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Investment in Electricity Networks with Transmission Switching

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Abstract

We consider the application of Dantzig-Wolfe decomposition to stochastic integer programming problems arising in the capacity planning of electricity transmission networks that have some switchable transmission elements. The decomposition enables a column-generation algorithm to be applied, which allows the solution of large problem instances. The methodology is illustrated by its application to a problem of determining the optimal investment in switching equipment and transmission capacity for an existing network. Computational tests on IEEE test networks with 73 nodes and 118 nodes confirm the efficiency of the approach.

Keywords: Stochastic programming, Electricity capacity planning, Transmission switching, Column generation, Branch and price

1. Introduction

In this paper we consider economic dispatch models for wholesale electricity supply through an AC transmission network as discussed in e.g. [1]. These models typically make use of a DC-load flow assumption in which reactive power is ignored, line resistance is assumed to be small in comparison to reactance, and voltage magnitudes are treated as constant throughout the system. In such models, Kirchhoff's laws are used to determine the flow on each line. The *voltage* law states that power flow on a transmission line is proportional to the difference in voltage phase angles at each endpoint, and the *current* law states that the total power flowing from the network into any location matches the demand minus supply at this point. Thus, given the optimal dispatch and demand for a tree network, the power flow is uniquely determined by the current law. The

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voltage phase angles that generate this flow can be uniquely determined up to an additive constant by applying the voltage law.

Most electricity transmission networks are designed as meshed networks (with cycles) for security reasons, so that if any line fails, the power can still flow from source to destination by alternative paths. When the network contains cycles, the voltage law and current law must be applied simultaneously to determine the line flows and voltage angles from the dispatch of flow and generation. The presence of cycles places additional constraints on the line flows that are absent in tree networks. In particular, for each cycle in a network the sum of voltage angle differences (with respect to the direction) around the cycle must equal zero. Hence, each cycle in the network gives rise to one additional constraint on the line flows. This leads to a paradox (see e.g. [2]) in which adding a new line to a transmission network might increase the cost of supplying electricity, even if the cost of the line itself is zero.

Based on these observations, it is easy to see that it may be beneficial in mesh networks to take some lines out of operation — to either decrease system cost or increase reliability [3, 4]. The process of taking out lines and bringing them back in is done by opening (respectively closing) a switch at the end of the line and is referred to as *switching*.

Recent interest in renewable intermittent energy sources and the call for intelligent transmission networks or smart grids have spurred a renewed interest in switching problems. Fisher et. al. presents in [5] the problem of optimal switching of transmission elements in an electricity transmission network to minimize the delivered cost of energy. They propose a mixed-integer program to solve the DC-loadflow economic dispatch model with switching decisions in a single time period. They note that the problem is NP-hard. Results are provided for a 118-node network with 186 transmission lines. Hedman et. al. [6] extends the model to consider reliability of the network. Reliability constraints are added to the problem to ensure that any line failure will not lead to an infeasible dispatch of generation. They note that in some cases adding reliability constraints increases the value of switching.

In [7] Hedman et. al. discuss a decomposition algorithm to solve the transmission switching problem with unit commitment decisions made heuristically over 24 time periods. It is noted that adding transmission switching may yield a cheaper unit commitment plan than what could be achieved without switching. In this model, it is assumed that a technology is available that makes it possible to switch lines instantaneously. That is, a line may be switched automatically from one moment to the next without delay. In this case, switching out lines will (in theory) not affect system security (disregarding failures on switching equipment), since all lines may be switched back in immediately, in case of any failure in the system. Khodaei and Shahidehpour [8] describe a Benders decomposition of the security constrained unit commitment problem with transmission switching that outperforms an integrated MIP-model, and Khodaei et. al. [9] provide a Benders decomposition approach for solving capacity expansion problems in electricity networks with active transmission switching.

The solution of the large-scale mixed-integer programming problems that

arise when switching is considered remains a challenging obstacle to their implementation in practice. Most of the literature in this area has focused on demonstrating the savings in cost that can be made by transmission switching, while acknowledging that there are still computational hurdles to be overcome when solving large real-life instances. Fisher et. al. [5] were unable to prove optimality of transmission switching in the IEEE 118-bus network with a single scenario and unrestricted number of open lines. The heuristic approach presented by Hedman et. al. [7] for the transmission switching and unit commitment problem with security constraints is unable to prove optimality for the IEEE 73-bus network over 24 time periods — even with extensive computer resources. Khodaei and Shahidehpour [8] limits the space of switchable lines to find solutions to the security constrained unit commitment problem with transmission switching using Benders decomposition. Even when the single-scenario problems are restricted to allowing a small number of switches, these are sufficiently hard to make a multi-period or multi-scenario model intractable.

Making it possible to switch lines instantaneously often requires that some hardware is installed in the network. Firstly, a switch needs to be installed at the line. Secondly, communications equipment between the switch and operating control center is required to ensure automatic remote operation of the switch. Moreover, the ability to profitably switch lines out might be enhanced by adding new transmission lines to the network to absorb increases in flow.This leads to a two-stage stochastic integer programming problem of determining an optimal capital provisioning plan that will satisfy demand almost surely at least expected cost. Note, that even though the fixed cost of enabling a line to be switched instantaneously may be small (e.g. if the switch is already present and only communication equipment needs to be installed) it may not be worthwhile to enable switching on all lines (unless this cost is 0 for all lines), since some lines may never be switched.

