Technical University of Denmark

Plasmonic nanoantenna based coupler for telecom range

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ABSTRACTS

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FOREWORD

The Seminars/Conferences “Days on Diffraction” are annually held since 1968 in late May or in June by the Faculty of Physics of St. Petersburg State University, St. Petersburg Branch of the Steklov’s Mathematical Institute and Euler International Mathematical Institute of the Russian Academy of Sciences.

This booklet contains the abstracts of 195 talks to be presented at oral and poster sessions in 5 days of the Conference. Author index can be found on the last page.

The full texts of selected talks will be published in the Proceedings of the Conference. The texts in LaTeX format are due by June 20, 2011 to e-mail `diffraction11@gmail.com`. Format file and instructions can be found on the Seminar Web site at `http://eimi.imi.ras.ru/~dd/proceedings.php`. The final judgement on accepting the paper for the Proceedings will be made by the Organizing Committee following the recommendations of the referees.

We are as always pleased to see in St. Petersburg active researchers in the field of Diffraction Theory from all over the world.

Organizing Committee

List of talks

Workshop on meta-materials

Ince–Gaussian beams and Hermite–Laguerre–Gaussian beams: a comparison

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Ince–Gaussian (IG) beams and Hermite–Laguerre–Gaussian (HLG) beams are two parametrical families of structurally stable solutions of the paraxial equation. Both families contain Hermite–Gaussian and Laguerre–Gaussian beams as partial representatives. Based on the theory of HLG beams, asymptotic expansions and general astigmatic transform of IG beams are found. We discuss the similarities and differences between the two families of beams.

Functionally invariant solutions of triple sine-Gordon equation

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The solutions $U(x,y,z,t)$ of triple sine-Gordon equation

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 U}{\partial t^2}=p_1\sin U+p_2\sin 2U+p_3\sin 3U \tag{1}$$

are obtained in the form of a composite function $U=Q(W)$. The function $W(x,y,z,t)$ satisfy simultaneously two equations – the homogeneous wave equation

$$\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}+\frac{\partial^2 W}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 W}{\partial t^2}=0 \tag{2}$$

and a nonhomogeneous eikonal type equation

$$\left(\frac{\partial W}{\partial x}\right)^2+\left(\frac{\partial W}{\partial y}\right)^2+\left(\frac{\partial W}{\partial z}\right)^2-\frac{1}{v^2}\left(\frac{\partial W}{\partial t}\right)^2=1. \tag{3}$$

The function $W(x,y,z,t)$ can be found by the method of functionally invariant solutions construction [1–5] of the equations (2), (3):

$$W=a_1x+a_2y+a_3z-\sigma v^2 t+F(\alpha). \tag{4}$$

Here (a_1,a_2,a_3,σ) are arbitrary constants, which satisfy condition

$$a_1^2+a_2^2+a_3^2=1+\sigma^2v^2, \tag{5}$$

$F(\alpha)$ is an arbitrary function, which argument is ansatz $\alpha(x,y,z,t)$. This ansatz may be found from the equation

$$x\,l(\alpha)+y\,m(\alpha)+z\,n(\alpha)-t\,v^2p(\alpha)+q(\alpha)=0. \tag{6}$$

The coefficients $(l(\alpha),\ m(\alpha),\ n(\alpha),\ p(\alpha),\ q(\alpha))$ are the arbitrary functions of the ansatz α. They coupled by two relationships

$$l^2+m^2+n^2=v^2p^2, \qquad a_1l+a_2m+a_3n=\sigma v^2 p\,. \tag{7}$$

Function $Q(W)$ is the solution of a nonlinear ordinary differential equations of the second order. The form of $Q(W)$ is determined by the parameters $(p_1,\ p_2\, p_3)$ and by the constants of integration.

In general case $Q(W)$ is an inversion of ultra-elliptic integral [6] of the genus two. In some cases ultra-elliptic integrals reduce to the elliptic integrals. The conditions for such reduction is found. Elliptic integrals take place also in the case, when one of the integration constants equals to zero. Such cases are analyzed in detail. Bifurcation lines and domains on plane, bounded by such lines, are found. Real solutions on the bifurcation lines, in points of bifurcation lines crossing and in the domains are received. This solutions are represented by the Jacobi elliptic integrals.

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Kinematic inverse problem for transversely isotropic medium

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The transversal isotropy is a typical approximation for models describing anisotropic formations. However, inversion for Thomsen-type anisotropic parameters in transversely isotropic media using surface seismic data proved to be a highly non-unique problem.

In this work we focus on estimation of information content for seismic signatures in 2D model with dipping reflector overlaid by homogeneous TTI medium. The following notations for angles are used: φ – reflector dip angle; θ – well inclination angle; ν – symmetry axis tilt angle.
We assume that the seismic data we possess consists of surface seismic reflection data (CMP gather) and checkshot survey along the deviated well. The seismic signatures we use are NMO velocity V_{nmo}, non-hyperbolic moveout A_4 and checkshot velocity measured along the well of arbitrary deviation. The former contains information about phase velocity $V_p(\theta)$ in the direction which is parallel to the well. In this work we studied two-parameter inversion for ϵ and δ, assuming know tilt angle ν. The function $F(\theta,\varphi,\nu;\epsilon,\delta)$ has three angle parameters (θ,φ,ν) and links seismic signatures with Thomsen parameters ϵ and δ (1). Expanding this function into a series in weak anisotropy approximation we obtain a matrix $M(\theta,\varphi,\nu)$ which is a Frechet derivative of $F(\theta,\varphi,\nu;\epsilon,\delta)$.

$$\begin{pmatrix} V_p \\ V_{nmo} \\ A_4 \end{pmatrix} = F(\theta,\varphi,\nu;\epsilon,\delta) + F(\theta,\varphi,\nu;0,0) + M(\theta,\varphi,\nu)\begin{pmatrix} \epsilon \\ \delta \end{pmatrix} + O(\epsilon,\delta) \tag{1}$$

Pseudo-inverse transform allows us to find estimates for Thomsen parameters (2).

$$\begin{pmatrix} \epsilon \\ \delta \end{pmatrix} = (M^TM)^{-1}M^T\left[\begin{pmatrix} V_p \\ V_{nmo} \\ A_4 \end{pmatrix} - F(\theta,\varphi,\nu;0,0)\right] \tag{2}$$

However, the aim is to analyze stability of the inverse problem solution. We utilize singular value decomposition for M-matrix and present it as a product of two unitary matrices and diagonal matrix with singular values (3).

$$M = U \begin{pmatrix} \sigma_1 & \\ & \sigma_k \end{pmatrix} V^T,\ UU^T = I,\ VV^T = I \tag{3}$$

To characterize inversion stability of the problem it is convenient to use a so called condition number (4) which is a ratio of maximal to minimal singular value. The smaller the condition number the more stable the solution is. Analysis of the condition number behaviour was performed for VTI and TTI models. In case of VTI media we considered two cases of narrow- and wide-azimuth data. The reflection seismic data is considered to be measured in dip and strike directions respectively. It was found that inverse problem stability decreases while reflector dip angle increases for both cases. It is reasonable to combine both datasets to constrain the solution. Utilizing the same approach for TTI medium, we studied the condition number behaviour for wide range of angles θ, φ and ν. This approach can be used to suggest acquisition design to obtain seismic data which can be inverted yielding the most accurate estimates for anisotropic parameters. In particular, checkshot along the deviated well often provides more information for inversion compared to vertical checkshot data.

