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pages + tables + illustrations

Abstract

The report describes a one-dimensional model of a U-tube steam generator. The main emphasis is placed on the derivation of the equations and a discursion of the problem arising by a pure digital and a hybrid solution. Only a few results are included.

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CONTENTS

1. INTRODUCTION

In connection with the development of an overall model of a PWR power station it has been necessary to develop a rather detailed model of a steam generator in order to be able to establish and check a more simple model for inclusion in the complete power station model. Once developed the detailed model may be used to investigate more accurately the transient behaviour of the steam generator, especially space dependent variables, for given input and load conditions.

The model is a one dimensional model of the Westinghouse type steam generator. A simplified diagram of the physical structure is shown in fig. 1. The steam generator is divided in 8 sections as shown. The central "core" and the downcomer are described by partial differential equations, while the other sections are described by ordinary differential equations. The partial differential equations are solved by sampling in time and division in subsections in space, thus transforming the equations to algebraic equations.

The main approximations used for the formulation of the equations are:

a. Uniform water and steam velocity in the primary and the secondary side and constant steam to water velocity ratio (slip factor) are used.
b. No subcooled boiling is included. The water is heated to saturation and afterwards all the energy is used for steam production.
c. Thermal equilibrium at the saturation point is assumed in the boiling part of the "core", the riser, the steam volume and the upper part of the feedwater chamber.
d. No heat conduction along the tubes takes place. The tubes are divided in two shells each with half part of the heat capacity and all the heat resistance between the shells.
e. No boiling is allowed in the downcomer and no heat transmission from the "core" takes place. This assumption limits the working range of the pressure derivative to values less than 1-2 bar/s dependent of the power level.
f. All heat exchange with the wall and the steel constructions in the steam volume is neglected.

The basic equations are derived in chapter 2. Chapter 3 and 4 contains a discussion of the problems arising by pure digital and by hybrid solutions together with an explanation of the usage of the basic equations.

The steady state values calculated by the model for 3 load levels may be found in a table in fig. 6 and a few transients at full load in fig. 7.

2. FORMULATION OF THE BASIC EQUATIONS

2A. The "core"

The temperature in the primary circuit and the nonboiling part of the secondary circuit is described by the two partial differential equations:

$$1. \qquad \frac{\partial T_p}{\partial z} = -\frac{1}{W_p}\left(\frac{q_p}{c_{pp}} + A_p \rho_f \dot{T}_p\right)$$

$$4a. \qquad \frac{\partial T_s}{\partial x} = \frac{1}{W_s}\left(\frac{q_s}{c_{ps}} - A_s \rho_{fs} \dot{T}_s\right)$$

The heat capacity c_p and the density ρ_f are taken as functions of temperature assuming constant pressure as the pressure influence is small at a certain distance from the saturation point. However, at the secondary side, where the temperature is quite near saturation c_{ps} is calculated from the curve for 50 bar prolonged with "near saturation values" up to 300 $^{\circ}$C. The two curves used for the primary and the secondary side are shown in fig. 2.

The z-axis goes along the U-tubes up and down and the x-axis from the bottom to the top of the "core". The U-tubes are substituted by two straight tubes, with an artificial connection in the top. This means, that the core can be treated as a parallel flow and a counter flow heat exchanger with common secondary flow and with the primary output from the first as the primary input to the second.

The two tube shell temperatures are described by ordinary differential equations for sections of unit length with constant temperature within a section.

$$2. \qquad \dot{T}_{r1} = \frac{2}{C_r}\left(q_p - q_r\right)$$

$$3. \qquad \dot{T}_{r2} = \frac{2}{C_r}\left(q_r - q_s\right)$$

In the boiling part of the secondary side is equation 4a substituted by two equations 4b and 4c giving the steam and water flow:

$$4b. \quad \frac{\partial}{\partial x}(v_g \alpha) = \frac{1}{h_{fg}\rho_{gs}}\left(\frac{q_s}{A_s} - \dot{p}\left(\alpha\left(\rho_{gs}\frac{dh_{gs}}{dp} + h_{fg}\frac{d\rho_{gs}}{dp}\right) + (1-\alpha)\rho_{fs}\frac{dh_{fs}}{dp} - 1\right)\right) - \dot{\alpha}$$

$$4c. \quad \frac{\partial}{\partial x}(v_f(1-\alpha)) = \dot{\alpha}\left(1 - \frac{\rho_{gs}}{\rho_{fs}}\right) - \dot{p}\left(\frac{\alpha}{\rho_{fs}}\frac{d\rho_{gs}}{dp} + \frac{1-\alpha}{\rho_{fs}}\frac{d\rho_{fs}}{dp}\right) - \frac{\rho_{gs}}{\rho_{fs}}\frac{\partial}{\partial x}(v_g\alpha)$$

Equation 4b and 4c are derived from the continuity equations for mass and energy which in the fundamental form are given in 4B and 4C.

$$\text{4C.} \quad \frac{\partial}{\partial t}\left(\alpha \rho_{gs} + (1-\alpha)\rho_{fs}\right) = -\rho_{fs}\frac{\partial}{\partial x}\left(v_f(1-\alpha)\right) - \rho_{gs}\frac{\partial}{\partial x}\left(v_g \alpha\right)$$

$$\text{4B.} \quad \frac{\partial}{\partial t}\left(\alpha \rho_{gs} h_{gs} + (1-\alpha)\rho_{fs} h_{fs}\right) = \frac{q_s}{A_s} - h_{fs}\rho_{fs}\frac{\partial}{\partial x}\left(v_f(1-\alpha)\right) - h_{gs}\rho_{gs}\frac{\partial}{\partial x}\left(v_g\alpha\right) + \dot{p}$$

Equation 4C leads directly to 4c and 4b is obtained by multiplying 4C with h_{fs} and subtracting from 4B.

The heat transfer is calculated according to the Dittus-Boelter equation or, in the boiling region, according to the Thom equation.

The Dittus-Boelter equation is:

$$\frac{h D_e}{\lambda} = 0.023 \left(\frac{D_e G}{\eta}\right)^{0.8} \left(\frac{c_p \eta}{\lambda}\right)^n$$

where n = 0.4 for cooling and 0.3 for heating. G is the mass flow rate per unit area.

The heat transfer coefficient h can be written as:

$$h = 0.023 \frac{G^{0.8}}{D_e^{0.2}} H(p,T)$$

where

$$H(p,T) = \frac{c_p^{\,n}\,\lambda^{1-n}}{\eta^{0.8-n}}$$

The function H(p, T) is shown in figs. 3 and 4 for n = 0.4 and 0.3 respectively corresponding to the primary and secondary side. The influence of the pressure is small except near saturation at 150 bar, and this area is not used. The functions may with a reasonable good accuracy be represented by straight lines or better by smooth curves independent of pressure.

The Thom equation gives the heat transfer in the following form:

$$q = 1.972 \exp\left(\frac{p}{43.4}\right)\left(T_{r2} - T_{sa}\right)^2$$

Due to the boiling heat transfer mechanisme the heat flow is proportional to the square of the difference between the metal surface and saturation temperature. This equation is used when it gives a higher heat flow than the Dittus-Boelter equation.

The heat transmission equation for the primary side giving the heat flow per unit length of the tubes will then be:

$$\text{5.} \qquad q_p = \frac{0.023}{D_{ep}^{0.2}}\, O_p \left(\frac{W_p}{A_p}\right)^{0.8} H_p(T)\left(T_p - T_{r1}\right)$$

And for the secondary side one of the two equation:

7a. $$q_s = \frac{0.023}{D_{es}^{0.2}} O_s \left(\frac{W_s}{A_s}\right)^{0.8} H_s(T)\,(T_{r2} - T_s)$$

7b. $$q_s = 1.972\, O_s \exp\left(\frac{p}{43.4}\right)(T_{r2} - T_{sc})^2$$

The equation, 7a or 7b, which gives the highest value of q is selected. The total q_s in equation 4 is the sum of two terms from the tube sections in the same height in the upstream and downstream leg of the U-tube. The units for q in eq. 5 and 7 are kW/m when MKS units are used for the other variables and H has the dimension in figs. 3 and 4.

The heat flow per unit length through the tube wall is given by:

6. $$q_r = \frac{\lambda_r}{\Delta r} O_r (T_{r1} - T_{r2})$$

In the boiling region is eq. 4b and 4c used to calculate the flows ($v_g\alpha$) and ($v_f(1-\alpha)$) along the tubes. When these flows are known the void fraction α may be calculated as:

8. $$\alpha = \frac{v_g\alpha/S}{v_g\alpha/S + v_f(1-\alpha)}$$

using the relation $v_g = S\, v_f$ where the slip factor S is taken as a constant.

The steam quality X is calculated as:

9. $$X = \frac{v_g\alpha\,\rho_{gs}}{v_g\alpha\,\rho_{gs} + v_f(1-\alpha)\rho_{fs}}$$

The steam quality is used to calculate the friction pressure drop in the boiling region. The two phase friction is calculated according to Becker as the one phase friction multiplied by the two phase friction multiplier R. The pressure drop across Δx is then:

$$\Delta p = f\,\frac{\Delta X}{2D_e}\,\rho v^2 R$$

where we for the friction coefficient f use:

$$f = 0.184\, R_e^{-0.2}$$

$$R_e = \frac{1}{\eta} D_e v \rho$$

and the two phase friction multiplier R is taken as:

$$R = 1 + 2400\,\frac{X}{p}$$

Introducing f, Re and R in the equation for Δp we obtain:

$$\Delta p = 0.092 \frac{\Delta x}{D_e^{1.2}} v^{1.8} \rho^{0.8} \eta^{0.2} \left(1 + 2400 \frac{X}{p}\right)$$

The calculation can be simplified by expressing the pressure drop relative to the density and introducing the function $F_f(T)$

$$F_f(T) = \left(\frac{\eta}{\rho}\right)^{0.2}$$

The pressure drop across a section of the "core" then takes the form:

$$10. \qquad \frac{\Delta p_1}{\rho_{fs}} = 0.092 \frac{\Delta x}{D_{ep}^{1.2}} v^{1.8} F_f(T) \left(1 + 2400 \frac{X}{p}\right)$$

F_f is the only factor dependent of temperature. It is shown in fig. 5 and appears to be independent of pressure and further it is fairly constant in a large temperature range. For simplification we will use $F_f(\tau) = 0.0425$ (MKS units). The velocity v should be the total mass flow divided by the flow area and the density.

$$v = \frac{W_g + W_f}{A_s \rho_{fs}} = \frac{1}{A_s}\left(v_f(1-\alpha) + v_g \alpha \frac{\rho_{gs}}{\rho_{fs}}\right)$$

2B. The hydraulic loop

The secondary circulation rate is governed by the void volume and the friction forces, which in the steady state neutralizes each other.

Besides the pressure drop $\sum^{\Delta x} \frac{\Delta p_1}{\rho_{fs}}$ in the "core" we have pressure drops in the downcomer, the riser, feedwater chamber and at the inlet to the "core". Only the first of these can be calculated, the others can only be estimated. So they are included in the total pressure drop by a multiplication factor in connection with the downcomer, assuming that all of them depends of the water velocity in the same way.

Using the same procedure as for the "core" the downcomer pressure drop becomes:

$$11. \qquad \frac{\Delta p_2}{\rho_{fs}} = 0.092 \frac{L'_d}{D_{ed}^{1.2}} F_f(T)\, v_d^{1.8}$$

The effective length of the downcomer is equal to L_d multiplied by the correction factor mentioned above. In lack of better knowledge L'_d is used as $2\,L_d$.

