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Luczak, Marcin; Manzato, Simone; Peeters, Bart; Branner, Kim; Berring, Peter; Haselbach, Philipp Ulrich

Published in:
Proceedings

Publication date:
2011

Document Version
Publisher's PDF, also known as Version of record

Experimental Verification of the Implementation of Bend-Twist Coupling in a Wind Turbine Blade

Marcin Luczak,
Simone Manzato, Bart Peeters

LMS International

marcin.luczak@lmsintl.com

Kim Branner,
Peter Berring, Philipp Haselbach

Wind Energy Division,
Risø DTU
kibr@risoe.dtu.dk

Abstract

This paper presents some results and aspects of the multidisciplinary and interdisciplinary research oriented for the experimental and numerical study in static and dynamic domains on the bend-twist coupling in the full scale section of a wind turbine blade structure.

The main goal of the conducted research is to confirm experimentally the numerical prediction of modification of the dynamic and static properties of the wind turbine blade. Bend-twist coupling was implemented by adding angled UD layers on the suction and pressure side of the blade.

Static and dynamic test were performed on a section of a full scale wind turbine blade provided by Vestas Wind Systems A/S. The results are presented and compared to the original and modified blade measurements.

Comparison analysis confirmed that UD layers introduce measurable bend-twist couplings, which the original blade did not have.

KEYWORDS: Wind turbine blade, bending, torsion, composite materials, bend-twist coupling, modal analysis, structural dynamics, finite element analysis

1. Introduction

Wind turbine blades must be designed to resist the extreme loads cases and fatigue loads from normal operation. Sudden wind gusts are often too quick for the active pitch control system to react and may shorten the fatigue life substantially. This problem may be overcome by an aero-elastic tailoring of the blades. Particular implementation of the anisotropic composite material can introduce the bend-twist coupling in the blade [7-15]. The coupling causes the feathering blade to twist under the bending load and as a result decreases the angle of attack. This paper presents the progress and results of a comprehensive long-term scientific research focused on the bend-twist coupling analysis, design and implementation in wind turbine blade made from composite materials. First three parts of the paper briefly recall the research activity carried out [1-3] while the main focus is put on measuring and modeling the dynamic behavior described within the fourth part.

The first part of the paper reports on the experimental and numerical studies of a standard wind turbine blade section. The wind turbine blade section made of composite material was statically tested and modeled. Different load configurations were applied at the tip of the blade section to assess the twist and bend behavior [1].

The second part of the paper presents the structural dynamics identification, which was performed by means of experimental modal analysis. A finite element method model [2] was developed, updated and validated against the static measurement results. Based on the validated model the modified design of existing blade was studied. A baseline concept of the modification was implementation for bend-twist coupling by means of application of additional composite material layers. The original blade section was modified with four layers of UD1200, which were laminated on the pressure and suction side of the blade, with an angle of 25 degrees to the blade axis, in order to create a measurable flapwise bend-twist coupling.

In the third part the static experimental and numerical analysis on the modified blade section is exposed to verify the design correctness of the bend-twist coupling.

Finally in the fourth part the dynamic behavior of the modified blade section is experimentally identified with the assessment of the coupling. Moreover the influence of the support structure dynamics on the test specimen is discussed.

2. Static investigation of original blade section

Static loads in bending, torsion and combined bending and torsion configurations were introduced with different loading force levels [1-3].

2.1. Object of the investigations

The object of investigation is an 8 meter long section cut from a 23 meter wind turbine blade. This section is mounted in the two root clamps with additional clamp at the tip for the hydraulic jack fastening (Figure 1).

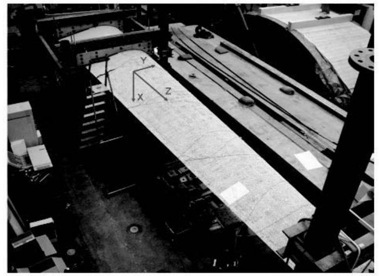


Figure 1 Wind turbine blade section under investigation with the coordinate system. For measurements of the original blade, axis system is rotated of 90° about z axis

Bending angles are computed considering the rotation of two consecutive measured cross sections about the x axis. Twist angles are computed as the rotation about the z axis of each cross section with respect to the unloaded configuration. A detailed description of the calculation can be found in [1]. Figure 2 shows the bend and twist angles of the original blade computed from the static bending measurement.