VTI modeling for TTI medium

Another question we have considered in this work was a possibility and validity of applying VTI model to fit traveltimes obtained for TTI model. We were interested in how far the obtained Thomsen parameters will move from their true values. We considered a model with vertical well here. Technically analysis is made in the following way: we define ϵ, δ and ν in TTI media, calculate phase and NMO velocity and quartic coefficient and use this data as input data for inverse problem where we set ν equal to zero. The principal calculation part is expressed in formula (4). Here ϵ and δ – Thomsen parameters in TTI media, M_ν and M_0 – Frechet derivatives for TTI and VTI models respectively, ϵ_0 and δ_0 – Thomsen parameters for VTI model with the same seismic response as TTI model.

$$\begin{pmatrix} \epsilon_0 \\ \delta_0 \end{pmatrix} = (M_0^T M_0)^{-1} M_0^T M_\nu \begin{pmatrix} \epsilon \\ \delta \end{pmatrix} \tag{4}$$

Several models were examined with different reflector dip angles and tilt angles. It appeared that while anisotropy remains close to elliptical ($\epsilon = \delta$), the relative error for ϵ_0 and δ_0 remains rather small even for large values of tilt angle ν. However if one of Thomsen parameters in TTI media is much greater than another the result of TTI seismic data inversion using VTI model will be unsatisfactory. In addition relative error for δ_0 proved to be more sensitive to dip angle changes compared to relative error for ϵ_0.

Problem of electromagnetic wave diffraction on the screen and transmission problem

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Waveguide structure is considered. One part of this structure is filled with one homogenius isotropic medium, the other part — with the other medium. There is a metal screen (α, β) at the junction of two dielectrics. The plane waveguide and plane are considered.

The diffraction problem of TE-polarized electromagnetic wave, which is falled on thin metal screen, is well studied. In diffraction problem the scaterring field $\overrightarrow{v}\,(x,z)$ and $\overleftarrow{u}\,(x,z)$ which will satisfy the conditions at infinity, are required to be calculated by an incident field $\overrightarrow{u}\,(x,z)$. The potential functions $\overrightarrow{u}\,(x,z)$, $\overleftarrow{u}\,(x,z)$ are the solution of Helmholtz equation at $z < 0$. All components of the electromagnetic waves propagating at $z<0$ to the left and right can be expressed through these functions respectively. All components of the electromagnetic waves propagating at $z>0$ to the right can be expressed through potential function $\overrightarrow{v}\,(x,z)$.

The ability to restore electromagnetic field at $z<0$ ($\overrightarrow{u}\,(x,z)$ and $\overleftarrow{u}\,(x,z)$ functions) by field at $z>0$ (function $\overrightarrow{v}\,(x,z)$) is investigated. We call this problem the transmission problem, inverse to diffraction problem.

The transmission problem is ill-posed. Additional conditions are necessary to obtain a unique solution of this problem.

In the plane waveguide unknown functions are sought as Fourier series by eigen functions $\{\varphi_n(x)\}$, $n=1,2,\ldots$. Trace $\overrightarrow{u}_0\,(x)$ of the function $\overrightarrow{u}\,(x,z)$ for $z\to 0-0$ satisfies $\int_0^h \overrightarrow{u}_0\,(x)\varphi_n(x)dx=0$, for $n>N_1$. By computer experiment it was shown, that when $(\beta-\alpha)<h/2$ the field $\overrightarrow{u}\,(x,z)$ and $\overleftarrow{u}\,(x,z)$ accurately restored by $\overrightarrow{v}\,(x,z)$.

On the plane the transmission problem is solved by the same way.

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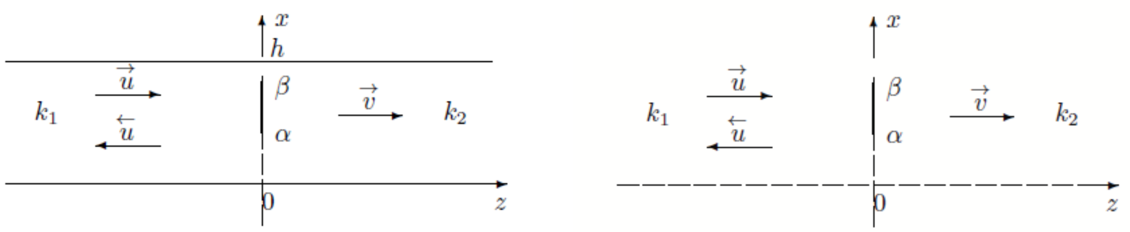

Scale-dependent corrections to Casimir effect

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The Casimir force, which attracts to each other two perfectly conducting parallel plates separated by the distance a in vacuum, is one of the blueprints of the reality of vacuum fluctuations. Following the recent conjecture, that quantum fields should be described in terms of the fields depending on the resolution of measurement (M. V. Altaisky, Phys. Rev. D81(2010)125003), we derive the correction to the Casimir energy depending on the ratio of the plate displacement amplitude to the distance between plates. The effects of random environments to the measurement are also discussed.
Reference: arXiv:1103.6018

High frequency diffraction by elongated bodies of revolution

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High frequency electromagnetic field diffracted by an elongted body is characterized by much smaller attenuation than predicted by V.A. Fock asymptotics. Accurate description of such fields

requires the transverse curvature of the surface to be taken into account. Asymptotic approach of [1], [2] which shows very good agreement with numerical test results for perfectly conducting spheroids, is extended to take into account the back going wave produced by creeping modes that are reflected by the shadowed extremity of the spheroid.

The same approach can be used for some other canonical shapes such as paraboloids, hyperboloids and cones.

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Modeling and analysis of wave scattering and generation on cubically polarisable layered structures

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In this paper we investigate the problem of scattering and generation of waves on an isotropic, non-magnetic, linearly polarised (E-polarisation), non-linear, layered, cubically polarisable, dielectric structure, which is excited by a packet of plane waves, in the domain of resonance frequencies. We consider wave packets consisting of both strong electromagnetic fields at the excitation frequency of the non-linear structure, leading to the generation of waves, and of weak fields at the multiple frequencies, which do not lead to the generation of harmonics but influence on the process of scattering and generation of waves by the non-linear structure. The electromagnetic waves for a non-linear layer with a cubic polarisability of the medium can be described by an infinite system of non-linear boundary-value problems. In the study of particular non-linear effects it proves to be possible to restrict this system to a finite number of problems, and also to leave certain terms in the representation of the polarisation coefficients, which characterize the physical problem under investigation [1–3]. The analysis of the quasi-homogeneous electromagnetic fields of the non-linear dielectric layered structure made it possible to derive a condition of phase synchronism of waves. If the classical formulation of the problem is supplemented by the condition of phase synchronism, we arrive at a rigorous formulation of a system of boundary-value problems with respect to the components of the scattered and generated fields [1]. This system is transformed to equivalent systems of non-linear problems, namely a system of one-dimensional non-linear Fredholm integral equations of the second kind and a system of non-linear boundary-value problems of Sturm–Liouville type. For each of these problems we obtain sufficient conditions for existence and uniqueness of the solution. The solutions of these problems are approximated numerically by the help of a quadrature method. The numerical algorithms of the solution of the non-linear problems are based on iterative procedures which require the solution of a linear system in each step. In this way the approximate solution of the non-linear problems is described by means of solutions of linear problems with an induced non-linear dielectric permeability. The analytical continuation of these linear problems into the region of complex values of the frequency parameter allows us to move towards the analysis of spectral problems. That is, we wish to find the eigenfrequencies and the corresponding eigenfields of the homogeneous linear problems with the induced non-linear dielectric permeability. The eigenfrequencies of the corresponding spectral problems form a discrete, countable set of points, with the only possible accumulation point at infinity, and lie on a complex two-sheeted Riemann surface. In the frequency domain, the resonant scattering and generation properties of non-linear structures are determined by the proximity of the excitation frequencies of the nonlinear structures to the complex eigenfrequencies of the corresponding homogeneous linear spectral problems with the induced non-linear dielectric permeability of the medium.

