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Pricing and Capacity Planning Problems in Energy Transmission Networks

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Resumé

Effektiv brug af energi er et stadig vigtigere emne. Miljø-og klima problemer samt bekymring for forsyningssikkerhed har gjort vedvarende energikilder til et reelt alternativ til traditionelle energikilder. Men den fluktuerende karakter af for eksempel vind- og solenergi nødvendiggør en radikal ændring i den måde vi planlægger og driver energisystemer. Et andet paradigmeskift, som begyndte i 1990'erne for el-systemer er markedsderegulering, hvilket har ført til en række forskellige markedsstrukturer forskellige steder i verden.

I denne afhandling diskuterer vi kapacitetsplanlægnings- og transmissionsprissætningsproblemer i energitransmissionsnetværk. Selv om modelleringen gælder for energinetværk i almindelighed vedrører de fleste af anvendelsesområderne transmission af elektricitet.

En række af de forelagte problemer indebærer *switching* af transmissionsnettet. Dette giver operatøren af et el-transmissionsnet mulighed for automatisk at tage transmissionslinjer ind og ud operationelt for at optimere flow af elektricitet i netværket. Vi viser, at *transmission switching* i systemer med storskala vindkraft kan reducere overbelastninger i nettet, hvilket kan føre til en højere udnyttelsesgrad af den installerede vindkraftkapacitet. Vi præsenterer formuleringer af — og effektive løsningsmetoder til — problemet at bestemme den optimale udbygning af transmissionskapacitet samt unit-commitment problemet i el-systemer med *transmission switching*. Vi viser også, at *transmission switching* radikalt kan ændre den optimale kapacitetsudbygningsstrategi for el-transmissionsnetværk.

I det nordiske elsystem er det vedtaget at el-markedet er opdelt i zoner, således at hver zone tildeles en bestemt markedspris. Vi formulerer problemet med at designe zoner på en optimal måde når usikkerhed omkring f.eks. udbud og efterspørgsel indgår. Endelig formulerer vi det integrerede problem at beslutte en optimal pipelineinvesteringsstrategi samt prissætning af transmissionstariffer i et naturgastransmissionsnet.

Summary

Efficient use of energy is an increasingly important topic. Environmental and climate concerns as well as concerns for security of supply has made renewable energy sources a viable alternative to traditional energy sources. However, the intermittent nature of for instance wind and solar energy necessitates a radical change in the way we plan and operate energy systems. Another paradigm change which began in the 1990's for electricity systems is that of deregulation. This has led to a variety of different market structures implemented across the world.

In this thesis we discuss capacity planning and transmission pricing problems in energy transmission networks. Although the modelling framework applies to energy networks in general, most of the applications discussed concern the transmission of electricity.

A number of the problems presented involves *transmission switching*, which allows the operator of an electricity transmission network to switch lines in and out in an operational context in order to optimise the network flow. We show that transmission switching in systems with large-scale wind power may alleviate network congestions and reduce curtailment of wind power leading to higher utilisation of installed wind power capacity. We present formulations of — and efficient solution methods for — the transmission line capacity expansion problem and the unit commitment problem with transmission switching. We also show that transmission switching may radically change the optimal line capacity expansion strategy.

In the Nordic electricity system a market with zonal prices is adopted. We consider the problem of designing zones in an optimal way explicitly considering uncertainty. Finally, we formulate the integrated problem of pipeline capacity expansion planning and transmission pricing in natural gas transmission networks.

Preface

This dissertation was prepared in partial fulfillment of the requirements for acquiring the Ph.D. degree and submitted to the Department of Management Engineering, Technical University of Denmark. The work was supervised by Professor Jens Clausen and Professor David Pisinger.

Acknowledgements

I would like to thank my supervisors Professor Jens Clausen, who encouraged me to begin the Ph.D. and Professor David Pisinger, who took over when Jens had to let go.

In particular, I would like to thank Professor Andy Philpott from University of Auckland for inviting me to New Zealand and letting me participate in his research. The many interesting discussions was very inspiring and an important source of motiviation for me. Also, I would like to thank Andy for showing me the true spirit of the *City of Sails* on the Waitemata Harbour and Hauraki Gulf.

I would like to thank Energinet.dk for supporting this project financially, and in particular Peter Børre Eriksen, Jens Pedersen, and the rest of the systems analysis group at Energinet for introducing me to energy systems optimisation and supporting me all the way. Also, thanks to Geir Brønmo for taking over when I left Energinet and for valuable feedback during the past three years.

Thanks to everyone at the operations research group at DTU for creating a great work atmosphere and in particular to Berit, Matias, and Simon for proof reading my thesis at the end.

My memories goes to my grandfather for stimulating my curiousity as a child with his words (freely translated):

there are still many things to be discovered ...

Finally, many thanks goes to my wife Pu and daughter Gan for always being there for me and keeping me company anywhere in the world!

Contents

Part I

Background and Synopsis

Chapter 1

Introduction

Efficient use of energy ressources is one of the most important issues in modern societies due to the economic importance of energy to a functioning society. The increasing population, level of development, globalisation and potentially scarce resources continually cause the use and cost of energy to increase. From 1973 to 2008 the global use of primary energy almost doubled from approximately 6.1 mio. kilo ton of oil equivalent to 11.9 mio. kilo ton of oil equivalent [72].

In recent years focus on environmental and climate issues has created an increased demand for renewable energy sources. Intermittent energy sources like wind and solar energy are available in most parts of the world and constitute feasible sources of energy to substitute fossil fuels in electricity generation. However, the intermittent nature of these energy carriers makes it difficult to integrate them into existing energy systems as supply and demand of electricity must balance at all times, since storage of electricity is not (yet) available on a large scale.

Several technologies exist or are being developed for the purpose of integrating large amounts of renewable energy in the electricity system. These include among others (and not necessarily disjoint):

- flexible demand
- storage of electricity
- linking energy carriers
- transmission expansion
- various smart grid applications

One particular smart grid application in electricity networks is the concept of *transmission switching* [27]. The idea of transmission switching is to view the topology of the transmission network as dynamic rather than static, as is the case in classic electricity transmission systems. This enables us to co-optimise the network topology with the changing patterns of supply and demand.

Transmission switching may alleviate congestions in the network arising from load flow constraints and may increase the potential of renewable energy sources when the network is congested. Another way of dealing with congestion in the transmission network is to expand the transmission capacity by building new lines. In the short term, start-up of existing power plants may incur large fixed start-up costs, while in the long term building new power plants requires large fixed investment costs. This leads us to the following design question:

How to optimally design and configure a transmission network?

Another recent development in energy systems is the policy push for deregulated energy systems in many parts of the world. This was first implemented for electricity systems in the 1990's (e.g. New Zealand, Scandinavia, U.S.A. [10], etc.). This meant that supply and transmission of energy was unbundled and as a consequence a central optimisation of the energy system was no longer possible. Instead, individual agents will optimise their own part of the system and the combined solution may not be optimal in a global sense. Regulatory institutions and frameworks, however, may be (and indeed have been) put in place to ensure that solutions obtained in deregulated markets are close to system optimality. (See e.g. Nagurney [48] for a discussion of user optimality and system optimality for transportation networks and how to model this using variational inequalities.)

The *design* of a particular energy market and the regulatory frameworks may indeed affect the efficiency of an energy system and are therefore important issues when planning efficient energy systems. Often, market design is a trade-off between simplicity of the market structure (to ensure transparency) and the ability to obtain efficient (globally near-optimal) solutions. This leads us to the following question:

How to design energy markets that are both simple and yield solutions that are close to system optimality?

In particular, this thesis deals with the issue of pricing the use of transmission services, when market participants are not subjected directly to network constraints.

In deregulated energy markets the planning of physical capacity and decisions on transmission pricing are interrelated problems as transmission prices affect the demand for capacity, while expansion of capacity may affect the optimal pricing strategy.

Throughout the thesis we assume that the design of the market and the transmission network is performed by a welfare maximising organisation. Total (expected) welfare of the system comprises benefit to all actors (producers, consumers, network operators, etc.) except total cost of the system. The amount of availabel renewable energy in the system may be imposed, exogenously, but is only dispatched if it is economically efficient under given circumstances.

Outline of thesis

Two parts constitute this thesis. Part I, that you are reading now, provides an overview and background as well as a unified framework for the problems discussed. Part II consists of a number of self contained scientific papers relating to the topic of the thesis.

The remaining part of Part I is laid out as follows: Chapter 2 gives an introduction to concepts and models used in the planning of energy systems today. In Chapter 3 a specific model is considered and its mathematical formulation is provided. In particular, this model makes it possible to formulate inter-temporal and strategic decisions for transmission pricing and capacity planning problems for energy networks in a unified approach. Several interesting applications of the model are discussed in Chapter 4. Chapter 5 provides a summary of papers included in Part II, while Chapter 6 discusses the results and contributions of the thesis as well as directions for future research.

Part II is organised as follows: Chapter 7 – 9 presents two different capacity planning problems in switchable electricity transmission networks. Efficient solution methods are presented and the problems are applied to realistically sized networks. In particular, Chapter 7 provides a Dantzig-Wolfe reformulation of a two-stage stochastic investment problem with integer variables in both stages, that seems to be very efficient compared to existing methods. Chapter 8 shows that changing the topology of the electricity transmission network in an operational context by *switching*, may lead to a higher utilisation of wind power, when the network is congested.

Chapter 10 considers the problem of partitioning electricity transmission nodes into zones in a zonal pricing scheme and shows that ensuring contiguous zones may lead to higher cost generation. In Chapter 11 a model for planning of pipeline capacity and design of transmission tariffs in natural gas transmission networks is provided.

A more detailed description of each of the papers are given in Chapter 5.

Chapter 2

Modeling Energy Systems in a Planning Context

This chapter describes and compares some relevant energy models used in the industry and draws on theory developed in the litteraure. Each section of the chapter focusses on one important concept in energy systems, namely supply, demand, storage, transmission, and conversion, respectively. For each section the concept is defined and it is briefly explained how the concept is implemented in various energy models. It is not the purpose of this chapter to give an exhaustive description of each model, but rather to highlight some important features of energy systems modelling as well as to facilitate a comparison between the different models.

The scope of this chapter is energy models that are used in the planning of a larger energy system, i.e. on the national or regional level. No choice has been made regarding the type of energy carrier although the majority of the examples are from electricity networks. The chosen models describe both technical and economical properties of the system. This means that there are associated costs and benefits of operation as well as technical restrictions limiting the operation of the system. Transient behaviour of the system is disregarded and thus all the models describe the steady state of the energy system.

The concepts targeted here are: Supply, demand, storage, transmission, and conversion of energy. Supply denotes the input of energy to the system and is associated with a cost and technical capacity of supply. Similarly, demand defines the extraction of energy from the system giving rise to a benefit to the consumer. For some energy carriers, storage of energy in time is an important property that is modelled to a greater or lesser extent by various models.

The capacity of the transmission network restricts the flow of energy spatially, which may cause congestions and bottlenecks in the system. To accomodate this limitation, one may model the complete transmission network in detail to make sure to capture all potential congestions. Alternatively, one may choose to

	model	MARS	SIVAEL	EMPS	*Balmorel*	*eTransport*	ELMOD	GASMARS
energy carrier(s)	electricity	+	+	+	+	+	+	
	natural gas				+	+		+
	district heat			+	+	+		
	others					+		
supply cost function	linear	+	+	+	+	+	+	+
	quadratic		+					
	unit-commitment		+		+		+	
demand utility function	piecewise constant		+	+	+	+		+
	linear						+	
	cobb-douglas	+						
market power		+						
transmission	link capacity	+	+	+	+			+
	load flow					+	+	
storage	hydro reservoirs			+	+		+	
	nat. gas reservoirs					+		+
	district heat		+		+	+		
conversion			+			+		

Table 2.1: Summary of the properties of the different models considered.

subdivide the geographical space into areas such that interconnections between areas reflect known bottlenecks in the network. When considering more than one energy carrier the conversion between two or more energy carriers must be accounted for.

The models considered in this chapter are: MARS [18], SIVAEL [51, 19], EFI's Multi-area Power-Market Simulator (EMPS) [21], *Balmorel* [55, 2], *eTransport* [5], ELMOD [40], and GAS-MARS. Not all of the models are discussed in each section. Rather, representative examples are discussed in order to illustrate the respective concept. The majority of the models considered in this paper are either used or developed (or both) by the Danish transmission system operator for electricity and natural gas Energinet.dk. Table 2.1 gives an overview of the models discussed and how the different concepts have been modelled.

2.1 Supply

In general, *supply* denotes the process of capturing energy outside the system and converting it to a form of energy inside the system that is to be modelled. In that respect the supply defines a system boundary of the model on the *input* side. As the name implies, *supply cost* denotes the cost of supplying energy to the market or the system under consideration from some energy supply source. In some cases supply costs include both *operating costs* and *investment costs*, while in other cases only operating costs are considered (when investments in production/conversion facilities are irrelevant).

Operating costs may include *start-up costs* associated with the start-up of a given supply unit. The *start-up* cost of a supply unit in time period t is positive if and only if supply in period $t-1$ is 0 and supply in period t is positive, otherwise start-up cost is 0. *The unit commitment problem* is the problem of determining an optimal pattern of start-ups and shut-downs of supply units over a discretised planning period. This problem may include inter-temporal constraints on the minimum and maximum up- and down-time of a unit, ramping constraints, and reserve requirements. See e.g. [61] and [50] for a formulation and litterature review of solution techniques for the unit commitment problem.

The supply from a source may be limited. For each source the supply in each period of time is limited by e.g. equipment (conversion units, extraction facilities, etc.), transport facilities (LNG shipping, natural gas pipelines, etc.) and natural phenomena (wind patterns, cloud cover, etc.). In addition, exhaustible resources may be limited in the total amount of energy available.

In deregulated energy markets it is often assumed that suppliers (and consumers) are price takers — thus the market is said to be perfectly competitive. Samuelson's seminal paper [57] describe spatial price equilibria and their relation to linear programming under this assumption. Under certain conditions, however, large suppliers may be able to affect market prices in deregulated energy markets. Harker [28] generalises spatial price equilibria to imperfectly competitive markets. This may lead to a lower value of welfare than in the case of perfect competition. In particular, large suppliers may exploit the spatial [23] and temporal [34, 70] structure of energy markets.

NATGAS [75] describes an equilibrium model of the European market for natural gas in which suppliers may assert strategic behaviour. This assumes a closed form expression for the supply price (as a function of the supplied quantity), which leads to a mixed complementarity problem (MCP) — a generalisation of non-linear complementarity problems (see e.g. [14] for a thorough introduction to linear complementarity problems).

When exactly one of the suppliers can assert market power we can model him as a leader in a Stackelberg game [71]. This leads to a mathematical program with equilibrium constraints (MPEC, see e.g. [42, 58]) in which the upper level problem maximises the profit of the strategic supplier subject to optimality and feasibility constraints of all the other players including technical feasibility of the system (see e.g. [41] for a definition of Stackelberg games and its corresponding MPEC).

This approach is presented in [33] for an oligopolistic electricity market with transmission network constraints. The authors use a penalty interior point algorithm to solve the MPEC. In [24] the authors model the European electricity market with one strategic supplier. They model the equilibrium constraints by introducing auxillary binary variables and disjunctive constraints [22].

When several strategic players are present this results in an equilibrium problem with equilibrium constraints [65], [41]. In [33] the single supplier problem is extended to a multi supplier problem and an algorithm for solving the resulting EPEC is presented and tested on a 30 node network with two suppliers. The paper [74] presents an equilibrium model of an electricity market with oligopoly suppliers settling the forward and spot market with demand uncertainty. The resulting EPEC consists of a number of MPEC's each corresponding to the profit maximisation of one of the strategi suppliers. The authors propose an EPEC-solution scheme in which the underlying MPEC's are solved iteratively fixing the decision variables of the other MPEC's.

A similar approach is used in MARS to solve the market equilibrium in the Nordic electricity market with several strategic suppliers. Each supply unit is assumed to have constant marginal unit operating cost, but producers may be given the possibility of exercising market power by adding a linear mark-up to the constant marginal cost function. Hence, dominant producers (i.e. producers, that are allowed to use mark-up) bid their supply capacity u at price $p = c + ax$, where c is the marginal cost and $0 \leq x \leq u$. See [68] for a description of the model. This is shown in Figure 2.1 (left). The mark-up a is adjusted iteratively for all dominant producers until a Nash-equilibrium is reached, i.e. until no dominant producer has an incentive to unilaterally change his strategy.

SIVAEL operates with two types of supply; supply with constant marginal cost and supply with piecewise linear marginal cost and unit commitment. The former being used for production units with constant marginal cost and no unit commitment cost (such as wind generators) as well as import from areas exogenous to the model, while the latter may be used to describe the operating costs of thermal production units. In Figure 2.1 (right), the marginal operating cost function for a thermal unit with three potential fuels is shown. The marginal fuel (i.e. the fuel necessary to increase supply marginally) is determined by the operating point of the supply unit. E.g. at a supply of x MWh fuel 1 is used fully and fuel 2 is used partly and the marginal supply cost is given by c. Note that the marginal cost function is not necessarily continous due to differences in fuel cost of different fuels.

Each all-electricity supply unit (i.e. unit supplying only electricity) i maximises its own profit given the market prices on electricity for each weekly period of 168 hours, i.e.

$$\max \sum_{t=1}^{168} \left(\pi^t x_i^{e,t} - c_i(x_i^{e,t})\right)$$

where π^t is the market price for electricity in time period t, $x_i^{e,t}$ is the amount of electricity supplied to the market by unit i in time period t, and c_i is the operating cost function of unit i.

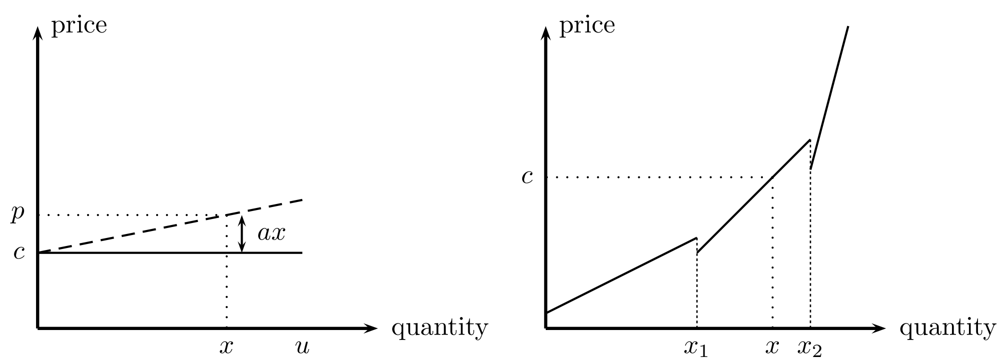

Figure 2.1: Left: Supply curve in MARS. The solid curve shows the supply curve when strategic behaviour is not applied, i.e. the marginal cost c is bidded. When strategic behaviour is applied a linear markup ax is added to the marginal cost (the dashed curve). Right: The marginal cost curve for a thermal unit with three potential types of fuel in SIVAEL. Fuel 1 is being used when the unit is active, i.e. when the level of supply is positive. Fuel 2 is being used at a level of supply greater than x_1, while fuel 3 is being used at a level of supply greater than x_2. At the indicated supply x the marginal fuel is fuel 2 and the marginal supply cost is c.

Combined heat and power (CHP) plants produce both electricity and heat for consumption in a local district heating network. In SIVAEL, two different CHP production technologies are modeled, namely extraction and backpressure technology. The production technology determines the feasible operating region of a CHP plant, i.e. a set of (x^e, x^h)-pairs, where x^e denotes the output of electricity and x^h denotes the output of heat. Figure 2.2 shows the feasible operating region of the two technologies graphically.

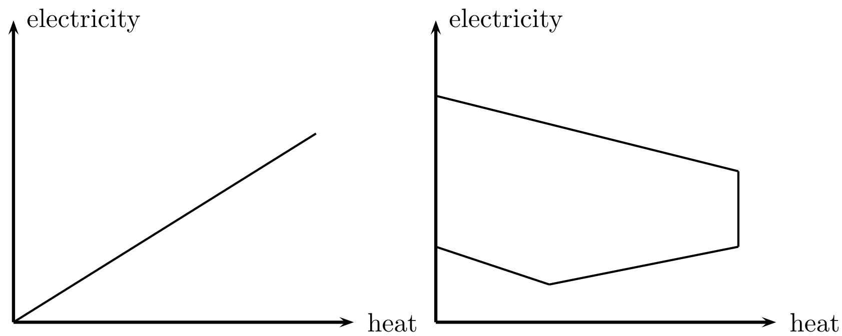

Figure 2.2: The feasible operating region for CHP plants with backpressure, resp., extraction technology in SIVAEL.

Piecewise constant marginal operating costs are used for all supply units in *Balmorel*, as well as in GAS-MARS. This is a prerequisite for the linearity of the two models that makes them computationally less complex compared to non-linear models. ELMOD [40] has piecewise constant marginal supply cost and fixed cost for start-up of supply units. Furthermore, minimum up- and down time constraints are incorporated.

In *eTransport* optimal capacity expansion strategies for supply units may be identified using dynamic programming, when the market is assumed to be perfectly competitive. *Balmorel* allows for a heuristic approach to investments in supply capacity. For supply capacity expansion in imperfect electricity markets see [47]. For a survey on generation capacity planning in centralised as well as deregulated electricity markets see [36].

2.2 Demand

Like supply, demand defines a system boundary — the interface where energy leaves the system. This may be due to consumption of energy (or conversion to an energy carrier that is not modelled) or simply by export to a geographical area or timeperiod outside the scope of the model. Demand and consumption are sometimes used interchangably. Here, consumption is defined as the amount of energy actually consumed (or exported), whereas demand is defined as *the* *ability and desire to purchase* energy [1]. By this definition, demand defines an upper bound on consumption.

For each demand, the associated utility is determined by a demand curve. In traditional engineering models, the demand curve is often assumed to be vertical and the utility set to infinity so that consumption equals demand at all times, irrespective of the cost of supplying the energy. In economic models, the demand curve is usually assumed to have some negative slope indicating, that some users are not willing to pay an arbitrarily high price for supply of energy. The standard choice in the economic litterature assumes a Cobb-Douglas demand function [12], where the price $p = kx^{\frac{1}{\beta}}$ is a function of the consumption x, and where k is a constant, and β is the elasticity of demand [1], which is also constant.

SIVAEL assumes a combination of fixed demand that must be satisfied and flexible demand that is only satisfied if the price is below a specified threshold. Hence, the demand curve is a piecewise constant curve going towards infinity at the fixed demand. An example of a demand curve with three flexible demands is shown in Figure 2.3 (left). Similarly, in EMPS, the demand curve is assumed to be piecewise constant, but with no fixed demand part as shown in Figure 2.3 (right).

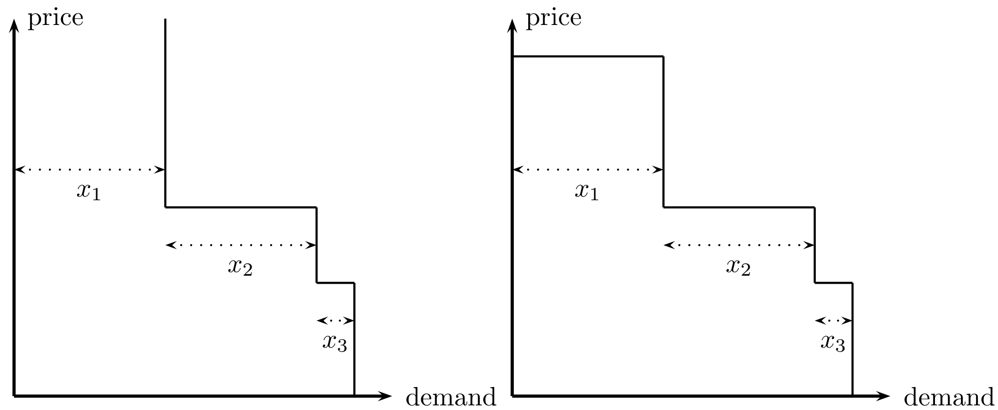

Figure 2.3: Demand curve in SIVAEL with a fixed demand and two flexible demand units (left) and in EMPS with three flexible demand units (right)

In MARS, the demand curve is assumed for each area to be a Cobb-Douglas function with a specified constant elasticity as shown in Figure 2.4 (left). *Balmorel* assumes a piecewise constant demand curve (resulting in a linear model. However, the Cobb-Douglas demand function may be approximated as detailed in [55].

[1]The elasticity of demand is defined as the relative change in consumption divided by the relative change in price, i.e. $\frac{\Delta x/x}{\Delta p/p}$ (see e.g. [39]). In general, the elasticity of demand will vary along the demand curve, however, for the Cobb-Douglas function elasticity is constant in the entire domain of the function.

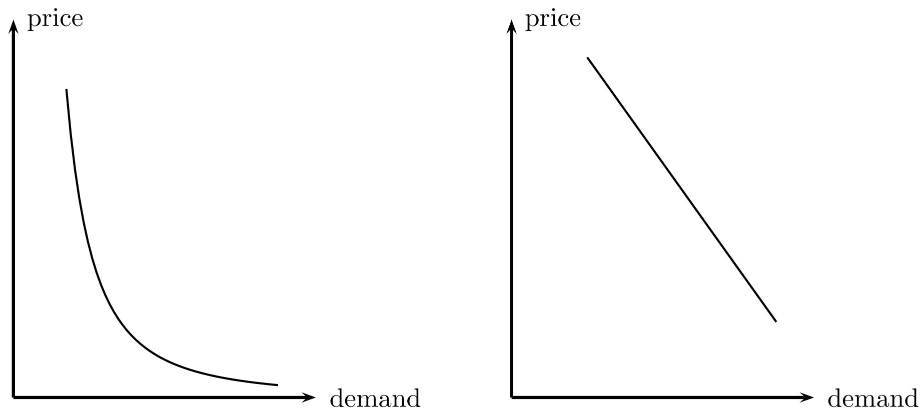

Figure 2.4: Cobb-Douglas demand curve used in MARS (left) and a linear inverse demand function (right) as used in many economic models.

2.3 Storage

A storage facility enables the system to keep energy supplied at one point in time for consumption at a later point in time. Energy storage may either be modelled endogenously by optimising the system over the entire planning horizon or by subdividing the planning horizon and performing an approximated optimisation of the system. In order to keep computational complexity low storage of energy is sometimes neglected or simplified in models of the power system.

Of course, the type and size of the storage(s) must be considered when determining the length of the planning horizon and subdivisions. For hydro reservoirs the seasonal variation in precipitation and the size of the reservoirs typically calls for optimisation over one year. Storages for natural gas are also typically optimised over the year. The varying demand pattern causes injection to the storage during low demand periods (e.g. summer) and extraction during high demand periods (e.g. winter). For district heating storages the storage size is much smaller and optimisation may be performed on a weekly basis considering the variation in heating demand pattern over the week.

In [53, 54] Quelhas et. al. applied differentiated time resolutions to a system of multiple energy carriers. The authors optimise the network flow of coal, natural gas, and electricity in the U.S.A. over one year, where the time resolution for the three energy carriers are 1 week, 1 day, and 1 hour, respectively, and where storage of coal and natural gas are possible (and storage of electricity is not possible).

Computational issues arise when the detail of the model becomes too large. This may happen when the planning horizon needed to describe the use of a storage facility is long, but the temporal resolution must still be kept relatively small in order to capture e.g. supply or demand variations in areas where storage is not available (or for energy carriers that cannot be stored).

In general, energy storages are used to accomodate temporal differences in supply and demand patterns, so that arbitrage may be performed between periods of high supply (or low demand) and periods of low supply (or high demand) yielding an economic trading surplus.

EMPS is a simulation model of a hydro-based power system, where efficient use of hydro reservoirs is of great importance. However, it is costly to run an optimisation of a large system (such as the nordic system) in detail for one entire year. Therefore, the optimisation is done in two steps. Firstly, an optimisation of the hydro reservoirs is carried out. This yields for each time period and each area a vector of *water values*. Secondly, an optimisation of the whole system is carried out for each time period using the water values as input. There is no feed-back loop to the first iteration, hence the water values are not guranteed to be optimal for the problem of the second iteration.

The water values are calculated using manual calibration and give an indication of the expected value of the water in the reservoir at a given point in time given the level of the reservoir. That is, the water value (v_{it}^r) is the minimum price that the respective hydro power producer will accept for energy from the reservoir r in time period t given that the reservoir level is i relative to the capacity of the reservoir. In other words, v_{it}^r indicates the (constant) supply price of reservoir r in time period t given the reservoir level i. The water value is the price a non-strategic hydro power producer will bid in the market corresponding to the marginal costs of thermal power plants, since he must also (apart from a marginal production cost close to zero) consider the scarcity of the water ressource. This supply price is used in the second iteration, where the supply, consumption and flows of energy are determined for the model area for each time period independently.

In MARS, the water values calculated by EMPS are used as supply prices for hydro power stations. As for the second iteration of EMPS, the market clearing are calculated for each time period (hour), independently, taking into account the level of the reservoir based on the supply of energy in previous time periods.

SIVAEL models the storage of heat in warm water tanks for consumption in local district heating networks. The optimisation of the heat storages are done for each weekly period of 168 hours. (In fact, optimisation is done over 192 hours of which the last 24 hours are discarded, so that only the first 168 hours are used in the solution. This is done in order to ensure a realistic level of storage at the end of the week). The input to the storage are supplied by combined heat and power (CHP) generation plants operating according to fixed prices for heat and variable market price for electricity.

2.4 Transmission

It may be necessary explicitly to model the transmission of energy, i.e. the movement of energy in space. This may either be due to technical restrictions (bottlenecks) in the system or because of high costs related to the transmission. When it is necessary to model transmission explicitly, the geographical space of the model is subdivided into areas, such that transmission within each area is costless and without restriction (i.e. there are no bottlenecks). The geographical segregation must be chosen so that the requirement of costless and unrestricted transmission holds (to a satisfactory degree), while minimising the number of areas and thereby the computational complexity.

The geographical resolution is analogous to the temporal resolution described in Section 2.3 and geographical arbitrage may be performed between adjacent areas, i.e. between areas that are directly connected by transmission links, yielding an economic trading surplus. Hence, energy will be bought in areas with excess supply (low price area) and sold in areas with excess demand (high price area). The trading surplus is the price difference between the two areas multiplied by the amount of energy traded except transaction costs. In a perfect market, price differences (larger than transaction costs) only occur when the capacity of transmission is reached. Assuming perfect competition between owners of transmission infrastructure, this potential difference would attract capital to increase the capacity of the transmission link in question. However, most energy transmission networks are natural monopolies and therefore not subject to the assumption of perfect competition.

The flows on transmission links of an energy network is in practice not free, but governed by some physical laws. The flow of energy along a link is determined by some variable potential at the end nodes of the respective link. E.g. for natural gas networks the flow of energy in a pipeline is determined by the end pressures of the pipeline. Conservation of energy implies that the sum of potential differences around a circuit in the transmission network is 0. For electrical networks this is referred to as *Kirchhoff's second law* or *Kirchhoff's voltage law*. These type of restrictions will be denoted *load flow constraints*. However, these physical laws are neglected in many economic macro-level models of energy systems. Instead, an estimated capacity level is used for the maximum flow of each transmission link. In the following, these constraints will be denoted *link capacity constraints*. The two types of models will be referred to as *load flow models* and *link capacity models*, respectively.

2.4.1 Link capacity models

MARS is an example of a link capacity model, where the areas are determined beforehand to reflect important bottlenecks of the power transmission system. Each link between two areas has an associated capacity and a constant unit flow cost. The link capacity is determined by the total maximum technical capacity or the available trading capacity between the two areas. The electrical network is not modelled in detail and Kirchhoff's second law is not considered. The transmission cost per unit of energy transmitted may be set arbitrarily and reflects e.g. loss of energy or transaction costs related to trading. Also, the transmission costs limit the number of optimal solutions to the market clearing problem.

SIVAEL is originally a one-area model, where transmission is not modelled. However, a heuristic approach to model a single transmission constraint has been implemented. The approach is outlined in Algorithm 1.

Algorithm 1 Transmission constraint heuristic used with SIVAEL.

```
find a solution to the problem where the transmission constraint is relaxed
for all timeperiods where the transmission constraint is violated do
  set the flow to the capacity of the transmission line
end for
for all timeperiods do
  fix flows on the transmission link
end for
for all areas do
  find a solution with fixed import respectively export corresponding to the
  fixed flow on the transmission line as determined previously
end for
```

This heuristic ensures that the solution is feasible with respect to the transmission constraint, but not necessarily optimal. When transmission is considered SIVAEL may be classified as a link capacity model, as only a fixed capacity on the transmission link is employed.

Balmorel defines geographical regions, such that transmission of electricity between regions is restricted. Distribution within one region (i.e. from generators to consumers) is assumed not to lead to bottlenecks and is therefore not restricted. However, transmission (between regions) and distribution (within regions) of electricity always cause a loss of electricity relative to the amount transmitted. Furthermore, a constant unit cost of transmission and distribution is implied. *Balmorel* may be classified as a link capacity model. An area in *Balmorel* is a geographical subdivision of a region. Heat is produced and consumed within the same area, i.e. transmission of heat between areas is not possible. Distribution of heat within an area from supply source to consumer lead to a constant unit loss of energy as well as a constant unit cost similar to electricity distribution. As for electricity distribution there is no capacity constraint on the distribution of heat.