The momentum equation for the hydraulic loop is used to calculate the velocity v_d. It takes the following form:

$$12. \qquad \sum L \dot{v} = g \sum^{\Delta x} \alpha \, \Delta x - \frac{\Delta p_2}{\rho_{fs}} - \sum^{\Delta x} \frac{\Delta p_f}{\rho_{fs}}$$

The summation on the left side is carried out for the closed loop over the different sections. The summations on the wright side is made over the "core". On the left side all the velocities can be referred to the downcomer by the relation $v_x A_x = v_d A_d$ for an arbitrary section x. For the "core" is used a velocity equal to $\frac{1}{1-\alpha_m}$ times the input velocity corresponding to a mean void fraction α_m. This approximation is allowable as the main part of the momentum comes from the downcomer.

The secondary flow is finally calculated from v_d as

$$13. \qquad W_s = A_d v_d \rho_{fs}$$

The saturation value of the density is used as the temperature always will be quite near the saturation point.

2C. The riser and the steam volume

Due to the complicated mechanism of the steam-water separators in the riser is a detailed description of the steam-water destribution not possible, so only a rough approximation is used. It is assumed that the void fraction is equal to the output value for the "core" throughout the riser in the steady state and follows dynamically with a time lag equal to the transit time for a steam particle:

$$14. \qquad \dot{\alpha}_r = \frac{1}{\tau_r} (\alpha_\sigma - \alpha_r)$$

$$15. \qquad \tau_r = \frac{V_r}{A_s v_{g\sigma}} = \frac{V_r \rho_{gs} \alpha_r}{W_{g\sigma}}$$

It is further assumed that the water and steam phase is in thermal equilibrium at the system pressure during transients. The water in the upper part of the feed water chamber is in this respect included in the water volume of the riser. Thermal equilibrium means that mass transfer between the two phases takes place.

The energy equation for steam volume and the riser leads to the following equation:

$$(V_e + \alpha_r V_r)\frac{d\rho_{gs}}{dp}\dot{p} = W_{gr} - W_l - V_r\dot{\alpha}_r\rho_{gs} - C1\cdot\dot{p}$$

$$C1 = \frac{1}{h_{fg}}\left[\rho_{fs}\frac{dh_{fs}}{dp}\left(V_{br} + V_r(1-\alpha_r)\right) + (V_e + \alpha_r V_r)\rho_{gs}\frac{dh_{gs}}{dp} - (V_{br} + V_r + V_e)\right]$$

The term $C1\cdot\dot{p}$ represents the mass exchange with the water during pressure variations. C1 may be somewhat simplified: $(V_e + \alpha V_r)$ can be used as a constant volume as the variable term αV_r always will be small compared with V_e; and $(V_{br} + V_r + V_e)$ can be reduced to the same volume as that used for $(V_e + \alpha V_r)$ because $\rho_{fs}\frac{dh_{fs}}{dp} \gg 1$ (Remark: $\rho\frac{dh}{dp}$ has the dimension 1). So $\dot{p}$ may be written as:

$$16. \quad \dot{p} = \left(W_{gr} - W_l - V_r\dot{\alpha}_r\rho_{gs}\right) \Big/ \left((V_e + \alpha_{rm}V_r)\frac{d\rho_{gs}}{dp} + C1\right)$$

$$C1 = \frac{1}{h_{fg}}\left[\rho_{fs}\frac{dh_{fs}}{dp}\left(V_{br} + V_r(1-\alpha_r)\right) + (V_e + \alpha_{rm}V_r)\left(\rho_{gs}\frac{dh_{gs}}{dp} - 1\right)\right]$$

The steam load W_l is taken as a load constant Cl multiplied by the pressure:

$$17. \quad W_l = Cl\cdot p$$

The steam load is varied by variation of Cl.

The water flow to the feed water chamber will be:

$$18. \quad W_b = W_{fr} + C1\cdot\dot{p} + V_r\dot{\alpha}_r\rho_{fs}$$

2D. The feedwater chamber

The energy equation for the lower part is used to calculate the temperature assuming complete mixture of the feed water with the recirculating water:

$$\dot{h}_b V_b \rho_f = W_b(h_{fs} - h_b) + W_i(h_i - h_b)$$

Introducing the heat capacity C_{ps} for the recirculating water at saturation, $C_{pm} = \frac{1}{2}(C_{pi} + C_{ps})$ for the feedwater, and as an approximation using C_{ps} for the mixture gives us:

$$19. \quad \dot{T}_b = \frac{1}{V_b\rho_{fs}}\left(W_b(T_{sa} - T_b) + W_i\frac{C_{pm}}{C_{ps}}(T_i - T_b)\right)$$

The ratio $\frac{C_{pm}}{C_{ps}}$ can be taken as a constant value equal to 0.94 with an error less than 3% in the pressure range 50-75 bar and an feedwater temperature of 210-240 °C.

The water level changes may be calculated as:

$$20. \qquad \Delta L_b = \frac{1}{A_b \cdot \rho_{fs}} \left(W_b + W_i - W_s + \dot{p} \frac{d\rho_{fs}}{dp} \sum V \right)$$

where ΣV is the sum of all water volumes outside the "core". It is assumed that the overall temperature dynamically on the whole follows the saturation temperature.

2E. The downcomer

The downcomer is separated from the "core" by a steel wall. As the temperature difference across the wall is less than 5 °C and the heating surface less than 5% of the U-tube surface the heat transmission through the wall can be neglected. It is further assumed that the pressure variations are so slow that boiling in the downcomer is avoided. It means that $\dot{p} < \frac{dp}{dT} \frac{\Delta T}{\Delta t}$, where $\Delta T \simeq 4$ °C, $\Delta t \simeq 2$ s and $\frac{dp}{dT} \simeq 1$ bar/s.

The energy equation then gives:

$$21. \qquad \frac{\partial T_d}{\partial x} = - \frac{1}{W_s} A_d \rho_{fs} \dot{T}_d$$

with the direction of the x-axis from the top to the bottom.

2F. The inlet and the outlet chamber

The inlet and the outlet chambers in the primary loop is not important being small and without heat exchanges to the surroundings. Both of them introduces a time lag of approximately 1 s at normal primary flow rate. The two equations are:

$$22. \qquad \dot{T}_{p1} = \frac{W_p}{V_{pi} \rho_f} (T_{pi} - T_{p1})$$

$$23. \qquad \dot{T}_{p2} = \frac{W_p}{V_{po} \rho_f} (T_{po} - T_{p2})$$

A summary of the equations is found in Appendix C.

The equations may be solved either by a digital program or by hybrid simulation. The solution involves different procedures in the two cases as discussed in the next two chapters. But a common feature in connection with the partial differential equations is the division of the space in subsections, 20 "core" and downcomer sections, and sampling in the time domain with a sampling rate of 10-20 per second. The time derivatives are substituted by first order differences in both cases, while the space derivatives are handled in different ways.

The most stable and accurate solution is obtained by calculation of the space derivative, for the next step in space and time, as functions of the variables taken as mean values for both the space and time step concerned, except the time derivative which always for stability reasons must be evaluated at the end of the space section. This procedure demands for feedback in the equations and parallel solution of several equations. This is easily done by hybrid computation, while pure digital programming leads to iterations or solution of many coupled equations.

Another problem common for the two technics is the simultaneous integration along the primary and secondary space axis which in principle is impossible, but never the less is needed for an mathematical exact solution. This problem is circumvented in different ways in the two methods.

3. SOLUTION BY FORTRAN PROGRAMMING

The equations with numerical constants is given in appendix D. A number of temperature and pressure dependent parameters is used. They are all approximated by polynomials given in appendix E.

The eq. 1 and 4 have extremely strong feedbacks from $\dot{T}$ and $\dot{\alpha}$ which is absolutely necessary to take into account by the solution. It is done quite simply in the temperature equations. We introduce the following approximations:

$$\dot{T}\left(j+1, n+\tfrac{1}{2}\right) = \frac{1}{\Delta t}\left(T(j+1, n+1) - T(j+1, n)\right)$$

$$\frac{\partial}{\partial x}\left(T\left(j+\tfrac{1}{2}, n+1\right)\right) = \frac{1}{\Delta x}\left(T(j+1, n+1) - T(j, n+1)\right)$$

The integers j and n stands for the space and time step respectively. Introducing in eq. 1 and solving with respect to $T_p(j+1, n+1)$ gives:

$$1. \quad T(j+1, n+1) = \left(-\frac{q_p}{W_p c_{pp}} + \frac{A_p \rho_f}{W_p \Delta t} T_p(j+1, n) + \frac{1}{\Delta z} T_p(j, n+1)\right) \Big/ \left(\frac{1}{\Delta z} + \frac{A_p \rho_f}{W_p \Delta t}\right)$$

where $A_p = 1.035\ m^2$ and $\Delta z = 0.5055$ m.

In a similar way is eq. 4a changed:

$$4a. \quad T_s(j+1, n+1) = \left(\frac{q_s}{W_s C_{ps}} + \frac{A_s \rho_{fs}}{W_s \Delta t} T_s(j+1, n) + \frac{1}{\Delta x} T_s(j, n+1) \right) \Big/ \left(\frac{1}{\Delta x} + \frac{A_s \rho_{fs}}{W_s \Delta t} \right)$$

where $A_s = 5.16$ m^2 and $\Delta x = 0.5055$ m. Eq. 4b and 4c are more complicated as $\dot{\alpha}$ not can be introduced as a simple function of one of the two variables $(v_g \alpha)$ or $(v_g(1-\alpha))$. We use eqs. 4b, 4c and 8 to find an explicit solution for $\alpha(j+1, n+1)$.

For convenience we introduce the following short notations:

$$C2 = h_{fg} \rho_{gs}$$

$$C3 = \rho_{gs} \frac{d h_{gs}}{dp} + h_{fg} \frac{d \rho_{gs}}{dp}$$

$$C4 = \rho_{fs} \frac{d h_{fs}}{dp}$$

$$C6 = \frac{1}{\rho_{fs}} \frac{d \rho_{gs}}{dp}$$

$$C7 = \frac{1}{\rho_{fs}} \frac{d \rho_{fs}}{dp}$$

$$C8 = \frac{\rho_{gs}}{\rho_{fs}}$$

$$\alpha' = \frac{1}{2} \left(\alpha(j, n+1) + \alpha(j+1, n) \right)$$

$$\alpha_g = \alpha(j+1, n)$$

$$\alpha = \alpha(j+1, n+1)$$

$$QS1 = \left(\frac{q_s}{A_s} - \dot{p} \left(\alpha' C3 + (1-\alpha') C4 - 0.1 \right) \right) \Big/ C2$$

$$QS2 = -\dot{p} \left(\alpha' \cdot C6 + (1-\alpha') C7 \right)$$

$$UG = v_g \alpha(j, n+1)$$

$$UF = v_f (1-\alpha)(j, n+1)$$

NB: The unit for power is here MW, therefore the constant 1 in QS1 has been changed to 0.1.