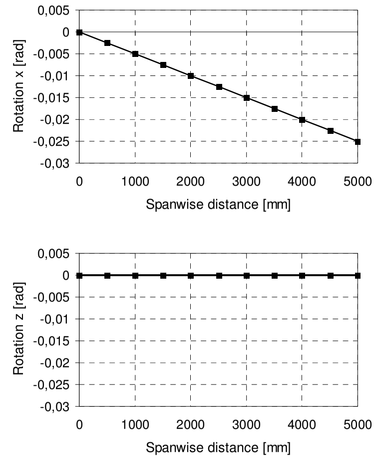


Figure 2 Bending slope angle (top) and cross sections twist angle (bottom) values under flapwise bending load. The blade section is bending but not twisting. That indicates that the bend-twist coupling is close to zero

Graph indicates that the bend-twist coupling is equal or close to zero, since the bending moment does not result in a twist angle (rotation about the z-axis) of the blade section.

3. Dynamic investigation of original blade section

Observations from the static investigations were verified in the structural dynamics of the tested original blade section [19]. Measurement points were defined on the leading and trailing edge at thirteen cross sections. Flapwise and edgewise direction accelerations were measured. Points on the support structure were not measured. As the drawback of this fact some of the natural frequencies of the support structure were indentified as the modes of the blade while they are modes of the support structure. Natural frequencies and corresponding mode shapes were estimated and some examples are presented on the Figure 3.

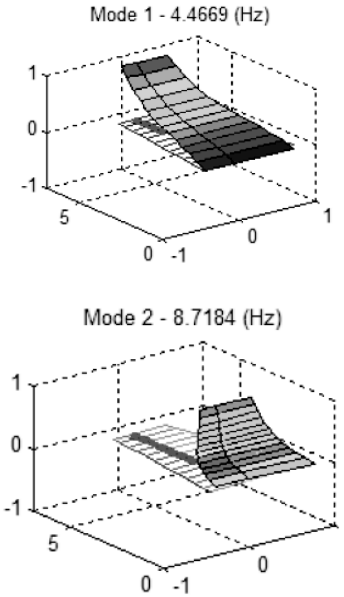


Figure 3 3D plots of mode shapes of original blade section. 1st flap wise bending (top) and first edgewise bending (bottom)

4. Static investigation of modified blade section

One of the primary aims of this long-term research is to study, design and implement desired bend-twist coupled behavior. The original blade section was modified with four layers of UD1200, which were laminated on the pressure and suction side of the blade with the fibers angle of 25° to create a measurable flapwise bend-twist coupling. The additional layers were laminated as indicated in Figure 4.

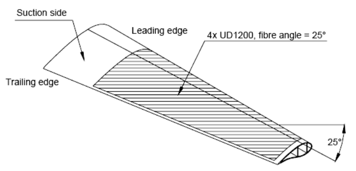


Figure 4 Fiber orientations of the extra UD layers

To verify the numerical prediction of the bend-twist coupling the static tests campaign was repeated such as on a modified blade. The twist angles for equidistant cross sections under flapwise bending load were calculated from measurement (Figure 5).

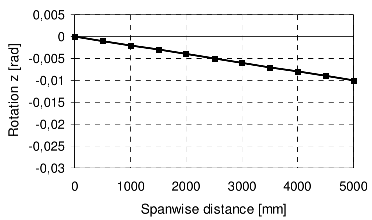


Figure 5 Twist angle under flapwise bending load. This indicates that the blade section has a measurable bend-twist coupling

Assuming that the shear center is located in the center of the spar, the rotation angles about the z axis for the modified blade section in Figure 5 show a measurable bend-twist coupling.