In this paper we show how one can attack the stochastic capital provisioning problem using Dantzig-Wolfe decomposition [10] and column generation to give provably optimal or close to optimal solutions. Our approach is based on the approach of Singh et al [11] for determining optimal discrete investments in the capacity of production facilities. They proposed a split-variable formulation and Dantzig-Wolfe reformulation resulting in a sub-problem for each node in the scenario tree, and showed how this could enable the solution of previously intractable instances of capacity planning problems for electricity distribution networks. Our contribution in this paper is to show how this methodology applies to a transmission switching model, to enable their solution in settings where there are many scenarios representing future uncertainty. With a limitation on the number of switches used in each scenario, the decomposition approach enables us to solve IEEE test problems with up to 256 scenarios, which appears to be well beyond the capapility of competing methods.

We begin the paper by recalling a mixed-integer programming formulation for transmission switching based on the model in [5]. In section 3, we address the problem of the planning of transmission networks under uncertainty considering both installation of switches and line capacity expansions. In particular,

we consider a two-stage stochastic program in which the first-stage decisions concern the investments in switch equipment and line capacity, while the second stage models operational decisions in different scenarios. The model is reformulated using Dantzig-Wolfe decomposition, and solved using column generation. In section 4 we study the structure of the master problem in order to provide some insights into the strength of the decomposition. We show that the master problem has naturally integer optimal solutions in some circumstances, and provide counter examples where this is not true. Computational results of the method applied to two standard test problems (the IEEE 73-bus network and the IEEE 118-bus network) are presented in section 5. We then draw some general conclusions about the effectiveness of the approach.

2. Optimal Transmission Switching

We model the electricity transmission system as a network where N denotes the set of nodes (or busses) and A denotes a set of arcs representing transmission lines (and transformers) connecting the nodes. Let $\mathcal{T}(i)$ denote the set of arcs incident with node i where i is the head of the incident arc, and let $\mathcal{F}(i)$ denote the set of arcs incident with node i, where i is the tail of the incident arcs. So an arc in $\mathcal{F}(i) \cap \mathcal{T}(j)$ is directed *from* node i *to* node j. Since power flow can flow in both directions in a transmission line we allow these flows to take negative values, indicating power flow in the opposite direction from the arc direction.

Many transmission systems consist of alternating current circuits, interlinked by high voltage direct current links. We shall ignore these interconnections in this paper, and assume that all lines carry alternating current. The methodologies can easily be adapted to treat direct current lines as special cases. Note, that even though we assume all lines to be alternating current lines, the models presented are based on the linear direct current optimal power flow approximation as discussed in the introduction.

Let G be the set of all generating units, where $G(i)$ is the set of generating units located in (and supplying electricity to) node i. For simplicity, we assume that each unit $g \in G$ offers its entire electricity capacity u_g to the system at its marginal cost c_g. (A model in which each unit offers a step supply curve is a straightforward extension.) We denote by q_g the dispatch of power of unit g.

At each node i the demand d_i must be met. Load shedding at node i may be modelled by introducing a dummy generator at each node offering d_i at a penalty price.

Each transmission line $a \in A$ is characterised by its reactance X_a and thermal capacity K_a. The flow on line a is denoted P_a, which can be negative in order to model power flows in the direction opposite to the orientation of a.

A subset of lines $S \subseteq A$ are considered to be switchable. Lines that are switchable may be taken out of operation in any given period of time. For each line $a \in S$, $z_a = 1$ denotes that the line has been switched out (opened), while $z_a = 0$ denotes that the switch is closed.

The economic dispatch problem of finding the minimum cost optimal DC-load flow may now be formulated as

EDP: $$\text{minimize} \sum_{g \in G} c_g q_g \tag{1}$$

$$\text{s.t.} \qquad 0 \le q_g \le u_g, \qquad\qquad\qquad g \in G \tag{2}$$

$$z_a = 0 \Rightarrow X_a P_a - \theta_i + \theta_j = 0, \qquad a = (i,j) \in A \tag{3}$$

$$\sum_{g \in G(i)} q_g - \sum_{a \in \mathcal{F}(i)} P_a + \sum_{a \in \mathcal{T}(i)} P_a = d_i, \qquad i \in N \tag{4}$$

$$-K_a(1 - z_a) \le P_a \le K_a(1 - z_a), \qquad a \in A \tag{5}$$

$$\sum_{a \in A} z_a \le k, \tag{6}$$

$$z_a = 0, \qquad a \in A \setminus S \tag{7}$$

$$z_a \in \{0,1\}, \qquad a \in S. \tag{8}$$

The objective (1) minimizes the total generation costs respecting generation capacities (2), flow conservation (4), and thermal line capacity (5). For lines that are not switched out, Kirchhoff's voltage law must be respected (3). Furthermore, we only allow k lines to be switched simultaneously (6) and only lines in S are switchable (7). Finally switching decisions are binary (8).

Note, that constraint (3) may be linearised using a big-M construction

$$-M z_a \le X_a P_a - \theta_i + \theta_j \le M z_a, \qquad a = (i,j) \in A \tag{9}$$

where M is some sufficiently large number. To give a strong linear programming relaxation, lower values of M are better. The choice of an appropriate value of M is discussed in [12], who observe that the difference in voltage angles between any two nodes i and j is bounded by

$$M_{ij} = \max\{\sum_{a \in R} X_a K_a \mid R \text{ is a path of edges joining } i \text{ and } j\}$$

and so choosing $M = \max_{i,j} M_{ij}$ will give the smallest value in general. This poses some difficulty in practice, since the computation of M_{ij} is a hard problem, and so its use is restricted to small networks where it can be found by enumeration (see [12]). The approach taken in [7] imposes a uniform bound on the magnitude of the voltage phase angle of 0.6 radians. This constraint allows a value of $M = 1.2$ to be chosen.