From eqs. 4b and 4c we find:

$$\Delta UG = \Delta x \frac{\partial}{\partial x}(v_g \alpha) = \Delta x \cdot QS1 - \frac{\Delta x}{\Delta t}(\alpha - \alpha_g)$$

$$\Delta UF = \Delta x \frac{\partial}{\partial x}(v_f(1-\alpha)) = (1 - C8)\frac{\Delta x}{\Delta t}(\alpha - \alpha_g) + \Delta x \; QS2 - C8 \cdot \Delta UG$$

Insertion in eq. 8 gives:

$$\begin{aligned} &\alpha\left(UG + \Delta x \cdot QS1 - \frac{\Delta x}{\Delta t}(\alpha - \alpha_g)\right) \\ &+ \alpha \cdot S \left(UF + (1 - C8)\frac{\Delta x}{\Delta t}(\alpha - \alpha_g) + \Delta x \cdot QS2 \right. \\ &\left. - C8 \cdot \Delta x \; QS1 + C8 \frac{\Delta x}{\Delta t}(\alpha - \alpha_g)\right) = \\ &\quad UG + \Delta x \cdot QS1 - \frac{\Delta x}{\Delta t}(\alpha - \alpha_g) \end{aligned}$$

Which after reduction takes the form:

$$A\alpha^2 + B\alpha + C = 0$$

where:

$$A = \frac{\Delta x}{\Delta t}(S - 1)$$

$$B = UG + S \cdot UF + \Delta x\left[QS1 + S \cdot (QS2 - C8 \cdot QS1) - \frac{1}{\Delta t}(\alpha_g(S-1) - 1)\right]$$

$$C = -\left[UG + \Delta x\left(QS1 + \frac{1}{\Delta t}\alpha_g\right)\right]$$

The solution gives:

$$\alpha = \left(-B + (B^2 - 4AC)^{0.5}\right)/2A$$

The shift from temperature calculation to void calculation takes place when a value of $T_s(j+1, n+1)$ exceeds the saturation value. For that section the output value of T_s is fixed to T_{sa} and the length of the boiling part of that section is calculated as:

$$\Delta X = 0.5055 \frac{T_s(j+1, n+1) - T_{sa}}{T_s(j+1, n+1) - T_s(j, n+1)}$$

and used in the calculation of α from the formulaes given above.

In the calculation of the temperature and void profiles for the time step n+1 by integration of eq. 1-4 we should use heat flows taken at time n+ $\frac{1}{2}$:

$$q(n+\tfrac{1}{2}) = \frac{1}{2}(q(n) + q(n+1))$$

but this is generally not possible. Therefore we perform the calculations for q taken at time n and store the results as temporary profiles. The calculations are then repeated with the heat flow calculated on the basis of thise temporary profiles, and finally we use a mean value of the two results. This procedure corresponds to the improved Euler integration method.

However, by the primary and the secondary temperature calculations, where it is possible to improve the stability and accuracy, by using the mean temperature

$$T_m = \frac{1}{2}(T(j, n+1) + T(j+1, n))$$

for the heat flow calculation this is done.

Along with the void calculations for the subsections of the "core" we calculate the friction pressure drop also as a mean value between two steps. Afterwards we calculate the driving force from the void distribution at time n+1, the pressure drop outside the "core" from the velocity v_d at time n, and finally by simple Euler integration the velocity $v_d(n+1)$.

The pressure is calculated straight forward from eq. 16 by Euler integration, but it must be mentioned that the internal feedback from mass exchange between steam and water has been taken into account. Only the small feedback term $\Delta W_1 = \Delta p \cdot \Delta Cl$ is neglected as it appears to have an effect much smaller than that coming from the uncertainty in the steam volume.

4. SOLUTION BY HYBRID COMPUTATION

The equations are those given in appendix D and with scaling factors in appendix F. Some of the parameters are calculated as polynomials as in the Fortran program and used in multiplying D/A converters, others are calculated on analog form by diode function generators. The last mentioned are: $C_{pp}(T)$, $C_{ps}(T)$, $H_p(T)$, $H_s(T)$ and $T_{sa}(p)$.

Parallel computations with analog components solve the problem with feedback in and interaction between the equations except the coupling between the secondary sides for the upstream and downstream U-tubes. To solve this problem we here use a second order prediction of the outside tube wall temperature for the integration of the primary flow and the inside wall temperature. Afterwards we integrate the secondary flow and the outside wall temperature on the basis of the newly calculated inside wall temperature.

The advantages obtained by parallel computations improve the stability and dynamic accuracy so a sampling time of 0.1 s can be used.

The same analog conponents are used for each "core" section during the primary and secondary integration, 40 and 20 times respectively, and in addition do we use the temperature calculation circuit for the primary side again for the secondary side in order to save computing components.

The hydraulic driving force and friction pressure drop in the "core" are calculated along with the secondary side integration, while the outside "core" pressure drop and the integration of $\dot{v}_d$ is done in continuous analog form.

The calculation for the riser, steam volume, feed water chamber and downcomer is done in pure digital form due to lack of analog components. However, as an exception, is the integration of $\dot{p}$ done in analog form in order to improve the accuracy in the A/D convertion of the steam production--steam load difference and the D/A convertion of the pressure, which appear to be critical.

The temperature and void profiles are calculated in analog form but converted to and stored in digital form. The full computing power of the 3 computer units: The EAI 680 hybrid machine, the PDP8 central processor and the Floating Point Processor are used in parallel computation. This makes it possible to run the simulation in true time scale, 20 times faster than the Fortran program.

The limited amount of analog components has not allowed to include water level calculation and feed water control or simulation of the primary inlet and outlet chamber. Further has some approximation been used for void and friction calculation. When more analog equipment and D/A converters become available the model will be improved and enlarged so it may be used as an universal model for the U-tube type of steam generators.

APPENDIX A

GEOMETRICAL AND PHYSICAL CONSTANTS

"core" :

Diameter		3.15 m
Height		10.11 "
U-tube:	number	3388
	diameter	22.23/19.69 mm
	wall thickness	Δr = 0.00127 m
Cross sections:	primary	A_p = 1.035 m^2
	secondary	A_s = 5.160 m^2
	tube wall	0.283 m^2
Heating surfaces:	primary	O_p = 210 m^2/m
	secondary	O_s = 237 "
	tube	O_r = 223 "
Hydraulic diameter:	primary	D_{ep} = 0.0197 m
	secondary	D_{es} = 0.0436 "
U-tube physics:	ρ_r = 8440 kg/m^3	
	c_r = 0.41 kJ/kg ^oC	
		λ_r = 0.014 kW/m ^oC
		C_r = 980 kJ/m ^oC

Riser:

Diameter	2.425 m	
Height	2.725 m	
Cross section	4.630 m	
Volume		V_r = 12.60 m^3

Steam volume: $V_e = 75 \ m^3$

Feed water chamber :

Height	2.725 m	
Cross section	14.40 - 4.63	$A_b = 9.77 \ m^2$
Volume	26.6 m^3	
Volume above inlet		$V_{br} = 7.8 \ m^3$
Volume below inlet		$V_b = 18.8 \ m^3$

Downcomer:

Diameter	3.286/3.150 m	
Cross section		$A_d = 0.687 \ m^2/m$
Volume		$V_d = 6.94 \ m^3$
Heating surface	10.1 m^2/m	

Primary inlet and outlet chamber:

Volume $V_{pi} = V_{po} = 4.5 \ m^3$

APPENDIX B

LIST OF SYMBOLS

A : cross sections (m^2)

A_p "core" primary

A_s "core" secondary

A_b feed water chamber

A_d downcomer

D_e : hydraulic diameters (m)

D_{ep} "core" primary

D_{es} "core" secondary

D_{ed} downcomer

V : volumes (m^3)

V_e steam volume

V_r riser

V_b feed water chamber below inlet

V_{br} feed water chamber above inlet

V_{pi} primary inlet chamber

V_{po} primary outlet chamber

L: length (m)

L_b feed water chamber water level

L_d downcomer

L_c "core"

$$\Delta x = \frac{L_c}{20}$$

$$\Delta z = \frac{L_c}{20}$$

O: Surfaces (m^2/m)

O_p "core" primary

O_s "core" secondary

O_r "core" tube between inside and outside shell

O_d downcomer inside

G: mass flow per m^2 (kg/s m^2)

W: mass flow (kg/s)

W_p "core" primary

W_s "core" secondary inlet

W_g "core" secondary steam phase

W_f "core" secondary water phase

W_{go} "core" secondary steam outlet

W_{fo} "core" secondary water outlet

W_l steam load

W_i feed water inlet

W_b water to feed water chamber

T: Temperature (^oC)

T_p "core" primary

T_s " secondary

T_{r1} " tube wall inside

T_{r2} " " " outside

T_b feed water chamber

T_d downcomer

T_{pi} primary inlet

T_{po} primary outlet

T_{p1} primary inlet chamber

T_{p2} primary outlet chamber

T_{si} "core" secondary inlet

T_{sa} saturation

q: Heat flow (MW/m)

- q_p from primary flow
- q_s to secondary flow
- q_r through tube wall

v: Velocities (m/s)

- v_f "core" water phase
- v_g " steam phase
- v_{go} " steam outlet
- v_d downcomer

h: Enthalpy (MJ/kg)

- h_i feed water inlet
- h_b feed water chamber
- h_{gs} saturated steam
- h_{fs} " water
- $h_{fg} = h_{gs} - h_{fs}$ evaporation heat

ρ : Densities (kg/m^3)

- ρ_f water
- ρ_g steam
- ρ_{gs} saturated steam
- ρ_{fs} saturated water

C_p: Heat capacities

- C_{pp} primary, specific (MJ/kg ^oC)
- C_{ps} secondary " "
- C_{pi} feed water inlet specific (MJ/kg ^oC)
- $C_{pm} = \frac{1}{2}(C_{ps} + C_{pi})$ "
- C_m tube wall (MJ/m ^oC)

α : Void fraction

- α_o "core" outlet
- α_r riser

η Dynamic viscosity (kg/m s)

λ Thermal conductivity (MW/m ^{o}C)

f Single phase friction coefficient

F_f " " " parameter

R Two phase friction multiplier

R_e Reynolds number

H Heat transfer parameter

S Slip factor = v_g/v_f

X Steam quality

Appendix C.

Summary of basic equation

1. $$\frac{\partial T_p}{\partial z} = -\frac{1}{W_p}\left(\frac{q_p}{C_{pp}} + A_p \rho_f \dot{T}_p\right)$$

2. $$\dot{T}_{r1} = \frac{2}{C_r}(q_p - q_r)$$

3. $$\dot{T}_{r2} = \frac{2}{C_r}(q_r - q_s)$$

4a. $$\frac{\partial T_s}{\partial x} = \frac{1}{W_s}\left(\frac{q_s}{C_{ps}} - A_s \rho_f \dot{T}_s\right)$$

4b. $$\frac{\partial}{\partial x}(V_g \alpha) = \frac{1}{h_{fg}\rho_{gs}}\left(\frac{q_s}{A_s} - \dot{p}\left(\alpha\left(\rho_{gs}\frac{dh_{gs}}{dp} + h_{fg}\frac{d\rho_{gs}}{dp}\right) + (1-\alpha)\rho_{fs}\frac{dh_{fs}}{dp} - 1\right)\right) - \dot{\alpha}$$

4c. $$\frac{\partial}{\partial x}(V_f(1-\alpha)) = \dot{\alpha}\left(1 - \frac{\rho_{gs}}{\rho_{fs}}\right) - \dot{p}\left(\frac{\alpha}{\rho_{fs}}\frac{d\rho_{gs}}{dp} + \frac{1-\alpha}{\rho_{fs}}\frac{d\rho_{fs}}{dp}\right) - \frac{\rho_{gs}}{\rho_{fs}}\frac{\partial}{\partial x}(V_g \alpha)$$

5. $$q_p = \frac{0.023}{De_p^{0.2}} O_p \left(\frac{W_p}{A_p}\right)^{0.8} H_p(T)(T_p - T_{r1})$$

6. $$q_r = \frac{\lambda_r}{\Delta r} O_r (T_{r1} - T_{r2})$$

7a. $$q_s = \frac{0.023}{De_s^{0.2}} O_s \left(\frac{W_s}{A_s}\right)^{0.8} H_s(T)(T_{r2} - T_s)$$

7b. $$q_s = 1.972\, O_s \exp(p/43.4)(T_{r2} - T_{sa})^2$$

8. $$\alpha = \frac{V_g \alpha / S}{V_g \alpha / S + V_f(1-\alpha)}$$

9. $$X = \frac{V_g \alpha \cdot \rho_{gs}}{V_g \alpha \rho_{gs} + V_f(1-\alpha)\rho_{fs}}$$