5. Dynamic investigation of modified blade section

The flapwise bend-twist coupling was also investigated by means of experimental modal analysis. The research was focused on the bend-twist coupling presence in the mode shapes of the blade section. Important aspect was also analysis of the influence of additional mass and stiffness introduced by extra layers. Finally the influence of the support structure on the correlation analysis between numerical and experimental modal models was studied. [16, 17].

Blade section was excited with two electro-dynamic shakers attached at the tip end in the flapwise and edgewise directions. Frequency Response Functions were measured and stored within 0 and 120 Hz frequency range.

For adequate identification of the blade dynamic displacement, accelerations of the vibrations were measured in 130 points. Thirteen equidistant measurement cross sections were defined along the span wise direction (Z) every 0.5 [m]. Each cross section contains five measurement points in which accelerations were acquired along the flapwise (X) and edgewise (Y) direction. These points are located at the leading edge, trailing edge, on the line of airfoil maximum thickness and in the mid-points between the previous three. Measurement directions were precisely defined based on the CAD geometry of the blade section.

Model quality assessment was an integrated part of the investigation. Except time invariance another conditions must be observed to satisfy of modal analysis assumptions: linearity, Maxwell's reciprocity principle and observability. Possible sources of nonlinearities within investigated structure are material properties, geometrical properties and the existence of bond connections Verification of a superposition rule is one of the methods of detecting nonlinearities. Linearity check was done for the level of driving voltage ranging from 0.5 [V] to 2 [V] with a step of 0.5 [V]. Results are presented on Figure 6. Frequency Response Function (FRF) between input signal and output spectrum defined as acceleration over force remain constant independently of excitation voltage level. This proves that the structure dynamic behavior is linear within bandwidth of interest.

The reciprocity check is based on Maxwell's principle, which states that the FRFs obtained by applying the force on point 1 and measuring the response in 2 and vice versa should be the same. The results for the two checks performed confirmed applicability of the reciprocity rule.

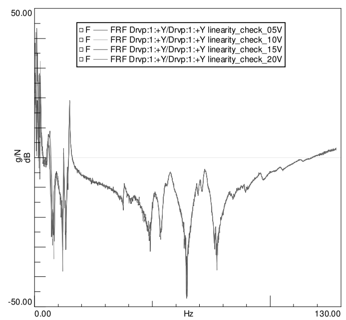


Figure 6 Linearity check for one of the points on the blade. Voltage values= 0.5V, 1V, 1,5V and 2V

During the processing of the data, some significant noise was observed in the acquired FRFs in the low frequency region. The driving point coherence functions show a small drop in this region, meaning a non-ideal excitation (Figure 7).

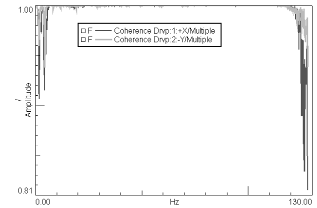


Figure 7 Coherence functions for the two driving points. It is used measure of the FRF quality. Ideally it should take value equal 1.

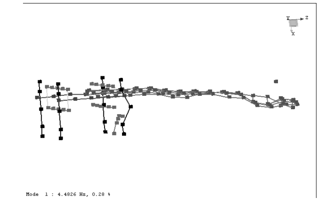
The modal parameter identification technique was not able to clearly stabilize modes in this region, possibly resulting in some local errors in the mode shapes below 7 Hz. The estimation provided natural frequencies, mode shapes and corresponding damping ratios in the frequency bandwidth 0-60 Hz. First five out of 12 identified mode shapes are provided on Figure 8.

