Technical University of Denmark

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Fernandez Grande, Efren; Jacobsen, Finn

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SEPARATION OF RADIATED SOUND FIELD COMPONENTS FROM WAVES SCATTERED BY A SOURCE UNDER NON-ANECHOIC CONDITIONS

Efren Fernandez Grande, Finn Jacobsen

Acoustic Technology. B. 352, Department of Electrical Engineering (Technical University of Denmark). 2800 Lyngby, Copenhagen, Denmark.
efg@elektro.dtu.dk , fja@elektro.dtu.dk

Abstract
A method of estimating the sound field radiated by a source under non-anechoic conditions has been examined. The method uses near field acoustic holography based on a combination of pressure and particle velocity measurements in a plane near the source for separating outgoing and ingoing wave components. The outgoing part of the sound field is composed of both radiated and scattered waves. The method compensates for the scattered components of the outgoing field on the basis of the boundary condition of the problem, exploiting the fact that the sound field is reconstructed very close to the source. Thus the radiated free-field component is estimated simultaneously with solving the inverse problem of reconstructing the sound field near the source. The method is particularly suited to cases in which the overall contribution of reflected sound in the measurement plane is significant.

Keywords: Near-field Acoustic Holography (NAH), sound field separation, sound radiation.

1 Introduction

In the original formulations of Near-field acoustic Holography (NAH) [1, 2], it is a requirement that all the sources are confined to one side of the microphone array. This requirement stems from the fact that it is not possible to determine whether the sound is coming from one or the other side of the array based on simple measurements of the sound pressure in one single plane. If the pressure in the measurement plane is contaminated by components coming from the other side, all the measured acoustic energy would be attributed to the primary source, leading to an erroneous reconstruction of the field [2–6].

Conventional NAH methods require that the measurement is performed under free-field conditions to avoid reflections from the source-free half space [3]. However, it is not always possible to perform the measurements under free-field conditions. This paper considers the case in which there is sound reflected from the source-free half space[1].

To minimize the influence of sound coming from the source-free half space separation techniques can be used. These techniques separate the outgoing sound field from the source and the incoming sound field from reflections or other sources. Some of these separation methods rely on a measurement of the field in two closely spaced parallel planes [7–11]. Other approaches are based on a combined measurement of the sound pressure and particle velocity of the field [12, 13]. The so called p-u method has been the subject of several research papers. It relies on the measurement of the pressure and particle velocity. It is a rather simple method that makes use of the fact that, unlike the pressure, the particle velocity is a vector that changes sign if the sound is coming from one or the other side of the array. Thus, by adding or subtracting the pressure and particle velocity based estimates of the sound field, the contribution of sound coming form each side of the array can be determined [13–16].

These techniques make it possible to estimate the outgoing field from the source, which is in general a good indicator of its acoustic radiation. However, the outgoing component of the sound field may be composed of both radiated and scattered waves by the source. The scattered sound propagates in the same direction as the radiated sound. It is therefore not trivial to separate the two components. Recently some techniques based on the Helmholtz integral formulation have been proposed to compensate for the scattered field and determine the free-field radiation by the source [17, 18].

This paper describes and examines a technique based on Statistically Optimized Near-field Acoustic Holography (SONAH) [3, 19, 20] and the p-u separation method for estimating the free field radiation by a source in the presence of reflections. The method compensates for the scattered component of the outgoing field based on the boundary conditions of the problem, and thus makes it possible to estimate the source's free field radiation.