10. $$\frac{\Delta p_1}{\rho_{fs}} = 0.092 \frac{\Delta x}{Dep^{1.2}} F_f(T) \sum V^{1.8}\left(1 + 2400\frac{X}{p}\right) \qquad V = \frac{W_g + W_f}{A_s \rho_{fs}}$$

11. $$\frac{\Delta p_2}{\rho_{fs}} = 0.092 \frac{L_d'}{De_d^{1.2}} F_f(T) V_d^{1.8}$$

12. $$\sum L\dot{V} = g\sum^{\Delta x}\alpha\Delta x + \sum^{\Delta x}\frac{\Delta p_1}{\rho_{fs}} - \frac{\Delta p_2}{\rho_{fs}}$$

13. $$W_s = A_d V_d \rho_{fs}$$

14. $\dot{\alpha}_r = \frac{1}{\tau_r}(\alpha_\sigma - \alpha_r)$

15. $\tau_r = \frac{V_r}{A_s v_{g\sigma}} = \frac{V_r \rho_{gs} \alpha_\sigma}{W_{g\sigma}}$

16. $\dot{p} = (W_{g\sigma} - W_l - V_r \dot{\alpha}_r \rho_{gs}) / ((V_e + \alpha_{rm} V_r) \frac{d\rho_{gs}}{dp} + C1)$

$C1 = \frac{1}{h_{fg}}\left[\rho_{fs}\frac{dh_{fs}}{dp}(V_{br} + V_r(1-\alpha_r)) + (V_e + \alpha_{rm} V_r)(\rho_{gs}\frac{dh_{gs}}{dp} - 1)\right)$

17. $W_l = C_l\, p$

18. $W_b = W_{f\sigma} + \dot{p}\, C1 + V_r \dot{\alpha}_r \rho_{fs}$

19. $\dot{T}_b = \frac{1}{V_b \rho_{fs}}\left(W_b(T_{sa} - T_b) + W_i \frac{c_{pin}}{c_{ps}}(T_i - T_b)\right)$

20. $\Delta \dot{L}_b = \frac{1}{A_b \rho_{fs}}\left(W_b + W_i - W_s + \dot{p}\,\frac{d\rho_{fs}}{dp}\sum V\right.$

21. $\frac{\partial T_d}{\partial x} = -\frac{1}{W_s} A_d \rho_{fs} \dot{T}_d$

22. $\dot{T}_{p1} = \frac{W_p}{V_{pi}\rho_f}(T_{pi} - T_{p1})$

23. $\dot{T}_{p2} = \frac{W_p}{V_{po}\rho_f}(T_{po} - T_{p2})$

Appendix D

Equations with numerical constants

1. $$\frac{\partial T_p}{\partial z} = -\frac{1}{W_p}\left(\frac{q_p}{c_{pp}} + 1.035\,\rho_f\,\dot{T}_p\right)$$

2. $$\dot{T}_{r1} = 2.04\,(q_p - q_r)$$

3. $$\dot{T}_{r2} = 2.04\,(q_r - q_s)$$

4a. $$\frac{\partial T_s}{\partial x} = \frac{1}{W_s}\left(\frac{q_s}{c_{ps}} - 5.16\,\rho_f\,\dot{T}_s\right)$$

4b. $$\frac{\partial}{\partial x}(v_g\alpha) = \frac{1}{h_{fg}\,\rho_{gs}}\left(0.1938\,q_s - \dot{p}\left(\alpha\left(\rho_{gs}\frac{dh_{gs}}{dp} + h_{fg}\frac{d\rho_{gs}}{dp}\right) + (1-\alpha)\rho_{fs}\frac{dh_{fs}}{dp} - 0.1\right)\right) - \dot{\alpha}$$

4b. $$\frac{\partial}{\partial x}(v_f(1-\alpha)) = \dot{\alpha}\left(1 - \frac{\rho_{gs}}{\rho_{fs}}\right) - \dot{p}\left(\frac{\alpha}{\rho_{fs}}\frac{d\rho_{gs}}{dp} + \frac{1-\alpha}{\rho_{fs}}\frac{d\rho_{fs}}{dp}\right) - \frac{\rho_{gs}}{\rho_{fs}}\frac{\partial}{\partial x}(v_g\alpha)$$

5. $$q_p = 10.34\cdot 10^{-3}\,H_p(T)\,W_p^{0.8}\,(T_p - T_{r1})$$

6. $$q_r = 2.43\,(T_{r1} - T_{r2})$$

7a. $$q_s = 2.74\cdot 10^{-3}\,H_s(T)\cdot W_s^{0.8}\,(T_s - T_{r2})$$

7b. $$q_s = 0.467\,\exp\left(\frac{p}{43.4}\right)(T_{r2} - T_{sa})^2$$

8. $$\alpha = \frac{v_g\alpha/S}{v_g\alpha/S + v_f(1-\alpha)}$$

9. $$X = \frac{v_g\alpha\,\rho_{gs}}{v_g\alpha\,\rho_{gs} + v_f(1-\alpha)\rho_{fs}}$$

10. $$\frac{\Delta p_1}{\rho_{fs}} = 0.08422\sum v^{1.8}\left(1 + 2400\,\frac{X}{p}\right),\quad v = 0.1938\left(v_f(1-\alpha) + v_g\alpha\frac{\rho_{gs}}{\rho_{fs}}\right)$$

11. $$\frac{\Delta p_2}{\rho_{fs}} = 0.85\,v_d^{1.8}$$

12. $$v_d = \left(4.96\sum\alpha - \frac{\Delta p_1}{\rho_{fs}} + \frac{\Delta p_2}{\rho_{fs}}\right)\Big/\left(10.5 + \frac{1.3}{1 + \alpha_m}\right)$$

13. $$W_s = 6.16\,v_d\,\rho_{fs}$$

14. $\dot{\alpha}_r = \frac{1}{\tau_r}(\alpha_o - \alpha_r)$

15. $\tau_r = \frac{2.44}{V_{go}} = 12.6 \frac{\rho_{gs}\,\alpha_o}{W_{go}}$

16. $\dot{p} = (W_{go} - W_l - 12.6\,\dot{\alpha}_r \rho_{gs}) / (80 \frac{d\rho_{gs}}{dp} + C1)$

$C1 = \frac{1}{h_{fg}}\left[\rho_{fs} \frac{dh_{fs}}{dp}\left(7.8 + 12.6(1-\alpha_r)\right) + 80\left(\rho_{gs}\frac{dh_{gs}}{dp} - 0.1\right)\right]$

17. $W_l = Cl\, p$

18. $W_b = W_{fo} + \dot{p}\,C1 + 12.6\,\dot{\alpha}_r \rho_{fs}$

19. $\dot{T}_b = \frac{0.0532}{\rho_{fs}}\left(W_b(T_{sa} - T_b) + 0.94\, W_i(T_i - T_b)\right)$

20. $\Delta\dot{L}_b = \frac{0.102}{\rho_{fs}}\left(W_b + W_i - W_s - 70\,\dot{p}\right)$

21. $\frac{\partial T_d}{\partial x} = -\frac{0.687}{W_s} \rho_{fs} \dot{T}_d$

22. $\dot{T}_{p1} = 0.222 \frac{W_p}{\rho_f}(T_{pi} - T_{p1})$

23. $\dot{T}_{p2} = 0.222 \frac{W_p}{\rho_f}(T_{po} - T_{p2})$

Some equations in the final form for digital simulation

$$\Delta z = \Delta x = 0.5055 \ m$$

1. $$[T_p]_{j+1}^{n+1} = \left(-\frac{q_p}{W_p c_{pp}} + 1.978[T_p]_j^{n+1} + \frac{1.035 \rho_f}{W_p \cdot \Delta t}[T_p]_{j+1}^{n}\right) \Big/ \left(1.978 + \frac{1.035 \rho_f}{W_p \cdot \Delta t}\right)$$

4a. $$[T_s]_{j+1}^{n+1} = \left(\frac{q_{s1} + q_{s2}}{W_s c_{ps}} + 1.978[T_s]_j^{n+1} + \frac{5.16 \rho_{fs}}{W_s \cdot \Delta t}[T_s]_{j+1}^{n}\right) \Big/ \left(1.978 + \frac{5.16 \rho_{fs}}{W_p \cdot \Delta t}\right)$$

12. $$[v_d]^{n+1} = [v_d]^n + \Delta t \left(4.96 \sum \alpha - \left(\frac{\Delta p_1}{\rho_{fs}} + \frac{\Delta p_2}{\rho_{fs}}\right)\right) \Big/ \left(10.5 + \frac{1.3}{1 - \alpha_m}\right)$$

14. $$[\alpha_r]^{n+1} = \left(\Delta t [\alpha_o]^{n+1} + \tau [\alpha_r]^n\right) \Big/ (\Delta t + \tau)$$

$$\dot{\alpha}_r = (\alpha_o - \alpha_r) / \tau$$

19. $$[T_b]^{n+1} = \left([T_b]^n - 0.0532 \frac{\Delta t}{\rho_{fs}}\left(W_b T_{sa} + 0.94 W_i T_i\right)\right) \Big/ \left(1 + 0.0532 \frac{\Delta t}{\rho_{fs}}\left(W_b + 0.94 W_i\right)\right)$$

20. $$[\Delta L_b]^{n+1} = [\Delta L_b]^n + 0.102 \frac{\Delta t}{\rho_{fs}}\left(W_b + W_i - W_s + 70 \dot{p}\right)$$

21. $$[T_d]_j^{n+1} = \left([T_d]_j^n + 2.88 W_s \frac{\Delta t}{\rho_{fs}}[T_d]_{j-1}^{n+1}\right) \Big/ \left(1 + 2.88 W_s \frac{\Delta t}{\rho_{fs}}\right)$$

APPENDIX E

WATER-STEAM DATA POLYNOMIALS

$$T_{sa} = a_0 + a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4 \qquad (^{o}c)$$

$a_0 = 137.88$

$a_1 = 5.0121$

$a_2 = -0.79614 \cdot 10^{-1}$

$a_3 = 0.72476 \cdot 10^{-3}$

$a_4 = -0.25717 \cdot 10^{-5}$

$$\rho_f = a_0 + a_1 T + a_2 T^2 + a_3 T^3 \quad \text{ved 150 bar} \qquad (kg/m^3)$$

$a_0 = 1740.9$

$a_1 = -9.4540$

$a_2 = 0.36496 \cdot 10^{-1}$

$a_3 = -0.54202 \cdot 10^{-4}$

$$C_{pp} = a_0 + a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4 + a_5 T^5 \qquad (MJ/kg\ ^{o}C)$$

$a_0 = -0.42044 \cdot 10^{-1}$

$a_1 = 0.20448 \cdot 10^{-3}$

$a_2 = 0.77403 \cdot 10^{-6}$

$a_3 = -0.28309 \cdot 10^{-8}$

$a_4 = -0.87750 \cdot 10^{-11}$

$a_5 = 0.26327 \cdot 10^{-13}$

$$C_{ps} = a_0 + a_1 T + a_2 T^2 + a_3 T^3 \qquad (MJ/kg\ ^{o}C)$$

$a_0 = 0.22556 \cdot 10^{-3}$

$a_1 = 0.61417 \cdot 10^{-4}$

$a_2 = -0.31531 \cdot 10^{-6}$

$a_3 = 0.57419 \cdot 10^{-9}$

$$H_p(T) = 0.925 + 0.0018 \cdot (T_p - 300)$$

$$H_s(T) = 0.875 + 0.0012 \cdot (T_s - 255)$$

$$\rho_{fs} = a_0 + a_1 T_{sa} + a_2 T_{sa}^2 \qquad (kg/m^3)$$

$a_0 = 922.02$

$a_1 = 0.54104$

$a_2 = -0.41304 \cdot 10^{-2}$

$$\rho_{gs} = a_0 + a_1 T_{sa} + a_2 T_{sa}^2 + a_3 T_{sa}^3 \qquad (kg/m^3)$$

$a_0 = -104.953$

$a_1 = 1.53481$

$a_2 = -0.768233 \cdot 10^{-2}$

$a_3 = 0.141607 \cdot 10^{-4}$

$$\frac{d\rho_{fs}}{dp} = a_0 + a_1 T_{sa} + a_2 T_{sa}^2 + a_3 T_{sa}^3 \qquad (kg/m^3 bar)$$

