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Economic Model Predictive Control for Smart Energy Systems

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Economic Model Predictive Control for Smart Energy Systems

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Model Predictive Control (MPC) can be used to control the energy distribution in a Smart Grid with a high share of stochastic energy production from renewable energy sources like wind. Heat pumps for heating residential buildings can exploit the slow heat dynamics of a building to store heat and hereby shift the heat pump power consumption to periods with both low electricity prices and a high fraction of green energy in the grid.

Heat pump and building model

A model of the heat dynamics of a house floor heating system connected to a geothermal heat pump can be formulated as three first order differential equations describing the energy balance of three thermal masses exchanging heat, namely the room air, the house floor and the water in the floor heating system modeled as a condenser tank. The heat pump enters the model as the compressor work W_c. Both outdoor temperature and solar radiation are included disturbances, see Fig. 1.

$$C_{p,r}\dot{T}_r = (UA)_{fr}(T_f - T_r) - (UA)_{ra}(T_r - T_a) + (1-p)\phi_s$$
$$C_{p,f}\dot{T}_f = (UA)_{wf}(T_w - T_f) - (UA)_{fr}(T_f - T_r) + p\phi_s$$
$$C_{p,w}\dot{T}_w = \eta W_c - (UA)_{wf}(T_w - T_f)$$

Economic MPC

An Economic MPC was formulated as a linear program with the heat pump model formulated as a discrete time state space model. The electricity prices is directly in the optimization problem as the cost coefficients cu.

$$
\begin{aligned}
\min_{\{x,u,y\}} \quad & \phi = \sum_{k\in\mathcal{N}} c_{y,k}^T y_k + c_{u,k}^T u_k + \rho_v^T v_k & \\
s.t. \quad & x_{k+1} = A_d x_k + B_d u_k + E_d d_k & k\in\mathcal{N} \\
& y_k = C x_k & k\in\mathcal{N} \\
& u_{\min} \le u_k \le u_{\max} & k\in\mathcal{N} \\
& \Delta u_{\min} \le \Delta u_k \le \Delta u_{\max} & k\in\mathcal{N} \\
& y_{\min} \le y_k + v_k & k\in\mathcal{N} \\
& y_{\max} \ge y_k - v_k & k\in\mathcal{N} \\
& v_k \ge 0 & k\in\mathcal{N}
\end{aligned}
$$

Bounds are put on the input, u, its rate of movement and the output, y, including a slack variable, v. At each sampling time, k, the optimization problem is solved to obtain the optimal input signal before it is applied and the process is repeated according to the moving horizon principle.

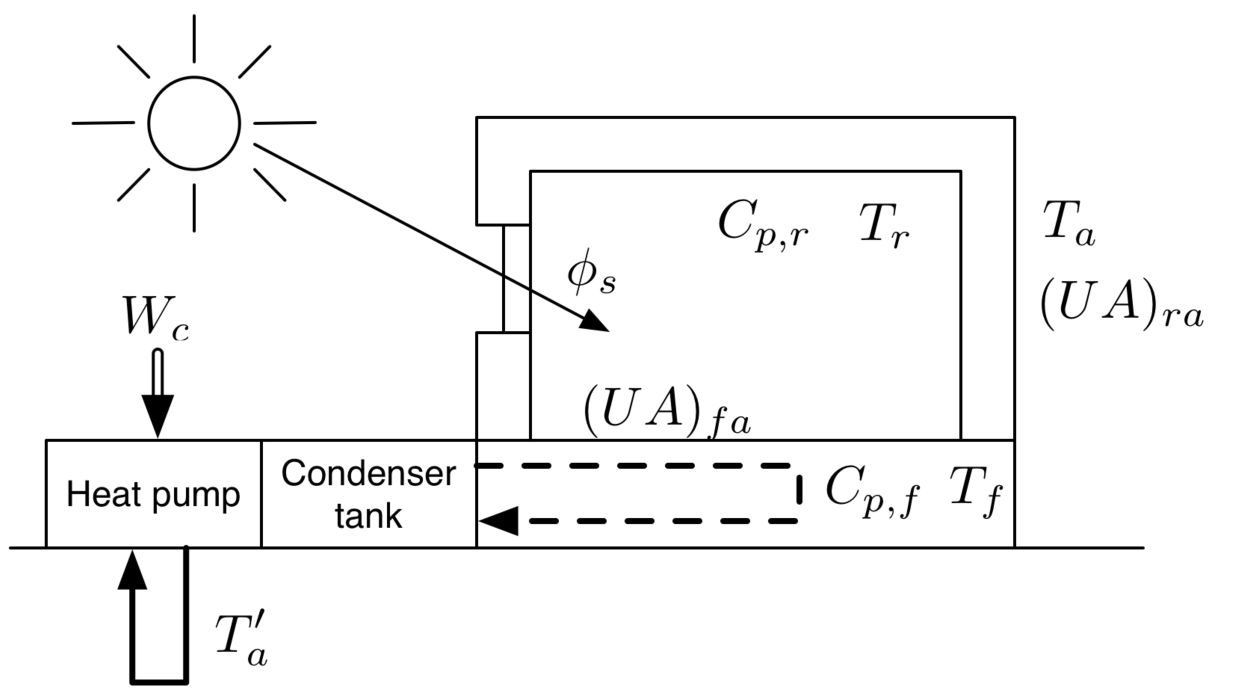

Fig. 1 House and heat pump floor heating system and its thermal properties. The dashed line represents the floor heating pipes. The term UA is a product of the heat conductivity and the surface area of the layer between two heat exchanging media. Its reciprocal value R = 1/(UA) is often used and is interpreted as a resistance against heat flow. Cp is the heat capacity and Ta' is the ground temperature.