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Scattering and generation properties of cubically polarisable layers with negative and positive cubic susceptibility of the medium

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In this paper results of calculations of characteristics of the scattered field of a plane wave are presented, taking into account the third harmonic generated by non-linear cubically polarisable layers with both negative as well as positive values of the cubic susceptibility of the medium. Within the framework of the closed system, which is given by a system of self-consistent boundaryvalue problems, we show the following. The variation of the imaginary part of the permittivity of the layer at the excitation frequency can take both positive and negative values along the height of the non-linear layer [1–3]. It is induced by the non-linear part of the permittivity and is caused by the loss of energy in the non-linear medium (at the frequency of the incident field), which is spent for the generation of the electromagnetic field of the third harmonic (at the triple frequency). The magnitude of this variation is determined by the amplitude and phase characteristics of the fields which are scattered and generated by the non-linear layer.

It is shown that layers with negative and positive values of the coefficient of cubic susceptibility of the non-linear medium have fundamentally different scattering and generation properties in the domain of resonance. For instance, in the case of negative values of the susceptibility, a decanalisation of the electromagnetic field can be detected. With the increase of intensity of the incident field, the maximal variations of the reflection and transmission coefficients can be observed in the vicinity of the normal incidence of the plane wave [3]. A previously transparent structure becomes semi-transparent, and the reflection and transmission coefficients become comparable. For the layer considered here, the maximal portion of the total energy generated in the third harmonic is observed in the direction normal to the structure and amounts to 3.9% of the total dissipated energy. For a layer with a positive value of the susceptibility an effect of energy canalisation is observed. The increase of intensity of the incident field leads to an increase of the angle of transparancy which increasingly deviates from the direction normal to the layer and which is responsible for a reflection coefficient close to zero. In this case, the maximal portion of energy generated in the third harmonic is observed near the angle of transparency of the non-linear layer. In the numerical experiments there have been reached intensities of the excitation field of the layer such that the relative portion of the total energy generated in the third harmonic is 36%. The paper also presents results of numerical calculations that describe properties of the non-linear dielectric permeabilities of the layers as well as their scattering and generation characteristics. The tests are illustrated by figures showing the dependence on the amplitudes and the angles of incidence of the plane wave for layers with negative and positive values of the coefficient of the cubic susceptibility of the non-linear medium.

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Towards an ultimate modified Smyshlyaev formula for the problem of diffraction by a quarter-plane

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The rehabilitation of the open-rotor engine leads to some noise concerns. In this talk, we will try to have a better understanding of the noise generated by the tip of the second contra-rotating propeller. In order to do so, we will explore a new technique developed by A.V. Shanin in 2005 to tackle diffraction problems. Our interest will be focused especially on the diffraction of acoustic waves by a quarter-plane. Shanin's theory, based on embedding formulae, the acoustic uniqueness theorem and spherical edge Green's functions, leads to three modified Smyshlyaev formulae, which partially solve the far-field problem of scattering of an incident plane wave by a quarter-plane in the Dirichlet case. In this talk, we will also present similar formulae in the Neumann case, and describe a numerical method allowing a fast computation of the diffraction coefficient using Shanin's third modified Smyshlyaev formula. The method requires knowledge of the eigenvalues of the Laplace-Beltrami operator on the unit sphere with a cut, and we also describe a way of computing these eigenvalues. Numerical results are given for different directions of incident plane wave in the Dirichlet and the Neumann cases, emphasising the superiority of the third modified Smyshlyaev formula over the other two. Finally, current work on the derivation of an ultimate modified Smyshlyaev formula, solving the far-field problem everywhere, shall be presented.

Diffraction a plane wave by a transparent wedge. Calculation of the diffraction coefficients of wave scattered by vertex of the wedge

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The problem of diffraction a plane wave by a transparent wedge is considered. We consider two dimensional scalar problem. The wave process is described by the classical Helmholtz equations. The wave velocities inside and outside the wedge are different. Conjugation boundary condition take place on the side of the wedge. The solution satisfies the radiation conditions on the infinity. The Meixner condition is true in the neighborhood of the vertex of the wedge. We suppose that a plane wave illuminates both sides of the wedge.

We seek the solution as a sum of layer potentials, where densities belong to a special class. This problem results in obtaining Fourier transform of the densities from integral equations system. This system can be solved numerically by using collocation method. Diffraction coefficients of the wave scattered by the vertex of the wedge can be present easy via Fourier transform of the densities. To calculate diffraction coefficients we need to analytically extend the solution into the domain where singularity is possible. The functional equation allows us to make this extent. Diffraction coefficients are calculated in a way, which is similar to the one from the book J.-P.Croisille, G.Lebeau [1].

Other approaches to the similar problem are considered in the books [2], [3] and paper [4].

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Even order periodic operators on the real line

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We consider the self-adjoint $2p \geq 4$ order differential operator $(-1)^p d^{2p} dt^{2p} + \sum_{j=0}^{p-1} d^j dt^j q_{j+1} d^j dt^j$, acting in $L^2(\mathbb{R})$, with 1-periodic coefficients $q_j \in L^1_{real}(0,1), j = 1, 2, ..., p$. The spectrum of this operator is absolutely continuous and is a union of spectral bands separated by gaps. We define a Lyapunov function, which is analytic on a p-sheeted Riemann surface. The Lyapunov function has real or complex branch points. We prove the following results: (1) The spectrum at high energy has the multiplicity two. (2) Endpoints of all gaps are 2-periodic (i.e. periodic or anti-periodic) eigenvalues or the real branch points. (3) The spectrum of the operator has an infinite number of open gaps and there exists only a finite number of the non-real branch points for some specific coefficients (the generic case). (4) The asymptotics of the periodic, anti-periodic spectrum and branch points are determined at high energy.

Typical applications of our operator are vibrations of beams, plates and shells. Moreover, higher order differential operators arise in the inverse problem method of integration of non-linear evolution equations. There exist the Lax pairs, where the self-adjoint operator is a higher order operator and the corresponding non-linear Lax equation is integrable by the inverse problem method. Many physically interesting equations have this form, see [AC].

The spectral theory for higher order operators with decreasing coefficients is well developed, see [BDT] and the references therein. The results for higher order periodic operators are still modest.

We describe our goal. In the case $p = 1$ the spectrum of the Hill operator $-d^2dt^2 + q_1$ is a union of spectral bands, where all endpoints of the bands are the 2-periodic eigenvalues of the equation $-y'' + q_1y = ly$. In the case $p = 2$ the spectrum is also a union of the spectral bands, but all endpoints of the bands are the 2-periodic eigenvalues of the equation $y'''' + (q_2y')' + q_1y = ly$ or the branch points of the Lyapunov function [BK]. Until now for the case $p > 2$ there are no any results about the multiplicity of the spectrum at high energy, number of the gaps (is it finite or infinite ?), asymptotics and type of endpoints of the gaps at high energy etc. Our main goal is to answer some of these questions.