2.4.2 Load flow models

Many electricity network models (e.g. *eTransport*, ELMOD, etc.) considers a direct current load flow (DCLF) approximation of the alternating current load flow (ACLF). DCLF assumes that voltage phase angle differences between neighbouring nodes are small. See e.g. [11] or [64] for a derivation of DCLF.

Under these assumptions the real power flow on a line $a = (i, j)$ may be described as a linear function of the voltage phase angle difference between the two end nodes.

$$x_a = \frac{w_i - w_j}{r_a}$$

where r_a is the reactance (typical for line type), w_i and w_j are the voltage phase angles of the two end nodes and the flow of power is in the direction from i to j if x_a is positive and in the direction from j to i if x_a is negative. Due to the resistance of a transmission line energy dissipates when flowing across a line. This has two consequences:

1. The amount of energy received at node j is less than the amount of energy sent at node i (assuming positive flows).
2. The dissipated energy causes the line to heat up.

Often energy losses (1.) is neglected. However, for *eTransport* [5] and the New Zealand market dispatch model [67], energy losses are considered explicitly for each power line and included in the energy conservation constraints for each network node. Due to (2.) an upper bound (thermal constraint) on the flow of a line is necessary to ensure secure operation of the system [73].

The DCLF approximation to the ACLF is generally considered to be acceptable for long term planning.

Several commercial simulation packages for electricity networks exist, including PowerWorld [52] and DIgSILENT's PowerFactory [16]. These typically have both an alternating current (AC) and a direct current (DC) load flow model. This makes it possible to simulate in detail the load of the network given the consumption and supplies.

Determining optimal investments in new transmission capacity is an important problem as new transmission capacity usually has a high fixed cost and a high impact on the existing system. Furthermore, the problem is computationally hard.

Villasana et. al. [69] uses linear programming to identify bottlenecks in electricity transmission networks. The static transmission capacity expansion problem is the problem of determining an an optimal expansion strategy for a target period. For electricity networks assuming a DC-approximated load flow formulation, the problem can be formulated as a mixed integer program using disjunctive constraints (see e.g. [4] and [3]).

The standard way of solving the static transmission capacity expansion problem is by use of Benders decomposition [6]. This yields a master problem for generating (possibly infeasible) capacity expansion candidates and a sub problem checking feasibility of the expansion plans generated in the master problem. The sub problem solution results in cuts that are added to the master problem. See [13] for a survey on solving fixed charge network design problems. [56] proposes a hierarchical decomposition based on Benders decomposition for the static transmission network expansion problem. In [49] Oliveira et. al. notes that the investment master problem is a computationally hard mixed integer program, that needs to be solved successively. They propose to use heuristics to solve the master problem of the Benders algorithm. Another approach based on Benders decomposition is proposed in [7]. The stochastic version of the transmission capacity expansion problem is presented in [25]. Here it is assumed that capacity expansion decisions are taken in the first stage subject to operational dispatch decisions of market players (who are all pricetakers) in a number of scenarios (second stage). In [62] the authors propose a Dantzig-Wolfe reformulation [15] and a column generation approach for design of survivable electricity distribution networks.

Strengthening transmission lines by reducing the reactance or increasing transmission capacity by adding new lines in electricity networks may, however, increase the cost of power generation. This paradox is due to Kirchoffs second law and is demonstrated in [73] and [9].

Transmission switching [27] may alleviate such effects by switching out transmission lines in periods when this is helpful. Ultimately, one may view the transmission network – not as a static network – but as a dynamic network, which is reconfigured in an operational context in order to match supply and demand. In general transmission switching may increase security of a network by the use of corrective (or post-contingency) rescheduling [46, 59, 60] and decrease cost of generation [20, 29, 30]. The transmission switching model has been extended to include unit commitment of suppliers with transmission switching and security constraints. A heuristic solution procedure [31] and an algorithm based on Benders decomposition [37] has been proposed for this problem. Khodaei and Shahidehpour [38] propose a Benders decomposition approach for solving the dynamic transmission capacity expansion problem with transmission switching.

2.5 Conversion

When multiple energy carriers are considered proper conversion between the energy carriers must be modelled. Examples of conversion facilities are hydro power stations (water to electricity), gas turbines (natural gas to electricity), compressed air energy storages (electricity and natural gas to compressed air to electricity). This conversion may incur some loss of energy, so that the energy input is higher than the energy output.

eTransport is an example of a model that considers multiple energy carriers and conversion. *eTransport* operates with the following types of conversion facilities: CHP plants, boilers, LNG-plants, LNG-regassification plants, and power plants with emission flows. Boilers convert fuels or electricity to heat for consumption in a district heating network. Heat output x_t^b from boiler b in time period t is a linear function of the input energy f_t^b and is bounded above by u^b, i.e.

$$x_t^b = k^b f_t^b \leq u^b$$

where $k^b \in [0;1]$ is the efficiency of boiler b and u^b is the capacity of the boiler [5].

Even for models that may not be classified here as considering multiple energy carriers, conversion between different forms of energy may be considered. In SIVAEL conversion from fuels to electricity and heat supply is modelled in the supply units as described in section 2.1.[2] The increasing marginal cost function for each fuel in Figure 2.1 (right) reflects a decreasing efficiency of conversion, while the difference in cost level (corresponding to an efficiency of 1) between different fuels reflects difference in fuel costs.

In [26] a general framework for considering multiple energy carriers is considered. In particular, the concept of an energy hub is developed. An energy hub may model a CHP-plant, an industrial plant, or an urban or rural area, and is characterised by the input and output energy carriers as well as the coupling between them. In Figure 2.5 an energy hub with three input energy carriers (electricity, natural gas, and district heat) and two output energy carriers (electricity and heat) is shown. The energy hub consists of an electrical transformer, a gas turbine, a gas furnace, and a heat exchanger. The coupling between the vector of power input x^{in} and the vector of power output x^{out} is determined by the coupling matrix C, such that $x^{out} = Cx^{in}$, where the ith element of x^{in}, respectively, x^{out} denotes the input, respectively, output of energy carrier i and C_{ij} denote the coupling between input energy carrier i and output energy carrier j (i.e. one unit of input i results in C_{ij} units of output j.

[2]SIVAEL is here not classified as considering multiple energy carriers as fuels are only modelled with one parameter, namely its price (which is constant). No technical restrictions are considered, which implies that any quantity of fuel is available at any supply unit at any time.

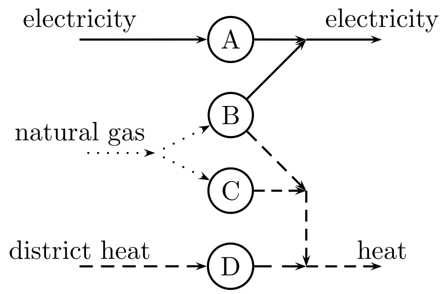

Figure 2.5: Energy hub with three input energy carriers and two output energy carriers. The hub consists of four converters, namely an electrical transformer (A), a gas turbine (B), a gas furnace (C), and a heat exchanger (D). The figure is derived from [26].

2.6 Summary

In this chapter some important concepts in energy systems modelling have been discussed covering supply, demand, storage, transmission, and conversion of energy. Examples from practical models have been used to illustrate the different concepts and to facilitate a comparison between the models.

Chapter 3

Formulation

This chapter provides a unified framework for modelling capacity planning and market design problems in energy transmission networks. In particular, the formulation presented applies to load flow formulations of energy transmission networks. For modelling specific energy systems additional side constraints may be necessary. For instance, pressure bounds are usually included when modelling natural gas networks [45, 66]. Section 3.1 provides an optimal load flow formulation of an energy system maximising total social welfare in a particular point in time. Section 3.2 introduces strategic or inter-temporal decisions linking operational decisions made at different points in time. Particular applications of the framework is presented in chapter 4.

Consider a directed multigraph $G = (\mathcal{N}, \mathcal{A})$ with source/sink node s. For each arc $a \in \mathcal{A}$, the cost, capacity and lowerbound are stochastic (time dependent) parameters given by c_a, u_a, and l_a, respectively. We wish to find a vector of flowvariables $x \in \mathbb{R}^{|\mathcal{A}|}$, a vector of node *potentials* $w \in \mathbb{R}^{|\mathcal{N}|}$, as well as, a *switching configuration* defined by a binary vector $z \in \{0,1\}^{|\mathcal{A}|}$. Each node $i \in \mathcal{N} \setminus \{s\}$ has a fixed demand represented by the stochastic (time dependent) variable d_i. Let the set of arcs $\mathcal{F}(i)$, respectively, $\mathcal{T}(i)$ denote the set of arcs with tail, resp. , head i. Let the set of supply and demand arcs $\mathcal{S} = \mathcal{F}(s) \cup \mathcal{T}(s) \subseteq \mathcal{A}$ be defined by having s as the tail, respectively, head. We allow cost coefficients to be negative. This allows us in particular to model positive benefit of energy consumption corresponding to flows on demand arcs $\mathcal{T}(s)$. An illustration of a five node instance is shown in Figure 3.1.

3.1 Operational Dispatch

We will now present a generic load flow formulation for an energy transmission network before we introduce strategic and inter-temporal decisions . This formulation applies in particular to direct current load flow (DCLF) formulations of electricity networks with switching capabilities.

For a single time period or single scenario ω we observe a realisation $c(\omega), u(\omega), l(\omega), d(\omega)$ of the stochastic parameters c, u, l, d.

3.1.1 A minimum cost network flow model

The globally optimal state (with maximum social surplus) of the system may be found by solving:

$$\text{MCNF}(\omega): \qquad \min \sum_{a \in \mathcal{A}} c_a(\omega) x_a \tag{3.1}$$

subject to

$$\begin{aligned}
x_a &\leq u_a(\omega) z_a && \forall a \in \mathcal{A} && (3.2)\\
x_a &\geq l_a(\omega) z_a && \forall a \in \mathcal{A} && (3.3)\\
\sum_{a \in \mathcal{F}(i)} x_a - \sum_{a \in \mathcal{T}(i)} x_a &= d_i(\omega) && \forall i \in \mathcal{N} \setminus \{s\} && (3.4)\\
z_a = 1 \;\Rightarrow\; x_a &= g_a(w_i, w_j) && \forall a = (i,j) \in \mathcal{A} \setminus \mathcal{S} && (3.5)\\
z_a &\in \{0,1\} && \forall a \in \mathcal{A} && (3.6)
\end{aligned}$$

where g_a is the power flow function, x_a is the flow on arc a, z_a is the *switching* decision indicating whether arc a is on or off, and d_i is the potential of node i.

The binary *switching* decision $z_a = 1$ may for instance represent the decision to turn on a supply unit (for a in $\mathcal{S}$) or switch a transmission element in (for a in $\mathcal{A} \setminus \mathcal{S}$). For potential investments $z_a = 1$ may represent a request for capacity on that particular arc.

For the DCLF model in electricity networks with linear generation costs and no line losses, we have for all arcs $a = (i,j)$ in $\mathcal{A} \setminus \mathcal{S}$, $g(w_i, w_j) = k_a(w_j - w_i)$, where k_a denotes the susceptance coefficient of line a and fixed z_a for all arcs a in $\mathcal{A}$. The node potentials w represents the physical voltage phase angles.

In natural gas transmission networks the flow on a pipeline may be approximated by the Weymouth equation [66], $g_a(w_i, w_j) = k_a\sqrt{w_i^2 - w_j^2}$, where w_i, w_j denotes the pressure at the end points of the pipeline.

In order to illustrate the model we will consider a small example with four transmission nodes, two generators, and two flexible demands. We assume a DCLF formulation with susceptance coefficient $k_a = -1$. The remaining parameters are shown in Figure 3.1. All transmission arcs have zero cost and infinite capacities except for the arc from node 2 to 4, that has capacity $u_a = -l_a = 5$. All supply and demand arcs $a \in \mathcal{S}$ have lower bound $l_a = 0$, while all transmission arcs $a \in \mathcal{A} \setminus \mathcal{S}$ have lower bound $l_a = -u_a$. Figure 3.2 shows the optimal flow and prices with fixed topology, that is $z_a = 1$ for all arcs a in $\mathcal{A}$. This solution has a cost of -777.5. (A negative value indicates that the benefit to consumers is larger than the total cost on arcs with positive cost coefficients).

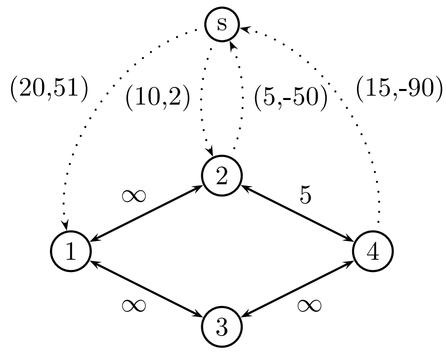

Figure 3.1: DCLF instance with five nodes, four transmission lines, two supply arcs, and two demand arcs. Arc labels (u_a, c_a) indicate arc capacities and costs for supply and demand arcs $a \in \mathcal{S}$ (dotted) and capacities for transmission arcs $a \in \mathcal{A} \setminus \mathcal{S}$ (solid). s denotes the source/sink node, while 1,2,3,4 denotes transmission nodes.

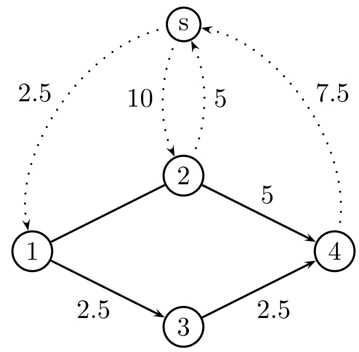

Figure 3.2: Optimal flow. Arc labels indicate flows.

In electricity networks Kirchhoff's voltage law may — together with thermal capacity constraints — impose additional restrictions on the network flow. In some cases this leads to a higher cost of generation, than what could be achieved with a simple link capacity model. Actively switching network elements may help to alleviate negative effects of Kirchhoff's voltage law. Figure 3.3 shows the optimal solution in a switched network, where z is free. The corresponding total cost of this solution is -1560.

In deregulated energy markets several players have conflicting interests and make decisions that may not maximise social welfare. In the following, we assume that each supply and demand arc may be operated by individual agents, each of which maximise their own profit.

3.1.2 An *arc* operator

Assume now that a supply or demand arc $a = (i, j)$ in $\mathcal{S}$ is operated by an independent operator purchasing energy in node i and selling energy in node j in order to maximise his profit. In a particular scenario or time period ω we can formulate his optimisation problem as follows, assuming that $z_a = 1$ is fixed exogenously,

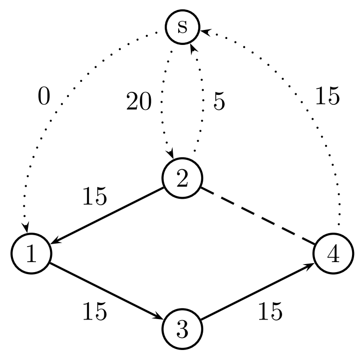

Figure 3.3: Optimal flow in switched network. Arc labels indicate flows. The dashed arc is switched out ($z_a = 0$).

$$\begin{aligned} \mathrm{AOP}_a(\omega): \qquad \max \qquad & (\rho_i - \rho_j - c_a(\omega))x_a && (3.7) \\ \text{s.t.} \qquad & x_a \le u_a(\omega) && (3.8) \\ & x_a \ge l_a(\omega) && (3.9) \end{aligned}$$

where ρ is a vector of market prices determined exogenously. Constraint set (3.8)-(3.9) is equivalent to

$$\begin{aligned} -x_a &\ge -u_a(\omega) && (\lambda_a) && (3.10) \\ x_a &\ge l_a(\omega) && (\mu_a) && (3.11) \end{aligned}$$

where λ_a and μ_a are the dual prices associated with constraints (3.10) and (3.11), respectively.

The corresponding dual problem is

$$\begin{aligned} \min \qquad & u_a(\omega)\lambda_a - l_a(\omega)\mu_a && (3.12) \\ \text{s.t.} \qquad & \lambda_a - \mu_a = c_a(\omega) + \rho_i - \rho_j && (3.13) \\ & \lambda_a, \mu_a \ge 0 && (3.14) \end{aligned}$$

The basic solutions x_a and (λ_a, μ_a) are said to be complementary if and only if

$$\begin{aligned} \lambda_a &\perp u_a(\omega) - x_a && (3.15) \\ \mu_a &\perp -l_a(\omega) + x_a && (3.16) \end{aligned}$$

are satisfied (see e.g. [32]). The $\perp$-operator ensures that at least one of the two operands must be 0. Furthermore, x_a is said to be an optimal solution to $\text{AOP}_a(\omega)$ if and only if x_a is feasible and there exist a complementary dual solution (λ_a, μ_a) satisfying (3.13)- (3.14) [32].

The corresponding mixed complementarity problem is then to find

$$(x_a^*, \lambda_a^*, \mu_a^*) \in \{(x_a, \lambda_a, \mu_a) | (3.10) - (3.11), (3.13) - (3.14), (3.15) - (3.16)\}$$

The complementarity constraints can be linearised using disjunctive constraints [22].

3.2 Strategic and Inter-Temporal Decisions

We now turn our attention to decision problems involving several time periods or scenarios. This allows us to model decisions that are dependent in time and decisions taken under uncertainty. Let the *planning horizon* be defined by a set Ω of time periods or scenarios, and — as before — let $c(\omega), u(\omega), l(\omega)$, and $d(\omega)$ be time dependent parameters or particular realisations of stochastic parameters in a particular time period or scenario ω in Ω.

Let y be a vector of strategic capacity decision variables and let f be the per unit cost vector associated with these decisions. A generic capacity planning problem may now be formulated by

$$\begin{aligned}
\text{CPP:}\min \quad & f^\top y + \sum_{\omega \in \Omega} p(\omega) c(\omega)^\top x(\omega) && (3.17)\\
\text{s.t.} \quad & (y, z) \in \mathcal{Y} && (3.18)\\
& (x(\omega), z(\omega), w(\omega)) \in \mathcal{X}(\omega) \qquad \forall \omega \in \Omega && (3.19)
\end{aligned}$$

where $\mathcal{X}(\omega) = \{(x, w, z) | (3.2) - (3.6)\}$ denotes the set of feasible operational decisions for scenario (time period) ω and $\mathcal{Y}$ denotes the set of constraints linking the scenarios (time periods). If CPP is a stochastic program, $p(\omega)$ denotes the expected probability of scenario ω and the objective maximises the expected social welfare. If CPP is a deterministic program, $p(\omega)$ denotes a weight on each time period. When $p = \{\mathbf{1}\}$, the objective maximises the total social welfare over the planning horizon.

In deregulated markets we need to ensure optimality for each of the arcs that are individually operated and so $\mathcal{X}(\omega)$ is replaced by $\mathcal{Q}(\omega) = \{(x, w, z, \rho, \lambda, \mu) | (3.2) - (3.6), (3.13) - (3.16)\}$ for all scenarios ω in Ω.

Now, let ψ be a vector of strategic pricing decisions. A generic transmission pricing problem maximising total social welfare may be formulated by

$$\text{TPP:} \min \quad \sum_{\omega \in \Omega} p(\omega) c(\omega)^\top x(\omega) \tag{3.20}$$

$$\text{s.t.} \quad (\psi, \rho) \in \mathcal{Y} \tag{3.21}$$

$$(x(\omega), z(\omega), w(\omega)) \in \mathcal{X}(\omega) \qquad \forall \omega \in \Omega \tag{3.22}$$

$$(x(\omega), \rho(\omega), \lambda(\omega), \mu(\omega)) \in \mathcal{Q}(\omega) \qquad \forall \omega \in \Omega \tag{3.23}$$

where $\mathcal{X}(\omega) = \{(x, w, z) | (3.2) - (3.6)\}$ denotes the set of feasible operational decisions for scenario (time period) ω and $\mathcal{Q}(\omega) = \{(x, \rho, \lambda, \mu) | (3.13) - (3.16)\}$ ensures optimality for each of the individually operated arcs for scenario ω.

In the next chapter we will discuss some important applications of CPP and TPP in energy systems.

Chapter 4

Applications

The modelling framework presented in chapter 3 is applied to a range of capacity planning and transmission pricing problems. Most of the problems presented below are strategic decision problems involving uncertainty, that can be modelled as two-stage stochastic programs. However, one example of a short-term capacity planning problem is given in section 4.2. Section 4.1 presents the node partitioning problem for markets with zonal pricing. In section 4.2 the unit commitment problem for power generators is presented, while section 4.3 considers network capacity expansions and transmission switching in electricity transmission networks. Finally, section 4.4 models optimal investments in wind power generation parks.

4.1 Optimal partitioning of nodes for zonal pricing

In deregulated electricity markets different transmission pricing schemes may be employed. A distinction is made between between *nodal* and *zonal pricing* schemes. Nodal pricing refers to a system with market prices for each physical node in the network, whereas in zonal pricing the network is partitioned into zones and a market price is assigned to each zone.

In the following, we will consider the design of zones under uncertainty for a deregulated electricity market employing zonal pricing with $|\mathcal{K}|$ zones maximising expected social welfare. See [8] for the corresponding deterministic version. Let ψ be a vector of binary variables and let $\psi_{ij} = 1$ if and only if node i in $\mathcal{N} \setminus \{s\}$ is located in the same zone as node j in $\mathcal{N} \setminus \{s\}$. The problem may now be defined as the strategic pricing problem (3.20)-(3.23) with

$$\mathcal{Y} = \{(\psi, \rho) \,|\, \psi_{ij} = 1 \Rightarrow \rho_i(\omega) = \rho_j(\omega), \forall i, j \in \mathcal{N} \setminus \{s\}, \omega \in \Omega\} \tag{4.1}$$

where $\psi_{ij} = 1$ forall $(i, j) \in \mathcal{H}_{\mathcal{K}}$ and 0 otherwise, and $\mathcal{H}_{\mathcal{K}}$ is a forest of $|\mathcal{K}|$

components spanning the complete graph with node set $\mathcal{N} \setminus \{s\}$. See [44] for a MIP formulation of the minimum spanning tree problem, that may easily be adapted to a minimum spanning forest problem.

This problem is considered in chapter 10, where a Dantzig-Wolfe reformulation based on a split variable approach [43] is also provided.

4.2 Unit Commitment of Power Generators

We will now consider the problem of finding the minimum cost dispatch and commitment of power generation units in a transmission network. First, we make the assumption that the planning horizon Ω is a cyclic ordered set of timeperiods and that $\Pi(\omega)$ denotes the timeperiod immediately preceding ω. Furthermore, we assume that a fixed cost $f_a(\omega)$ is incurred by the start-up of a generation unit corresponding to supply arc a in $\mathcal{S}$. Let $y_a(\omega) = 1$ denote the decision to start-up unit a in time period ω and, correspondingly, let $z_a(\omega) = 1$ if and only if unit a is on in time period ω.

The unit commitment problem can now be formulated as the capacity planning problem (3.17) - (3.19), with

$$\mathcal{Y} = \Big\{ (y,z) \,|\, z(\omega) - z(\Pi(\omega)) \leq y(\omega) \in \{0,1\}^{|\mathcal{A}|}, \forall \omega \in \Omega \Big\} \tag{4.2}$$

Assuming a non-negative fixed cost vector, we can relax the integrality requirements on y.

The unit commitment problem with transmission switching and a Dantzig-Wolfe reformulation is presented in chapter 9 together with computational results for the IEEE 118-bus network.

4.3 Line Capacity Expansion and Transmission Switching

The line capacity expansion or network design problem is an important problem in the design of efficient electricity systems. In this application, we consider the possibility of actively switching transmission lines in an operational context. We assume that the switching of a transmission line (or transformer) is conditioned on the line having an advanced switch already installed incurring a fixed cost.

The problem is to determine an optimal strategy for investing in new line capacity and switches. Let y_1 be the binary decision vector of line capacity investments and let y_2 be the binary decision vector of switch investments so that $e_a^\top y_1 = 1$ if and only if line a is installed and, similarly, $e_a^\top y_2 = 1$ if and only if line a is equipped with a switch, where e_a is the unit vector of all zeros except the ath element which is equal to 1.

Let $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$. We may now formulate the problem by (3.17)-(3.19), with

$$\mathcal{Y} = \Big\{(y, z) \,|\, y_1 - y_2 \leq z(\omega) \leq y_1, y \in \{0, 1\}^{2|\mathcal{A}|} \forall \omega \in \Omega\Big\} \tag{4.3}$$

This yields a two-stage stochastic program in which first stage decisions identify investments in transmission capacity and switches and the second stage models operational decisions after the stochastic realisations of demand, generation cost, generation capacity, transmission capacity, etc.

A Dantzig-Wolfe reformulation and its integrality properties are discussed in chapter 7. In chapter 8 we apply the model to the Danish transmission network and show that switching may increase wind power generation considerably in systems with large-scale wind.

Switching to increase network reliability

Now, we turn to the problem of increasing the reliability of a transmission network by investing in transmission switches. We begin by defining the concept of *N-1 reliability.*

Definition 1 *A network defined by the multi-graph $G = (\mathcal{N}, \mathcal{A})$ is N-1 reliable in scenario (time period) ω if and only if forall arcs $a \in \mathcal{A}$: MCNF(ω) with fixed $z = \{\mathbf{1}\}$ is feasible for the network defined by $G = (\mathcal{N}, \mathcal{A} \setminus \{a\})$.*

Consider a network, which is not $N-1$ reliable, i.e. in at least one case a failure of a single transmission line will cause load to be shedded. Traditionally, unreliable networks are made reliable by adding new (and often expensive) line capacity to the network. An alternative is to employ active switching of lines in a post-contingency corrective scheduling framework to increase the reliability of the network [59, 60]. Or in general: What is the optimal strategy for making a network $N-1$ reliable, when we can choose to install switches on existing lines as well as reinforce existing lines?

In practice the capacity planning model (3.17)-(3.19) with $\mathcal{Y}$ defined by (4.3) is applied with large penalties on load shedding. That is, we set $d_i = 0$ for all nodes $i \in \mathcal{N}$ and instead introduce flexible demands with a very high benefit (negative cost). We use the IEEE 118-bus network, with data described in [10]. This network has 185 lines, total peak load of 4519 MW, and a total thermal generator capacity of 5859 MW.

Each of the scenarios correspond to a contingency, where exactly one of the lines fail. In total there are 185 existing lines, however, only 7 of these lines cause load to be shedded when failing. We consider expansion of line capacity on 7 lines by installing new lines in parallel with existing lines. The base case refer to the situation without line failure and without switching and has total operational dispatch cost of 2074 $/hour.

Each of these potential lines have thermal capacity 220 MW and reactance coefficient equal to the existing parallel line. Cost of installing a new line is 10000 $. Switches may be installed across the network with a fixed cost of 100 $, however, we only allow 2 switches to be open in each of the failure scenarios (in addition to the one failing) in order to reduce computational complexity [20].

In the optimal solution one new line and five switches are installed incurring a total investment cost of 10500. The optimal switching configuration is shown in Table 4.1. The operational cost is lower than in the base case in most scenarios. If switching is not possible we need to install two new lines incurring a total investment cost of 20000 $.

ω	Line failure	Shedded load (MW)	Opt. switch conf.
1	E77-82	4.33	E83-85, E89-90
2	E82-83	149.88	E77-82, E89-90
3	E83-85	0.16	E77-82, E89-90
4	E85-88	6.20	E77-82, E89-90
5	E89-90	87.31	E77-82, E83-85
6	E89-92	30.00	E77-82, E83-85
7	E91-92	67.97	E89-91, E95-96

Table 4.1: Failure scenarios for IEEE 118-bus network.

4.4 Positioning and connection of wind parks

The supply capacity of wind power turbines vary over time according to the speed and direction of the wind and hence large fluctuations in capacity may occur. When installing off-shore wind parks it is usually worthwhile to invest in very large wind parks due to the high fixed investment costs. Together with the extreme wind speeds at sea this exacerbates the sudden changes in electricity supply. Since electricity cannot (yet) be stored in large amounts these variations may cause three related problems:

- Electricity supplied by wind power does not match demand,
- other technologies (such as thermal power plants) does not support this fluctuating supply (i.e. cannot fill in the missing supply at a rate fast enough), and

- large supplies from a single point in the network may cause congestion in the transmission network.

Hence, when planning an energy system with large amount of wind power it may be beneficial to consider the location of wind parks and their connection to the transmission network in order to alleviate (partly) some of these problems.

What is the optimal locations and connection points for large off-shore wind parks with respect to social welfare?

For this purpose we may consider the network design problem (3.17) - (3.19), where $z_a(\omega) = 1$ is fixed for all transmission lines $a \in \mathcal{A} \setminus \mathcal{S}$ and scenarios $\omega \in \Omega$ and $\mathcal{Y} = \{(y, z) | z(\omega) = y, \forall \omega \in \Omega\}$ ensures that the chosen investment strategy is implemented in all scenarios.

Here the upper problem considers a number of potential wind park locations and connection points minimising the total operating and investment costs, while the lower problem describes the equilibrium flow of the network given the upper level investment decisions y.

Consider the network in Figure 4.1, where the solid edges represent the existing electricity transmission network ignoring existing generation units. The dotted arcs represent potential locations and connection points for three new wind power parks. It is assumed that exactly one connection point must be chosen for each wind park.

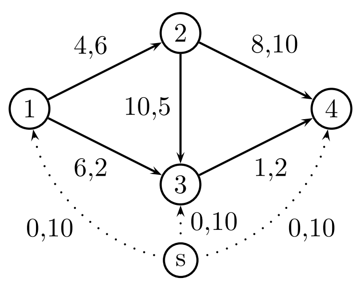

Figure 4.1: A small example of a network with three potential windpower parks and their connection (dotted edges) to the transmission network. Arc labels indicate cost c and capacity u, respectively of the arc flow x.

The model presented in chapter 8 allows for expansion of an electricity transmission network in systems with large-scale wind power. However, the model can easily be adapted to also identify optimal expansion plans for wind power generators in an integrated approach.

In this chapter we have shown how the modelling framework presented in chapter 3 can be applied to decision problems in energy transmission networks involving strategic or inter-temporal constraints. Some of these applications are discussed in more detail in Part II. The next chapter provides a summary and discussion of each of the chapters in Part II.

Chapter 5

Summary of Papers

This chapter summarises and discusses the papers presented in Part II. The first three papers are concerned with capacity planning of electricity networks assuming a centralised optimal dispatch, while the last three papers are concerned with pricing problems in deregulated energy markets.

In terms of energy carriers, the first four papers consider electricity only. Paper 5 considers natural gas only, while paper 6 considers transmission of electricity with hydro storage.

5.1 Investment in Electricity Networks with Tranmission Switching

Submitted to *European Journal of Operational Research*, 2011.

This paper presents the strategic transmission capacity and switch provisioning problem as a two-stage stochastic program minimising total investment cost and expected cost of power generation. First stage decisions determine an optimal investment strategy subject to uncertainty of demand and supply. In the second stage economic dispatch of power is determined assuming transmission elements may be switched if a switch is installed.

A Dantzig-Wolfe reformulation is presented and its integrality properties are discussed. The Dantzig-Wolfe reformulation leads to a column generation approach embedded in a branch and bound framework for finding provably optimal solutions. The approach is based on the approach provided by Singh et. al. [63] for stochastic capacity planning problems.

The solution approach is tested on the IEEE 118-bus and the IEEE 73-bus test cases. We were able to solve instances with up to 256 scenarios, which seems to be well beyond the capability of competing methods. In all but one instance the LP-relaxation of the Dantzig-Wolfe reformulation yields optimal integral solutions demonstrating the strength of the reformulation.

In most of the instances studied the number of active switches allowed in each scenario (that is, the number of transmission elements that may be switched out in any single scenario) are limited to at most $k \leq 3$. Increasing k leads to significantly more difficult sub problems to be solved in the column generation approach, which increases the solution time. Furthermore, one would expect that the nice integrality properties of the LP-relaxation reduce. On the other hand, when solving the compact (original) formulation using a commercial MIP solver (CPLEX) increasing k seems to help convergence of the branch and bound. However, this is a trend we have not explored — additional experiments are necessary to confirm this.

The column generation approach includes fixed charge network design as a subproblem and hence is $\mathcal{NP}$-hard [35]. Therefore, strong formulations of the subproblem is crucial in order to solve instances with many switches and many scenarios. In our experiments we applied cuts that would prohibit augmentation of the network into disconnected components by switching out transmission elements. In particular, the following cut was implemented

$$\sum_{a \in \mathcal{P}} (1 - z_a) \leq 1 \tag{5.1}$$

for any path $\mathcal{P}$ such that all nodes in $\mathcal{P}$ except the first and last node are two-connected. Constraints (5.1) provided some improvement for sparse networks and may be explored further.

The main contribution of the paper is on the applicability of a Dantzig-Wolfe reformulation to a strategic stochastic planning problem involving switching of an electricity network. The paper illustrates the strength of the reformulation by numerical examples and computational results on standard test cases.

5.2 Line Capacity Expansion in a Power System with Large-Scale Wind Power

Submitted to *IEEE Transactions on Power Systems*, 2011 and presented at the 19^{th} *Triennial Conference of the International Federation of Operational Research Societies*, 2011.

In this paper we apply the problem and solution methodology presented in paper 1, to the problem of finding optimal line capacity expansion and switch provisioning plans for electricity networks with large-scale wind power. While paper 1 was concerned with the methodological framework showing the computational efficiency of the Dantzig-Wolfe reformulation, this paper is concerned with the application to a real network and the value of switching — in particular in connection with wind power. Computational results are provided for the IEEE 118-bus network (only switch provisioning) and the projected Danish transmission network for year 2025 (line capacity expansion and switch provisioning).

