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Angular spectrum approach for fast simulation of pulsed non-linear ultrasound fields

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***Abstract*—The paper presents an Angular Spectrum Approach (ASA) for simulating pulsed non-linear ultrasound fields. The source of the ASA is generated by Field II, which can simulate array transducers of any arbitrary geometry and focusing. The non-linear ultrasound simulation program - Abersim, is used as the reference. A linear array transducer with 64 active elements is simulated by both Field II and Abersim. The excitation is a 2-cycle sine wave with a frequency of 5 MHz. The second harmonic field in the time domain is simulated using ASA. Pulse inversion is used in the Abersim simulation to remove the fundamental and keep the second harmonic field, since Abersim simulates non-linear fields with all harmonic components. ASA and Abersim are compared for the pulsed fundamental and second harmonic fields in the time domain at depths of 30 mm, 40 mm (focal depth) and 60 mm. Full widths at -6 dB (FWHM) are {0.97, 0.95} mm at the focal depth for the fundamental fields for ASA and Abersim, and {0.56, 0.55} mm for the second harmonic fields. Full widths at -12 dB are {1.27, 1.26} mm for the fundamental fields for ASA and Abersim, and {0.77, 0.73} mm for the second harmonic fields. The calculation time, for the second harmonic fields, using ASA is 12 minutes and for all harmonic fields using Abersim is 14 hours. Compared to Abersim, the error of ASA for calculating the second harmonic fields is 1.5% at -6 dB and 6.4% at -12 dB, and the calculation time is reduced by a factor of 70.**

I. Introduction

An efficient optimization of non-linear ultrasound imaging can be carried out using simulation programs, which should be fast and yield accurate results. Simulation of non-linear wave equation is usually solved by numerically integrating the KZK [1]–[4] or Burgers equation [5] based on the operator splitting method [6]. This makes the simulation slow and inefficient, since a small stepsize in the propagating direction has to be used each time in calculation of the forward non-linear acoustic pressure. Thus, hundreds of steps are needed, if the desired simulated points are far from the original acoustic source.

Our previous studies [7], [8] presented an analytical solution to the non-linear Westervelt equation [9], where an Angular Spectrum Approach (ASA) has been used to solve it in one iteration step. This makes the simulation of non-linear ultrasound fields hundreds of times faster than using numerical solutions. The previous studies focused on simulating the monochromatic field. However, in an ultrasound imaging system, a short pulse is usually emitted from the transducer. This yields a high bandwidth signal, and calculation with a single temporal frequency is insufficient. The purpose of this paper is to develop the ASA for simulating pulsed non-linear ultrasound fields. The source for the ASA will be created by Field II [10], [11] that can simulate array transducers of any arbitrary geometry, focusing, and excitation. A released non-linear ultrasound simulation program - Abersim [12], [13] is used as the reference to validate the accuracy of ASA.

The solution for the pulsed non-linear ultrasound field is given and the implementation is described in Section II. The simulated results using both ASA and Abersim are illustrated in Section III.

II. Method

A solution to the Westervelt equation for monochromatic fields using the ASA can be expressed by [14], [15]

$$\begin{aligned}\hat{P}_2(k_x,k_y,z_1) &= \frac{\beta k^2}{2\pi^2\rho_0 c_0^2}\iint \frac{\hat{P}_0(k_x',k_y',z_0)}{k_{z2}^2-(k_z'+k_z'')^2} \\ &\times\left[e^{-j(z_1-z_0)(k_z'+k_z'')}-e^{-j(z_1-z_0)k_{z2}}\right] \\ &\times\hat{P}_0(k_x-k_x',k_y-k_y',z_0)dk_x'dk_y', \quad (1)\end{aligned}$$

where

$$\begin{aligned}k_{z2} &= \sqrt{4k^2-k_x^2-k_y^2}, \\ k_z' &= \sqrt{k^2-(k_x')^2-(k_y')^2}, \\ k_z'' &= \sqrt{k^2-(k_x-k_x')^2-(k_y-k_y')^2}.\end{aligned}$$

$\hat{P}_0$ and $\hat{P}_2$ are the pressures of the source and second harmonic components in k-space. c_0 is sound speed, ρ_0 is medium density, β is the coefficient of nonlinearity, and k is the wave number. k_x and k_y are wave numbers along the x-axis and y-axis as shown in Fig. 1, which also shows the acoustic propagation and calculation of the second harmonic fields p_2 using the ASA. The first plane at $z = z_0$ is the source, which is close to the transducer surface and, thus, non-linear effect at $z = z_0$ can be neglected. The source plane is calculated using Field II. The second harmonic component at $z = z_1$ is then obtained after propagation using the ASA as formulated in (1).