MAC (Modal Assurance Criterion) can be used to compare two modal models. The MAC between two mode shape vectors, ϕ_i and $\phi_i(s)$, is defined as:

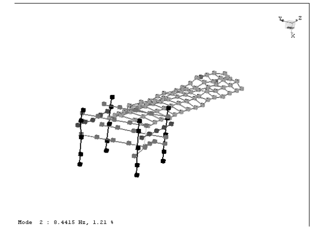
$$MAC(s) = \frac{\left|\phi_i^T \phi_i(s)\right|^2}{\phi_i^T \phi_i \phi_i(s)^T \phi_i(s)} \qquad (1)$$

If a linear relationship exists between the two complex vectors ϕ_i and $\phi_i(s)$, the MAC value will be near to 100. If they are linearly independent, the MAC value will be small (near zero). Figure 9 shows a comparison between the AutoMAC of the modal model obtained by considering only the sensors on the blade and the one where also the response of the supporting structure is included.

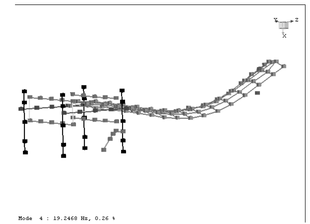
Low valued off-diagonal terms for the blade only model ensure linear independence of estimated modal vectors. The correlation between off-diagonal terms is increased when including the supporting structure in the analysis. This is due to the fact that the clamping is not perfectly rigid and the support has its own dynamic behavior which influences the measured response of the blade.



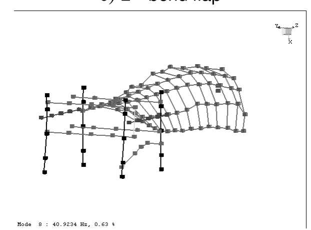
a) 1st bend flap



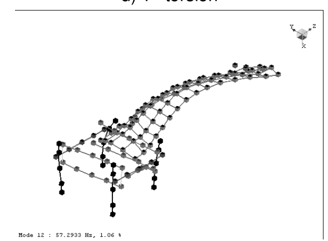
b) 1st bend edge



c) 2nd bend flap



d) 1st torsion



e) 2nd bend edge

Figure 8 a-e Estimated experimental mode shapes of the modified blade section and support structure

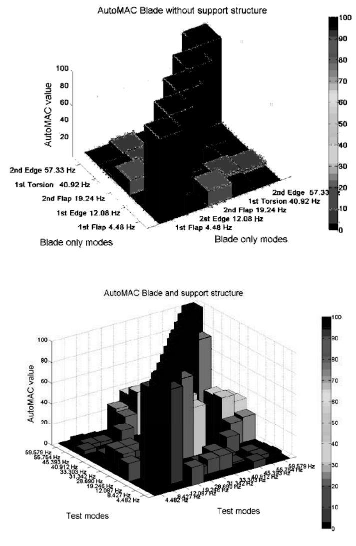


Figure 9 AutoMAC matrices for experimental modal model with sensors only on the modified blade section (top) and blade section with it's support structure (bottom)

On Figure 9, red color corresponds to MAC value equal 100. Dark Blue color reflects the MAC value 0. Modes corresponding to frequencies 8 Hz, 28 Hz, 31 Hz and 33 Hz are related to dynamic properties of the supporting structure.

5.1 Correlation analysis for the simulation and test results

Based on the estimated experimental modal model and two developed FEM models (modeling the original and modified blades) the correlation analysis can be applied. The FE model should be characterized by good consistency of the natural frequency values and mode shapes obtained from measurement. Modal Assurance Criterion is used as the original-modified blade simulation and also test-simulation correlation metrics.

The global axis system used to define the test model differs from that used for the FE model. In order to make the models match it is necessary to apply geometric correlation by translation and rotation of the test model (Figure 10). Next step is node mapping. The number of measurement nodes is much less than the FE nodes. Modal vectors are compared only for the nodes from FE which are located closest to the measurement points. Only the portion of the blade after the clamp is considered.

For tests described in section 2 to 4, the support structure was not taken into account and measured data reduced to obtain a perfectly rigid boundary condition on the clamped section [1-5]. Since the FEM model of the blade is modeled using the same assumption, some differences in frequency values and mode shapes from experimental results for the modified blade are expected.