2 Theoretical background

2.1 Statistically Optimized Near-field Acoustic Holography

The method examined in this paper is based on Statistically Optimized Near-field Acoustic Holography (SONAH). A detailed description of the SONAH method can be found in [20]. In SONAH, the measured sound field is expressed as a decomposition of plane elementary waves with different weightings. In matrix form it can be expressed as

$$\mathbf{p}(\mathbf{r}_h) = \mathbf{B}\mathbf{c}, \tag{1}$$

where $\mathbf{p}$ is a column vector with the measured pressures, $\mathbf{c}$ is a column vector with the n coefficients of the elementary functions, and $\mathbf{B}$ is a matrix with the elementary wave functions $\alpha(\mathbf{r}_{hm})$ at the measurement positions,

$$\alpha_n(\mathbf{r}) = e^{-j(k_{x,n}x + k_{y,n}y + k_{z,n}(z - z^+))}. \tag{2}$$

The regularized solution to the inversion of equation 1 is

$$\mathbf{c} = (\mathbf{B}\mathbf{B}^H + \lambda\mathbf{I})^{-1}\mathbf{B}^H\mathbf{p}(\mathbf{r}_h), \tag{3}$$

[1]For simplicity, the source-free half space will be also referred as the "wrong" side of the array

where λ is the regularization parameter. Once the coefficients are known, the sound field at each position of the reconstruction plane can be obtained as $p(\mathbf{r}_s) = \alpha(\mathbf{r}_s)\mathbf{c}$:

$$p(\mathbf{r}_s) = \alpha(\mathbf{r}_s)(\mathbf{B}\mathbf{B}^H + \lambda\mathbf{I})^{-1}\mathbf{B}^H\mathbf{p}(\mathbf{r}_h), \tag{4}$$

so that the sound pressure in the reconstruction plane can be expressed in terms of the measured pressure in the hologram plane. The reconstruction of the field can as well be done based on measurement of the particle velocity,

$$\mathbf{u}_z(\mathbf{r}_s) = \alpha(\mathbf{r}_s)(\mathbf{B}\mathbf{B}^H + \lambda\mathbf{I})^{-1}\mathbf{B}^H\mathbf{u}_z(\mathbf{r}_h). \tag{5}$$

The pressure can be estimated from the normal velocity making use of Euler's equation of motion,

$$p(\mathbf{r}_s) = \gamma(\mathbf{r}_s)(\mathbf{B}\mathbf{B}^H + \lambda\mathbf{I})^{-1}\mathbf{B}^H\mathbf{u}_z(\mathbf{r}_h), \tag{6}$$

where $\gamma(\mathbf{r}) = (-j\omega\rho)\int \alpha(\mathbf{r})dz$:

$$\gamma(\mathbf{r}) = \rho c \frac{k}{k_z}\alpha(\mathbf{r}). \tag{7}$$

In order to describe the separation method in a simple way it is convenient to introduce a simplified notation of SONAH, in which the acoustic quantities in the hologram and in the reconstruction plane are related through a transfer matrix that accounts for the propagation of the elementary waves in which the field is decomposed. This is expressed as

$$\mathbf{p}(\mathbf{r}_h) = \mathbf{C}_{pp}\mathbf{p}(\mathbf{r}_s),$$
$$\mathbf{u}_z(\mathbf{r}_h) = \mathbf{C}_{pu}\mathbf{p}(\mathbf{r}_s).$$

The matrix $\mathbf{C}_{pp}$ relates the pressure in the measurement plane to the pressure in the reconstruction plane and $\mathbf{C}_{pu}$ relates the pressure in the reconstruction plane with the particle velocity in the measurement plane. Based on this notation, the reconstruction equations can be expressed in a simple way. For instance, the pressure in the reconstruction plane is[2] $\mathbf{p}(\mathbf{r}_s) = \mathbf{C}_{pp}^{-1}\mathbf{p}(\mathbf{r}_h)$ or from the normal velocity $\mathbf{p}(\mathbf{r}_s) = \mathbf{C}_{pu}^{-1}\mathbf{u}(\mathbf{r}_h)$.