$a_0 = -33.314$

$a_1 = 0.29584$

$a_2 = -0.93865 \cdot 10^{-3}$

$a_3 = 0.10129 \cdot 10^{-5}$

$$\frac{d\rho_{gs}}{dp} = a_0 + a_1 T_{sa} + a_2 T_{sa}^2 \qquad (kg/m^3 bar)$$

$a_0 = 1.0923$

$a_1 = -0.59817 \cdot 10^{-2}$

$a_2 = 0.14787 \cdot 10^{-4}$

$$\frac{dh_{fs}}{dp} = a_0 + a_1 T_{sa} + a_2 T_{sa}^2 + a_3 T_{sa}^3 \qquad \text{(kJ/kg bar)}$$

$a_0 = 180.65$

$a_1 = -1.7121$

$a_2 = 0.56767 \cdot 10^{-2}$

$a_3 = -0.64176 \cdot 10^{-5}$

$$\frac{dh_{gs}}{dp} = a_0 + a_1 T_{sa} + a_2 T_{sa}^2 + a_3 T_{sa}^3 \qquad \text{(kJ/kg bar)}$$

$a_0 = 64.714$

$a_1 = -0.63723$

$a_2 = 0.20824 \cdot 10^{-2}$

$a_3 = -0.23142 \cdot 10^{-5}$

$$h_{fg} = a_0 + a_1 T_{sa} + a_2 T_{sa}^2 \qquad \text{(kJ/kg)}$$

$a_0 = 1991.2$

$a_1 = 3.2023$

$a_2 = -0.17199 \cdot 10^{-1}$

Appendix F

Equations with scale factors for hybrid simulation

1. $$\frac{\partial}{\partial z}\left[\frac{T_p}{50}\right]^{n+1}_{j+\frac{1}{2}} = \frac{-0.1}{\left[\frac{W_p}{5000}\right]}\left(0.8\,\frac{\left[\frac{q_p}{200}\right]^{n+\frac{1}{2}}}{[100\,c_{pp}]} + 2.07\left[\frac{\rho_f}{1000}\right]\left[\frac{T_p^{\,n+1} - T_p^{\,n}}{6}\right]\right)$$

$$\left[\frac{T_p}{50}\right]^{n+1}_{j+1} = \left[\frac{T_p}{50}\right]^{n+1}_{j} + 0.5055\,\frac{\partial}{\partial z}\left[\frac{T_p}{50}\right]^{n+1}_{j+\frac{1}{2}}$$

2. $$\left[\frac{T_{r1}}{50}\right]^{n+1} = \left[\frac{T_{r1}}{50}\right]^{n} + 0.408\left(\left[\frac{q_p}{100}\right]^{n+\frac{1}{2}} - \left[\frac{q_r}{100}\right]^{n+\frac{1}{2}}\right)$$

3. $$\left[\frac{T_{r2}}{50}\right]^{n+1} = \left[\frac{T_{r2}}{50}\right]^{n} + 0.408\left(\left[\frac{q_r}{100}\right]^{n+\frac{1}{2}} - \left[\frac{q_s}{100}\right]^{n+\frac{1}{2}}\right)$$

4a. $$\frac{\partial}{\partial x}\left[\frac{T_s}{50}\right]^{n+1}_{j+\frac{1}{2}} = \frac{0.1}{\left[\frac{W_s}{5000}\right]}\left(0.8\,\frac{\left[\frac{q_s}{200}\right]^{n+\frac{1}{2}}}{[100\,c_{ps}]} - 10.32\left[\frac{\rho_{fs}}{1000}\right]\left[\frac{T_s^{\,n+1} - T_s^{\,n}}{5}\right]\right)$$

$$\left[\frac{T_s}{50}\right]^{n+1}_{j+1} = \left[\frac{T_s}{50}\right]^{n+1}_{j} + 0.5055\,\frac{\partial}{\partial z}\left[\frac{T_s}{50}\right]^{n+1}_{j+\frac{1}{2}}$$

4b1. $$\frac{\partial}{\partial x}\left[\frac{\rho_{gs} v_g \alpha}{50}\right]^{n+1}_{j+\frac{1}{2}} = \left[\frac{1}{h_{fg}}\right]\left(0.7752\left[\frac{q_{s1}+q_{s2}}{200}\right]^{n+\frac{1}{2}} - 0.4\left[\frac{\dot{p}}{2}\right]\left(0.1\,[\alpha]^{n+\frac{1}{2}}_{j+\frac{1}{2}}\left[\rho_{gs}\frac{dh_{gs}}{dp}\right.\right.\right.$$
$$\left.\left.\left. + h_{fg}\frac{d\rho_{gs}}{dp}\right] + [1-\alpha]^{n+\frac{1}{2}}_{j+\frac{1}{2}}\left[\frac{\rho_{fs}}{10}\frac{dh_{fs}}{dp}\right] - 0.01\right)\right) - \left[10\left(\alpha^{n+1}_{j+1} - \alpha^{n}_{j+1}\right)\right]\left[\frac{\rho_{gs}}{50}\right]$$

4b2. $$\frac{\partial}{\partial x}\left[\frac{\rho_{fs} v_f (1-\alpha)}{500}\right]^{n+1}_{j+\frac{1}{2}} = 2\left[10\left(\alpha^{n+1}_{j+1} - \alpha^{n}_{j+1}\right)\right]\left[\frac{\rho_{fs}-\rho_{gs}}{1000}\right] - 0.04\left[\frac{\dot{p}}{2}\right]\cdot$$
$$\left([\alpha]^{n+\frac{1}{2}}_{j+\frac{1}{2}}\left[0.1\,\frac{d\rho_{gs}}{dp}\right] + [1-\alpha]^{n+\frac{1}{2}}_{j+\frac{1}{2}}\left[0.1\,\frac{d\rho_{fs}}{dp}\right]\right) - 0.1\,\frac{\partial}{\partial x}\left[\frac{\rho_{gs} v_g \alpha}{50}\right]^{n+\frac{1}{2}}_{j+\frac{1}{2}}$$

$$\left[\frac{W_g}{750}\right]^{n+1}_{j+1} = \left[\frac{W_g}{750}\right]^{n+1}_{j} + 0.1739\,\frac{\partial}{\partial x}\left[\frac{\rho_{gs} v_g \alpha}{50}\right]^{n+\frac{1}{2}}_{j+\frac{1}{2}}$$

$$\left[\frac{W_f}{7500}\right]^{n+1}_{j+1} = \left[\frac{W_f}{7500}\right]^{n+1}_{j} + 0.1739\,\frac{\partial}{\partial x}\left[\frac{\rho_{fs} v_f (1-\alpha)}{500}\right]^{n+\frac{1}{2}}_{j+\frac{1}{2}}$$

5. $$\left[\frac{q_p}{100}\right]^{n+\frac{1}{2}} = 2.35\left[\left(\frac{W_p}{5000}\right)^{0.8}\right][H_p]\left[\frac{T_p - T_{r1}}{25}\right]^{n+\frac{1}{2}}_{j+\frac{1}{2}}$$

6. $$\left[\frac{q_r}{100}\right]^{n+\frac{1}{2}} = 1.23\left[\frac{T_{r1} - T_{r2}}{50}\right]^{n+\frac{1}{2}}_{j+\frac{1}{2}}$$

7b. $$\left[\frac{q_s}{100}\right]^{n+\frac{1}{2}} = 0.467\left[\exp(P/43.4)\right]\left[\left(\frac{T_{r2} - T_{sa}}{10}\right)^2\right]$$

8. $$[\alpha] = \left[0.03\,\frac{\rho_{fs}}{S\,\rho_{gs}}\right]\left[\frac{W_g}{750}\right] \Big/ \left[\frac{A_s v_f \rho_{fs}}{25000}\right]$$

$$\left[\frac{A_s v_f \rho_{fs}}{25000}\right] = \left[0.03\,\frac{\rho_{fs}}{S\,\rho_{gs}}\right]\left[\frac{W_g}{750}\right] + 0.3\left[\frac{W_f}{7500}\right]$$

9. $$[4X] = \frac{0.4\left[\frac{W_g}{750}\right]}{0.1\left[\frac{W_g}{750}\right] + \left[\frac{W_f}{7500}\right]}$$

10. $$\left[\frac{\Delta p_1}{\rho_{ts}}\right] = 1.76\left[\frac{V}{2}\right]^{1.8} \sum\left(\frac{1}{6} + \frac{[4X]}{\left[\frac{\rho}{100}\right]}\right)$$

$$\left[\frac{V}{2}\right] = 0.7267\left[\frac{W_g + W_f}{7500}\right] \Big/ \left[\frac{\rho_{ts}}{1000}\right]$$

11. $$\left[\frac{\Delta p_2}{\rho_{ts}}\right] = 53.6\left[\frac{V_d}{10}\right]^{1.8}$$

12. $$\left[\frac{\dot{V}_d}{10}\right] = 0.390\sum(0.1\,\alpha) - 0.00788\left(\frac{\Delta p_1}{\rho_{ts}} + \frac{\Delta p_2}{\rho_{ts}}\right)$$

$$= 0.390\sum\left(0.1[\alpha] - 0.0355\left(\left[\frac{V}{2}\right]^{1.8}\left(\frac{1}{6} + \frac{[4X]}{\left[\frac{\rho}{100}\right]}\right)\right)\right) - 0.422\left[\frac{V_d}{10}\right]^{1.8}$$

13. $$\left[\frac{W_s}{5000}\right] = 1.374\left[\frac{\rho_{ts}}{1000}\right]\left[\frac{V_d}{10}\right]$$

Equations 14 - 21 are not scaled as they are solved by digital calculations.

Equations 22 - 23 are not yet included.