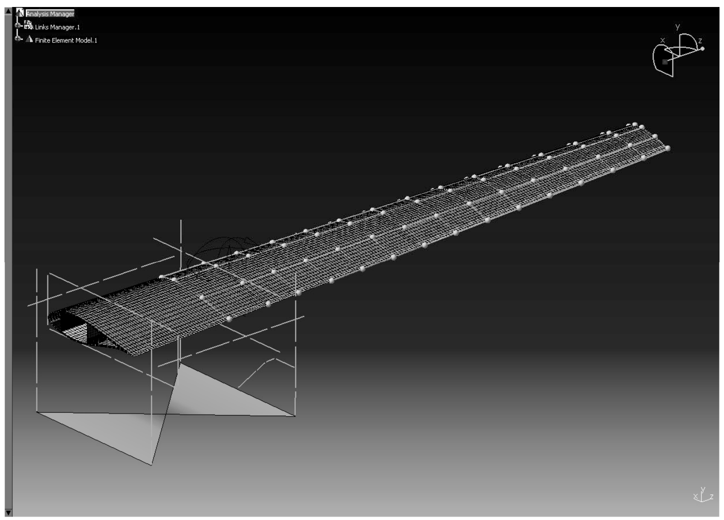


Figure 10 Test and FE geometry correlation with node mapping.

The blade model was solved to compute mode shapes in the 0-60 Hz frequency bandwidth and computations were performed on a 50Tflop cluster.

First Modal Assurance Criterions were calculated for the corresponding modes in order to associate the closest numerical and experimental mode shapes (Figure 11). The procedure accounted for both natural frequency value and the mode shape consistency.

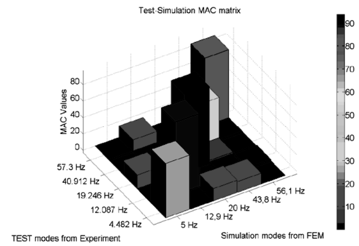


Figure 11 MAC matrix for test and FE simulation modal vectors of modified blade.

The following modes were investigated: 1st and 2nd flapwise bending, 1st and 2nd edgewise bending and 1st torsional (Figure 8). The MAC matrix in Figure 11 clearly shows that the off-diagonal terms are low valued which confirms linear independence of estimated modal vectors. The best test and simulation modal vectors consistency can be observed for the 2nd flapwise mode. The consistency of the results can be recognized as satisfactory, however the present differences need to be further investigated. Observing the values of the MAC criterion between test and simulation modes (Figure 9), differences can be notified. They are caused by the influence of the support structure and not perfectly excited 1st bending mode. Comparison of natural frequencies for experimental and simulation for the original and modified blade is presented in Table 1.

Original blade FE	Original blade Test	Modified blade FE	Modified blade Test
4.7 Hz	4.5 Hz	5.01 Hz	4.48 Hz
1st bend flap			
10.85 Hz	8.7 Hz	12.9 Hz	12.08 Hz
1st bend edge			
18.56 Hz	18.9 Hz	20.03 Hz	19.24 Hz
2nd bend flap			
42.99 Hz	39.5 Hz	43.75 Hz	40.92 Hz
1st torsion			

Table 1 Comparison of the natural frequencies for the experimental and numerical results obtained for the original and modified blade.

The difference between the Test and FE frequencies can be explained by the modeling of the boundary condition as rigid in FE. Moreover, further differences in the frequency values between original and modified blade results are introduced by additional mass and stiffness implemented from angled UD layers on the suction and pressure side of the blade. These UD layers introduce measurable bend-twist couplings, which the original blade did not have both in terms of static and dynamic response of the investigated section of the blade. While in static response there was clear indication of the coupling, the modification of dynamic stiffness is not fully recognized which can be observed from the comparison of the simulation of original and modified blade structural dynamics (Figure 12).

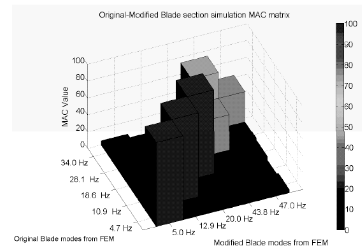


Figure 12 MAC matrix for original and modified blade Finite Element models

100% MAC matrix terms for the first three modes may indicate that these particular mode shape sensitivity towards the implemented additional layers is not significant.