2.2 Sound field separation

In the sound field separation technique used in this investigation, it is assumed that the pressure measured in the hologram plane is due to a superposition of outgoing waves from the primary source and incoming waves corresponding to the reflected sound from the source-free half space. In the conventional formulation of SONAH, a single set of elementary wave functions is used to model the sound field radiated by the source. In this case, two sets of elementary wave functions must be used. The first set of elementary wave functions (α and γ) models the outgoing field from the primary source, and an additional set of functions models the incoming sound field. Let the new set of elementary functions be

$$\psi_n(\mathbf{r}) = e^{-j(k_{x,n}x + k_{y,n}y - k_{z,n}(z-z^-))}, \tag{8}$$

[2] Note that the inversion of the transfer matrices needs regularization

$$\vartheta(\mathbf{r}) = (-j\omega\rho)\int \psi(\mathbf{r})dz. \tag{9}$$

The $\psi(\mathbf{r})$ wave functions are analogous to α, and they are used for pressure-to-pressure and velocity-to-velocity predictions. The $\vartheta(\mathbf{r})$ are analogous to γ, and they are used for estimating the sound pressure from the normal component of the particle velocity.

The sound field measured in the hologram plane can be expressed as the contribution from the outgoing and incoming components as:

$$\mathbf{p}(\mathbf{r}_h) = \mathbf{C}_{ppo}\mathbf{p}_o(\mathbf{r}_s) + \mathbf{C}_{ppi}\mathbf{p}_i(\mathbf{r}_s), \tag{10}$$

$$\mathbf{u}_z(\mathbf{r}_h) = \mathbf{C}_{puo}\mathbf{p}_o(\mathbf{r}_s) - \mathbf{C}_{pui}\mathbf{p}_i(\mathbf{r}_s), \tag{11}$$

where the subscripts o and i of the transfer matrices indicate whether they refer to the outgoing or incoming fields, thus requiring the use of different elementary wave functions. The transfer matrices C_{ppo}, C_{puo}, use the outgoing elementary wave functions α and γ respectively, and the transfer matrices C_{ppi}, C_{pui}, use the incoming elementary wave functions ψ and ϑ , and $\mathbf{p}_o(\mathbf{r}_s)$ and $\mathbf{p}_i(\mathbf{r}_s)$ are the outgoing and incoming pressure fields at the reconstruction positions, which can be calculated from (10) and (11) as:

$$\mathbf{p}_o(\mathbf{r}_s) = \left(\mathbf{C}_{puo} + \mathbf{C}_{pui}\mathbf{C}_{ppi}^{-1}\mathbf{C}_{ppo}\right)^{-1}\left(\mathbf{u}_z(\mathbf{r}_h) + \mathbf{C}_{pui}\mathbf{C}_{ppi}^{-1}\mathbf{p}(\mathbf{r}_h)\right), \tag{12}$$

$$\mathbf{p}_i(\mathbf{r}_s) = \left(\mathbf{C}_{pui} + \mathbf{C}_{puo}\mathbf{C}_{ppo}^{-1}\mathbf{C}_{ppi}\right)^{-1}\left(\mathbf{C}_{puo}\mathbf{C}_{ppo}^{-1}\mathbf{p}(\mathbf{r}_h) - \mathbf{u}_z(\mathbf{r}_h)\right). \tag{13}$$

However, the outgoing field is composed of both radiated and scattered waves,

$$p_o(\mathbf{r}) = p_f(\mathbf{r}) + p_s(\mathbf{r}), \tag{14}$$

where $(p_f,\ u_{zf})$ is the free field sound radiated by the source, and $(p_s,\ u_{zs})$ is the scattered sound. If $\mathbf{r}_s$ is sufficiently close to the boundary of the source, and if the source can be regarded as rigid, the boundary conditions apply,

$$p_s(\mathbf{r}_s) = p_i(\mathbf{r}_s), \tag{15}$$

where p_s is the scattered sound pressure and p_i is the incident sound pressure. Making use of eqs. (14) and (15),

$$p_f(\mathbf{r}_s) = p_o(\mathbf{r}_s) - p_i(\mathbf{r}_s). \tag{16}$$

From equations (12), (13) and (16) the free field radiation by the source can be estimated as:

$$\begin{aligned}\mathbf{p}_f(\mathbf{r}_s) = &\left(\mathbf{C}_{puo} + \mathbf{C}_{pui}\mathbf{C}_{ppi}^{-1}\mathbf{C}_{ppo}\right)^{-1}\left(\mathbf{u}_z(\mathbf{r}_h) + \mathbf{C}_{pui}\mathbf{C}_{ppi}^{-1}\mathbf{p}(\mathbf{r}_h)\right) - \\ &\left(\mathbf{C}_{pui} + \mathbf{C}_{puo}\mathbf{C}_{ppo}^{-1}\mathbf{C}_{ppi}\right)^{-1}\left(\mathbf{C}_{puo}\mathbf{C}_{ppo}^{-1}\mathbf{p}(\mathbf{r}_h) - \mathbf{u}_z(\mathbf{r}_h)\right).\end{aligned} \tag{17}$$

3 Numerical results

The method has been tested by means of a numerical simulation study. The study consists of a simply supported baffled plate radiating in the presence of a disturbing monopole. The dimensions of the plate are 0.3 x 0.3 m, it is 3 mm thick, made of aluminum and driven at the center. The plate is centered at the origin of coordinates and the monopole is placed at (x=0,y=0.1,z=3) m. The sound from the monopole that is reflected by the baffle is modeled by means of a virtual source, assuming a perfect reflection ($R = 1$). The measurement aperture is 0.3 x 0.3 m, with a measurement grid of 11 x 11 positions uniformly spaced. The measurement plane is at $z_h = 4\ cm$, and the reconstruction plane at $z_s = 1\ cm$. The measurement noise corresponds to a signal-to-noise ratio (SNR) of 25 dB. Using the method described in the previous section, the free field radiation by the primary source (the baffled plate) can be estimated.

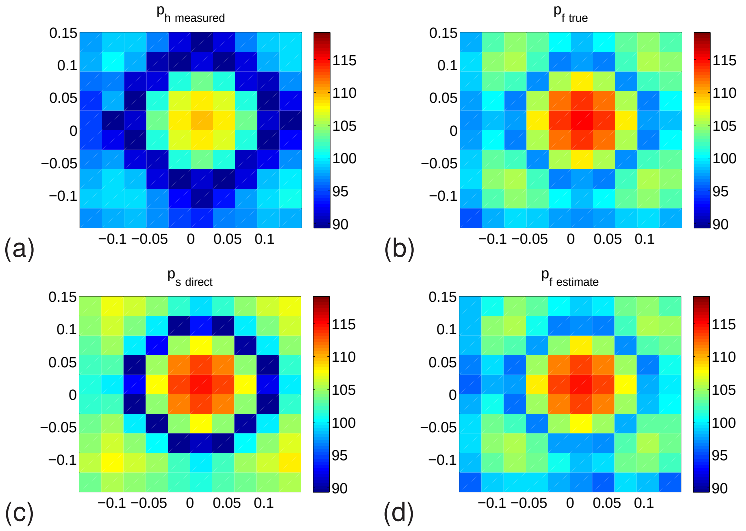

Figure 1 – Pressure field of a baffled plate driven at 850 Hz in the presence of a disturbing monopole radiating from the opposite side of the array. (a) Pressure in the hologram plane. (b) True free field pressure in the reconstruction plane. (c) Reconstruction with the direct formulation of SONAH. (d) Estimation of the free field with equation (17)

Figure 1 shows the pressure in the hologram plane, the true pressure in the reconstruction plane, the reconstruction with eq. (4) where no separation of the sound field is used, and finally the estimation of the free field pressure produced by the primary source using eq. (17).