Results for the IEEE 118-bus network with a single large wind power farm show actively switching the network allows for an increase in generation of wind power by 49 MW (or app. 10 %) in off-peak demand periods and 340 MW (or app. 64 %) in peak demand periods with maximum wind availability. With three equally sized wind power farms the corresponding increase in wind power is 430 MW (app. 23 %) in peak demand and 293 MW (app. 20 %) in off-peak demand periods. For the Danish network we see an increase of up to 187 MW (app. 3 %) in peak demand with maximum availability of wind, but no increase in off-peak periods when allowing to actively switch the transmission network. These results confirm our intuition that switching yields the highest benefits in periods where the network is congested.

More interestingly perhaps, the optimal transmission capacity expansion strategy is highly sensitive to the level of switching allowed in the network. For instance, for the Danish network the optimal strategy invests in 6 new lines when no switching is allowed; 10 new lines when only new lines may be switched; and 8 new lines when all new lines and one additional line may be switched at any point in time. Contrary to our expectations, introducing a switching regime generally increases the number of new lines in the optimal expansion plan.

The paper applies active switching to electricity networks with large-scale wind power. To our knowledge, it is the first time that the benefit of switched networks for the integration of large-scale wind power has been quantified. This is an important application in many parts of the world as wind power and other intermittent power sources are becoming increasingly more dominant in electric power systems.

5.3 Column Generation for Transmission Switching of Electricity Networks with Unit Commitment

Presented at *IAENG International Conference on Operations Research*, 2011, and published in the proceedings *Lecture Notes in Engineering and Computer Science*. The paper was awarded *Certificate of Merit*.

This paper considers the economic dispatch and unit commitment of power generators connected by a switchable electricity transmission network over a short planning horizon. The application departs from the previous two papers in that the problem is a deterministic short-term capacity planning problem. For this problem the capacity decisions refer to the decision to start-up a generation unit at a particular point in time. The paper derives a Dantzig-Wolfe reformulation based on a decomposition of the planning horizon which is quite similar to the one derived in paper 1.

Although integrality of the LP-relaxation of the Dantzig-Wolfe reformulation is not guaranteed, results for the IEEE 118-bus network yield optimal integral solutions without branching for limited switching.

The model presented here is rather simplified. Many constraints often included for unit commitment of power generators have been disregarded. This includes minimum and maximum up- and down time constraints, ramp rate constraints, and security constraints. Adding these constraints are likely to affect the computational complexity of the column generation scheme. We expect that up- and down time constraints will make the algorithm converge faster as the feasible space is limited considerably. In a column generation context, this means that once a *good* column (with low cost) is added to the restricted master problem, the shadow price for the up/down time constraints will impose a high cost on having units turned on or off in certain time periods. This is likely to improve convergence for the branch and bound algorithm used to solve the sub problem as well as reduce the optimality gap for the relaxed master problem.

The contribution of this paper is on the Dantzig-Wolfe reformulation of the simplified unit commitment problem with transmission switching. The more general problem has been treated before and solved using heuristics [31] and Benders decomposition [37]. However, computational results suggest that column generation for the Dantzig-Wolfe reformulation has potential in future algorithms yielding optimal or near optimal solutions for realistic data sets.

5.4 Modelling Zonal Pricing Design Under Uncertainty in Electricity Markets

Submitted, 2011.
Technical report, Technical University of Denmark, 2011.

In deregulated electricity markets, market prices are used to determine the dispatch of generation (and to a certain extent consumption). In some markets, such as the Nordic electricity market, zonal pricing is employed restricting the price for all generators within the same zone to be equal. In this paper we present the problem of optimally designing price zones in electricity markets with zonal pricing under uncertainty assuming a fixed number of zones. The deterministic non-linear version of this zonal design problem is presented in [8].

A novel formulation of the stochastic problem is presented in the form of a linear mixed integer two-stage stochastic program. In general, two-stage stochastic programs are #P-hard under the assumption that the stochastic parameters are independently distributed even if all variables are continuous [17]. Under the same assumption we show that the stochastic zonal design problem is also #P-hard. This motivates a Dantzig-Wolfe reformulation of the problem, as we hope that decomposing the problem will lead to a stronger formulation and a more efficient solution procedure. However, it remains to be shown that the reformulation does in fact lead to efficient solution algorithms. One may argue that for practical instances, the stochastic parameters are highly correlated. For instance, if uncertainty is due to the intermittency of wind generators.

Finally, we show that contiguous zones may be ensured by embedding a spanning forest formulation, so that each tree in the forest correspond to a zone. We show that with the same number of zones, ensuring contiguity of zones may yield solutions with higher cost of generation compared to the non-contiguous case.

To our knowledge, it is the first time a stochastic version of the zonal design problem is presented. Also, we are not aware of any previous formulations for this problem that ensures contiguous zones.

5.5 Capacity Expansion and Transmission Pricing in Natural Gas Networks

Working paper.

In this paper we combine strategic capacity planning decisions with pricing decisions at the tactical level (short to medium term) for a natural gas transmission network. This leads to a three stage stochastic program where strategic decisions regarding the expansion of the network are made in the first stage, while decisions regarding the level of transmission tariffs are made at the second stage after realisation of some of the stochastic parameters. The third stage models the economic dispatch of natural gas in the network. Strategic and tactical decisions are assumed to be made by a welfare maximising organisation, while dispatch of supply and consumption is made by an independent operator, who is not subjected to network constraints.

It is shown that different pricing strategies affect the social welfare of the system and the profit of the independent operator. Furthermore, expanding the capacity of the network may influence the optimal pricing strategy chosen. Hence, an integrated approach is needed.

To our knowledge it is the first time that the problem of determining optimal transmission tariffs for a natural gas network has been proposed. The main contribution of the paper is on the modelling framework integrating strategic capacity planning and tactical tariff decisions in a stochastic programming framework.

Since capacity decisions are often long-term strategic decisions, while market designs or regulatory frameworks may in principle be changed with very short notice, it may not be optimal to rely on a single specific market framework when choosing capacity investments. We may accommodate this by letting stage two model different market regimes.

5.6 Modelling Hydro-electric Power Producers Strategic Use of Water Reservoirs

Presented at the *8th Conference on Applied Infrastructure Research (INFRA-DAY)*, 2009.

In all the papers of Part II markets are assumed to be perfectly competitive. In this paper, however, we are concerned with the exercise of market power by hydro power suppliers in electricity transmission networks. The paper models an electricity market with oligopolistic power producers and storage of hydro energy.

A numerical example shows that market power of a single hydro-power supplier may lead to spilling of water (letting water run through the dam without producing power) and lower social welfare. Even though, power flows in the transmission network are not affected by market power, it leads to generally higher prices and a lower reservoir level at the end of the planning horizon compared to the case with perfect competition.

The non-linear, non-convex nature of this type of problems make them notoriously hard to solve. One option is to discretise the supply quantities and introduce conjunctive constraints for the equilibrium constraints as described in [24]. This leads to a mixed integer linear program. The advantage is that this allows for modelling additional discrete decisions. Unfortunately, discretising continuous variables may lead to sub-optimal solutions when the chosen discretisation is too coarse, while a fine discretisation leads to a MIP with a large number of integer variables, that may yield the problem intractable.

Chapter 6
Conclusion

This chapter concludes Part I. First a brief summary of the main content of the thesis is given. Then the main scientific contribution of the papers are highlighted, and finally, some ideas and directions for further research is provided.

6.1 Summary

This thesis is concerned with transmission pricing and capacity planning problems in energy tranmission networks. Various models are provided and efficient solution methods are suggested in many cases. A generic modelling framework is presented and several applications are suggested. Although the modelling framework applies generally to energy transmission networks most of the problems considered in this thesis describe applications to electricity transmission networks. Each of the applications involve the *design* or *configuration* of an optimal transmission network or transmission constrained energy market.

All of the applications involve finding the optimal dispatch and equilibrium flow in the transmission network in a number of discrete time periods or scenarios given *upper level* decisions taken by a social welfare maximising system operator anticipating the dispatch and network flow in all time periods or scenarios. In most of the applications the upper level decisions refer to capacity expansion or pricing decisions taken under uncertainty in a stochastic programming framework.

The special multi-period or multi-scenario structure of the problems considered is amenable to a decomposition by time period (scenario) due to the block structure of the corresponding mixed integer program. A Dantzig-Wolfe reformulation is provided for several of the problems leading to efficient solution procedures using column generation. The following briefly summarises the chapters of Part II.

In chapter 7 a stochastic capacity planning model for determining optimal transmission network expansion and switch provisioning in an electricity transmission network is presented. A Dantzig-Wolfe reformulation of the problem is provided and its integrality properties are discussed. The model is applied to the problem of investing in advanced switches under uncertainty allowing the transmission network to be switched in an operational context. Branch and price and column generation algorithms are used to obtain provably optimal solutions for two IEEE test networks with a large number of scenarios.

Chapter 8 considers the application of transmission switching to electricity networks with large-scale wind power. The model presented in chapter 7 is applied to two different networks. Firstly, the switch provisioning problem is considered on an IEEE network with 118 busses. These results indicates that switching increases the throughput of wind power in particular in congested networks. Secondly, the projected Danish transmission network for 2025 with large amount of installed wind power capacity and potential line capacity expansions is considered. Results for the Danish network show that switching may reduce curtailment of wind power in periods with peak demand and high wind power significantly.

Chapter 9 presents a simplified unit-commitment problem for power generation units in a switchable electricity transmission network. Whereas the preceeding two chapters considers strategic capacity planning under uncertainty, the problem considered here is a short-term, deterministic capacity planning problem determining optimal commitment patterns for generating units. A Dantzig-Wolfe reformulation of the problem is provided and applied to an IEEE test case.

In the first three chapters of Part II we looked at capacity planning problems in electricity networks. We did not make any assumptions regarding the type of market setting — or rather, we took a central planning approach assuming that the behaviour of individual agents will not deviate significantly.

In chapter 10 we treat the problem of finding a partition of the transmission nodes into price zones in an electricity market employing zonal pricing. Since, the zonal design must be static we embed the problem into a two-stage stochastic programming framework. A Dantzig-Wolfe reformulation for the problem is also provided.

Chapter 11 combines capacity plannning and transmission pricing design for a natural gas transmission network with entry-exit tariffs. A three-stage stochastic model is presented in which first stage decisions model investments in pipeline capacity, second stage model pricing decisions, while the operational dispatch of natural gas is modelled in the third stage.

6.2 Main Contributions

The main contributions of this thesis are highlighted below. The contributions fall into three categories: Novel model formulations, new applications of existing models, and efficient solution methods and model reformulations.

Three novel model formulations are described and discussed.

- A formulation of the network capacity expansion problem with switch provisioning under uncertainty is provided as a two-stage stochastic mixed integer program.
- We give a linearised stochastic version of the zonal pricing problem and introduce a clustering method based on a minimum spanning forest formulation to ensure spatially contiguous zones.
- An integrated approach is presented for determining an optimal pipeline capacity expansion strategy and transmission pricing decisions in natural gas transmission networks with entry/exit tariffs.

Transmission switching in electricity networks has given rise to some interesting results.

- Switching principles are applied to electricity networks with large-scale supply of wind power. We have shown that introducing switching may reduce curtailment of wind power, especially in peak demand periods and may support integration of large supplies of wind power into existing electricity transmission systems.
- When considering expansion of electricity transmission networks by constructing new lines, introducing a regime where transmission lines may be switched alters the optimal line capacity expansion plan considerably. The results suggest that allowing to switch new lines increases the number of lines to be installed, while allowing to switch existing lines has the opposite effect. However, this should be confirmed on more test instances.

Eventhough the data for these problems have been selected carefully, the results should not be seen as a recommendation for introducing active switching, but rather as an indication of a promising area for research and development within the industry. The model does not capture for instance security and stability issues. Also, the scenarios described are rather coarse — capturing only major fluctuations. Hence, further analyses are needed.

For several of the problems considered a Dantzig-Wolfe reformulation enables efficient solution methods.

- We derive a Dantzig-Wolfe reformulation of the stochastic network capacity expansion problem with switch provisioning and show that this gives rise to efficient column generation and branch and price algorithms. These algorithms are tested on two IEEE test networks as well as the Danish transmission network showing superiority of the algorithms over standard branch and bound on the original formulation. For the switch provisioning problem we are able to solve instances with up to 256 scenarios in less than 9 hours, which seems to be well beyond the capability of competing methods.

- A Dantzig-Wolfe reformulation for the simplified unit-commitment problem with network constraints and transmission switching is provided. Computational results seem promising.

6.3 Some Ideas for Future Research

In this section we outline some interesting ideas and directions for future research.

Long term stochastic capacity planning problems usually involve several capacity expansion decisions to be taken over an extended period of time (the planning horizon). The two-stage stochastic capacity planning models presented in this thesis may be naturally extended to multi-stage models where each stage refers to a particular point in time in a discretised planning horizon.

Multi-stage formulations may also be used to ensure that the topology of the switched network is $N-1$ reliable in any scenario. This would result in a three-stage stochastic program in which investment decisions are taken in the first stage, switching decisions in a number of scenarios in the second stage, while redispatch decisions are taken in the third stage representing contigencies corresponding to failure of each of the transmission lines.

Similarly, one can extend the two-stage stochastic capacity planning problem presented in chapter 7 with unit-commimtment decisions over a number of time periods in each scenario. In this way the operational dispatch problem for each scenario is itself a multi-time period problem, that may be solved using column generation as outlined in chapter 9 in a nested approach.

The network capacity expansion and switch provisioning problem is described for electricity transmission networks in chapter 7 and 8. The approach may also be applied to distribution networks, in which lines are required to be switched into a radial topology in each scenario.

The scenarios discussed in chapter 8 are greatly simplified. The relative wind power capacity is uniformly spread geographically, so that all off-shore wind power plants has the same capacity coefficients across the network in each scenario. The power of switchable networks stems from the ability to provide a network topology that is optimised for the generation and demand patterns at any point in time. It would be interesting to include in the analysis scenarios that have different levels of wind in different parts of the network. For instance, one may construct two scenarios; one with high wind capacity in the eastern part of the network and no wind in the western part and vice versa. Two interesting questions arise: *Does the benefit from transmission switching increase when considering more versatile scenarios?* and *how does this influence solution times and the LP-integrality of the Dantzig-Wolfe reformulation?*

The model considered in chapter 9 is rather simplified. Furhter effort should be directed at introducing additional constraints to make the formulation more realistic. These include ramp rate constraints, minimum and maximum up- and down time of generation units, and security constraints. See e.g. [31] for a description of these constraints. Inclusion of these additional constraints are likely to influence the proposed column generation algorithm as they further couple time periods. However, it is yet to be seen if they have a positive or negative effect on the convergence of the algorithm. Also, solutions to the Dantzig-Wolfe reformulation of the unit commitment problem are not guaranteed to be optimal. Embedding the column generation in a branch and bound framework would make it possible to achieve optimal integral solutions in general.

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Part II

Scientific Papers

Chapter 7

Investment in Electricity Networks with Transmission Switching

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We consider the application of Dantzig-Wolfe decomposition to stochastic integer programming problems arising in the capacity planning of electricity transmission networks that have some switchable transmission elements. The decomposition enables a column-generation algorithm to be applied, which allows the solution of large problem instances. The methodology is illustrated by its application to a problem of determining the optimal investment in switching equipment and transmission capacity for an existing network. Computational tests on IEEE test networks with 73 nodes and 118 nodes confirm the efficiency of the approach.

Keywords: Stochastic programming, Electricity capacity planning, Transmission switching, Column generation, Branch and price

7.1 Introduction

In this paper we consider economic dispatch models for wholesale electricity supply through an AC transmission network as discussed in e.g. [4]. These models typically make use of a DC-load flow assumption in which reactive power is ignored, line resistance is assumed to be small in comparison to reactance, and voltage magnitudes are treated as constant thoughout the system. In such models, Kirchhoff's laws are used to determine the flow on each line. The *voltage* law states that power flow on a transmission line is proportional to the difference in voltage phase angles at each endpoint, and the *current* law states that the total power flowing from the network into any location matches the demand minus supply at this point. Thus, given the optimal dispatch and demand for a tree network, the power flow is uniquely determined by the current law. The voltage phase angles that generate this flow can be uniquely determined up to an additive constant by applying the voltage law.

Most electricity transmission networks are designed as meshed networks (with cycles) for security reasons, so that if any line fails, the power can still flow from source to destination by alternative paths. When the network contains cycles, the voltage law and current law must be applied simultaneously to determine the line flows and voltage angles from the dispatch of flow and generation. The presence of cycles places additional constraints on the line flows that are absent in tree networks. In particular, for each cycle in a network the sum of voltage angle differences (with respect to the direction) around the cycle must equal zero. Hence, each cycle in the network gives rise to one additional constraint on the line flows. This leads to a paradox (see e.g. [2]) in which adding a new line to a transmission network might increase the cost of supplying electricity, even if the cost of the line itself is zero.

Based on these observations, it is easy to see that it may be beneficial in mesh networks to take some lines out of operation — to either decrease system cost or increase reliability [10, 17]. The process of taking out lines and bringing them back in is done by opening (respectively closing) a switch at the end of the line and is referred to as *switching*.

Recent interest in renewable intermittent energy sources and the call for intelligent transmission networks or smart grids have spurred a renewed interest in switching problems. Fisher et. al. presents in [8] the problem of optimal switching of transmission elements in an electricity transmission network to minimize the delivered cost of energy. They propose a mixed-integer program to solve the DC-loadflow economic dispatch model with switching decisions in a single time period. They note that the problem is NP-hard. Results are provided for a 118-node network with 186 transmission lines. Hedman et. al. [12] extends the model to consider reliability of the network. Reliability constraints are added to the problem to ensure that any line failure will not lead to an infeasible dispatch of generation. They note that in some cases adding reliability constraints increases the value of switching.

In [13] Hedman et. al. discuss a decomposition algorithm to solve the transmission switching problem with unit commitment decisions made heuristically over 24 time periods. It is noted that adding transmission switching may yield a cheaper unit commitment plan than what could be achieved without switching. In this model, it is assumed that a technology is available that makes it possible to switch lines instantaneously. That is, a line may be switched automatically from one moment to the next without delay. In this case, switching out lines will (in theory) not affect system security (disregarding failures on switching equipment), since all lines may be switched back in immediately, in case of any failure in the system. Khodaei and Shahidehpour [14] describe a Benders decomposition of the security constrained unit commitment problem with transmission switching that outperforms an integrated MIP-model, and Khodaei et. al. [15] provide a Benders decomposition approach for solving capacity expansion problems in electricity networks with active transmission switching.

The solution of the large-scale mixed-integer programming problems that arise when switching is considered remains a challenging obstacle to their implementation in practice. Most of the literature in this area has focused on demonstrating the savings in cost that can be made by transmission switching, while acknowledging that there are still computational hurdles to be overcome when solving large real-life instances. Fisher et. al. [8] were unable to prove optimality of transmission switching in the IEEE 118-bus network with a single scenario and unrestricted number of open lines. The heuristic approach presented by Hedman et. al. [13] for the transmission switching and unit commitment problem with security constraints is unable to prove optimality for the IEEE 73-bus network over 24 time periods — even with extensive computer resources. Khodaei and Shahidehpour [14] limits the space of switchable lines to find solutions to the security constrained unit commitment problem with transmission switching using Benders decomposition. Even when the single-scenario problems are restricted to allowing a small number of switches, these are sufficiently hard to make a multi-period or multi-scenario model intractable.

Making it possible to switch lines instantaneously often requires that some hardware is installed in the network. Firstly, a switch needs to be installed at the line. Secondly, communications equipment between the switch and operating control center is required to ensure automatic remote operation of the switch. Moreover, the ability to profitably switch lines out might be enhanced by adding new transmission lines to the network to absorb increases in flow.This leads to a two-stage stochastic integer programming problem of determining an optimal capital provisioning plan that will satisfy demand almost surely at least expected cost. Note, that even though the fixed cost of enabling a line to be switched instantaneously may be small (e.g. if the switch is already present and only communication equipment needs to be installed) it may not be worthwhile to enable switching on all lines (unless this cost is 0 for all lines), since some lines may never be switched.

In this paper we show how one can attack the stochastic capital provisioning problem using Dantzig-Wolfe decomposition [6] and column generation to give provably optimal or close to optimal solutions. Our approach is based on the approach of Singh et al [19] for determining optimal discrete investments in the capacity of production facilities. They proposed a split-variable formulation and Dantzig-Wolfe reformulation resulting in a sub-problem for each node in the scenario tree, and showed how this could enable the solution of previously intractable instances of capacity planning problems for electricity distribution networks. Our contribution in this paper is to show how this methodology applies to a transmission switching model, to enable their solution in settings where there are many scenarios representing future uncertainty. With a limitation on the number of switches used in each scenario, the decomposition approach enables us to solve IEEE test problems with up to 256 scenarios, which appears to be well beyond the capapility of competing methods.

We begin the paper by recalling a mixed-integer programming formulation for transmission switching based on the model in [8]. In section 7.3, we address the problem of the planning of transmission networks under uncertainty considering both installation of switches and line capacity expansions. In particular, we consider a two-stage stochastic program in which the first-stage decisions concern the investments in switch equipment and line capacity, while the second stage models operational decisions in different scenarios. The model is reformulated using Dantzig-Wolfe decomposition, and solved using column generation. In section 4 we study the structure of the master problem in order to provide some insights into the strength of the decomposition. We show that the master problem has naturally integer optimal solutions in some circumstances, and provide counter examples where this is not true. Computational results of the method applied to two standard test problems (the IEEE 73-bus network and the IEEE 118-bus network) are presented in section 7.5. We then draw some general conclusions about the effectiveness of the approach.

7.2 Optimal Transmission Switching

We model the electricity transmission system as a network where N denotes the set of nodes (or busses) and A denotes a set of arcs representing transmission lines (and transformers) connecting the nodes. Let $\mathcal{T}(i)$ denote the set of arcs incident with node i where i is the head of the incident arc, and let $\mathcal{F}(i)$ denote the set of arcs incident with node i, where i is the tail of the incident arcs. So an arc in $\mathcal{F}(i) \cap \mathcal{T}(j)$ is directed *from* node i *to* node j. Since power flow can flow in both directions in a transmission line we allow these flows to take negative values, indicating power flow in the opposite direction from the arc direction.

Many transmission systems consist of alternating current circuits, interlinked by high voltage direct current links. We shall ignore these interconnections in this paper, and assume that all lines carry alternating current. The methodologies can easily be adapted to treat direct current lines as special cases. Note, that even though we assume all lines to be alternating current lines, the models presented are based on the linear direct current optimal power flow approximation as discussed in the introduction.

Let G be the set of all generating units, where $G(i)$ is the set of generating units located in (and supplying electricity to) node i. For simplicity, we assume that each unit $g \in G$ offers its entire electricity capacity u_g to the system at its marginal cost c_g. (A model in which each unit offers a step supply curve is a straightforward extension.) We denote by q_g the dispatch of power of unit g.

At each node i the demand d_i must be met. Load shedding at node i may be modelled by introducing a dummy generator at each node offering d_i at a penalty price.

Each transmission line $a \in A$ is characterised by its reactance X_a and thermal capacity K_a. The flow on line a is denoted P_a, which can be negative in order to model power flows in the direction opposite to the orientation of a.

A subset of lines $S \subseteq A$ are considered to be switchable. Lines that are switchable may be taken out of operation in any given period of time. For each line $a \in S$, $z_a = 1$ denotes that the line has been switched out (opened), while $z_a = 0$ denotes that the switch is closed.

The economic dispatch problem of finding the minimum cost optimal DC-load flow may now be formulated as

$$
\begin{aligned}
\text{EDP:} \quad & \text{minimize} \sum_{g \in G} c_g q_g && && (7.1)\\
\text{s.t.} \quad & 0 \le q_g \le u_g, && g \in G && (7.2)\\
& z_a = 0 \ \Rightarrow \ X_a P_a - \theta_i + \theta_j = 0, && a = (i,j) \in A && (7.3)\\
& \sum_{g \in G(i)} q_g - \sum_{a \in \mathcal{F}(i)} P_a + \sum_{a \in \mathcal{T}(i)} P_a = d_i, && i \in N && (7.4)\\
& -K_a(1 - z_a) \le P_a \le K_a(1 - z_a), && a \in A && (7.5)\\
& \sum_{a \in A} z_a \le k, && && (7.6)\\
& z_a = 0, && a \in A \setminus S && (7.7)\\
& z_a \in \{0, 1\}, && a \in S && (7.8)
\end{aligned}
$$

The objective (9.1) minimizes the total generation costs respecting generation capacities (9.2), flow conservation (9.3), and thermal line capacity (9.4). For lines that are not switched out, Kirchhoff's voltage law must be respected (9.5). Furthermore, we only allow k lines to be switched simultaneously (9.8) and only lines in S are switchable (7.7). Finally switching decisions are binary (7.8).

Note, that constraint (9.5) may be linearised using a big-M construction

$$
-Mz_a \le X_a P_a - \theta_i + \theta_j \le M z_a, \qquad a = (i,j) \in A \tag{7.9}
$$

where M is some sufficiently large number. To give a strong linear programming relaxation, lower values of M are better. The choice of an appropriate value of

M is discussed in [1], who observe that the difference in voltage angles between any two nodes i and j is bounded by

$$M_{ij} = \max\{\sum_{a \in R} X_a K_a \mid R \text{ is a path of edges joining } i \text{ and } j\}$$

and so choosing $M = \max_{i,j} M_{ij}$ will give the smallest value in general. This poses some difficulty in practice, since the computation of M_{ij} is a hard problem, and so its use is restricted to small networks where it can be found by enumeration (see [1]). The approach taken in [13] imposes a uniform bound on the magnitude of the voltage phase angle of 0.6 radians. This constraint allows a value of $M = 1.2$ to be chosen.

7.3 Switch and transmission provision under uncertainty

We now consider the problem of installing switches and new lines in an electricity transmission network to minimize the capital cost and expected operating cost averaged over a number of scenarios denoted $\omega \in \Omega$. In each scenario ω we have a realization of demand $d(\omega)$ and generation cost $c(\omega)$ and generation capacity $u(\omega)$. This enables us to vary parameters according to climatic conditions (e.g. high costs could model shortage of water in hydro stations, and low capacity model low wind outcomes for wind farms). We assume that transmission switching and economic dispatch is carried out after these random outcomes are realized. In each scenario ω we have the switching and dispatch problem:

$$\begin{aligned}
\text{EDP}(\omega): \quad & \text{minimize} \sum_{g \in G} c_g(\omega) q_g && && (7.10)\\
\text{s.t.} \quad & 0 \le q_g \le u_g(\omega), && g \in G && (7.11)\\
& X_a P_a - \theta_i + \theta_j + M z_a \ge 0, && a = (i,j) \in A && (7.12)\\
& X_a P_a - \theta_i + \theta_j - M z_a \le 0, && a = (i,j) \in A && (7.13)\\
& \sum_{g \in G(i)} q_g - \sum_{a \in \mathcal{F}(i)} P_a + \sum_{a \in \mathcal{T}(i)} P_a = d_i(\omega), && i \in N && (7.14)\\
& -K_a(1 - z_a) \le P_a \le K_a(1 - z_a), && a \in A && (7.15)\\
& \sum_{a \in A} z_a \le k, && && (7.16)\\
& z_a = 0, && a \in A \setminus S && (7.17)\\
& z_a \in \{0,1\}, && a \in S. && (7.18)
\end{aligned}$$

We now consider the problem of installing switches and new transmission lines prior to the realization of ω. We assume that a fixed cost is associated with installing switching equipment at each line and that this cost covers all the actual costs of making it possible to perform instantaneous switching of that particular line. Furthermore, we assume a fixed cost of installing new lines from a fixed set of possible line expansions.

This gives a two-stage stochastic model, where the first-stage decisions involve investments in switching equipment y_S and line capacity y_L, while the second-stage problem EDP(ω) models operational decisions (q, P, θ, z) for dispatch and switching in each scenario ω occuring with probability $p(\omega)$. For each scenario $\omega \in \Omega$, let

$$\mathcal{Q}(\omega) = \{(q, P, \theta, z) \mid (7.11\text{-}7.18)\}.$$

The model may now be formulated as

$$\min \; f_S^\top y_S + f_L^\top y_L + \sum_{\omega \in \Omega} p(\omega) c(\omega)^\top q(\omega) \tag{7.19}$$

$$\begin{aligned}
\text{s.t.} \quad & y_L - y_S + z(\omega) \leq 1, && \omega \in \Omega && (7.20)\\
& y_L + z(\omega) \geq 1, && \omega \in \Omega && (7.21)\\
& (q(\omega), P(\omega), \theta(\omega), z(\omega)) \in \mathcal{Q}(\omega), && \omega \in \Omega && (7.22)\\
& y_L, y_S \in \{0,1\}^{|A|}. && && (7.23)
\end{aligned}$$

The capital costs f_S and f_L are amortized to give a per period capital charge that is traded off against the expected economic dispatch cost per period, as expressed by objective (9.11). We set $e_a^\top y_L = 1$ and $e_a^\top f_L = 0$ for existing lines. The constraints (9.12) ensure that switching of installed lines is only possible if a switch is also installed. Constraints (8.12) allow lines to be switched in only if they have non-zero capacity. Note, that not installing a line corresponds to having the line switched out (i.e. $z(\omega) = 1$) in all scenarios $\omega \in \Omega$.

We can decompose (9.11)-(8.14) following the approach in [19]. The idea is to decompose the stochastic problem into a master problem and a number of subproblems — one for each scenario. We let the binary vector $z(\omega)$ define a *feasible switching plan* (FSP) for scenario ω if there exists $q(\omega), P(\omega), \theta(\omega)$ such that $(q(\omega), P(\omega), \theta(\omega), z(\omega)) \in \mathcal{Q}(\omega)$. Now, let $Z(\omega) = \{\hat{z}^j(\omega) | j \in J(\omega)\}$ be the set of all FSP's for scenario ω, where $J(\omega)$ is the index set for $Z(\omega)$. We can write any element in $Z(\omega)$ as

$$\begin{aligned}
z(\omega) &= \sum_{j \in J(\omega)} \varphi^j(\omega) \hat{z}^j(\omega)\\
\sum_{j \in J(\omega)} \varphi^j(\omega) &= 1, \quad \varphi^j(\omega) \in \{0,1\}, \; \forall j \in J(\omega).
\end{aligned}$$

Assume that for each feasible switching plan $\hat{z}^j(\omega)$ the corresponding optimal dispatch of generation and load shedding is given by $\hat{q}^j(\omega)$. The master problem can now be written in terms of $\hat{z}$ and $\hat{q}$ as

$$\text{MP: } \min \; f_L^\top y_L + f_S^\top y_S + \sum_{\omega\in\Omega}\sum_{j\in J_\omega} p(\omega)c(\omega)^\top \hat{q}^j(\omega)\varphi^j(\omega) \tag{7.24}$$

$$\begin{aligned}
\text{s.t.} \quad y_L - y_S + \sum_{j\in J(\omega)} \hat{z}^j(\omega)\varphi^j(\omega) &\le 1, && [\pi(\omega)]\,,\ \omega\in\Omega && (7.25)\\
y_L + \sum_{j\in J(\omega)} \hat{z}^j(\omega)\varphi^j(\omega) &\ge 1, && [\rho(\omega)]\,,\ \omega\in\Omega && (7.26)\\
\sum_{j\in J(\omega)} \varphi^j(\omega) &= 1, && [\mu(\omega)]\,,\ \omega\in\Omega && (7.27)\\
\varphi^j(\omega) &\in \{0,1\}, && j\in J(\omega) && (7.28)\\
y_L, y_S &\in \{0,1\}^{|A|} && && (7.29)
\end{aligned}$$

where $\mu(\omega)$, $\pi(\omega)$ and $\rho(\omega)$ denote the dual prices associated with the respective constraints.

The master problem MP is a two-stage stochastic integer program with integer variables in both stages. Although in general these are difficult to solve, the structure of MP is such that integer extreme point solutions are common. To help understand the reasons for this we examine some special cases of MP in the following section.

7.4 The structure of MP

In this section we investigate the structure of MP. We first assume that we do not install new lines, so that $e_a^\top y_L = 1$, $a \in A$. This simplifies MP since the constraints (7.26) can be removed from the formulation. The constraints (7.25) become

$$-y_S + \sum_{j\in J(\omega)} \hat{z}^j(\omega)\varphi^j(\omega) \le 0.$$

Suppose now that there is at most one switch allowed in each scenario. The master problem matrix with $|S|$ possible locations for switches takes the form

$$A = \begin{bmatrix} -I & I & & & & & & \\ -I & & I & & & & & \\ \vdots & & & \ddots & & & & \\ -I & & & & I & & & \\ & e^\top & & & & 1 & & \\ & & e^\top & & & & 1 & \\ & & & \ddots & & & & \ddots \\ & & & & e^\top & & & & 1 \end{bmatrix}$$

where I is the $|S| \times |S|$ identity matrix and $e \in \{0,1\}^{|S|}$ is a vector of 1's, and there is a copy of $-I$ and I for each scenario $\omega \in \Omega$. If switches are permitted only on a small subset of lines then we are guaranteed an integer optimal solution to MP.