3. Switch and transmission provision under uncertainty

We now consider the problem of installing switches and new lines in an electricity transmission network to minimize the capital cost and expected operating cost averaged over a number of scenarios denoted $\omega \in \Omega$. In each scenario ω

we have a realization of demand $d(\omega)$ and generation cost $c(\omega)$ and generation capacity $u(\omega)$. This enables us to vary parameters according to climatic conditions (e.g. high costs could model shortage of water in hydro stations, and low capacity model low wind outcomes for wind farms). We assume that transmission switching and economic dispatch is carried out after these random outcomes are realized. In each scenario ω we have the switching and dispatch problem:

$$\text{EDP}(\omega): \qquad \text{minimize} \sum_{g \in G} c_g(\omega) q_g \qquad (10)$$

$$\text{s.t.} \qquad 0 \le q_g \le u_g(\omega), \qquad g \in G \qquad (11)$$

$$X_a P_a - \theta_i + \theta_j + M z_a \ge 0, \qquad a = (i,j) \in A \qquad (12)$$

$$X_a P_a - \theta_i + \theta_j - M z_a \le 0, \qquad a = (i,j) \in A \qquad (13)$$

$$\sum_{g \in G(i)} q_g - \sum_{a \in \mathcal{F}(i)} P_a + \sum_{a \in \mathcal{T}(i)} P_a = d_i(\omega), \qquad i \in N \qquad (14)$$

$$-K_a(1 - z_a) \le P_a \le K_a(1 - z_a), \quad a \in A \qquad (15)$$

$$\sum_{a \in A} z_a \le k, \qquad (16)$$

$$z_a = 0, \qquad a \in A \setminus S \qquad (17)$$

$$z_a \in \{0, 1\}, \qquad a \in S. \qquad (18)$$

We now consider the problem of installing switches and new transmission lines prior to the realization of ω. We assume that a fixed cost is associated with installing switching equipment at each line and that this cost covers all the actual costs of making it possible to perform instantaneous switching of that particular line. Furthermore, we assume a fixed cost of installing new lines from a fixed set of possible line expansions.

This gives a two-stage stochastic model, where the first-stage decisions involve investments in switching equipment y_S and line capacity y_L, while the second-stage problem $\text{EDP}(\omega)$ models operational decisions (q, P, θ, z) for dispatch and switching in each scenario ω occuring with probability $p(\omega)$. For each scenario $\omega \in \Omega$, let

$$\mathcal{Q}(\omega) = \{(q, P, \theta, z) \mid (11\text{-}18)\}.$$

The model may now be formulated as

$$\min \ f_S^\top y_S + f_L^\top y_L + \sum_{\omega \in \Omega} p(\omega) c(\omega)^\top q(\omega) \qquad (19)$$

$$\text{s.t.} \qquad y_L - y_S + z(\omega) \leq 1, \qquad\qquad \omega \in \Omega \qquad (20)$$

$$y_L + z(\omega) \geq 1, \qquad\qquad \omega \in \Omega \qquad (21)$$

$$(q(\omega), P(\omega), \theta(\omega), z(\omega)) \in \mathcal{Q}(\omega), \qquad\qquad \omega \in \Omega \qquad (22)$$

$$y_L, y_S \in \{0,1\}^{|A|}. \qquad (23)$$

The capital costs f_S and f_L are amortized to give a per period capital charge that is traded off against the expected economic dispatch cost per period, as expressed by objective (19). We set $e_a^\top y_L = 1$ and $e_a^\top f_L = 0$ for existing lines. The constraints (20) ensure that switching of installed lines is only possible if a switch is also installed. Constraints (21) allow lines to be switched in only if they have non-zero capacity. Note, that not installing a line corresponds to having the line switched out (i.e. $z(\omega) = 1$) in all scenarios $\omega \in \Omega$.

We can decompose (19)-(23) following the approach in [11]. The idea is to decompose the stochastic problem into a master problem and a number of subproblems — one for each scenario. We let the binary vector $z(\omega)$ define a *feasible switching plan* (FSP) for scenario ω if there exists $q(\omega), P(\omega), \theta(\omega)$ such that $(q(\omega), P(\omega), \theta(\omega), z(\omega)) \in \mathcal{Q}(\omega)$. Now, let $Z(\omega) = \{\hat{z}^j(\omega) | j \in J(\omega)\}$ be the set of all FSP's for scenario ω, where $J(\omega)$ is the index set for $Z(\omega)$. We can write any element in $Z(\omega)$ as

$$z(\omega) = \sum_{j \in J(\omega)} \varphi^j(\omega) \hat{z}^j(\omega)$$

$$\sum_{j \in J(\omega)} \varphi^j(\omega) = 1, \quad \varphi^j(\omega) \in \{0,1\}, \ \forall j \in J(\omega).$$

Assume that for each feasible switching plan $\hat{z}^j(\omega)$ the corresponding optimal dispatch of generation and load shedding is given by $\hat{q}^j(\omega)$. The master problem can now be written in terms of $\hat{z}$ and $\hat{q}$ as

$$\text{MP: } \min \ f_L^\top y_L + f_S^\top y_S + \sum_{\omega \in \Omega} \sum_{j \in J_\omega} p(\omega) c(\omega)^\top \hat{q}^j(\omega) \varphi^j(\omega) \qquad (24)$$

$$\text{s.t.} \qquad y_L - y_S + \sum_{j \in J(\omega)} \hat{z}^j(\omega) \varphi^j(\omega) \leq 1, \qquad [\pi(\omega)], \ \omega \in \Omega \qquad (25)$$

$$y_L + \sum_{j \in J(\omega)} \hat{z}^j(\omega) \varphi^j(\omega) \geq 1, \qquad [\rho(\omega)], \ \omega \in \Omega \qquad (26)$$

$$\sum_{j \in J(\omega)} \varphi^j(\omega) = 1, \qquad [\mu(\omega)], \ \omega \in \Omega \qquad (27)$$

$$\varphi^j(\omega) \in \{0,1\}, \qquad j \in J(\omega) \qquad (28)$$

$$y_L, y_S \in \{0,1\}^{|A|} \qquad (29)$$

where $\mu(\omega)$, $\pi(\omega)$ and $\rho(\omega)$ denote the dual prices associated with the respective constraints.

