

## Energy Dynamics of an Infinitely Large Offshore Wind Farm

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## Abstract

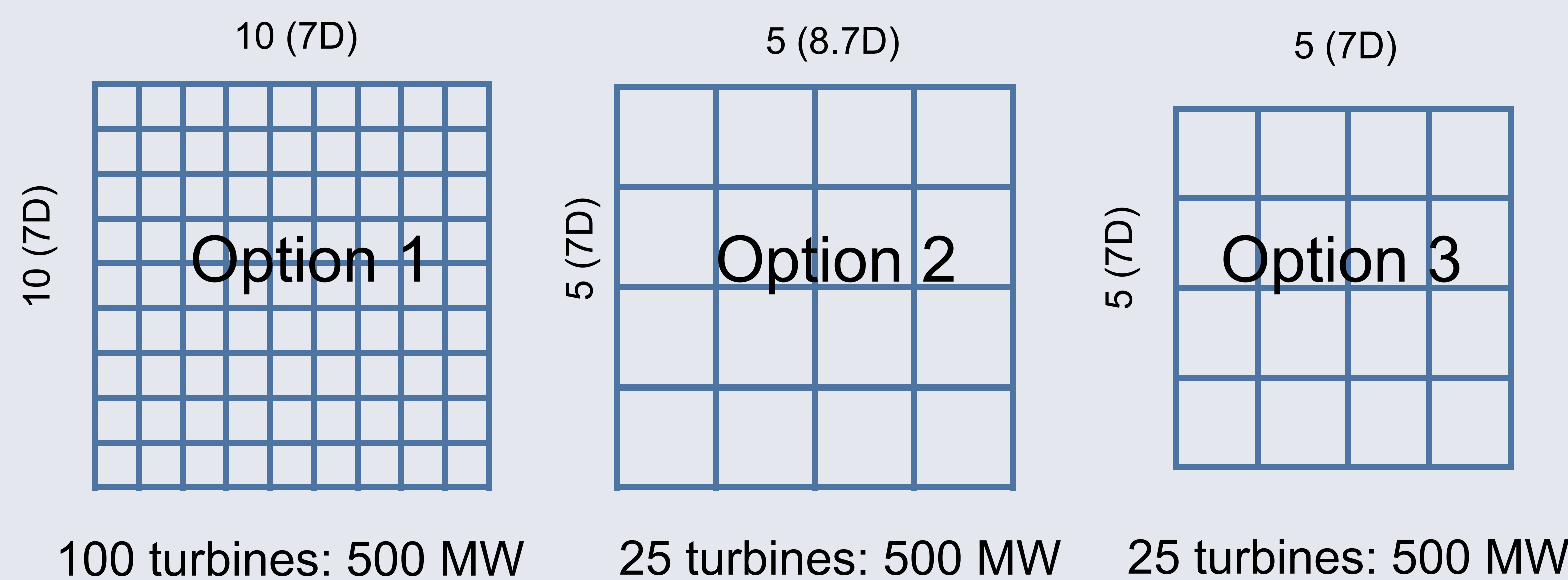
Plans for offshore wind energy development by 2030 are up to 150 GW capacity in the waters of the European Union with individual targets of e.g. 20-25 GW in Germany, 33 GW in the UK and 4.6 GW in Denmark. With this scale of development, one of the research challenges is to evaluate how to model interactions between the individual turbines, the atmosphere and neighbouring wind farms so as to accurately predict power output.

Here we consider whether the size of wind turbines and the scale of wind farms impacts the amount of energy that can be extracted from a given area. This research indicates that increasing turbine size from 2 MW to 20 MW should increase energy capture from about 2.7 to at least 4.0 W m<sup>-2</sup>. Energy capture on this scale requires approximately 2.4 times the standard transfer of energy from the top of the atmosphere into the boundary-layer. However, consideration of wind farm wakes suggest that for offshore areas a limit of 1.8 Wm<sup>-2</sup> will be reached for the 'infinite' wind farm.