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Schrödinger operator on armchair nano-tube

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We are interested in the Schrödinger operator on a specific periodic graph. From the physics point of view our operator can be considered as a quantum network model Hamiltonain for π-electrons in armchair single-wall carbon nano-tubes. The spectrum of this operator consists of an absolutely continuous part (intervals separated by gaps) plus an infinite number of eigenvalues with infinite multiplicity. We describe all eigenfunctions with the same eigenvalue. We define a Lyapunov function which is analytic on some Riemann surface. On each sheet, the Lyapunov function has the same properties as in the scalar case, but it has ramification points. We describe the absolutely continuous spectrum of the Schrödinger operator: 1) the multiplicity, 2) endpoints of the gaps, they are given by periodic or antiperiodic eigenvalues or the ramifications. We determine the asymptotics of the gaps at high energy.

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Characterization of dynamical data in one-dimensional BC-method

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Let $u(x,t)$ be the (generalized) solution of the inhomogeneous string equation $\rho u_{tt} - u_{xx} = 0$ for $x > 0,\ t > 0$ with a smooth positive function $\rho = \rho(x)$, provided $u|_{t=0} = u_t|_{t=0} = 0$ for $x \geq 0$ and $u|_{x=0} = \delta(t)$. The representation

$$u_x(0,t) = \alpha\delta'(t) + \beta\delta(t) + r(t)\,, \qquad t \geq 0$$

holds, where δ is the Dirac delta-function, $\alpha := \sqrt{\rho(0)}, \beta := -\frac{1}{4}\frac{\rho'(0)}{\rho(0)}$; the function $r(t)$ is said to be a reply function of the string.

Define a function $\tau(x) := \int_0^x \sqrt{\rho(s)}\,ds$; let $x(\tau)$ be its inverse function.

The dynamical inverse problem is: *for a fixed* $T > 0$, *given* $r|_{[0,2T]}$ *to recover* $\rho|_{[0,x(T)]}$.

Introduce a family of operators $\{C^\xi\}_{\xi\in(0,T]}$ acting in $L_2(0,T)$ by

$$(C^\xi f)(t) \;=\; \alpha f(t) + \int_0^\xi \left[\frac{1}{2}\int_{|t-s|}^{2\xi-t-s} r(\eta)\,d\eta\right] f(s)\,ds\,, \qquad 0 \leq t \leq \xi\,;$$

let $f = f^\xi(t)$ be the solutions of the integral equations

$$(C^\xi f)(t) \;=\; -\alpha - \beta(\xi - t) + \int_t^\xi (\xi-\eta) r(\eta)\,d\eta\,, \qquad 0 \leq t \leq \xi\,. \tag{1}$$

We show that a set $\{\alpha, \beta, r|_{[0,T]}\}$ is the data of a string *if and only if*

- $(C^T g, g)_{L_2(0,T)} > 0$ for all nonzero $g \in L_2(0,T)$
- the solutions of the equation (1) satisfy $f^\xi(0) \neq 0$.

Under these conditions, the density is recovered by $\rho\,(x(\xi)) = \alpha[f^\xi(0)]^{-4}$. Such a characterization is a relevant version of the classical result by A.S.Blagoveschenskii [2], [3], the version being derived in the framework of the BC-method [1]. Note that in [1] (Theorem 3) the condition $f^\xi(0) \neq 0$ is missed by mistake.

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Andreev reflection and the semiclassical Bogoliubov–de Gennes Hamiltonian: resonant states

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Bogoliubov–de Gennes Hamiltonian is a 2×2 matrix Hamiltonian $\mathcal{P}(x, hD_x)$ which describes the dynamics of a pair of quasi-particles (hole/electron) in a 1-D metallic lead connecting 2 superconducting contacts. Diagonal terms are of the form $\pm(\xi^2 + \mu(x))$, where $\mu(x)$ stands for the chemical potential, while the off-diagonal interaction with the supraconducting bulk is modeled through a complex potential, or superconducting gap, $\Delta_0 e^{i\phi_\pm/2}$ at the boundary $\pm L$; due to the finite range of the junction, this interaction continues to a function $x \mapsto \Delta(x)e^{i\phi(x)/2}$, which vanishes rapidly inside the interval $[-L, L]$. In Bogoliubov-de Gennes Hamiltonian we ignore the self-consistency relations and consider $\Delta(x)$ as an "effective potential".

An electron e^- moving in the metallic lead with energy $E \leq \Delta$ (measured with respect to Fermi level E_F) and kinetic energy $K_+(x) = \xi^2 = \mu(x) + \sqrt{E^2 - \Delta(x)^2}$ is reflected back as a hole e^+ from the supraconductor, injecting a Cooper pair into the bulk.

When $\inf_{[-L,L]} \mu(x) \geq E$, and $\phi \neq 0$, this process yields so called phase-sensitive Andreev states, carrying supercurrents proportional to the ϕ-derivative of the eigen-energies $E_k(h)$ of $\mathcal{P}(x, hD_x)$.

Extending previous results by N. Chtchelkatchev, G. Lesovik, and G. Blatter [1] in the case where $\Delta(x)$ is a "hard-wall" potential, we derived in [2] semi-classical quantization rules for Andreev states near energy E, from a microlocal study of the Hamiltonian in the "inner region" $\Delta(x) \leq E$ alone.

Here we want to take also into account the "outer region" $\Delta(x) \geq E$ of the junction, entering the supraconducting bulk ; to this end, we make the assumption that the junction is extended, and the quasi-particle turns into a resonant state before creating a new Cooper pair, so that the dynamics is still governed by Bogoliubov–de Gennes Hamiltonian. As a matter of fact, the microlocal solutions, purely oscillating in $\Delta(x) \leq E$, acquire a complex phase in $\Delta(x) \geq E$, giving raise to a small complex correction to the real eigen-energies $E_k(h)$ of $\mathcal{P}(x, hD_x)$. This is of course related to phase-space tunneling, though the complex gap $\Delta(x)$ is "transparent" to the wave function.

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Method of discrete sources for elliptic problems arising from collider simulation. Numerical aspects

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An electron accelerating section of a supercollider is synthesized. A proper structure might represent a periodic set of coaxial radial-corrugated discs, which compose a Bragg reflection cavity [1]. The principal problem is to optimize parameters of the system considered. The aim is to accumulate minimum of the RF energy in a paraxial domain under a given accelerating gradient. The principal problem generates a lot of particular ones (boundary, inverse spectral, scattering) [2], [3].

For 3D azimuthally symmetrical case all model problems are governed by a scalar equation of Helmholtz type for the azimuthal component of a magnetic field. Both the longitudinal component of an electric field and the radial one are expressed in terms of the azimuthal component of a magnetic field. Boundary conditions reflect the absence of the current on a metallic surface as well as some periodicity and symmetry of the structure. One more boundary condition depends on the certain problem.

To solve the Helmholtz type equation the method of discrete sources is used. The way of source placement (both on smooth parts of the boundary and in a neighborhood of any edge) is suggested. Moreover, two sequences of source placements ensuring convergence for smooth boundary profiles are discovered. The convergence is verified using a test scattering problem.

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Differential equations for the simplest N-symmetric generalized Chebyshev polynomials

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Investigation of the simplest N-symmetric generalized Chebyshev polynomials is continued. These polynomials constitute the basis of composite model for generalized Chebyshev oscillator generated by N-periodic Jacobi matrix. This model was discussed in our talk on DD-2010. In previous works we consider in some details the cases $N = 2, 3, 4, 5$ and shaw that such polynomials are absent for $N \geq 6$. The polynomials of considered type are orthogonal on some subsets in complex plane which are symmetric with respect to rotations on the angle $2\pi/N$. These systems of polynomials splits into N series of polynomials connected with generalized Chebyshev oscillators.

In reported work we obtain second order differential equations for the simplest 3-symmetric generalized Chebyshev polynomials. These equations have polynomial coefficients which are different for different series.