Figure 2 shows a comparison between the direct reconstruction of the pressure field based in the direct SONAH formulation, and the estimated free field radiation based on eq. (17). It shows the reconstruction for two cases: one in which the influence of the disturbing monopole is strong (3 dB higher than the source), and another in which the pressure produced by the monopole in the measurement aperture is about 7 dB less than the one produced by the plate. It is apparent that the method can estimate the sound pressure successfully even if the the disturbance is not very strong. It should however be noted that this result is based in simulated measurements, in which the agreement between the reconstructions based on pressure and particle velocity is almost perfect.

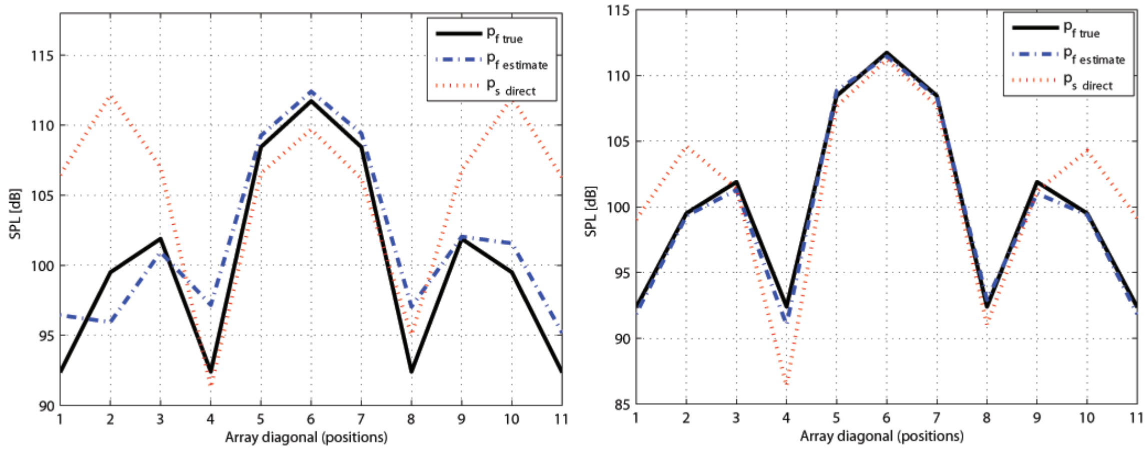

Figure 2 – Sound pressure level across the diagonal of a baffled plate driven at 850 Hz, radiating in the presence of a disturbing monopole at the opposite side of the array. True pressure, reconstructed pressure with eq. (17), and direct reconstruction with eq. (4) without separating the sound field. Left: the monopole radiation is 3 dB higher than the plate. Right: the monopole radiation is about 6 dB less than the plate.

4 Experimental results

An experimental study has been conducted to investigate the applicability of technique described in this paper. The experimental setup consists of a primary source radiating in the presence of a large reflecting panel. The primary source is a vibrating plate mounted on a rigid wooden box. The dimensions of the plate are $45 \times 45\ cm^2$, it is 3 mm thick, made of aluminum and driven acoustically by a loudspeaker inside the box. The origin of the coordinate system is at the bottom left corner of the plate. A large reflecting panel is positioned parallel to the plate, at $0.6\ m$ distance $(x, y, 0.6)\ m$, of dimensions $1.2 \times 1.5\ m$. The pressure and the normal component of the particle velocity were measured in 10×10 positions uniformly spaced $5\ cm$ from each other. The measurement aperture is thus $50 \times 50\ cm^2$, at $z_h = 6\ cm$, and the reconstruction plane at $z_s = 2\ cm$. The set-up of the experiment is sketched in Figure 3.

