.05398 *
5	4	4	1	-1	0	-0.10242 *
5	4	4	1	0	-1	0.10242 *
5	4	4	1	1	-2	0.05398 *
5	4	4	1	2	-3	-0.14282 *
5	4	4	1	3	-4	-0.10796 *
5	4	4	0	-4	4	-0.05575 *
5	4	4	0	-3	3	-0.15331 *
5	4	4	0	-2	2	-0.05575 *
5	4	4	0	-1	1	0.12544 *
5	4	4	0	1	-1	-0.12544 *
5	4	4	0	2	-2	0.05575 *
5	4	4	0	3	-3	0.15331 *
5	4	4	0	4	-4	0.05575 *
5	4	4	-1	-3	4	0.10796 *
5	4	4	-1	-2	3	0.14282 *
5	4	4	-1	-1	2	-0.05398 *
5	4	4	-1	0	1	-0.10242 *
5	4	4	-1	1	0	0.10242 *
5	4	4	-1	2	-1	0.05398 *
5	4	4	-1	3	-2	-0.14282 *
5	4	4	-1	4	-3	-0.10796 *
5	4	4	-2	-2	4	-0.15268 *
5	4	4	-2	-1	3	-0.07634 *
5	4	4	-2	0	2	0.12774 *
5	4	4	-2	1	1	0.00000 *
5	4	4	-2	2	0	-0.12774 *
5	4	4	-2	3	-1	0.07634 *
5	4	4	-2	4	-2	0.15268 *
5	4	4	-3	-1	4	0.17630 *
5	4	4	-3	0	3	-0.02787 *
5	4	4	-3	1	2	-0.11661 *
5	4	4	-3	2	1	0.11661 *
5	4	4	-3	3	0	0.02787 *
5	4	4	-3	4	-1	-0.17630 *
5	4	4	-4	0	4	-0.16725 *
5	4	4	-4	1	3	0.13222 *
5	4	4	-4	2	2	-0.00000 *
5	4	4	-4	3	1	-0.13222 *
5	4	4	-4	4	0	0.16725 *
5	4	4	-5	1	4	0.11826 *
5	4	4	-5	2	3	-0.17739 *

j_1	j_2	j_3	m_1	m_2	m_3		
5	4	4	-5	3	2	0.17739	*
5	4	4	-5	4	1	-0.11826	*
5	5	0	5	-5	0	0.30151	
5	5	0	4	-4	0	-0.30151	
5	5	0	3	-3	0	0.30151	
5	5	0	2	-2	0	-0.30151	
5	5	0	1	-1	0	0.30151	
5	5	0	0	0	0	-0.30151	
5	5	0	-1	1	0	0.30151	
5	5	0	-2	2	0	-0.30151	
5	5	0	-3	3	0	0.30151	
5	5	0	-4	4	0	-0.30151	
5	5	0	-5	5	0	0.30151	
5	5	1	5	-5	0	0.27524	*
5	5	1	5	-4	-1	-0.12309	*
5	5	1	4	-5	1	-0.12309	*
5	5	1	4	-4	0	-0.22019	*
5	5	1	4	-3	-1	0.16514	*
5	5	1	3	-4	1	0.16514	*
5	5	1	3	-3	0	0.16514	*
5	5	1	3	-2	-1	-0.19069	*
5	5	1	2	-3	1	-0.19069	*
5	5	1	2	-2	0	-0.11010	*
5	5	1	2	-1	-1	0.20597	*
5	5	1	1	-2	1	0.20597	*
5	5	1	1	-1	0	0.05505	*
5	5	1	1	0	-1	-0.21320	*
5	5	1	0	-1	1	-0.21320	*
5	5	1	0	1	-1	0.21320	*
5	5	1	-1	0	1	0.21320	*
5	5	1	-1	1	0	-0.05505	*
5	5	1	-1	2	-1	-0.20597	*
5	5	1	-2	1	1	-0.20597	*
5	5	1	-2	2	0	0.11010	*
5	5	1	-2	3	-1	0.19069	*
5	5	1	-3	2	1	0.19069	*
5	5	1	-3	3	0	-0.16514	*
5	5	1	-3	4	-1	-0.16514	*
5	5	1	-4	3	1	-0.16514	*
5	5	1	-4	4	0	0.22019	*
5	5	1	-4	5	-1	0.12309	*
5	5	1	-5	4	1	0.12309	*
5	5	1	-5	5	0	-0.27524	*
5	5	2	5	-5	0	0.22901	
5	5	2	5	-4	-1	-0.17739	
5	5	2	5	-3	-2	0.08362	
5	5	2	4	-5	1	-0.17739	
5	5	2	4	-4	0	-0.09161	
5	5	2	4	-3	-1	0.18511	
5	5	2	4	-2	-2	-0.12955	
5	5	2	3	-5	2	0.08362	
5	5	2	3	-4	1	0.18511	
5	5	2	3	-3	0	-0.01527	
5	5	2	3	-2	-1	-0.15268	

j_1	j_2	j_3	m_1	m_2	m_3		
5	5	2	3	-1	-2	0.16158	
5	5	2	2	-4	2	-0.12955	
5	5	2	2	-3	1	-0.15268	
5	5	2	2	-2	0	0.09161	
5	5	2	2	-1	-1	0.09895	
5	5	2	2	0	-2	-0.18065	
5	5	2	1	-3	2	0.16158	
5	5	2	1	-2	1	0.09895	
5	5	2	1	-1	0	-0.13741	
5	5	2	1	0	-1	-0.03414	
5	5	2	1	1	-2	0.18699	
5	5	2	0	-2	2	-0.18065	
5	5	2	0	-1	1	-0.03414	
5	5	2	0	0	0	0.15268	
5	5	2	0	1	-1	-0.03414	
5	5	2	0	2	-2	-0.18065	
5	5	2	-1	-1	2	0.18699	
5	5	2	-1	0	1	-0.03414	
5	5	2	-1	1	0	-0.13741	
5	5	2	-1	2	-1	0.09895	
5	5	2	-1	3	-2	0.16158	
5	5	2	-2	0	2	-0.18065	
5	5	2	-2	1	1	0.09895	
5	5	2	-2	2	0	0.09161	
5	5	2	-2	3	-1	-0.15268	
5	5	2	-2	4	-2	-0.12955	
5	5	2	-3	1	2	0.16158	
5	5	2	-3	2	1	-0.15268	
5	5	2	-3	3	0	-0.01527	
5	5	2	-3	4	-1	0.18511	
5	5	2	-3	5	-2	0.08362	
5	5	2	-4	2	2	-0.12955	
5	5	2	-4	3	1	0.18511	
5	5	2	-4	4	0	-0.09161	
5	5	2	-4	5	-1	-0.17739	
5	5	2	-5	3	2	0.08362	
5	5	2	-5	4	1	-0.17739	
5	5	2	-5	5	0	0.22901	
5	5	3	5	-5	0	0.17312	*
5	5	3	5	-4	-1	-0.18964	*
5	5	3	5	-3	-2	0.14135	*
5	5	3	5	-2	-3	-0.07068	*
5	5	3	4	-5	1	-0.18964	*
5	5	3	4	-4	0	0.03462	*
5	5	3	4	-3	-1	0.11308	*
5	5	3	4	-2	-2	-0.16423	*
5	5	3	4	-1	-3	0.11826	*
5	5	3	3	-5	2	0.14135	*
5	5	3	3	-4	1	0.11308	*
5	5	3	3	-3	0	-0.12695	*
5	5	3	3	-2	-1	-0.00816	*
5	5	3	3	-1	-2	0.13656	*
5	5	3	3	0	-3	-0.15268	*
5	5	3	2	-5	3	-0.07068	*
5	5	3	2	-4	2	-0.16423	*
5	5	3	2	-3	1	-0.00816	*
5	5	3	2	-2	0	0.13272	*
5	5	3	2	-1	-1	-0.07933	*

j_1	j_2	j_3	m_1	m_2	m_3		
5	5	3	2	0	-2	-0.07634	*
5	5	3	2	1	-3	0.17070	*
5	5	3	1	-4	3	0.11826	*
5	5	3	1	-3	2	0.13656	*
5	5	3	1	-2	1	-0.07933	*
5	5	3	1	-1	0	-0.08079	*
5	5	3	1	0	-1	0.12774	*
5	5	3	1	1	-2	0.00000	*
5	5	3	1	2	-3	-0.17070	*
5	5	3	0	-3	3	-0.15268	*
5	5	3	0	-2	2	-0.07634	*
5	5	3	0	-1	1	0.12774	*
5	5	3	0	1	-1	-0.12774	*
5	5	3	0	2	-2	0.07634	*
5	5	3	0	3	-3	0.15268	*
5	5	3	-1	-2	3	0.17070	*
5	5	3	-1	-1	2	0.00000	*
5	5	3	-1	0	1	-0.12774	*
5	5	3	-1	1	0	0.08079	*
5	5	3	-1	2	-1	0.07933	*
5	5	3	-1	3	-2	-0.13656	*
5	5	3	-1	4	-3	-0.11826	*
5	5	3	-2	-1	3	-0.17070	*
5	5	3	-2	0	2	0.07634	*
5	5	3	-2	1	1	0.07933	*
5	5	3	-2	2	0	-0.13272	*
5	5	3	-2	3	-1	0.00816	*
5	5	3	-2	4	-2	0.16423	*
5	5	3	-2	5	-3	0.07068	*
5	5	3	-3	0	3	0.15268	*
5	5	3	-3	1	2	-0.13656	*
5	5	3	-3	2	1	0.00816	*
5	5	3	-3	3	0	0.12695	*
5	5	3	-3	4	-1	-0.11308	*
5	5	3	-3	5	-2	-0.14135	*
5	5	3	-4	1	3	-0.11826	*
5	5	3	-4	2	2	0.16423	*
5	5	3	-4	3	1	-0.11308	*
5	5	3	-4	4	0	-0.03462	*
5	5	3	-4	5	-1	0.18964	*
5	5	3	-5	2	3	0.07068	*
5	5	3	-5	3	2	-0.14135	*
5	5	3	-5	4	1	0.18964	*
5	5	3	-5	5	0	-0.17312	*
5	5	4	5	-5	0	0.11826	
5	5	4	5	-4	-1	-0.16725	
5	5	4	5	-3	-2	0.16725	
5	5	4	5	-2	-3	-0.12774	
5	5	4	5	-1	-4	0.06828	
5	5	4	4	-5	1	-0.16725	
5	5	4	4	-4	0	0.11826	
5	5	4	4	-3	-1	-0.00000	
5	5	4	4	-2	-2	-0.10796	
5	5	4	4	-1	-3	0.15268	
5	5	4	4	0	-4	-0.11826	
5	5	4	3	-5	2	0.16725	
5	5	4	3	-4	1	-0.00000	
5	5	4	3	-3	0	-0.11826	
5	5	4	3	-2	-1	0.10796	
5	5	4	3	-1	-2	-0.00000	
5	5	4	3	0	-3	-0.11826	
5	5	4	3	1	-4	0.15268	
5	5	4	2	-5	3	-0.12774	
5	5	4	2	-4	2	-0.10796	
5	5	4	2	-3	1	0.10796	
5	5	4	2	-2	0	0.01971	
5	5	4	2	-1	-1	-0.11661	
5	5	4	2	0	-2	0.09032	
5	5	4	2	1	-3	0.04407	
5	5	4	2	2	-4	-0.16491	
5	5	4	1	-5	4	0.06828	
5	5	4	1	-4	3	0.15268	
5	5	4	1	-3	2	-0.00000	
5	5	4	1	-2	1	-0.11661	
5	5	4	1	-1	0	0.07884	
5	5	4	1	0	-1	0.04828	
5	5	4	1	1	-2	-0.12466	
5	5	4	1	2	-3	0.04407	
5	5	4	1	3	-4	0.15268	
5	5	4	0	-4	4	-0.11826	
5	5	4	0	-3	3	-0.11826	
5	5	4	0	-2	2	0.09032	
5	5	4	0	-1	1	0.04828	
5	5	4	0	0	0	-0.11826	
5	5	4	0	1	-1	0.04828	
5	5	4	0	2	-2	0.09032	
5	5	4	0	3	-3	-0.11826	
5	5	4	0	4	-4	-0.11826	
5	5	4	-1	-3	4	0.15268	
5	5	4	-1	-2	3	0.04407	
5	5	4	-1	-1	2	-0.12466	
5	5	4	-1	0	1	0.04828	
5	5	4	-1	1	0	0.07884	
5	5	4	-1	2	-1	-0.11661	
5	5	4	-1	3	-2	0.00000	
5	5	4	-1	4	-3	0.15268	
5	5	4	-1	5	-4	0.06828	
5	5	4	-2	-2	4	-0.16491	
5	5	4	-2	-1	3	0.04407	
5	5	4	-2	0	2	0.09032	
5	5	4	-2	1	1	-0.11661	
5	5	4	-2	2	0	0.01971	
5	5	4	-2	3	-1	0.10796	
5	5	4	-2	4	-2	-0.10796	
5	5	4	-2	5	-3	-0.12774	
5	5	4	-3	-1	4	0.15268	
5	5	4	-3	0	3	-0.11826	
5	5	4	-3	1	2	0.00000	
5	5	4	-3	2	1	0.10796	
5	5	4	-3	3	0	-0.11826	
5	5	4	-3	4	-1	0.00000	
5	5	4	-3	5	-2	0.16725	
5	5	4	-4	0	4	-0.11826	
5	5	4	-4	1	3	0.15268	
5	5	4	-4	2	2	-0.10796	
5	5	4	-4	3	1	-0.00000	
5	5	4	-4	4	0	0.11826	
5	5	4	-4	5	-1	-0.16725	

			m_1	m_2	m_3		
5	5	4	-5	1	4	0.06828	
5	5	4	-5	2	3	-0.12774	
5	5	4	-5	3	2	0.16725	
5	5	4	-5	4	1	-0.16725	
5	5	4	-5	5	0	0.11826	
5	5	5	5	-5	0	0.07242	*
5	5	5	5	-4	-1	-0.12544	*
5	5	5	5	-3	-2	0.15645	*
5	5	5	5	-2	-3	-0.15645	*
5	5	5	5	-1	-4	0.12544	*
5	5	5	5	0	-5	-0.07242	*
5	5	5	4	-5	1	-0.12544	*
5	5	5	4	-4	0	0.14484	*
5	5	5	4	-3	-1	-0.09349	*
5	5	5	4	-2	-2	-0.00000	*
5	5	5	4	-1	-3	0.09349	*
5	5	5	4	0	-4	-0.14484	*
5	5	5	4	1	-5	0.12544	*
5	5	5	3	-5	2	0.15645	*
5	5	5	3	-4	1	-0.09349	*
5	5	5	3	-3	0	-0.02414	*
5	5	5	3	-2	-1	0.10796	*
5	5	5	3	-1	-2	-0.10796	*
5	5	5	3	0	-3	0.02414	*
5	5	5	3	1	-4	0.09349	*
5	5	5	3	2	-5	-0.15645	*
5	5	5	2	-5	3	-0.15645	*
5	5	5	2	-4	2	-0.00000	*
5	5	5	2	-3	1	0.10796	*
5	5	5	2	-2	0	-0.09656	*
5	5	5	2	-1	-1	0.00000	*
5	5	5	2	0	-2	0.09656	*
5	5	5	2	1	-3	-0.10796	*
5	5	5	2	2	-4	0.00000	*
5	5	5	2	3	-5	0.15645	*
5	5	5	1	-5	4	0.12544	*
5	5	5	1	-4	3	0.09349	*
5	5	5	1	-3	2	-0.10796	*
5	5	5	1	-2	1	0.00000	*
5	5	5	1	-1	0	0.09656	*
5	5	5	1	0	-1	-0.09656	*
5	5	5	1	1	-2	0.00000	*
5	5	5	1	2	-3	0.10796	*
5	5	5	1	3	-4	-0.09349	*
5	5	5	1	4	-5	-0.12544	*
5	5	5	0	-5	5	-0.07242	*
5	5	5	0	-4	4	-0.14484	*
5	5	5	0	-3	3	0.02414	*
5	5	5	0	-2	2	0.09656	*
5	5	5	0	-1	1	-0.09656	*
5	5	5	0	1	-1	0.09656	*
5	5	5	0	2	-2	-0.09656	*
5	5	5	0	3	-3	-0.02414	*
5	5	5	0	4	-4	0.14484	*
5	5	5	0	5	-5	0.07242	*
5	5	5	-1	-4	5	0.12544	*
5	5	5	-1	-3	4	0.09349	*
5	5	5	-1	-2	3	-0.10796	*

			m_1	m_2	m_3		
5	5	5	-1	-1	2	0.00000	*
5	5	5	-1	0	1	0.09656	*
5	5	5	-1	1	0	-0.09656	*
5	5	5	-1	2	-1	-0.00000	*
5	5	5	-1	3	-2	0.10796	*
5	5	5	-1	4	-3	-0.09349	*
5	5	5	-1	5	-4	-0.12544	*
5	5	5	-2	-3	5	-0.15645	*
5	5	5	-2	-2	4	0.00000	*
5	5	5	-2	-1	3	0.10796	*
5	5	5	-2	0	2	-0.09656	*
5	5	5	-2	1	1	0.00000	*
5	5	5	-2	2	0	0.09656	*
5	5	5	-2	3	-1	-0.10796	*
5	5	5	-2	4	-2	0.00000	*
5	5	5	-2	5	-3	0.15645	*
5	5	5	-3	-2	5	0.15645	*
5	5	5	-3	-1	4	-0.09349	*
5	5	5	-3	0	3	-0.02414	*
5	5	5	-3	1	2	0.10796	*
5	5	5	-3	2	1	-0.10796	*
5	5	5	-3	3	0	0.02414	*
5	5	5	-3	4	-1	0.09349	*
5	5	5	-3	5	-2	-0.15645	*
5	5	5	-4	-1	5	-0.12544	*
5	5	5	-4	0	4	0.14484	*
5	5	5	-4	1	3	-0.09349	*
5	5	5	-4	2	2	0.00000	*
5	5	5	-4	3	1	0.09349	*
5	5	5	-4	4	0	-0.14484	*
5	5	5	-4	5	-1	0.12544	*
5	5	5	-5	0	5	0.07242	*
5	5	5	-5	1	4	-0.12544	*
5	5	5	-5	2	3	0.15645	*
5	5	5	-5	3	2	-0.15645	*
5	5	5	-5	4	1	0.12544	*
5	5	5	-5	5	0	-0.07242	*
6	3	3	6	-3	-3	0.27735	
6	3	3	5	-3	-2	-0.19612	
6	3	3	5	-2	-3	-0.19612	
6	3	3	4	-3	-1	0.13222	
6	3	3	4	-2	-2	0.20484	
6	3	3	4	-1	-3	0.13222	
6	3	3	3	-3	0	-0.08362	
6	3	3	3	-2	-1	-0.17739	
6	3	3	3	-1	-2	-0.17739	
6	3	3	3	0	-3	-0.08362	
6	3	3	2	-3	1	0.04828	
6	3	3	2	-2	0	0.13656	
6	3	3	2	-1	-1	0.18699	
6	3	3	2	0	-2	0.13656	
6	3	3	2	1	-3	0.04828	
6	3	3	1	-3	2	-0.02414	
6	3	3	1	-2	1	-0.09349	
6	3	3	1	-1	0	-0.17070	
6	3	3	1	0	-1	-0.17070	
6	3	3	1	1	-2	-0.09349	
6	3	3	1	2	-3	-0.02414	