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Quantum dynamics of particles with position dependent mass via Feynman formulae

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We review a new mathematical method to describe the dynamics of quantum (quasi) particles whose mass is variable. The considered results can be useful for investigations of inhomogeneous semiconductor devices and liquid crystals. This method also allows to describe quantum evolution on curved spaces, which can be useful for the investigation of nonplanar nano-structures.

We consider a class of evolutionary equations with position dependent coefficients that describe either a diffusion or a quantum evolution of (quasi)particles with position dependent masses. We represent some solutions of Cauchy–Dirichlet and Cauchy problems for such equations in bounded and unbounded domains of Euclidean spaces and Riemannian manifolds in the form of a limit of finite dimensional integrals.

Such representations are called the Feynman formulae (cf. [6]). The limits in the Feynman formulae coincide with integrals with respect to suitable measures or pseudomeasures on a set of functions taking values either in the configuration space or in the phase space of the corresponding physical system. These integrals are usually called either the Feynman path integrals, or functional integrals, or integrals over trajectories (in the configuration and in the phase space respectively).

The representations of solutions of evolutionary equations by functional integrals are usually called Feynman–Kac formulae. Hence, getting Feynman formulae is one of the ways to get Feynman–Kac formulae. The significance and importance of the Feynman–Kac formulae are due to several reasons: they allow to connect investigating evolutionary equations with stochastic analysis, they are convenient for obtaining quasiclassical asymptotics, they help to investigate some geometrical and transformation properties of mathematical models in quantum field theory etc.

On the other hand, Feynman formulae give approximations for integrals from Feynman–Kac formulae (and hence for solutions for the corresponding evolutionary equations, or, what is the same, for the semigroups resolving the equations) by some relatively simple integrals, containing only elementary functions. (The direct calculation of integrals in the corresponding Feynman–Kac formulae usually leads to some limits of finite dimensional integrals, containing either transitional probabilities or transitional amplitudes which are not expressible through elementary functions.) Just this situation takes place in the case of evolution in domains with boundary conditions, or evolution on manifolds, or evolution of (quasi) particles with variable mass. Hence, Feynman formulae can be useful for direct calculations and simulations of the described dynamics.

We refer to the papers [1]–[8] for the detailed discussion.

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Homogenisation in finite elasticity for composites with a high contrast in the vicinity of rigid-body motions

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A multiscale asymptotic framework is proposed for analysing the overall behaviour of high-contrast nonlinear periodic elastic composites in the regime when the deformations are situated in the vicinity of a rigid-body motion. The corresponding minimising sequences are shown to retain the multiscale behaviour in the homogenisation limit, which is determined rigorously via a new multiscale version of Γ-convergence.

High-frequency spectral analysis of thin periodic acoustic strips

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The asymptotic behaviour of the high-frequency spectrum of the wave equation with periodic coefficients in a "thin" elastic strip $\Sigma_\eta = (0,1) \times (-\eta/2, \eta/2)$, $\eta > 0$, is discussed. The main geometric assumption is that the structure period equals the strip thickness η and is chosen in such a way that η^{-1} is a large positive integer. On the boundary $\partial\Sigma_\eta$ we set Dirichlet (clamped) or Neumann (traction-free) boundary condition. Aiming to describe sequences of eigenvalues of order η^{-2} in the above problem, which correspond to oscillations of high frequencies of the order η^{-1}, we study an appropriately rescaled limit of the spectrum. Using suitable a suitable notion of two-scale convergence for bounded operators acting on two-scale spaces, we show that the limiting spectrum consists of two parts: the Bloch (or band) spectrum and a "boundary" spectrum. The latter corresponds to sequences of eigenvectors concentrating on the vertical boundaries of Σ_η, and is characterised by a problem set in a semi-infinite periodic strip with either clamped or stress-free boundary conditions.

Energy control for Frenkel–Kontorova model

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The Frenkel–Kontorova coupled oscillator system [1] is a fundamental model in patterning, synchronization [2] and solid state theory, this system exhibits a number of interesting phenomena, in particular, soliton motion. In this paper we investigate the energy stability of this system under random fluctuations.

Let us consider a system of $N >> 1$ coupled oscillators defined by the following equations [3, 4]:

$$\ddot{\phi}_i + {\omega_0}^2 \sin\phi_i = d\triangle\phi_i + \delta_{i1}u(t) + \alpha_i\xi(t), \tag{1}$$

where $i = 1, 2, 3, ..., N$, $\triangle$ is a difference diffusion operator, defined by $(\triangle\phi)_i = \phi_{i+1} - 2\phi_i + \phi_{i-1}$, $\xi(t)$ is a noise, u is a control acting only on the first oscillator. If $d = 0$, the oscillators are independent. Let $\xi(t)$ be a Markov random process simulating fluctuations. We assume that the coefficients α_i are random parameters: $\alpha_i = \bar{\alpha} + R\tilde{\alpha}_i$, $\tilde{\alpha}_i$ are random numbers from $[-1, 1]$ with zero mean, R is a parameter. For $d = 0, u = 0$ and $\alpha_i = 0$ the energy of the system is a sum $E(\phi) = \sum_{i=1}^N E_i(\phi)$, where

$$\begin{cases} E_i(\phi, \dot{\phi}) &= 0.5\dot{\phi}_i^2 + {\omega_0}^2(1 - \cos\phi_i) + 0.5k(\phi_{i+1} - \phi_i)^2 \\ E_N(\phi, \dot{\phi}) &= 0.5\dot{\phi}_i^2 + {\omega_0}^2(1 - \cos\phi_N) + 0.5k(\phi_1 - \phi_N)^2, \end{cases} \tag{2}$$

We use the control to synchronize our cyclic oscillator chain on some time interval $[0, T]$. Let us define the control target function:

$$Q_E(\phi) = 0.5(E(\phi, \dot{\phi}) - E^*)^2, \tag{3}$$

where $E(\phi(t))$ is the system energy, E^* is a prescribed energy level. The speed gradient method for the control, proposed by [4] for the target function Q_E, is defined by

$$u(t) = -\gamma(E(\phi, \dot{\phi}) - E^*)\dot{\phi}_1(t)H(T - t). \tag{4}$$

Also we use a hybrid control of the following form

$$u(t) = -\gamma sign((E(\phi, \dot{\phi}) - E^*)\dot{\phi}_1(t))H(T - t), \tag{5}$$

where H is a Heaviside step function. Such a control on $[0, T)$ allows us to create a stable oscillator state with the prescribed energy E^*. On $[T, \infty)$ the noise $\xi(t)$ can destroy this stable state.

We compare speed gradient control and hybrid control by numerical simulations and some analytical methods. We have studied our system under a noise environment and with dissipative effects. We have concluded that the hybrid control is a more effective for a support of prescribed energy level than the speed gradient control.

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The new laws of diffuse scattering and oscillations of indicatrix envelope in the short-wavelength limit

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Solution of the Rayleigh wave [1] scattering problem in the Born (Rayleigh–Born [2]) approximation of perturbation theory (in roughness amplitude) for three-dimensional deterministic roughness $x_3 = f(x_1, x_2) = \delta_0 f_0(x_{\shortparallel})$ of isotropic solid [3, 4] is considered in the short-wavelength limit $d/\bar{\lambda} \gg 1$, i.e. for the diffuse scattering (d is radius of the surface rough part, $\bar{\lambda} = \lambda/(2\pi)$ – wavelength). Asymptotic formulas for the displacement field in the scattered Rayleigh waves are derived.