## Energy density of existing wind farms

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Area of wind farm (km <sup>2</sup> )	23	25
Number of turbines/Turbine capacity (MW)	72/2.3	80/2.0
Density of turbines (# km <sup>-2</sup> )	0.3	0.3
Production density – no wake losses (W m <sup>-2</sup> )	7.2	6.4
Annual production (GWh a <sup>-1</sup> )	595 <sup>(2)</sup>	600 <sup>(3)</sup>
Estimated capacity factor	41%	43%
Production density (W m <sup>-2</sup> )	3.0	2.7

## Upscaling with WAsP



	Option 1	Option 2 "Equal area"	Option 3 "Equal spacing"
Turbine	5 MW	20 MW	20 MW
Hub height/rotor diameter (m)	90/126	153/252	153/252
Installed capacity (MW)	500	500	500
Area of installation (km <sup>2</sup> )	8.8 × 8.8 = 77.4	8.8 × 8.8 = 77.4	8.8 × 7.1 = 62.1
Area capacity (W m <sup>-2</sup> )	6.5	6.5	8.1
Turbine wake losses (%) (WAsP k=0.04, U=8.6 ms <sup>-1</sup> )	14.5	6.5	9.0
Annual production (GWh a <sup>-1</sup> ) (WAsP k=0.04, U=8.6 ms <sup>-1</sup> )	2197	2211	2152
Production density (W m <sup>-2</sup> )	3.2	3.3	4.0



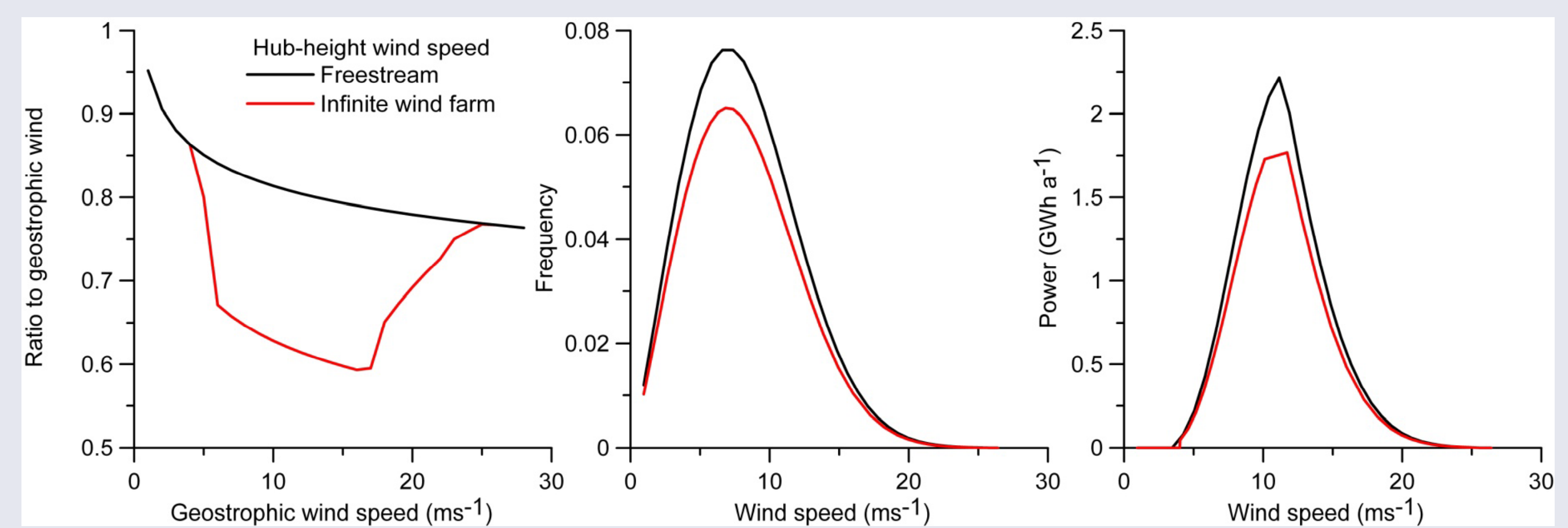
## Upscaling to the 'infinite' wind farm (Frandsen method)