j_1	j_2	j_3	m_1	m_2	m_3	
6	3	3	0	-3	3	0.00912
6	3	3	0	-2	2	0.05474
6	3	3	0	-1	1	0.13686
6	3	3	0	0	0	0.18248
6	3	3	0	1	-1	0.13686
6	3	3	0	2	-2	0.05474
6	3	3	0	3	-3	0.00912
6	3	3	-1	-2	3	-0.02414
6	3	3	-1	-1	2	-0.09349
6	3	3	-1	0	1	-0.17070
6	3	3	-1	1	0	-0.17070
6	3	3	-1	2	-1	-0.09349
6	3	3	-1	3	-2	-0.02414
6	3	3	-2	-1	3	0.04828
6	3	3	-2	0	2	0.13656
6	3	3	-2	1	1	0.18699
6	3	3	-2	2	0	0.13656
6	3	3	-2	3	-1	0.04828
6	3	3	-3	0	3	-0.08362
6	3	3	-3	1	2	-0.17739
6	3	3	-3	2	1	-0.17739
6	3	3	-3	3	0	-0.08362
6	3	3	-4	1	3	0.13222
6	3	3	-4	2	2	0.20484
6	3	3	-4	3	1	0.13222
6	3	3	-5	2	3	-0.19612
6	3	3	-5	3	2	-0.19612
6	3	3	-6	3	3	0.27735
6	4	2	6	-4	-2	0.27735
6	4	2	5	-4	-1	-0.16013
6	4	2	5	-3	-2	-0.22646
6	4	2	4	-4	0	0.08362
6	4	2	4	-3	-1	0.19312
6	4	2	4	-2	-2	0.18065
6	4	2	3	-4	1	-0.03740
6	4	2	3	-3	0	-0.12955
6	4	2	3	-2	-1	-0.19789
6	4	2	3	-1	-2	-0.13993
6	4	2	2	-4	2	0.01247
6	4	2	2	-3	1	0.07052
6	4	2	2	-2	0	0.16158
6	4	2	2	-1	-1	0.18657
6	4	2	2	0	-2	0.10430
6	4	2	1	-3	2	-0.02787
6	4	2	1	-2	1	-0.10430
6	4	2	1	-1	0	-0.18065
6	4	2	1	0	-1	-0.16491
6	4	2	1	1	-2	-0.07375
6	4	2	0	-2	2	0.04828
6	4	2	0	-1	1	0.13656
6	4	2	0	0	0	0.18699
6	4	2	0	1	-1	0.13656
6	4	2	0	2	-2	0.04828
6	4	2	-1	-1	2	-0.07375
6	4	2	-1	0	1	-0.16491
6	4	2	-1	1	0	-0.18065

j_1	j_2	j_3	m_1	m_2	m_3		
6	4	2	-1	2	-1	-0.10430	
6	4	2	-1	3	-2	-0.02787	
6	4	2	-2	0	2	0.10430	
6	4	2	-2	1	1	0.18657	
6	4	2	-2	2	0	0.16158	
6	4	2	-2	3	-1	0.07052	
6	4	2	-2	4	-2	0.01247	
6	4	2	-3	1	2	-0.13993	
6	4	2	-3	2	1	-0.19789	
6	4	2	-3	3	0	-0.12955	
6	4	2	-3	4	-1	-0.03740	
6	4	2	-4	2	2	0.18065	
6	4	2	-4	3	1	0.19312	
6	4	2	-4	4	0	0.08362	
6	4	2	-5	3	2	-0.22646	
6	4	2	-5	4	1	-0.16013	
6	4	2	-6	4	2	0.27735	
6	4	3	6	-4	-2	0.20966	*
6	4	3	6	-3	-3	-0.18157	*
6	4	3	5	-4	-1	-0.19139	*
6	4	3	5	-3	-2	-0.04280	*
6	4	3	5	-2	-3	0.19612	*
6	4	3	4	-4	0	0.14135	*
6	4	3	4	-3	-1	0.14427	*
6	4	3	4	-2	-2	-0.06828	*
6	4	3	4	-1	-3	-0.17739	*
6	4	3	3	-4	1	-0.08940	*
6	4	3	3	-3	0	-0.16423	*
6	4	3	3	-2	-1	-0.05913	*
6	4	3	3	-1	-2	0.13222	*
6	4	3	3	0	-3	0.14484	*
6	4	3	2	-4	2	0.04712	*
6	4	3	2	-3	1	0.13696	*
6	4	3	2	-2	0	0.13656	*
6	4	3	2	-1	-1	-0.02787	*
6	4	3	2	0	-2	-0.15768	*
6	4	3	2	1	-3	-0.10796	*
6	4	3	1	-4	3	-0.01825	*
6	4	3	1	-3	2	-0.08955	*
6	4	3	1	-2	1	-0.15582	*
6	4	3	1	-1	0	-0.07634	*
6	4	3	1	0	-1	0.09855	*
6	4	3	1	1	-2	0.15331	*
6	4	3	1	2	-3	0.07242	*
6	4	3	0	-3	3	0.04181	*
6	4	3	0	-2	2	0.12774	*
6	4	3	0	-1	1	0.14282	*
6	4	3	0	1	-1	-0.14282	*
6	4	3	0	2	-2	-0.12774	*
6	4	3	0	3	-3	-0.04181	*
6	4	3	-1	-2	3	-0.07242	*
6	4	3	-1	-1	2	-0.15331	*
6	4	3	-1	0	1	-0.09855	*
6	4	3	-1	1	0	0.07634	*
6	4	3	-1	2	-1	0.15582	*
6	4	3	-1	3	-2	0.08955	*
6	4	3	-1	4	-3	0.01825	*
6	4	3	-2	-1	3	0.10796	*

j_1	j_2	j_3	m_1	m_2	m_3		
6	4	3	-2	0	2	0.15768	*
6	4	3	-2	1	1	0.02787	*
6	4	3	-2	2	0	-0.13656	*
6	4	3	-2	3	-1	-0.13696	*
6	4	3	-2	4	-2	-0.04712	*
6	4	3	-3	0	3	-0.14484	*
6	4	3	-3	1	2	-0.13222	*
6	4	3	-3	2	1	0.05913	*
6	4	3	-3	3	0	0.16423	*
6	4	3	-3	4	-1	0.08940	*
6	4	3	-4	1	3	0.17739	*
6	4	3	-4	2	2	0.06828	*
6	4	3	-4	3	1	-0.14427	*
6	4	3	-4	4	0	-0.14135	*
6	4	3	-5	2	3	-0.19612	*
6	4	3	-5	3	2	0.04280	*
6	4	3	-5	4	1	0.19139	*
6	4	3	-6	3	3	0.18157	*
6	4	3	-6	4	2	-0.20966	*
6	4	4	6	-4	-2	0.14322	
6	4	4	6	-3	-3	-0.18947	
6	4	4	6	-2	-4	0.14322	
6	4	4	5	-4	-1	-0.17541	
6	4	4	5	-3	-2	0.08771	
6	4	4	5	-2	-3	0.08771	
6	4	4	5	-1	-4	-0.17541	
6	4	4	4	-4	0	0.16725	
6	4	4	4	-3	-1	0.02644	
6	4	4	4	-2	-2	-0.13993	
6	4	4	4	-1	-3	0.02644	
6	4	4	4	0	-4	0.16725	
6	4	4	3	-4	1	-0.13656	
6	4	4	3	-3	0	-0.10796	
6	4	4	3	-2	-1	0.09032	
6	4	4	3	-1	-2	0.09032	
6	4	4	3	0	-3	-0.10796	
6	4	4	3	1	-4	-0.13656	
6	4	4	2	-4	2	0.09656	
6	4	4	2	-3	1	0.14484	
6	4	4	2	-2	0	-0.00000	
6	4	4	2	-1	-1	-0.12774	
6	4	4	2	0	-2	0.00000	
6	4	4	2	1	-3	0.14484	
6	4	4	2	2	-4	0.09656	
6	4	4	1	-4	3	-0.05713	
6	4	4	1	-3	2	-0.14035	
6	4	4	1	-2	1	-0.08569	
6	4	4	1	-1	0	0.09032	
6	4	4	1	0	-1	0.09032	
6	4	4	1	1	-2	-0.08569	
6	4	4	1	2	-3	-0.14035	
6	4	4	1	3	-4	-0.05713	
6	4	4	0	-4	4	0.02493	
6	4	4	0	-3	3	0.10596	
6	4	4	0	-2	2	0.13713	
6	4	4	0	-1	1	-0.00623	
6	4	4	0	0	0	-0.12466	
6	4	4	0	1	-1	-0.00623	

j_1	j_2	j_3	m_1	m_2	m_3	
6	4	4	0	2	-2	0.13713
6	4	4	0	3	-3	0.10596
6	4	4	0	4	-4	0.02493
6	4	4	-1	-3	4	-0.05713
6	4	4	-1	-2	3	-0.14035
6	4	4	-1	-1	2	-0.08569
6	4	4	-1	0	1	0.09032
6	4	4	-1	1	0	0.09032
6	4	4	-1	2	-1	-0.08569
6	4	4	-1	3	-2	-0.14035
6	4	4	-1	4	-3	-0.05713
6	4	4	-2	-2	4	0.09656
6	4	4	-2	-1	3	0.14484
6	4	4	-2	0	2	0.00000
6	4	4	-2	1	1	-0.12774
6	4	4	-2	2	0	0.00000
6	4	4	-2	3	-1	0.14484
6	4	4	-2	4	-2	0.09656
6	4	4	-3	-1	4	-0.13656
6	4	4	-3	0	3	-0.10796
6	4	4	-3	1	2	0.09032
6	4	4	-3	2	1	0.09032
6	4	4	-3	3	0	-0.10796
6	4	4	-3	4	-1	-0.13656
6	4	4	-4	0	4	0.16725
6	4	4	-4	1	3	0.02644
6	4	4	-4	2	2	-0.13993
6	4	4	-4	3	1	0.02644
6	4	4	-4	4	0	0.16725
6	4	4	-5	1	4	-0.17541
6	4	4	-5	2	3	0.08771
6	4	4	-5	3	2	0.08771
6	4	4	-5	4	1	-0.17541
6	4	4	-6	2	4	0.14322
6	4	4	-6	3	3	-0.18947
6	4	4	-6	4	2	0.14322
6	5	1	6	-5	-1	0.27735
6	5	1	5	-5	0	-0.11323
6	5	1	5	-4	-1	-0.25318
6	5	1	4	-5	1	0.03414
6	5	1	4	-4	0	0.15268
6	5	1	4	-3	-1	0.22901
6	5	1	3	-4	1	-0.05913
6	5	1	3	-3	0	-0.17739
6	5	1	3	-2	-1	-0.20484
6	5	1	2	-3	1	0.08362
6	5	1	2	-2	0	0.19312
6	5	1	2	-1	-1	0.18065
6	5	1	1	-2	1	-0.10796
6	5	1	1	-1	0	-0.20197
6	5	1	1	0	-1	-0.15645
6	5	1	0	-1	1	0.13222
6	5	1	0	0	0	0.20484
6	5	1	0	1	-1	0.13222
6	5	1	-1	0	1	-0.15645
6	5	1	-1	1	0	-0.20197

j_1	j_2	j	m_1	m_2	m	
6	5	1	-1	2	-1	-0.10796
6	5	1	-2	1	1	0.18065
6	5	1	-2	2	0	0.19312
6	5	1	-2	3	-1	0.08362
6	5	1	-3	2	1	-0.20484
6	5	1	-3	3	0	-0.17739
6	5	1	-3	4	-1	-0.05913
6	5	1	-4	3	1	0.22901
6	5	1	-4	4	0	0.15268
6	5	1	-4	5	-1	0.03414
6	5	1	-5	4	1	-0.25318
6	5	1	-5	5	0	-0.11323
6	5	1	-6	5	1	0.27735
6	5	2	6	-5	-1	0.23440 *
6	5	2	6	-4	-2	-0.14825 *
6	5	2	5	-5	0	-0.16575 *
6	5	2	5	-4	-1	-0.12839 *
6	5	2	5	-3	-2	0.18157 *
6	5	2	4	-5	1	0.08656 *
6	5	2	4	-4	0	0.17880 *
6	5	2	4	-3	-1	0.03871 *
6	5	2	4	-2	-2	-0.18964 *
6	5	2	3	-5	2	-0.03161 *
6	5	2	3	-4	1	-0.12994 *
6	5	2	3	-3	0	-0.15581 *
6	5	2	3	-2	-1	0.03462 *
6	5	2	3	-1	-2	0.18321 *
6	5	2	2	-4	2	0.05997 *
6	5	2	2	-3	1	0.15549 *
6	5	2	2	-2	0	0.11308 *
6	5	2	2	-1	-1	-0.09161 *
6	5	2	2	0	-2	-0.16725 *
6	5	2	1	-3	2	-0.08940 *
6	5	2	1	-2	1	-0.16423 *
6	5	2	1	-1	0	-0.05913 *
6	5	2	1	0	-1	0.13222 *
6	5	2	1	1	-2	0.14484 *
6	5	2	0	-2	2	0.11826 *
6	5	2	0	-1	1	0.15645 *
6	5	2	0	1	-1	-0.15645 *
6	5	2	0	2	-2	-0.11826 *
6	5	2	-1	-1	2	-0.14484 *
6	5	2	-1	0	1	-0.13222 *
6	5	2	-1	1	0	0.05913 *
6	5	2	-1	2	-1	0.16423 *
6	5	2	-1	3	-2	0.08940 *
6	5	2	-2	0	2	0.16725 *
6	5	2	-2	1	1	0.09161 *
6	5	2	-2	2	0	-0.11308 *
6	5	2	-2	3	-1	-0.15549 *
6	5	2	-2	4	-2	-0.05997 *
6	5	2	-3	1	2	-0.18321 *
6	5	2	-3	2	1	-0.03462 *
6	5	2	-3	3	0	0.15581 *
6	5	2	-3	4	-1	0.12994 *
6	5	2	-3	5	-2	0.03161 *
6	5	2	-4	2	2	0.18964 *
6	5	2	-4	3	1	-0.03871 *

j_1	j_2	j	m_1	m_2	m	
6	5	2	-4	4	0	-0.17880 *
6	5	2	-4	5	-1	-0.08656 *
6	5	2	-5	3	2	-0.18157 *
6	5	2	-5	4	1	0.12839 *
6	5	2	-5	5	0	0.16575 *
6	5	2	-6	4	2	0.14825 *
6	5	2	-6	5	1	-0.23440 *
6	5	3	6	-5	-1	0.18157
6	5	3	6	-4	-2	-0.18157
6	5	3	6	-3	-3	0.10483
6	5	3	5	-5	0	-0.18157
6	5	3	5	-4	-1	0.00000
6	5	3	5	-3	-2	0.14825
6	5	3	5	-2	-3	-0.14825
6	5	3	4	-5	1	0.13410
6	5	3	4	-4	0	0.12241
6	5	3	4	-3	-1	-0.09995
6	5	3	4	-2	-2	-0.07742
6	5	3	4	-1	-3	0.16725
6	5	3	3	-5	2	-0.07742
6	5	3	3	-4	1	-0.15484
6	5	3	3	-3	0	-0.03161
6	5	3	3	-2	-1	0.13410
6	5	3	3	-1	-2	-0.00000
6	5	3	3	0	-3	-0.16725
6	5	3	2	-5	3	0.03161
6	5	3	2	-4	2	0.12241
6	5	3	2	-3	1	0.12774
6	5	3	2	-2	0	-0.05161
6	5	3	2	-1	-1	-0.11826
6	5	3	2	0	-2	0.06828
6	5	3	2	1	-3	0.15268
6	5	3	1	-4	3	-0.06321
6	5	3	1	-3	2	-0.14599
6	5	3	1	-2	1	-0.07068
6	5	3	1	-1	0	0.10796
6	5	3	1	0	-1	0.06828
6	5	3	1	1	-2	-0.11826
6	5	3	1	2	-3	-0.12774
6	5	3	0	-3	3	0.09656
6	5	3	0	-2	2	0.14484
6	5	3	0	-1	1	0.00000
6	5	3	0	0	0	-0.12774
6	5	3	0	1	-1	0.00000
6	5	3	0	2	-2	0.14484
6	5	3	0	3	-3	0.09656
6	5	3	-1	-2	3	-0.12774
6	5	3	-1	-1	2	-0.11826
6	5	3	-1	0	1	0.06828
6	5	3	-1	1	0	0.10796
6	5	3	-1	2	-1	-0.07068
6	5	3	-1	3	-2	-0.14599
6	5	3	-1	4	-3	-0.06321
6	5	3	-2	-1	3	0.15268
6	5	3	-2	0	2	0.06828
6	5	3	-2	1	1	-0.11826
6	5	3	-2	2	0	-0.05161
6	5	3	-2	3	-1	0.12774

