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Single Gradientless Light Beam Drags Particles as Tractor Beams

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Usually a light beam pushes a particle when the photons act upon it. We investigate the optical forces by nonparaxial gradientless beams and find that the forces can drag suitable particles all the way towards the light source. The major criterion of realizing the backward dragging force is the strong nonparaxiality of the light beam, which contributes to the pulling force owing to momentum conservation. The non-paraxiality of the Bessel beam can be manipulated to possess a dragging force along both the radial longitudinal directions, i.e., a "tractor beam" with stable trajectories is achieved.

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In spite of the well established theory, experiment, and applications in particle-light interaction [1–5], there are many unknowns yet to be explored in the field of radiation pressure and optical micromanipulation by particular light beams. In ordinary optical tweezers, the transportation of the particles along the 3D trajectories can be achieved using the spatial light modulators [6]. In this case, the consecutive change of the computer-generated holograms for the beam produces the necessary effect—trapping the particle due to the field gradient. However, the radiation pressure on the particle is positive everywhere owing to the focused gradient beam. In this Letter, we aim to propose a gradientless light beam to achieve a continuous and stable dragging force on the particle, which drives it along the beam path towards the light source, i.e., a "tractor beam" *per se.*

Light usually results in a repelling force on the object [7–10] due to positive radiation pressure and momentum conservation. It is correct for the electric-dipole approximation and paraxial light beams when the particle's size is much less than the wavelength [2]. Even in this case, the optical force exerted upon the particle can still be manipulated to be *negative* provided that two oppositely directed beams are used simultaneously to illuminate the particle [11]. However, such a two-beam configuration (requiring exactly opposite directions) is not necessary. As shown in Ref. [12], two beams with different longitudinal wave numbers can locally operate as a tractor beam. Another possibility of getting attractive force is to use gain media [13], but the exotic media strongly confine the possible applications of the negative force. In Ref. [14] it was suggested, without proof, that the negative Poynting vector could be the reason for the attractive force. Though the negative energy flux density is not the reason for the particle being dragged towards the light source, negative

force (acting as a tractor beam) is quite possible for the nonparaxial beams if more photons can be transferred into the forward direction than the backward. The resulting dragging force can thus be obtained from the momentum conservation owing to the creation of an asymmetric scattering pattern and associated momentum transfer [15], which is justified by the aerodynamic analog [16] and acoustic analog [17].

Therefore, the tractor beam in classic optics is believed to be feasible and this Letter investigates the physical realization of such a special beam as well as the necessary conditions. As suggested by the acoustic analog, the pulling force has a wave-based nature [17], so the methodology of realizing the pulling force can be applied over any frequency range. The particle can be large compared to the wavelength. We find that a nonparaxial gradientless beam is a good candidate for a tractor beam in terms of transferring more photons to the forward direction of the particle. The experimental generation of a strongly non-paraxial vector Bessel beam is still challenging. Nevertheless, the recent technique of hypergeometric laser beam generation by axicons [18] paves a promising path toward realizing a tractor beam in practice at least within a certain propagating length where the Bessel beam possesses enough nonparaxiality.

The Bessel beam propagating along the z axis is described as [14]

$$\mathbf{E} = e^{im\varphi+i\beta z}\Big(J_m(qr)c_2\mathbf{e}_z - \frac{k_0}{q}c_1(\mathbf{e}_z \times \mathbf{b}) + \frac{\beta}{q}c_2\mathbf{b}\Big),$$

$$\mathbf{H} = e^{im\varphi+i\beta z}\Big(J_m(qr)c_1\mathbf{e}_z + \frac{\beta}{q}c_1\mathbf{b} + \frac{k_0}{q}c_2(\mathbf{e}_z \times \mathbf{b})\Big),$$