Proposition 2 *If $|S| \leq 2$ then A is totally unimodular.*

Proof. If $|S| = 1$, then $A^\top$ (after multiplying its last $|\Omega|$ rows by -1) is a node-arc incidence matrix which is totally unimodular. For the case $|S| = 2$, we use the fact that the total unimodularity of $\begin{bmatrix} L & M \end{bmatrix}$ implies that $\begin{bmatrix} L & M \\ 0 & I \end{bmatrix}$ is totally unimodular. It suffices to show that the transpose of the first $2\,(|\Omega|+1)$ columns of A is totally unimodular. This matrix is

$$B = \begin{bmatrix} -I & -I & \dots & -I & & & & \\ I & & & & e & & & \\ & I & & & & e & & \\ & & \ddots & & & & \ddots & \\ & & & I & & & & e \end{bmatrix}$$

which can be transformed into a node-arc incidence matrix by multiplying the first row of B by -1, and then multiplying by -1 each row of B corresponding to the first row in each occurrence of I. ■

For larger values of $|S|$, we cannot guarantee that A is totally unimodular, even if only at most one switch is allowed in each scenario.

Example 1

Consider the network in Figure 1.

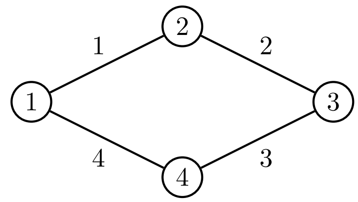

Figure 7.1: Network for Example 1, showing node and line indices. We seek optimal switch investments on lines 1, 2, and 3

Suppose that all lines have equal reactance and lines 1, 2, and 3 have capacity 5 while line 4 has capacity 1. Suppose that there are three scenarios $\omega = 1, 2, 3$. In scenario ω, zero cost power of 5 units is available at node ω and there is a demand of 5 and unlimited power at cost 2 at node $\omega + 1$ (one could imagine these being different wind scenarios). We consider installing switches on lines 1, 2, and 3, each with a cost of 1.

In scenario 1, without any switches we can only send 4 units from 1 to 2 through the network (3 directly from 1 to 2 and 1 unit from 1 to 4 to 3 to 2). Given the extra unit of generation required at node 2, this has cost 2, which is more expensive than switching out either lines 2 or 3 in this scenario, enabling 5 units to be sent directly from 1 to 2 at zero cost. If we switch out line 1, then we can send only one unit and the cost of generating the shortfall is 8.

The other scenarios are essentially the same. In scenario 2, we can switch out lines 1 or 3 to get a zero cost dispatch, and in scenario 3, we can switch out lines 1 or 2 to get a zero cost dispatch. Note that line 4 is unable to be switched.

If we consider the single switch options in each scenario, then we get a master problem constraint matrix of the following form:

$$A = \begin{bmatrix}
-1 & & & 1 & & & & & & & & & & & \\
& -1 & & & 1 & & & & & & & & & & \\
& & -1 & & & 1 & & & & & & & & & \\
-1 & & & & & & 1 & & & & & & & & \\
& -1 & & & & & & 1 & & & & & & & \\
& & -1 & & & & & & 1 & & & & & & \\
-1 & & & & & & & & & 1 & & & & & \\
& -1 & & & & & & & & & 1 & & & & \\
& & -1 & & & & & & & & & 1 & & & \\
& & & 1 & 1 & 1 & & & & & & & 1 & & \\
& & & & & & 1 & 1 & 1 & & & & & 1 & \\
& & & & & & & & & 1 & 1 & 1 & & & 1
\end{bmatrix}$$

This is not totally unimodular. Moreover MP has a fractional solution given by

$$\begin{aligned} y_S &= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}^\top \\ \varphi(1) &= \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}^\top \\ \varphi(2) &= \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}^\top \\ \varphi(3) &= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}^\top \end{aligned}$$

This corresponds to installing half a switch on each of lines 1,2, and 3, giving a total cost of $\frac{3}{2}$. The optimal integer solution will place a switch on any two of the lines to give total cost of 2.

We now look at the case where we have only two scenarios but more than one binary decision variable in MP. This means that we now allow more than one switch to occur in each scenario.

Example 2

Consider the following two-stage two-scenario switch investment problem with two investment options, that may be chosen in the first stage only. As in Example 1, we also consider investment only in switches. That is we assume $e_a^\top y_L = 1$ for all arcs a in A as before. If we enumerate all the possible switching plans, these make up our columns of switch requests that may be chosen for each scenario. The corresponding constraint matrix is as follows:

$$A = \begin{bmatrix} -1 & & 1 & 1 & & & & & & \\ & -1 & 1 & & 1 & & & & & \\ -1 & & & & & & 1 & 1 & & \\ & -1 & & & & & 1 & & 1 & \\ & & 1 & 1 & 1 & 1 & & & & \\ & & & & & & 1 & 1 & 1 & 1 \end{bmatrix}$$

where the first four rows correspond to the capacity constraints (7.25) and the last two rows corresponds to the convexity constraints (7.27). Columns 3 to 6 (respectively 7 to 10) represent feasible switching patterns in scenario 1 (respectively 2). Note that the submatrix consisting of the first five rows and columns 1, 2, 4, 5, and 7 has determinant 2. Assume, that the vector of cost coefficients is represented by $c = (3, 3, 10, 0.9, 1, 10, 1.5, 10, 10, 7.3)$. Then the optimal integer solution minimizing $c\begin{pmatrix} y_S \\ \varphi \end{pmatrix}$ is

$$\left(y_S^\top, \varphi^\top\right)_{IP}^* = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

yielding a cost of 8.4, while the optimal LP relaxed solution

$$\left(y_S^\top, \varphi^\top\right)_{LP}^* = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

yields the (slightly) lower value of 8.35.

The third column in A represents a request for switching capacity on both lines in scenario 1 incurring an operational cost of 10. So, in our example, switching both lines would incur a high operational cost in scenario 1, but a very low cost in scenario 2. Also, in scenario 1, switching exactly one line incurs a much smaller cost. The result is that the solution to the relaxed master problem saves enough by installing only half a switch on each line to compensate for the extra operating cost that is incurred by only admitting half the operational benefits of switching to accrue. This results in a fractional optimal solution.

Although the examples above show how fractional optimal solutions to the master problem might arise, in practice we obtain fractional solutions to MP very rarely. We conjecture that this is because it is unlikely with realistic electricity network data that the symmetrical situations as in the examples above will generate subproblem solutions with the specific cost structure needed to give fractions at optimality.

Fractional solutions are also prevented by the fact that MP has inequality constraints rather than equations. Observe in our model that every possible line expansion involves a switch as well. This means that the use of the line in any dispatch scenario is optional. As we have observed above, the mandatory use of an additional line might increase the dispatch cost in some scenarios. If such a situation occurs then a fractional expansion, that trades off good and bad dispatch outcomes, might become more likely.

If this does occur then we need to apply a branch-and-price procedure. This is easy to implement owing to the following result.

Proposition 3 *If y_L and y_S are chosen to be fixed vectors of binary integers, then the linear programming relaxation of MP has integer extreme points.*

Proof. When y_L and y_S are fixed, the constraints of the linear programming relaxation of MP decouple by scenario to give

$$\sum_{j \in J} \varphi^j = 1, \tag{7.30}$$

$$y_L - y_S + \sum_{j \in J} \hat{z}^j \varphi^j \leq 1, \tag{7.31}$$

$$y_L + \sum_{j \in J} \hat{z}^j \varphi^j \geq 1, \tag{7.32}$$

$$\varphi^j \geq 0, \tag{7.33}$$

where we suppress the dependence on ω for notational simplicity. Let

$$\begin{aligned} I_0 &= \{i \mid e_i^\top y_L = 0\}, \\ I_1 &= \{i \mid e_i^\top y_L = e_i^\top y_S = 1\}, \\ I_2 &= \{i \mid e_i^\top y_L = 1, e_i^\top y_S = 0\}. \end{aligned}$$

This gives constraints

$$-e_i^\top y_S + e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \le 1, \quad i \in I_0 \tag{7.34}$$

$$e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \ge 1, \quad i \in I_0 \tag{7.35}$$

$$e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \le 1, \quad i \in I_1 \tag{7.36}$$

$$e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \ge 0, \quad i \in I_1 \tag{7.37}$$

$$e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \le 0, \quad i \in I_2 \tag{7.38}$$

$$e_i^\top \sum_{j \in J} \hat{z}^j \varphi^j \ge 0, \quad i \in I_2 \tag{7.39}$$

$$\sum_{j \in J} \varphi^j = 1, \tag{7.40}$$

$$\varphi^j \ge 0, \tag{7.41}$$

Constraints (7.34) and (7.36) are dominated by (7.40) and can be removed. Similarly constraints (7.37) and (7.39) are redundant.

Constraint (7.35) must be satisfied as an equation and subtracting from (7.40) implies that $\varphi^j = 0$ for all columns j with $e_i^\top \hat{z}^j = 0$, for some $i \in I_0$. Similarly constraint (7.38) implies $\varphi^j = 0$ for all columns j with $e_i^\top \hat{z}^j = 1$, for some $i \in I_2$. Let

$$Z = \{j \in J \mid e_i^\top \hat{z}^j = 0 \text{ for some } i \in I_0, \text{ or } e_i^\top \hat{z}^j = 1, \text{ for some } i \in I_2\}.$$

Then $\varphi^j = 0$ for all columns $j \in Z$, which may be removed. This results in the system

$$\begin{aligned} e_i^\top \sum_{j \in J \setminus Z} 1 \varphi^j &= 1, \quad i \in I_0, \\ e_i^\top \sum_{j \in J \setminus Z} 0 \varphi^j &\le 0, \quad i \in I_2, \\ \sum_{j \in J \setminus Z} \varphi^j &= 1, \\ \varphi^j &\ge 0, \end{aligned}$$

which clearly has integer extreme points. ■

This means that any branch-and-price scheme may branch in the master problem simply by imposing constraints on the variables y_L and y_S. In other words to solve this problem, we do not need to construct a specific constraint-branching methodology, where columns generated on each side of the branch are constrained to meet certain conditions (see e.g. [16]).

It is convenient to consider only a subset $Z(\omega)' \subseteq Z(\omega)$ of feasible switching plans for each scenario ω in the master problem. We define this restricted master problem (RMP) by (8.15) - (8.20) with $J(\omega)$ replaced by $J(\omega)'$ the index set of $Z(\omega)'$. A column generation algorithm is applied to dynamically add feasible switching plans to the linear relaxation of the master problem. The algorithm is initialised by letting $Z(\omega)' = \{\hat{z}^0(\omega)\} = \{\mathbf{0}\}$, for all scenarios $\omega \in \Omega$. That is, initially no line may be switched out in either scenario. The corresponding operational costs $c(\omega)^T \hat{q}^0(\omega)$ can easily be found by solving a linear program for each scenario. In each iteration of the algorithm, the linear relaxation (RMP-LP) of RMP is solved yielding the dual prices μ, π, and ρ. A new column $(p(\omega)c(\omega)^\top \hat{q}^j(\omega), 1, \hat{z}^j(\omega))$ may improve the solution of RMP-LP if and only if the associated reduced cost $\bar{c}(\omega) = p(\omega)c(\omega)^\top \hat{q}^j(\omega) + \pi(\omega)^\top \hat{z}^j(\omega) - \rho(\omega)^\top \hat{z}^j(\omega) - \mu(\omega)$ is negative.

A column for scenario ω may therefore be constructed by solving the subproblem:

$$\begin{array}{ll} \min & p(\omega)c(\omega)^\top q + \pi(\omega)^\top z - \rho(\omega)^\top z - \mu(\omega) \\ \text{s.t.} & (q, P, \theta, z) \in \mathcal{Q}(\omega), \end{array}$$

where $\pi(\omega)$, $\rho(\omega)$, and $\mu(\omega)$ are the dual prices returned from RMP-LP.

Any feasible solution $(q, P, \theta, z) \in \mathcal{Q}(\omega)$ with negative objective function gives rise to a potential candidate column for RMP-LP. If no columns with negative reduced cost exist then we have solved the relaxed master problem (MP-LP) to optimality. Furthermore, if the solution (φ^*, y^*) to MP-LP is integral then (φ^*, y^*) is an optimal solution to the master problem (8.15) - (8.20) and y^* is the optimal line and switch investment strategy. Otherwise, we may resort to a branch-and-price framework for finding optimal integral solutions. Note that a fractional solution will always have at least one fractional y-value (see Proposition 7). Hence, we branch on one of the fractional y-variables and hope that this will resolve the fractionality. If not one may continue branching on y-variables until the fractionality is resolved.

7.5 Computational Results

In this section we apply the column generation technique to the problem of investing in switching equipment to minimize total investment and expected generation cost over a number of scenarios with varying demand and supply.

Computational experiments are performed on two different IEEE electricity transmission networks — the IEEE 73-bus network and the IEEE 118-bus network. The computational results are compared to solving the original formulation with a commercial MIP-solver (CPLEX).

The IEEE 73-bus network is based on the three area reliability test system 1996 [11]. Data for this network can be found in [7]. The transmission network was modified as described in [13] The resulting network has 117 lines, 99 generators, total generation capacity of 8998 MW, and total peak demand of 8550 MW. The IEEE 118-bus network is described in [3]. This network has 185 lines, 20 generators, total peak load of 4519 MW, and a total thermal generator capacity of 5859 MW. These networks were modified to accommodate varying supply and demand scenarios.

For the IEEE 118-node network a 1600 MW intermittent wind-power generator with varying supply capacity is located at node 91 supplying power at marginal generation cost 0. Nodal demands were scaled uniformly in the interval 0.535 to 1.0. In each instance half the scenarios had no wind power while the other half had full wind-power capacity.

The original IEEE 73-bus network has 18 hydro units (six in each area) each with capacity 50 MW and marignal cost 0. In our model, four of the hydro units in nodes 222 and 322 (area 2 and 3) were modified to represent wind generators with marginal generation cost 0 and varying generation capacity over the scenarios. Similarly, all six hydro units in node 122 (area 1) were modified to have varying marginal cost but constant generation capacity of 50 MW over the scenarios. Nodal demands were scaled uniformly by a factor in the interval 0.5 to 1.0. Table 7.1 gives a summary of the values of the stochastic parameters used in the different instances of the problem. For both networks the stochastic parameters are all assumed to be independent of each other and scenarios are assumed to be equally likely to occur.

	$\lvert\Omega\rvert = 16$			
	$\lvert\Omega\rvert = 81$			
stochastic parameter	$\lvert\Omega\rvert = 256$			
demand factor	1	0.67	0.5	0.84
wind capacity factor, node 222	1	0	0.33	0.67
wind capacity factor, node 322	1	0	0.67	0.33
hydro price factor, node 122	0	30	5	15

Table 7.1: Summary of stochastic demand, generation capacity, and marginal cost factors.

First stage decisions include only investment decisions in switching equipment. That is, we assume $e_a^\top y_L = 1$ for all arcs a in A. The fixed amortized switch investment costs are set to \$5/h for each switch.

Computational experiments were performed on a 2.26 GHz Core 2 Duo computer with 4 GB RAM.

7.5.1 Experiments with branch and price

In order to solve large instances, the Dantzig-Wolfe reformulation described above was implemented in a branch-and-price framework using the DIP software framework [9]. DIP (Decomposition for Integer Programming) is a general open source framework developed under COIN-OR for solving discrete optimization problems using various decomposition algorithms. DIP allows the user to formulate mixed integer programs in the original space and to provide the problem structure needed for decomposition. DIP then handles the reformulation and provides methods for solving the problem using decomposition algorithms. The code is implemented in C++ and is designed and maintained by Matthew Galati and Ted Ralphs at Lehigh University [5].

Instances with the IEEE 118-bus network and the IEEE 73-bus network and different number of scenarios and values of k are constructed. These are solved using DIP's branch and price algorithm with default parameters, except that each node is solved to optimality before branching (`TailOffPercent = 0`), compression of columns are turned off (`CompressColumns = 0`), and the master problems are solved to optimality (`MasterGapLimit = 0`) using interior point method (CPLEX 12.2 barrier). Sub-problems are solved using the CPLEX 12.2 MIP-solver. For comparison the instances are also solved using the CPLEX 12.2 branch-and-bound solver with default settings. Computational results are shown in Table 7.2, while Table 7.3 and Table 7.5 shows the objective function values and number of installed switches in the optimal solution of the corresponding instances for the IEEE 118-bus network and the IEEE 73 bus network.

Results show that the CPLEX MIP solver performs well on instances with a small number of scenarios. With more scenarios, however, the CPLEX solver exhausts the memory, while DIP solves to optimality in reasonable time. DIP outperforms CPLEX for 11 out of the 15 instances investigated. In general, it seems that DIP scales well with the number of scenarios, while CPLEX handles large k-values better.

For branch and price all instances except the 118-node, 32-scenario instance are solved to integer optimality in the root node and hence no branching is needed. For the 32-scenario instance a fractional solution is returned in the root node. However, integrality is obtained by branching only once. (The fractional solution has a strictly lower value than the optimal integer solution obtained.)

The decomposition relies on solving a large number of sub-problems with feasible set $\mathcal{Q}(\omega)$. For large k the computational complexity of the sub-problems is high and solving them to optimality is hard. This can be seen from Table 7.2 that shows that only a small fraction of the time is spent solving the master problems, while the majority of time is spent solving the sub-problems. This makes the branch-and-price algorithm perform less well on instances with large k. Hence, further research is needed to strengthen the sub-problems in order to solve instances with large k. On the other hand, as shown in section 7.4, the master problem matrices have some nice properties resulting in shallow branching trees.

Instance			Branch and price				Branch and bound		
			time (s)		price-	no. of	time (s)	gap	lower
$\|N\|$	$\|\Omega\|$	k	total	master	passes	nodes			bound
118	2	3	**96**	0	10	1	547	0.00	1351.22
118	2	5	1427	1	20	1	**330**	0.00	1338.55
118	4	3	**257**	4	16	1	2310	0.00	1036.93
118	4	5	**3172**	6	38	1	7133	0.00	1033.95
118	4	10	22444	30	98	1	**3055**	0.00	1009.60
118	8	3	**978**	8	26	1	2070 †	2.26	846.17
118	16	3	**3213**	11	36	1	3722 †	0.00	775.01
118	32	3	**25477**	279	129	3	4968 †	9.68	690.81
118	64	3	**11126**	72	38	1	8589 †	28.84	678.44
73	4	1	**12**	1	6	1	401	0.00	65297.22
73	16	1	**53**	2	9	1	5013 †	2.05	66270.18
73	81	1	**2037**	36	18	1	14414 ‡	6.07	52884.46
73	4	3	972	3	35	1	**42**	0.00	65266.08
73	16	3	3888	17	69	1	**490**	0.00	66266.34
73	81	3	**36793**	163	93	1	6732 †	0.08	52885.56

Table 7.2: Results for the switch investment problem on the IEEE 118-bus network and IEEE 73-bus network. Solve times and gaps are reported for the branch-and-price algorithm (DIP) and standard branch-and-bound (CPLEX) for problem instances with at most k open switches and $|\Omega|$ scenarios. All instances was solved to optimality using branch-and-price. Branch-and-bound was terminated with the CPLEX default optimality tolerance except for ‡ which was terminated manually after 14400 s. For branch-and-price the total solve time for the master problems, number of pricepasses, and the number of nodes in the branching tree are also reported. For branch-and-bound the lower bound is reported. Gaps reported are absolute gap to best known solution. For the branch-and-bound the best lower bound is also reported. The fastest solution time is highlighted in bold face. † denotes that the optimization was terminated due to lack of memory.

7.5.2 Experiments with column generation for the 73 node network

In this subsection we consider the column-generation algorithm without branching. The motivation for this study comes from the need to solve stochastic models with many scenarios. To investigate how the decomposition algorithm scales with scenarios, we restrict attention to the smaller IEEE 73-bus network. The decomposition and models in the following results are formulated using the AMPL modelling language and all master problems and subproblems are solved

Instance		Optimal solution	
$\|\Omega\|$	k	obj. function value	no. of switches
2	3	1351.36	5
2	5	1338.68	7
4	3	1037.03	5
4	5	1034.06	6
4	10	1009.70	9
8	3	898.48	7
16	3	871.10	3
32	3	734.04	3
64	3	763.97	3

Table 7.3: Objective function value and number of switches installed in the optimal solution for instances of the switch investment problem for the IEEE 118-bus network.

with CPLEX 12.2. The relaxed master problems are solved using CPLEX barrier algorithm without crossover, while the subproblems are solved using the CPLEX standard branch-and-bound algorithm. For the branch-and-bound algorithm CPLEX 12.2 was applied with default parameters.

Computational results are shown in Table 7.4, while Table 7.5 shows the objective function values and number of switches installed in the corresponding optimal solutions. These results show that the time taken to solve the Dantzig-Wolfe reformulation is approximately proportional to the number of scenarios, and we are able to solve up to 256 scenarios in reasonable time. Note that all instances are solved to optimality in the root node of the branch and bound tree and hence no branching is necessary. For $k = 1$ solving the compact formulation using CPLEX is much more time consuming than solving the Dantzig-Wolfe reformulation. However, for $k = 3$ solving the compact formulation is faster for a small number of scenarios, but performs worse with an increasing number of scenarios.

The computational results presented in this section show that solving the Dantzig-Wolfe reformulation by column generation is faster for small values of k compared to solving the original formulation in CPLEX. When k is small, column generation scales well with the number of scenarios. For large values of k, however, the subproblems become intractable. The master problem — except for one instance — always yields an optimal integer solution in the root node.

7.6 Conclusion

In this paper we consider decomposition methods for stochastic investment problems involving transmission switching in electricity networks. In particular, we look at determining optimal switch investment and line capacity expansion

Instance			Column Generation			Branch-and-bound		
				price-	no. of			lower
$\|N\|$	$\|\Omega\|$	k	time (s)	passes	nodes	time (s)	gap	bound
73	4	1	**16**	4	1	401	0.00	65297.22
73	16	1	**54**	6	1	5013 †	2.05	66270.18
73	81	1	**286**	8	1	14414 ‡	6.07	52884.46
73	256	1	**1518**	12	1	48373 †	8.15	54648.50
73	4	3	1153	31	1	**42**	0.00	65266.08
73	16	3	1326	30	1	**490**	0.00	66266.34
73	81	3	**25056**	52	1	6732 †	0.08	52885.56
73	256	3	**30880**	39	1	68670 ‡	4.72	54647.76

Table 7.4: Computational results for the switch investment problem on the IEEE73-bus network. All instances was solved to optimality using column generation. Branch-and-bound was terminated with the CPLEX default optimality tolerance. Gaps reported are absolute gap to the optimal solution. For the branch-and-bound the best lower bound is also reported. † denotes that the optimization was terminated due to lack of memory. The fastest solution time is highlighted in bold face. ‡ denotes that CPLEX was manually terminated before reaching optmimum.

Instance		Optimal solution	
$\|\Omega\|$	k	obj. function value	no. of switches
4	1	65303.75	1
16	1	66301.56	1
81	1	52905.05	1
256	1	54668.09	1
4	3	65270.02	2
16	3	66272.91	2
81	3	52890.74	2
256	3	54656.02	2

Table 7.5: Objective function value and number of switches installed in the optimal solution for instances of the switch investment problem for the IEEE 73-bus network.

strategies and we propose a Dantzig-Wolfe reformulation of a two-stage stochastic mixed integer program.

A column-generation approach is outlined to solve the Dantzig-Wolfe reformulation. The approach is tested on two IEEE test networks. When the number of allowed switching actions is small, the proposed algorithm turns out to be significantly more efficient than solving the compact formulation directly, and it enables us to solve instances with up to 256 scenarios.

In general, the linear programming relaxation of the reformulation does not have integer extreme points, but in practice this often happens to be the case. In the rare instances where the relaxed master problem has fractional solutions, our formulation admits a simple branch-and-price scheme that can be used to resolve these with very few iterations.

The approach is limited by the complexity of the subproblems. Solving large-scale problems requires a strong formulation of the subproblem, especially when many switching actions are allowed. In our experiments, we attempted to apply some strengthening to the subproblems by adding the constraint

$$\sum_{a \in P} z_a \leq 1$$

for any path P in which all nodes except the first and last nodes are two-connected, and setting $z_a = 0$ for any arc a such that the network $(N, A \setminus \{a\})$ is not connected. These provided some improvement on subproblems in sparse networks and/or with larger values of k, but gave no improvement in computation time for the instances discussed in this paper. This indicates that further research should be directed at providing stronger formulations and more efficient solution methods for the subproblems in order to improve efficiency of the algorithm.

The methodology described in this paper can be applied to other stochastic programming problems in which switching is allowed. For example, one might construct a multi-stage plan for investing in switches and transmission line expansions using the approach explored in [19] for distribution networks (in which switching to a radial structure is required in each scenario and stage). The approach can also be used to investigate the optimal investment in switches to ensure the $N-1$ reliability of an existing network. In this setting the scenarios represent failures of single lines or units. This approach is explored for distribution networks in [18], and described for transmission networks in [20].

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Chapter 8

Line Capacity Expansion and Transmission Switching in Power Systems with Large-Scale Wind Power

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In 2025 electricity production from wind power should constitute nearly 50 % of electricity demand in Denmark. In this paper we look at optimal expansion of the transmission network in order to integrate 50 % wind power in the system, while minimising total fixed investment cost and expected cost of power generation. We allow for active switching of transmission elements to eliminate negative effects of Kirchhoffs voltage law. Results show that actively switching transmission lines may yield a better utilisation of transmission networks with large-scale wind power and increased wind power penetration. Furthermore, transmission switching is likely to affect the optimal line capacity expansion plan.

8.1 Introduction

In 2025 electricity generation from wind power is planned to constitute nearly 50 % of demand for electricity in Denmark. This will primarily be achieved through a huge increase in the number of offshore wind farms.

As a consequence, massive changes are expected in the Danish electricity system in the years to come. Conventional power plants will close down, new market structures are expected to emerge as balancing needs and the requirement for flexible demand are going to increase. Transmission flows will change and the need for transmission capacity will increase.

In addition to the wind power development, it has been decided that all 132/150 kV overhead lines in Denmark shall be replaced by underground cables during the next 30 years. This constitutes a huge challenge for the Danish transmission system operator, Energinet.dk, however, it is also a great opportunity to redesign this part of the transmission grid as a whole.

Energinet.dk, is state owned and operates the grid on a non-profit basis. In general, transmission investments are carried out at lowest cost while maintaining a certain high level of security of supply. Connections abroad and large domestic grid investments are considered to have significant societal economical impact, and hence the socio-economic welfare effect of such investments must be evaluated. Only investments that provide positive overall socio-economic impacts are promoted. To ensure robust decisions, the impact of any larger investment is evaluated in a context of different future scenarios, each describing likely developments of the society twenty years ahead in time.

Traditionally, in Denmark, investments have mainly been considered incrementally. However, considering the underground cabling and the rapid wind power development there might be huge gains by coordinating investments to find an optimal future grid. In this paper, we consider the stochastic line capacity expansion problem with transmission switching. This model can be applied to the development of the Danish grid and we show that active switching of transmission lines increases the utilisation of the transmission grid.

The combinatorial complexity of the line capacity expansion problem makes it hard to solve. The deterministic version of this problem has been solved succesfully using Benders decomposition [17, 2] and a commercial MIP solver (CPLEX) [1]. In [7] a stochastic scenario based formulation of the line capacity expansion problem for competitive markets is presented.

Adding new lines to an electricity network may increase the cost of power generation. This paradox is due to Kirchhoff's *voltage law* and is demonstrated in [3]. Transmission switching [14] may alleviate such effects by switching out transmission lines. In general transmission switching may increase security of a network [18] and decrease cost of generation [12, 15]. Khodaei and Shahidehpour [16] propose a Benders decomposition approach for solving the dynamic line capacity expansion problem with transmission switching.

In this paper we apply the model proposed in [21] for the two-stage stochastic line capacity and switch investment problem with uncertain future generation capacities and demand. In the first stage decisions on line capacity expansions and switch investments are made. Line capacity expansions and switch investments are chosen from a candidate set of potential line upgrades. In the second stage operational decisions on power generation, line flows, and switching decisions are made in a number of scenarios reflecting variations in demand (e.g. peak/off-peak) and generation capacity (e.g. level of wind).

The contribution of this paper is two-fold. Firstly, we demonstrate that the line capacity expansion problem with transmission switching modeled as a two-stage stochastic mixed integer program can be solved efficiently for realistically sized networks using column generation. Secondly, we show that actively switching transmission elements in congested networks with large-scale wind power may reduce generation cost and increase the amount of wind power that can be integrated into the system. Furthermore, actively switching transmission lines may alter the optimal line capacity expansion plan.

We begin the paper by recalling a mixed integer progamming formulation of the direct current approximated optimal power flow problem with transmission switching [12]. In section 8.3 we state the line capacity expansion and switch investment problem as a two-stage stochastic program and provide the Dantzig-Wolfe reformulation [6] as proposed in [21]. Section 9.4 provides computational results for the IEEE 118-bus network and the Danish transmission network with large-scale wind power. Finally, concluding remarks and directions for future research is given in section 9.5.

8.2 Operational Dispatch

We assume a linear DC-approximation of the optimal power flow (see e.g. [5], [20]) with linear generation costs and no line losses.

Consider the directed graph $G = (\mathcal{N}, \mathcal{A})$ with a source/sink node s. For each arc $a \in \mathcal{A}$, the cost, capacity, and reactance coefficients are given by c_a, u_a, and r_a, respectively. The flow on each arc $a \in \mathcal{A}$ is denoted by x_a, while w_i denotes the voltage phase angle for each node $i \in \mathcal{N}$. Let the set of arcs $\mathcal{F}(i)$, respectively, $\mathcal{T}(i)$ denote the set of arcs with tail, resp. , head i. Let the set of supply and demand arcs $\mathcal{S} = \mathcal{F}(s) \cup \mathcal{T}(s) \subseteq \mathcal{A}$ be defined by having s as the tail or head, and let $r_a = 0$ for all $a \in \mathcal{S}$. The supply arcs $\mathcal{F}(s)$ represent generation units with marginal generation cost c_a and generation capacity u_a, while the demand arcs $\mathcal{T}(s)$ represent flexible demand with value of consumption $-c_a$ and maximum consumption u_a. Inflexible demand d_i must be met at all nodes i in

$\mathcal{N}$.

Furthermore, let z_a be a binary variable indicating whether line e is active ($z_a = 0$) or not. An active line is one which is installed and not switched out. Define by $\mathcal{D}$ the set of direct current (DC) lines, while $\mathcal{E}$ denotes the set of existing lines and $\mathcal{H}$ the set of switchable lines.

For a single time period (snapshot) the globally optimal state (with maximum social surplus) of the system may be found by solving the mixed integer linear program

$$\min \sum_{a \in \mathcal{A}} c_a x_a \tag{8.1}$$

subject to

$$\begin{aligned}
x_a &\leq u_a z_a && \forall a \in \mathcal{A} && (8.2)\\
x_a &\geq l_a z_a && \forall a \in \mathcal{A} && (8.3)\\
\sum_{a \in \mathcal{T}(i)} x_a - \sum_{a \in \mathcal{F}(i)} x_a &= d_i && \forall i \in \mathcal{N} \setminus \{s\} && (8.4)\\
z_a = 0 \Rightarrow r_a x_a - w_i + w_j &= 0 && \forall a = (i,j) \in \mathcal{A} \setminus (\mathcal{S} \cap \mathcal{D}) && (8.5)\\
z_a &\in \{0,1\} && \forall a \in \mathcal{A} && (8.6)\\
z_a &= 0 && \forall a \in \mathcal{A} \setminus \mathcal{H} && (8.7)
\end{aligned}$$

where the objective (10.1) minimises the total cost of operation subject to the following constraints. Capacity limits on arc flows (10.2)-(10.4). Conservation of flow (10.5) at each node except the source node s. Kirchhoffs *voltage law* (8.5) for all active arcs. The *switching variables* z_a are binary (8.6) and fixed for lines that are not switchable (8.7).

Furthermore, we may restrict the number of *existing* lines that are being switched.

$$\sum_{a \in \mathcal{E}} z_a \leq k \tag{8.8}$$

Note, that constraints (8.5) may be linearised using a big-M construction

$$-M z_a \leq r_a x_a - w_i + w_j \leq M z_a, \qquad \forall a = (i,j) \in \mathcal{A} \setminus (\mathcal{S} \cap \mathcal{D}) \tag{8.9}$$

where M is some sufficiently large number.

8.3 A Two-Stage Stochastic Model with Transmission Switching

We now consider a two-stage stochastic model, where the first stage decisions involve investment in switching equipment y_S and line capacity y_L, while the second stage models operational decisions (x, w, z) for dispatch and switching in a number of scenarios $\omega \in \Omega$ each occuring with probability $p(\omega)$. For each scenario $\omega \in \Omega$, let

$$\mathcal{Q}(\omega) = \{(x(\omega), w(\omega), z(\omega)) \mid (10.2) \text{ - } (8.8)\}.$$

The model may now be formulated as

$$\min \; f_S^\top y_S + f_L^\top y_L + \sum_{\omega \in \Omega} p(\omega) c(\omega)^\top x(\omega) \tag{8.10}$$

$$\begin{aligned}
\text{s.t.} \quad && y_L - y_S + z(\omega) &\leq 1 && \forall \omega \in \Omega && (8.11)\\
&& y_L + z(\omega) &\geq 1 && \forall \omega \in \Omega && (8.12)\\
&& (x(\omega), w(\omega), z(\omega)) &\in \mathcal{Q}(\omega) && \forall \omega \in \Omega && (8.13)\\
&& y_L, y_S &\in \{0,1\}^{|\mathcal{A}|} && && (8.14)
\end{aligned}$$

The objective (9.11) minimizes the hourly fixed and operational cost, while (9.12) ensures that switching of installed lines is only possible if a switch is also installed. Constraint (8.12) allows lines to be switched in only if they are also installed. We set $e_a^\top y_L = 1$ and $e_a^\top f_L = 0$ for existing lines a in $\mathcal{E}$, where e_a is the binary unit vector of $|\mathcal{A}|$ elements with the ath element being 1.