The new laws of diffuse scattering $d/\bar{\lambda} \gg 1$ are obtained: a frequency dependence of the scattering indicatrix envelope is determined by the roughness form

$$I^{(R)}_{\shortparallel,3} \sim (k_R d)^{2-2n} \quad , \quad n = 0, 1, 2, 3, \; \ldots \; , \tag{1}$$

where $I^{(R)}_{\shortparallel,3}$ is indicatrix of scattering [3, 4], $k_R = \omega/c_R$ — absolute value of the wave vector, ω — frequency, c_R — velocity of the Rayleigh wave. The integer n is determined by the roughness form. Known classical law [1] of diffuse scattering gives for the scattering indicatrix envelope $I^{(R)}_{\shortparallel,3} \sim (k_R d)^2$ (Fig. 1 — classical law; Fig. 2 — one of the new laws (1) $I^{(R)}_{\shortparallel,3} \sim (k_R d)^{-2}$).

Appropriate superposition of the roughnesses with definite forms, corresponding to the new laws (1), leads to the oscillations of scattering indicatrix envelope in the short-wavelength limit $d/\bar{\lambda} \gg 1$, i.e. for the diffuse scattering (Figs. 3, 4).

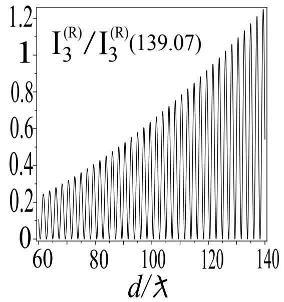

Fig. 1

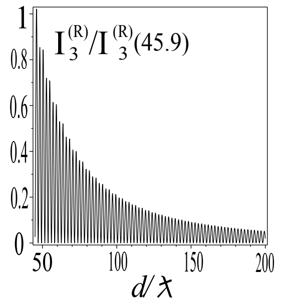

Fig. 2

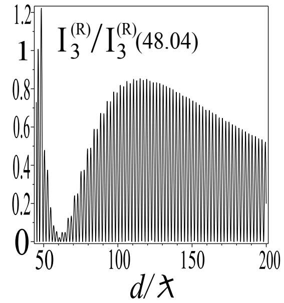

Fig. 3

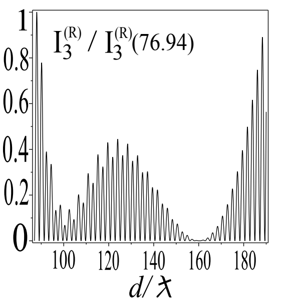

Fig. 4

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On violation of Bragg law of scattering

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Solution of Rayleigh wave [1] scattering problem by deterministic three-dimensional continuous roughness with cylindrical symmetry, occupying finite size region of homogeneous isotropic solid surface is considered in Rayleigh–Born approximation of perturbation theory in roughness amplitude at big distances from rough region.

The next two type of scattering are investigated.

1. Classical diffuse scattering $d/\bar{\lambda} \gg 1$, where d is radius of rough region of surface, $\lambda = 2\pi\bar{\lambda}$ is wavelength. In this scattering indicatrix envelope is proportional to the second power of the Rayleigh wave frequency

$$I^{(R)}_{\shortparallel,3} \sim (k_R d)^2 \ , \tag{1}$$

where $I^{(R)}_{\shortparallel,3}$ is indicatrix of scattering [2], $k_R = \omega/c_R$ – absolute value of the wave vector, ω – frequency, c_R – velocity of the Rayleigh wave, and the Bragg law of scattering is valid: positions of scattering indicatrix zeroes

$$k_R d\sqrt{2(1-\cos\varphi_s)} = \pi\left(m+\frac{1}{4}\right) \ , \quad m = 0, 1, 2, 3, \ \ldots \tag{2}$$

and maxima

$$k_R d\sqrt{2(1-\cos\varphi_s)} = \pi\left(m+\frac{3}{4}\right) \ , \quad m = 0, 1, 2, 3, \ \ldots \tag{3}$$

are defined only by Bragg parameter $k_R d$ and by the angle of scattering φ_s. Amplitudes of the maxima are defined by the amplitude of the roughness in point $x_{\shortparallel}/d = 1$, besides $k_R d$ and φ_s. All these physical values do not depend on roughness form (Fig. 1, curve 1, $F(x_{\shortparallel}/d)$ – function, describing roughness profile; Fig. 2).

2. Diffuse scattering $d/\bar{\lambda} \gg 1$ corresponding to the new laws, when frequency dependence of scattering indicatrix envelope is defined by the roughness form

$$I^{(R)}_{\shortparallel,3} \sim (k_R d)^{2-2n} \quad , \quad n = 0, 1, 2, 3, \ \dots \ , \tag{4}$$

where the integer n is determined by the roughness form.

It is obtained that appropriate superposition of surface roughnesses of definite form, each of which corresponds to the one of new laws of diffuse scattering (4) [2] can violate the Bragg law of scattering. That is, positions of any finite number of scattering indicatrix zeroes and maxima as well as amplitudes of these maxima are defined by roughness form at constant roughness amplitude, but not only by Bragg parameter $k_R d$ and angle of scattering φ_s (Fig. 1, curves 2, 3; Fig. 3, Fig. 4). The number of such extraordinary scattering indicatrix zeroes and maxima is defined by roughness form as well. Positions and amplitudes of remaining infinite number of scattering indicatrix zeroes and maxima for such superposition of roughnesses are defined by the Bragg law of scattering (2), (3).

It is obtained that reduction and straightening of Bragg oscillations are possible (Fig. 5, Fig. 6).

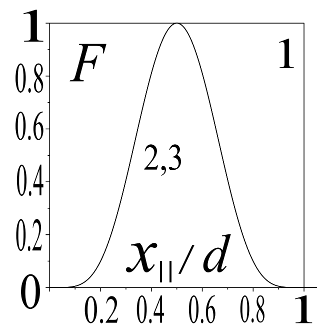

Fig. 1

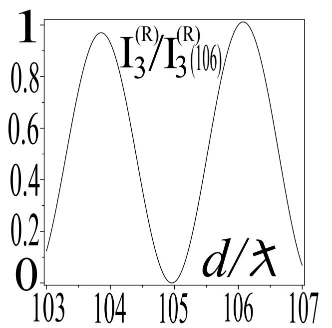

Fig. 2

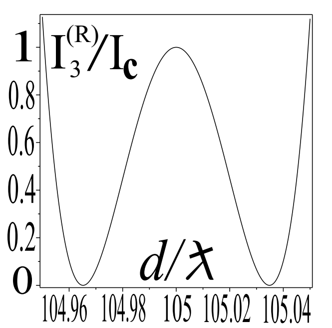

Fig. 3

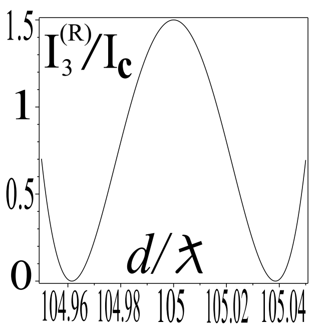

Fig. 4

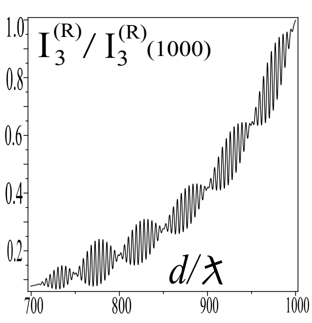

Fig. 5

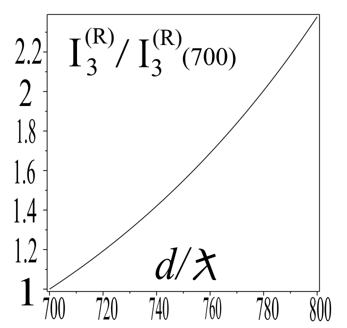

Fig. 6

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1-D linear and nonlinear run-up problem: focal points in nonlinear wave equation with degenerating velocity

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We discuss the exact and asymptotic solutions of the Cauchy problem for 1-D Shallow water equation on the half of axis $x > 0$ with variable velocity $c(x) > 0$, such that for small x $c \sim \sqrt{x}$. The initial data have $au((x-a)/h)$, where $u(y)$ is a decaying function as $|y| \to \infty$, $a > 0$, $h << 1$. We give our interpretation of results of Stoker, Carier and Greenspane, Pelinovski and Mazova, based on focal points appearance, and also construct new exact and asymptotic solutions of the original problem. This work was done together with B.Tirozzi.