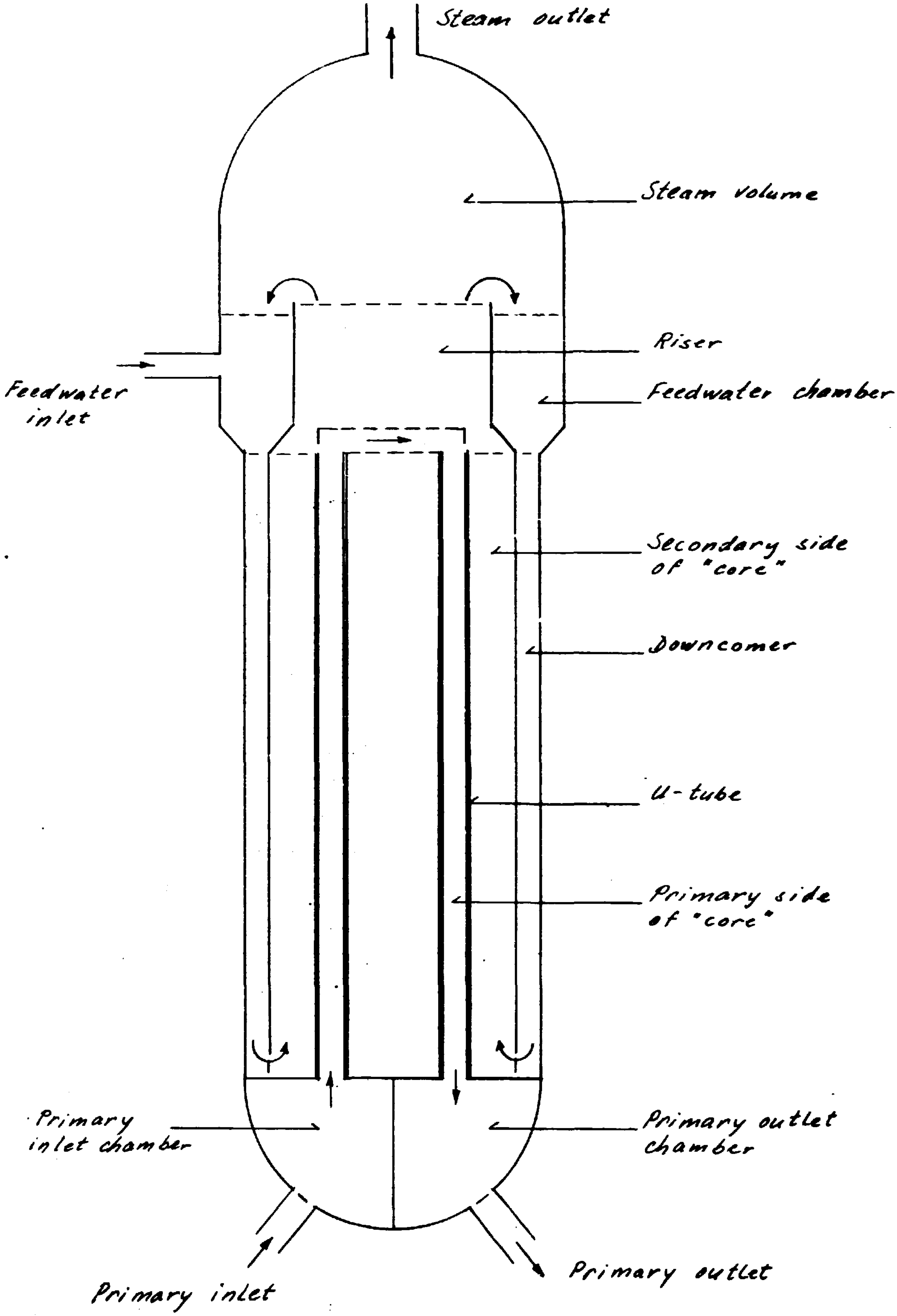
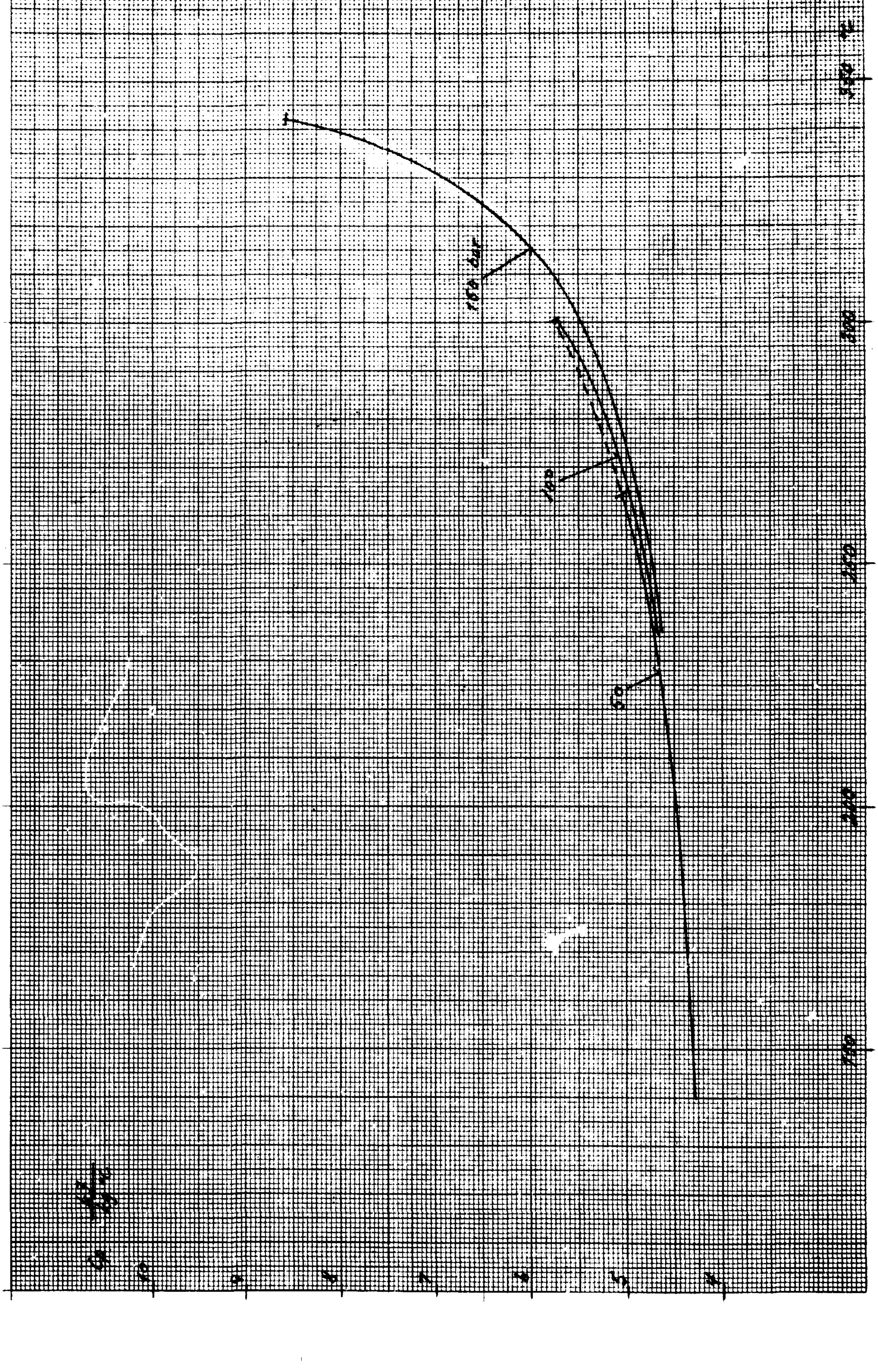