(1)

where $J_m(qr)$ is the Bessel function of the first kind, $k_0 = \omega/c$ is the wave number in vacuum, $q = k_0 \sin\alpha$ is the
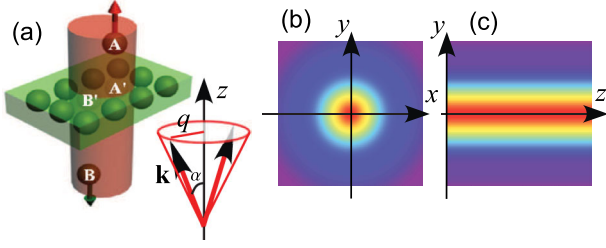
FIG. 1 (color online). (a) Light beam pushes a particle from position B' to position B while it drags another particle from the original position A' to the new position A (it will keep dragging if the light keeps shining). On the right, wave vectors of the gradientless Bessel beam lying on the cone are sketched. (b) and (c) denote the cross-sectional intensity of such a beam propagating along the z axis ($c_1 = 1$, $c_2 = i$, $q/k_0 = 0.9$, $m = 1$).

transverse wave number [α is the cone angle, see Fig. 1(a)], $\beta = \sqrt{k_0^2 - q^2}$ is the longitudinal wave number, m is the beam order, $\mathbf{b} = iJ'_m(qr)\mathbf{e}_r - \frac{m}{qr}J_m(qr)\mathbf{e}_\varphi$, and $J'_m(qr) = \frac{dJ_m}{d(qr)}$. The intensity distribution $|\mathbf{E}|^2$ of the beam is illustrated in Figs. 1(b) and 1(c). The intensity is actually propagation invariant (z independent) and azimuth invariant (φ independent). Coefficients c_1 and c_2 of the vector Bessel beam (1), which are, respectively, associated with TE-polarized ($c_2 = 0$) and TM-polarized ($c_1 = 0$) waves, provide a propagation invariant field with complex polarizations. In general, coefficients c_1 and c_2 can be complex numbers, which means that TE and TM beams are phase shifted. Strong nonparaxial Bessel beams have already demonstrated a number of exciting properties like local negative direction of the Poynting vector [14] and intensity transformation on reflection [19].

The backward dragging force can be justified by the conservation of photon momentum and Newton's third law regarding reaction force. When the light strongly scatters upon the back (forward) hemisphere, the photons transfer more forward (backward) momentum to the particle [20], resulting in a pulling (pushing) force. Such a pulling force has been verified in acoustic analog by eliminating backward scattering [17]. The concept of a tractor beam can be realized when the Bessel beam is sufficiently nonparaxial.

If the size of a magnetodielectric particle is much less than the wavelength, the imaginary part of the complex polarizability $\alpha_e = \alpha_e^{(0)} + i2k_0^3\alpha_e^{(0)2}/3$ (one has similar expressions for the magnetic polarizability α_e) is much smaller than the real part [3], where $\alpha_e^{(0)} = a^3(\varepsilon - 1)/(\varepsilon + 2)$. Thus, we derive the z component of the force

$$\langle F_z \rangle = \frac{\beta}{2}[\text{Im}(\alpha_e)|\mathbf{E}|^2 + \text{Im}(\alpha_m)|\mathbf{H}|^2]$$
$$- \frac{k_0^4}{3}\text{Re}(\alpha_e)\text{Re}(\alpha_m)\text{Re}(P_z), \qquad (2)$$

where $P_z = \mathbf{e}_z(\mathbf{E} \times \mathbf{H}^*)$.

F_z is fully nonconservative [12], because the z component of the tractor beam is gradientless due to the nondiffractive nature (z independent). The negative nonconservative force explains the pulling effect. For nonmagnetic small particles ($\alpha_m = 0$), the force is always *pushing*, based on Eq. (2). However, the negative force for the Rayleigh magnetic particle is feasible due to the *small* longitudinal wave number β of the light beam and *large positive* Poynting vector (quantity $\text{Re}(P_z)$). Nevertheless, note that nonmagnetic non-Rayleigh particles can be attracted owing to the contributions of higher-order electrical moments, which will be discussed further.

If β is not small, e.g., the plane-wave case ($\beta = 1$ and $|\mathbf{E}|^2 = |\mathbf{H}|^2 = \text{Re}(P_z) = 1$), we have

$$\langle F_z \rangle = \frac{k_0^4 a^6}{3}\left(\frac{(\varepsilon - 1)^2}{(\varepsilon + 2)^2} + \frac{(\mu - 1)^2}{(\mu + 2)^2} - \frac{(\varepsilon - 1)(\mu - 1)}{(\varepsilon + 2)(\mu + 2)}\right).$$

Then, force F_z cannot be attractive for any dielectric permittivity or magnetic permeability under the illumination of a plane wave.