5.2 Twist and bend angles computation

Using the modal vectors identified from the experimental modal analysis results or computed from the FE model, twisting and bending angles for particular mode shapes can be computed. For experimental mode shapes, a fitting is applied to smooth out some odd local behavior due to inherent errors in the measuring process and excitation limitations shown in Figure 7.

For twisting angles, a similar approach to the one used in the static computation is applied [1]. For each cross-section, displacements from the undeformed configuration are associated to the estimated modal amplitudes. Only the leading and trailing edge results are used in the computation. Both relative angles for each section and the sum of these angles along the blade are computed.

Bending angles are computed in a slightly different way than for static measurements. The modal displacements obtained from the sensors located on the point of maximum thickness of the airfoil for each section are considered. By considering two consecutive cross-sections, relative bending is evaluated as the difference between the modal displacements in the x direction (see Figure 1). The tangent of the relative angle for the section is then computed by dividing the difference for the length of the section. On each cross-section, the global bending angle is computed by summing all the relative angles for the previous cross-sections. Computed angles are modal angles, so they depend on the scaling applied to the modal vectors. Moreover, torsional angles are computed assuming that the mass and shear centers are in the same locations.

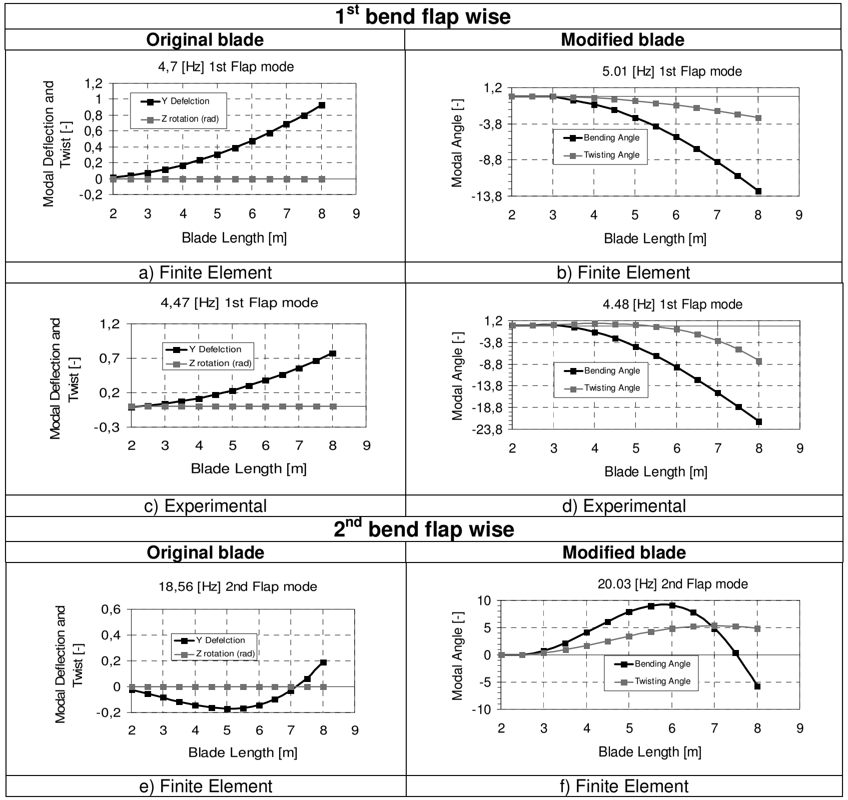
5.3 Numerical and experimental twist and bend angles

The methodology described in section 5.2 is applied both to numerical and experimental modal analysis results. In the present paper only a limited part of the experimental results are presented in details. The results for first and second flap modes will be discussed.

Original blade simulation flap bending translation values (Figure 13 a, e) are plotted with the dark blue, edge bending with red and rotation with light blue.