Asymptotic solutions of 3-D Laplace equation in the layer with fast oscillating boundary.

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We consider emission effects for a massive of nanotubes with a given potential drop. The massive is a close-packed set of nanotubes in $\mathbf{R}^3 = (x_1, x_2, z)$ space. Nanotubes are oriented along the z axis. Length of nanotubes varies, so the lower ends of nanotube are fixed on the plate $z = 0$ and the upper ends lie of the surface $z = h \cdot g(x_1/S, x_2/S) > 0$. Emitted electrons goes to a target that is the plate $z = D > 0$. So, we need to calculate an electric field E in the area $\Omega = \Omega_{1x_1x_2} \times [h \cdot g(x_1/S, x_2/S), D] \subset \mathbf{R}^3$. Parameter $h > 0$ characterize the lengths of nanotubes and parameter $S > 0$ characterize distance between them. The both parameters h, S are considered to be small as compared with the distance of the target: $h \sim S \ll D$.

We consider in the layer Ω the following problem for the Laplace equation:

$$\triangle u = 0, \quad u|_{z=h\cdot g(x_1/S, x_2/S)} = 0, \quad u|_{z=D} = U, \tag{1}$$

where $u(x_1, x_2, z)$ is a scalar potential of electric field $E = \mathrm{grad}\ u$; U is a certain parameter; $h, S \ll D$ are parameters. Function g defines the array of nanotubes, and their width d is considered to be small as compared with their height $d \ll h$.

The desired quantity is a net current $\mathcal{J}$, which can be found by Fowler-Nordheim formula:

$$\mathcal{J} = \int_{\Omega_{1x_1x_2}} J|_{z=h\cdot g(x_1/S, x_2/S)} dx_1 dx_2, \quad J = C_1 E_z^2 \exp(-\frac{C_2}{E_z}), \tag{2}$$

where $E_z(x_1, x_2, z) = \frac{\partial u(x_1, x_2, z)}{\partial z}$ is the z-component of the electric field $\vec{E} = (E_{x_1}, E_{x_2}, E_z) = \mathrm{grad}\ u$; C_1 and C_2 are physical constants.

We construct an analytical-numerical algorithm for constructing asymptotic solution to (1) and analyze the dependence of the net current (2) on nanotubes width d and distance S between them.

The abnormal electromagnetic skin effect and fields diffraction in fractal medium

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In this presentation we discuss the geometric and physical aspects of the so-called $\underset{\cdot}{\alpha}$-characteristics for studying the behavior of electromagnetic field diffraction components and the abnormal skin effect in the medium with fractal properties. The mathematical techniques developed in [2], [3] are also applicable to the problems considered in [1].

The select of adequate physical and mathematical model of a fractal structure of a medium allows to consider interaction of the structured substance with an electromagnetic field, interaction with loops, surfaces and skew fields is artificial the constructed radiants of a field.

We have received the Maxwell equations in the terms of α - forms in fractal medium

$$d^{\alpha}\vec{E}^{(\alpha)} = -\frac{\partial}{\partial t}\vec{B}^{(\alpha)} - \vec{j}_m^{(\alpha)}, d^{\alpha}\vec{H}^{(\alpha)} = \frac{\partial}{\partial t}\vec{D}^{(\alpha)} + \vec{j}_e^{(\alpha)};$$

$$d^{\alpha}\vec{D}^{(\alpha)} = \rho_e^{(\alpha)}, \quad d^{\alpha}\vec{B}^{(\alpha)} = \rho_m^{(\alpha)},$$

are received $\alpha-$characters of the field.

For electromagnetic field modelling in it is fractal the structured nonmagnetic metal we will consider following approximations: $D_r^{\alpha}\vec{B} = D_r^{\alpha}\vec{H}$, displacement current $\partial(D_r^{\alpha}\vec{D}(\vec{r},t))/\partial t$ is much less than conduction current $D_r^{\alpha}\vec{j}$ in wide (to optical) a frequency band, and a requirement of an electrical quasi-neutrality looks like $D_r^{\alpha}(\rho(\vec{r},t)) = 0$.

Using B.Pippard's idea that in the conditions of the abnormal skin effect the basic role in formation of a screening surface current is played by electrons which move under small angles to a surface (and this appearance can be magnified or loosened by fractal structure), we spot a current density in a skin layer as

$$D_z^{\alpha}\vec{j}(z) = \sigma_{eff}\vec{E}(z) = \sigma_{\mu}D_z^{\alpha}\vec{E}(z).$$

To estimate the α-characteristics, possible algorithms are formulated, namely a geometric one involving evaluation of the Hausdorff measure and an analytical algorithm permitting the Hausdorff measure to be evaluated through application of fractional derivatives and integrals. The fractal portion of the contour we approximate to with a segmented line with the links $\Delta x_{i(k)}$ of constant length and the ends lying on the contour (k-number of covering generation). To represent the fractal contour approximately with points of the segmented line, let us cover it with a segmented line with links of a smaller length, $\Delta x_{i(k+1)} < \Delta x_{i(k)}$. Introducing the notation $u = \Delta x_{i(k)}/\Delta x_{i(k+1)}$ and $v = \Delta x_{i(k)}/\Delta x_{i(k+2)}$ we arrive at a functional equation $f(x)f(y/x) = f(y)$ whose smooth solution is unique in the form of a power law, $f(x) = x^{\alpha}$.

Thus, for a task in view solution it is used specificity differinegral models of a fractal medium: such medium is always nonuniform, and the theoretical model in terms α-differinegral, corresponds to shapes of mathematical model for a homogeneous environment (α=0).

We have constructed Green's function for delta-source in an alpha-form and have shown the decision of wave diffraction problem on a fractal artificial barrier.

Examples of the strict decision of a problem of wave diffraction on a cylindrical fractal surface are resulted. The methods presented in this paper enable one to construct asymptotic forms of the eigenfunctions [4].

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Effect of the bed unevenness on soliton propagation in a channel

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Nonlinear waves in wide horizontal channels with uneven bottom are studied. The wave's amplitudes are supposed to be small. The channel bed topography weakly depends on the transversal coordinate. The waves are described by the two-dimensional Boussinesq equations in the form derived in [1]. Stationary solutions in the form of a soliton with a tail of a system of sinusoidal waves are found. The phase velocity of the tail waves equals the soliton velocity while their group velocity is less than the latter. The dependence of the length and amplitude of the tail waves on the width of the channel, soliton amplitude and the degree of the bottom unevenness is investigated. The loss of the soliton energy due to emission of tail waves is estimated.