We first assume that the incident vector Bessel beam is scattered by the spherical particle of radius R situated at the beam axis. The scattered fields can be calculated, for example, using the matrix approach [21,22], which is valid for any incident electromagnetic wave. The fields at the boundary of the sphere can be written as the sum of the incident and scattered fields: $\mathbf{E} = \mathbf{E}^{\text{inc}} + \mathbf{E}^{\text{sc}}$ and $\mathbf{H} = \mathbf{H}^{\text{inc}} + \mathbf{H}^{\text{sc}}$. These fields are sufficient to compute the time-averaged force on the particle:

$$\mathbf{F} = \int_0^\pi \int_0^{2\pi}(\hat{T}\mathbf{n})R^2\sin\theta d\theta d\varphi, \qquad (3)$$

where $\mathbf{n}$ denotes the normal direction to the surface of the dielectric object [15], and $\hat{T}$ denotes the time-averaged Maxwell's stress tensor

$$\hat{T} = \frac{1}{8\pi}\text{Re}\left(\mathbf{E} \otimes \mathbf{E}^* + \mathbf{H} \otimes \mathbf{H}^* - \frac{1}{2}(|\mathbf{E}|^2 + |\mathbf{H}|^2)\right). \qquad (4)$$

Here $\mathbf{E} \otimes \mathbf{E}^*$ defines the dyad, which is $(\mathbf{E} \otimes \mathbf{E}^*)_{ij} = E_iE_j^*$ in index form. Because of the axial symmetry of the Bessel beam and the position of a spherical particle, the only nonzero component of the force is the F_z. If a particle is not placed exactly at the axis, the radial force will arise. In particular, we investigate the situation of $F_z < 0$ herein, i.e., when the particle is pulled by the beam.

When the particle is located at the beam axis, angle φ for the Bessel beam in Eq. (1) is identical to the azimuthal angle of the particle's spherical coordinates. Therefore, this single term $\exp(im\varphi)$ will be summed over the integer azimuthal number to calculate the fields. It is not the case for the particle situated away from the axis. Then we need to take into account the terms with integer azimuthal numbers not equal to m.

The configuration of the beam strongly defines the possibility of the backward dragging force. However, the intensity of the light is not related to the dragging force. Even identical intensity patterns result in different dependencies of the force [e.g., the insets of Figs. 2(b) and 2(d)]. It should also be noted that the negative longitudinal component of the Poynting vector S_z is not responsible for the appearance of the negative force, because the large negative S_z exists for $c_2 = -i$ [14], while the force is positive in this case [Fig. 2(f)].