Fig. 1. Simplified diagram of U-tube steam generator

Fig. 2 Specific heat capacity c_p



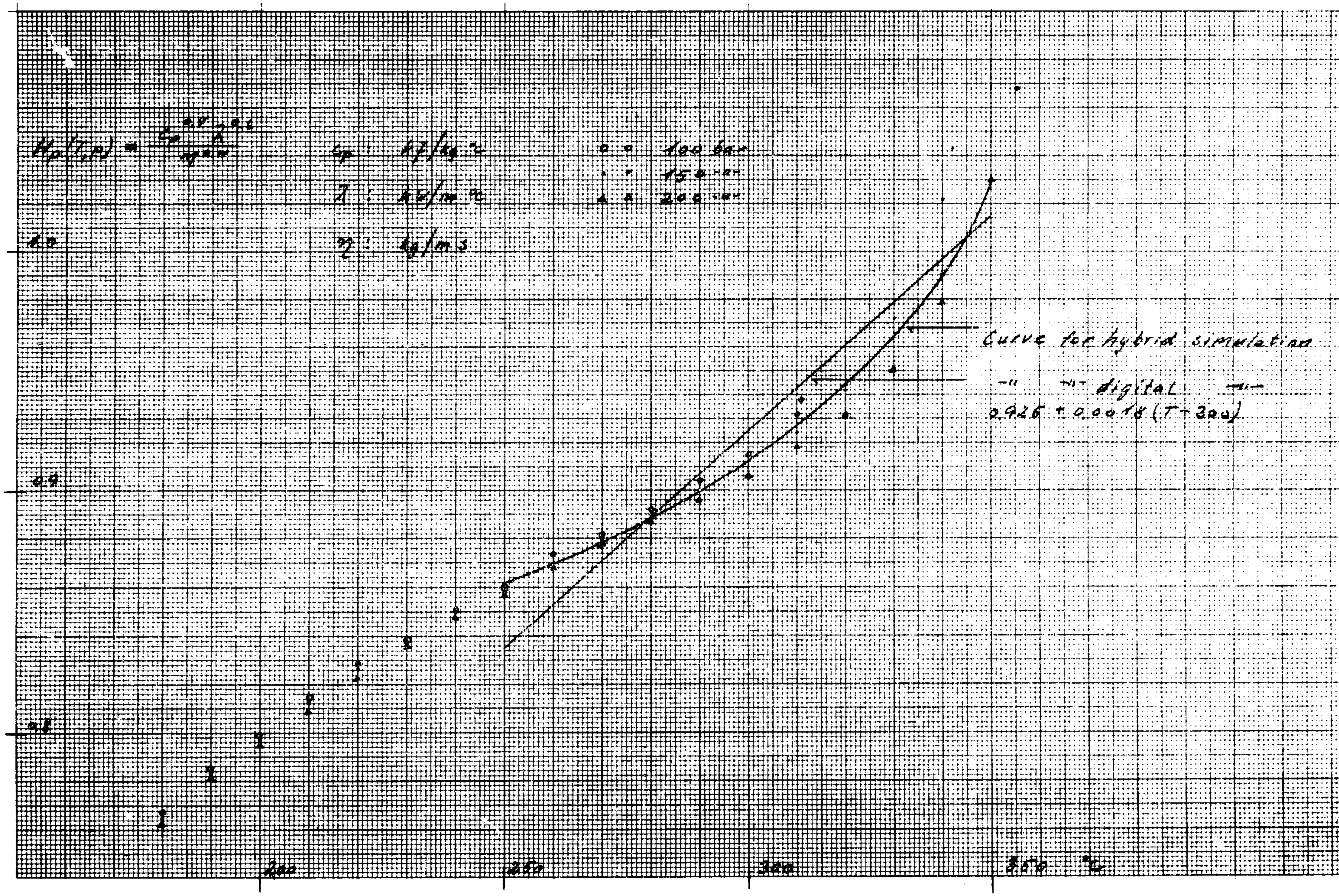


Fig. 3 Heat transfer parameter H_p

Fig. 4 Heat transfer parameter H_i

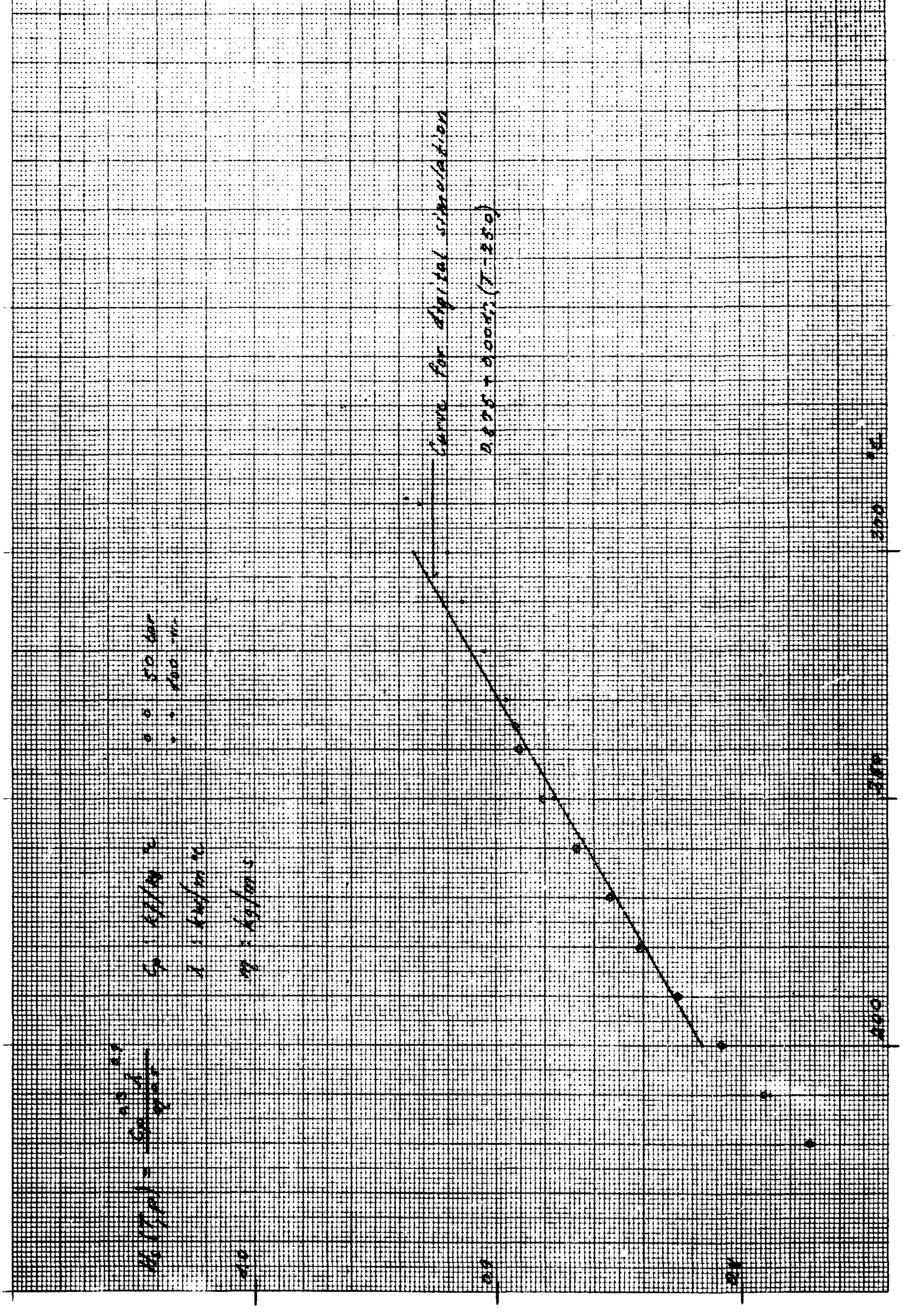


Fig. 5 Friction parameter F_f

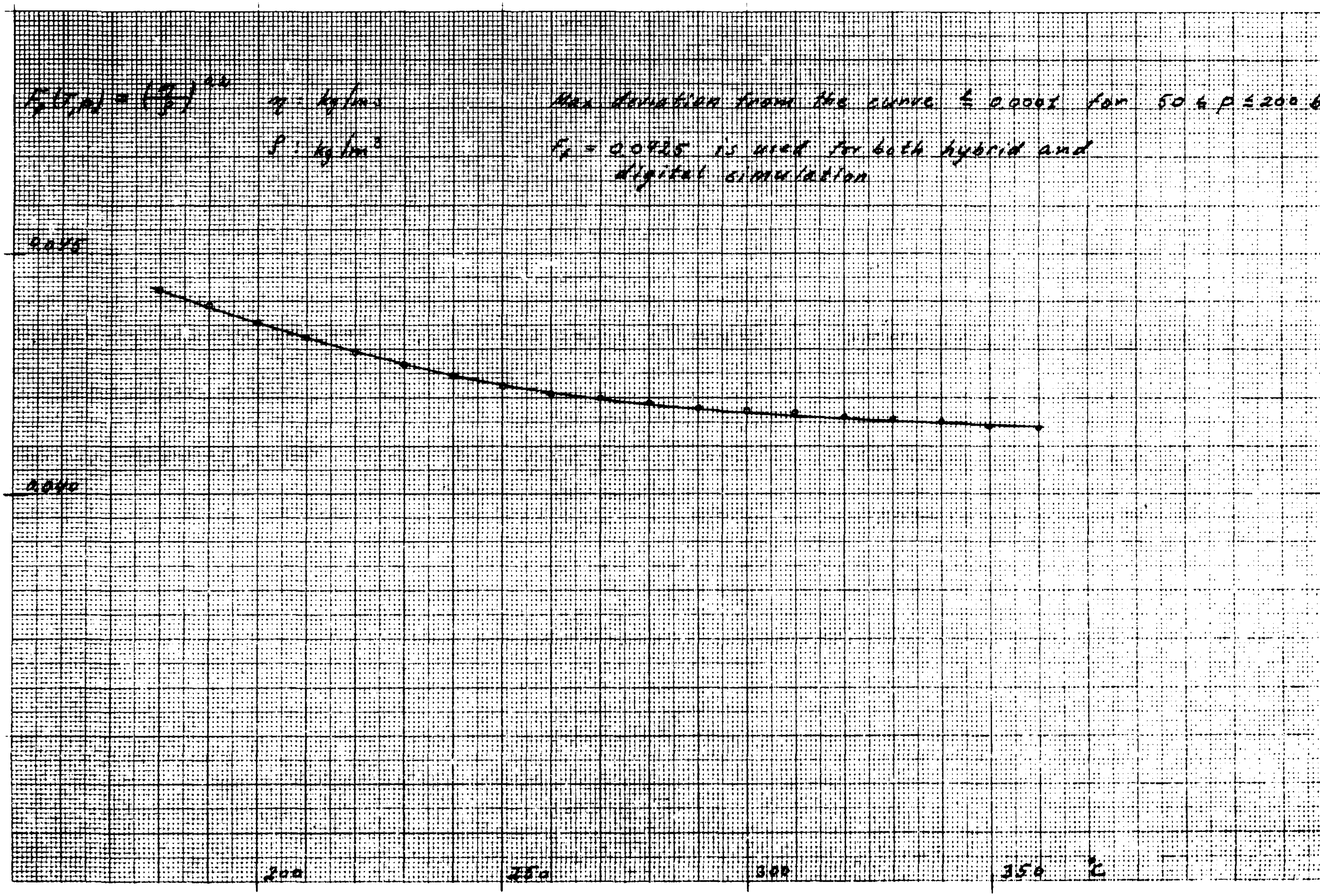
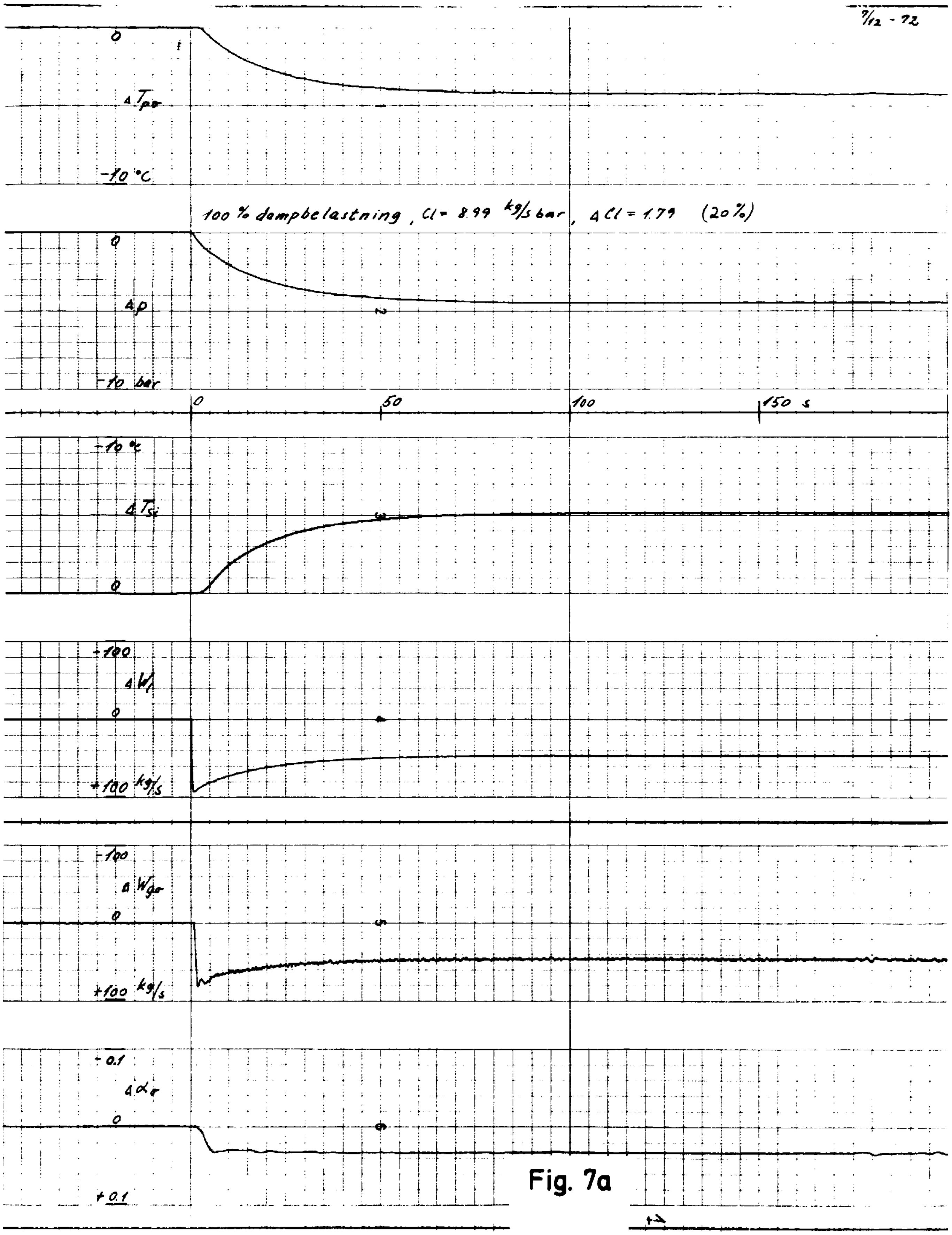