Note, that not installing a line corresponds to having the line switched out (i.e. $z(\omega) = 1$) in all scenarios $\omega \in \Omega$.

8.3.1 Dantzig-Wolfe Reformulation

The mathematical program (9.11)-(8.14) may be reformulated using Dantzig-Wolfe [6] and a branch-and-price algorithm may be applied to obtain optimal solutions to this reformulation.

The idea is to decompose the stochastic problem into a master problem and a number of subproblems — one for each scenario. We let the binary vector $z(\omega)$ define a *feasible switching plan* (FSP) for scenario ω if there exists $x(\omega), w(\omega)$ such that $(x(\omega), w(\omega), z(\omega)) \in \mathcal{Q}(\omega)$. Now, let $Z(\omega) = \{\hat{z}^j(\omega) | j \in J(\omega)\}$ be the set of all FSP's for scenario ω, where $J(\omega)$ is the index set for $Z(\omega)$. We can write any element in $Z(\omega)$ as

$$\begin{aligned} z(\omega) &= \sum_{j\in J(\omega)} \varphi^j(\omega)\hat{z}^j(\omega) \\ \sum_{j\in J(\omega)} \varphi^j(\omega) &= 1, \quad \varphi^j(\omega) \in \{0,1\},\ \forall j \in J(\omega). \end{aligned}$$

Assume that for each feasible switching plan $\hat{z}^j(\omega)$ the corresponding optimal dispatch of generation and load shedding is given by $\hat{x}^j(\omega)$. The master problem can now be written in terms of $\hat{z}$ and $\hat{x}$ as

$$\text{MP: } \min\ f_L^\top y_L + f_S^\top y_S + \sum_{\omega\in\Omega}\sum_{j\in J_\omega} p(\omega)c(\omega)^\top \hat{x}^j(\omega)\varphi^j(\omega) \tag{8.15}$$

$$\text{s.t.} \quad \sum_{j\in J(\omega)} \varphi^j(\omega) = 1 \qquad [\mu(\omega)]\,,\ \forall\omega\in\Omega \tag{8.16}$$

$$y_L - y_S + \sum_{j\in J(\omega)} \hat{z}^j(\omega)\varphi^j(\omega) \le 1 \qquad [\pi(\omega)]\,,\ \forall\omega\in\Omega \tag{8.17}$$

$$y_L + \sum_{j\in J(\omega)} \hat{z}^j(\omega)\varphi^j(\omega) \ge 1 \qquad [\rho(\omega)]\,,\ \forall\omega\in\Omega \tag{8.18}$$

$$\varphi^j(\omega) \in \{0,1\}, \qquad \forall j \in J(\omega) \tag{8.19}$$

$$y_L, y_S \in \{0,1\}^{|E|} \tag{8.20}$$

where $\mu(\omega)$, $\pi(\omega)$ and $\rho(\omega)$ denote the dual prices associated with the respective constraints.

The master problem MP is a two-stage stochastic integer program with integer variables in both stages. Although in general these are difficult to solve, the structure of MP is amenable to a branch-and-bound procedure by virtue of the following result.

Proposition 4 *If y_L and y_S are chosen to be fixed binary integers, then the linear programming relaxation of MP has integer extreme points.*

For a proof we refer the reader to [21].

It is convenient to consider only a subset $Z(\omega)' \subseteq Z(\omega)$ of feasible switching plans for each scenario ω in the master problem. We define this restricted master problem (RMP) by (8.15) - (8.20) with $J(\omega)$ replaced by $J(\omega)'$ the index set of $Z(\omega)'$. A column generation algorithm is applied to dynamically add feasible switching plans to the linear relaxation of the master problem. The algorithm is initialised by letting $Z(\omega)' = \{\hat{z}^0(\omega)\} = \{\mathbf{0}\}$, for all scenarios $\omega \in \Omega$. That is, initially no line may be switched out in either scenario. The corresponding operational costs $c(\omega)^T \hat{q}^0(\omega)$ can easily be found by solving a linear program for each scenario. In each iteration of the algorithm, the linear relaxation (RMP-LP) of RMP is solved yielding the dual prices μ, π, and ρ. A new column $(p(\omega)c(\omega)^T \hat{x}^j(\omega), 1, \hat{z}^j(\omega))$ may improve the solution of RMP-LP if and only if the associated reduced cost $\bar{c}(\omega) = p(\omega)c(\omega)^\top \hat{x}^j(\omega) + \pi(\omega)^\top \hat{z}^j(\omega) - \rho(\omega)^\top \hat{z}^j(\omega) - \mu(\omega)$ is negative.

A column for scenario ω may therefore be constructed by solving the subproblem:

$$\begin{array}{ll} \min & p(\omega)c(\omega)^\top x + \pi(\omega)^\top z - \rho(\omega)^\top z - \mu(\omega) \\ \text{s.t.} & (x, w, z) \in \mathcal{Q}(\omega), \end{array}$$

where $\mu(\omega)$, $\pi(\omega)$ and $\rho(\omega)$ are the dual prices returned from RMP-LP.

Any feasible solution $(x, w, z) \in \mathcal{Q}(\omega)$ with negative objective function gives rise to a potential candidate column for RMP-LP. If no columns with negative reduced cost exist then we have solved the relaxed master problem (MP-LP) to optimality. Furthermore, if the solution (φ^*, y^*) to MP-LP is integral then (φ^*, y^*) is an optimal solution to the master problem (8.15) - (8.20) and y^* is the optimal switch investment strategy. Otherwise, we may resort to a branch-and-price framework for finding optimal integral solutions. Note that a fractional solution will always have at least one fractional y-value (see Proposition 7). Hence, we branch on one of the fractional y-variables and hope that this will resolve the fractionality. If not one may continue branching on y-variables until the fractionality is resolved.

8.4 Computational Results

In this section experiments are performed on two different networks — the IEEE 118 bus network and the Danish transmission network. Experiments with the IEEE 118 bus network with four scenarios suggests that transmission switching is beneficial for the integration of large-scale wind power. Also, these results justify the use of stochastic programming. Results for the Danish network with the expected development of off-shore wind power generation by 2025 confirm that allowing switching may reduce generation cost and increase the amount of wind power integrated in the system.

8.4.1 The IEEE 1118 Bus Network

We will first study the IEEE 118 bus network [8] with network data described in [4]. This network has 185 lines, total peak load of 4519 MW, and a total thermal generator capacity of 5859 MW. We will consider a four scenario instance of the switch investment problem presented in section 8.3 with uncertain outcomes of demand and wind generation capacity. First stage decisions include only investment decisions in switching equipment. That is we assume $y_L = \mathbf{1}$ to be fixed. The results justify the use of stochastic programming and indicates that transmission switching is particularly beneficial in systems with large-scale wind power.

Four scenarios are defined with respect to the load level (peak/off-peak) and amount of wind power (high/low). The scenarios have equal probabilities and are summarised in Table 8.1. The fixed amortized switch investment costs are arbitrarily set to \$5/h for each switch.

A 1600 MW intermittent wind power generator with varying supply capacity and 0 marginal cost is located at node 91. Generation from the windpower generator is not fixed so wind generation may be curtailed.

Scenario	Probability	Load		Windpower	
ω	$p(\omega)$		% of peak		capacity, MW
1	0.25	off-peak	59%	high	1600
2	0.25	peak	100%	high	1600
3	0.25	off-peak	64%	low	0
4	0.25	peak	99%	low	0

Table 8.1: Summary of scenarios for a small instance of the switch investment problem.

Without switches the total generation cost incurred is \$1031.55/h. In the optimal switch investment strategy with $k = 3$ five switches are installed incurring a total investment cost of \$25/h and generation cost \$910.25/h. The total savings from switching is thus approximately 9%. With optimal switching the dispatched windpower is increased from 499 MW to 648 MW in scenario 1 and from 535 MW to 875 MW in scenario 2. Thus by employing active switching one can increase the amount of windpower in the system and decrease system cost. The optimal switching configurations and the corresponding saved operational costs for each scenario are shown in Table 8.2.

ω	Switching configuration					Saved costs
1	E77-80	E89-90		E89-92		7.6
2	E77-80	E89-90		E89-92		36.0
3	E77-80	E89-90			E94-96	2.2
4	E77-80		E89-91		E94-96	75.4
Total						121.2

Table 8.2: Optimal switching configurations and saved operational costs for a small four-scenario instance of the switch investment problem on the IEEE 118 bus network.

Since in general (10.1)-(8.8) is a difficult mixed integer program for $k \geq 1$ one might consider to decouple the scenarios and solve each scenario separately with amortized investment costs and subsequently piece the solutions together. While this approach might yield good solutions for some instances, we cannot rely on this in general. Applying this approach to our four-scenario instance described above by solving four smaller mixed integer programs, we obtain an investment strategy with nine switches and total operational cost of \$904/h. The net benefit (including switch installation costs) of switching is only \$82.6/h as opposed to \$96.3/h for the optimal switch investment strategy obtained by solving the integrated model. Hence, the value of switching is clearly lower when decoupling the scenarios completely.

We now consider an instance of the problem where we — in addition to have a wind power park at node 91 — also have wind power parks in node 5 and node 26. All wind power parks are assumed to be relatively large (1600 MW installed capacity). The scenarios and network are unchanged.

Without switching the total expected generation cost is \$881.56/h. With switching ($k = 3$) this is decreased to \$750.45/h with a total of five installed switches leading to a net benefit of \$106.12/h or approximately 12 %. This reduction in costs covers an increase in wind power on the three parks by a total of 430 MW in scenario 2 and 293 MW in scenario 1 (see Table 8.3). The corresponding optimal switching configuration is shown in Table 8.4.

Scenario	$k = 0$			$k = 3$		
ω	91	5	26	91	5	26
1	499.7	479	453	633.13	697.49	395.28
2	538	918	383	737.43	937.25	595.08
3	0	0	0	0	0	0
4	0	0	0	0	0	0

Table 8.3: Wind power generation in different scenarios without ($k = 0$) and with ($k = 3$) switching.

ω	Switching configuration				
1	E77-80			E23-25	
2	E77-80		E94-96	E23-25	
3		E89-91			E38-37
4				E23-25	

Table 8.4: Optimal switching configurations for a small four-scenario instance of the switch investment problem on the IEEE 118 bus network.

8.4.2 The Danish Transmission Network

We will now consider the current Danish transmission network and potential line capacity expansions in a future setting with development of many new off-shore wind power plants.

Potential off-shore windpower development in Denmark in the period 2010 -

2025 is described in [9] and [11]. The projects considered are summarised in table 8.5. We assume that all projects are realised.

Network and generation data is obtained from the Danish transmission system operator Energinet.dk. This data is confidential, but aggregated values are available in Table 8.6. Potential line capacity expansion projects are likewise based on confidential data form Energinet.dk. Line investment costs for 400kV lines are based on underground cable costs - overhead lines are not considered. This is in accordance with future expansion guidelines for the Danish electricity transmission grid [10]. The potential candidates for new transmission lines are limited to a set of 10 lines on the 400 kV level and 5 lines on 132 kV level.

Neighbouring areas (Norway, Sweden, Germany, and The Netherlands) are modeled in a very simplistic way with no demand and a generator with fixed marginal cost and capacity. This assumes that there is always excess generation capacity in the respective areas which can be supplied at constant cost.

The ability to switch a particular line incurs a fixed investment cost. This is assumed to cover any equipment needed to perform automatic switching of that line including the switch itself (if it needs to be upgraded) and any communication equipment if necessary. Here, we arbitrarily assumes a relatively small fixed cost of 1 DKK/h. In the experiments swithcing was allowed on high voltage (>100 kV) transmission lines only.

name	capacity (MW)	area
Djursland	400	DK1
Horns Rev	1000	DK1
Læsø	600	DK1
Jammerbugt	800	DK1
Ringkøbing	1000	DK1
Kriegers Flak	800	DK2
Rønne Banke	400	DK2
S. Middelgrund	200	DK2

Table 8.5: Summary of projected installed off-shore wind power capacity in the year 2025.

no. of nodes (busses)	610
no. of transmission lines	529
no. of transformers	302
no. of generators	418
total gen. capacity (DK)	13530
total peak demand	6945

Table 8.6: Summary of network data.

We shall now consider a particular six-scenario instance of the problem. In Table 8.7 a summary of the scenarios are given. The scenarios and their probabilities are for illustration only, and do not reflect our true expectations for the future. Nevertheless, they do give some valuable insights – in particular with regard to the value of switching and its impact on investments in line capacity.

		Demand (MW)		Wind capacity	
ω	$p(\omega)$	DK1	DK2	on-shore	off-shore
0	0.16	4076	2869	0.90	0.95
1	0.16	4076	2869	0.50	0.50
2	0.17	4076	2869	0.00	0.00
3	0.17	1448	934	0.90	0.95
4	0.17	1448	934	0.50	0.50
5	0.17	1448	934	0.00	0.00

Table 8.7: Summary of scenarios. Wind capacity is the share of installed capacity.

Five instances of the problem with different levels of switching is investigated: *no switch* and $k = 0, 1, ..., 3$. The *no switch* instance has fixed $y_S = 0$ and allows for investments in new lines only. The $k = 0$ instance allows switching on new lines only — no switches on existing lines is allowed. For the instances with $k = 1, 2, 3$ switching on all new lines and at most k existing lines are allowed. Table 8.8 gives a summary of results for the different instances, while Table 8.9 gives an overview of the benefit of switching.

For the six scenarios described above, the optimal solution contains investments in 5 of the possible 15 new lines without switching (total cost of 466970 DKK/h of which 141 DKK/h are investment costs). Allowing to switch new lines ($k = 0$), results in a total of 10 new lines installed of which 7 may be switched (Table 8.8). This gives a net-benefit of 12366 DKK/h (Table 8.9).

Figure 8.1 shows part of the 400kV network topology in Eastern Denmark, with proposed network expansions, while Figure 8.2 shows the partial optimal line capacity expansion plan for that part of the network when no switching is allowed. The network shown in Figure 8.3 depicts the optimal line capacity expansion plan when switching is allowed on new lines only ($k = 0$).

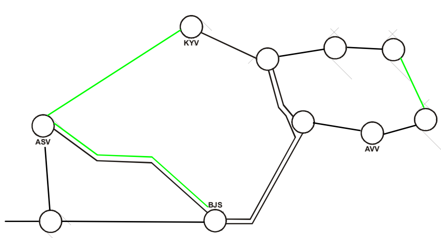

Figure 8.1: Part of the existing Eastern 400kV topology (black) and potential expansion options (green).

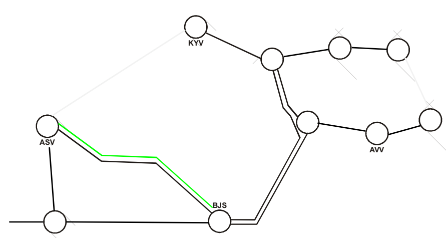

Figure 8.2: Partial line capacity expansion plan without switching.

In the following experiment, we – in addition to allowing switching on new lines – also allow for switching of one existing line in each scenario ($k = 1$). This yields an investment plan with 8 new lines and 12 switches. The total net benefit of the solution is 32706 DKK/h or 7% compared to the solution without switching. Figure 8.4 depicts (part of) the corresponding optimal line capacity expansion plan.

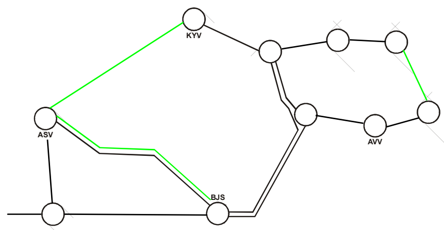

Figure 8.3: Partial line capacity expansion plan ($k = 0$).

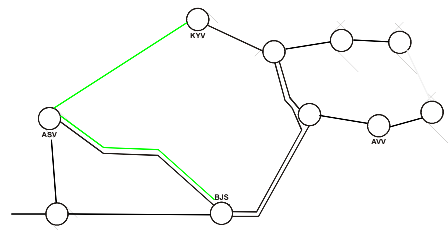

Figure 8.4: Partial line capacity expansion plan ($k = 1$).

By allowing switching the wind generation is increased by up to 251 MW in scenario 0 ($k = 3$). Also, operational costs are reduced significantly in the peak demand without wind (scenario 2) by switching to lower cost (thermal) generation.

	no switch	$k=0$	$k=1$	$k=2$	$k=3$
no. of installed lines	5	10	8	11	10
no. of installed switches	-	7	12	13	15
wind (MWh/h)	2649	2661	2687	2687	2689
fixed cost (DKK/h)	141	311	283	320	319
op. cost (DKK/h)	466829	454293	433982	430916	427294
total cost (DKK/h)	466970	454604	434265	431237	427613

Table 8.8: Summary of results for different levels of switching.

	$k=0$		$k=1$		$k=2$		$k=3$	
	abs	rel	abs	rel	abs	rel	abs	rel
op. cost, DKK/h	-12536	-3	-32847	-7	-35913	-8	-39535	-8
fixed cost, DKK/h	170	120	142	100	179	127	178	126
total cost, DKK/h	-12366	-3	-32706	-7	-35734	-8	-39357	-8
wind (avg.), MWh/h	12	0.46	38	1.42	38	1.42	40	1.52
wind ($\omega = 0$), MWh/h	76	1.13	235	3.53	236	3.53	251	3.77

Table 8.9: Benefit of switching. Values are absolute and relative (in %) difference as compared to the non-switched network ($y_S = 0$).

The results obtained from experiments with the Danish transmission network with large-scale wind power suggests that transmission switching may reduce generation cost and increase wind power generation. As transmission switching acts to reduce congestion in the network, this reduction in cost is entirely due to relief of congestion in the peak demand scenarios.

More interestingly, the optimal line expansion plan is highly sensitive to the level of switching allowed. Actively switching transmission elements increases the number of installed transmission lines (roughly by a factor of 2). This is due to the fact that some lines may be beneficial in some scenarios, but restrictive in others.

The previous experiments include only a few *extreme* scenarios with equal probabilities. In addition, we now introduce a *medium* scenario with a high probability in order to investigate the proportionality of cost and wind power generation. The scenarios are summarised in Table 8.10. Also, the cost of adding a switch has been quadrupled. Both of these changes are expected to discourage the use of transmission switching.

		Demand (MW)		Wind capacity	
ω	$p(\omega)$	DK1	DK2	on-shore	off-shore
0	0.0833	4076	2869	0.90	0.95
1	0.0833	4076	2869	0.50	0.50
2	0.0833	4076	2869	0.00	0.00
3	0.0833	1448	934	0.90	0.95
4	0.0833	1448	934	0.50	0.50
5	0.0833	1448	934	0.00	0.00
6	0.5000	2869	1902	0.30	0.30

Table 8.10: Summary of scenarios for the instances with 7 scenarios. Wind capacity is the share of installed capacity.

Results for the seven scenario instances are summarised in Table 8.11. We see that increasing the cost of switches and introducing a new scenario, results in different investment strategies for $k > 0$ with fewer (or the same) switches and line expansions.

	no switch	$k = 0$	$k = 1$	$k = 2$	$k = 3$
no. of installed lines	6	10	8	8	7
no. of installed switches	-	7	10	10	12
wind (MWh/h)	2860	2861	2874	2874	2875
fixed cost (DKK/h)	254	332	300	300	323
op. cost (DKK/h)	339693	332965	322614	321094	319265
total cost (DKK/h)	339947	333297	322914	321394	319588

Table 8.11: Summary of results for different levels of switching with seven scenarios.

Table 8.12 shows the benefit of allowing to switch transmission lines in the instances with seven scenarios. It is seen that increasing the cost of switches and introducing a *medium* scenario does reduce the benefit of switching considerably. However, the benefit is still significant. Switching allows a reduction in total cost of up to 6% and increases wind power generation in scenario $\omega = 0$ by up to 187 MW.

	$k=0$		$k=1$		$k=2$		$k=3$	
	abs	rel	abs	rel	abs	rel	abs	rel
op. cost, DKK/h	-6728	-1.98	-17080	-5.03	-18599	-5.48	-20428	-6.01
fixed cost, DKK/h	78	30.76	46	18.11	46	18.11	69	27.15
total cost, DKK/h	-6650	-1.96	-17034	-5.01	-18553	-5.46	-20359	-5.99
wind (avg.), MWh/h	1	0.03	14	0.50	14	0.50	16	0.55
wind ($\omega = 0$), MWh/h	12	0.17	171	2.55	172	2.55	187	2.78

Table 8.12: Benefit of switching. Values are absolute and relative (in %) difference as compared to the non-switched network ($y_S = 0$).

We acknowledge that the scenarios described here are not truly representative and that more work is necessary to identify a set of scenarios representing our true expectation of the future.

8.4.3 Running times

Optimal solutions for the six-scenario instances described above was obtained using column generation. The model was implemented using the COIN-OR DIP framework [13] and instances were solved using default parameters except that each node was solved to optimality before branching (`TailOffPercent = 0`), compression of columns was turned off (`CompressColumns = 0`), and the master problems were solved to optimality (`MasterGapLimit = 0`) using interior point method (CPLEX 12.2 barrier). Subproblems were solved using CPLEX 12.2 MIP-solver. Table 8.13 gives a summary of running times for different instances of the problem with branch-and-price (DIP) and CPLEX.

Instance		Branch-and-price					CPLEX	
		time (s)		price-	no.			
$\lvert\Omega\rvert$	k	total	master	passes	nodes	gap	time (s)	gap (%)
6	-	740	131	134	3	0	8.4	0.00
6	0	126	11	20	1	0	431	0.00
6	1	965	68	35	1	0	2239	† 0.00
6	2	2592	58	32	1	0	8999	† 0.02
6	3	4795	57	31	1	0	10006	† 0.02
12	1	4094	201	62	1	0	-	-
24	1	11982	178	62	1	0	-	-

Table 8.13: Computational results for solving the Dantzig-Wolfe reformulation using branch-and-price and the compact formulation using CPLEX. Gap is relative (in %) from best known solution.

Except for the instance without switching all instances were solved to optimality in the root node — that is no branching was needed. For all instances with switching column generation seems to be superior to solving the compact formulation using a commercial MIP solver (CPLEX). We were able to solve for 24 scenarios with $k = 1$ in less than 12000 s. Solution times for the column generation approach seem to scale relatively well with the number of scenarios. However, the majority of the solution time is used to solve subproblems and this is prohibitive for the number of scenarios that can be solved in reasonable time — especially for values of k larger than 1.

8.5 Conclusion

In this paper we have treated the line capacity expansion problem with transmission switching under future uncertainty in demand and wind generation capacity. The problem is formulated as a two-stage stochastic program and the Dantzig-Wolfe decomposition is solved using column generation.

Results indicate that the topology of the transmission network is important for the dispatch of wind energy and that intermittent generation calls for a dynamically optimised topology. This can be achieved by actively switching transmission lines. Our results show that transmission switching may reduce curtailment of wind power with up to 250 MW in peak demand for the Danish network under study. Also, switching of transmission elements may influence the optimal line capacity expansion strategy, making it worthwhile to install more new transmission capacity. Solving the decomposed model makes it possible to solve instances for real networks in reasonable time and is superior to solving the compact formulation using a commercial MIP solver (CPLEX). Furthermore, the decomposition approach seems to scale well with increased number of scenarios.

The Danish network presented in this paper is isolated from the remaining European electricity transmission network. In order to obtain more realistic results further work is needed to represent the neighbouring areas in a better way. This is important as large scale wind power generation is also under way in other parts of Northern Europe.

The results presented here are only for a limited number of scenarios, that may not reflect our true expectation of the future. Further work is needed to identify realistic and representative scenarios. Other stochastic parameters may be relevant such as generation prices (depending on water values of hydro power generation units, oil prices, etc.). Also, geographically dependent wind power generation time series is highly relevant in order to capture periods of high wind power in one part of the network and low wind power in other parts of the network. Even though such outcomes may occur only with low probability (e.g. only for short periods of time), this may increase further the need for a dynamic network topology and the value of transmission switching.

In practice, expansion of transmission line capacity and investment in new offshore wind power plants is performed over a planning period of many years. At each stage of the planning period the expectation of the future is changed as more information becomes available and so the optimal expansion plan may change as well. This model can be extended to a multi-stage formulation following the approach in [19]. In a multi-stage setting decisions on line capacity expansions may be made at any stage, while the expansion of wind power capacity may be subject to uncertainty.

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Chapter 9

Column Generation for Transmission Switching of Electricity Networks with Unit Commitment

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This paper presents the problem of finding the minimum cost dispatch and commitment of power generation units in a transmission network with *active switching*. We use the term *active switching* to denote the use of switches to optimize network topology in an operational context. We propose a Dantzig-Wolfe reformulation and a novel column generation framework to solve the problem efficiently. Preliminary results are presented for the IEEE-118 bus network with 19 generator units. Active switching is shown to reduce total cost by up to 15 % for a particular 24-hour period. Furthermore, the need for generator startups is reduced by 1. Instances with limited switching, some of which are intractable for commercial solvers, are shown to solve to optimality in reasonable time.

9.1 Introduction

In meshed electricity transmission networks Kirchhoff's laws constrain the flow on each line in a cycle. In the DC-load flow approximation, the power flow on any line must be proportional to the voltage phase angle difference for the two end nodes. For power flow in a tree network, the power flows are uniquely determined by flow conservation at the nodes. For any feasible flow the voltage phase angles are then uniquely determined up to an additive constant, and so they do not affect the economic dispatch.

When the network contains cycles, the voltage phase angles affect the dispatch. This is important in practice, since most electricity transmission networks are designed as meshed networks (with cycles) for security reasons, so that if any line fails, the power can still flow from source to destination by alternative paths. In a meshed network, voltage phase angles become important, since they result in additional constraints on the line flows. In particular, for each cycle in a network the sum of voltage angle differences (with respect to the direction) around the cycle must equal zero. Hence, each cycle in the network gives rise to one additional constraint on the line flows. This leads to a paradox (see e.g. [1]) in which adding a new line to a transmission network might increase the cost of supplying electricity, even if the cost of the line itself is zero.

Based on these observations, it is easy to see that it may be beneficial in mesh networks to take some lines out of operation — to either decrease system cost or increase reliability [5, 10]. The process of taking out lines and bringing them back in is done by opening (respectively closing) a switch at the end of the line and is referred to as *switching*.

Recent interest in renewable intermittent energy sources and the call for intelligent transmission networks or smart grids have spurred a renewed interest in switching problems. Fisher et. al. presents in [4] the problem of optimal switching of transmission elements in an electricity transmission network to minimize the delivered cost of energy. They propose a mixed-integer program to solve the DC-approximated loadflow with switching decisions in a single time period. They note that the problem is NP-hard. Results are provided for a 118-node network with 186 transmission lines. Hedman et. al. [6] extends the model to consider reliability of the network. Reliability constraints are added to the problem to ensure that any line failure will not lead to an infeasible dispatch of generation. They note that in some cases adding reliability constraints increases the value of switching.

In [7] Hedman et. al. discuss a decomposition algorithm to solve the security constrained transmission switching problem with unit commitment decisions made heuristically over 24 time periods. The master problem handles unit commitment decisions over the planning horizon given a fixed switching configuration of the lines, while sub-problems — one for each time period — optimizes the switching configuration given a fixed unit commitment plan. It is noted that adding transmission switching may yield a cheaper unit commitment plan with fewer start-ups than what could be achieved without switching. Khodaei and Shahidehpour [8] propose a Benders decomposition of the security constrained unit commitment and transmission switching problem.

In this paper, we assume that a technology is available that makes it possible to switch lines instantaneously. That is, a line may be switched automatically from one moment to the next without delay. In this case, switching out lines will (in theory) not affect system security (disregarding failures on switching equipment), since all lines may be switched back in immediately, in case of any failure in the system.

We propose a Dantzig-Wolfe reformulation and column generation framework for the transmission switching and unit commitment problem. In this approach, each subproblem generates a feasible switching and unit commitment plan for a single time period, while the master problem makes a selection from the set of generated plans so as to minimize the total cost of generation. In this paper we disregard security constraints and other special constraints such as minimum up- and down time, ramp rate, and reserve constraints. Results show that employing active switching may reduce generation cost by up to 15 % and save generator startup costs. This is in line with results obtained in [7].

The paper is laid out as follows. Section 9.2 describes a deterministic minimum cost dispatch DC-approximated load flow model with transmission switching for a single time period. In section 9.3 we look at the multi-period problem with start-up costs and propose a Dantzig-Wolfe reformulation and column generation framework for finding (near-) optimal solutions. In section 9.4 we present some results for the IEEE 118-bus network. Section 9.5 concludes the paper and gives some directions for future research.

9.2 Optimal Dispatch with Transmission Switching and Unit Commitment

Consider an electricity transmission network, where N denotes the set of nodes (or buses) and E denotes the set of transmission lines (and transformers) connecting the nodes. Let $\mathcal{T}(i)$ denote the set of lines incident with node i where i is the head of the incident lines, and let $\mathcal{F}(i)$ denote the set of lines incident with node i, where i is the tail of the incident lines. So a line in $\mathcal{F}(i) \cap \mathcal{T}(j)$ is oriented *from i to j*.

Many transmission systems consist of alternating current circuits, interlinked by high voltage direct current links. We shall ignore these in this paper, and assume that all lines carry alternating current. The methodologies can easily be adapted to treat high voltage direct current lines as special cases.

Let G be the set of all generating units, that may offer electricity to the market. Furthermore, let $G(i)$ be the set of generating units located in (and supplying electricity to) node i. Each generating unit g offers a price c_g and a quantity u_g of energy to be generated. If the offer is accepted, unit g will deliver the quantity q_g to the market (assuming that a generator will never offer more than its generating capacity).

At each node i the demand d_i must be met. Load shedding at node i may be modelled by introducing a dummy generator g' offering the quantities $q_{g'}$ at the corresponding (sufficiently high) price $c_{g'}$ such that $q_{g'} \leq \max(0, d_i) = u^q_{g'}$.

Each transmission line $e \in E$ is characterized by its reactance R_e and thermal capacity u_e. The flow on line e is denoted x_e, which can be negative to model power flows in the direction opposite to the orientation of e. All lines are assumed to be switchable and may be taken out of operation in any given period of time. For each line $e \in E$, $z_e = 0$ denotes that the line has been switched out (opened), while $z_e = 1$ denotes that the switch is closed.

The minimum cost dispatch problem for a single period assuming no start-up cost may now be formulated as,

$$\min \sum_{g \in G} c_g q_g \tag{9.1}$$

s.t.

$$\begin{aligned}
l_g z_g \ &\leq q_g \leq u^q_g z_g, && \forall g \in G && (9.2)\\
\sum_{e \in \mathcal{T}(i)} x_e - \sum_{e \in \mathcal{F}(i)} x_e + \sum_{g \in G(i)} &q_g = d_i, && \forall i \in N && (9.3)\\
-u_e z_e \ &\leq x_e \leq u_e z_e, && \forall e \in E && (9.4)\\
z_e = 1 \ \Rightarrow \ &R_e x_e = \theta_i - \theta_j, && \forall e = (i,j) \in E && (9.5)\\
&q_g \geq 0, && \forall g \in G && (9.6)\\
&z_g, z_e \in \{0,1\}, && \forall g \in G, e \in E && (9.7)
\end{aligned}$$

The objective (9.1) minimizes the total generation costs respecting generation capacities and minimum generation on committed units (9.2), flow conservation (9.3), and thermal line capacity (9.4). For lines that are not switched out Kirchhoff's voltage law must be respected (9.5). Generation quantities are non-negative (9.6), and switching and unit commitment decisions are binary (9.7).

Since (9.2) - (9.7) is NP-hard [4], we may — for computational reasons — limit the number of lines to be switched simultaneously to at most k:

$$\sum_{e \in E} (1 - z_e) \leq k, \tag{9.8}$$

Note, that constraints (9.5) may be linearized using the big-M notation,

$$-M(1 - z_e) \leq R_e x_e - \theta_i + \theta_j, \ \forall e = (i,j) \in E \tag{9.9}$$
$$M(1 - z_e) \geq R_e x_e - \theta_i + \theta_j, \ \forall e = (i,j) \in E \tag{9.10}$$

where M is some sufficiently large number.

9.3 Multi-Period Formulation and Dantzig-Wolfe Reformulation

We now consider the problem of finding a minimum cost dispatch of generation and commitment of generator units in an electricity transmission network with active switching over several timeperiods.

Consider the discretized planning horizon Ω as a set of discrete time periods. For each period $\omega \in \Omega$, let $\mathcal{Q}(\omega)$ denote the set of feasible operational decisions (q, x, θ, z) satisfying constraints (9.2) - (9.8) . The multi-period model may now be formulated as,

$$\min \sum_{\omega \in \Omega} \left(f^\top y(\omega) + c(\omega)^\top q(\omega) \right) \tag{9.11}$$

s.t.

$$z_g(\omega) - z_g(\omega - 1) \ \leq \ y_g(\omega), \qquad \forall g \in G, \omega \in \Omega \tag{9.12}$$
$$(q(\omega), x(\omega), \theta(\omega), z(\omega)) \in \mathcal{Q}(\omega), \qquad \forall \omega \in \Omega \tag{9.13}$$
$$y(\omega) \in \{0,1\}^{|G|}, \qquad \forall \omega \in \Omega \tag{9.14}$$

The objective (9.11) minimizes the hourly fixed and operational cost, while (9.12) ensures that fixed unit commitment cost is incurred if the unit is on in the current time period ω and off in the previous time period $\omega - 1$. We assume, that the planning period is cyclic so that for the first element ω' of Ω, $\omega' - 1$ refer to the last element of Ω.