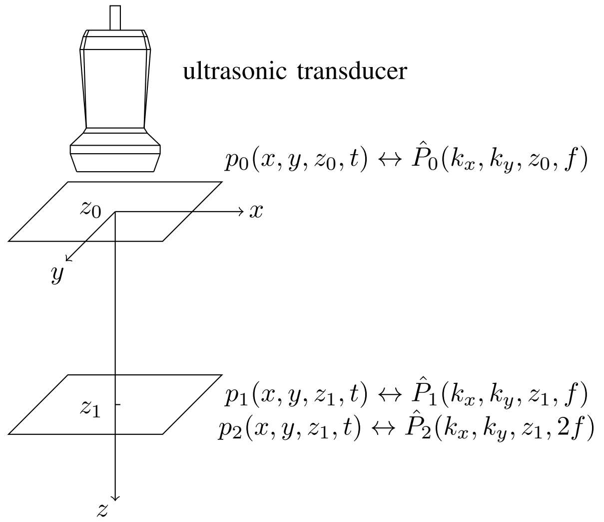

Fig. 1: Schematic view of the acoustic propagation using ASA - The planes are parallel to the transducer surface. Source plane at $z = z_0$; Simulated plane at $z = z_1$.

The previous solution using (1) is for simulating monochromatic fields. To expand it to a high bandwidth ultrasound field, each temporal frequency component is supposed to be calculated individually. In this case, (1) can be transferred and given by

$$\hat{P}_2(k_x,k_y,z_1,2f) = \frac{2\beta f^2}{\rho_0 c_0^4} \iint \frac{\hat{P}_0(k_x-k'_x,k_y-k'_y,z_0,f)}{k_{z2}^2-(k'_z+k''_z)^2} \times \left[e^{-j(z_1-z_0)(k'_z+k''_z)} - e^{-j(z_1-z_0)k_{z2}}\right] \times \hat{P}_0(k'_x,k'_y,z_0,f)dk'_x dk'_y, \quad (2)$$

where f is the temporal frequency variable. $\hat{P}_0(k_x,k_y,z_0,f)$ is the Fourier transform of $p_0(x,y,z_0,t)$ that is the pressure for the ASA source in time domain and calculated by Field II using the function "calc_hp" [16]. A linear array transducer is configured by Field II using the function "xdc_focused_array". The transducer parameters, and impulse and frequency responses used in the simulation are shown in Table I and Fig. 2, respectively. These data refer to a commercially available linear array transducer. The size of a kerf for the transducer is too small and not specified by Abersim. To match it, the kerf is set to zero in Field II. The fundamental and second harmonic components are obtained using the ASA by implementing (2) as shown in Fig. 1, where the bandwidth of the second harmonic components are twice of the fundamental one. The time domain pressure p_1 and p_2 at the simulated plane are calculated using a 3D (2D for space and 1D for time) inverse Fourier transform.

The transducer with the same setup as shown in Table I is also simulated in Abersim [17], [18] as a reference. A pulse inversion method [19], [20] is used to remove the fundamental component. This will be made by sending two inverted excitations in turn and adding the reflected simulated signals resulting in the cancelation of odd harmonic components. Then a bandpass filter is applied to remove the 4th and higher even harmonic components.