The master problem MP is a two-stage stochastic integer program with integer variables in both stages. Although in general these are difficult to solve, the structure of MP is such that integer extreme point solutions are common. To help understand the reasons for this we examine some special cases of MP in the following section.

4. The structure of MP

In this section we investigate the structure of MP. We first assume that we do not install new lines, so that $e_a^\top y_L = 1$, $a \in A$. This simplifies MP since the constraints (26) can be removed from the formulation. The constraints (25) become

$$-y_S + \sum_{j \in J(\omega)} \hat{z}^j(\omega)\varphi^j(\omega) \le 0.$$

Suppose now that there is at most one switch allowed in each scenario. The master problem matrix with $|S|$ possible locations for switches takes the form

$$
A = \begin{bmatrix}
-I & I & & & & & & \\
-I & & I & & & & & \\
\vdots & & & \ddots & & & & \\
-I & & & & I & & & \\
& e^\top & & & & 1 & & \\
& & e^\top & & & & 1 & \\
& & & \ddots & & & & \ddots \\
& & & & e^\top & & & & 1
\end{bmatrix}
$$

where I is the $|S| \times |S|$ identity matrix and $e \in \{0, 1\}^{|S|}$ is a vector of 1's, and there is a copy of $-I$ and I for each scenario $\omega \in \Omega$. If switches are permitted only on a small subset of lines then we are guaranteed an integer optimal solution to MP.

Proposition 4.1. *If $|S| \le 2$ then A is totally unimodular.*

Proof 4.1. *If $|S| = 1$, then $A^\top$ (after multiplying its last $|\Omega|$ rows by -1) is a node-arc incidence matrix which is totally unimodular. For the case $|S| = 2$, we use the fact that the total unimodularity of $\begin{bmatrix} L & M \end{bmatrix}$ implies that $\begin{bmatrix} L & M \\ 0 & I \end{bmatrix}$ is totally unimodular. It suffices to show that the transpose of the first $2(|\Omega| + 1)$ columns of A is totally unimodular. This matrix is*

$$
B = \begin{bmatrix}
-I & -I & \ldots & -I & & & & \\
I & & & & & e & & \\
& I & & & & & e & \\
& & \ddots & & & & & \ddots \\
& & & I & & & & & e
\end{bmatrix}
$$

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which can be transformed into a node-arc incidence matrix by multiplying the first row of B by -1, and then multiplying by -1 each row of B corresponding to the first row in each occurrence of I.

For larger values of $|S|$, we cannot guarantee that A is totally unimodular, even if only at most one switch is allowed in each scenario.

Example 1
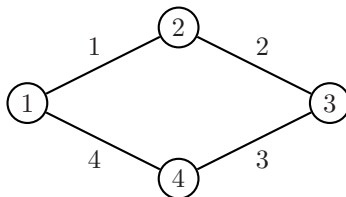
Consider the network in Figure 1.

Figure 1: Network for Example 1, showing node and line indices. We seek optimal switch investments on lines 1, 2, and 3

Suppose that all lines have equal reactance and lines 1, 2, and 3 have capacity 5 while line 4 has capacity 1. Suppose that there are three scenarios $\omega = 1, 2, 3$. In scenario ω, zero cost power of 5 units is available at node ω and there is a demand of 5 and unlimited power at cost 2 at node $\omega + 1$ (one could imagine these being different wind scenarios). We consider installing switches on lines 1, 2, and 3, each with a cost of 1.

In scenario 1, without any switches we can only send 4 units from 1 to 2 through the network (3 directly from 1 to 2 and 1 unit from 1 to 4 to 3 to 2). Given the extra unit of generation required at node 2, this has cost 2, which is more expensive than switching out either lines 2 or 3 in this scenario, enabling 5 units to be sent directly from 1 to 2 at zero cost. If we switch out line 1, then we can send only one unit and the cost of generating the shortfall is 8.

The other scenarios are essentially the same. In scenario 2, we can switch out lines 1 or 3 to get a zero cost dispatch, and in scenario 3, we can switch out lines 1 or 2 to get a zero cost dispatch. Note that line 4 is unable to be switched.

If we consider the single switch options in each scenario, then we get a master problem constraint matrix of the following form:

$$A = \begin{bmatrix}
-1 & & & & 1 & & & & & & & \\
& -1 & & & & 1 & & & & & & \\
& & -1 & & & & 1 & & & & & \\
-1 & & & & & & & 1 & & & & \\
& -1 & & & & & & & 1 & & & \\
& & -1 & & & & & & & 1 & & \\
-1 & & & & & & & & & & 1 & \\
& -1 & & & & & & & & & & 1 \\
& & -1 & & & & & & & & & \\
& & & 1 & 1 & 1 & & & & & 1 & \\
& & & & & & 1 & 1 & 1 & & & 1 \\
& & & & & & & & & 1 & 1 & 1 \\
\end{bmatrix}$$

This is not totally unimodular. Moreover MP has a fractional solution given by

$$
\begin{aligned}
y_S &= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}^\top \\
\varphi(1) &= \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}^\top \\
\varphi(2) &= \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}^\top \\
\varphi(3) &= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}^\top
\end{aligned}
$$

This corresponds to installing half a switch on each of lines 1,2, and 3, giving a total cost of $\frac{3}{2}$. The optimal integer solution will place a switch on any two of the lines to give total cost of 2.