j_1	j_2	j_3	m_1	m_2	m_3	
6	5	3	-2	4	-2	0.12241
6	5	3	-2	5	-3	0.03161
6	5	3	-3	0	3	-0.16725
6	5	3	-3	1	2	0.00000
6	5	3	-3	2	1	0.13410
6	5	3	-3	3	0	-0.03161
6	5	3	-3	4	-1	-0.15484
6	5	3	-3	5	-2	-0.07742
6	5	3	-4	1	3	0.16725
6	5	3	-4	2	2	-0.07742
6	5	3	-4	3	1	-0.09995
6	5	3	-4	4	0	0.12241
6	5	3	-4	5	-1	0.13410
6	5	3	-5	2	3	-0.14825
6	5	3	-5	3	2	0.14825
6	5	3	-5	4	1	0.00000
6	5	3	-5	5	0	-0.18157
6	5	3	-6	3	3	0.10483
6	5	3	-6	4	2	-0.18157
6	5	3	-6	5	1	0.18157
6	5	4	6	-5	-1	0.12839 *
6	5	4	6	-4	-2	-0.17225 *
6	5	4	6	-3	-3	0.15191 *
6	5	4	6	-2	-4	-0.08771 *
6	5	4	5	-5	0	-0.16575 *
6	5	4	5	-4	-1	0.09376 *
6	5	4	5	-3	-2	0.04688 *
6	5	4	5	-2	-3	-0.14322 *
6	5	4	5	-1	-4	0.13397 *
6	5	4	4	-5	1	0.15803 *
6	5	4	4	-4	0	0.02235 *
6	5	4	4	-3	-1	-0.12722 *
6	5	4	4	-2	-2	0.06529 *
6	5	4	4	-1	-3	0.08079 *
6	5	4	4	0	-4	-0.15645 *
6	5	4	3	-5	2	-0.12241 *
6	5	4	3	-4	1	-0.10949 *
6	5	4	3	-3	0	0.08656 *
6	5	4	3	-2	-1	0.06321 *
6	5	4	3	-1	-2	-0.11826 *
6	5	4	3	0	-3	-0.00000 *
6	5	4	3	1	-4	0.15645 *
6	5	4	2	-5	3	0.07634 *
6	5	4	2	-4	2	0.14194 *
6	5	4	2	-3	1	0.01290 *
6	5	4	2	-2	0	-0.11779 *
6	5	4	2	-1	-1	0.02787 *
6	5	4	2	0	-2	0.10796 *
6	5	4	2	1	-3	-0.07375 *
6	5	4	2	2	-4	-0.13797 *
6	5	4	1	-5	4	-0.03414 *
6	5	4	1	-4	3	-0.12214 *
6	5	4	1	-3	2	-0.10387 *
6	5	4	1	-2	1	0.07330 *
6	5	4	1	-1	0	0.07884 *
6	5	4	1	0	-1	-0.09656 *
6	5	4	1	1	-2	-0.04986 *
6	5	4	1	2	-3	0.12341 *
6	5	4	1	3	-4	0.10687 *
6	5	4	0	-4	4	0.06997 *
6	5	4	0	-3	3	0.13993 *
6	5	4	0	-2	2	0.03054 *
6	5	4	0	-1	1	-0.11425 *
6	5	4	0	1	-1	0.11425 *
6	5	4	0	2	-2	-0.03054 *
6	5	4	0	3	-3	-0.13993 *
6	5	4	0	4	-4	-0.06997 *
6	5	4	-1	-3	4	-0.10687 *
6	5	4	-1	-2	3	-0.12341 *
6	5	4	-1	-1	2	0.04986 *
6	5	4	-1	0	1	0.09656 *
6	5	4	-1	1	0	-0.07884 *
6	5	4	-1	2	-1	-0.07330 *
6	5	4	-1	3	-2	0.10387 *
6	5	4	-1	4	-3	0.12214 *
6	5	4	-1	5	-4	0.03414 *
6	5	4	-2	-2	4	0.13797 *
6	5	4	-2	-1	3	0.07375 *
6	5	4	-2	0	2	-0.10796 *
6	5	4	-2	1	1	-0.02787 *
6	5	4	-2	2	0	0.11779 *
6	5	4	-2	3	-1	-0.01290 *
6	5	4	-2	4	-2	-0.14194 *
6	5	4	-2	5	-3	-0.07634 *
6	5	4	-3	-1	4	-0.15645 *
6	5	4	-3	0	3	0.00000 *
6	5	4	-3	1	2	0.11826 *
6	5	4	-3	2	1	-0.06321 *
6	5	4	-3	3	0	-0.08656 *
6	5	4	-3	4	-1	0.10949 *
6	5	4	-3	5	-2	0.12241 *
6	5	4	-4	0	4	0.15645 *
6	5	4	-4	1	3	-0.08079 *
6	5	4	-4	2	2	-0.06529 *
6	5	4	-4	3	1	0.12722 *
6	5	4	-4	4	0	-0.02235 *
6	5	4	-4	5	-1	-0.15803 *
6	5	4	-5	1	4	-0.13397 *
6	5	4	-5	2	3	0.14322 *
6	5	4	-5	3	2	-0.04688 *
6	5	4	-5	4	1	-0.09376 *
6	5	4	-5	5	0	0.16575 *
6	5	4	-6	2	4	0.08771 *
6	5	4	-6	3	3	-0.15191 *
6	5	4	-6	4	2	0.17225 *
6	5	4	-6	5	1	-0.12839 *
6	5	5	6	-5	-1	0.08239
6	5	5	6	-4	-2	-0.13786
6	5	5	6	-3	-3	0.15918
6	5	5	6	-2	-4	-0.13786
6	5	5	6	-1	-5	0.08239
6	5	5	5	-5	0	-0.13026
6	5	5	5	-4	-1	0.13537
6	5	5	5	-3	-2	-0.05628
6	5	5	5	-2	-3	-0.05628
6	5	5	5	-1	-4	0.13537

j_1	j_2	j_3	m_1	m_2	m_3		
6	5	5	5	0	-5	-0.13026	
6	5	5	4	-5	1	0.15211	
6	5	5	4	-4	0	-0.07026	
6	5	5	4	-3	-1	-0.05896	
6	5	5	4	-2	-2	0.11756	
6	5	5	4	-1	-3	-0.05896	
6	5	5	4	0	-4	-0.07026	
6	5	5	4	1	-5	0.15211	
6	5	5	3	-5	2	-0.14696	
6	5	5	3	-4	1	-0.01756	
6	5	5	3	-3	0	0.11338	
6	5	5	3	-2	-1	-0.06085	
6	5	5	3	-1	-2	-0.06085	
6	5	5	3	0	-3	0.11338	
6	5	5	3	1	-4	-0.01756	
6	5	5	3	2	-5	-0.14696	
6	5	5	2	-5	3	0.11999	
6	5	5	2	-4	2	0.09294	
6	5	5	2	-3	1	-0.09108	
6	5	5	2	-2	0	-0.03703	
6	5	5	2	-1	-1	0.10732	
6	5	5	2	0	-2	-0.03703	
6	5	5	2	1	-3	-0.09108	
6	5	5	2	2	-4	0.09294	
6	5	5	2	3	-5	0.11999	
6	5	5	1	-5	4	-0.08049	
6	5	5	1	-4	3	-0.13199	
6	5	5	1	-3	2	0.01386	
6	5	5	1	-2	1	0.10262	
6	5	5	1	-1	0	-0.06196	
6	5	5	1	0	-1	-0.06196	
6	5	5	1	1	-2	0.10262	
6	5	5	1	2	-3	0.01386	
6	5	5	1	3	-4	-0.13199	
6	5	5	1	4	-5	-0.08049	
6	5	5	0	-5	5	0.03928	
6	5	5	0	-4	4	0.12568	
6	5	5	0	-3	3	0.07593	
6	5	5	0	-2	2	-0.09426	
6	5	5	0	-1	1	-0.03142	
6	5	5	0	0	0	0.10474	
6	5	5	0	1	-1	-0.03142	
6	5	5	0	2	-2	-0.09426	
6	5	5	0	3	-3	0.07593	
6	5	5	0	4	-4	0.12568	
6	5	5	0	5	-5	0.03928	
6	5	5	-1	-4	5	-0.08049	
6	5	5	-1	-3	4	-0.13199	
6	5	5	-1	-2	3	0.01386	
6	5	5	-1	-1	2	0.10262	
6	5	5	-1	0	1	-0.06196	
6	5	5	-1	1	0	-0.06196	
6	5	5	-1	2	-1	0.10262	
6	5	5	-1	3	-2	0.01386	
6	5	5	-1	4	-3	-0.13199	
6	5	5	-1	5	-4	-0.08049	
6	5	5	-2	-3	5	0.11999	
6	5	5	-2	-2	4	0.09294	
6	5	5	-2	-1	3	-0.09108	
6	5	5	-2	0	2	-0.03703	
6	5	5	-2	1	1	0.10732	
6	5	5	-2	2	0	-0.03703	
6	5	5	-2	3	-1	-0.09108	
6	5	5	-2	4	-2	0.09294	
6	5	5	-2	5	-3	0.11999	
6	5	5	-3	-2	5	-0.14696	
6	5	5	-3	-1	4	-0.01756	
6	5	5	-3	0	3	0.11338	
6	5	5	-3	1	2	-0.06085	
6	5	5	-3	2	1	-0.06085	
6	5	5	-3	3	0	0.11338	
6	5	5	-3	4	-1	-0.01756	
6	5	5	-3	5	-2	-0.14696	
6	5	5	-4	-1	5	0.15211	
6	5	5	-4	0	4	-0.07026	
6	5	5	-4	1	3	-0.05896	
6	5	5	-4	2	2	0.11756	
6	5	5	-4	3	1	-0.05896	
6	5	5	-4	4	0	-0.07026	
6	5	5	-4	5	-1	0.15211	
6	5	5	-5	0	5	-0.13026	
6	5	5	-5	1	4	0.13537	
6	5	5	-5	2	3	-0.05628	
6	5	5	-5	3	2	-0.05628	
6	5	5	-5	4	1	0.13537	
6	5	5	-5	5	0	-0.13026	
6	5	5	-6	1	5	0.08239	
6	5	5	-6	2	4	-0.13786	
6	5	5	-6	3	3	0.15918	
6	5	5	-6	4	2	-0.13786	
6	5	5	-6	5	1	0.08239	
6	6	0	6	-6	0	0.27735	
6	6	0	5	-5	0	-0.27735	
6	6	0	4	-4	0	0.27735	
6	6	0	3	-3	0	-0.27735	
6	6	0	2	-2	0	0.27735	
6	6	0	1	-1	0	-0.27735	
6	6	0	0	0	0	0.27735	
6	6	0	-1	1	0	-0.27735	
6	6	0	-2	2	0	0.27735	
6	6	0	-3	3	0	-0.27735	
6	6	0	-4	4	0	0.27735	
6	6	0	-5	5	0	-0.27735	
6	6	0	-6	6	0	0.27735	
6	6	1	6	-6	0	0.25678	*
6	6	1	6	-5	-1	-0.10483	*
6	6	1	5	-6	1	-0.10483	*
6	6	1	5	-5	0	-0.21398	*
6	6	1	5	-4	-1	0.14194	*
6	6	1	4	-5	1	0.14194	*
6	6	1	4	-4	0	0.17118	*
6	6	1	4	-3	-1	-0.16575	*
6	6	1	3	-4	1	-0.16575	*
6	6	1	3	-3	0	-0.12839	*
6	6	1	3	-2	-1	0.18157	*
6	6	1	2	-3	1	0.18157	*

j_1	j_2	j_3	m_1	m_2	m_3		
6	6	1	2	-2	0	0.08559	*
6	6	1	2	-1	-1	-0.19139	*
6	6	1	1	-2	1	-0.19139	*
6	6	1	1	-1	0	-0.04280	*
6	6	1	1	0	-1	0.19612	*
6	6	1	0	-1	1	0.19612	*
6	6	1	0	1	-1	-0.19612	*
6	6	1	-1	0	1	-0.19612	*
6	6	1	-1	1	0	0.04280	*
6	6	1	-1	2	-1	0.19139	*
6	6	1	-2	1	1	0.19139	*
6	6	1	-2	2	0	-0.08559	*
6	6	1	-2	3	-1	-0.18157	*
6	6	1	-3	2	1	-0.18157	*
6	6	1	-3	3	0	0.12839	*
6	6	1	-3	4	-1	0.16575	*
6	6	1	-4	3	1	0.16575	*
6	6	1	-4	4	0	-0.17118	*
6	6	1	-4	5	-1	-0.14194	*
6	6	1	-5	4	1	-0.14194	*
6	6	1	-5	5	0	0.21398	*
6	6	1	-5	6	-1	0.10483	*
6	6	1	-6	5	1	0.10483	*
6	6	1	-6	6	0	-0.25678	*
6	6	2	6	-6	0	0.21989	
6	6	2	6	-5	-1	-0.15549	
6	6	2	6	-4	-2	0.06630	
6	6	2	5	-6	1	-0.15549	
6	6	2	5	-5	0	-0.10995	
6	6	2	5	-4	-1	0.17225	
6	6	2	5	-3	-2	-0.10483	
6	6	2	4	-6	2	0.06630	
6	6	2	4	-5	1	0.17225	
6	6	2	4	-4	0	0.01999	
6	6	2	4	-3	-1	-0.15645	
6	6	2	4	-2	-2	0.13410	
6	6	2	3	-5	2	-0.10483	
6	6	2	3	-4	1	-0.15645	
6	6	2	3	-3	0	0.04998	
6	6	2	3	-2	-1	0.12241	
6	6	2	3	-1	-2	-0.15484	
6	6	2	2	-4	2	0.13410	
6	6	2	2	-3	1	0.12241	
6	6	2	2	-2	0	-0.09995	
6	6	2	2	-1	-1	-0.07742	
6	6	2	2	0	-2	0.16725	
6	6	2	1	-3	2	-0.15484	
6	6	2	1	-2	1	-0.07742	
6	6	2	1	-1	0	0.12994	
6	6	2	1	0	-1	0.02644	
6	6	2	1	1	-2	-0.17138	
6	6	2	0	-2	2	0.16725	
6	6	2	0	-1	1	0.02644	
6	6	2	0	0	0	-0.13993	
6	6	2	0	1	-1	0.02644	
6	6	2	0	2	-2	0.16725	
6	6	2	-1	-1	2	-0.17138	
6	6	2	-1	0	1	0.02644	
6	6	2	-1	1	0	0.12994	
6	6	2	-1	2	-1	-0.07742	
6	6	2	-1	3	-2	-0.15484	
6	6	2	-2	0	2	0.16725	
6	6	2	-2	1	1	-0.07742	
6	6	2	-2	2	0	-0.09995	
6	6	2	-2	3	-1	0.12241	
6	6	2	-2	4	-2	0.13410	
6	6	2	-3	1	2	-0.15484	
6	6	2	-3	2	1	0.12241	
6	6	2	-3	3	0	0.04998	
6	6	2	-3	4	-1	-0.15645	
6	6	2	-3	5	-2	-0.10483	
6	6	2	-4	2	2	0.13410	
6	6	2	-4	3	1	-0.15645	
6	6	2	-4	4	0	0.01999	
6	6	2	-4	5	-1	0.17225	
6	6	2	-4	6	-2	0.06630	
6	6	2	-5	3	2	-0.10483	
6	6	2	-5	4	1	0.17225	
6	6	2	-5	5	0	-0.10995	
6	6	2	-5	6	-1	-0.15549	
6	6	2	-6	4	2	0.06630	
6	6	2	-6	5	1	-0.15549	
6	6	2	-6	6	0	0.21989	
6	6	3	6	-6	0	0.17384	*
6	6	3	6	-5	-1	-0.17384	*
6	6	3	6	-4	-2	0.11720	*
6	6	3	6	-3	-3	-0.05241	*
6	6	3	5	-6	1	-0.17384	*
6	6	3	5	-5	0	0.00000	*
6	6	3	5	-4	-1	0.12839	*
6	6	3	5	-3	-2	-0.14825	*
6	6	3	5	-2	-3	0.09078	*
6	6	3	4	-6	2	0.11720	*
6	6	3	4	-5	1	0.12839	*
6	6	3	4	-4	0	-0.09482	*
6	6	3	4	-3	-1	-0.04998	*
6	6	3	4	-2	-2	0.14223	*
6	6	3	4	-1	-3	-0.12241	*
6	6	3	3	-6	3	-0.05241	*
6	6	3	3	-5	2	-0.14825	*
6	6	3	3	-4	1	-0.04998	*
6	6	3	3	-3	0	0.12643	*
6	6	3	3	-2	-1	-0.02737	*
6	6	3	3	-1	-2	-0.10949	*
6	6	3	3	0	-3	0.14484	*
6	6	3	2	-5	3	0.09078	*
6	6	3	2	-4	2	0.14223	*
6	6	3	2	-3	1	-0.02737	*
6	6	3	2	-2	0	-0.11062	*
6	6	3	2	-1	-1	0.08656	*
6	6	3	2	0	-2	0.05913	*
6	6	3	2	1	-3	-0.15645	*
6	6	3	1	-4	3	-0.12241	*
6	6	3	1	-3	2	-0.10949	*
6	6	3	1	-2	1	0.08656	*
6	6	3	1	-1	0	0.06321	*

j_1	j_2	j_3	m_1	m_2	m_3	
6	6	3	1	0	-1	-0.11826 *
6	6	3	1	1	-2	0.00000 *
6	6	3	1	2	-3	0.15645 *
6	6	3	0	-3	3	0.14484 *
6	6	3	0	-2	2	0.05913 *
6	6	3	0	-1	1	-0.11826 *
6	6	3	0	1	-1	0.11826 *
6	6	3	0	2	-2	-0.05913 *
6	6	3	0	3	-3	-0.14484 *
6	6	3	-1	-2	3	-0.15645 *
6	6	3	-1	-1	2	0.00000 *
6	6	3	-1	0	1	0.11826 *
6	6	3	-1	1	0	-0.06321 *
6	6	3	-1	2	-1	-0.08656 *
6	6	3	-1	3	-2	0.10949 *
6	6	3	-1	4	-3	0.12241 *
6	6	3	-2	-1	3	0.15645 *
6	6	3	-2	0	2	-0.05913 *
6	6	3	-2	1	1	-0.08656 *
6	6	3	-2	2	0	0.11062 *
6	6	3	-2	3	-1	0.02737 *
6	6	3	-2	4	-2	-0.14223 *
6	6	3	-2	5	-3	-0.09078 *
6	6	3	-3	0	3	-0.14484 *
6	6	3	-3	1	2	0.10949 *
6	6	3	-3	2	1	0.02737 *
6	6	3	-3	3	0	-0.12643 *
6	6	3	-3	4	-1	0.04998 *
6	6	3	-3	5	-2	0.14825 *
6	6	3	-3	6	-3	0.05241 *
6	6	3	-4	1	3	0.12241 *
6	6	3	-4	2	2	-0.14223 *
6	6	3	-4	3	1	0.04998 *
6	6	3	-4	4	0	0.09482 *
6	6	3	-4	5	-1	-0.12839 *
6	6	3	-4	6	-2	-0.11720 *
6	6	3	-5	2	3	-0.09078 *
6	6	3	-5	3	2	0.14825 *
6	6	3	-5	4	1	-0.12839 *
6	6	3	-5	5	0	0.00000 *
6	6	3	-5	6	-1	0.17384 *
6	6	3	-6	3	3	0.05241 *
6	6	3	-6	4	2	-0.11720 *
6	6	3	-6	5	1	0.17384 *
6	6	3	-6	6	0	-0.17384 *
6	6	4	6	-6	0	0.12649
6	6	4	6	-5	-1	-0.16329
6	6	4	6	-4	-2	0.14770
6	6	4	6	-3	-3	-0.10090
6	6	4	6	-2	-4	0.04757
6	6	4	5	-6	1	-0.16329
6	6	4	5	-5	0	0.08432
6	6	4	5	-4	-1	0.04020
6	6	4	5	-3	-2	-0.12455
6	6	4	5	-2	-3	0.13593
6	6	4	5	-1	-4	-0.08684
6	6	4	4	-6	2	0.14770
6	6	4	4	-5	1	0.04020