A strong dragging force exists for the non-phase-shifted superposition of TE- and TM-polarized beams [c_2 is a positive real number as in Figs. 2(b) and 2(c)] and for $\pi/2$-shifted beams [c_2 is an imaginary number with $\mathrm{Im}c_2 > 0$ as in Fig. 2(e)]. In the latter case, the negative force arises over a wide selection of the particle's permittivity. Because of the nonparaxial regime, large particle size, and/or magnetic moments, the force owing to the dipole term can become a dragging force. When we introduce the loss δ into the permittivity $\varepsilon + i\delta$, the pulling force reduces as expected as shown in Fig. 2(e). This can be explained by the momentum transfer from the photons to the particle creating the additional pushing force as the result of nonelastic interaction.
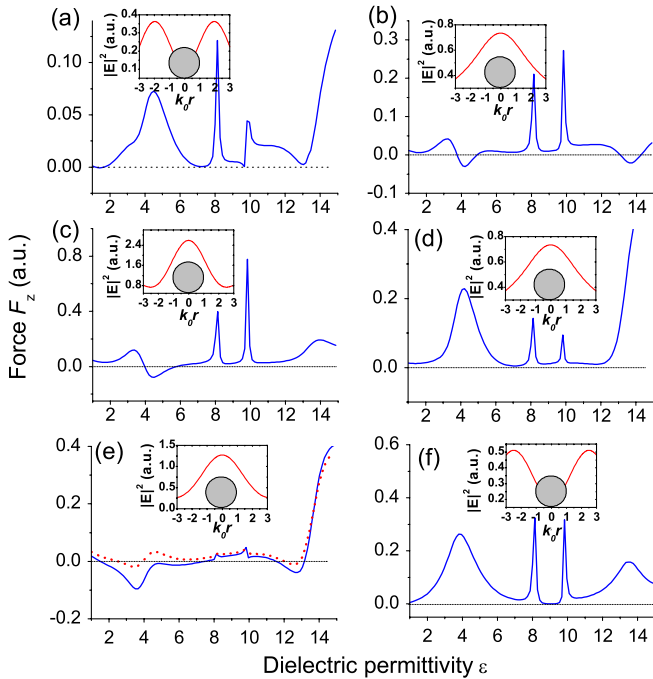
Thus, the Bessel beams with $c_2 = 1$ and $c_2 = i$ are preferred based on their wide ranges of permittivity for dragging force, but they are quite distinct. In Figs. 3(a) and 3(c) we show the force acting on the particle for $c_2 = 1$ and for $c_2 = i$ in Figs. 3(b) and 3(d). Asymmetry in the case of $c_2 = 1$ implies that, in order to induce the dragging force, the permittivity should be greater than the permeability; i.e., the excited electric moments play the dominant part. In the symmetric case [Figs. 3(b) and 3(d)], the dragging force becomes largest at the impedance matching condition of $\varepsilon = \mu$. It is interesting that the region of the negative force is really wide in Figs. 3(b) and 3(d). The value of the force can be enlarged by adjusting the radius of the sphere [see Fig. 4(a)]. From Fig. 3(d) we can see that the force can be attractive even for $\mu = 1$ (it is out of the range of the plot). This is actually the case for the black solid line in Fig. 4(a). So the dragging force can act even on the common glass particles if it has an appropriate size. The only limitation of realizing our tractor beam is the large nonparaxiality. As shown in Figs. 4(b) and 4(c), the variation of the parameters does not substantially reduce the cone angle $\alpha = \arcsin(q/k_0)$ (for $q = 0.9$ the angle α equals $\approx 64°$), beyond which the backward dragging force will be pronounced. The angle α can be reduced for the large particles [e.g., see $k_0R = 1.8$ in Fig. 4(c)]. The beam should have a small longitudinal wave number β so as to create the dragging force. Figure 4(d) clearly shows that in the shaded region, both F_z and F_r [projection of the total force in Eq. (4) onto the radial direction] are negative when a specific particle ($\epsilon = \mu = 3$) is within a certain distance
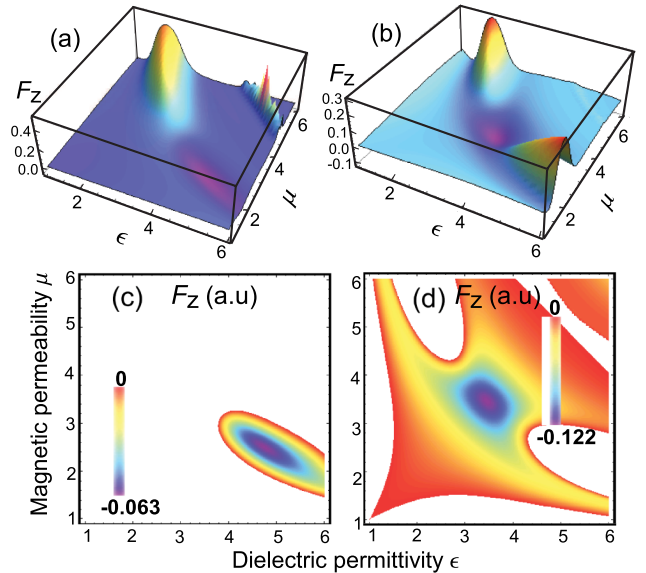

FIG. 2 (color online). Force F_z on the spherical particle ($k_0R = 1$, $\mu = 3$) versus dielectric permittivity ε for (a) $c_2 = 0$, (b) $c_2 = 1$, (c) $c_2 = 2$, (d) $c_2 = -1$, (e) $c_2 = i$, and (f) $c_2 = -i$. In the insets, the field intensities are demonstrated (grey circle stands for the particle). Beam parameters are $c_1 = 1$, $q/k_0 = 0.9$, and $m = 1$. Red dashed curve in subfigure (e) shows the force F_z for lossy particles $\varepsilon + i0.1$.