We propose a Dantzig-Wolfe reformulation of the multi-period model following the approach in [11]. First, let the binary vector $z(\omega)$ define a feasible switching and unit commitment plan (FSUP) for time period ω, if and only if, there exists $q(\omega), x(\omega), \theta(\omega)$ such that $(q(\omega), x(\omega), \theta(\omega), z(\omega)) \in \mathcal{Q}(\omega)$. The idea is to decompose the multi-period problem into a master problem and a number of subproblems — one for each time period. Each of the subproblems generate feasible switching and unit commitment plans for the corresponding time period, while the master problem chooses among the generated FSUP's and determines the optimal unit commitment strategy.

Now, let $Z(\omega) = \{\hat{z}^j(\omega) | j \in J(\omega)\}$ be the set of FSUP's in time period ω, where $J(\omega)$ is the index set for $Z(\omega)$. We can now write any element in $Z(\omega)$ as

$$\begin{aligned} z(\omega) &= \sum_{j \in J(\omega)} \varphi^j(\omega) \hat{z}^j(\omega), \\ \sum_{j \in J(\omega)} \varphi^j(\omega) &= 1, \quad \varphi^j(\omega) \in \{0,1\}, \ \forall j \in J(\omega) \end{aligned}$$

Assume that for each feasible switching and unit commitment plan $\hat{z}^j(\omega)$ the corresponding optimal dispatch of generation and load shedding is given by $\hat{q}^j(\omega)$. The master problem can now be written in terms of $\hat{z}$ and $\hat{q}$, that is

$$\min \quad \sum_{\omega \in \Omega} \left(f^\top y(\omega) + \sum_{j \in J(\omega)} c(\omega)^\top \hat{q}^j(\omega) \varphi^j(\omega) \right) \tag{9.15}$$

s.t.

$$\sum_{j \in J(\omega)} \varphi^j(\omega) = 1, \qquad [\mu(\omega)] \ \forall \omega \in \Omega \tag{9.16}$$

$$\sum_{j \in J(\omega)} \hat{z}^j(\omega) \varphi^j(\omega) - \sum_{j \in J(\omega-1)} \hat{z}^j(\omega-1) \varphi^j(\omega-1) \leq y(\omega), \quad [\pi(\omega)] \ \forall \omega \in \Omega \tag{9.17}$$

$$\varphi^j(\omega) \in \{0,1\}, \qquad \forall j \in J(\omega), \omega \in \Omega \tag{9.18}$$

$$y(\omega) \in \{0,1\}^{|G|}, \qquad \forall \omega \in \Omega \tag{9.19}$$

where $\mu(\omega)$ and $\pi(\omega)$ denote the dual prices for constraints (9.16) respectively (9.17).

It is convenient to consider only a subset $Z'(\omega) \subseteq Z(\omega)$ of feasible switching and unit commitment plans for each time period ω in the master problem. We define this restricted master problem (RMP) by (9.15) - (9.19) with $J(\omega)$ replaced by $J'(\omega)$ the index set of $Z'(\omega)$. A column generation algorithm is applied to dynamically add FSUP's to the linear relaxation of the master problem. The algorithm is initialized by letting $Z'(\omega) = \{z^0(\omega)\} = \{\mathbf{1}\}$, for all time periods $\omega \in \Omega$. That is, initially no line may be switched and all units are comitted in all time periods. The corresponding operational costs $c^\top(\omega)\hat{q}^0(\omega)$ can easily be found by solving a linear program for each time period. In each iteration of the algorithm the linear relaxation (RMP-LP) of RMP is solved yielding the dual prices μ, π. A new column $(c^\top(\omega)\hat{q}^j(\omega), 1, \hat{z}^j(\omega))$ may improve the solution of RMP-LP if and only if the associated reduced cost $\bar{c}(\omega) = c^\top(\omega)\hat{q}^j(\omega) - (\hat{\pi}^\top(\omega) - \hat{\pi}^\top(\omega+1))\hat{z}^j(\omega) - \hat{\mu}(\omega)$ is negative.

A column for time period ω may be constructed by solving the subproblem,

$$\begin{aligned} \min \quad & c^\top(\omega)q(\omega) - (\hat{\pi}^\top(\omega) - \hat{\pi}^\top(\omega+1))z(\omega) - \hat{\mu}(\omega) && (9.20)\\ \text{s.t.} \quad & (q(\omega), x(\omega), \theta(\omega), z(\omega)) \in \mathcal{Q}(\omega) && (9.21) \end{aligned}$$

where $\hat{\pi}(\omega)$ and $\hat{\mu}(\omega)$ are the dual prices returned from RMP-LP. Note, that each subproblem is in fact a network design problem with single commodity flow and Kirchhoff's voltage requirements (9.5) — see eg. [9].

Any feasible solution $(\hat{q}(\omega), \hat{x}(\omega), \hat{\theta}(\omega), \hat{z}(\omega)) \in \mathcal{Q}_\omega$ with negative objective function gives rise to a potential candidate column for RMP-LP. Hence, we do not rely on finding optimal solutions to the subproblems. Since our subproblems are NP-hard mixed integer programs (and potentially large for realistic size transmission networks) we may settle with suboptimal solutions in favour of generating more columns. In fact, it is not necessary to generate solutions to all the subproblems in each iteration, and hence we may postpone the generation of columns for subproblems, where a solution with negative reduced cost is not easily obtained. When all subproblems return solutions with non-negative reduced costs $\bar{c}$ we resolve all subproblems to optimality if necessary.

If no columns with negative reduced cost exist we have solved the relaxed master problem (MP-LP) to optimality. Furthermore, if the solution (φ^*, y^*) to MP-LP is integral, (φ^*, y^*) is an optimal solution to the master problem (9.15) - (9.19) and y^* is the optimal unit commitment strategy. Otherwise, we may resort to a branch-and-bound framework for finding optimal integral solutions or simply solve the integral RMP in the hope of finding a good feasible integer solution.

When the relaxed master problem MP-LP yields fractional solutions one may resort to branching to attain integral solutions. In this paper a crude one-level branching scheme, where we branch on one of the y-variables, is proposed to resolve fractionality of the relaxation. This does not guarantee optimal integral solutions in general, but yields feasible near-optimal solutions in practice. A branch-and-price scheme — in which one continue to branch on fractional variables until fractionality is resolved — may be employed to guarantee optimal integral solutions in general.

9.4 Computational Results

In this section, we apply the column generation algorithm proposed above to the IEEE 118-bus network [3] with network data described in [2]. This network has 185 lines, 19 generator units, total peak load of 4519 MW, and a total thermal generator capacity of 5859 MW. We consider the demand data [3] for day 2 (winter, weekend) with 24 hourly time periods and assume a minimum generation level of 20 % of capacity and start-up cost of \$ 10 for each of the 19 generator units.

The decomposition and models are formulated using the AMPL modelling language and all master- and subproblems are solved with CPLEX 12.2. The relaxed master problems are solved using CPLEX barrier algorithm (without applying crossover at the end), while the subproblems are solved using the CPLEX standard branch-and-bound algorithm. Computational experiments are performed on a 2.26 GHz Core 2 Duo computer with 4 GB RAM.

The power dispatch problem with unit commitment is solved for different values of k, where $k = 0$ characterizes the instance without switching. Table 9.1 shows solutions and running times for the column generation and branch-and-bound algorithm for problem instances with 24 (hourly) time periods. For the branch-and-bound algorithm CPLEX 12.2 was applied with default parameters.

Instance		Column generation					Branch-and-bound	
network	k	time (s)	time, sub (s)	abs. gap	ite.	col.	time (s)	abs. gap
IEEE118	0	46.6	46.0	0	16 $\star$	84	8	0
IEEE118	1	1996.1	1995.5	0	18 $\star$	80	-	1353.52
IEEE118	2	5531.3	5531.0	0	7 $\star$	70	-	2467.23
IEEE118	3	15681.4	15680.9	0	9 $\star$	69	-	2434.40
IEEE118	4	29665.3	29664.8	0	13 $\star$	83	-	2779.21
IEEE118	8	45553.0	45552.4	-	17 †	106	-	-

Table 9.1: Solution times and absolute gap to the optimal solution for problem instances with 24 time periods for the proposed column generation algorithm and standard branch-and-bound (CPLEX). For the column generation algorithm the time used in the subproblems, the number of iterations and the number of columns added is also shown. Instances marked by $\star$ are solved to integrality in the root node, while † denotes that optimisation was terminated without proving optimality. For $k > 0$ the branch-and-bound algorithm was terminated due to lack of memory.

The column generation algorithm solved all instances with $k \leq 4$ to optimality in the root node and hence no branching was needed. For $k = 8$ the algorithm terminated without proving optimality or even providing a lower bound. In general, integrality is not guaranteed and branching on fractional variables may be necessary to obtain integral optimal solutions. Only a small fraction of the total solve time used by the column generation algorithm is spent solving the master problems. The majority of the time is spent in the subproblems. Future research should be directed at solving the subproblems efficiently.

Without switching ($k = 0$) the branch-and-bound algorithm proved more efficient than column generation. However, for $k > 0$ the branch-and-bound algorithm ran out of memory and the best feasible solution returned was considerably worse than the solution returned by the column generation algorithm.

Table 9.2 shows the objective function value and number of start-ups in the

Instance		Best known solution	
network	k	value	start-ups
IEEE118	0	28128.35	8
IEEE118	1	26580.22	6
IEEE118	2	25898.55	7
IEEE118	3	25215.01	7
IEEE118	4	24978.30	7
IEEE118	8	23884.08 ‡	7

Table 9.2: Solution values and number of start-ups for the best known solution to instances with $k = 0, 1, 2, 3, 4, 8$. ‡ indicates that optimality was not proven.

best known solution for each instance. In the situation without switching the total generation and unit commitment cost incurred is \$ 28128.35 and a total of 8 start-ups are required. When allowing switching of at most four switches in each time period the total cost is reduced to \$ 24978.3. For $k = 8$ the total cost is further reduced to \$ 23884.08 with 7 generator start-ups.

9.5 Conclusion

In this paper we consider the problem of determining an optimal dispatch and unit commitment of power generation in a transmission network with active switching. We propose a Dantzig-Wolfe reformulation of the multi-period formulation into a master problem handling start-ups over the entire planning horizon and a number of subproblems each of which generates feasible unit commitment and switching patterns for a single time period. A column generation approach is outlined to solve the Dantzig-Wolfe reformulation.

The effect of allowing active switching in a setting with start-up costs on generator units is evaluated on the IEEE-118 bus network. Computational results show that over a particular 24-hour period total cost is reduced by up to 15 % and the number of start-ups are reduced by 1.

Furthermore, the proposed column generation algorithm is shown to be significantly more efficient than solving the problem using CPLEX standard branch-and-bound with default options. However, due to the computational complexity of the subproblems the algorithm spends the majority of the time solving the subproblems. Hence, further research should be directed at providing stronger formulations and more efficient solution methods for the subproblems, in order to improve the overall efficiency of the algorithm.

The model in this paper disregards security constraints, ramp rate constraints, and other generation specific constraints. Security constraints have been noted by Hedman et. al. [7] to increase the computational complexity of the problem significantly. Future research should investigate the impact of such constraints on the running times of the column generation algorithm.

The algorithm employed in this paper is a first step showing proof of concept — and it may not always return optimal integer solutions. Integrality may be ensured by employing a general branch-and-price scheme, where we continue branching on fractional variables until an integer solution is obtained. Optimizing the algorithm design may speed up solution times as well as ensure optimal solutions in general.

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Chapter 10

Modelling Zonal Pricing Design under Uncertainty in Electricity Markets

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In deregulated electricity markets with zonal pricing the market is partitioned into a number of zones, each of which is assigned a market price to which market participants react at any given point in time . We discuss the problem of designing such zones for a market subject to uncertainty. A two-stage stochastic program is presented and its complexity is discussed. In particular, we show that, when the stochastic parameters are independently distributed, the problem is #P-hard. Furthermore, the stochastic program contains integer variables. Hence, the problem is potentially difficult to solve. This motivates a Dantzig-Wolfe reformulation of the problem based on scenario decomposition, as we conjecture that for large instances decomposing the problem will lead to more efficient solution procedures. Finally, we present a formulation ensuring spatially contiguous zones.

10.1 Introduction

Deregulated electricity markets may employ different transmission pricing mechanisms. *Nodal pricing* refers to a system with individual market prices for each physical node in the network, whereas in *zonal pricing* the network is partitioned into zones and a market price is assigned to each zone. The partitioning of the network may be based on physical characteristics of the network (e.g. capacity constraints) as well as political (national borders) and organisational divisions. In this paper we shall not delve into the discussion on nodal versus zonal pricing (see e.g. [8] for a discussion), but rather assume that a zonal pricing regime is chosen exogenously. However, we may note that while nodal pricing may be optimal in a perfect market, zonal pricing may offer greater transparency to market participants and a greater sense of *fairness*.

We refer to the problem of determining optimal price zones as the zonal design problem. This involves allocating each node in the tranmsission network to a particular zone. We will here assume that the number of zones is fixed. The zonal design problem for a single period with linear marginal generation cost and demand curves has been treated in [2].

In general, the resulting zonal design must be static in the short to medium term, but may be changed in the medium to long term. Johnsen et. al. [9] report in 1999 that the Norwegian zonal system may be changed on a weekly basis. The nordic market pool operator, NordPool, announced that Sweden will change from a single zone to four zones (to better reflect bottlenecks in the transmission network) in 2011 following a 17 months notice [12]. Figure 10.1 illustrates the current (September 2011) zonal design of the Nord Pool Spot electricity market [13]. The stochastic nature of electricity systems means that a particular zonal design must accommodate a variety of supply and load conditions in the network as well as potential line failures. This leads us to propose a stochastic version of the zonal design problem, that maximises the expected social welfare of the system. That is, the total generation cost (and potential transmission cost) except total consumer benefits is minimised.

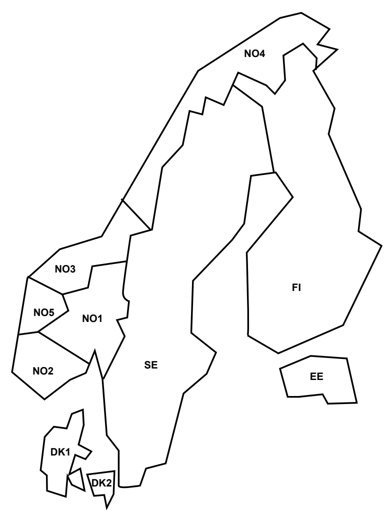


Figure 10.1: Schematic outline of the 10 price zones in the Nord Pool Spot market area comprising Norway (NO), Sweden (SE), Finland (FI), Estonia (EE), and Denmark (DK) as of September 2011.

Another attribute of price zones is contiguity. Often price zones are required to be spatially contiguous. That is, two nodes in the same zone must be connected by at least one path, that does not go through any other zone. When requiring that zones are contiguous with respect to the transmission network, the resulting problem is a graph clustering problem with an underlying equilibrium dispatch of electricity generation and network flow. Graph clustering problems have been studied within various applications for many years. For example, Augustson and Minker [1] explores clustering techniques for information retrieval systems.

The contribution of this paper is three-fold. Firstly, we present a linear mixed integer formulation of the deterministic zonal design problem. Secondly, a two-stage stochastic formulation based on scenario decomposition using a split variable approach (see e.g. [10]) is presented and we show that the stochastic problem is #P-hard when the stochastic parameters are independent. Thirdly, we provide a formulation that ensures spatially contiguous zones based on a minimum spanning forest formulation suggested by Martin [11] and show that this may lead to higher total generation cost.

We begin the paper by introducing the deterministic zonal design problem in section 10.2 and motivate the need for considering uncertainty. Subsequently, we state the stochastic version of the problem in section 10.3 and discuss its complexity. A Dantzig-Wolfe reformulation [5] and column generation framework for solving the stochastic problem more efficiently is suggested. In section 10.4 we provide a formulation ensuring spatially contiguous zones. Finally, some concluding remarks are given in section 10.5.

10.2 Model Formulation

We assume a linear direct current approximation of the optimal alternating current power flow (see e.g. [4, 14]) with linear generation costs and no line losses.

Consider the directed graph $G = (\mathcal{N}, \mathcal{A})$ with a source/sink node s. For each arc $a \in \mathcal{A}$, the cost, lower-, and upper bound on power flows, as well as reactance coefficients are given by c_a, l_a, u_a, and r_a, respectively. The flow on each arc $a \in \mathcal{A}$ is denoted by x_a, while w_i denotes the voltage phase angle for each node $i \in \mathcal{N}$. Let the set of arcs $\mathcal{F}(i)$, respectively, $\mathcal{T}(i)$ denote the set of arcs with tail, resp. , head i. Let the set of supply and demand arcs $\mathcal{S} = \mathcal{F}(s) \cup \mathcal{T}(s) \subseteq \mathcal{A}$ be defined by having s as the tail, respectively, head. Let the set of transmission arcs be denoted by $\mathcal{R} = \mathcal{A} \setminus \mathcal{S}$.

The economic dispatch of generation, consumption, and flows in the network, at any given time, may be found by solving the following linear program,

$$\min \sum_{a \in \mathcal{A}} c_a x_a \tag{10.1}$$

subject to

$$
\begin{aligned}
-x_a &\geq -u_a && (\lambda_a) && \forall a \in \mathcal{A} && (10.2)\\
x_a &\geq l_a && (\mu_a) && \forall a \in \mathcal{A} && (10.3)\\
\sum_{a \in \mathcal{F}(i)} x_a - \sum_{a \in \mathcal{T}(i)} x_a &= 0 && (\pi_i) && \forall i \in \mathcal{N} && (10.4)\\
r_a x_a + w_j - w_i &= 0 && (\gamma_a) && \forall a = (i,j) \in \mathcal{A} \setminus \mathcal{S} && (10.5)
\end{aligned}
$$

where symbols in parenthesis denotes dual prices and in particular π is a vector of nodal prices. The objective (10.2) maximises total social welfare. Constraints (10.2) and (10.3) provides upper, respectively, lower bounds on the arc flows, constraints (10.4) ensures conservation of energy, and constraints (10.5) is Kirchhoff's voltage constraints.

We now introduce a set of zones $\mathcal{K}$ and we wish to restrict the market prices so that the price in two nodes belonging to the same zone is equal. Let $\rho \in \mathbb{R}^{\mathcal{N}}$ denote the vector of market prices. In a nodal pricing regime we have $\rho = \pi$.

A vector $z \in \{0,1\}^{(|\mathcal{N}|-1)|\mathcal{K}|}$ of binary variables denotes the allocation of nodes to zones, such that $z_{ik} = 1$ if and only if node i belongs to zone k. That is,

$$
\begin{aligned}
z_{ik} = z_{jk} = 1 \Rightarrow \rho_i - \rho_j &= 0 && \forall i \neq j \in \mathcal{N} \setminus \{s\}, k \in \mathcal{K} && (10.6)\\
\sum_{k \in \mathcal{K}} z_{ik} &= 1 && \forall i \in \mathcal{N} \setminus \{s\} && (10.7)\\
z_{ik} &\in \{0,1\} && \forall i \in \mathcal{N} \setminus \{s\}, k \in \mathcal{K} && (10.8)
\end{aligned}
$$

Introducing some sufficiently large number M, we can rewrite (10.6) in a linear form as,

$$-M(2 - z_{ik} - z_{jk}) \leq \rho_i - \rho_j \leq M(2 - z_{ik} - z_{jk}) \quad \forall i \neq j \in \mathcal{N} \setminus \{s\}, k \in \mathcal{K} \tag{10.9}$$

We may without loss of generality allocate the first transmission node to the first zone.

For the market dispatch of generation (and consumption) to be feasible, we must ensure that if generation (consumption) is at the lower bound, then the cost is at least the price difference beween the end nodes (and profit is non-positive). Similarly, if generation (consumption) is at capacity, then the cost is at most the price difference and the corresponding profit is non-negative. Also, we must ensure that for a generator (demand segment) producing (consuming) strictly in the interval $]l_a, u_a[$ the price difference must equal the cost. Otherwise the generator (demand segment) would either increase or decrease generation (consumption). That is,

$$\begin{aligned} x_a = l_a \quad &\Rightarrow \quad c_a \geq \rho_j - \rho_i \qquad \forall a = (i,j) \in \mathcal{S} \\ l_a < x_a < u_a \quad &\Rightarrow \quad c_a = \rho_j - \rho_i \qquad \forall a = (i,j) \in \mathcal{S} \\ x_a = u_a \quad &\Rightarrow \quad c_a \leq \rho_j - \rho_i \qquad \forall a = (i,j) \in \mathcal{S} \end{aligned}$$

We can write this using the shadow prices λ_a and μ_a as,

$$c_a - \lambda_a + \mu_a = \rho_j - \rho_i \qquad \forall a = (i,j) \in \mathcal{S} \tag{10.10}$$

and complementarity constraints,

$$0 \leq \lambda_a \perp u_a - x_a \geq 0 \qquad \forall a \in \mathcal{S} \tag{10.11}$$

$$0 \leq \mu_a \perp x_a - l_a \geq 0 \qquad \forall a \in \mathcal{S} \tag{10.12}$$

In a nodal pricing scheme, constraint set (10.10) corresponds to the set of dual constraints associated with the flow variables x on supply and demand arcs with $\rho = \pi$. For simplicity, we may without loss of generality assume that $\rho_s = 0$.

The complementarity conditions (10.11) - (10.12) may be linearised (due to Fortuny-Amat [7]) by introducing new auxillary binary variables v_a^+ and v_a^- for each a in $\mathcal{S}$ and a sufficiently large constant M. That is, we can replace (10.11) - (10.12) by

$$u_a - x_a \leq M v_a^+ \qquad \forall a \in \mathcal{S} \tag{10.13}$$

$$\lambda_a \leq M(1 - v_a^+) \qquad \forall a \in \mathcal{S} \tag{10.14}$$

$$x_a - l_a \leq M v_a^- \qquad \forall a \in \mathcal{S} \tag{10.15}$$

$$\mu_a \leq M(1 - v_a^-) \qquad \forall a \in \mathcal{S} \tag{10.16}$$

$$\lambda_a, \mu_a \geq 0 \qquad \forall a \in \mathcal{S} \tag{10.17}$$

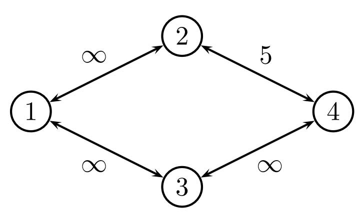
$$v_a^+, v_a^- \in \{0, 1\} \qquad \forall a \in \mathcal{S} \tag{10.18}$$

Now we can formulate the problem of finding optimal zones by minimising $\sum_{a \in \mathcal{A}} c_a x_a$ subject to the constraints $(10.2) - (10.5), (10.6) - (10.8), (10.10) - (10.12)$ or as the equivalent mixed integer linear program

$$\min \sum_{a \in \mathcal{A}} c_a x_a \text{ s.t. } (10.1) - (10.5), (10.7) - (10.9), (10.10), (10.13) - (10.18)$$

For notational convenience define the vector of binary variables $v = \begin{pmatrix} v^- \\ v^+ \end{pmatrix}$.

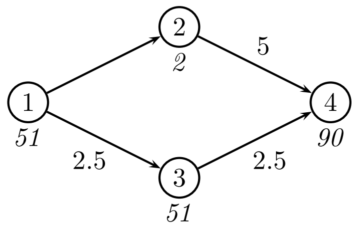
We will now look at a tiny instance with four transmission nodes, two generators, and two demands. The parameters are shown in Figure 10.2. All transmission arcs have reactance coefficient 1, zero cost and infinite capacities except for the arc from node 2 to 4, that has capacity 5. Figure 10.3 shows the optimal flow and prices in a nodal pricing scheme, while Figure 10.4 shows the flows and price for zonal pricing scheme with only one zone. Having a single price zone reduces the total social surplus of the system as well as the consumption in node 2.



	supply		demand	
node	1	2	2	4
c	51	2	-50	-90
u	20	10	5	15

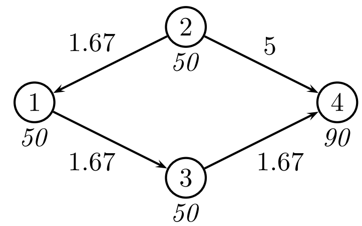
Figure 10.2: Small instance with four transmission nodes, two supply arcs, and two demand arcs. Arc labels $u = -l$ indicate capacities of the transmission arcs. All transmission cost and reactance coefficients are 0 respectively 1.

Usually, the zonal design is static in the short to medium term, while costs and capacities may vary over time. For instance, the capacity of a wind power generator varies from hour to hour with the wind velocity etc., while the cost of generation from a natural gas turbine varies with the market price on natural gas. Also, thermal transmission line capacities may vary over the year due to temperature differences. Hence, a good zonal design must be robust to such changes. The following two-period example illustrates the problem. The parameters of the example are shown in Figure 10.5. The transmission network consists of four nodes and four lines.



	supply		demand	
node	1	2	2	4
x	2.5	10	5	7.5

Figure 10.3: Optimal flow with nodal pricing (four zones). Arc labels indicate flows, while node labels indicate prices. Solution value is -777.50.



	supply		demand	
node	1	2	2	4
x	0	10	3.33	6.67

Figure 10.4: Optimal flow with two price zones consisting of the nodes $1, 2, 3$ respectively node 4. Arc labels indicate flows, while node labels indicate prices. Solution value is -746.67.

An optimal solution with two zones for each of the two periods is depicted in Figure 10.6. However, these solutions dictates a dynamic zonal allocation, since the low price zone consists of node 2 and 3 in period 1, while in period 2 it consists of node 1 and 2. Also, imposing the optimal zonal design obtained for period 1 will yield a suboptimal flow for period 2.

When we require the zonal allocation to be identical in the two scenarios, the total surplus decreases in scenario (a), while it remains the same in scenario (b). This is due to a reduction of consumption in node 2 and 4 and a reduction of generation in node 1. The result is shown in Figure 10.7

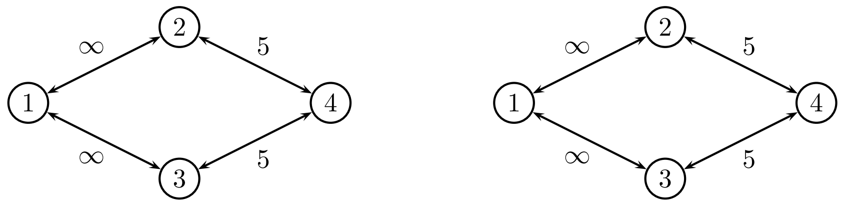
Based on these observations the optimal design of zones is not obvious. In the following section, we propose a two-stage stochastic programming formulation for the zonal design problem minimising the total expected cost over a number of scenarios.

10.3 A Two-Stage Stochastic Model

Now, we extend the problem of identifying optimal zones to a stochastic setting in which costs and capacities are not constant.

Let $z_{ik}(\omega)$ denote a *request* for node i to belong to zone k in scenario ω.

Consider a two-stage stochastic model, where the first stage decisions determine the zonal design, while the second stage models operational decisions (x, w, z, v) of dispatch and zonal allocation requests in a number of scenarios $\omega \in \Omega$, each occurring with probability $p(\omega)$. For each scenario $\omega \in \Omega$ let



	supply		demand	
node	1	2	2	4
c	51	2	-50	-90
u	20	10	5	15

(a)

	supply			demand		
node	1	2	3	1	2	4
c	2	2	51	-50	-50	-90
u	5	10	5	3	5	15

(b)

Figure 10.5: Small instance with two scenarios (a) and (b) and four transmission nodes. Arc labels show transmission capacities $u = -l$. All transmission reactance coefficients are 1. The tables show supply and demand arc coefficients. Lower bound on supply and demand is 0.

$$\begin{aligned}\mathcal{Q}(\omega) = \{&(x(\omega), w(\omega), z(\omega), v(\omega)) \mid \\ &(10.2) - (10.5), (10.7) - (10.9), (10.10), (10.13) - (10.18)\} \end{aligned} \tag{10.19}$$

be the set of feasible dispatch (and zonal design) solutions for a particular scenario ω.

The problem of identifying an optimal zonal design may now be formulated as

$$\text{SZDP:} \quad \min \quad \sum_{\omega \in \Omega} p(\omega) c(\omega)^\top x(\omega) \tag{10.20}$$

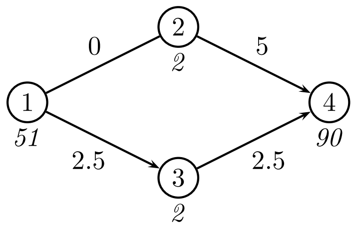
$$\text{s.t.} \quad z(\omega) = y \qquad \forall \omega \in \Omega \tag{10.21}$$

$$(x(\omega), w(\omega), z(\omega), v(\omega)) \in \mathcal{Q}(\omega) \qquad \forall \omega \in \Omega \tag{10.22}$$

The objective (10.20) minimises the expected operational costs, while (10.21) ensures that zone allocation is static (over all scenarios).

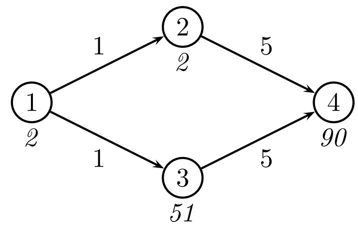
10.3.1 Complexity

Two-stage stochastic programs are in general #P-hard even when efficient algorithms exist for solving the single scenario problem. This is shown by Dyer and Stougie by reduction from the graph reliability problem for discrete probability



	supply		demand	
node	1	2	2	4
x	2.5	10	5	7.5

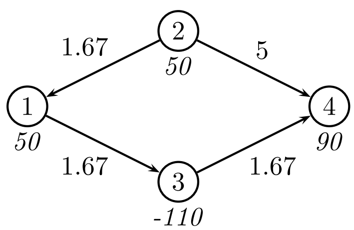
(a)



	supply			demand		
node	1	2	3	1	2	4
x	5	9	4	3	5	10

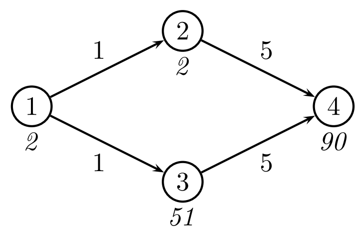
(b)

Figure 10.6: Resulting flows and prices when the scenarios are optimised separately with 3 price zones. Arc labels indicate transmission flows, while node labels indicate prices. Scenario (a) has an optimal cost of -777.50, while scenario (b) has optimal cost -1068.00.



	supply		demand	
node	1	2	2	4
x	0	10	3.33	6.67

(a)



	supply			demand		
node	1	2	3	1	2	4
x	5	9	4	3	5	10

(b)

Figure 10.7: Resulting flows and prices when the scenarios are co-optimised with 3 price zones. Arc labels indicate transmission flows, while node labels indicate prices. Scenario (a) has an optimal cost of -746.67, while scenario (b) has optimal cost -1068.00.

distributions and by reduction from the *volume of a knapsack polytope problem* for continuous probability distributions [6].

In the following we show that the stochastic zonal design problem is also #P-hard, which motivates the decomposition of the problem presented in the succeeding section. The proof relies on the stochastic parameters being independently distributed, which will lead to an exponential number of scenarios.

In many practical cases this may not hold. E.g. if the stochastic parameters represent capacity of wind turbines or level of water in hydro reservoirs across the network, these are likely to be highly correlated.

Define the graph $G' = (\mathcal{N}', \mathcal{R})$, where $\mathcal{N}' = \mathcal{N} \setminus \{s\}$.

Consider the problem of finding a path from i to j in G' following a random event which renders each arc in $\mathcal{R}$ unusable with probability 1/2 corresponding to the failure of a transmission arc. Furthermore, assume that arc failures are independently and identically distributed.

This corresponds to the following two-stage stochastic program SP with $2^{|\mathcal{R}|}$ scenarios, where each scenario ω in Ω corresponds to an outcome of the random event occuring with equal probabilities $p(\omega) = (1/2)^{|\mathcal{R}|}$.

$$\text{SP:} \quad \max \quad Z = \sum_{\omega \in \Omega} p(\omega) x_{a'}(\omega) \tag{10.23}$$

$$\text{s.t.} \quad 0 \leq x_a(\omega) \leq u_a(\omega) \quad \forall \omega \in \Omega, a \in \mathcal{A} \tag{10.24}$$

$$\sum_{a \in \mathcal{T}(i)} x_a(\omega) - \sum_{a \in \mathcal{F}(i)} x_a(\omega) = 0 \quad \forall i \in \mathcal{N}, \omega \in \Omega \tag{10.25}$$

SP is obtained from SZDP by setting the number of zones to the number of transmission nodes $|\mathcal{K}| = |\mathcal{N}'|$ so that each transmission node constitutes its own price zone. This makes the zonal pricing constraints (10.7) - (10.9), and the equilibrium constraints (10.10), (10.13) - (10.18) redundant. Furthermore, we let the reactance coefficients $r_a = 0$ for all transmission arcs a in $\mathcal{R}$. This eliminates the constraints (10.5), as the flows are decoupled from the voltage phase angles. For each scenario $\omega \in \Omega$, we set the lower bound on arc flows to $l_a(\omega) = 0$. Finally, the objective function is defined by a negative unit cost on supply $c_{a'}(\omega) = -1$, and $c_a(\omega) = 0$ for all $a \neq a' \in \mathcal{A}$ and $\omega \in \Omega$.