j_1	j_2	j_3	m_1	m_2	m_3	
6	6	4	4	-4	0	-0.12265
6	6	4	4	-3	-1	0.06572
6	6	4	4	-2	-2	0.05090
6	6	4	4	-1	-3	-0.13092
6	6	4	4	0	-4	0.11999
6	6	4	3	-6	3	-0.10090
6	6	4	3	-5	2	-0.12455
6	6	4	3	-4	1	0.06572
6	6	4	3	-3	0	0.06899
6	6	4	3	-2	-1	-0.11142
6	6	4	3	-1	-2	0.03066
6	6	4	3	0	-3	0.09294
6	6	4	3	1	-4	-0.14197
6	6	4	2	-6	4	0.04757
6	6	4	2	-5	3	0.13593
6	6	4	2	-4	2	0.05090
6	6	4	2	-3	1	-0.11142
6	6	4	2	-2	0	0.01405
6	6	4	2	-1	-1	0.09576
6	6	4	2	0	-2	-0.09108
6	6	4	2	1	-3	-0.03346
6	6	4	2	2	-4	0.14965
6	6	4	1	-5	4	-0.08684
6	6	4	1	-4	3	-0.13092
6	6	4	1	-3	2	0.03066
6	6	4	1	-2	1	0.09576
6	6	4	1	-1	0	-0.08177
6	6	4	1	0	-1	-0.03703
6	6	4	1	1	-2	0.11313
6	6	4	1	2	-3	-0.03346
6	6	4	1	3	-4	-0.14197
6	6	4	0	-4	4	0.11999
6	6	4	0	-3	3	0.09294
6	6	4	0	-2	2	-0.09108
6	6	4	0	-1	1	-0.03703
6	6	4	0	0	0	0.10732
6	6	4	0	1	-1	-0.03703
6	6	4	0	2	-2	-0.09108
6	6	4	0	3	-3	0.09294
6	6	4	0	4	-4	0.11999
6	6	4	-1	-3	4	-0.14197
6	6	4	-1	-2	3	-0.03346
6	6	4	-1	-1	2	0.11313
6	6	4	-1	0	1	-0.03703
6	6	4	-1	1	0	-0.08177
6	6	4	-1	2	-1	0.09576
6	6	4	-1	3	-2	0.03066
6	6	4	-1	4	-3	-0.13092
6	6	4	-1	5	-4	-0.08684
6	6	4	-2	-2	4	0.14965
6	6	4	-2	-1	3	-0.03346
6	6	4	-2	0	2	-0.09108
6	6	4	-2	1	1	0.09576
6	6	4	-2	2	0	0.01405
6	6	4	-2	3	-1	-0.11142
6	6	4	-2	4	-2	0.05090
6	6	4	-2	5	-3	0.13593
6	6	4	-2	6	-4	0.04757
6	6	4	-3	-1	4	-0.14197
6	6	4	-3	0	3	0.09294

j_1	j_2	j_3	m_1	m_2	m_3	
6	6	4	-3	1	2	0.03066
6	6	4	-3	2	1	-0.11142
6	6	4	-3	3	0	0.06899
6	6	4	-3	4	-1	0.06572
6	6	4	-3	5	-2	-0.12455
6	6	4	-3	6	-3	-0.10090
6	6	4	-4	0	4	0.11999
6	6	4	-4	1	3	-0.13092
6	6	4	-4	2	2	0.05090
6	6	4	-4	3	1	0.06572
6	6	4	-4	4	0	-0.12265
6	6	4	-4	5	-1	0.04020
6	6	4	-4	6	-2	0.14770
6	6	4	-5	1	4	-0.08684
6	6	4	-5	2	3	0.13593
6	6	4	-5	3	2	-0.12455
6	6	4	-5	4	1	0.04020
6	6	4	-5	5	0	0.08432
6	6	4	-5	6	-1	-0.16329
6	6	4	-6	2	4	0.04757
6	6	4	-6	3	3	-0.10090
6	6	4	-6	4	2	0.14770
6	6	4	-6	5	1	-0.16329
6	6	4	-6	6	0	0.12649
6	6	5	6	-6	0	0.08432 *
6	6	5	6	-5	-1	-0.13333 *
6	6	5	6	-4	-2	0.15041 *
6	6	5	6	-3	-3	-0.13453 *
6	6	5	6	-2	-4	0.09513 *
6	6	5	6	-1	-5	-0.04757 *
6	6	5	5	-6	1	-0.13333 *
6	6	5	5	-5	0	0.12649 *
6	6	5	5	-4	-1	-0.04923 *
6	6	5	5	-3	-2	-0.04757 *
6	6	5	5	-2	-3	0.11651 *
6	6	5	5	-1	-4	-0.13026 *
6	6	5	5	0	-5	0.08899 *
6	6	5	4	-6	2	0.15041 *
6	6	5	4	-5	1	-0.04923 *
6	6	5	4	-4	0	-0.06899 *
6	6	5	4	-3	-1	0.11115 *
6	6	5	4	-2	-2	-0.06085 *
6	6	5	4	-1	-3	-0.03928 *
6	6	5	4	0	-4	0.11999 *
6	6	5	4	1	-5	-0.12295 *
6	6	5	3	-6	3	-0.13453 *
6	6	5	3	-5	2	-0.04757 *
6	6	5	3	-4	1	0.11115 *
6	6	5	3	-3	0	-0.04216 *
6	6	5	3	-2	-1	-0.06298 *
6	6	5	3	-1	-2	0.10539 *
6	6	5	3	0	-3	-0.04647 *
6	6	5	3	1	-4	-0.07099 *
6	6	5	3	2	-5	0.14197 *
6	6	5	2	-6	4	0.09513 *
6	6	5	2	-5	3	0.11651 *
6	6	5	2	-4	2	-0.06085 *
6	6	5	2	-3	1	-0.06298 *
6	6	5	2	-2	0	0.09966 *
6	6	5	2	-1	-1	-0.02656 *
6	6	5	2	0	-2	-0.07589 *
6	6	5	2	1	-3	0.10039 *
6	6	5	2	2	-4	-0.00000 *
6	6	5	2	3	-5	-0.14197 *
6	6	5	1	-6	5	-0.04757 *
6	6	5	1	-5	4	-0.13026 *
6	6	5	1	-4	3	-0.03928 *
6	6	5	1	-3	2	0.10539 *
6	6	5	1	-2	1	-0.02656 *
6	6	5	1	-1	0	-0.07666 *
6	6	5	1	0	-1	0.09070 *
6	6	5	1	1	-2	-0.00000 *
6	6	5	1	2	-3	-0.10039 *
6	6	5	1	3	-4	0.07099 *
6	6	5	1	4	-5	0.12295 *
6	6	5	0	-5	5	0.08899 *
6	6	5	0	-4	4	0.11999 *
6	6	5	0	-3	3	-0.04647 *
6	6	5	0	-2	2	-0.07589 *
6	6	5	0	-1	1	0.09070 *
6	6	5	0	1	-1	-0.09070 *
6	6	5	0	2	-2	0.07589 *
6	6	5	0	3	-3	0.04647 *
6	6	5	0	4	-4	-0.11999 *
6	6	5	0	5	-5	-0.08899 *
6	6	5	-1	-4	5	-0.12295 *
6	6	5	-1	-3	4	-0.07099 *
6	6	5	-1	-2	3	0.10039 *
6	6	5	-1	-1	2	0.00000 *
6	6	5	-1	0	1	-0.09070 *
6	6	5	-1	1	0	0.07666 *
6	6	5	-1	2	-1	0.02656 *
6	6	5	-1	3	-2	-0.10539 *
6	6	5	-1	4	-3	0.03928 *
6	6	5	-1	5	-4	0.13026 *
6	6	5	-1	6	-5	0.04757 *
6	6	5	-2	-3	5	0.14197 *
6	6	5	-2	-2	4	-0.00000 *
6	6	5	-2	-1	3	-0.10039 *
6	6	5	-2	0	2	0.07589 *
6	6	5	-2	1	1	0.02656 *
6	6	5	-2	2	0	-0.09966 *
6	6	5	-2	3	-1	0.06298 *
6	6	5	-2	4	-2	0.06085 *
6	6	5	-2	5	-3	-0.11651 *
6	6	5	-2	6	-4	-0.09513 *
6	6	5	-3	-2	5	-0.14197 *
6	6	5	-3	-1	4	0.07099 *
6	6	5	-3	0	3	0.04647 *
6	6	5	-3	1	2	-0.10539 *
6	6	5	-3	2	1	0.06298 *
6	6	5	-3	3	0	0.04216 *
6	6	5	-3	4	-1	-0.11115 *
6	6	5	-3	5	-2	0.04757 *
6	6	5	-3	6	-3	0.13453 *
6	6	5	-4	-1	5	0.12295 *
6	6	5	-4	0	4	-0.11999 *
6	6	5	-4	1	3	0.03928 *

j_1	j_2	j_3	m_1	m_2	m_3		
6	6	5	-4	2	2	0.06085	*
6	6	5	-4	3	1	-0.11115	*
6	6	5	-4	4	0	0.06899	*
6	6	5	-4	5	-1	0.04923	*
6	6	5	-4	6	-2	-0.15041	*
6	6	5	-5	0	5	-0.08899	*
6	6	5	-5	1	4	0.13026	*
6	6	5	-5	2	3	-0.11651	*
6	6	5	-5	3	2	0.04757	*
6	6	5	-5	4	1	0.04923	*
6	6	5	-5	5	0	-0.12649	*
6	6	5	-5	6	-1	0.13333	*
6	6	5	-6	1	5	0.04757	*
6	6	5	-6	2	4	-0.09513	*
6	6	5	-6	3	3	0.13453	*
6	6	5	-6	4	2	-0.15041	*
6	6	5	-6	5	1	0.13333	*
6	6	5	-6	6	0	-0.08432	*
6	6	6	6	-6	0	0.05118	
6	6	6	6	-5	-1	-0.09575	
6	6	6	6	-4	-2	0.12911	
6	6	6	6	-3	-3	-0.14144	
6	6	6	6	-2	-4	0.12911	
6	6	6	6	-1	-5	-0.09575	
6	6	6	6	0	-6	0.05118	
6	6	6	5	-6	1	-0.09575	
6	6	6	5	-5	0	0.12796	
6	6	6	5	-4	-1	-0.10608	
6	6	6	5	-3	-2	0.04083	
6	6	6	5	-2	-3	0.04083	
6	6	6	5	-1	-4	-0.10608	
6	6	6	5	0	-5	0.12796	
6	6	6	5	1	-6	-0.09575	
6	6	6	4	-6	2	0.12911	
6	6	6	4	-5	1	-0.10608	
6	6	6	4	-4	0	0.01861	
6	6	6	4	-3	-1	0.06882	
6	6	6	4	-2	-2	-0.10446	
6	6	6	4	-1	-3	0.06882	
6	6	6	4	0	-4	0.01861	
6	6	6	4	1	-5	-0.10608	
6	6	6	4	2	-6	0.12911	
6	6	6	3	-6	3	-0.14144	
6	6	6	3	-5	2	0.04083	
6	6	6	3	-4	1	0.06882	
6	6	6	3	-3	0	-0.10004	
6	6	6	3	-2	-1	0.04523	
6	6	6	3	-1	-2	0.04523	
6	6	6	3	0	-3	-0.10004	
6	6	6	3	1	-4	0.06882	
6	6	6	3	2	-5	0.04083	
6	6	6	3	3	-6	-0.14144	
6	6	6	2	-6	4	0.12911	
6	6	6	2	-5	3	0.04083	
6	6	6	2	-4	2	-0.10446	
6	6	6	2	-3	1	0.04523	
6	6	6	2	-2	0	0.05118	
6	6	6	2	-1	-1	-0.09536	
6	6	6	2	0	-2	0.05118	
6	6	6	2	1	-3	0.04523	
6	6	6	2	2	-4	-0.10446	
6	6	6	2	3	-5	0.04083	
6	6	6	2	4	-6	0.12911	
6	6	6	1	-6	5	-0.09575	
6	6	6	1	-5	4	-0.10608	
6	6	6	1	-4	3	0.06882	
6	6	6	1	-3	2	0.04523	
6	6	6	1	-2	1	-0.09536	
6	6	6	1	-1	0	0.04653	
6	6	6	1	0	-1	0.04653	
6	6	6	1	1	-2	-0.09536	
6	6	6	1	2	-3	0.04523	
6	6	6	1	3	-4	0.06882	
6	6	6	1	4	-5	-0.10608	
6	6	6	1	5	-6	-0.09575	
6	6	6	0	-6	6	0.05118	
6	6	6	0	-5	5	0.12796	
6	6	6	0	-4	4	0.01861	
6	6	6	0	-3	3	-0.10004	
6	6	6	0	-2	2	0.05118	
6	6	6	0	-1	1	0.04653	
6	6	6	0	0	0	-0.09306	
6	6	6	0	1	-1	0.04653	
6	6	6	0	2	-2	0.05118	
6	6	6	0	3	-3	-0.10004	
6	6	6	0	4	-4	0.01861	
6	6	6	0	5	-5	0.12796	
6	6	6	0	6	-6	0.05118	
6	6	6	-1	-5	6	-0.09575	
6	6	6	-1	-4	5	-0.10608	
6	6	6	-1	-3	4	0.06882	
6	6	6	-1	-2	3	0.04523	
6	6	6	-1	-1	2	-0.09536	
6	6	6	-1	0	1	0.04653	
6	6	6	-1	1	0	0.04653	
6	6	6	-1	2	-1	-0.09536	
6	6	6	-1	3	-2	0.04523	
6	6	6	-1	4	-3	0.06882	
6	6	6	-1	5	-4	-0.10608	
6	6	6	-1	6	-5	-0.09575	
6	6	6	-2	-4	6	0.12911	
6	6	6	-2	-3	5	0.04083	
6	6	6	-2	-2	4	-0.10446	
6	6	6	-2	-1	3	0.04523	
6	6	6	-2	0	2	0.05118	
6	6	6	-2	1	1	-0.09536	
6	6	6	-2	2	0	0.05118	
6	6	6	-2	3	-1	0.04523	
6	6	6	-2	4	-2	-0.10446	
6	6	6	-2	5	-3	0.04083	
6	6	6	-2	6	-4	0.12911	
6	6	6	-3	-3	6	-0.14144	
6	6	6	-3	-2	5	0.04083	
6	6	6	-3	-1	4	0.06882	
6	6	6	-3	0	3	-0.10004	
6	6	6	-3	1	2	0.04523	
6	6	6	-3	2	1	0.04523	
6	6	6	-3	3	0	-0.10004	

j_1	j_2	j_3	m_1	m_2	m_3	
6	6	6	-3	4	-1	0.06882
6	6	6	-3	5	-2	0.04083
6	6	6	-3	6	-3	-0.14144
6	6	6	-4	-2	6	0.12911
6	6	6	-4	-1	5	-0.10608
6	6	6	-4	0	4	0.01861
6	6	6	-4	1	3	0.06882
6	6	6	-4	2	2	-0.10446
6	6	6	-4	3	1	0.06882
6	6	6	-4	4	0	0.01861
6	6	6	-4	5	-1	-0.10608
6	6	6	-4	6	-2	0.12911
6	6	6	-5	-1	6	-0.09575
6	6	6	-5	0	5	0.12796
6	6	6	-5	1	4	-0.10608
6	6	6	-5	2	3	0.04083
6	6	6	-5	3	2	0.04083
6	6	6	-5	4	1	-0.10608
6	6	6	-5	5	0	0.12796
6	6	6	-5	6	-1	-0.09575
6	6	6	-6	0	6	0.05118
6	6	6	-6	1	5	-0.09575
6	6	6	-6	2	4	0.12911
6	6	6	-6	3	3	-0.14144
6	6	6	-6	4	2	0.12911
6	6	6	-6	5	1	-0.09575
6	6	6	-6	6	0	0.05118