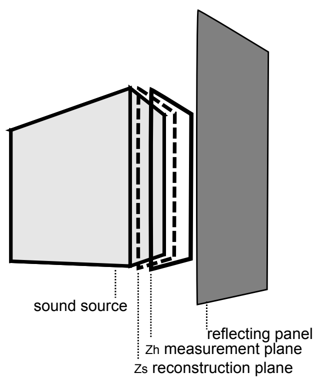

Figure 3 – Set-up of the measurement. The plate (primary source) is at $z = 0$, the hologram plane is at $z_h = 6$ cm, the reconstruction plane at $z_s = 2$ cm, and the reflecting panel at $z = 60$ cm

The pressure and the normal component of the particle velocity fields were measured in the hologram plane, both in the presence of the reflecting panel and without it. Also the true acoustic field radiated by the source was measured in the reconstruction plane under free field conditions, without the influence of the reflecting panel.

Figure 4 shows the measured pressure field in the hologram plane, the true pressure in the reconstruction plane, the reconstructed pressure without separating the sound field using eq. (4), and the estimation of the free field pressure produced by the primary source using eq. (17).

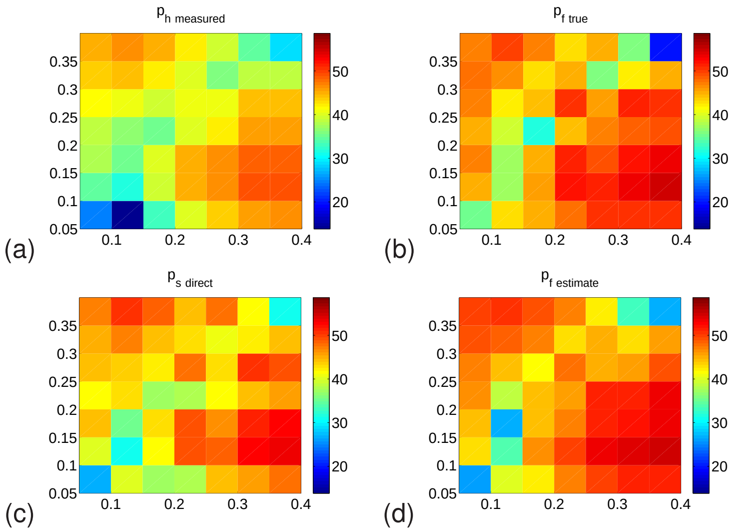

Figure 4 – SPL at 500 Hz of the primary source radiating in the presence of a reflecting panel. (a) Measured pressure in the hologram plane. (b) Measured free field pressure in the reconstruction plane. (c) Reconstruction with the direct formulation, equation (4) . (d) Estimation of the free field pressure with equation (17)

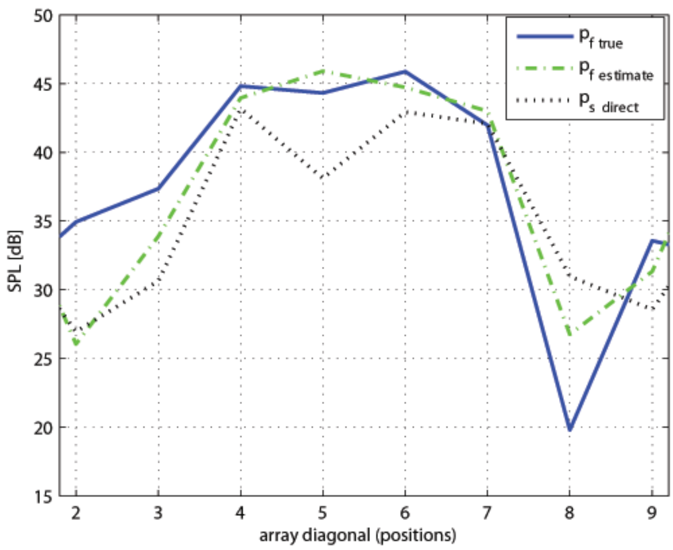

Figure 5 – SPL across the diagonal of the aperture at 500 Hz. The primary source is radiating in the presence of a reflecting panel at the opposite side of the array. True pressure, reconstructed pressure with eq. (17), and direct reconstruction with eq. (4) without separating the sound field.