Fig. 6 Steady state values as calculated by hybrid simulation

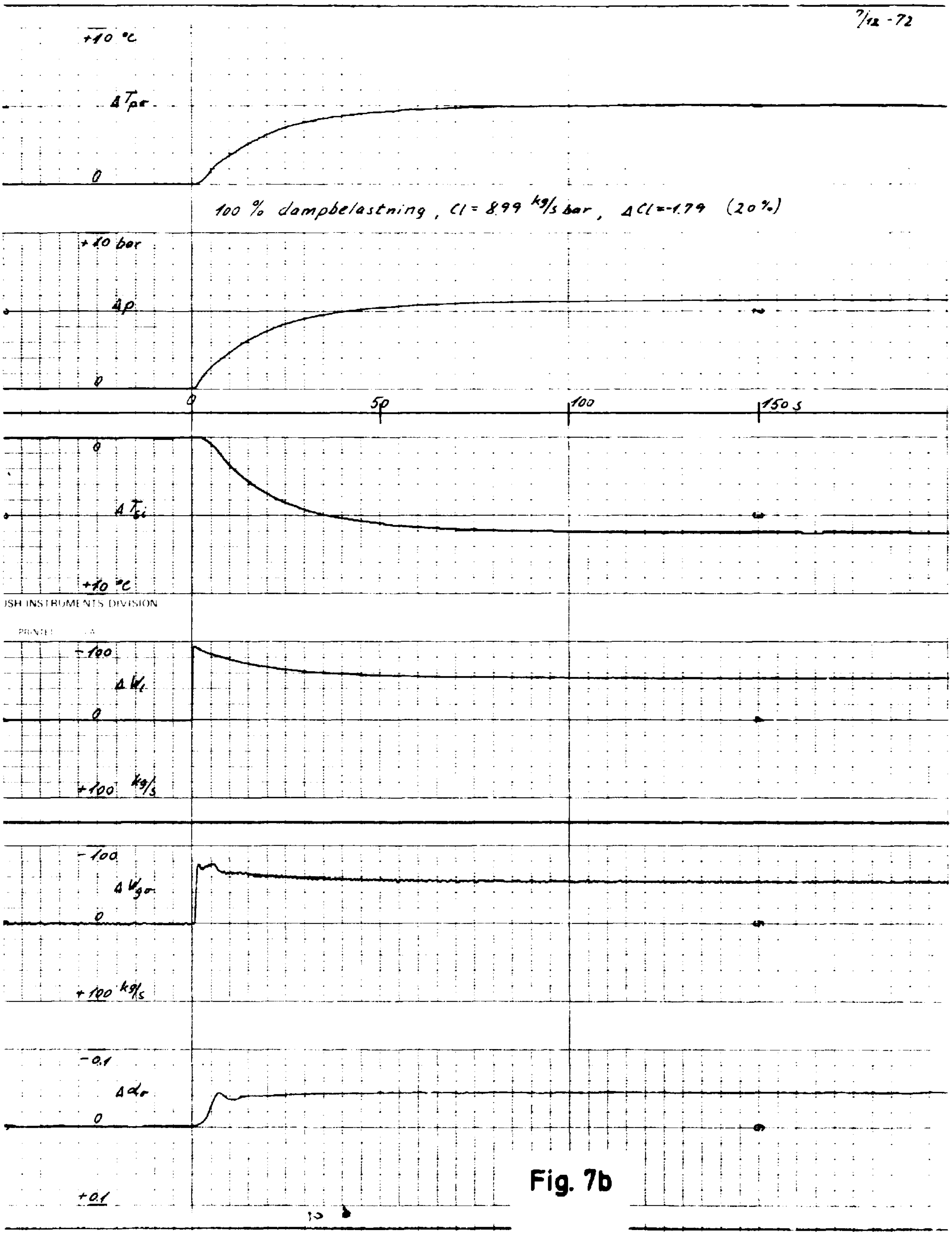
		100 %	50 %	25 %
T_{pi}	°C	319.0	309.7	305.2
T_{po}	"	281.3	291.6	296.2
T_{si}	"	263.2	280.1	289.2
T_{sa}	"	267.3	284.0	292.1
p	bar	52.9	68.3	76.2
W_p	kg/s	4230	4230	4230
W_s	"	4230	3315	2555
W_l	"	475	237	118.5
v_d	m/s	8.18	6.48	5.11
α_o		0.695	0.507	0.356
T_{fi}	°C	226.0	226.0	226.0
Cl	kg/s bar	8.99	3.47	1.555

T_{pi}, T_{fi} and W_p are given input values
Cl are adjusted to the value that gives the desired load.



9/12 - 72
0
ΔT_{pv}
-10 °C
100 % dampbelastning, Cl = 8.99 kg/s bar, ΔCl = 1.79 (20%)
0
Δp
-10 bar
0
50
100
150 s
-10 °C
ΔT_{si}
0
+100
ΔW_f
0
+100 kg/s
-100
ΔW_{gv}
0
+100 kg/s
-0.1
$\Delta\alpha_r$
0
+0.1

Fig. 7a



7/12 -72
+10 °C
ΔTpe
0
100 % dampbelastning , Cl = 8.99 kg/s bar , ΔCl = -1.79 (20 %)
+10 bar
Δp
0
0
50
100
150 s
0
ΔTsi
+10 °C
+100
ΔWl
0
+100 kg/s
-100
ΔWgo
0
+100 kg/s
-0.1
Δαr
0
+0.1

Fig. 7b

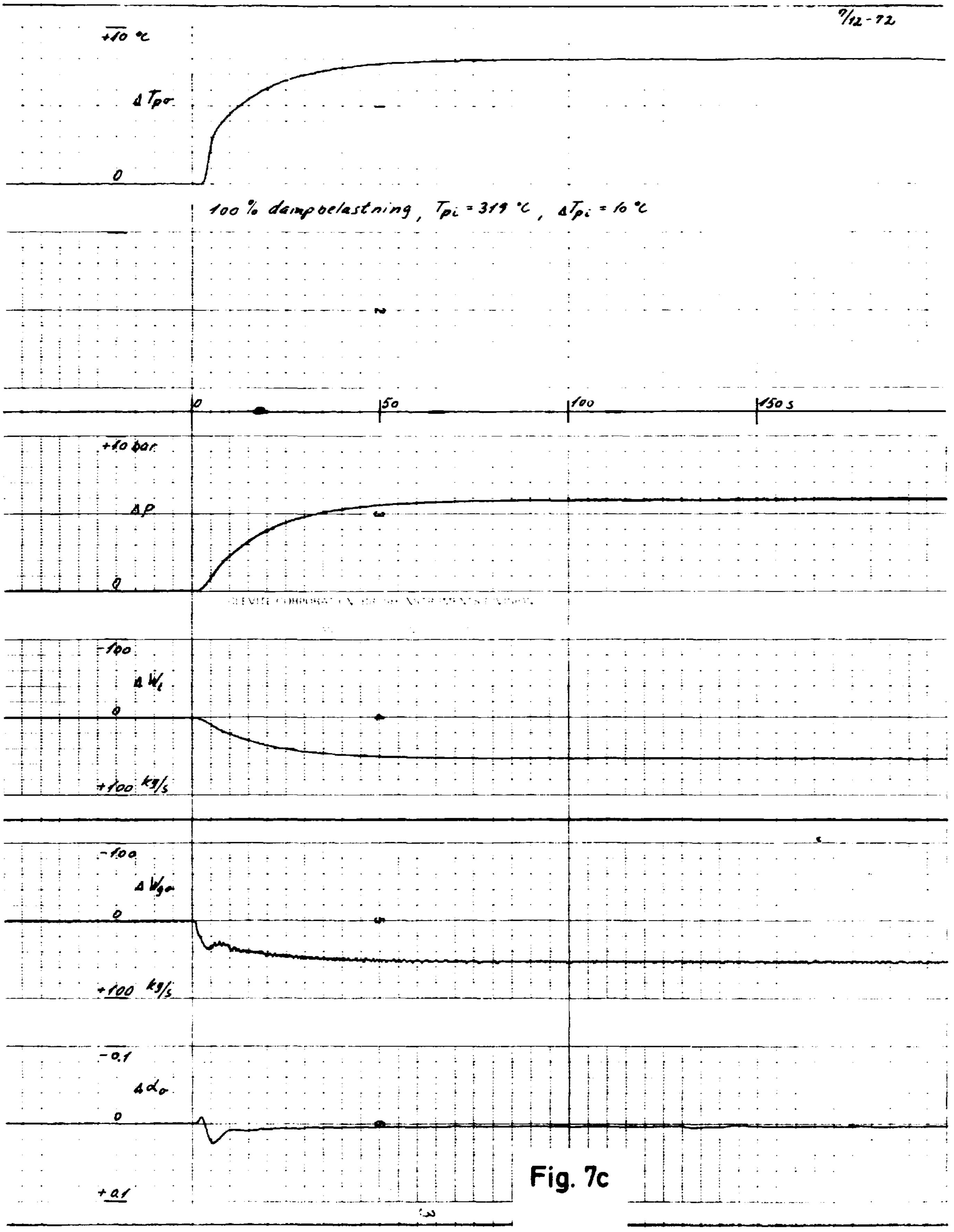

9/12-72
+10 °C
ΔT_{po}
0
100 % dampbelastning, T_{pi} = 319 °C, ΔT_{pi} = 10 °C
0
50
100
150 s
+10 bar
ΔP
0
-100
ΔW_l
0
+100 kg/s
-100
ΔW_{go}
0
+100 kg/s
-0.1
$\Delta \alpha_o$
0
+0.1

Fig. 7c

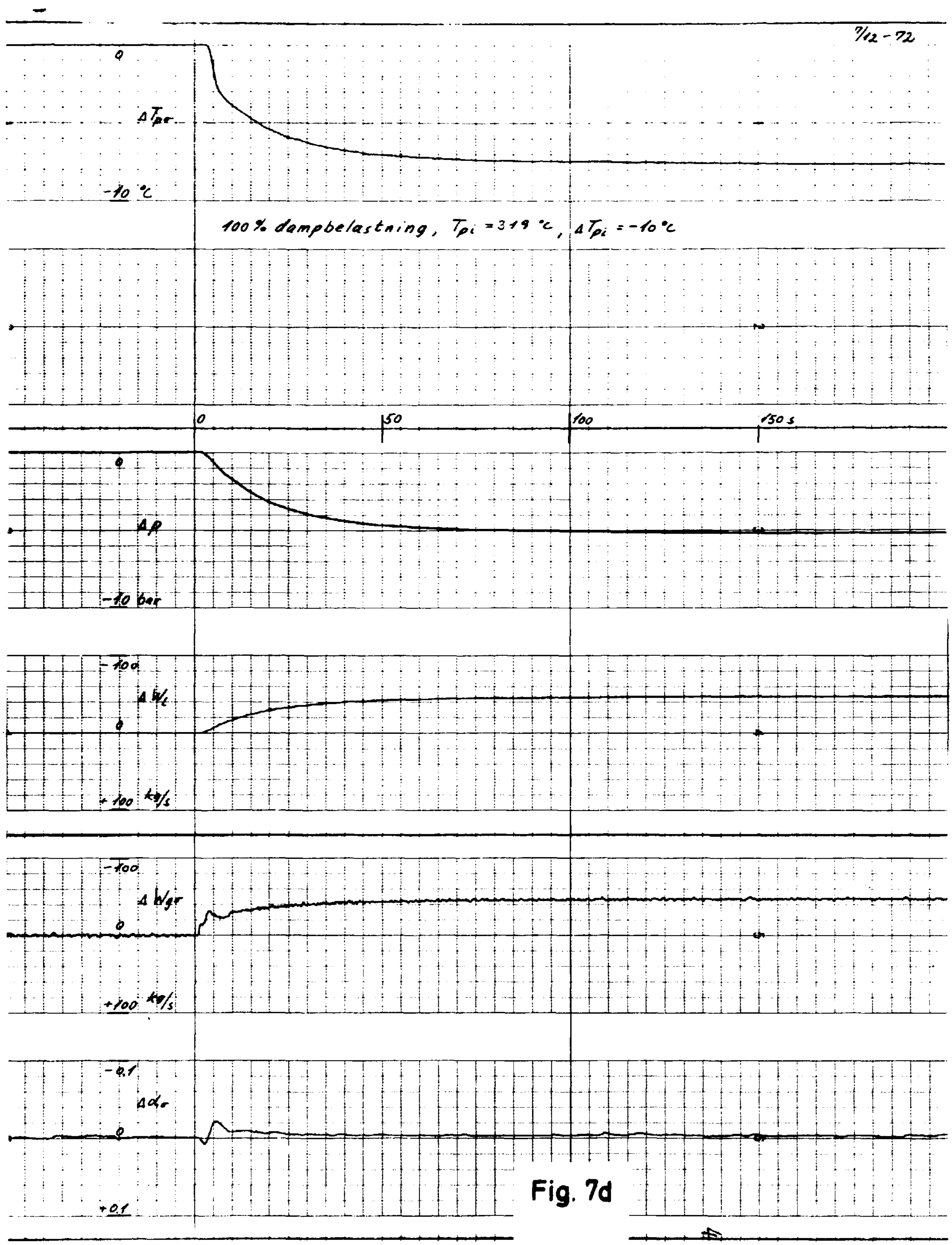

9/12 - 72
0
ΔTpr
-10 °C
100 % dampbelastning, Tpi = 319 °C, ΔTpi = -10 °C
0
50
100
150 s
0
Δp
-10 bar
-100
ΔWt
0
+100 kg/s
-100
ΔWgr
0
+100 kg/s
-0.1
Δαr
0
+0.1

Fig. 7d