Table 12. 6j-symbols

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ l_1 & l_2 & l_3 \end{Bmatrix}$$

j_1	j_2	j_3	l_1	l_2	l_3	
1	1	0	0	0	1	0.57735
1	1	0	1	1	0	0.33333
1	1	1	1	1	0	-0.33333
1	1	1	1	1	1	0.16667
2	1	1	0	1	1	0.33333
2	1	1	1	1	1	0.16667
2	1	1	1	2	0	0.25820
2	1	1	1	2	1	-0.22361
2	1	1	2	1	1	0.03333
2	2	0	0	0	2	0.44721
2	2	0	1	1	1	0.25820
2	2	0	1	1	2	-0.25820
2	2	0	2	2	0	0.20000
2	2	1	1	0	2	-0.25820
2	2	1	1	1	1	-0.22361
2	2	1	1	1	2	0.07454
2	2	1	2	1	1	-0.10000
2	2	1	2	1	2	0.15275
2	2	1	2	2	0	-0.20000
2	2	1	2	2	1	0.16667
2	2	2	1	1	1	0.15275
2	2	2	2	1	1	0.15275
2	2	2	2	2	0	0.20000
2	2	2	2	2	1	-0.10000
2	2	2	2	2	2	-0.04286
3	2	1	0	1	2	0.25820
3	2	1	1	0	3	0.21822
3	2	1	1	1	2	0.14907
3	2	1	1	1	3	-0.17817
3	2	1	1	2	1	0.20000
3	2	1	1	2	2	-0.16330
3	2	1	1	2	3	0.10690
3	2	1	2	1	2	0.04364
3	2	1	2	1	3	-0.09759
3	2	1	2	2	1	0.06667
3	2	1	2	2	2	-0.10690
3	2	1	2	3	0	0.16903
3	2	1	2	3	1	-0.15936
3	2	1	3	2	1	0.00952
3	2	2	0	2	2	-0.20000
3	2	2	1	2	1	-0.16330
3	2	2	1	2	2	0.00000
3	2	2	1	3	1	-0.09759
3	2	2	1	3	2	0.13093
3	2	2	1	3	3	-0.11066
3	2	2	2	2	1	-0.10690
3	2	2	2	2	2	0.11429
3	2	2	2	3	0	-0.16903
3	2	2	2	3	1	0.11952
3	2	2	2	3	2	-0.03499
3	2	2	3	2	1	-0.02857
3	2	2	3	2	2	0.07143
3	2	2	3	3	1	0.07825
3	3	0	0	0	3	0.37796
3	3	0	1	1	2	0.21822
3	3	0	1	1	3	-0.21822
3	3	0	2	2	1	0.16903
3	3	0	2	2	2	-0.16903
3	3	0	3	3	0	0.14286
3	3	1	1	0	3	-0.21822
3	3	1	1	1	2	-0.17817
3	3	1	1	1	3	0.04454
3	3	1	2	1	2	-0.09759
3	3	1	2	1	3	0.13363
3	3	1	2	2	1	-0.15936
3	3	1	2	2	2	0.11952
3	3	1	2	2	3	-0.05976
3	3	1	3	2	1	-0.04762
3	3	1	3	2	2	0.07825
3	3	1	3	3	0	-0.14286
3	3	1	3	3	1	0.13095
3	3	2	1	1	2	0.10690

j_1	j_2	j_3	l_1	l_2	l_3	
3	3	2	1	1	3	0.13363
3	3	2	2	0	3	0.16903
3	3	2	2	1	2	0.13093
3	3	2	2	1	3	-0.05976
3	3	2	2	2	1	0.13997
3	3	2	2	2	2	-0.03499
3	3	2	2	2	3	-0.06415
3	3	2	3	1	2	0.05714
3	3	2	3	1	3	-0.10102
3	3	2	3	2	1	0.07825
3	3	2	3	2	2	-0.10000
3	3	2	3	2	3	0.07377
3	3	2	3	3	0	0.14286
3	3	2	3	3	1	-0.10714
3	3	2	3	3	2	0.04524
3	3	3	2	2	1	-0.11066
3	3	3	2	2	2	-0.05533
3	3	3	3	2	1	-0.10102
3	3	3	3	2	2	0.07377
3	3	3	3	3	0	-0.14286
3	3	3	3	3	1	0.07143
3	3	3	3	3	2	0.02381
3	3	3	3	3	3	-0.07143
4	2	2	0	2	2	0.20000
4	2	2	1	2	2	0.13333
4	2	2	1	3	1	0.16903
4	2	2	1	3	2	-0.12599
4	2	2	1	3	3	0.07063
4	2	2	2	2	2	0.05714
4	2	2	2	3	1	0.06901
4	2	2	2	3	2	-0.10102
4	2	2	2	3	3	0.10595
4	2	2	2	4	0	0.14907
4	2	2	2	4	1	-0.13608
4	2	2	2	4	2	0.11168
4	2	2	3	2	2	0.01429
4	2	2	3	3	1	0.01506
4	2	2	3	3	2	-0.04124
4	2	2	3	4	1	-0.05143
4	2	2	4	2	2	0.00159
4	3	1	0	1	3	0.21822
4	3	1	1	0	4	0.19245
4	3	1	1	1	3	0.13363
4	3	1	1	1	4	-0.15215
4	3	1	1	2	2	0.16903
4	3	1	1	2	3	-0.13363
4	3	1	1	2	4	0.08529
4	3	1	2	1	3	0.04454
4	3	1	2	1	4	-0.09129
4	3	1	2	2	2	0.05901
4	3	1	2	2	3	-0.09960

j_1	j_2	j_3	l_1	l_2	l_3	
4	3	1	2	2	4	0.11443
4	3	1	2	3	1	0.14286
4	3	1	2	3	2	-0.13041
4	3	1	2	3	3	0.11169
4	3	1	3	2	2	0.01506
4	3	1	3	2	3	-0.03367
4	3	1	3	3	1	0.03571
4	3	1	3	3	2	-0.05952
4	3	1	3	4	0	0.12599
4	3	1	3	4	1	-0.12199
4	3	1	4	3	1	0.00397
4	3	2	0	2	3	-0.16903
4	3	2	1	1	3	-0.13363
4	3	2	1	1	4	-0.09129
4	3	2	1	2	2	-0.12599
4	3	2	1	2	3	-0.01992
4	3	2	1	2	4	0.11443
4	3	2	1	3	2	-0.09528
4	3	2	1	3	3	0.11168
4	3	2	1	3	4	-0.08650
4	3	2	2	0	4	-0.14907
4	3	2	2	1	3	-0.09960
4	3	2	2	1	4	0.09526
4	3	2	2	2	2	-0.10102
4	3	2	2	2	3	0.08748
4	3	2	2	2	4	-0.01117
4	3	2	2	3	1	-0.13041
4	3	2	2	3	2	0.07143
4	3	2	2	3	3	-0.00000
4	3	2	2	3	4	-0.06317
4	3	2	2	4	1	-0.05143
4	3	2	2	4	2	0.07897
4	3	2	2	4	3	-0.09118
4	3	2	3	1	3	-0.03367
4	3	2	3	1	4	0.07629
4	3	2	3	2	2	-0.04124
4	3	2	3	2	3	0.07377
4	3	2	3	2	4	-0.08518
4	3	2	3	3	1	-0.05952
4	3	2	3	3	2	0.08333
4	3	2	3	3	3	-0.07897
4	3	2	3	4	0	-0.12599
4	3	2	3	4	1	0.10572
4	3	2	3	4	2	-0.06839
4	3	2	4	2	2	-0.00794
4	3	2	4	2	3	0.02354
4	3	2	4	3	1	-0.01190
4	3	2	4	3	2	0.03254
4	3	2	4	4	1	0.04672
4	3	3	0	3	3	0.14286
4	3	3	1	2	2	0.07063
4	3	3	1	3	2	0.11168
4	3	3	1	3	3	-0.02381
4	3	3	1	4	2	0.05832
4	3	3	1	4	3	-0.09221
4	3	3	1	4	4	0.08595

j_1	j_2	j_3	l_1	l_2	l_3	
4	3	3	2	2	2	0.10595
4	3	3	2	3	1	0.11168
4	3	3	2	3	2	-0.00009
4	3	3	2	3	3	-0.07143
4	3	3	2	4	1	0.07629
4	3	3	2	4	2	-0.09518
4	3	3	2	4	3	0.04303
4	3	3	2	4	4	0.02862
4	3	3	3	2	2	0.07063
4	3	3	3	3	1	0.07897
4	3	3	3	3	2	-0.07897
4	3	3	3	3	3	0.02381
4	3	3	3	4	0	0.12599
4	3	3	3	4	1	-0.08133
4	3	3	3	4	2	0.01243
4	3	3	3	4	3	0.04634
4	3	3	4	2	2	0.02354
4	3	3	4	3	1	0.02381
4	3	3	4	3	2	-0.05556
4	3	3	4	3	3	0.06999
4	3	3	4	4	1	-0.06299
4	3	3	4	4	2	0.07384
4	4	0	0	0	4	0.33333
4	4	0	1	1	3	0.19245
4	4	0	1	1	4	-0.19245
4	4	0	2	2	2	0.14907
4	4	0	2	2	3	-0.14907
4	4	0	2	2	4	0.14907
4	4	0	3	3	1	0.12599
4	4	0	3	3	2	-0.12599
4	4	0	4	4	0	0.11111
4	4	1	1	0	4	-0.19245
4	4	1	1	1	3	-0.15215
4	4	1	1	1	4	0.03043
4	4	1	2	1	3	-0.09129
4	4	1	2	1	4	0.11941
4	4	1	2	2	2	-0.13608
4	4	1	2	2	3	0.09526
4	4	1	2	2	4	-0.04082
4	4	1	3	2	2	-0.05143
4	4	1	3	2	3	0.07629
4	4	1	3	2	4	-0.09258
4	4	1	3	3	1	-0.12199
4	4	1	3	3	2	0.10572
4	4	1	3	3	3	-0.08133
4	4	1	4	3	1	-0.02778
4	4	1	4	3	2	0.04672
4	4	1	4	4	0	-0.11111
4	4	1	4	4	1	0.10556
4	4	2	1	1	3	0.08529
4	4	2	1	1	4	0.11941
4	4	2	2	0	4	0.14907
4	4	2	2	1	3	0.11443
4	4	2	2	1	4	-0.04082
4	4	2	2	2	2	0.11168
4	4	2	2	2	3	-0.01117
4	4	2	2	2	4	-0.06599
4	4	2	3	1	3	0.05832
4	4	2	3	1	4	-0.09258
4	4	2	3	2	2	0.07897
4	4	2	3	2	3	-0.08518
4	4	2	3	2	4	0.05169
4	4	2	3	3	1	0.11398
4	4	2	3	3	2	-0.06839
4	4	2	3	3	3	0.01243
4	4	2	3	3	4	0.03896
4	4	2	4	2	2	0.02381
4	4	2	4	2	3	-0.04762
4	4	2	4	2	4	0.06945
4	4	2	4	3	1	0.04672
4	4	2	4	3	2	-0.06905
4	4	2	4	3	3	0.07384
4	4	2	4	4	0	0.11111
4	4	2	4	4	1	-0.09444
4	4	2	4	4	2	0.06371
4	4	3	2	1	3	-0.08650
4	4	3	2	1	4	-0.09258
4	4	3	2	2	2	-0.07897
4	4	3	2	2	3	-0.06317
4	4	3	2	2	4	0.05169
4	4	3	3	0	4	-0.12599
4	4	3	3	1	3	-0.09221
4	4	3	3	1	4	0.04880
4	4	3	3	2	2	-0.09118
4	4	3	3	2	3	0.04303
4	4	3	3	2	4	0.03896
4	4	3	3	3	1	-0.10195
4	4	3	3	3	2	0.02039
4	4	3	3	3	3	0.04634
4	4	3	3	3	4	-0.05746
4	4	3	4	1	3	-0.03968
4	4	3	4	1	4	0.07571
4	4	3	4	2	2	-0.04762
4	4	3	4	2	3	0.07143
4	4	3	4	2	4	-0.05788
4	4	3	4	3	1	-0.06299
4	4	3	4	3	2	0.07384
4	4	3	4	3	3	-0.04545
4	4	3	4	3	4	-0.00780
4	4	3	4	4	0	-0.11111
4	4	3	4	4	1	0.07778
4	4	3	4	4	2	-0.02410
4	4	3	4	4	3	-0.02756
4	4	4	2	2	2	0.04244
4	4	4	3	2	2	0.08489
4	4	4	3	3	1	0.06585
4	4	4	3	3	2	0.02862
4	4	4	3	3	3	-0.05983

j_1	j_2	j_3	l_1	l_2	l_3	
4	4	4	4	2	2	0.06945
4	4	4	4	3	1	0.07571
4	4	4	4	3	2	-0.05788
4	4	4	4	3	3	-0.00780
4	4	4	4	4	0	0.11111
4	4	4	4	4	1	-0.05556
4	4	4	4	4	2	-0.01659
4	4	4	4	4	3	0.05267
4	4	4	4	4	4	-0.02592
5	3	2	0	2	3	0.16903
5	3	2	1	1	4	0.14907
5	3	2	1	2	3	0.11952
5	3	2	1	2	4	-0.10541
5	3	2	1	3	2	0.14286
5	3	2	1	3	3	-0.10102
5	3	2	1	3	4	0.05429
5	3	2	2	0	5	0.13484
5	3	2	2	1	4	0.06667
5	3	2	2	1	5	-0.12060
5	3	2	2	2	3	0.05832
5	3	2	2	2	4	-0.09258
5	3	2	2	2	5	0.09489
5	3	2	2	3	2	0.07143
5	3	2	2	3	3	-0.09221
5	3	2	2	3	4	0.08921
5	3	2	2	3	5	-0.06276
5	3	2	2	4	1	0.12599
5	3	2	2	4	2	-0.10911
5	3	2	2	4	3	0.08585
5	3	2	2	4	4	-0.05864
5	3	2	3	1	4	0.01699
5	3	2	3	1	5	-0.05096
5	3	2	3	2	3	0.01844
5	3	2	3	2	4	-0.04414
5	3	2	3	2	5	0.07502
5	3	2	3	3	2	0.02381
5	3	2	3	3	3	-0.04762
5	3	2	3	3	4	0.06945
5	3	2	3	4	1	0.03984
5	3	2	3	4	2	-0.06299
5	3	2	3	4	3	0.07678
5	3	2	3	5	0	0.11396
5	3	2	3	5	1	-0.10811
5	3	2	3	5	2	0.09685
5	3	2	4	2	3	0.00321
5	3	2	4	2	4	-0.01097
5	3	2	4	3	2	0.00476
5	3	2	4	3	3	-0.01436
5	3	2	4	4	1	0.00698
5	3	2	4	4	2	-0.01974
5	3	2	4	5	1	-0.03178
5	3	2	5	3	2	0.00043
5	3	3	0	3	3	-0.14286
5	3	3	1	3	2	-0.10102
5	3	3	1	3	3	-0.03571
5	3	3	1	4	2	-0.08909
5	3	3	1	4	3	0.04598
5	3	3	1	4	4	-0.06557
5	3	3	2	3	2	-0.09221
5	3	3	2	3	3	0.05952
5	3	3	2	4	1	-0.11269
5	3	3	2	4	2	0.04880
5	3	3	2	4	3	0.01920
5	3	3	2	4	4	-0.06557
5	3	3	2	5	1	-0.05096
5	3	3	2	5	2	0.07502
5	3	3	2	5	3	-0.08103
5	3	3	2	5	4	0.06848
5	3	3	3	3	2	-0.04762
5	3	3	3	3	3	0.07143
5	3	3	3	4	1	-0.06006
5	3	3	3	4	2	0.07597
5	3	3	3	4	3	-0.05788
5	3	3	3	4	4	0.01192
5	3	3	3	5	0	-0.11396
5	3	3	3	5	1	0.09009
5	3	3	3	5	2	-0.04842
5	3	3	3	5	3	0.00000
5	3	3	4	3	2	-0.01436
5	3	3	4	3	3	0.03680
5	3	3	4	4	1	-0.01551
5	3	3	4	4	2	0.03923
5	3	3	4	4	3	-0.05858
5	3	3	4	5	1	0.04842
5	3	3	4	5	2	-0.06607
5	3	3	5	3	2	-0.00216
5	3	3	5	3	3	0.00974
5	3	3	5	4	2	0.00923
5	4	1	0	1	4	0.19245
5	4	1	1	0	5	0.17408
5	4	1	1	1	4	0.12172
5	4	1	1	1	5	-0.13484
5	4	1	1	2	3	0.14907
5	4	1	1	2	4	-0.11547
5	4	1	1	2	5	0.07247
5	4	1	2	1	4	0.04342
5	4	1	2	1	5	-0.09529
5	4	1	2	2	3	0.06667
5	4	1	2	2	4	-0.09211
5	4	1	2	2	5	0.10249
5	4	1	2	3	2	0.12599
5	4	1	2	3	3	-0.11269
5	4	1	2	3	4	0.09489
5	4	1	2	3	5	-0.07247
5	4	1	3	2	3	0.01699
5	4	1	3	2	4	-0.03482
5	4	1	3	2	5	0.05455
5	4	1	3	3	2	0.03984
5	4	1	3	3	3	-0.06006
5	4	1	3	3	4	0.07502

j_1	j_2	j_3	l_1	l_2	l_3	
5	4	1	3	4	1	0.11111
5	4	1	3	4	2	-0.10541
5	4	1	3	4	3	0.09685
5	4	1	4	3	2	0.00694
5	4	1	4	3	3	-0.01551
5	4	1	4	4	1	0.02222
5	4	1	4	4	2	-0.03761
5	4	1	4	5	0	0.10050
5	4	1	4	5	1	-0.09847
5	4	1	5	4	1	0.00202
5	4	2	0	2	4	-0.14907
5	4	2	1	1	4	-0.11547
5	4	2	1	1	5	-0.08528
5	4	2	1	2	3	-0.10541
5	4	2	1	2	4	-0.02722
5	4	2	1	2	5	0.10249
5	4	2	1	3	3	-0.08909
5	4	2	1	3	4	0.09813
5	4	2	1	3	5	-0.07247
5	4	2	2	0	5	-0.13484
5	4	2	2	1	4	-0.09211
5	4	2	2	1	5	0.08040
5	4	2	2	2	3	-0.09258
5	4	2	2	2	4	0.07107
5	4	2	2	2	5	0.00000
5	4	2	2	3	2	-0.10911
5	4	2	2	3	3	0.04880
5	4	2	2	3	4	0.01370
5	4	2	2	3	5	-0.06276
5	4	2	2	4	2	-0.05556
5	4	2	2	4	3	0.07571
5	4	2	2	4	4	-0.08103
5	4	2	2	4	5	0.07146
5	4	2	3	1	4	-0.03482
5	4	2	3	1	5	0.07247
5	4	2	3	2	3	-0.04414
5	4	2	3	2	4	0.07035
5	4	2	3	2	5	-0.07385
5	4	2	3	3	2	-0.05299
5	4	2	3	3	3	0.07597
5	4	2	3	3	4	-0.06326
5	4	2	3	3	5	0.02659
5	4	2	3	4	1	-0.10541
5	4	2	3	4	2	0.07778
5	4	2	3	4	3	-0.04083
5	4	2	3	4	4	0.00000
5	4	2	3	5	1	-0.03178
5	4	2	3	5	2	0.05125
5	4	2	3	5	3	-0.06464
5	4	2	4	2	3	-0.01097
5	4	2	4	2	4	0.02701
5	4	2	4	2	5	-0.04855
5	4	2	4	3	2	-0.01974
5	4	2	4	3	3	0.03923
5	4	2	4	3	4	-0.05663
5	4	2	4	4	1	-0.03761
5	4	2	4	4	2	0.05758
5	4	2	4	4	3	-0.06607
5	4	2	4	5	0	-0.10050
5	4	2	4	5	1	0.09027
5	4	2	4	5	2	-0.07081
5	4	2	5	3	2	-0.00303
5	4	2	5	3	3	0.00923
5	4	2	5	4	1	-0.00606
5	4	2	5	4	2	0.01717
5	4	2	5	5	1	0.03090
5	4	3	0	3	4	0.12599
5	4	3	1	2	3	0.05479
5	4	3	1	2	4	0.09813
5	4	3	1	2	5	0.05685
5	4	3	1	3	3	0.09598
5	4	3	1	3	4	-0.00813
5	4	3	1	3	5	-0.08409
5	4	3	1	4	3	0.05952
5	4	3	1	4	4	-0.00333
5	4	3	1	4	5	0.07107
5	4	3	2	1	4	0.09489
5	4	3	2	1	5	0.07247
5	4	3	2	2	3	0.08921
5	4	3	2	2	4	0.01370
5	4	3	2	2	5	-0.07385
5	4	3	2	3	2	0.08585
5	4	3	2	3	3	0.01920
5	4	3	2	3	4	-0.06590
5	4	3	2	3	5	0.02659
5	4	3	2	4	2	0.07571
5	4	3	2	4	3	-0.06746
5	4	3	2	4	4	0.02124
5	4	3	2	4	5	0.03746
5	4	3	2	5	2	0.02686
5	4	3	2	5	3	-0.05025
5	4	3	2	5	4	0.06654
5	4	3	2	5	5	-0.06776
5	4	3	3	0	5	0.11396
5	4	3	3	1	4	0.07502
5	4	3	3	1	5	-0.06607
5	4	3	3	2	3	0.06945
5	4	3	3	2	4	-0.06326
5	4	3	3	2	5	-0.00223
5	4	3	3	3	2	0.07678
5	4	3	3	3	3	-0.05788
5	4	3	3	3	4	0.00185
5	4	3	3	3	5	0.04936
5	4	3	3	4	1	0.09685
5	4	3	3	4	2	-0.04083
5	4	3	3	4	3	-0.01659
5	4	3	3	4	4	0.05154
5	4	3	3	4	5	-0.04553
5	4	3	3	5	1	0.04842
5	4	3	3	5	2	-0.06607
5	4	3	3	5	3	0.06061
5	4	3	3	5	4	-0.03306
5	4	3	4	1	4	0.02686
5	4	3	4	1	5	-0.00155
5	4	3	4	2	3	0.02989