TABLE I: Parameters of the transducer

Center frequency f_0	7 MHz
Bandwidth bw	60%
Sampling frequency f_s	100 MHz
Excitation	2-cycle sine wave
Excitation's frequency	5 MHz
Electronic focus	40 mm
Elevation focus	20 mm
Number of elements	64
Pitch	0.208 mm
Height of element	4.5 mm

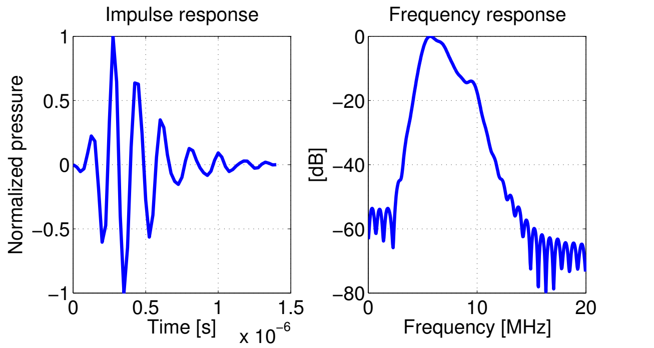

Fig. 2: Impulse and frequency responses of the simulated transducer

III. Results

The emitted pressure fields are calculated by the ASA and Abersim. Fig. 3 presents the fundamental and second harmonic fields in the time domain at the focal depth 40 mm from the transducer surface. A visibly good agreement can be found from the comparison of the results. The calculation time, for Abersim, which can simulate all harmonic fields, is 14 hours, and for ASA, it is 12 minutes for simulating the second harmonic fields. The simulations are made by Matlab 7.11.0 (R2010b) using a computer with 2.4 GHz Q6600 CPU and 4 GB memory. To further investigate the ASA results, the fundamental and second harmonic fields at different depths are calculated by ASA and Abersim as shown in Figs. 4 and 5. The full widths at -6 dB and -12 dB of the results using different methods are compared and shown in Table II.

TABLE II: Comparison of the full width (FW)

Depth = 30 mm		ASA	Abersim	Error
Fundamental	FW at -6 dB	2.97 mm	2.98 mm	0.6%
	FW at -12 dB	4.40 mm	4.41 mm	0.2%
2nd harmonic	FW at -6 dB	2.56 mm	2.62 mm	2.5%
	FW at -12 dB	3.37 mm	3.45 mm	2.4%
Depth = 40 mm		**ASA**	**Abersim**	**Error**
Fundamental	FW at -6 dB	0.97 mm	0.95 mm	1.7%
	FW at -12 dB	1.27 mm	1.26 mm	1.2%
2nd harmonic	FW at -6 dB	0.56 mm	0.55 mm	1.5%
	FW at -12 dB	0.77 mm	0.73 mm	6.4%
Depth = 60 mm		**ASA**	**Abersim**	**Error**
Fundamental	FW at -6 dB	6.06 mm	5.99 mm	1.1%
	FW at -12 dB	8.90 mm	8.82 mm	1.0%
2nd harmonic	FW at -6 dB	5.29 mm	5.21 mm	1.4%
	FW at -12 dB	7.01 mm	6.89 mm	1.6%