The graph reliability problem is defined as follows [6],

Definition 5 *Given a directed graph G and a pair of vertices (i, j). $R_{ij}(G)$ is an instance of the graph reliability problem defined by the problem of finding the probability that i and j are connected, if each arc fails independently with probability 1/2.*

Proposition 6 *SP is equivalent to the graph reliability problem.*

Proof. Take any instance $R_{ij}(G')$ of the graph reliability problem on the graph $G' = (\mathcal{N}', \mathcal{R})$. Add to G' the node s and the arcs $a' = (s, i)$ and $a'' = (j, s)$ and assign to them the fixed capacities $u_{a'} = u_{a''} = 1$. For all arcs a in $\mathcal{R}$ assign random capacities u_a, that are independent and identically distributed with discrete probability distribution $p(u_a = 0) = p(u_a = 1) = 1/2$. Define a set of scenarios Ω, such that each scenario ω in Ω corresponds to an outcome of the random vector u occurring with probability $p(\omega) = (1/2)^{|\mathcal{R}|}$. Let $u(\omega)$ denote the realisation of arc capacities u in scenario ω.

Suppose, that for a realisation of arc failures in the graph reliability instance corresponding to the scenario ω, there exist a path $\mathcal{P}$ from i to j. The corresponding partial solution $x(\omega)$ to SP is constructed by letting $x_{a'}(\omega) = x_{a''}(\omega) = 1$ and $x_a(\omega) = 1$ for all a in the path $\mathcal{P}$ and $x_a(\omega) = 0$ for all remaining arcs a in $\mathcal{R} \setminus \mathcal{P}$. Similarly, if for a realisation of arc failures in the graph reliability instance corresponding to the scenario ω, there does not exist a path $\mathcal{P}$ from i to j, the corresponding partial solution $x(\omega)$ is constructed by letting $x_a(\omega) = 0$ for all a in $\mathcal{R} \cup \{a', a''\}$. The combined solution for all realisations of arc failures yields the optimal solution x^* with value Z^* being the realibility of the graph reliability instance $R_{ij}(G')$.

Conversely, an optimal solution x^* to SP will have for each scenario ω in Ω, corresponding to some realisation of arc failures in the graph reliability problem, $x_{a'}(\omega) = 1$ if and only if the graph G' contains a path from i to j and $x_{a'}(\omega) = 0$, otherwise. Hence, the optimal value Z^* is the realiability of the graph G'. ■

It follows from Proposition 6 and the fact that the graph reliability problem is #P-hard [15], that SP is also #P-hard. Hence, SZDP is #P-hard.

10.3.2 Dantzig-Wolfe reformulation

We have shown in section 10.3.1 that the stochastic zonal design problem is #P-hard, and hence potentially hard to solve. In the following, we provide a Dantzig-Wolfe reformulation of the problem, that allows us to decompose the problem based on scenarios and solve it using column generation and branch-and-price. We conjecture that for large instances a decomposition of the problem will lead to more efficient solution procedures.

The Dantzig-Wolfe reformulation follows in the line of [16]. Let the binary vector $z(\omega)$ define a feasible zonal design (FZD) for scenario ω if there exists $x(\omega), w(\omega), v(\omega)$ such that $(x(\omega), w(\omega), z(\omega), v(\omega)) \in \mathcal{Q}(\omega)$. Now, let $Z(\omega) = \{\hat{z}^j(\omega) | j \in J(\omega)\}$ be the set of all FZD's for scenario ω, where $J(\omega)$ is the index set for $Z(\omega)$. We can write any element in $Z(\omega)$ as

$$
\begin{aligned}
z(\omega) &= \sum_{j \in J(\omega)} \varphi^j(\omega) \hat{z}^j(\omega) \\
\sum_{j \in J(\omega)} \varphi^j(\omega) &= 1, \quad \varphi^j(\omega) \in \{0,1\},\ \forall j \in J(\omega).
\end{aligned}
$$

Assume that for each feasible zonal design $\hat{z}^j(\omega)$ the corresponding optimal dispatch is given by $\hat{x}^j(\omega)$. The master problem can now be written in terms of $\hat{z}$ and $\hat{x}$ as

$$\text{MP: } \min \sum_{\omega\in\Omega} p(\omega) \sum_{j\in J_\omega} c(\omega)^\top \hat{x}^j(\omega)\varphi^j(\omega) \tag{10.26}$$

$$\text{s.t.} \quad \sum_{j\in J(\omega)} \varphi^j(\omega) = 1 \qquad [\nu(\omega)]\,,\ \forall\omega\in\Omega \tag{10.27}$$

$$y - \sum_{j\in J(\omega)} \hat{z}^j(\omega)\varphi^j(\omega) = 0 \qquad [\rho(\omega)]\,,\ \forall\omega\in\Omega \tag{10.28}$$

$$\varphi^j(\omega) \in \{0,1\}, \qquad j\in J(\omega) \tag{10.29}$$

$$y \in \{0,1\}^{|\mathcal{A}||\mathcal{K}|} \tag{10.30}$$

where $\nu(\omega)$ and $\rho(\omega)$ denote the dual prices associated with the respective constraints.

The master problem MP is a two-stage stochastic integer program with integer variables in both stages.

Proposition 7 *If y is chosen to be a fixed vector of binary integers, then the linear programming relaxation of MP has integer extreme points.*

For a proof we refer the reader to [16].

It is convenient to consider only a subset $Z(\omega)' \subseteq Z(\omega)$ of feasible zonal designs for each scenario ω in the master problem. We define this restricted master problem (RMP) by (10.26) - (10.30) with $J(\omega)$ replaced by $J(\omega)'$ the index set of $Z(\omega)'$. A column generation algorithm is applied to dynamically add FZD's to the linear relaxation of the master problem.

In each iteration of the algorithm, the linear relaxation (RMP-LP) of RMP is solved yielding the dual prices ν and ρ. A new column $(p(\omega)c(\omega)^\top\hat{x}^j(\omega), 1, \hat{z}^j(\omega))$ may improve the solution of RMP-LP if and only if the associated reduced cost $\bar{c}(\omega) = p(\omega)c(\omega)^\top\hat{x}^j(\omega) + \rho(\omega)^\top\hat{z}^j(\omega) - \nu(\omega)$ is negative.

A column for scenario ω may therefore be constructed by solving the subproblem:

$$\begin{array}{ll} \min & p(\omega)c(\omega)^\top x + \rho(\omega)^\top z - \nu(\omega) \\ \text{s.t.} & (x,w,z,v) \in \mathcal{Q}(\omega), \end{array}$$

where $\nu(\omega)$ and $\rho(\omega)$ are the dual prices returned from RMP-LP.

Any feasible solution $(x,w,z,v) \in \mathcal{Q}(\omega)$ with negative objective function gives rise to a potential candidate column for RMP-LP. If no columns with negative reduced cost exist then we have solved the relaxed master problem (MP-LP) to optimality. Furthermore, if the solution (φ^*, y^*) to MP-LP is integral then (φ^*, y^*) is an optimal solution to the master problem (10.26) - (10.30) and y^*

is the optimal zonal design. Otherwise, we may resort to a branch-and-price framework for finding optimal integral solutions. Note that a fractional solution will always have at least one fractional y-value (see Proposition 7). Hence, we branch on one of the fractional y-variables and hope that this will resolve the fractionality. If not, one may continue branching on y-variables until the fractionality is resolved.

10.4 Contiguous Zones

The formulation presented so far does not restrict zones to be spatially contiguous. This means that a feasible zone may consist of nodes that are separated by nodes from another zone. In this section we provide a spanning forest formulation that requires zones to be contiguous. The formulation is based on the minimum spanning tree formulation by Martin [11].

Let $\mathcal{H}_{\mathcal{K}}$ be a spanning forest of $|\mathcal{K}|$ trees on the graph $(\mathcal{N} \setminus \{s\}, \mathcal{A} \setminus \mathcal{S})$. We can now replace (10.6)-(10.8) by

$$a \in \mathcal{H}_{\mathcal{K}} \Rightarrow \rho_i - \rho_j = 0 \qquad \forall a = (i,j) \in \mathcal{A} \setminus \mathcal{S} \tag{10.31}$$

Let χ be binary vector defining a spanning forest on the transmission network. The following is due to Martin [11]. Arc a belongs to $\mathcal{H}_{\mathcal{K}}$ if and only if $\chi_a = 1$ and (χ, q) is a feasible solution to,

$$\sum_{a \in \mathcal{R}} \chi_a = |\mathcal{N}| - |\mathcal{K}| - 1 \tag{10.32}$$

$$\chi_a = q_{hij} + q_{hji} \qquad \forall h \in \mathcal{N}, a = (i,j) \in \mathcal{R} \tag{10.33}$$

$$\sum_{j \neq h} q_{hhj} \leq 0 \qquad \forall h \in \mathcal{N} \tag{10.34}$$

$$\sum_{j \neq i} q_{hij} \leq 1 \qquad \forall h \neq i \in \mathcal{N} \tag{10.35}$$

$$\chi_a, q_{hij}, q_{hji} \ \{0,1\} \qquad \forall h \in \mathcal{N}, a = (i,j) \in \mathcal{R} \tag{10.36}$$

where q is vector of binary auxillary variables.

We can now write (10.31) as,

$$-M(1-\chi_a) \leq \rho_i - \rho_j \leq M(1-\chi_a) \qquad \forall a = (i,j) \in \mathcal{A} \setminus \mathcal{S} \tag{10.37}$$

Example

For the purpose of illustration we consider in the following a single scenario instance of the zonal design problem on a network with 13 transmission nodes. The network is described in [3], however the generation data is modified to give interesting zonal designs. The topology of the transmission network is shown in Figure 10.8. All transmission line capacities are set to 55, that is $u_a = -l_a = 55$ for all arcs a in $\mathcal{R}$. Reactance coefficients are given in Table 10.1, while demand and generation is summarised in Table 10.2. Lower bound on generation for all generators is 0, that is $l_a = 0$ for all a in $\mathcal{S}$. We wish to find a partition of the nodes into three zones, that minimises the total generation cost of the system.

transmission arc		reactance
from	to	r_a
1	2	0.1515
1	5	0.1515
2	5	0.1887
2	4	0.1563
2	3	0.1020
3	4	0.1333
4	5	0.1515
4	7	0.1961
4	8	0.5263
7	8	0.1695
8	9	0.1099
9	10	0.1667
6	10	0.1887
5	6	0.2326
6	12	0.2703
12	13	0.3448
6	13	0.3125
11	12	0.4545
8	11	0.2632

Table 10.1: Reactance coefficients for 13 node transmission network.

node	demand	supply	
i	d_i	capacity	marg. cost
1	0.0	65	10
2	77.6		
3	7.8		
4	94.7		
5	7.6	200	20
6	11.2		
7	0.0		
8	29.5	200	40
9	9.0		
10	3.5		
11	6.1		
12	13.5	200	10
13	14.9		

Table 10.2: Supply and demand coefficients for the 13 node network.

An optimal zonal design for the 13 node instance, when zones are not required to be contiguous is shown in Figure 10.9 with a total generation cost of 3926.77, while Figure 10.10 shows an optimal design when contiguity is enforced yielding a total cost of 4150.24. When not requiring contiguous zones the optimal solution involves generation strictly within bounds for all generators (that is, $l_a < x_a < u_a$ for all a in $\mathcal{S}$), which requires that the corresponding zonal price equals the marginal generation cost. If zones must be contiguous, this is no longer possible (with only three zones). Hence, the generation pattern is changed shifting generation to nodes with higher cost generation, yielding a solution at a considerably higher cost.

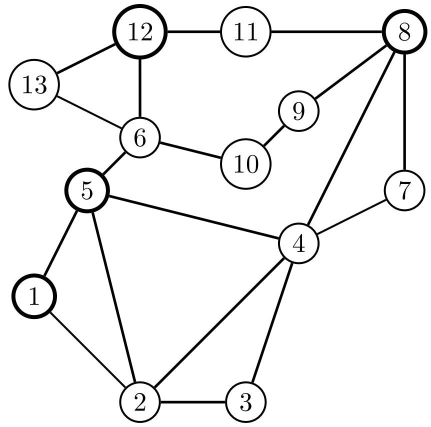


Figure 10.8: 13 node network with four generators located in the four emphasised (bold) nodes (1,5,8,12). All transmission line capacities are $u_a = -l_a = 55$. Supply arc capacities are all 200 except for the arc into node 1 which have $u_a = 65$.

10.5 Conclusion

In this paper, we have presented a linearised version of the stochastic zonal design problem and we have shown that when the stochastic parameters are independently distributed the problem is #P-hard. The complexity of the problem motivated a Dantzig-Wolfe reformulation based on a split variable approach. Finally, a formulation ensuring spatially contiguous zones based on a spanning forest is provided.

The Dantzig-Wolfe reformulation is prone to the symmetry of the zonal requests and we do not expect a column generation algorithm based on this formulation to be efficient unless this symmetry is broken. A similar Dantzig-Wolfe reformulation for the stochastic model with contiguous zones may be deduced based on using tree variables as master problem variables. However, this construction exhibits similar symmetry problems, as many trees may represent the same zone. One approach may be to define linking constraints between scenarios based on the arcs belonging to the cuts between zones as these will be uniquely determined.

Two-stage stochastic programs with integer variables are in general hard to solve due to both the non-convexities and potential explosion in the number of scenarios (as shown). However, it remains to be shown whether the integrality constraints yields the problem NP-hard. In practice, scenarios may be correlated and the resulting zonal design problem may not be #P-hard. Hence, further research should be dedicated to computational experiments to verify the efficiency of an algorithm based on the Dantzig-Wolfe reformulation to practical instances.

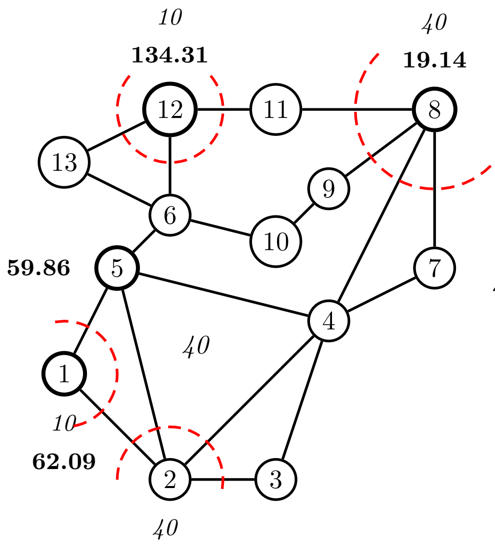


Figure 10.9: Resulting zonal design with three zones when zones are not required to be contiguous. Total generation cost is 3926.77. Dashed lines indicate zonal borders and italic font zonal prices.

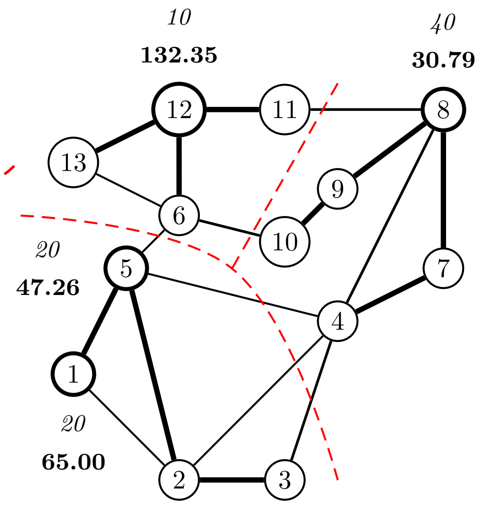


Figure 10.10: Resulting zonal design with 3 zones when zones must be contiguous. Bold lines indicate the forest defining the zonal design, while dashed lines indicate the corresponding zonal borders. Total generation cost is 4150.24.

If, indeed, the stochastic problem is difficult to solve for large instances with many scenarios, further research should be directed towards stronger formulations of the stochastic zonal design problem.

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Chapter 11

Capacity Expansion and Transmission Pricing in Natural Gas Networks

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For natural gas transmission networks in liberalised markets investments in pipeline capacity and decisions on the level of transmission tariffs collected are important and interrelated problems. In this paper we present a three-stage stochastic program determining an optimal investment strategy and transmission tariffs under uncertainty of supply, demand, transport costs, and pipeline capacity. The problem is formulated as a mixed integer linear program with equilibrium constraints and a numerical example justifies the model.

11.1 Introduction

In this paper we discuss the problem of determining an optimal pipeline capacity expansion strategy and transmission tariffs for natural gas transmission networks.

We will assume here that the natural gas transmission network is owned and operated by one or several transmission system operators (TSO's). If several TSO's exists each TSO exclusively own and operate his own part of the network. Producers deliver gas at entry points of the network, while consumers demand gas at exit points of the network. Transport customers purchase gas from producers at an entry point and deliver to consumers at an exit point. Transportation of gas across the transmission network accrues a fee for the use of the transmission network infrastructure payable to the TSO. The TSO's have two different, but interrelated decision problems: 1) *What should the physical transmission capacity be on each of the pipelines?* and 2) *What to charge transport customers for providing this capacity?*

New transmission capacity reduces congestion in the network and changes the cost of transporting gas across the network. Hence the decision in problem 1 affect the optimal decision in problem 2. Conversely, the price of transmission capacity determined in problem 2 might influence transport customers choice of where to purchase and sell natural gas and hence the need for capacity in different parts of the transmission network (problem 1). Thus, an integrated decision approach is desirable.

If the current capacity level is insufficient, capacity may be increased by one of the following means: Doubling of existing pipelines, installing compressors along an existing pipeline, or by construction of completely new pipelines. Each of these tasks involves a large fixed investment cost, which must be recovered by increased social welfare (aggregated demand utility except cost of the system).

A transmission pricing scheme may help to reduce congestion in the network and increase social welfare. In Europe, the most common transmission pricing scheme is that of entry-exit tariffs. Here, transport customers pay a tariff per unit energy entering the network at the entry point and another tariff per unit energy leaving the network at the exit point. The tariffs may differ between points in the network, but are not directly related to the distance or route of the gas being transported. Not knowing the network structure in detail, transport customers may choose combinations of entry and exit points that causes congestions in the network without being charged for this congestion. Furthermore, if entry and exit points are located in parts of the transmission network belonging to different TSO's, any compensation between the two TSO's and any intermediate TSO's must be worked out by the TSO's themselves and does not concern the transport customer.

In this paper, we assume – for simplicity – that there is only one TSO operating the entire transmission network and one transport customer handling all supplies and demands.

In recent years a number of equilibrium and optimisation models for deregulated natural gas markets has been proposed. In [5] Gabriel et. al. describe a mixed complementarity problem for modelling deregulated natrual gas markets. The North American and European markets are modelled and studied by Gabriel et. al. in [6] respectively by Egging et. al. in [2]. In [3] Egging et. al. presents a global natural gas model. Zhuang and Gabriel extends in [10] the equilibrium problem of natural gas market by introducing uncertainty. A stochastic portfolio optimisation model for a natural gas producer considering network effects and uncertainty in spot prices is presented by Midthun et. al. in paper 3 of Midthun's PhD. thesis [7]. In paper 4 of [7] Midthun et. al. propose an equilibrium model for transport booking in natural gas transmission networks. This is modelled using a stochastic complementarity problem, where each producer solves a two-stage stochastic optimisation problem. NATGAS [11] models the equilibrium of a European natural gas market, and allows continuous investments in pipeline capacity, LNG capacity, and storage by the network owner.

In this paper we consider a particular market structure in which transport customers are charged entry- and exit tariffs for transporting gas across the network. We present an optimisation model for a system operator and network owner for determining optimal discrete investments in pipeline capacity and transmission tariffs given equilibrium conditions governing demand for transport capacity. Our model differs from most of the existing models in that it provides optimal decisions for a system operator maximising total social welfare. To our knowledge it is the first model that considers the problem of choosing optimal tariffs — and to explore the interdependence between the tariffs chosen and the optimal capacity expansion strategy in deregulated natural gas markets.

We assume that the cost and capacity associated with supply and transport of natural gas – as well as the demand for natural gas – are subject to uncertainty. In the short term, demand varies seasonally with temperature (among others) while supply and transport capacity may be subject to contigencies (e.g. disruption of supply from a well or from a pipeline connecting neighbouring markets). In the long term (and perhaps to a lesser extent in the short term), supply and transport cost is subject to the general trend of the oil price on global markets (and the general global situation), while supply capacity is subject to the discovery of new natural gas reservoirs, and development of new — or delay or cancellation of already planned — pipelines for supplying gas from new markets.

This leads to a three-stage stochastic program in which the stochastic parameters are realised partly in the second stage, partly in the third stage. We assume that decisions on investments in pipeline capacity are taken in the first stage, while decisions on transmission tariffs are taken in the second stage, and the third stage models the economic dispatch of natural gas. The first and second stage decisions are assumed to be taken by a system operator (who also owns the network).

In the third stage operational dispatch decisions are taken by transport customers to maximise their profit of trading with gas and by the system operator minimising total transport cost. Figure 11.1 shows a scenario tree with uncertainty in the supply price and demand and with a number of contingency scenarios representing e.g. pipeline unavailability. The resulting deterministic equivalent is a mathematical program with equilibrium constraints, which can be formulated as a mixed integer linear program.

The model is described in section 11.2, while section 11.3 presents a numerical example for a small network. Section 11.4 provides some concluding remarks.

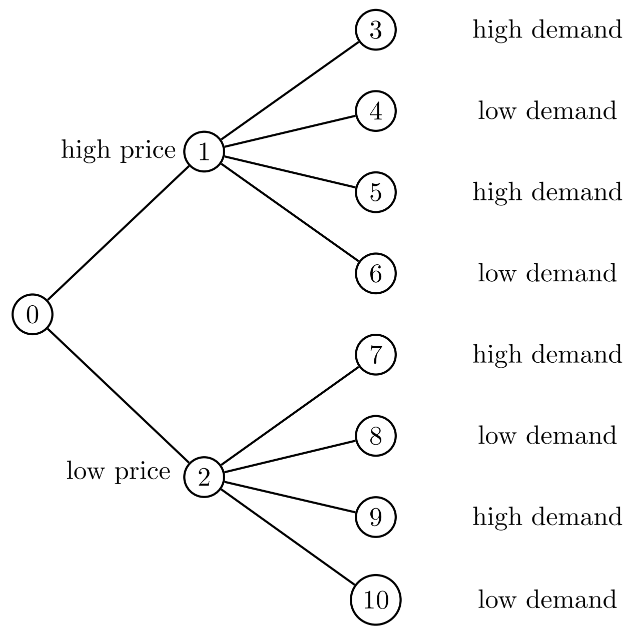


Figure 11.1: Example of a scenario tree. Branch (0,1) and (0,2) represents the realisation of a high respectively low natural gas supply price in the long term. Scenario 3,5,7, and 9 represents relisations of a high demand for natural gas e.g. winter. Scenario 3,4,7,8 may represent normal seasons, while 5,6,9,10 may represent contingency scenarios. Investment decisions are taken in the root node (0), while decisions on the transmission tariffs are taken in node 1 and 2.

11.2 Capacity Planning and Transmission Pricing under Uncertainty

Consider the network $G = (\mathcal{N}, \mathcal{A})$. Where $\mathcal{N}$ is the node set representing physical nodes in a natural gas transmission network and a source/sink node s. Let the arcs in $\mathcal{A}$ be partitioned into two exclusive subsets $\mathcal{S}$ and $\mathcal{T}$, where $\mathcal{S} = \{a = (i,j) \in \mathcal{A} | i = s \vee j = s\}$ is the set of supply and demand arcs representing supply of – and demand for – natural gas and the transmission arcs $\mathcal{T}$ represents physical properties of the transmission network (pipelines, compressors, etc.). Define by $\mathcal{F}(i) \subseteq \mathcal{A}$ and $\mathcal{H}(i) \subseteq \mathcal{A}$ the set of edges with tail, respectively head, i.

Denote by x_a for all $a \in \mathcal{A}$ the physical flow of energy in the transmission network (for $a \in \mathcal{T}$) and supply and consumption of energy (for $a \in \mathcal{S}$). For all $a \in \mathcal{S}$, let c_a be the cost (negative benefit) of supplying (consuming) one unit of energy along supply (demand) arc a in S, and let u_a be the maximum supply (consumption). This allow us to model a piecewise linear generation cost (inverse demand) function at each node. For all $a \in \mathcal{T}$, let c_a denote the cost of transporting one unit of energy along arc a (e.g. cost for use of compressors along a pipeline).

The flow x_a along a pipeline $a = (i, j)$ depends on the pressure w_i, w_j at the two end nodes, and is often modelled by the Weymouth equation (see e.g. [1, 8]):

$$x_a = k_a \sqrt{w_i^2 - w_j^2} \qquad \forall a = (i, j) \in \mathcal{T} \tag{11.1}$$

where k_a is the Weymouth constant describing physical properties of the pipeline (including length and diameter).

As in [9] we approximate the capacity of each pipeline $a \in \mathcal{T}$ by a set $\mathcal{L}$ of linear inequality constraints with additional parameters $k'_{a,m}, k''_{a,m}$ based on selected pairs of input and output pressures. This linearisation, however, relaxes the equality of (11.1) to an inequality so that the flow on a pipeline is not uniquely determined by the pressure difference. Let $a = (i, j)$ denote an arc representing a pipeline, as explained in paper 2 of [7] the relaxation is equivalent to allowing a pressure drop between the tail node i and the inlet of the pipeline. We refer the reader to [9] for a description of the linearisation.

For all transmission nodes i in $\mathcal{N} \setminus \{s\}$, let w_i denote the pressure and let l_i and u_i denote lower and upper bounds on the pressure.

Assume for simplicity of the model that sets of entry and exit nodes are disjunct and let $\mathcal{M} = \{i \in \mathcal{N} \setminus \{s\} | (\mathcal{F}(i) \cup \mathcal{H}(i)) \cap \mathcal{S} \neq \emptyset\}$ denote the set of entry and exit nodes in the network and the source node s. Let t_i denote the tariff for entry respectively exit of energy at node i.

In the following we consider the problem of installing new pipeline capacity and determine optimal transmission tariffs in a natural gas network to minimise capital cost and expected operating cost over a number of scenarios denoted $\omega \in \Omega_3$.

We assume that investment decisions y are taken first. Subsequently, some of the stochastic parameters are realised. Then decisions on transmission tariffs t are taken prior to the realisation of the remaining stochastic parameters. Finally, the economic dispatch of natural gas is carried out. This leads to a three-stage scenario tree in which the root node corresponds to the first stage. We denote by Ω_2 the set of scenario tree nodes at the second stage, and by Ω_3 the leaf nodes of the scenario tree. Let $\rho(\omega) \in \Omega_2$ denote the predecessor node of node ω for all nodes ω except the root node. For $\omega \in \Omega_2$, $t(\omega)$ denotes the transmission tariffs at node ω.

11.2.1 Transport Customer

For the purpose of simplicity, we assume that all transport customers are represented by one aggregate transport customer maximising total profit.

In each leaf node $\omega \in \Omega_3$ of the scenario tree the problem of the transport customer is to determine for all supply arcs $a \in \{(i,j) \in \mathcal{S} | i = s\}$ the amount of energy x_a to be purchased at entry node j and for all demand arcs $a \in \{(i,j) \in \mathcal{S} | j = s\}$ the amount of energy x_a to be delivered at exit node i in the network given the transmission tariffs $t(\rho(\omega))$ of the predecessor node of ω. Let $k(\omega)$ denote the realisation of supply arc capacities and maximum demand k and let $c(\omega)$ be the realisation of supply costs and negative demand utility c in leaf node ω. We assume that $t_s(\omega) = 0$ is fixed for all $\omega \in \Omega_2$.

$$\mathrm{TC}(\omega): \ \min \sum_{a=(i,j)\in\mathcal{S}} (c_a(\omega) + t_i(\rho(\omega)) + t_j(\rho(\omega)))x_a \tag{11.2}$$

subject to

$$-x_a \geq -k_a(\omega) \qquad (\lambda_a) \qquad \forall a \in \mathcal{S} \tag{11.3}$$

$$\sum_{a\in(\mathcal{R}\cup\mathcal{S})\cap\mathcal{F}(i)} x_a - \sum_{a\in(\mathcal{R}\cup\mathcal{S})\cap\mathcal{H}(i)} x_a = 0 \qquad (\pi_i) \qquad \forall i \in \mathcal{M} \tag{11.4}$$

$$0 \leq x_a \qquad \forall a \in \mathcal{R}\cup\mathcal{S} \tag{11.5}$$

where λ and π denote dual variables associated with the respective constraints. The transport customer must minimise his total cost of purchase, delivery and transport (11.2), respecting supply capacity and maximum demand (11.3), and conservation of energy in entry and exit nodes (11.4). We assume without loss of generality, that arc flows are non-negative (11.5).

We may now write the conditions for optimality and feasibility of the transport customers problem (11.2) - (11.5) as,

$$0 \leq x_a \perp \ \lambda_a + \pi_i - \pi_j + c_a(\omega) + t_i(\rho(\omega)) + t_j(\rho(\omega)) \geq 0 \quad \forall a = (i,j) \in \mathcal{S} \tag{11.6}$$

$$0 \leq x_a \perp \qquad \pi_i - \pi_j \geq 0 \quad \forall a = (i,j) \in \mathcal{R} \tag{11.7}$$

$$0 \leq \lambda_a \perp \qquad k_a(\omega) - x_a \geq 0 \quad \forall a \in \mathcal{S} \tag{11.8}$$

$$\sum_{a\in(\mathcal{R}\cup\mathcal{S})\cap\mathcal{F}(i)} x_a - \sum_{a\in(\mathcal{R}\cup\mathcal{S})\cap\mathcal{H}(i)} x_a = 0 \qquad \forall i \in \mathcal{M} \tag{11.9}$$

Using Fortuny-Amat [4] we can linearise (11.6) - (11.8) by introducing binary variables v and z indicating the binary decision to set the left, respectively right, operand of the $\perp$-operator to zero. For instance, for constraints (11.8) we have $v_a = 0 \Rightarrow \lambda_a = 0$ and $v_a = 1 \Rightarrow x_a = k_a(\omega)$.

$$
\begin{aligned}
&0 \le x_a && \le k_a(\omega) z_a && \forall a \in \mathcal{S} && (11.10)\\
&0 \le x_a && \le M z_a && \forall a \in \mathcal{R} && (11.11)\\
&0 \le \lambda_a + \pi_i - \pi_j + && && && \\
&\qquad t_i(\rho(\omega)) + t_j(\rho(\omega)) + c_a(\omega) && \le M(1 - z_a) && \forall a = (i,j) \in \mathcal{S} && (11.12)\\
&0 \le \pi_i - \pi_j && \le M(1 - z_a) && \forall a = (i,j) \in \mathcal{R} && (11.13)\\
&0 \le \lambda_a && \le M v_a && \forall a \in \mathcal{S} && (11.14)\\
&0 \le k_a(\omega) - x_a && \le k_a(\omega)(1 - v_a) && \forall a \in \mathcal{S} && (11.15)
\end{aligned}
$$

where M is a sufficiently large number.

11.2.2 System Operator

In each leaf node $\omega \in \Omega_3$ of the scenario tree we have a realisation of transport costs $c(\omega)$ and the Weymouth constant $k(\omega)$. Note, that k is normally deterministic. However, we may choose $k_a(\omega) = 0$ to model disruption of pipeline capacity in a contingency scenario. The problem of the system operator is to determine energy flows x_a on all transmission arcs $a \in \mathcal{T}$ and pressure w_i in all transmission nodes $i \in \mathcal{N} \setminus \{s\}$ given flows x_a on all supply and demand arcs $a = (i,j) \in \mathcal{S}$ and the current configuration of the transmission network y.

$$
\mathrm{SO}(\omega): \ \min \sum_{a \in \mathcal{T}} c_a(\omega) x_a \tag{11.16}
$$

subject to

$$
x_a \le k_a(\omega) \left(k'_{a,m} w_i - k''_{a,m} w_j \right) y_a \qquad \forall a = (i,j) \in \mathcal{T}, m \in \mathcal{L} \tag{11.17}
$$

$$
\begin{aligned}
\sum_{a \in (\mathcal{S} \cup \mathcal{T}) \cap \mathcal{F}(i)} x_a &= \sum_{a \in (\mathcal{S} \cup \mathcal{T}) \cap \mathcal{H}(i)} x_a && \forall i \in \mathcal{N} \setminus \{s\} && (11.18)\\
l_i \le w_i &\le u_i && \forall i \in \mathcal{N} && (11.19)\\
0 &\le x_a && \forall a \in \mathcal{T} && (11.20)
\end{aligned}
$$

The system operator must minimise operational transport cost (11.16) respecting transmission arc capacities (11.17), conservation of energy (11.18), and bounds on pressure (11.19). We assume that the pipeline capacity is linearised and is a function of the pressures in the two end nodes and that $\mathcal{L}$ is the index set of the linearisation constraints for each pipeline. See e.g. [9] for an example on how to linearise the Weymouth equation. Furthermore, we assume without loss of generality that pipeline flows are non-negative (11.20).