We now look at the case where we have only two scenarios but more than one binary decision variable in MP. This means that we now allow more than one switch to occur in each scenario.

Example 2

Consider the following two-stage two-scenario switch investment problem with two investment options, that may be chosen in the first stage only. As in Example 1, we also consider investment only in switches. That is we assume $e_a^\top y_L = 1$ for all arcs a in A as before. If we enumerate all the possible switching plans, these make up our columns of switch requests that may be chosen for each scenario. The corresponding constraint matrix is as follows:

$$A = \begin{bmatrix}
-1 & & 1 & 1 & & & & \\
& -1 & 1 & & 1 & & & \\
-1 & & & & & 1 & 1 & \\
& -1 & & & & 1 & & 1 \\
& & 1 & 1 & 1 & 1 & & \\
& & & & & 1 & 1 & 1 & 1 \\
\end{bmatrix}$$

where the first four rows correspond to the capacity constraints (25) and the last two rows corresponds to the convexity constraints (27). Columns 3 to 6

(respectively 7 to 10) represent feasible switching patterns in scenario 1 (respectively 2). Note that the submatrix consisting of the first five rows and columns 1, 2, 4, 5, and 7 has determinant 2. Assume, that the vector of cost coefficients is represented by $c = (3, 3, 10, 0.9, 1, 10, 1.5, 10, 10, 7.3)$ Then the optimal integer solution minimizing $c \begin{pmatrix} y_S \\ \varphi \end{pmatrix}$ is

$$\left(y_S^\top, \varphi^\top\right)^*_{IP} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

yielding a cost of 8.4, while the optimal LP relaxed solution

$$\left(y_S^\top, \varphi^\top\right)^*_{LP} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

yields the (slightly) lower value of 8.35.

The third column in A represents a request for switching capacity on both lines in scenario 1 incurring an operational cost of 10. So, in our example, switching both lines would incur a high operational cost in scenario 1, but a very low cost in scenario 2. Also, in scenario 1, switching exactly one line incurs a much smaller cost. The result is that the solution to the relaxed master problem saves enough by installing only half a switch on each line to compensate for the extra operating cost that is incurred by only admitting half the operational benefits of switching to accrue. This results in a fractional optimal solution.

Although the examples above show how fractional optimal solutions to the master problem might arise, in practice we obtain fractional solutions to MP very rarely. We conjecture that this is because it is unlikely with realistic electricity network data that the symmetrical situations as in the examples above will generate subproblem solutions with the specific cost structure needed to give fractions at optimality.

Fractional solutions are also prevented by the fact that MP has inequality constraints rather than equations. Observe in our model that every possible line expansion involves a switch as well. This means that the use of the line in any dispatch scenario is optional. As we have observed above, the mandatory use of an additional line might increase the dispatch cost in some scenarios. If such a situation occurs then a fractional expansion, that trades off good and bad dispatch outcomes, might become more likely.

If this does occur then we need to apply a branch-and-price procedure. This is easy to implement owing to the following result.

Proposition 4.2. *If y_L and y_S are chosen to be fixed vectors of binary integers, then the linear programming relaxation of MP has integer extreme points.*

Proof 4.2. *When y_L and y_S are fixed, the constraints of the linear programming relaxation of MP decouple by scenario to give*

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$$\sum_{j \in J} \varphi^j = 1, \tag{30}$$

$$y_L - y_S + \sum_{j \in J} \hat{z}^j \varphi^j \leq 1, \tag{31}$$

$$y_L + \sum_{j \in J} \hat{z}^j \varphi^j \geq 1, \tag{32}$$

$$\varphi^j \geq 0, \tag{33}$$

where we suppress the dependence on ω for notational simplicity. Let

$$
\begin{aligned}
I_0 &= \{i \mid e_i^\top y_L = 0\}, \\
I_1 &= \{i \mid e_i^\top y_L = e_i^\top y_S = 1\}, \\
I_2 &= \{i \mid e_i^\top y_L = 1, e_i^\top y_S = 0\}.
\end{aligned}
$$

This gives constraints

$$-e_i^\top y_S + e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \leq 1, \quad i \in I_0 \tag{34}$$

$$e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \geq 1, \quad i \in I_0 \tag{35}$$

$$e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \leq 1, \quad i \in I_1 \tag{36}$$

$$e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \geq 0, \quad i \in I_1 \tag{37}$$

$$e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \leq 0, \quad i \in I_2 \tag{38}$$

$$e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \geq 0, \quad i \in I_2 \tag{39}$$

$$\sum_{j \in J} \varphi^j = 1, \tag{40}$$

$$\varphi^j \geq 0, \tag{41}$$

Constraints (34) and (36) are dominated by (40) and can be removed. Similarly constraints (37) and (39) are redundant.