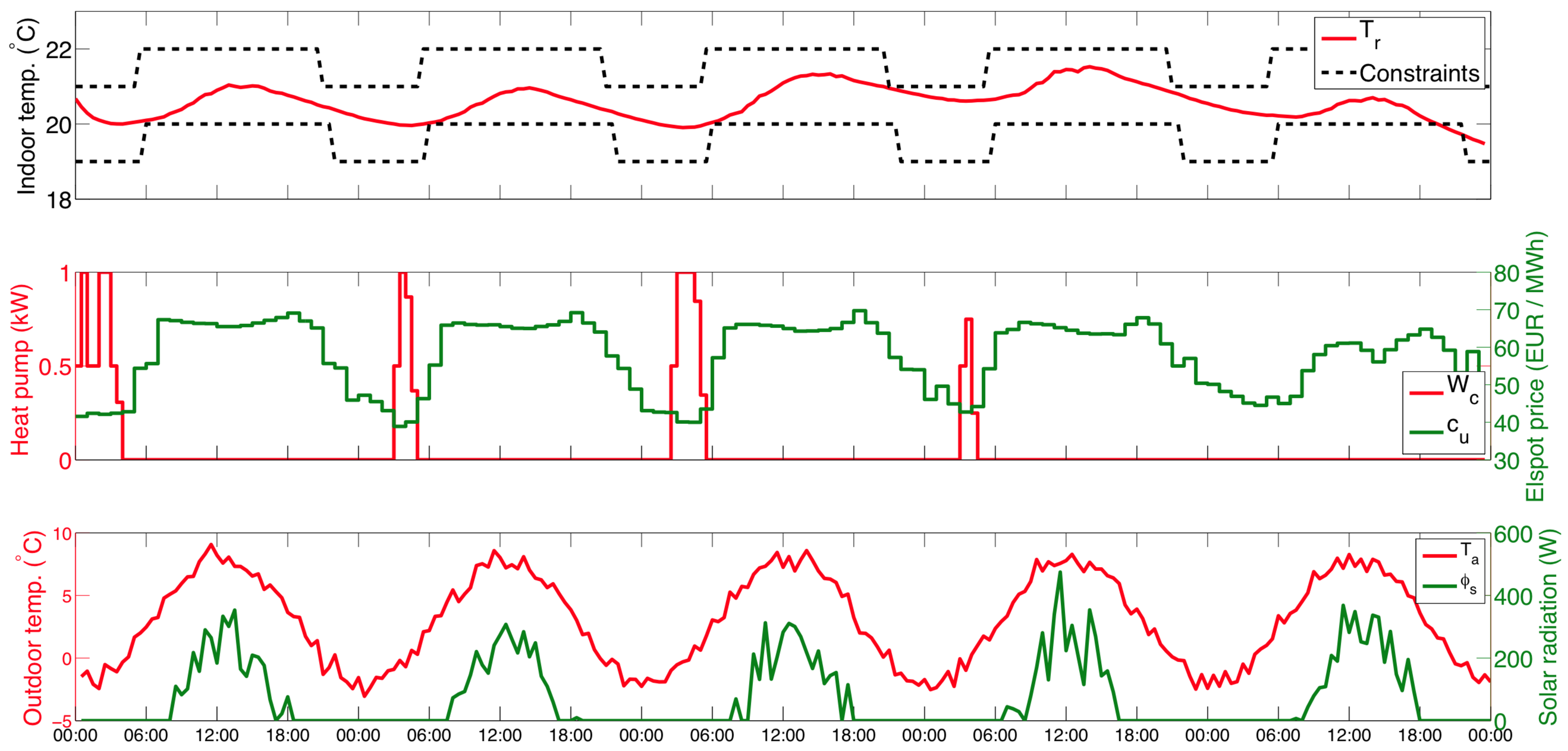

Fig. 2 Temperature in a house with time varying soft constraints, time varying electricity prices, and time varying outdoor temperatures. The time is the time since 18 JAN 2011 00:00. The upper figure shows the indoor temperature, the middle figure contains the electricity spot price and the optimal schedule for the heat pump, and the lower figure contains the ambient outdoor temperature and sun radiation. The compressor is on when the electricity spot price is low. The temperature constraints indicate that night time temperatures are allowed lower than at day time.

The simulation demonstrates the MPC load shifting ability that reduces the total heat pump power consumption and minimizes the electricity cost while keeping the house indoor temperature requirements.

Simulation

A long prediction horizon of 5 days is simulated with perfect model predictions. The open loop profile is shown in Fig. 2. The middle plot shows the actual electricity prices in Western Denmark along with the computed optimal heat pump power input. Clearly the power consumption is moved to periods with low electricity prices and the thermal capacity of the house floor is able to store enough energy such that the heat pump can be left off during day time and still maintain acceptable indoor temperatures. Notice that the time varying constraints on room temperature are soft in this case and are consequently violated slightly a few times. The penalties on the slack variables causing this behavior can be set sufficiently large, such that the output constraints are met whenever possible.

The above simulation with varying electricity price shows economic savings around 33% compared to a scenario with constant electricity prices and therefore no load shifting. Using hard constraints the savings decrease to 26%.

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