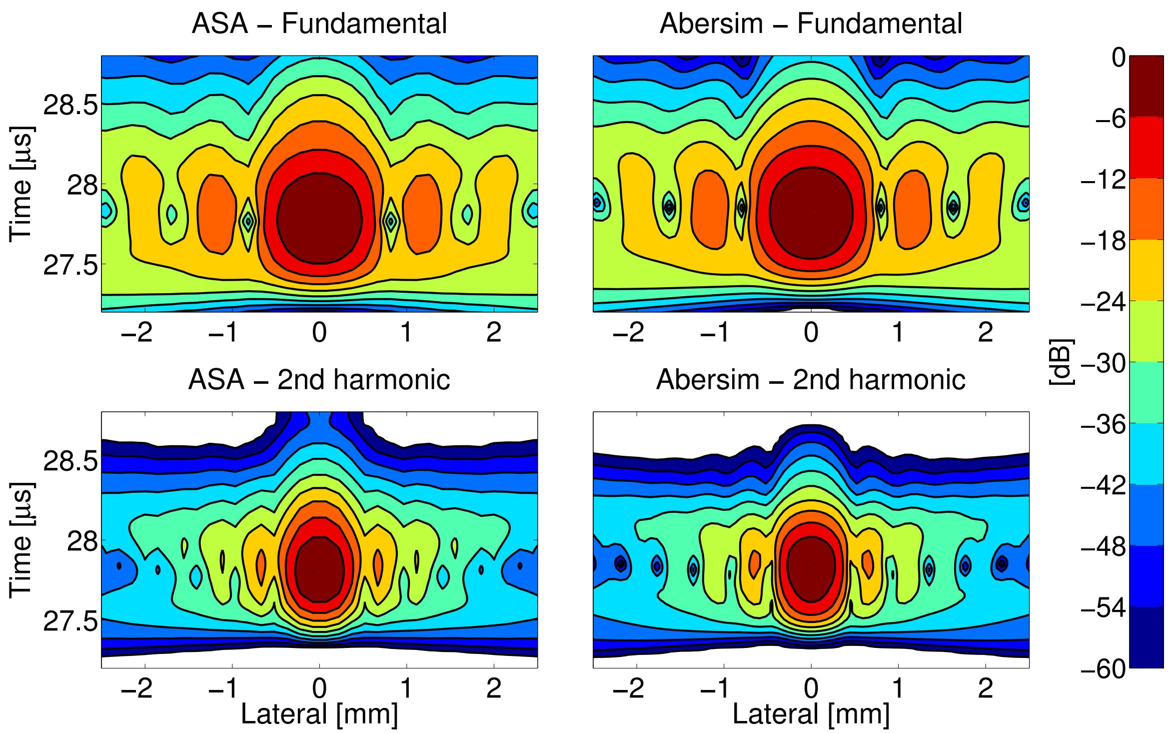

Fig. 3: Emitted ultrasound fields calculated by ASA and Abersim - Fundamental and second harmonic fields at the focal depth (40 mm) are shown in the figure with 6 dB between two adjacent color lines.