Equations (11.17) have bi-linear terms. We may linearise the capacity constraints by replacing (11.17) by

$$x_a \leq k_a(\omega)\left(k'_{a,m}w_i - k''_{a,m}w_j\right) \qquad \forall a = (i,j) \in \mathcal{T}, m \in \mathcal{L} \tag{11.21}$$

$$x_a \leq M y_a \qquad \forall a \in \mathcal{T} \tag{11.22}$$

where $M \geq k_a(\omega)\left(k'_{a,m}w_i - k''_{a,m}w_j\right)\ \forall a = (i,j) \in \mathcal{T}, m \in \mathcal{L}$.

Prior to the operational dispatch of gas the system operator may invest in new pipelines at a fixed cost and set transmission tariffs at entry and exit nodes. We assume that investment decisions are taken first and decisions on transmission tariffs are taken secondly after the realisation of some of the stochastic parameters. This leads to a three-stage stochastic model, where the first stage decisions involve pipeline investments y and second stage decisions involve tariffs t for each scenario tree node $\omega \in \Omega_2$ occuring with probability $p(\omega)$. The third stage problem SO(ω) models operational decisions (x, w) for dispatch of natural gas in each leaf node $\omega \in \Omega_3$ occuring with probability $p(\omega)$. For each leaf node $\omega \in \Omega_3$, let

$$\mathcal{X}(\omega) = \{(x, w, \lambda, \pi) | (11.16) - (11.20), (11.6) - (11.9)\}$$

be the feasible set of arc flows (representing supply, demand, and pipeline flows), node pressures, capacity-, and nodal prices.

The system operators problem of determining an optimal investment strategy and optimal transmission tariffs under uncertainty, may now be formulated as

$$\text{SO1}: \qquad \min f^\top y + \sum_{\omega \in \Omega} p(\omega)p(\rho(\omega)) \sum_{a \in \mathcal{T}} c_a(\omega) x_a(\omega) \tag{11.23}$$

$$\text{s.t.} \quad (x(\omega), w(\omega), \lambda(\omega), \pi(\omega)) \in \mathcal{X}(\omega) \qquad \forall \omega \in \Omega_3 \tag{11.24}$$

$$t_i(\omega) \geq 0 \qquad \forall i \in \mathcal{M} \setminus \{s\}, \omega \in \Omega_2 \tag{11.25}$$

$$y \in \{0,1\}^{|\mathcal{T}|} \tag{11.26}$$

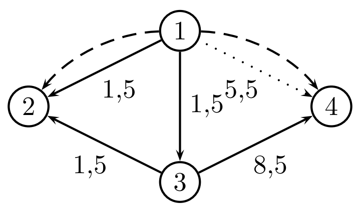
11.3 Numerical Example

We will now turn our attention to a small example illustrating the problem. For the sake of simplicity we will assume that there is no uncertainty in the second stage. So all the stochastic parameters are realised in the third stage after decisions on transmission tariffs are already made, that is $|\Omega_2| = 1$ and $p(\omega) = 1$ for $\omega \in \Omega_2$.

Consider the transmission network shown in Figure 11.2 with four pipelines, one entry point (and one supplier) and two exit points (and three demand segments). For simplicity of the example we assume that all pipeline capacities are approximated by one linear constraint with $k'_a = k''_a = 1$ for all pipelines $a \in T$, i.e. the capacity of a pipeline $a = (i, j)$ is the Weymouth constant k_a multiplied by the pressure difference between the two nodes i and j.

We consider two scenarios $\omega = 1, 2$ with variable supply capacity and demand utility. In scenario 1, supply capacity is 10, and demand utility is 9 in node 4, and 20 (high value demand), respectively, 5 (low value demand) in node 2. In all examples we assume the entry tariff $t_1 = 0$ is fixed.

First, let us look at the case without any potential transmission network improvements. The problem of the TSO is to determine the optimal entry tariff t_1 and exit tariffs t_2, t_4 to charge transport customers in order to minimise total transport cost of the network.



	supply	demand		
node	1	2	2	4
$c(1)$	1	-5	-20	-9
$u(1)$	10	5	5	5
$c(2)$	1	-4	-20	-10
$u(2)$	15	5	5	5

Figure 11.2: An illustrative example of a small natural gas network with one entry point (node 1) and two exit points (nodes 2 and 4). Cost (benefit) and capacity of injection (extraction) are given in the table for scenario $\omega = 1, 2$. Solid arcs represent transmission pipelines, while dashed arcs represent trading arcs. The dotted arc represents a potential new pipeline. Transmission arc labels indicate acutal system cost and Weymouth coefficient, respectively, of the arc flow.

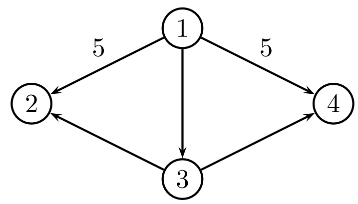
We first consider only scenario 1. In Table 11.1, three different pricing strategies are considered: No tariffs, single exit-zone with same tariff ($t_2 = t_4$), and differentiated exit tariffs. For simplicity of the example entry tariff is assumed to be 0. Without any transmission tariffs, the optimal strategy of the transport customer is to purchase 10 units and deliver 5 units to node 4, and 5 units to node 2, which will yield a profit of $5(9-1)+5(20-1) = 135$. The corresponding social welfare (total utility less actual supply and transport costs) is 85. Since, transport costs to node 4 is higher than the profit margin of the transport customer, it is not desirable from a system perspective to have flow on this path. Introducing a high exit tariff in node 4 forces the transport customer to consider a strategy only serving demand node 2. In a naive strategy where all exit tariffs are equal (to 8), the transport customer chooses to send 5 units to node 2 and nothing to node 4. This reduces his profit to 55, but increases social welfare to 90. In a situation with differentiated exit tariffs it may be possible to increase flow to 10 units (delivering 5 units to a low value demand segment), which increases transport customers profits to 115 and social welfare to 100.

tariffs			transport customer		
t_1	t_2	t_4	strategy	profit	social welfare
0	0	0	(0,5,5)	135	85
0	8	8	(0,5,0)	55	90
0	0	8	(5,5,0)	115	100
0	0	8	(5,5,0)	115	100
0	0	0	(0,5,5)	135	105
0	0	4	(0,5,5)	115	105
0	4	4	(0,5,5)	95	105
0	4	8	(0,5,5)	75	105

Table 11.1: Different entry-exit pricing strategies and their effect on a transport customer buying energy at node 1 and selling it at node 2 and 4 in scenario 1. For three different pricing strategies, the corresponding optimal strategy of the transport customer (the amount to sell at node 2 respectively node 4) and his profit is given along with the total social welfare of the system. The first three rows correspond to the network without network expansion, while the bottom five rows correspond to a situation with a new pipeline $(1, 4)$ (social welfare does not include investment costs).

Now, assume that the system operator may invest in new pipelines. Consider the option of constructing a new pipeline from node 1 to node 4 with operational unit cost 5. This will increase the throughput capacity, but also change the optimal transmission pricing strategy. Consider the best pricing strategy of Table 11.1 without the new pipeline ($t_2 = 0$, $t_4 = 8$). The best response of the transport customer (5,5,0) now yields a suboptimal solution. Changing the tariffs may increase social welfare to 105. The resulting flows are shown in Figure 11.3.

Consider now scenario 2. Table 11.2 shows the effect of different transmission pricing strategies for scenario 2 with and without network expansion of line $(1, 4)$.



	supply	demand		
node	1	2	2	4
x	10	0	5	5

Figure 11.3: Flows in the network with new pipeline from node 1 to node 4 and exit tariffs ($t_2 = 4$, $t_4 = 8$).

Without capacity expansion, the optimal pricing strategy for scenario 1 $(t_2, t_4) = (0, 8)$ is suboptimal for scenario 2. Increasing the exit tariff t_2 to 3 results in the social optimal welfare 100. A pricing strategy with $t_2 = 0$ and $t_4 < 11$ is infeasible as the optimal response of the transport customer is to purchase 15 units in node 1, and the maximum transport capacity out of node 1 is 10.

tariffs			transport customer		
t_1	t_2	t_4	strategy	profit	social welfare
0	0	0	(5,5,5)	-	inf.
0	0	11	(5,5,0)	110	95
0	3	3	(5,0,5)	120	100
0	3	11	(5,0,5)	80	100
0	0	0	(5,5,5)	165	125
0	0	11	(5,5,5)	110	125
0	4	11	(5,0,5)	75	120

Table 11.2: Different entry-exit pricing strategies and their effect on a transport customer buying energy at node 1 and selling it at node 2 and 4 in scenario 2. For different pricing strategies, the corresponding optimal strategy of the transport customer (the amount to sell at node 2 respectively node 4) and his profit is given along with the total social welfare of the system. The first four rows correspond to the network without network expansion, while the bottom five rows correspond to a situation with a new pipeline $(1, 4)$ (social welfare does not include investment costs).

These examples illustrate the need for an integrated approach when determining optimal transmission investments and pricing strategies for a natural gas transmission network.

11.4 Conclusion

This paper proposes a three-stage stochastic model to determine optimal investment strategies in pipeline capacity and transmission tariffs in natural gas transmission networks. Uncertainty in supply- and transport costs and capacities, as well as demand, are modelled by stochastic parameters. In this way, one may model seasonal variations of demand, long term uncertainty about the price and availability of natural gas, and even disruptions in supply and pipeline capacity. We assume a market structure in which a system operator collects entry-exit tariffs from transport customers maximising total social welfare. A numerical example justifies the use of an integrated model for determination of pipeline capacity and transmission tariffs.

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Appendix A

Modelling Hydroelectric Power Producers Strategic Use of Water Reservoir

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Strategic behaviour of suppliers in an electricity system may influence market prices and the level and distribution of social welfare. We formulate a model of an oligopolistic electricity market with energy storage in which suppliers determine their bid prices in order to optimise their profit over several time periods. In particular, a hydropower producer's strategic use of water reservoirs is analysed. Preliminary results suggests that the use of market power at a hydro power storage may alter the equilibrium and social welfare of the system significantly. The value of the social welfare is the basis for many transmission infrastructure decisions in the nordic area, where hydro power is dominant. We therefore believe that temporal market power deserves a more thorough investigation.

Keywords: Market power, Electricity Networks, Hydro power, Game theory, Mathematical Program with Equilibrium Constraints

JEL Classification: L94, C72

A.1 Introduction

Changes in total social welfare is used by several (public) electricity transmission network owners as indicator for investments in transmission infrastructure. Also, private production companies anticipating changes to their porfolio of production units need to analyse changes in their profit, given changed market conditions. Models calculating market prices and social welfare given assumptions about the infrastructure, composition of supply, and demand characteristics are therefore essential in order to identify optimal investments.

Because of the complexity of electricity systems, traditional models often exclude dominant electricity suppliers potential use of strategic behaviour to influence market prices (e.g. [7]). However, recent advances in solution procedures ([3], [4], [6]) has spurred a renewed interest in the area and several papers propose models that include strategic behaviour of dominant suppliers in a single time period [2], [4], [8], [9]. Johnsen [5] presents a numerical two-period model of strategic behaviour in a hydro power system with transmission constraints and stochastic water inflows, while Bushnell [1] presents a more rigorous modelling framework, but excludes transmission constraints.

In this paper, we propose a model with multiple time periods that is able to capture dominant suppliers use of market power across transmission lines (spatial marketpower), as well as across different time periods (temporal marketpower). The model allows for suppliers to bid their price in the market following the approach in [4] maximising their total profit over the planning horizon. This makes it possible to analyse e.g. a hydropower producer's strategic use of water reservoirs for storage of energy and the influence on market prices and social welfare.

We assume that supply units have constant marginal costs, while demand is modelled by a linear downward sloping inverse demand function. Energy may be transported between price areas observing a simple constant capacity. At certain areas energy may furthermore be stored between time periods.

Preliminary results on a small generated case show that including temporal market power in the model yields significantly different results with respect to the market prices, total social welfare, and the distribution of welfare between suppliers and consumers. However, further analyses are needed to investigate this result in a realistic setting.

A.2 Model Description

We model the electricity system by a network of nodes representing price areas, transmission links connecting the nodes, and supply and demand units located at each node. The supply units represent, for example, power plants supplying energy to the system, while demand units represent end consumers. The model considers multiple time periods (e.g. hours) and at certain nodes temporal storage of energy is possible.

A.2.1 Notation

Each supply unit s produces in time period t a maximum of u^S_{st} units of electricity x^S_{st} at a constant unit cost c^S_s. The set of supply units is denoted by S, while the set of supply units located at node $i \in N$ is given by $S_i \subseteq S$. Each supply unit is owned by exactly one firm $h \in H$, where H is the set of all firms, and S_h denotes the set of supply units owned by h.

The benefit of consumers at demand unit d is given by the constant utility f_{dt}. x^D_{dt} is the consumption of demand unit d in time period t and is limited by the maximum consumption u^D_{dt}. The set of demand units is denoted D and the set of demand units in node i is given by $D_i \subseteq D$.

In each time period t, each directed transmission link (i, j) connecting node i with node j transports x^A_{ijt} units of energy at a unit cost of c^A_{ij} and has a capacity of u^A_{ij}. The set A denotes the set of all transmission links. A link may be owned by the system operator or a firm. The set of links owned by the system operator is denoted by A_Q, while A_h denotes the set of links owned by firm $h \in H$.

x^R_{it} denotes the amount of energy stored at node i from time period t to $t+1$ at a cost of c^R_i per unit energy. The capacity of the storage is denoted by u^R_i. The set of nodes where storage is possible is denoted by $R \subseteq N$, and the set $R_Q \subseteq R$ ($R_h \subseteq R$) denotes the set of nodes at which the storage is owned by the system operator (firm h).

The initial level of each storage reservoir i is assumed to be fixed and is denoted by x^R_{i0}.

A.2.2 The system operator problem

In this paper, we assume that a system operator operates part of the transmission network including storages that is not owned by supply firms. That is, the system operator purchases energy at nodes (markets) in the network and delivers energy to the consumers maximising social welfare ensuring that the market clears in each node and that transmission capacity is respected.

The optimisation problem of the system operator is thus the sum of consumer surplus (net benefit) and trading surplus on links and storages owned by the system operator, i.e.

$$\max \sum_{t\in T}\left[\sum_{i\in N}\sum_{d\in D_i}\left(f_{dt}-\pi_{it}\right)x_{dt}^{D}+\sum_{(i,j)\in A_Q}x_{ijt}^{A}\left(\pi_{jt}-\pi_{it}-c_{ij}^{A}\right)\right.$$
$$\left.+\sum_{i\in R_Q}x_{it}^{R}\left(\pi_{i,t+1}-\pi_{it}-c_{i}^{R}\right)\right] \qquad \text{(A.1)}$$

subject to the capacity constraints,

$$\begin{aligned}
x_{ijt}^{A} &\leq u_{ij}^{A}, && (\lambda_{ijt}^{A}) && \forall (i,j)\in A_Q, t\in T && \text{(A.2)}\\
x_{it}^{R} &\leq u_{i}^{R}, && (\lambda_{it}^{R}) && \forall i\in R_Q, t\in T && \text{(A.3)}\\
x_{dt}^{D} &\leq u_{dt}^{D}, && (\lambda_{dt}^{D}) && \forall d\in D, t\in T && \text{(A.4)}
\end{aligned}$$

market clearing constraints,

$$\sum_{s\in S_i}x_{st}^{S}+\sum_{j\in N}\left(x_{jit}^{A}-x_{ijt}^{A}\right)-\sum_{d\in D_i}x_{dt}^{D}+x_{it}^{R}-x_{i,t+1}^{R}=0,\ (\pi_{it})\ \forall i\in N, t\in T \quad \text{(A.5)}$$

and non-negative x_{it}^{D} for all $i\in N$ and $t\in T$, x_{ijt}^{A} for all $(i,j)\in A_Q$ and $t\in T$, x_{it}^{R} for all $i\in R_Q$ and $t\in T$.

Here π_{it} denotes the market clearing price in node i in time period t and λ_{ijt}^{A} (λ_{it}^{R},λ_{dt}^{D}) denotes the shadow price on capacity of link (i,j) (storage i, demand unit d) in time period t.

Deriving the KKT-conditions for the above problem yields the following mixed complementarity problem:

$$\begin{aligned}
0 &\leq x_{ijt}^{A} & \perp && c_{ij}^{A}+\lambda_{ijt}^{A}+\pi_{it}-\pi_{jt} &\geq 0 && \forall (i,j)\in A_Q, t\in T && \text{(A.6)}\\
0 &\leq x_{dt}^{D} & \perp && -f_{dt}+\pi_{it}+\lambda_{dt}^{D} &\geq 0, && \forall i\in N, d\in D_i && \text{(A.7)}\\
0 &\leq x_{it}^{R} & \perp && c_{i}^{R}+\lambda_{it}^{R}+\pi_{it}-\pi_{i,t+1} &\geq 0, && \forall i\in R_Q, t\in T && \text{(A.8)}\\
0 &\leq \lambda_{ijt}^{A} & \perp && u_{ij}^{A}-x_{ijt}^{A} &\geq 0, && \forall (i,j)\in A_Q, t\in T && \text{(A.9)}\\
0 &\leq \lambda_{dt}^{D} & \perp && u_{d}^{D}-x_{dt}^{D} &\geq 0, && \forall d\in D, t\in T && \text{(A.10)}\\
0 &\leq \lambda_{it}^{R} & \perp && u_{i}^{R}-x_{it}^{R} &\geq 0, && \forall i\in R_Q, t\in T && \text{(A.11)}
\end{aligned}$$

This problem optimises the demand and part of the transmission and storage network owned by the system operator, maximising total social welfare. In the following subsection, the supply side of the network and the part of the transmission and storage network owned by supply firms is modelled.

A.2.3 The supplier problem

The suppliers of electricity may own one or more supply units located across the transmission network as well as transmission links and storages. S_h denote the set of supply units, while A_h denote the set of transmission links, and $R_h \subset N$ denote the set of storages owned by firm h. Supply units are represented by a constant marginal cost c^S and a supply capacity u^S. It is assumed that a supply unit may always be turned on and off without incurring extra costs.

The problem of a supply firm h is to maximise its profit z_h respecting capacity constraints on supply units, transmission links, and storages, i.e.:

$$\max z_h = \sum_{t \in T} \Bigg[\sum_{i \in N, s \in S_h \cap S_i} x^S_{st}(\pi_{it} - c^S_s) + \sum_{(i,j) \in A_h} x^A_{ijt}(\pi_{jt} - \pi_{it} - c^A_{ij}) + \sum_{i \in R_h} x^R_{it}(\pi_{i,t+1} - \pi_{it} - c^R_{it}) \Bigg] \quad \text{(A.12)}$$

subject to

$$x^S_{st} \le u^S_{st}, \qquad \forall s \in S_h, t \in T \quad \text{(A.13)}$$
$$x^A_{ijt} \le u^A_{ij}, \qquad \forall (i,j) \in A_h, t \in T \quad \text{(A.14)}$$
$$x^R_{it} \le u^R_{i}, \qquad \forall i \in R_h, t \in T \quad \text{(A.15)}$$

and $0 \le x^S_{st}$ for all $s \in S_h, t \in T$, $0 \le x^A_{ijt}$ for all $(i,j) \in A_h, t \in T$, $0 \le x^R_{it}$ for all $i \in R_h, t \in T$.

The following KKT-conditions ensures stationarity and feasibility of the single-firm problem P_h

$$0 \le x^A_{ijt} \quad \perp \quad c^A_{ij} + \lambda^A_{ijt} + \mu^A_{ijt} + \pi_{it} - \pi_{jt} \ge 0, \qquad \forall (i,j) \in A_h, t \in T \quad \text{(A.16)}$$
$$0 \le x^S_{st} \quad \perp \quad c^S_{st} + \lambda^S_{st} + \mu^S_{st} - \pi_{it} \ge 0, \qquad \forall s \in S_h, i \in N : s \in S_i \quad \text{(A.17)}$$
$$0 \le x^R_{it} \quad \perp \quad c^R_{i} + \lambda^R_{it} + \pi_{it} - \pi_{i,t+1} \ge 0, \qquad \forall i \in R_h, t \in T \quad \text{(A.18)}$$
$$0 \le \lambda^A_{ijt} \quad \perp \quad u^A_{ij} - x^A_{ijt} \ge 0, \qquad \forall (i,j) \in A_h, t \in T \quad \text{(A.19)}$$
$$0 \le \lambda^S_{st} \quad \perp \quad u^S_{s} - x^S_{st} \ge 0, \qquad \forall s \in S_h, t \in T \quad \text{(A.20)}$$
$$0 \le \lambda^R_{it} \quad \perp \quad u^R_{i} - x^R_{it} \ge 0, \qquad \forall i \in R_h, t \in T \quad \text{(A.21)}$$

where the firms strategic decisions μ^S_{st}, μ^A_{ijt} denote the firms price markup on supply and transmission respectively.

Furthermore, due to regulatory concerns, we assume that the price markups are bounded above by $\bar{\mu}^S_{st}$ and $\bar{\mu}^A_{ijt}$, respectively. In the remainder of the paper, we also assume that the supplier does not perform market power on its transmission links, ie. $\bar{\mu}^A_{ijt} = 0$ for all time periods and all $(i, j) \in A_h$.

Firm h anticipates the reaction of the system operator and the other supply firms, assuming that the other firms have fixed price markups. Hence, the problem of supply firm h may be formulated as the mathematical program with equilibrium constraints given by,

$$\max z_h \tag{A.22}$$

subject to the equilibrium constraints,

$$
\begin{array}{lclr}
0 \leq x^A_{ijt} & \perp & c^A_{ij} + \lambda^A_{ijt} + \pi_{it} - \pi_{jt} \geq 0, & \forall (i,j) \in A, t \in T \quad \text{(A.23)} \\
0 \leq x^S_{st} & \perp & c^S_{st} + \lambda^S_{st} + \mu^S_{st} - \pi_{it} \geq 0, & \forall s \in S, i \in N : s \in S_i \quad \text{(A.24)} \\
0 \leq x^R_{it} & \perp & c^R_i + \lambda^R_{it} + \pi_{it} - \pi_{i,t+1} \geq 0, & \forall i \in R, t \in T \quad \text{(A.25)} \\
0 \leq x^D_{dt} & \perp & -f_{dt}(x^D_{dt}) + \pi_{it} + \lambda^D_{dt} \geq 0, & \forall i \in N, d \in D_i \quad \text{(A.26)} \\
0 \leq \lambda^A_{ijt} & \perp & u^A_{ij} - x^A_{ijt} \geq 0, & \forall (i,j) \in A, t \in T \quad \text{(A.27)} \\
0 \leq \lambda^S_{st} & \perp & u^S_s - x^S_{st} \geq 0, & \forall s \in S, t \in T \quad \text{(A.28)} \\
0 \leq \lambda^R_{it} & \perp & u^R_i - x^R_{it} \geq 0, & \forall i \in R, t \in T \quad \text{(A.29)} \\
0 \leq \lambda^D_{dt} & \perp & u^D_d - x^D_{dt} \geq 0, & \forall d \in D, t \in T \quad \text{(A.30)}
\end{array}
$$

and bounds on strategic variables,

$$0 \leq \mu^S_{st} \leq \bar{\mu}^S_s, \ \forall s \in S_h, t \in T \tag{A.31}$$

where μ^S_{st} is fixed for all $s \in S \setminus S_h$ and $t \in T$.

Note that a solution to the problem of a perfectly competitive market, where all the firms are price takers, may be obtained by solving the mixed complementarity problem (A.23) - (A.30) with $\mu^S = 0$.

Using this modelling framework, a hydroelectric power supplier h may be modelled by adding a node $i \in R_h$ representing the reservoir owned by h and a transmission line $(i, j) \in A_h$ connecting the reservoir to an existing node j. The capacity of the hydropower unit is represented by the capacity of transmission line (i, j), while inflow to the reservoir is modelled by a supply unit s with marginal cost 0 and time dependent capacity u^S_{st} corresponding to the natural inflow of water (from e.g. precipitation, melting snow, rivers etc.). The final level of each storage reservoir is variable and is modelled by consumption of one or more demand units with $u^D_{dt} = 0$ in all time periods t except the last period

t'. The utility $f_{dt'}$ of this demand unit represents the expected future (long-term) value of water in the reservoir. A high value of $f_{dt'}$ gives the hydro power supplier an incentive not to empty the reservoir withing the planning horizon.

is sufficiently low, and if sufficiently high prevents emptying the reservoir.

A.2.4 Multi-firm equilibrium

In this section we consider the case when more than one supply firm can manipulate prices by adjusting their price markups.

A Nash equilibrium is defined as an equilibrium, where no firm has an incentive to change its decisions unilaterally.

We implement a simple algorithm to identify a Nash equilibrium among the firms. That is, we solve an Equilibrium Problem with Equilibrium Constraints in which the upper level problem is a game between firms that can exert market power, while the lower level describe the equilibrium of the players who do not exert market power. In each iteration of the algorithm solutions to each of the suppliers problem P_h are computed using GAMS/NLPEC [3]. If in two consecutive iterations the markups μ^S are constant for all firms, or if the maximum number of iterations $\bar{k}$ has been reached, the algorithm stops and returns the solution. The algorithm is outlined in Algorithm 2.

Algorithm 2 Outline of multi-firm equilibrium solution procedure.

```
while not equilibrium and k < k̄ do
  for all h ∈ H do
    Compute the solution to P_h and let μ*_k(h) be the corresponding optimal
    strategic decision variables
  end for
  if μ*_k(h) = μ*_{k−1}(h)  then
    equilibrium = true
  end if
  k ← k + 1
end while
```

Note that convergence of the algorithm or even existence of an equilibrium solution has not been established.

A.3 Analysis

We now use the modelling framework presented in section A.2 to analyse the potential use of water reservoirs by hydropower suppliers to exert market power in hydro-based electricity networks.

A.3.1 Results

A small problem instance with four nodes, one storage, and seven supply units, is constructed to illustrate the problem. One dominant hydropower producer is considered, while the remaining suppliers are assumed to be pricetakers. Two time periods are considered. The parameters are given in Tables A.1, A.2, and A.3.

Node A, B, and C represent price areas, while node H represents the reservoir of the hydro-power supplier. Each pair of price area A, B, and C are connected by transmission links. The hydro-power reservoir supplies energy to node B (modelled by a transmission link from node H to node B), while the remaining supply units are distributed across the network. Note that supply unit s_3 in node H (Table A.1) represents inflow to the reservoir. The demand in node H is 0 except for the last time period and represents the expected long-term value of water in the reservoir.

supply unit	marg. cost	mark-up	capacity		area	owner	comment
s	c_s^S	$\bar{\mu}_s^S$	$t=1$	$t=2$			
s_1	6.25	0	2289	2289	A	h_1	nuclear
s_2	19	0	572	572	A	h_3	thermal
s_3	1	50	19500	195	H	h_2	hydro
s_4	22	0	4579	4579	C	h_3	thermal
s_5	0	0	1456	1575	C	h_1	wind
s_6	45	0	114	114	B	h_1	thermal
s_7	65	0	1145	1145	C	h_2	thermal

Table A.1: Data for supply units of example 1.

	demand price f		max. demand u^D	
node	$t=1$	$t=2$	$t=1$	$t=2$
A	5259.5	5089	5219.5	5049
B	3551.3	3436.6	3511.3	3396.6
C	5828.9	5639.8	5788.9	5599.8
H	50	50	0	2000
H	30	30	0	2000
H	10	10	0	1000

Table A.2: Demand parameters of example 1.

The solution of the instance in which the hydropower producer is not allowed respectivley allowed to use the storage for strategically maximising his profit is shown in Figure A.1 respectively Figure A.2. It is seen that with marketpower, prices are significantly higher in the second time period and in node B of the first time period. Inflow to the reservoir in the second time period is reduced by 165, while maintaining the same production (5030). Hence, the storage level is reduced from 2165 to 2000 in the market power scenario. This is due to the

node	node	transmision cost		transmission capacity		owner
i	j	c_{ij}^A	c_{ji}^A	u_{ij}^A	u_{ji}^A	
A	B	1	1	1000	1000	Q
B	C	1	1	1000	1000	Q
A	C	1	1	3000	3000	Q
H	B	0	-	6000	0	h_2

Table A.3: Transmission link parameters of example 1.

long term value of water, which is assumed to be 50 for the first 2000 units and 30 for the next 2000 units (Table A.2).

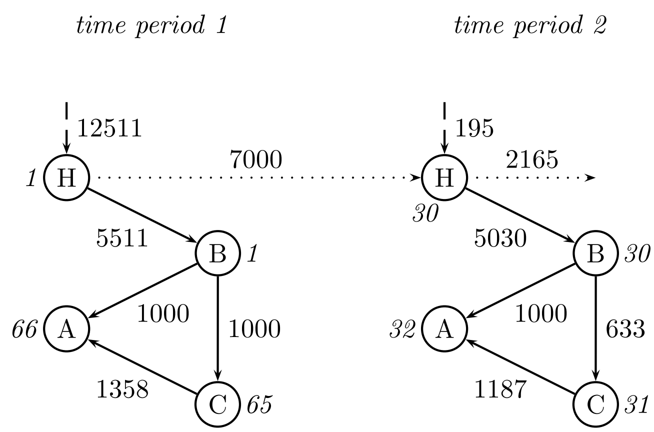

Figure A.1: Solution of a small example assuming perfect competition. The left part of the figure shows solution values for the first time period, while the right part shows values for the second period. Solid arcs represent transmission lines, while dotted arcs represent storage. Arc labels indicate transmission and storage flows, respectively. Node labels (in italic) indicate nodal prices.

Furthermore, allowing the use of market power causes a change in the total social welfare, and in the distribution of social welfare, as shown in Table A.4. While the suppliers gain, the consumers loose, and the total social welfare decreases.

	Perfect competition	Marketpower
supplier	617300	1212460
operator	128000	48000
storage	203000	0
consumer	141704200	141387200
total	142652500	142647700

Table A.4: Comparison of surplus values for different segments. Left column shows values when no the hydropower producer is not allowed to perform marketpower, while the right column shows values when marketpower is allowed.

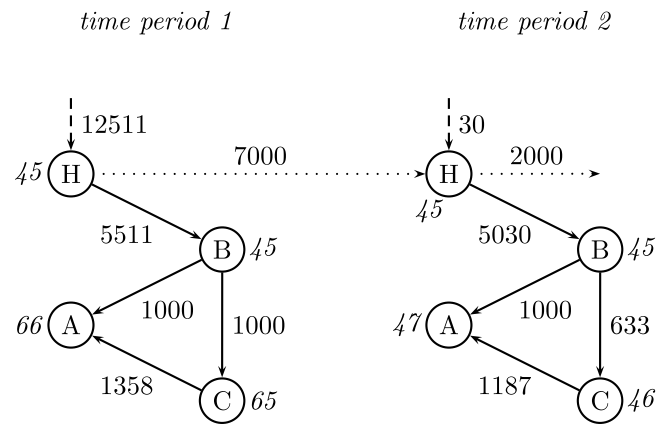

Figure A.2: Solution of a small example with market power. The left part of the figure shows solution values for the first time period, while the right part shows values for the second period. Solid arcs represent transmission lines, while dotted arcs represent storage. Arc labels indicate transmission and storage flows, respectively. Node labels (in italic) indicate nodal prices.

A.3.2 Computational Challenges

The single firm equilibrium is solved using GAMS and the MPEC solver NLPEC [3]. Two approaches have been tested. In the first, the problem is specified with a linear inverse demand function, while in the second the problem assumes constant utility for each demand segment.

In the first approach optimal solutions to the single firm problem given by equations (A.22) - (A.31) may be obtained immediately for small problems (e.g. single period) using NLPEC with default settings. However, solving large problems is not immediately possible within reasonable computation time. In fact, we may not even be able to obtain a feasible solution.

Considering constant utility of consumers (i.e. a piecewise constant demand curve) the single firm problem without market power gives rise to a linear optimisation problem maximising total social welfare. This makes it possible to easily find a feasible solution to the single firm problem - namely the solution in which all markups μ^S are 0 - using an LP solver. The MPEC solver may then be started with the initial solution being the socially optimal solution given by the LP solver.

For a large instance of the nordic area consisting of 10 price areas, 13 reservoirs and 24 time periods, the second approach yields a feasible locally optimal solution relatively fast (within a few minutes). However, the solution obtained is not guaranteed to be globally optimal. Indeed the solution is to close to the social optimum as calculated by the LP solver.

More research is necessary to develop solution procedures that can generate optimal or near-optimal solutions to large (multi-period) instances. It may be worthwile to investigate different candidates for initial solutions supplied to the MPEC solver as well as alternative solver parameters.

A.4 Conclusion

This paper introduces a modelling framework to investigate the potential use of market power in electricity networks. In particular, it allows to analyse the use of market power by e.g. hydropower producers with reservoirs. A small instance of the problem is constructed and a hydropower producers potential strategic use of a water reservoir to perform marketpower is analysed.

Preliminary results suggest that the use of market power at a hydro power storage may alter the equilibrium of the system and reduce social welfare significantly. The value of the social welfare is the basis for many transmission infrastructure decisions in the nordic area, where hydro power is dominant. We therefore believe that temporal market power deserves a more thorough investigation.

More work is needed in the investigation of temporal marketpower on realistic data of the nordic electricity system. Does temporal market power have the potential to influence the level and distribution of social welfare? Ultimately, a planning horizon of one year is preferable due to the size of the hydro power reservoirs and seasonal variations of water inflows.

From a computational point of view, more research is needed to develop efficient solution procedures that can solve large problem instances in reasonable time.

Further work include the extension of the model with Kirchoffs voltage law for direct current networks as well as stochastic modelling of windpower and water reservoir inflow.

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