j_1	j_2	j_3	l_1	l_2	l_3	
5	4	3	4	2	4	-0.05663
5	4	3	4	2	5	0.06362
5	4	3	4	3	2	0.03627
5	4	3	4	3	3	-0.05858
5	4	3	4	3	4	0.05783
5	4	3	4	3	5	-0.02568
5	4	3	4	4	1	0.05125
5	4	3	4	4	2	-0.06607
5	4	3	4	4	3	0.05325
5	4	3	4	4	4	-0.01747
5	4	3	4	5	0	0.10050
5	4	3	4	5	1	-0.07796
5	4	3	4	5	2	0.03929
5	4	3	4	5	3	0.00388
5	4	3	5	2	3	0.00649
5	4	3	5	2	4	-0.01956
5	4	3	5	2	5	0.03987
5	4	3	5	3	2	0.00923
5	4	3	5	3	3	-0.02489
5	4	3	5	3	4	0.04356
5	4	3	5	4	1	0.01212
5	4	3	5	4	2	-0.03131
5	4	3	5	4	3	0.04872
5	4	3	5	5	1	-0.04242
5	4	3	5	5	2	0.05824
5	4	4	0	4	4	-0.11111
5	4	4	1	3	3	-0.06557
5	4	4	1	4	3	-0.08333
5	4	4	1	4	4	0.02778
5	4	4	1	5	3	-0.04103
5	4	4	1	5	4	0.07107
5	4	4	1	5	5	-0.06998
5	4	4	2	3	2	-0.05864
5	4	4	2	3	3	-0.06557
5	4	4	2	4	2	-0.08103
5	4	4	2	4	3	0.02124
5	4	4	2	4	4	0.04798
5	4	4	2	5	2	-0.04855
5	4	4	2	5	3	0.06487
5	4	4	2	5	4	-0.04009
5	4	4	2	5	5	-0.01750
5	4	4	3	3	2	-0.07908
5	4	4	3	3	3	0.01192
5	4	4	3	4	1	-0.08541
5	4	4	3	4	2	0.00000
5	4	4	3	4	3	0.05154
5	4	4	3	4	4	-0.03535
5	4	4	3	5	1	-0.06155
5	4	4	3	5	2	0.06362
5	4	4	3	5	3	-0.02568
5	4	4	3	5	4	-0.02521
5	4	4	3	5	5	0.04778
5	4	4	4	3	2	-0.05331
5	4	4	4	3	3	0.05961
5	4	4	4	4	1	-0.06268
5	4	4	4	4	2	0.06061
5	4	4	4	4	3	-0.01747
5	4	4	4	4	4	-0.03225

j_1	j_2	j_3	l_1	l_2	l_3	
5	4	4	4	5	0	-0.10050
5	4	4	4	5	1	0.06155
5	4	4	4	5	2	-0.00454
5	4	4	4	5	3	-0.03882
5	4	4	4	5	4	0.04317
5	4	4	5	3	2	-0.01956
5	4	4	5	3	3	0.04356
5	4	4	5	4	1	-0.02020
5	4	4	5	4	2	0.04545
5	4	4	5	4	3	-0.05420
5	4	4	5	4	4	0.03283
5	4	4	5	5	1	0.05249
5	4	4	5	5	2	-0.05833
5	4	4	5	5	3	0.03163
5	5	0	0	0	5	0.30151
5	5	0	1	1	4	0.17408
5	5	0	1	1	5	-0.17408
5	5	0	2	2	3	0.13484
5	5	0	2	2	4	-0.13484
5	5	0	2	2	5	0.13484
5	5	0	3	3	2	0.11396
5	5	0	3	3	3	-0.11396
5	5	0	3	3	4	0.11396
5	5	0	4	4	1	0.10050
5	5	0	4	4	2	-0.10050
5	5	0	5	5	0	0.09091
5	5	1	1	0	5	-0.17408
5	5	1	1	1	4	-0.13484
5	5	1	1	1	5	0.02247
5	5	1	2	1	4	-0.08528
5	5	1	2	1	5	0.10871
5	5	1	2	2	3	-0.12060
5	5	1	2	2	4	0.08040
5	5	1	2	2	5	-0.03015
5	5	1	3	2	3	-0.05096
5	5	1	3	2	4	0.07247
5	5	1	3	2	5	-0.08528
5	5	1	3	3	2	-0.10811
5	5	1	3	3	3	0.09009
5	5	1	3	3	4	-0.06607
5	5	1	3	3	5	0.03604
5	5	1	4	3	2	-0.03178
5	5	1	4	3	3	0.04842
5	5	1	4	3	4	-0.06155
5	5	1	4	4	1	-0.09847
5	5	1	4	4	2	0.09027
5	5	1	4	4	3	-0.07796
5	5	1	5	4	1	-0.01818
5	5	1	5	4	2	0.03090
5	5	1	5	5	0	-0.09091
5	5	1	5	5	1	0.06788
5	5	2	1	1	4	0.07247
5	5	2	1	1	5	0.10871
5	5	2	2	0	5	0.13484

j_1	j_2	j_3	l_1	l_2	l_3	
5	5	2	2	1	4	0.10249
5	5	2	2	1	5	-0.03015
5	5	2	2	2	3	0.09489
5	5	2	2	2	4	-0.00000
5	5	2	2	2	5	-0.06387
5	5	2	3	1	4	0.05685
5	5	2	3	1	5	-0.08528
5	5	2	3	2	3	0.07502
5	5	2	3	2	4	-0.07385
5	5	2	3	2	5	0.03862
5	5	2	3	3	2	0.09685
5	5	2	3	3	3	-0.04842
5	5	2	3	3	4	-0.00223
5	5	2	3	3	5	0.04321
5	5	2	4	2	3	0.02686
5	5	2	4	2	4	-0.04855
5	5	2	4	2	5	0.06596
5	5	2	4	3	2	0.05125
5	5	2	4	3	3	-0.06607
5	5	2	4	3	4	0.06362
5	5	2	4	3	5	-0.04407
5	5	2	4	4	1	0.09441
5	5	2	4	4	2	-0.07081
5	5	2	4	4	3	0.03929
5	5	2	4	4	4	-0.00454
5	5	2	5	3	2	0.01212
5	5	2	5	3	3	-0.02535
5	5	2	5	3	4	0.03987
5	5	2	5	4	1	0.03090
5	5	2	5	4	2	-0.04848
5	5	2	5	4	3	0.05824
5	5	2	5	5	0	0.09091
5	5	2	5	5	1	-0.08182
5	5	2	5	5	2	0.06457
5	5	3	2	1	4	-0.07247
5	5	3	2	1	5	-0.08528
5	5	3	2	2	3	-0.06276
5	5	3	2	2	4	-0.06276
5	5	3	2	2	5	0.03862
5	5	3	3	0	5	-0.11396
5	5	3	3	1	4	-0.08409
5	5	3	3	1	5	0.03604
5	5	3	3	2	3	-0.08103
5	5	3	3	2	4	0.02659
5	5	3	3	2	5	0.04321
5	5	3	3	3	2	-0.08103
5	5	3	3	3	3	0.00000
5	5	3	3	3	4	0.04986
5	5	3	3	3	5	-0.04541
5	5	3	4	1	4	-0.04103
5	5	3	4	1	5	0.07107
5	5	3	4	2	3	-0.05025
5	5	3	4	2	4	0.06487
5	5	3	4	2	5	-0.04407
5	5	3	4	3	2	-0.06464
5	5	3	4	3	3	0.06061
5	5	3	4	3	4	-0.02568

j_1	j_2	j_3	l_1	l_2	l_3	
5	5	3	4	3	5	-0.02065
5	5	3	4	4	1	-0.06831
5	5	3	4	4	2	0.04416
5	5	3	4	4	3	0.00388
5	5	3	4	4	4	-0.03882
5	5	3	4	4	5	0.04610
5	5	3	5	2	3	-0.01515
5	5	3	5	2	4	0.03388
5	5	3	5	2	5	-0.05316
5	5	3	5	3	2	-0.02535
5	5	3	5	3	3	0.04545
5	5	3	5	3	4	-0.05559
5	5	3	5	3	5	0.04765
5	5	3	5	4	1	-0.04242
5	5	3	5	4	2	0.05824
5	5	3	5	4	3	-0.05431
5	5	3	5	4	4	0.03163
5	5	3	5	5	0	-0.09091
5	5	3	5	5	1	0.07273
5	5	3	5	5	2	-0.04103
5	5	3	5	5	3	0.00430
5	5	4	2	2	3	0.03063
5	5	4	2	2	4	0.07146
5	5	4	2	2	5	0.06596
5	5	4	3	1	4	0.07107
5	5	4	3	1	5	0.07107
5	5	4	3	2	3	0.06848
5	5	4	3	2	4	0.03746
5	5	4	3	2	5	-0.04407
5	5	4	3	3	2	0.06194
5	5	4	3	3	3	0.04130
5	5	4	3	3	4	-0.04553
5	5	4	3	3	5	-0.02065
5	5	4	4	0	5	0.10050
5	5	4	4	1	4	0.07107
5	5	4	4	1	5	-0.04103
5	5	4	4	2	3	0.06654
5	5	4	4	2	4	-0.04009
5	5	4	4	2	5	-0.02723
5	5	4	4	3	2	0.07053
5	5	4	4	3	3	-0.03306
5	5	4	4	3	4	-0.02521
5	5	4	4	3	5	0.04610
5	5	4	4	4	1	0.08017
5	5	4	4	4	2	-0.01336
5	5	4	4	4	3	-0.03730
5	5	4	4	4	4	0.04317
5	5	4	4	4	5	-0.00514
5	5	4	5	1	4	0.03030
5	5	4	5	1	5	-0.06061
5	5	4	5	2	3	0.03388
5	5	4	5	2	4	-0.05556
5	5	4	5	2	5	0.04754
5	5	4	5	3	2	0.03987
5	5	4	5	3	3	-0.05559
5	5	4	5	3	4	0.04002
5	5	4	5	3	5	0.00233
5	5	4	5	4	1	0.05249

j_1	j_2	j_3	l_1	l_2	l_3	
5	5	4	5	4	2	-0.05833
5	5	4	5	4	3	0.03163
5	5	4	5	4	4	0.01049
5	5	4	5	4	5	-0.03903
5	5	4	5	5	0	0.09091
5	5	4	5	5	1	-0.06061
5	5	4	5	5	2	0.01399
5	5	4	5	5	3	0.02681
5	5	4	5	5	4	-0.04157
5	5	5	3	3	2	-0.04130
5	5	5	3	3	3	-0.06194
5	5	5	4	3	2	-0.06776
5	5	5	4	3	3	-0.00794
5	5	5	4	4	1	-0.06998
5	5	5	4	4	2	-0.01750
5	5	5	4	4	3	0.04778
5	5	5	4	4	4	-0.00740
5	5	5	5	3	2	-0.05316
5	5	5	5	3	3	0.04765
5	5	5	5	4	1	-0.06061
5	5	5	5	4	2	0.04754
5	5	5	5	4	3	0.00233
5	5	5	5	4	4	-0.03903
5	5	5	5	5	0	-0.09091
5	5	5	5	5	1	0.04545
5	5	5	5	5	2	0.01282
5	5	5	5	5	3	-0.04196
5	5	5	5	5	4	0.02331
5	5	5	5	5	5	0.01923
6	3	3	0	3	3	0.14286
6	3	3	1	3	3	0.10714
6	3	3	1	4	2	0.12599
6	3	3	1	4	3	-0.08333
6	3	3	1	4	4	0.04066
6	3	3	2	3	3	0.05952
6	3	3	2	4	2	0.06901
6	3	3	2	4	3	-0.08333
6	3	3	2	4	4	0.07319
6	3	3	2	5	1	0.11396
6	3	3	2	5	2	-0.09535
6	3	3	2	5	3	0.07107
6	3	3	2	5	4	-0.04495
6	3	3	2	5	5	0.02112
6	3	3	3	3	3	0.02381
6	3	3	3	4	2	0.02686
6	3	3	3	4	3	-0.05025
6	3	3	3	4	4	0.06654
6	3	3	3	5	1	0.04029
6	3	3	3	5	2	-0.06155
6	3	3	3	5	3	0.07107
6	3	3	3	5	4	-0.06776
6	3	3	3	6	0	0.10483
6	3	3	3	6	1	-0.09806
6	3	3	3	6	2	0.08535
6	3	3	3	6	3	-0.06828
6	3	3	4	3	3	0.00649
6	3	3	4	4	2	0.00694
6	3	3	4	4	3	-0.01956
6	3	3	4	4	4	0.03697
6	3	3	4	5	1	0.00833
6	3	3	4	5	2	-0.02290
6	3	3	4	5	3	0.03987
6	3	3	4	6	1	-0.03269
6	3	3	4	6	2	0.05121
6	3	3	5	3	3	0.00108
6	3	3	5	4	2	0.00100
6	3	3	5	4	3	-0.00470
6	3	3	5	5	2	-0.00460
6	3	3	6	3	3	0.00008
6	4	2	0	2	4	0.14907
6	4	2	1	1	5	0.13484
6	4	2	1	2	4	0.10887
6	4	2	1	2	5	-0.09211
6	4	2	1	3	3	0.12599
6	4	2	1	3	4	-0.08607
6	4	2	1	3	5	0.04495
6	4	2	2	0	6	0.12403
6	4	2	2	1	5	0.06356
6	4	2	2	1	6	-0.10939
6	4	2	2	2	4	0.05685
6	4	2	2	2	5	-0.08528
6	4	2	2	2	6	0.08362
6	4	2	2	3	3	0.06901
6	4	2	2	3	4	-0.08409
6	4	2	2	3	5	0.07785
6	4	2	2	3	6	-0.05289
6	4	2	2	4	2	0.11111
6	4	2	2	4	3	-0.09296
6	4	2	2	4	4	0.07107
6	4	2	2	4	5	-0.04738
6	4	2	2	4	6	0.02423
6	4	2	3	1	5	0.01763
6	4	2	3	1	6	-0.04942
6	4	2	3	2	4	0.02010
6	4	2	3	2	5	-0.04425
6	4	2	3	2	6	0.07068
6	4	2	3	3	3	0.02686
6	4	2	3	3	4	-0.04855
6	4	2	3	3	5	0.06596
6	4	2	3	3	6	-0.07299
6	4	2	3	4	2	0.04444
6	4	2	3	4	3	-0.06268
6	4	2	3	4	4	0.07107
6	4	2	3	4	5	-0.06953
6	4	2	3	5	1	0.10050
6	4	2	3	5	2	-0.09211
6	4	2	3	5	3	0.08017
6	4	2	3	5	4	-0.06546
6	4	2	4	2	4	0.00416
6	4	2	4	2	5	-0.01247

j_1	j_2	j_3	l_1	l_2	l_3	
6	4	2	4	2	6	0.02787
6	4	2	4	3	3	0.00694
6	4	2	4	3	4	-0.01738
6	4	2	4	3	5	0.03219
6	4	2	4	4	2	0.01212
6	4	2	4	4	3	-0.02535
6	4	2	4	4	4	0.03987
6	4	2	4	5	1	0.02595
6	4	2	4	5	2	-0.04242
6	4	2	4	5	3	0.05477
6	4	2	4	6	0	0.09245
6	4	2	4	6	1	-0.08932
6	4	2	4	6	2	0.08320
6	4	2	5	3	3	0.00100
6	4	2	5	3	4	-0.00343
6	4	2	5	4	2	0.00202
6	4	2	5	4	3	-0.00620
6	4	2	5	5	1	0.00376
6	4	2	5	5	2	-0.01089
6	4	2	5	6	1	-0.02159
6	4	2	6	4	2	0.00016
6	4	3	0	3	4	-0.12599
6	4	3	1	2	4	-0.08607
6	4	3	1	2	5	-0.08323
6	4	3	1	3	3	-0.08333
6	4	3	1	3	4	-0.04066
6	4	3	1	3	5	0.08494
6	4	3	1	4	3	-0.08333
6	4	3	1	4	4	0.08333
6	4	3	1	4	5	-0.05372
6	4	3	2	1	5	-0.10050
6	4	3	2	1	6	-0.04942
6	4	3	2	2	4	-0.08409
6	4	3	2	2	5	0.03604
6	4	3	2	2	6	0.07068
6	4	3	2	3	3	-0.08333
6	4	3	2	3	4	0.04352
6	4	3	2	3	5	0.02686
6	4	3	2	3	6	-0.07299
6	4	3	2	4	2	-0.09296
6	4	3	2	4	3	0.02778
6	4	3	2	4	4	0.02973
6	4	3	2	4	5	-0.06229
6	4	3	2	4	6	0.05792
6	4	3	2	5	2	-0.05505
6	4	3	2	5	3	0.07107
6	4	3	2	5	4	-0.07042
6	4	3	2	5	5	0.05588
6	4	3	2	5	6	-0.03208
6	4	3	3	0	6	-0.10483
6	4	3	3	1	5	-0.05830
6	4	3	3	1	6	0.07938
6	4	3	3	2	4	-0.04855
6	4	3	3	2	5	0.06870
6	4	3	3	2	6	-0.03658
6	4	3	3	3	3	-0.05025
6	4	3	3	3	4	0.06487
6	4	3	3	3	5	-0.04407
6	4	3	3	3	6	-0.00975
6	4	3	3	4	2	-0.06268
6	4	3	3	4	3	0.06566
6	4	3	3	4	4	-0.04009
6	4	3	3	4	5	-0.00280
6	4	3	3	4	6	0.04434
6	4	3	3	5	1	-0.09401
6	4	3	3	5	2	0.06155
6	4	3	3	5	3	-0.02143
6	4	3	3	5	4	-0.01750
6	4	3	3	5	5	0.04584
6	4	3	3	6	1	-0.03269
6	4	3	3	6	2	0.05121
6	4	3	3	6	3	-0.06176
6	4	3	3	6	4	0.06327
6	4	3	4	1	5	-0.01707
6	4	3	4	1	6	0.04785
6	4	3	4	2	4	-0.01738
6	4	3	4	2	5	0.04097
6	4	3	4	2	6	-0.06164
6	4	3	4	3	3	-0.01956
6	4	3	4	3	4	0.04062
6	4	3	4	3	5	-0.05576
6	4	3	4	3	6	0.05023
6	4	3	4	4	2	-0.02535
6	4	3	4	4	3	0.04545
6	4	3	4	4	4	-0.05559
6	4	3	4	4	5	0.04765
6	4	3	4	5	1	-0.03983
6	4	3	4	5	2	0.05714
6	4	3	4	5	3	-0.05833
6	4	3	4	5	4	0.04274
6	4	3	4	6	0	-0.09245
6	4	3	4	6	1	0.07975
6	4	3	4	6	2	-0.05646
6	4	3	4	6	3	0.02658
6	4	3	5	2	4	-0.00343
6	4	3	5	2	5	0.01167
6	4	3	5	2	6	-0.02762
6	4	3	5	3	3	-0.00470
6	4	3	5	3	4	0.01421
6	4	3	5	3	5	-0.02920
6	4	3	5	4	2	-0.00620
6	4	3	5	4	3	0.01729
6	4	3	5	4	4	-0.03205
6	4	3	5	5	1	-0.00840
6	4	3	5	5	2	0.02262
6	4	3	5	5	3	-0.03791
6	4	3	5	6	1	0.03332
6	4	3	5	6	2	-0.04952
6	4	3	6	3	3	-0.00058
6	4	3	6	3	4	0.00256
6	4	3	6	4	2	-0.00078
6	4	3	6	4	3	0.00364
6	4	3	6	5	2	0.00432
6	4	4	0	4	4	0.11111