Constraint (35) must be satisfied as an equation and subtracting from (40) implies that $\varphi^j = 0$ for all columns j with $e_i^\top \hat{z}^j = 0$, for some $i \in I_0$. Similarly constraint (38) implies $\varphi^j = 0$ for all columns j with $e_i^\top \hat{z}^j = 1$, for some $i \in I_2$. Let

$$Z = \{j \in J \mid e_i^\top \hat{z}^j = 0 \text{ for some } i \in I_0, \text{ or } e_i^\top \hat{z}^j = 1, \text{ for some } i \in I_2\}.$$

Then $\varphi^j = 0$ for all columns $j \in Z$, which may be removed. This results in the system

$$e_i^\top \sum_{j \in J \setminus Z} 1\varphi^j = 1, \quad i \in I_0,$$

$$e_i^\top \sum_{j \in J \setminus Z} 0\varphi^j \leq 0, \quad i \in I_2,$$

$$\sum_{j \in J \setminus Z} \varphi^j = 1,$$

$$\varphi^j \geq 0,$$

which clearly has integer extreme points.

This means that any branch-and-price scheme may branch in the master problem simply by imposing constraints on the variables y_L and y_S. In other words to solve this problem, we do not need to construct a specific constraint-branching methodology, where columns generated on each side of the branch are constrained to meet certain conditions (see e.g. [13]).

It is convenient to consider only a subset $Z(\omega)' \subseteq Z(\omega)$ of feasible switching plans for each scenario ω in the master problem. We define this restricted master problem (RMP) by (24) - (29) with $J(\omega)$ replaced by $J(\omega)'$ the index set of $Z(\omega)'$. A column generation algorithm is applied to dynamically add feasible switching plans to the linear relaxation of the master problem. The algorithm is initialised by letting $Z(\omega)' = \{\hat{z}^0(\omega)\} = \{\mathbf{0}\}$, for all scenarios $\omega \in \Omega$. That is, initially no line may be switched out in either scenario. The corresponding operational costs $c(\omega)^T \hat{q}^0(\omega)$ can easily be found by solving a linear program for each scenario. In each iteration of the algorithm, the linear relaxation (RMP-LP) of RMP is solved yielding the dual prices μ, π, and ρ. A new column $(p(\omega)c(\omega)^\top \hat{q}^j(\omega), 1, \hat{z}^j(\omega))$ may improve the solution of RMP-LP if and only if the associated reduced cost $\bar{c}(\omega) = p(\omega)c(\omega)^\top \hat{q}^j(\omega) + \pi(\omega)^\top \hat{z}^j(\omega) - \rho(\omega)^\top \hat{z}^j(\omega) - \mu(\omega)$ is negative.

A column for scenario ω may therefore be constructed by solving the subproblem:

$$\begin{aligned} \min \quad & p(\omega)c(\omega)^\top q + \pi(\omega)^\top z - \rho(\omega)^\top z - \mu(\omega) \\ \text{s.t.} \quad & (q, P, \theta, z) \in \mathcal{Q}(\omega), \end{aligned}$$

where $\pi(\omega)$, $\rho(\omega)$, and $\mu(\omega)$ are the dual prices returned from RMP-LP.

Any feasible solution $(q, P, \theta, z) \in \mathcal{Q}(\omega)$ with negative objective function gives rise to a potential candidate column for RMP-LP. If no columns with negative reduced cost exist then we have solved the relaxed master problem (MP-LP) to optimality. Furthermore, if the solution (φ^*, y^*) to MP-LP is integral then (φ^*, y^*) is an optimal solution to the master problem (24) - (29) and y^* is the optimal line and switch investment strategy. Otherwise, we may resort to a branch-and-price framework for finding optimal integral solutions. Note that a fractional solution will always have at least one fractional y-value (see Proposition 4.2). Hence, we branch on one of the fractional y-variables and

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hope that this will resolve the fractionality. If not one may continue branching on y-variables until the fractionality is resolved.

5. Computational Results

In this section we apply the column generation technique to the problem of investing in switching equipment to minimize total investment and expected generation cost over a number of scenarios with varying demand and supply. Computational experiments are performed on two different IEEE electricity transmission networks — the IEEE 73-bus network and the IEEE 118-bus network. The computational results are compared to solving the original formulation with a commercial MIP-solver (CPLEX).

The IEEE 73-bus network is based on the three area reliability test system 1996 [14]. Data for this network can be found in [15]. The transmission network was modified as described in [7] The resulting network has 117 lines, 99 generators, total generation capacity of 8998 MW, and total peak demand of 8550 MW. The IEEE 118-bus network is described in [16]. This network has 185 lines, 20 generators, total peak load of 4519 MW, and a total thermal generator capacity of 5859 MW. These networks were modified to accommodate varying supply and demand scenarios.

For the IEEE 118-node network a 1600 MW intermittent wind-power generator with varying supply capacity is located at node 91 supplying power at marginal generation cost 0. Nodal demands were scaled uniformly in the interval 0.535 to 1.0. In each instance half the scenarios had no wind power while the other half had full wind-power capacity.

The original IEEE 73-bus network has 18 hydro units (six in each area) each with capacity 50 MW and marignal cost 0. In our model, four of the hydro units in nodes 222 and 322 (area 2 and 3) were modified to represent wind generators with marginal generation cost 0 and varying generation capacity over the scenarios. Similarly, all six hydro units in node 122 (area 1) were modified to have varying marginal cost but constant generation capacity of 50 MW over the scenarios. Nodal demands were scaled uniformly by a factor in the interval 0.5 to 1.0. Table 1 gives a summary of the values of the stochastic parameters used in the different instances of the problem. For both networks the stochastic parameters are all assumed to be independent of each other and scenarios are assumed to be equally likely to occur.

First stage decisions include only investment decisions in switching equipment. That is, we assume $e_a^\top y_L = 1$ for all arcs a in A. The fixed amortized switch investment costs are set to \$5/h for each switch.