Original blade experimental results are presented on Figure 13 c, g. Deflection in the flap direction is plotted with the red line, edgewise direction with blue and rotation around the blade axis is marked with green. The angles values are assumed to be 0 in the clamped section.

For the modified blade section bending angle values are plotted in dark blue and twist in red. To have an overview of the overall behavior, the absolute angles are presented. Results are obtained by processing the fitted mode shapes.



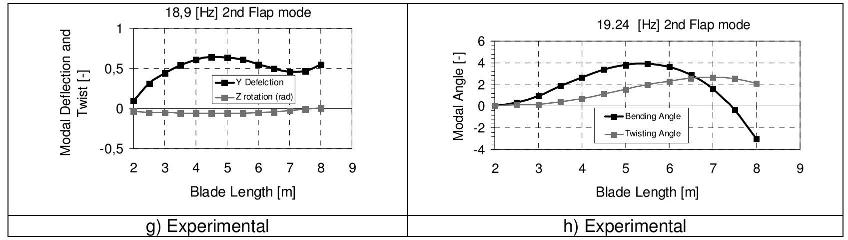


Figure 13 Twist and bend angle computation from numerical and experimental results for original (left column) and modified (right column) blade section for 1st and 2nd flapwise bending mode shapes. Blade segment between 2 clamps is not plotted.

Figure 13 a,c,e,g shows the original blade twist angle and bend translation computed for the 1st and 2nd flapwise modes for experimental and numerical results.

Figure 13 b,d,f,h shows the modified blade twist and bend angle computed for the 1st and 2nd flapwise modes for experimental and numerical results.

By graphically comparing the different plots, it can be immediately observed how the computed torsional angle is higher, as expected, for the modified blade than for the original one. Moreover, the trend of bending is the same in all the plots, confirming that the identified mode shapes in the different processing are consistent. Even if Figure 13 b,d,f,h show bending modal angles instead of displacements, they can be directly related.

By comparing Figure 13b with 13d, and 13f with 13h, some difference can be observed. Modal angle values depends on the scaling applied and the specific values is not important. Moreover, modeling clamping as rigid in the FE model introduces a different than measured boundary condition which modifies the mode shapes. Finally, fitting applied to experimental results of course gives a smoother behavior but can also introduce some numerical error. Despite these local problems, a good agreement between the overall trend of the twist and bend angle for measured and simulated results can be observed.

This observation would confirm some discrepancies in between test and simulations which were already spotted for the 1st mode (Figure 11).

For the original blade the bending was decoupled from twisting which was measured (13g) and simulated (13e). Introduced coupling is clearly visible in the increased torsional response for the simulated (13f) and measured results (13h).

5.4 Bend-twist coupling index

One of the main objectives is to investigate the amount of twisting introduced by the additional UD layers implemented on the modified blade. To obtain a quantitative measure of the coupling between twisting and bending angle, an index is introduced. For each considered blade cross-section, the ratio between the computed relative twisting and bending angles is evaluated. A coupling index value close to zero means that the twisting is negligible with respect to the bending for the considered mode. On the contrary, an high index value means that twisting is dominant. If the index is close to one, twisting and bending are of the same order of magnitude. Figure 14 shows the computed coupling index for the 1st and 2nd flapwise modes both for experimental and numerical results.

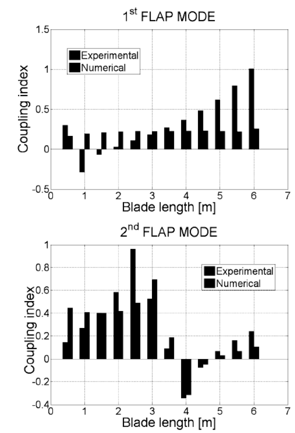


Figure 14 : bend-twist coupling index for 1st (top) and 2nd (bottom) flapwise mode