FIG. 3 (color online). Diagram of the force F_z as a function of the dielectric permittivity ε and magnetic permeability μ for (a), (c) $c_2 = 1$ and (b),(d) $c_2 = i$. In the bottom subfigures, only negative values of the force are shown. Parameters: $k_0R = 1$, $c_1 = 1$, $q/k_0 = 0.9$, $m = 1$.
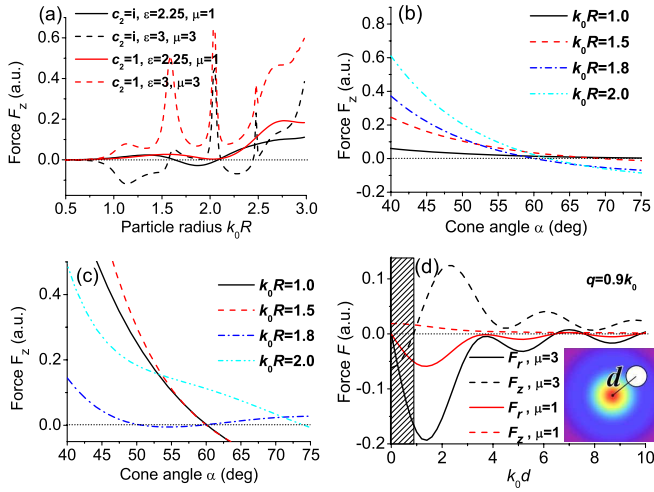
FIG. 4 (color online). (a)–(c) Force on the particle at beam axis. (d) Trajectories of dragging force. F_z versus (a) the particle radius $k_0 R$ ($q/k_0 = 0.9$); (b) cone angle $\alpha = \arcsin(q/k_0)$ ($c_2 = i$, $\varepsilon = 2.25$, $\mu = 1$); and (c) cone angle α ($c_2 = i$, $\varepsilon = 3$, $\mu = 3$). In (d), $k_0 d$ denotes the radial distance from the beam axis, and the shaded region represents that the dragging force can pull the particle ($k_0 R = 1$, $c_2 = i$, $\varepsilon = 3$) along stable trajectories because of $F_z < 0$ and $F_r < 0$. A multimedia file, corresponding to (c) is created to show the significance of the cone angle (nonparaxiality effect) and the particle's radius (size effect), in order to achieve dragging forces (see Media 1 [23]). Other parameters: $c_1 = 1$, $m = 1$.

from the beam axis of the nonparaxial laser. Therefore, such particles will be pulled along stable trajectories (toward the beam axis and light source), while others will be pushed away from the beam by $F_z > 0$ and/or $F_r > 0$. Then one can design the tractor beam *per se*, to realize dragging a targeted class of particles while pushing away others, as shown in Fig. 1(a).

In conclusion, we have demonstrated the possibility of realizing the tractor beam by a single gradientless light beam, which can drag the particle toward the source. Compared to the previously reported tractor beams [12], we have the broadband negative force and do not need two opposite beams with different wave numbers [11] or a gradient beam [6]. The proposed gradientless beam with the proper manipulation of the nonparaxiality can pull particles, which can be large or small and made of any materials. The tractor beam has a finite region of stable trajectory near the beam axis. Therefore, even if the particles are initially off the beam center, they can be shifted to the axis and attracted to the light source.

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[23] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.107.203601. In the Media file, we show the relation between the axial force and nonparaxial angle at varying sizes of the particle. R denotes $k_0 r$, i.e., the electric dimension of the particle. Dragging force is dependent on both cone angle and particle's size.