Computational experiments were performed on a 2.26 GHz Core 2 Duo computer with 4 GB RAM.

5.1. Experiments with branch and price

In order to solve large instances, the Dantzig-Wolfe reformulation described above was implemented in a branch-and-price framework using the DIP software

| | $|\Omega| = 16$ | | | |
| | | $|\Omega| = 81$ | | |
stochastic parameter			$|\Omega| = 256$	
demand factor	1	0.67	0.5	0.84
wind capacity factor, node 222	1	0	0.33	0.67
wind capacity factor, node 322	1	0	0.67	0.33
hydro price factor, node 122	0	30	5	15

Table 1: Summary of stochastic demand, generation capacity, and marginal cost factors.

framework [17]. DIP (Decomposition for Integer Programming) is a general open source framework developed under COIN-OR for solving discrete optimization problems using various decomposition algorithms. DIP allows the user to formulate mixed integer programs in the original space and to provide the problem structure needed for decomposition. DIP then handles the reformulation and provides methods for solving the problem using decomposition algorithms. The code is implemented in C++ and is designed and maintained by Matthew Galati and Ted Ralphs at Lehigh University [18].

Instances with the IEEE 118-bus network and the IEEE 73-bus network and different number of scenarios and values of k are constructed. These are solved using DIP's branch and price algorithm with default parameters, except that each node is solved to optimality before branching (`TailOffPercent = 0`), compression of columns are turned off (`CompressColumns = 0`), and the master problems are solved to optimality (`MasterGapLimit = 0`) using interior point method (CPLEX 12.2 barrier). Sub-problems are solved using the CPLEX 12.2 MIP-solver. For comparison the instances are also solved using the CPLEX 12.2 branch-and-bound solver with default settings. Computational results are shown in Table 2, while Table 3 and Table 5 shows the objective function values and number of installed switches in the optimal solution of the corresponding instances for the IEEE 118-bus network and the IEEE 73 bus network.

Results show that the CPLEX MIP solver performs well on instances with a small number of scenarios. With more scenarios, however, the CPLEX solver exhausts the memory, while DIP solves to optimality in reasonable time. DIP outperforms CPLEX for 11 out of the 15 instances investigated. In general, it seems that DIP scales well with the number of scenarios, while CPLEX handles large k-values better.

For branch and price all instances except the 118-node, 32-scenario instance are solved to integer optimality in the root node and hence no branching is needed. For the 32-scenario instance a fractional solution is returned in the root node. However, integrality is obtained by branching only once. (The fractional solution has a strictly lower value than the optimal integer solution obtained.)

The decomposition relies on solving a large number of sub-problems with feasible set $\mathcal{Q}(\omega)$. For large k the computational complexity of the sub-problems

	Instance		Branch and price				Branch and bound		
			time (s)		price-	no. of	time (s)	gap	lower
$|N|$	$|\Omega|$	k	total	master	passes	nodes			bound
118	2	3	**96**	0	10	1	547	0.00	1351.22
118	2	5	1427	1	20	1	**330**	0.00	1338.55
118	4	3	**257**	4	16	1	2310	0.00	1036.93
118	4	5	**3172**	6	38	1	7133	0.00	1033.95
118	4	10	22444	30	98	1	**3055**	0.00	1009.60
118	8	3	**978**	8	26	1	2070 †	2.26	846.17
118	16	3	**3213**	11	36	1	3722 †	0.00	775.01
118	32	3	**25477**	279	129	3	4968 †	9.68	690.81
118	64	3	**11126**	72	38	1	8589 †	28.84	678.44
73	4	1	**12**	1	6	1	401	0.00	65297.22
73	16	1	**53**	2	9	1	5013 †	2.05	66270.18
73	81	1	**2037**	36	18	1	14414 ‡	6.07	52884.46
73	4	3	972	3	35	1	**42**	0.00	65266.08
73	16	3	3888	17	69	1	**490**	0.00	66266.34
73	81	3	**36793**	163	93	1	6732 †	0.08	52885.56

Table 2: Results for the switch investment problem on the IEEE 118-bus network and IEEE 73-bus network. Solve times and gaps are reported for the branch-and-price algorithm (DIP) and standard branch-and-bound (CPLEX) for problem instances with at most k open switches and $|\Omega|$ scenarios. All instances was solved to optimality using branch-and-price. Branch-and-bound was terminated with the CPLEX default optimality tolerance except for ‡ which was terminated manually after 14400 s. For branch-and-price the total solve time for the master problems, number of pricepasses, and the number of nodes in the branching tree are also reported. For branch-and-bound the lower bound is reported. Gaps reported are absolute gap to best known solution. For the branch-and-bound the best lower bound is also reported. The fastest solution time is highlighted in bold face. † denotes that the optimization was terminated due to lack of memory.

is high and solving them to optimality is hard. This can be seen from Table 2 that shows that only a small fraction of the time is spent solving the master problems, while the majority of time is spent solving the sub-problems. This makes the branch-and-price algorithm perform less well on instances with large k. Hence, further research is needed to strengthen the sub-problems in order to solve instances with large k. On the other hand, as shown in section 4, the master problem matrices have some nice properties resulting in shallow branching trees.

5.2. Experiments with column generation for the 73 node network

In this subsection we consider the column-generation algorithm without branching. The motivation for this study comes from the need to solve stochastic models with many scenarios. To investigate how the decomposition algorithm scales with scenarios, we restrict attention to the smaller IEEE 73-bus network.