j_1	j_2	j_3	l_1	l_2	l_3	
6	4	4	1	3	3	0.04066
6	4	4	1	4	3	0.08333
6	4	4	1	4	4	0.00556
6	4	4	1	5	3	0.05803
6	4	4	1	5	4	-0.07521
6	4	4	1	5	5	0.05771
6	4	4	2	3	3	0.07319
6	4	4	2	4	2	0.07107
6	4	4	2	4	3	0.02973
6	4	4	2	4	4	-0.05707
6	4	4	2	5	2	0.07107
6	4	4	2	5	3	-0.05505
6	4	4	2	5	4	0.00606
6	4	4	2	5	5	0.04328
6	4	4	2	6	2	0.02787
6	4	4	2	6	3	-0.04986
6	4	4	2	6	4	0.06199
6	4	4	2	6	5	-0.05799
6	4	4	2	6	6	0.03747
6	4	4	3	3	3	0.06654
6	4	4	3	4	2	0.07107
6	4	4	3	4	3	-0.04009
6	4	4	3	4	4	-0.01717
6	4	4	3	5	1	0.08535
6	4	4	3	5	2	-0.02607
6	4	4	3	5	3	-0.02723
6	4	4	3	5	4	0.04892
6	4	4	3	5	5	-0.02830
6	4	4	3	6	1	0.04785
6	4	4	3	6	2	-0.06164
6	4	4	3	6	3	0.05023
6	4	4	3	6	4	-0.01852
6	4	4	3	6	5	-0.01998
6	4	4	4	3	3	0.03697
6	4	4	4	4	2	0.03987
6	4	4	4	4	3	-0.05559
6	4	4	4	4	4	0.04002
6	4	4	4	5	1	0.05121
6	4	4	4	5	2	-0.05980
6	4	4	4	5	3	0.03774
6	4	4	4	5	4	0.00233
6	4	4	4	5	5	-0.03607
6	4	4	4	6	0	0.09245
6	4	4	4	6	1	-0.06699
6	4	4	4	6	2	0.02555
6	4	4	4	6	3	0.01595
6	4	4	4	6	4	-0.04078
6	4	4	5	3	3	0.01280
6	4	4	5	4	2	0.01329
6	4	4	5	4	3	-0.03205
6	4	4	5	4	4	0.04681
6	4	4	5	5	1	0.01456
6	4	4	5	5	2	-0.03516
6	4	4	5	5	3	0.04853
6	4	4	5	5	4	-0.04317
6	4	4	5	6	1	-0.04318
6	4	4	5	6	2	0.05467
6	4	4	5	6	3	-0.04291
6	4	4	6	3	3	0.00256
6	4	4	6	4	2	0.00233

j_1	j_2	j_3	l_1	l_2	l_3	
6	4	4	6	4	3	-0.00991
6	4	4	6	4	4	0.02335
6	4	4	6	5	2	-0.00932
6	4	4	6	5	3	0.02374
6	4	4	6	6	2	0.02431
6	5	1	0	1	5	0.17408
6	5	1	1	0	6	0.16013
6	5	1	1	1	5	0.11237
6	5	1	1	1	6	-0.12230
6	5	1	1	2	4	0.13488
6	5	1	1	2	5	-0.10299
6	5	1	1	2	6	0.06397
6	5	1	2	1	5	0.04181
6	5	1	2	1	6	-0.08006
6	5	1	2	2	4	0.06356
6	5	1	2	2	5	-0.08569
6	5	1	2	2	6	0.09349
6	5	1	2	3	3	0.11396
6	5	1	2	3	4	-0.10050
6	5	1	2	3	5	0.08362
6	5	1	2	3	6	-0.06321
6	5	1	3	2	4	0.01763
6	5	1	3	2	5	-0.03462
6	5	1	3	2	6	0.05474
6	5	1	3	3	3	0.04029
6	5	1	3	3	4	-0.05830
6	5	1	3	3	5	0.07068
6	5	1	3	3	6	-0.07742
6	5	1	3	4	2	0.10059
6	5	1	3	4	3	-0.09401
6	5	1	3	4	4	0.08535
6	5	1	3	4	5	-0.07450
6	5	1	4	3	3	0.00833
6	5	1	4	3	4	-0.01707
6	5	1	4	3	5	0.02787
6	5	1	4	4	2	0.02595
6	5	1	4	4	3	-0.03983
6	5	1	4	4	4	0.05121
6	5	1	4	5	1	0.09091
6	5	1	4	5	2	-0.08783
6	5	1	4	5	3	0.08320
6	5	1	5	4	2	0.00376
6	5	1	5	4	3	-0.00840
6	5	1	5	5	1	0.01515
6	5	1	5	5	2	-0.02584
6	5	1	5	6	0	0.00362
6	5	1	5	6	1	-0.08245
6	5	1	6	5	1	0.00117
6	5	2	0	2	5	-0.13484
6	5	2	1	1	5	-0.10299
6	5	2	1	1	6	-0.08006
6	5	2	1	2	4	-0.09211
6	5	2	1	2	5	-0.03015
6	5	2	1	2	6	0.09349
6	5	2	1	3	4	-0.08323
6	5	2	1	3	5	0.08827

j_1	j_2	j_3	l_1	l_2	l_3	
6	5	2	1	3	6	-0.06321
6	5	2	2	0	6	-0.12403
6	5	2	2	1	5	-0.08569
6	5	2	2	1	6	0.07032
6	5	2	2	2	4	-0.08528
6	5	2	2	2	5	0.06022
6	5	2	2	2	6	0.00597
6	5	2	2	3	3	-0.09535
6	5	2	2	3	4	0.03604
6	5	2	2	3	5	0.01999
6	5	2	2	3	6	-0.06044
6	5	2	2	4	3	-0.05505
6	5	2	2	4	4	0.07107
6	5	2	2	4	5	-0.07299
6	5	2	2	4	6	0.06230
6	5	2	3	1	5	-0.03462
6	5	2	3	1	6	0.06863
6	5	2	3	2	4	-0.04425
6	5	2	3	2	5	0.06621
6	5	2	3	2	6	-0.06543
6	5	2	3	3	3	-0.06155
6	5	2	3	3	4	0.06870
6	5	2	3	3	5	-0.05244
6	5	2	3	3	6	0.01689
6	5	2	3	4	2	-0.09211
6	5	2	3	4	3	0.06155
6	5	2	3	4	4	-0.02607
6	5	2	3	4	5	-0.00975
6	5	2	3	4	6	0.04022
6	5	2	3	5	2	-0.03636
6	5	2	3	5	3	0.05249
6	5	2	3	5	4	-0.06176
6	5	2	3	5	5	0.06416
6	5	2	4	2	4	-0.01247
6	5	2	4	2	5	0.02827
6	5	2	4	2	6	-0.04779
6	5	2	4	3	3	-0.02290
6	5	2	4	3	4	0.04097
6	5	2	4	3	5	-0.05474
6	5	2	4	3	6	0.05877
6	5	2	4	4	2	-0.04242
6	5	2	4	4	3	0.05714
6	5	2	4	4	4	-0.05980
6	5	2	4	4	5	0.05023
6	5	2	4	5	1	-0.08783
6	5	2	4	5	2	0.07273
6	5	2	4	5	3	-0.05167
6	5	2	4	5	4	0.02658
6	5	2	4	6	1	-0.02159
6	5	2	4	6	2	0.03566
6	5	2	4	6	3	-0.04681
6	5	2	5	3	3	-0.00460
6	5	2	5	3	4	0.01167
6	5	2	5	3	5	-0.02201
6	5	2	5	4	2	-0.01089
6	5	2	5	4	3	0.02762
6	5	2	5	4	4	-0.03516
6	5	2	5	5	1	-0.02584
6	5	2	5	5	2	0.04126
6	5	2	5	5	3	-0.05115
6	5	2	5	6	0	-0.08262
6	5	2	5	6	1	0.07774
6	5	2	5	6	2	-0.06638
6	5	2	6	4	2	-0.00140
6	5	2	6	4	3	0.00432
6	5	2	6	5	1	-0.00350
6	5	2	6	5	2	0.01009
6	5	2	6	6	1	0.02192
6	5	3	0	3	5	0.11396
6	5	3	1	2	4	0.04495
6	5	3	1	2	5	0.08827
6	5	3	1	2	6	0.05474
6	5	3	1	3	4	0.08494
6	5	3	1	3	5	0.00000
6	5	3	1	3	6	-0.07742
6	5	3	1	4	4	0.05803
6	5	3	1	4	5	-0.07597
6	5	3	1	4	6	0.06143
6	5	3	2	1	5	0.08362
6	5	3	2	1	6	0.06863
6	5	3	2	2	4	0.07785
6	5	3	2	2	5	0.01999
6	5	3	2	2	6	-0.06543
6	5	3	2	3	3	0.07107
6	5	3	2	3	4	0.02686
6	5	3	2	3	5	-0.05960
6	5	3	2	3	6	0.01689
6	5	3	2	4	3	0.07107
6	5	3	2	4	4	-0.05505
6	5	3	2	4	5	0.00942
6	5	3	2	4	6	0.04022
6	5	3	2	5	3	0.03030
6	5	3	2	5	4	-0.05071
6	5	3	2	5	5	0.06199
6	5	3	2	5	6	-0.05940
6	5	3	3	0	6	0.10483
6	5	3	3	1	5	0.07068
6	5	3	3	1	6	-0.05603
6	5	3	3	2	4	0.06596
6	5	3	3	2	5	-0.05244
6	5	3	3	2	6	-0.00975
6	5	3	3	3	3	0.07107
6	5	3	3	3	4	-0.04407
6	5	3	3	3	5	-0.00890
6	5	3	3	3	6	0.04877
6	5	3	3	4	2	0.08017
6	5	3	3	4	3	-0.02143
6	5	3	3	4	4	-0.02723
6	5	3	3	4	5	0.04924
6	5	3	3	4	6	-0.03500
6	5	3	3	5	2	0.05249
6	5	3	3	5	3	-0.06061
6	5	3	3	5	4	0.04754
6	5	3	3	5	5	-0.01852
6	5	3	3	5	6	-0.01715
6	5	3	3	6	2	0.01456

j_1	j_2	j_3	l_1	l_2	l_3	
6	5	3	3	6	3	−0.02911
6	5	3	3	6	4	0.04298
6	5	3	3	6	5	−0.05264
6	5	3	4	1	5	0.02787
6	5	3	4	1	6	−0.05906
6	5	3	4	2	4	0.03219
6	5	3	4	2	5	−0.05474
6	5	3	4	2	6	0.05552
6	5	3	4	3	3	0.03987
6	5	3	4	3	4	−0.05576
6	5	3	4	3	5	0.04765
6	5	3	4	3	6	−0.01395
6	5	3	4	4	2	0.05477
6	5	3	4	4	3	−0.05833
6	5	3	4	4	4	0.03774
6	5	3	4	4	5	−0.00259
6	5	3	4	4	6	−0.03144
6	5	3	4	5	1	0.08320
6	5	3	4	5	2	−0.05167
6	5	3	4	5	3	0.01399
6	5	3	4	5	4	0.02014
6	5	3	4	5	5	−0.04078
6	5	3	4	6	1	0.03332
6	5	3	4	6	2	−0.04952
6	5	3	4	6	3	0.05417
6	5	3	4	6	4	−0.04614
6	5	3	5	2	4	0.00840
6	5	3	5	2	5	−0.02201
6	5	3	5	2	6	0.04058
6	5	3	5	3	3	0.01329
6	5	3	5	3	4	−0.02920
6	5	3	5	3	5	0.04447
6	5	3	5	3	6	−0.05073
6	5	3	5	4	2	0.02068
6	5	3	5	4	3	−0.03791
6	5	3	5	4	4	0.04853
6	5	3	5	4	5	−0.04610
6	5	3	5	5	1	0.03566
6	5	3	5	5	2	−0.05115
6	5	3	5	5	3	0.05228
6	5	3	5	5	4	−0.03863
6	5	3	5	6	0	0.06362
6	5	3	5	6	1	−0.07068
6	5	3	5	6	2	0.04731
6	5	3	5	6	3	−0.01824
6	5	3	6	3	3	0.00233
6	5	3	6	3	4	−0.00737
6	5	3	6	3	5	0.01625
6	5	3	6	4	2	0.00432
6	5	3	6	4	3	−0.01232
6	5	3	6	4	4	0.02374
6	5	3	6	5	1	0.00699
6	5	3	6	5	2	−0.01894
6	5	3	6	5	3	0.03230
6	5	3	6	6	1	−0.03037
6	5	3	6	6	2	0.04499
6	5	4	0	4	5	−0.10050
6	5	4	1	3	4	−0.05372
6	5	4	1	3	5	−0.07597
6	5	4	1	3	6	−0.04080
6	5	4	1	4	4	−0.07521
6	5	4	1	4	5	0.01641
6	5	4	1	4	6	0.06636
6	5	4	1	5	4	−0.04242
6	5	4	1	5	5	0.06611
6	5	4	1	5	6	−0.06002
6	5	4	2	2	4	−0.04738
6	5	4	2	2	5	−0.07299
6	5	4	2	2	6	−0.04779
6	5	4	2	3	3	−0.04495
6	5	4	2	3	4	−0.06229
6	5	4	2	3	5	0.00942
6	5	4	2	3	6	0.05877
6	5	4	2	4	3	−0.07042
6	5	4	2	4	4	0.00606
6	5	4	2	4	5	0.04824
6	5	4	2	4	6	−0.02878
6	5	4	2	5	3	−0.05071
6	5	4	2	5	4	0.05657
6	5	4	2	5	5	−0.02593
6	5	4	2	5	6	−0.02485
6	5	4	2	6	3	−0.01722
6	5	4	2	6	4	0.03616
6	5	4	2	6	5	−0.05211
6	5	4	2	6	6	0.05621
6	5	4	3	1	5	−0.07450
6	5	4	3	1	6	−0.05906
6	5	4	3	2	4	−0.06953
6	5	4	3	2	5	−0.00975
6	5	4	3	2	6	0.05552
6	5	4	3	3	3	−0.06776
6	5	4	3	3	4	−0.00280
6	5	4	3	3	5	0.04924
6	5	4	3	3	6	−0.01395
6	5	4	3	4	2	−0.06546
6	5	4	3	4	3	−0.01750
6	5	4	3	4	4	0.04892
6	5	4	3	4	5	−0.02052
6	5	4	3	4	6	−0.03144
6	5	4	3	5	2	−0.06176
6	5	4	3	5	3	0.04754
6	5	4	3	5	4	−0.00591
6	5	4	3	5	5	−0.03342
6	5	4	3	5	6	0.04036
6	5	4	3	6	2	−0.02762
6	5	4	3	6	3	0.04603
6	5	4	3	6	4	−0.04958
6	5	4	3	6	5	0.03329
6	5	4	3	6	6	−0.00000
6	5	4	4	0	6	−0.09745
6	5	4	4	1	5	−0.06022
6	5	4	4	1	6	0.05104
6	5	4	4	2	4	−0.05316
6	5	4	4	2	5	0.05023
6	5	4	4	2	6	0.00543
6	5	4	4	3	3	−0.05425