Comparing the numerical and experimental results, some difference can be observed. These differences are directly related to the observation made for Figure 11. For the first mode shape the MAC value between experiment and numerical model is 65.5. Explanation of the lower MAC value for the 1^{st} mode can be found in the coherence plot presented on Figure 7. It is a ratio of the maximum energy in a combined output signal due to its various components, and the total amount of energy in the output signal. Coherence is used as a measure of the power in the output channel that is caused by the power in the input or reference channel. As such it is useful in assessing the accuracy of the frequency response function measurements. The coherence function can take values that range between 0 and 1. A high value (near 1) indicates that the output is due almost entirely to the input and one can feel confident in the frequency response function measurements. A low value (near 0) indicates problems such as extraneous input signals not being measured, noise, nonlinearities or time delays in the system. 1^{st} mode is located in the frequency range of relatively poor coherence leading to decreased quality of it's estimation. Shakers which were used in the measurement have a low frequency limit around 2 Hz. The excitation signal was random that provides homogenous distribution of injected energy over the excited bandwidth. This could lead to the insufficient energy exciting the 1^{st} mode. Moreover shakers were hung from support cables. In such case at very low frequency in the sub-10 Hz range there is a problem to provide more inertia to push against the structure being excited for improved performance. To improve the excitation of the 1^{st} mode shape additional mass was applied to increase the inertia of the shakers apparently bringing not much improvement. In the higher frequency where the displacements become lower as is the case for the 2^{nd} mode the consistency between test and simulation is much better since the MAC value is 88,5

Bend and twist angles calculations are based on the experimental and numerical modal vectors. The difference between experimental and numerical models bend-twist indexes is caused by the relatively weak excitation of the 1^{st} mode due to the abovementioned reasens.

Coupling index for the numerical 1^{st} flapwise bending has approximately a constant value throughout the blade, while a different behavior is observed from experiments. In addition to some odd behavior from local mode shapes which could influence the results, it should also be noted that the measured boundary conditions are different than modeled.

The latter observation can be applied also for the 2^{nd} flap wise bending mode, which on the other hand shows a very similar trend between numerical and experimental results.

To be able to improve the correlation between the presented results, the same boundary conditions must be used both for the experimental and numerical models.

6. Conclusions

This paper presents some results and aspects of the multidisciplinary and interdisciplinary research oriented for the experimental and numerical study in static and dynamic domains on the bend-twist coupling in the full scale section of the wind turbine blade structure.

An extensive test campaign performed on the original and modified wind turbine blade section was presented. Test setups included different load configurations, excitation and measurement techniques of contact and non-contact type. Experimental test data examples were shown and used for two purposes. Firstly to evaluate the ability of different test method to measure the bend-twist coupling and secondly to use the test results for FE models updating. The common observation from displayed comparisons is that the original blade section did not have a measurable bend-twist coupling. Because one of the primary aims of this work is to develop and use a FE-model capable of modeling correct bend-twist coupling behavior, the original blade section was modified. For this purpose four UD layers were laminated on the pressure and suction side of the blade section to introduce a measurable flapwise bend-twist coupling.

The successful implementation of the bend-twist coupling was confirmed by extensive static and dynamic measurement campaigns. In both experimental methods the comparison of original and modified blade properties clearly show the presence of the bend-twist coupling.

Further research should introduce two clamps in the FE model in order to get a realistic structural behavior of the clamps. In detail the plywood plates and steel profiles should be included and contact elements should be applied to model the contact between the clamps and the blade section. It is expected that the more sophisticated support structure FE representation will improve the consistency in between test and simulations.

7. Acknowledgements

Vestas Wind Systems A/S has provided and modified the blade sections presented in this study. The work is partly supported by the Danish Energy Authority through the 2007 Energy Research Programme (EFP 2007). The supported EFP-project is titled "Anisotropic beam model for analysis and design of passive controlled wind turbine blades" and has journal no. 33033-0075. The support is gratefully acknowledged and highly appreciated.

Research presented in section 5 was conducted in the context of the FP7 project PROND Ref No. 239191. Computations were performed on a 50Tflop cluster in TASK Academic Computer Centre in Gdansk, Poland.

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