Instance		Optimal solution	
$|\Omega|$	k	obj. function value	no. of switches
2	3	1351.36	5
2	5	1338.68	7
4	3	1037.03	5
4	5	1034.06	6
4	10	1009.70	9
8	3	898.48	7
16	3	871.10	3
32	3	734.04	3
64	3	763.97	3

Table 3: Objective function value and number of switches installed in the optimal solution for instances of the switch investment problem for the IEEE 118-bus network.

The decomposition and models in the following results are formulated using the AMPL modelling language and all master problems and subproblems are solved with CPLEX 12.2. The relaxed master problems are solved using CPLEX barrier algorithm without crossover, while the subproblems are solved using the CPLEX standard branch-and-bound algorithm. For the branch-and-bound algorithm CPLEX 12.2 was applied with default parameters.

Computational results are shown in Table 4, while Table 5 shows the objective function values and number of switches installed in the corresponding optimal solutions. These results show that the time taken to solve the Dantzig-Wolfe reformulation is approximately proportional to the number of scenarios, and we are able to solve up to 256 scenarios in reasonable time. Note that all instances are solved to optimality in the root node of the branch and bound tree and hence no branching is necessary. For $k = 1$ solving the compact formulation using CPLEX is much more time consuming than solving the Dantzig-Wolfe reformulation. However, for $k = 3$ solving the compact formulation is faster for a small number of scenarios, but performs worse with an increasing number of scenarios.

The computational results presented in this section show that solving the Dantzig-Wolfe reformulation by column generation is faster for small values of k compared to solving the original formulation in CPLEX. When k is small, column generation scales well with the number of scenarios. For large values of k, however, the subproblems become intractable. The master problem — except for one instance — always yields an optimal integer solution in the root node.

6. Conclusion

In this paper we consider decomposition methods for stochastic investment problems involving transmission switching in electricity networks. In particular,

Instance			Column Generation			Branch-and-bound		
				price-	no. of			lower
$|N|$	$|\Omega|$	k	time (s)	passes	nodes	time (s)	gap	bound
73	4	1	**16**	4	1	401	0.00	65297.22
73	16	1	**54**	6	1	5013 †	2.05	66270.18
73	81	1	**286**	8	1	14414 ‡	6.07	52884.46
73	256	1	**1518**	12	1	48373 †	8.15	54648.50
73	4	3	1153	31	1	**42**	0.00	65266.08
73	16	3	1326	30	1	**490**	0.00	66266.34
73	81	3	**25056**	52	1	6732 †	0.08	52885.56
73	256	3	**30880**	39	1	68670 ‡	4.72	54647.76

Table 4: Computational results for the switch investment problem on the IEEE73-bus network. All instances was solved to optimality using column generation. Branch-and-bound was terminated with the CPLEX default optimality tolerance. Gaps reported are absolute gap to the optimal solution. For the branch-and-bound the best lower bound is also reported. † denotes that the optimization was terminated due to lack of memory. The fastest solution time is highlighted in bold face. ‡ denotes that CPLEX was manually terminated before reaching optmimum.

we look at determining optimal switch investment and line capacity expansion strategies and we propose a Dantzig-Wolfe reformulation of a two-stage stochastic mixed integer program.

A column-generation approach is outlined to solve the Dantzig-Wolfe reformulation. The approach is tested on two IEEE test networks. When the number of allowed switching actions is small, the proposed algorithm turns out to be significantly more efficient than solving the compact formulation directly, and it enables us to solve instances with up to 256 scenarios.

In general, the linear programming relaxation of the reformulation does not have integer extreme points, but in practice this often happens to be the case. In the rare instances where the relaxed master problem has fractional solutions, our formulation admits a simple branch-and-price scheme that can be used to resolve these with very few iterations.

The approach is limited by the complexity of the subproblems. Solving large-scale problems requires a strong formulation of the subproblem, especially when many switching actions are allowed. In our experiments, we attempted to apply some strengthening to the subproblems by adding the constraint

$$\sum_{a \in P} z_a \leq 1$$

for any path P in which all nodes except the first and last nodes are two-connected, and setting $z_a = 0$ for any arc a such that the network $(N, A \setminus \{a\})$ is not connected. These provided some improvement on subproblems in sparse networks and/or with larger values of k, but gave no improvement in computation time for the instances discussed in this paper. This indicates that

Instance		Optimal solution	
$|\Omega|$	k	obj. function value	no. of switches
4	1	65303.75	1
16	1	66301.56	1
81	1	52905.05	1
256	1	54668.09	1
4	3	65270.02	2
16	3	66272.91	2
81	3	52890.74	2
256	3	54656.02	2

Table 5: Objective function value and number of switches installed in the optimal solution for instances of the switch investment problem for the IEEE 73-bus network.

further research should be directed at providing stronger formulations and more efficient solution methods for the subproblems in order to improve efficiency of the algorithm.

The methodology described in this paper can be applied to other stochastic programming problems in which switching is allowed. For example, one might construct a multi-stage plan for investing in switches and transmission line expansions using the approach explored in [11] for distribution networks (in which switching to a radial structure is required in each scenario and stage). The approach can also be used to investigate the optimal investment in switches to ensure the $N-1$ reliability of an existing network. In this setting the scenarios represent failures of single lines or units. This approach is explored for distribution networks in [19], and described for transmission networks in [20].

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We consider the application of Dantzig-Wolfe decomposition to stochastic integer programming problems arising in the capacity planning of electricity transmission networks that have some switchable transmission elements. The decomposition enables a column-generation algorithm to be applied, which allows the solution of large problem instances. The methodology is illustrated by its application to a problem of determining the optimal investment in switching equipment and transmission capacity for an existing network. Computational tests on IEEE test networks with 73 nodes and 118 nodes confirm the efficiency of the approach.