Figure 5 shows the true pressure and the reconstructed pressures across the diagonal of the aperture at 500 Hz. It seems that it is somewhat more accurate to reconstruct the field using the free field estimation (eq. (17)), where the outgoing field is estimated and the scattered components compensated for, than the direct formulation (eq. (4)).

However, it should be remarked that there is a significant error associated with the estimation of the free field sound radiation using eq. (17). This mis-estimation is illustrated in Figure 6. The figure shows the reconstruction of the sound field in the case where there is no reflected sound from the opposite side of the primary source. Thus, the field measured in the hologram plane is only the one radiated by the source. The figure illustrates the error implicit in the method due to the fact that the pressure and particle velocity based estimates of the sound field are in practice not identical. Therefore, if there is just a small disturbance, or no disturbance at all from the side of the array opposite the primary source, it is consistently more accurate and straight forward to reconstruct the acoustic field based on the direct formulation.

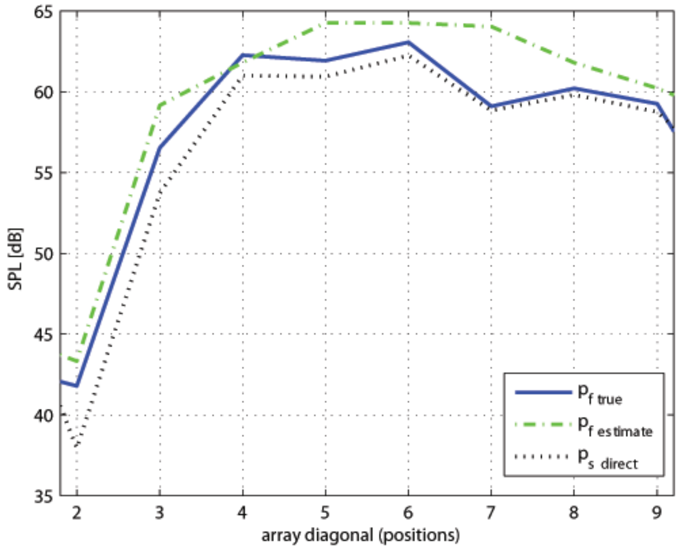

Figure 6 – True and reconstructed pressure with eq.(17) and eq.(4) when the primary source is radiating without the disturbance from any source or reflection from the opposite side of the array. SPL across the aperture diagonal at 400 Hz

5 Discussion

The study indicates that based on eq. (17), the sound pressure field radiated by the primary source can be estimated satisfactorily, particularly in the presence of extraneous noise from the opposite side of the array. However, there are limitations to the accuracy of the technique.

The method relies on the assumption that the sound pressure and particle velocity based estimates of the sound field are identical. This assumption is not completely true in practice, and there is an important source of error associated to it. Therefore, eq. (17) gives a more accurate reconstruction than eq. (4) provided that the disturbing sound is sufficiently strong. Otherwise, if the disturbance is not very significant, the latter yields a more accurate and straightforward reconstruction of the sound field. This result is in agreement with previous studies [14, 15]. This observation explains as well the accurate results obtained in the numerical study, since simulated measurements do obviously not suffer from the errors and uncertainty encountered in actual measurements.

It should also be noted that the set of elementary wave functions ψ used to model the incoming sound field (see eq. (8)) are scaled in the virtual plane z^-. This investigation revealed that the correct modeling of the incoming field depends strongly on the position of this virtual plane. Throughout the study, the best results were consistently found when the virtual plane was set at $z^- = 2z_h - z^+$.

6 Conclusion

A method of estimating the sound field radiated by a source under non-anechoic conditions has been described in this paper. A numerical and experimental study of the technique reveal that the technique can reconstruct the sound pressure field radiated by the primary source satisfactorily, particularly when there is a strong disturbance by sound coming from the wrong side of the array. If the disturbance is not very significant it is more accurate to reconstruct the sound field based on the conventional direct formulation.

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