One of the main issues of modelling with WAsP is that using standard parameters does not accurately capture wake losses in very large wind farms offshore. To assess power production in the 'infinite wind farm' (inf) the following procedure is used. Wind speed ( $U_z$ ) at hub-height ( $h$ ) is calculated from the geostrophic wind speed  $G$ , the modified Coriolis parameter  $f'$  and the wind farm thrust coefficient ( $c_t$ ) according to Frandsen<sup>(3)</sup>.  $c_t$  is based on the lateral and crosswind spacing ( $s_r, s_f$ ) and the turbine thrust coefficient  $C_T$ .

$$U_z = \frac{G}{1 + \ln\left(\frac{G}{f'h}\right)R} \quad R_{free\ stream} = \frac{1}{\ln(h/z_0)} \quad z_{0wind\ farm} = h \exp\left(-\frac{\kappa}{\sqrt{c_t + [\kappa/\ln(h/z_0)]^2}}\right)$$

$$R_{inf} = \frac{\sqrt{C_T}}{\kappa} \quad c_t = \frac{\pi C_T}{8s_r s_f}$$

For a given freestream wind speed distribution, the wind speed distribution within the wind farm can be calculated and the resulting power production summed. For Option 1, each turbine produces 15.9 GWh a<sup>-1</sup> (~1.8 W m<sup>-1</sup>).



## The "infinite" wind farm e.g. North Sea

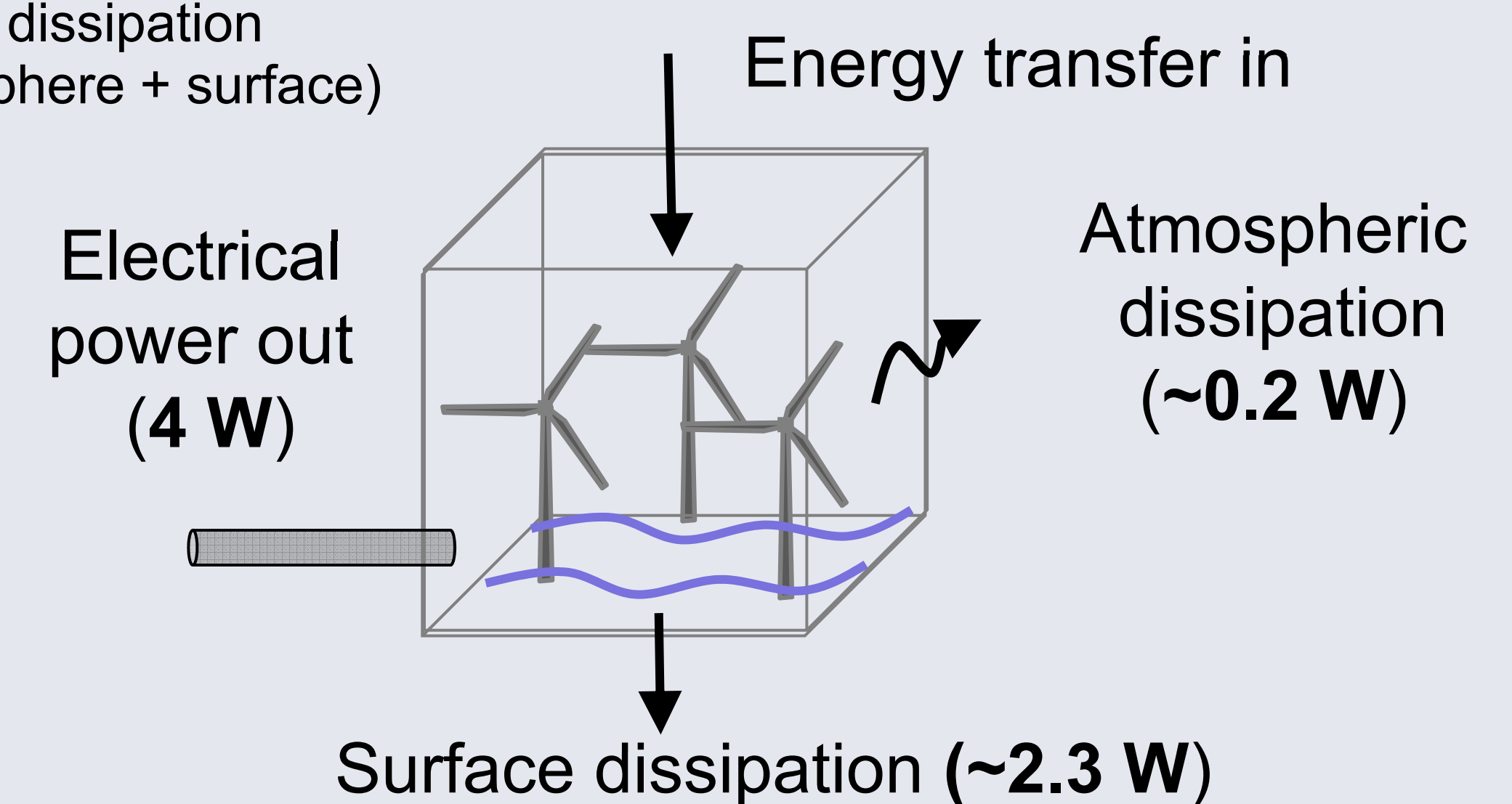
Turbine size	2 MW (Obs)	5 MW (WAsP)	5 MW (Frandsen)
Area of the North Sea (km <sup>2</sup> )	427100	427100	427100
Production density (W m <sup>-2</sup> )	2.9	3.2	1.8
Production in the North Sea (TWh a <sup>-1</sup> )	10678	11956	6734
% of North Sea needed to meet EU electricity demand (3,568 TWh a <sup>-1</sup> )	33	30	53

