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The core of traveling salesman games: computational experiments

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Let $G = (V, E)$ be a complete, undirected graph with vertices $V = \{0, 1, \ldots, n\}$ and edges E. A weight or cost c_e is associated with each edge $e \in E$. The aim of the *traveling salesman problem* (TSP) is to find a tour with minimum cost, visiting all vertices in V exactly once.

Now consider the traveling salesman game (TSG) from cooperative game theory, introduced in Potters et al. [1992] and Tamir [1989]. In the version of the TSG considered in this talk, vertex 0 is denoted the *home* vertex and $V_u = \{1, \ldots, n\}$ are *user* vertices. For any set $S \subseteq V_u$, let $c(S)$ be the cost of the minimum TSP tour on the vertices $S \cup \{0\}$. In the TSG one wants to distribute the cost of the TSP tour visiting all vertices in V among the user vertices. An application (from Potters et al. [1992]) of the problem is the following: a researcher is giving talks at a number of universities and wants to distribute his travel costs among the universities in a fair way. For the distribution to be fair it makes sense to require that no subset S of V_u would be better off by leaving the coalition and arranging a separate trip for the researcher that only visits the universities in S. To formalize the concepts, let x_i be the cost charged to user $i \in V_u$. The vector x should satisfy

$$x(V_u) = c(V_u) \tag{1}$$
$$x(S) \leq c(S) \quad \forall S \subset V_u \tag{2}$$
$$x \in \mathbb{R}^n \tag{3}$$

where $x(S) = \sum_{i \in S} x_i$. Here (1) ensures that all costs are distributed to the users and (2) ensures that the distribution is fair in the sense described earlier. The constraints define a polyhedron in $\mathbb{R}^n$ and vectors belonging to this polyhedron are said to be in the *core*. For some cooperative games it can happen that the core is empty. In other words, there is no way to fairly allocate the cost of the large tour to the users: no matter how x is chosen some subset of users $S \subset V_u$ would be better off by forming a sub-coalition compared to staying in the grand coalition. For the TSG it is known that the core is non-empty for $n \leq 5$ (Kuipers [1993]) and for $n = 6$ it is known that even instances where the cost matrix c is Euclidean can have an empty core (Faigle et al. [1998]).

In this talk we explore how often it happens that the core of a TSG is empty. Does it happen frequently "in practice" or only for specifically constructed instances? We do this through computational experiments on a large set of instances. We construct a computer program that given a TSG instance returns a vector in the core if the core is non-empty or reports that the core is empty otherwise. We test this on instances from the TSPLIB (Reinelt [1991]) as well on a large set of randomly generated instances. Finding a vector in the core amounts to solving the linear program (LP) defined by (1)–(3) with any objective function. The number of constraints in the LP grows exponentially in n, but the LP can be solved in a cutting-planes fashion. The separation problem for inequality (2) amounts to the following: given a vector x^* one has to find a subset $S \subset V_u$ such that $\sum_{i \in S} x_i^* > c(S)$ or, in other words solve

$$\min_{S \subset V_u} \{c(S) - \sum_{i \in S} x_i^*\},$$

which is equivalent to the *profitable tour problem* from Dell'Amico et al. [1995] and is closely related to the orienteering problem and the prize collecting TSP (see e.g. Fischetti et al. [1998] and Balas [1989], respectively). Let $y_i, i \in V_u$ be a binary variable that is one if and only if user i is included in the set S and let $z_e, e \in E$ be a binary variable that takes value one if and only if edge e is used in the cheapest tour visiting vertices in $S \cup \{0\}$, with S being defined by y_i. Define $z(E') = \sum_{e \in E'} z_e$

for all $E' \subseteq E$ and $\delta(T), T \subset V$ as the set of edges with exactly one endpoint in T. The separation problem for inequality (2) can now be stated as an integer programming problem:

$$\min \sum_{e \in E} c_e z_e - \sum_{i \in V_u} x^* y_i \tag{4}$$

subject to

$$z(\delta(\{0\})) = 2 \tag{5}$$
$$z(\delta(\{i\})) = 2y_i \qquad \forall i \in V_u \tag{6}$$
$$z(\delta(T)) \geq 2y_i \qquad \forall T \subset V, 0 \in T, i \in V \setminus T \tag{7}$$
$$z_e \in \{0,1\} \quad \forall e \in E \tag{8}$$
$$y_i \in \{0,1\} \quad \forall i \in V_u \tag{9}$$

where (5) ensures that the home vertex is visited by the tour, (6) establishes the connection between y and z variables and (7) are generalized subtour elimination constraints (see e.g. Fischetti et al. [1998]). We note that the tours found by (4)-(9) will contain at least two user vertices so we should a priori generate all constraints (2) with $|S| = 1$. We solve (4)-(9) using a branch-and-cut algorithm and we also propose an adaptive large neighborhood search heuristic (Ropke and Pisinger [2006]) for solving (4)−(9) in order to speed up the algorithm for solving (1)−(3).

The real life applications of the TSG may be limited, but the idea of allocating costs or profits among a number of participants in a fair way have many applications within transportation, e.g. in truck based transportation (Frisk et al. [2010], Krajewska et al. [2008]) and in liner shipping (Agarwal and Ergun [2010]) and we believe that it is a subject that enjoys a growing popularity.

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