j_1	j_2	j_3	l_1	l_2	l_3	
6	5	4	4	3	4	0.04765
6	5	4	4	3	5	-0.00259
6	5	4	4	3	6	-0.04063
6	5	4	4	4	2	-0.06201
6	5	4	4	4	3	0.04278
6	5	4	4	4	4	0.00233
6	5	4	4	4	5	-0.03759
6	5	4	4	4	6	0.03269
6	5	4	4	5	1	-0.07703
6	5	4	4	5	2	0.02658
6	5	4	4	5	3	0.02014
6	5	4	4	5	4	-0.04196
6	5	4	4	5	5	0.02936
6	5	4	4	5	6	0.00727
6	5	4	4	6	1	-0.04318
6	5	4	4	6	2	0.05467
6	5	4	4	6	3	-0.04291
6	5	4	4	6	4	0.01359
6	5	4	4	6	5	0.01922
6	5	4	5	1	5	-0.02224
6	5	4	5	1	6	0.05140
6	5	4	5	2	4	-0.02344
6	5	4	5	2	5	0.04603
6	5	4	5	2	6	-0.05093
6	5	4	5	3	3	-0.02606
6	5	4	5	3	4	0.04552
6	5	4	5	3	5	-0.04610
6	5	4	5	3	6	0.01791
6	5	4	5	4	2	-0.03185
6	5	4	5	4	3	0.04832
6	5	4	5	4	4	-0.04317
6	5	4	5	4	5	0.01480
6	5	4	5	4	6	0.02189
6	5	4	5	5	1	-0.04447
6	5	4	5	5	2	0.05417
6	5	4	5	5	3	-0.03863
6	5	4	5	5	4	0.00622
6	5	4	5	5	5	0.02543
6	5	4	5	6	0	-0.08362
6	5	4	5	6	1	0.06125
6	5	4	5	6	2	-0.02472
6	5	4	5	6	3	-0.01222
6	5	4	5	6	4	0.03519
6	5	4	6	2	4	-0.00548
6	5	4	6	2	5	0.01665
6	5	4	6	2	6	-0.03412
6	5	4	6	3	3	-0.00737
6	5	4	6	3	4	0.02020
6	5	4	6	3	5	-0.03596
6	5	4	6	3	6	0.04559
6	5	4	6	4	2	-0.00932
6	5	4	6	4	3	0.02374
6	5	4	6	4	4	-0.03823
6	5	4	6	4	5	0.04328
6	5	4	6	5	1	-0.01166
6	5	4	6	5	2	0.02897
6	5	4	6	5	3	-0.04229
6	5	4	6	5	4	0.04241
6	5	4	6	6	1	0.03810
6	5	4	6	6	2	-0.04950
6	5	4	6	6	3	0.05136
6	5	5	0	5	5	0.09091
6	5	5	1	4	4	0.05771
6	5	5	1	5	4	0.06611
6	5	5	1	5	5	-0.02727
6	5	5	1	6	4	0.03145
6	5	5	1	6	5	-0.05777
6	5	5	1	6	6	0.05903
6	5	5	2	3	3	0.02112
6	5	5	2	4	3	0.05568
6	5	5	2	4	4	0.04328
6	5	5	2	5	3	0.06199
6	5	5	2	5	4	-0.02593
6	5	5	2	5	5	-0.03473
6	5	5	2	6	3	0.03514
6	5	5	2	6	4	-0.05208
6	5	5	2	6	5	0.03596
6	5	5	2	6	6	0.01181
6	5	5	3	3	3	0.05280
6	5	5	3	4	2	0.04901
6	5	5	3	4	3	0.04584
6	5	5	3	4	4	-0.02830
6	5	5	3	5	2	0.06416
6	5	5	3	5	3	-0.01852
6	5	5	3	5	4	-0.03342
6	5	5	3	5	5	0.03450
6	5	5	3	6	2	0.04058
6	5	5	3	6	3	-0.05073
6	5	5	3	6	4	0.02731
6	5	5	3	6	5	0.01511
6	5	5	3	6	6	-0.03935
6	5	5	4	3	3	0.06092
6	5	5	4	4	2	0.06327
6	5	5	4	4	3	-0.01387
6	5	5	4	4	4	-0.03607
6	5	5	4	5	1	0.06931
6	5	5	4	5	2	-0.00000
6	5	5	4	5	3	-0.04078
6	5	5	4	5	4	0.02936
6	5	5	4	5	5	0.01390
6	5	5	4	6	1	0.05140
6	5	5	4	6	2	-0.05093
6	5	5	4	6	3	0.01791
6	5	5	4	6	4	0.02189
6	5	5	4	6	5	-0.03634
6	5	5	4	6	6	0.01181
6	5	5	5	3	3	0.04061
6	5	5	5	4	2	0.04298
6	5	5	5	4	3	-0.04805
6	5	5	5	4	4	0.01665
6	5	5	5	5	1	0.05198
6	5	5	5	5	2	-0.04928
6	5	5	5	5	3	0.01359
6	5	5	5	5	4	0.02543
6	5	5	5	5	5	-0.03438
6	5	5	5	6	0	0.08362

j_1	j_2	j_3	l_1	l_2	l_3	
6	5	5	5	6	1	-0.04947
6	5	5	5	6	2	0.00107
6	5	5	5	6	3	0.03320
6	5	5	5	6	4	-0.03385
6	5	5	5	6	5	0.00453
6	5	5	6	3	3	0.01625
6	5	5	6	4	2	0.01665
6	5	5	6	4	3	-0.03596
6	5	5	6	4	4	0.04328
6	5	5	6	5	1	0.01748
6	5	5	6	5	2	-0.03846
6	5	5	6	5	3	0.04429
6	5	5	6	5	4	-0.02541
6	5	5	6	5	5	-0.00871
6	5	5	6	6	1	-0.04491
6	5	5	6	6	2	0.04811
6	5	5	6	6	3	-0.02364
6	5	5	6	6	4	-0.01125
6	6	0	0	0	6	0.27735
6	6	0	1	1	5	0.16013
6	6	0	1	1	6	-0.16013
6	6	0	2	2	4	0.12403
6	6	0	2	2	5	-0.12403
6	6	0	2	2	6	0.12403
6	6	0	3	3	3	0.10483
6	6	0	3	3	4	-0.10463
6	6	0	3	3	5	0.10483
6	6	0	3	3	6	-0.10483
6	6	0	4	4	2	0.09245
6	6	0	4	4	3	-0.09245
6	6	0	4	4	4	0.09245
6	6	0	5	5	1	0.08362
6	6	0	5	5	2	-0.08362
6	6	0	6	6	0	0.07692
6	6	1	1	0	6	-0.16013
6	6	1	1	1	5	-0.12230
6	6	1	1	1	6	0.01747
6	6	1	2	1	5	-0.08006
6	6	1	2	1	6	0.10037
6	6	1	2	2	4	-0.10939
6	6	1	2	2	5	0.07032
6	6	1	2	2	6	-0.02344
6	6	1	3	2	4	-0.04942
6	6	1	3	2	5	0.06863
6	6	1	3	2	6	-0.07924
6	6	1	3	3	3	-0.09806
6	6	1	3	3	4	0.07938
6	6	1	3	3	5	-0.05603
6	6	1	3	3	6	0.02802
6	6	1	4	3	3	-0.03269
6	6	1	4	3	4	0.04785
6	6	1	4	3	5	-0.05906
6	6	1	4	3	6	0.06669
6	6	1	4	4	2	-0.08932
6	6	1	4	4	3	0.07975
6	6	1	4	4	4	-0.06699
6	6	1	4	4	5	0.05104
6	6	1	5	4	2	-0.02159
6	6	1	5	4	3	0.03332
6	6	1	5	4	4	-0.04318
6	6	1	5	5	1	-0.08245
6	6	1	5	5	2	0.07774
6	6	1	5	5	3	-0.07068
6	6	1	6	5	1	-0.01282
6	6	1	6	5	2	0.02192
6	6	1	6	6	0	-0.07692
6	6	1	6	6	1	0.07509
6	6	2	1	1	5	0.06387
6	6	2	1	1	6	0.10037
6	6	2	2	0	6	0.12403
6	6	2	2	1	5	0.09349
6	6	2	2	1	6	-0.02344
6	6	2	2	2	4	0.08362
6	6	2	2	2	5	0.00597
6	6	2	2	2	6	-0.06093
6	6	2	3	1	5	0.05478
6	6	2	3	1	6	-0.07924
6	6	2	3	2	4	0.07068
6	6	2	3	2	5	-0.06543
6	6	2	3	2	6	0.03022
6	6	2	3	3	3	0.08535
6	6	2	3	3	4	-0.03658
6	6	2	3	3	5	-0.00975
6	6	2	3	3	6	0.04384
6	6	2	4	2	4	0.02787
6	6	2	4	2	5	-0.04779
6	6	2	4	2	6	0.06230
6	6	2	4	3	3	0.05121
6	6	2	4	3	4	-0.06164
6	6	2	4	3	5	0.05552
6	6	2	4	3	6	-0.03483
6	6	2	4	4	2	0.08320
6	6	2	4	4	3	-0.05646
6	6	2	4	4	4	0.02555
6	6	2	4	4	5	0.00543
6	6	2	4	4	6	-0.03141
6	6	2	5	3	3	0.01456
6	6	2	5	3	4	-0.02762
6	6	2	5	3	5	0.04058
6	6	2	5	3	6	-0.05145
6	6	2	5	4	2	0.03566
6	6	2	5	4	3	-0.04952
6	6	2	5	4	4	0.05467
6	6	2	5	4	5	-0.05093
6	6	2	5	5	1	0.08011
6	6	2	5	5	2	-0.06638
6	6	2	5	5	3	0.04731
6	6	2	5	5	4	-0.02472
6	6	2	6	4	2	0.00699
6	6	2	6	4	3	-0.01495
6	6	2	6	4	4	0.02431
6	6	2	6	5	1	0.02192
6	6	2	6	5	2	-0.03546

j_1	j_2	j_3	l_1	l_2	l_3	
6	6	2	6	5	3	0.04499
6	6	2	6	6	0	0.07692
6	6	2	6	6	1	-0.07143
6	6	2	6	6	2	0.06088
6	6	3	2	1	5	-0.06321
6	6	3	2	1	6	-0.07924
6	6	3	2	2	4	-0.05289
6	6	3	2	2	5	-0.06044
6	6	3	2	2	6	0.03022
6	6	3	3	0	6	-0.10483
6	6	3	3	1	5	-0.07742
6	6	3	3	1	6	0.02802
6	6	3	3	2	4	-0.07299
6	6	3	3	2	5	0.01689
6	6	3	3	2	6	0.04389
6	6	3	3	3	3	-0.06828
6	6	3	3	3	4	-0.00975
6	6	3	3	3	5	0.04977
6	6	3	3	3	6	-0.03658
6	6	3	4	1	5	-0.04089
6	6	3	4	1	6	0.06669
6	6	3	4	2	4	-0.04986
6	6	3	4	2	5	0.05877
6	6	3	4	2	6	-0.03483
6	6	3	4	3	3	-0.06176
6	6	3	4	3	4	0.05023
6	6	3	4	3	5	-0.01395
6	6	3	4	3	6	-0.02625
6	6	3	4	4	2	-0.07442
6	6	3	4	4	3	0.02658
6	6	3	4	4	4	0.01595
6	6	3	4	4	5	-0.04063
6	6	3	4	4	6	0.03892
6	6	3	5	2	4	-0.01722
6	6	3	5	2	5	0.03514
6	6	3	5	2	6	-0.05145
6	6	3	5	3	3	-0.02911
6	6	3	5	3	4	0.04603
6	6	3	5	3	5	-0.05073
6	6	3	5	3	6	0.03858
6	6	3	5	4	2	-0.04681
6	6	3	5	4	3	0.05417
6	6	3	5	4	4	-0.04291
6	6	3	5	4	5	0.01791
6	6	3	5	4	6	0.01229
6	6	3	5	5	1	-0.07660
6	6	3	5	5	2	0.05034
6	6	3	5	5	3	-0.01828
6	6	3	5	5	4	-0.01222
6	6	3	5	5	5	0.03320
6	6	3	6	3	3	-0.00699
6	6	3	6	3	4	0.01665
6	6	3	6	3	5	-0.02883
6	6	3	6	3	6	0.04114
6	6	3	6	4	2	-0.01495
6	6	3	6	4	3	0.02897
6	6	3	6	4	4	-0.04061

j_1	j_2	j_3	l_1	l_2	l_3	
6	6	3	6	4	5	0.04559
6	6	3	6	5	1	-0.03037
6	6	3	6	5	2	0.04499
6	6	3	6	5	3	-0.04895
6	6	3	6	5	4	0.04136
6	6	3	6	6	0	-0.07692
6	6	3	6	6	1	0.06593
6	6	3	6	6	2	-0.04595
6	6	3	6	6	3	0.02073
6	6	4	2	2	4	0.02423
6	6	4	2	2	5	0.06230
6	6	4	2	2	6	0.06230
6	6	4	3	1	5	0.06143
6	6	4	3	1	6	0.06669
6	6	4	3	2	4	0.05792
6	6	4	3	2	5	0.04022
6	6	4	3	2	6	-0.03483
6	6	4	3	3	3	0.04901
6	6	4	3	3	4	0.04434
6	6	4	3	3	5	-0.03500
6	6	4	3	3	6	-0.02625
6	6	4	4	0	6	0.07245
6	6	4	4	1	5	0.06636
6	6	4	4	1	6	-0.03190
6	6	4	4	2	4	0.06199
6	6	4	4	2	5	-0.02878
6	6	4	4	2	6	-0.03141
6	6	4	4	3	3	0.06327
6	6	4	4	3	4	-0.01852
6	6	4	4	3	5	-0.03148
6	6	4	4	3	6	0.03892
6	6	4	4	4	2	0.06343
6	6	4	4	4	3	0.00453
6	6	4	4	4	4	-0.04079
6	6	4	4	4	5	0.03269
6	6	4	4	4	6	0.00623
6	6	4	5	1	5	0.03145
6	6	4	5	1	6	-0.05771
6	6	4	5	2	4	0.03616
6	6	4	5	2	5	-0.05208
6	6	4	5	2	6	0.03812
6	6	4	5	3	3	0.04298
6	6	4	5	3	4	-0.04988
6	6	4	5	3	5	0.02731
6	6	4	5	3	6	0.01229
6	6	4	5	4	2	0.05437
6	6	4	5	4	3	-0.04614
6	6	4		4	4	0.01359
6	6	4	5	4	5	0.02189
6	6	4	5	4	6	-0.03707
6	6	4	5	5	1	0.07192
6	6	4	5	5	2	-0.03082
6	6	4	5	5	3	-0.01027
6	6	4	5	5	4	0.03510
6	6	4	5	5	5	-0.03385
6	6	4	5	5	6	0.00907
6	6	4	6	2	4	0.01088

j_1	j_2	j_3	l_1	l_2	l_3	
6	6	4	6	2	5	-0.02617
6	6	4	6	2	6	0.04311
6	6	4	6	3	3	0.01665
6	6	4	6	3	4	-0.03341
6	6	4	6	3	5	0.04441
6	6	4	6	3	6	-0.04032
6	6	4	6	4	2	0.02431
6	6	4	6	4	3	-0.04061
6	6	4	6	4	4	0.04429
6	6	4	6	4	5	-0.03053
6	6	4	6	4	6	0.00276
6	6	4	6	5	1	0.03810
6	6	4	6	5	2	-0.04950
6	6	4	6	5	3	0.04136
6	6	4	6	5	4	-0.01759
6	6	4	6	5	5	-0.01125
6	6	4	6	6	0	0.07692
6	6	4	6	6	1	-0.05861
6	6	4	6	6	2	0.02797
6	6	4	6	6	3	0.00475
6	6	4	6	6	4	-0.02823
6	6	5	3	2	4	-0.03208
6	6	5	3	2	5	-0.05940
6	6	5	3	2	6	-0.05145
6	6	5	3	3	3	-0.03901
6	6	5	3	3	4	-0.05573
6	6	5	3	3	5	-0.01715
6	6	5	3	3	6	0.03458
6	6	5	4	1	5	-0.06002
6	6	5	4	1	6	-0.05771
6	6	5	4	2	4	-0.05799
6	6	5	4	2	5	-0.02495
6	6	5	4	2	6	0.03812
6	6	5	4	3	3	-0.05583
6	6	5	4	3	4	-0.01998
6	6	5	4	3	5	0.04036
6	6	5	4	3	6	0.01229
6	6	5	4	4	2	-0.05086
6	6	5	4	4	3	-0.04098
6	6	5	4	4	4	0.03818
6	6	5	4	4	5	0.00727
6	6	5	4	4	6	-0.03707
6	6	5	5	0	6	-0.08362
6	6	5	5	1	5	-0.05777
6	6	5	5	1	6	0.03534
6	6	5	5	2	4	-0.05211
6	6	5	5	2	5	0.03596
6	6	5	5	2	6	0.02060
6	6	5	5	3	3	-0.05264
6	6	5	5	3	4	0.03329
6	6	5	5	3	5	0.01511
6	6	5	5	3	6	-0.03830
6	6	5	5	4	2	-0.05767
6	6	5	5	4	3	0.02669
6	6	5	5	4	4	0.01922
6	6	5	5	4	5	-0.03634
6	6	5	5	4	6	0.00907
6	6	5	5	5	1	-0.06696

j_1	j_2	j_3	l_1	l_2	l_3	
6	6	5	5	5	2	0.00944
6	6	5	5	5	3	0.03146
6	6	5	5	5	4	-0.03461
6	6	5	5	5	5	0.00453
6	6	5	5	5	6	0.02748
6	6	5	6	1	5	-0.02448
6	6	5	6	1	6	0.05055
6	6	5	6	2	4	-0.02617
6	6	5	6	2	5	0.04545
6	6	5	6	2	6	-0.04032
6	6	5	6	3	3	-0.02883
6	6	5	6	3	4	0.04441
6	6	5	6	3	5	-0.03497
6	6	5	6	3	6	0.00000
6	6	5	6	4	2	-0.03412
6	6	5	6	4	3	0.04559
6	6	5	6	4	4	-0.03053
6	6	5	6	4	5	-0.00350
6	6	5	6	4	6	0.03093
6	6	5	6	5	1	-0.04491
6	6	5	6	5	2	0.04811
6	6	5	6	5	3	-0.02368
6	6	5	6	5	4	-0.01125
6	6	5	6	5	5	0.03198
6	6	5	6	5	6	-0.02257
6	6	5	6	6	0	-0.07692
6	6	5	6	6	1	0.04945
6	6	5	6	6	2	-0.00849
6	6	5	6	6	3	-0.02499
6	6	5	6	6	4	0.03408
6	6	5	6	6	5	-0.01626
6	6	6	3	3	3	0.01371
6	6	6	4	3	3	0.04114
6	6	6	4	4	2	0.03747
6	6	6	4	4	3	0.04684
6	6	6	4	4	4	-0.00937
6	6	6	5	3	3	0.05495
6	6	6	5	4	2	0.05621
6	6	6	5	4	3	-0.00000
6	6	6	5	4	4	-0.03747
6	6	6	5	5	1	0.05903
6	6	6	5	5	2	0.01181
6	6	6	5	5	3	-0.03935
6	6	6	5	5	4	0.01181
6	6	6	5	5	5	0.02697
6	6	6	6	3	3	0.04114
6	6	6	6	4	2	0.04311
6	6	6	6	4	3	-0.04032
6	6	6	6	4	4	0.00276
6	6	6	6	5	1	0.05055
6	6	6	6	5	2	-0.04032
6	6	5	6	5	3	-0.00000
6	6	6	6	5	4	0.03093
6	6	6	6	5	5	-0.02257
6	6	6	6	6	0	0.07692
6	6	6	6	6	1	-0.03946
6	6	6	6	6	2	-0.01049
6	5	6	6	6	3	0.03497
6	6	6	6	6	4	-0.02057
6	6	6	6	6	5	-0.01327
6	6	6	6	6	6	0.02943