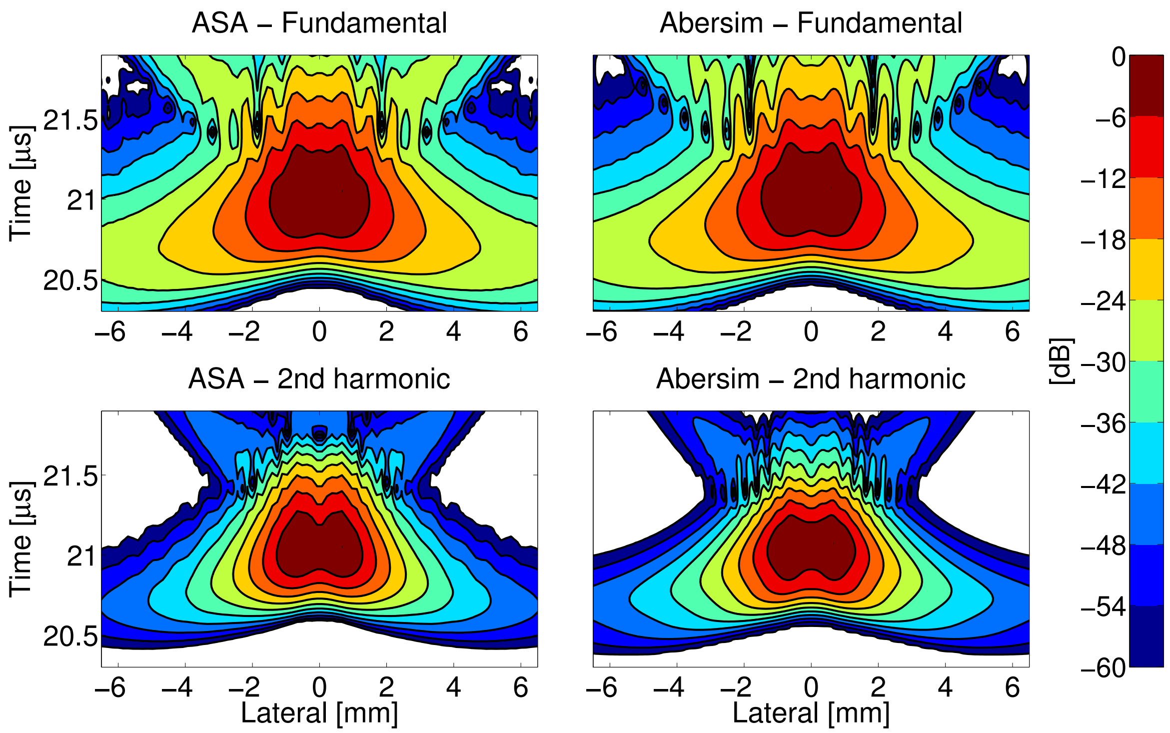

Fig. 4: Emitted fields calculated by ASA and Abersim - Fundamental and second harmonic fields are at 30 mm from the transducer.

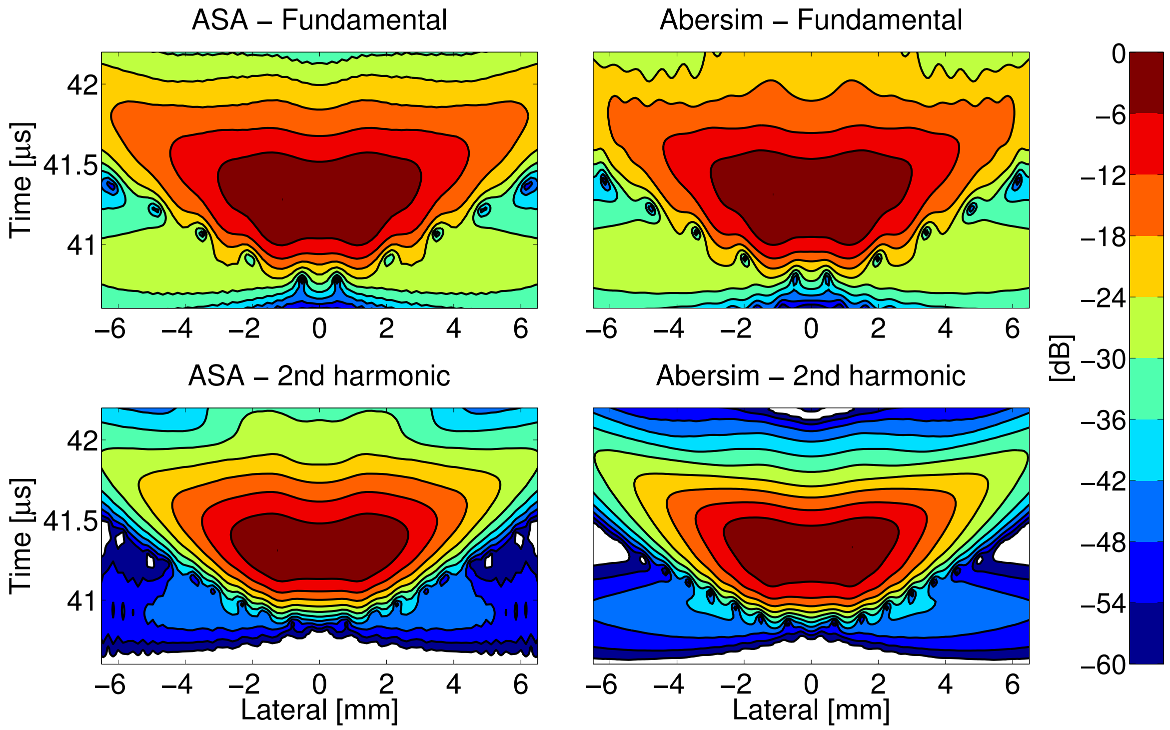

Fig. 5: Emitted fields calculated by ASA and Abersim - Fundamental and second harmonic fields are at 60 mm from the transducer.

IV. Conclusion

The pulsed non-linear ultrasound fields are successfully simulated by the ASA, whose accuracy is investigated and compared to Abersim. The ASA using the source generated by Field II, makes the non-linear ultrasound simulation flexible to any kind of transducer with arbitrary focus and excitation. The calculation speed using the ASA is 70 times faster than using Abersim for simulating the second harmonic fields.

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