A major issue in this type of calculation is to understand whether wind turbine wake losses can be calculated as they have been for small to medium wind farms. Once wakes have merged laterally, the only source of momentum to replace the energy extracted is to transfer momentum downwards. In addition, energy is lost in the atmosphere and at the surface through dissipation. Dissipation has been estimated as 2.5 W m<sup>-2</sup>, 90% of which is at the surface<sup>(6)</sup>. We depict the infinite wind farm as below and calculate the approximate value of the energy balance for each m<sup>2</sup>.

To be sustainable:

$$\text{Energy in} = \text{Electrical power out} + \text{Total dissipation (Atmosphere + surface)}$$

$$\text{Energy in} = 4.0 \text{ W} + 2.5 \text{ W}$$



Can a rate of 2.4 times natural dissipation (~6.5 W m<sup>-2</sup>) be transferred in to replenish the energy of the atmosphere over large areas? Understanding the limits of this transfer should indicate the scales of wind farms that can be sustained.

## Conclusions

A comparison has been made of the energy density of wind farms from observations (2.9 W m<sup>-2</sup>), using upscaling with the WAsP model (3.2-4.0 W m<sup>-2</sup>) and considerations of limits on the infinite wind farm from Frandsen<sup>(1)</sup> (1.8 W m<sup>-2</sup>).

Within large wind farms wind turbine wakes are laterally merged meaning that to sustain the same rate of power production, momentum must be transferred into the boundary layer to replenish this power and energy lost through dissipation. This rate of transfer must be the ultimate limit on the scales of power production.

## References

- <sup>(1)</sup> www.eon.com      <sup>(3)</sup> Frandsen 2005 Risø-R-1188(EN)      <sup>(5)</sup> Hendriks et al. 2008 EWEC  
<sup>(2)</sup> www.hornsrev.dk      <sup>(4)</sup> www.wasp.dk      <sup>(6)</sup> Gustavson Science 204 13-17

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