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SVD of seismic data inversion problem in case of viscoelastic media

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In this work we consider a two-dimensional inverse problem of retrieving elastic properties of the Earth's interior using seismic data recorded on its surface. A common mathematical model used to define this problem is a standard linear system of isotropic elastodynamics satisfied Hooke's law. However, attenuation and dispersion effects, which can not be described by this model, are commonly observed in the recorded seismic wavefields. To explain these effects and to take them into account one need to introduce viscoelasticity in the model and to assume that a stress state in some moment is determined by a deformation "history". Mathematically it is expressed as time-convolution of deformations with some kernel function. In current work we use Generalised Standart Solid model to describe the viscoelastic material and to pass from the system of integro-differential equations to system of differential equations. To simplify the corresponded equations, a $\tau-$method is considered, and attenuation parameters τ^P and τ^S are introduced for pressure and for shear waves correspondingly. So, the ultimate goal of this work is to identify five elastic parameters that define the isotropic elastic medium with attenuation.

To solve this problem we consider a numerical method, in which a misfit between the recorded and calculated data is minimized in L_2 norm[4]. In recent work[3] it was shown, that such approach allows us to identify the required parameters, however, it has several drawbacks ought to be studied. Coupling of parameters[1] is one of the most critical of them. Due to this effect an inhomogeneity in the media with respect to one parameter can wrongly be inverted as a inhomogeneity of another parameter. As a result, a totally incorrect solution may be obtained. Therefore, a choice of the uncoupled parameters should be made prior to developing and implementing of inversion algorithm.

In [4] a method for obtaining the correct parametrization was proposed based on diffraction patterns for point scatterers. These patterns represent amplitudes of scattered waves depending on scattering angles. Differences between diagrams for various parameters indicate that parameters are uncoupled and may be resolved. In this study we use this approach for the case of viscoelastic medium. We obtain that the most acceptable parametrization is the following: density, elastic impedances (IP, IS) and attenuation parameters (τ^p, τ^s). Density perturbation scatterers almost all incidence energy in the forward direction, thus it can not be identified reliably. The diffraction patterns shapes for elastic impedances and for attenuation parameters are similar for the single temporal frequency. Nevertheless, their amplitudes are different when several frequencies are involved, and it may be used, at least theoretically, to resolve the perturbations of different parameters independently.

To study the ability to resolve these parameters in practice, we perform a singular values decomposition analysis (SVD) of the considered problem. We construct the corresponded linear operator explicitly and study the SVD truncation based on a theory of r-solution for compact operators in Hilbert spaces developed in work of Cheverda and Kostin [2]. The r-solution is a projection of the true solution onto right singular vectors corresponded to largest singular values. Number r of the involved singular vectors controls the condition number of the problem and allows us to obtain the solution of the inverse problem with an acceptable accuracy. We note, that this technique may be numerically expensive, and in this work we use it only for study the inverse problem, while other numerical methods and regularization approaches may be used for its direct solution. By constructing the $r-$solutions for different perturbations we confirm, that there is a strong coupling between impedances and attenuation parameters. The coupling effect becomes weaker, when the number r of the considered singular vectors is increased. However, the condition number of the problem is also increased in that case, and therefore the seismic data with very low noise level is required. The broadening of the registered frequency range also mitigates the coupling of the parameters. So, the attenuation parameters may be in principle obtained while inverting the seismic data with very high quality. To apply the least-squares data fitting technique for the less quality data in case of viscoelastic medium, some updates to the standard L_2 minimization ought to be make. Developing of such modifications will be the topic of our future research.

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Dispersion of Elastic Waves in Microinhomogeneous Media and Structures

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The propagation of elastic waves in periodic stratified media with arbitrary local anisotropy and in anisotropic plates and bars inhomogeneous in thickness is considered under the condition that the ratio ε of the scale characterizing the inhomogeneity of the medium or the inhomogeneity of a plate or bar in thickness to the typical wavelength is small. The propagation of long waves is described using the effective averaged equations up to the second order of accuracy in ε, which are derived by the method of two-scale asymptotic expansions in ε. The results of analytic and numerical studies of their principal terms responsible for the dispersion of waves are presented. Generally, for locally anisotropic and inhomogeneous structures both the terms with the third and fourth derivatives of displacements over coordinates and time are present in the equations. The matrix of the coefficients at the third derivatives is antisymmetric. This matrix is equal to zero for some structures, e.g., this is true for two-layer media with arbitrary anisotropy of the layers. However it is not equal to zero, e.g., for two-layer plates even if the layers are homogeneous and isotropic. The form of the dependence of the wave velocity on the wavelength is studied for structures with different types of symmetry analytically and numerically. It is found that for any direction of the waves propagation at least one of the possible type of waves displays negative dispersion, i.e. the waves velocities decrease as their frequencies increase.

Spectrum and eigenfunctions of the operator of an induction of a magnetic field on a two-dimensional surface of rotation

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The magnetic field in a conducting liquid (in particular, magnetic fields of some planets and galaxies) is described by the operator of an induction L:

$$LB = \varepsilon\Delta B + V, B = \varepsilon\Delta B + (V,\nabla)B - (B,\nabla)V,$$

where B — magnetic field, V — field of velocities and ε — small parameter (resistance). We describe the asymptotic of the spectrum and eigenfunctions of this operator on any two-dimensional surface of rotation. Also we describe spatial structure of a magnetic field.

The energy analysis of vibrations problem of different shells

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The problem of oscillations of elastic constructions is one of the actual problems of modern techniques. The vibrations of such objects as different supports, tubes need the perfect analyses and it is important to estimate the parameters of vibrations and acoustical fields in such systems.

It is useful to explore the processes in these systems taking as an example more simple mechanical systems. The cylindrical shells and plates are the examples of these often used models. The problem of the shells oscillations is considered in the rigorous mathematical statement. Boundary contact conditions and energy streams are analyzed.

Generalized modes of optical fiber

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Let the three-dimensional space be occupied by an isotropic source-free medium, and let the refractive index of the fiber be prescribed as a positive real-valued function $n = n(x_1, x_2)$ independent of the longitudinal coordinate x_3 and equal to a constant n_∞ outside a cylinder. The axis of the cylinder is parallel to the x_3–axis, and its cross-section is a bounded domain Ω with a Lipschitz boundary Γ on the plane R^2. Let the function n belongs to the space of real-valued twice continuously differentiable in Ω functions. Denote by n_+ the maximum of the function n in the domain Ω, let $n_+ > n_\infty$. By U denote the space of twice continuously differentiable in Ω and $\Omega_\infty = R^2\backslash\overline{\Omega}$, continuous and continuously differentiable in $\overline{\Omega}$ and $\overline{\Omega}_\infty$ real-valued functions. The eigenvalue problem for generalized natural modes of weakly guiding optical fiber is formulated as the problem for Helmholtz equation with Reichardt condition at infinity in the cross-sectional plane:

$$\left[\Delta + \left(k^2n^2 - \beta^2\right)\right] u = 0, \quad x \in R^2\backslash\Gamma, \tag{1}$$

$$u^+ = u^-, \quad \frac{\partial u^+}{\partial \nu} = \frac{\partial u^-}{\partial \nu}, \quad x \in \Gamma, \tag{2}$$

$$u = \sum_{l=-\infty}^{\infty} a_l H_l^{(1)}(\chi r) \exp(il\varphi), \quad |x| > R_0. \tag{3}$$

Here $k = \omega\sqrt{\varepsilon_0 \mu_0}$, $\chi = \sqrt{k^2 n_\infty^2 - \beta^2}$; ε_0, μ_0 are the free-space dielectric and magnetic constants, respectively; $H_l^{(1)}$ is the Hankel function of the first kind and index l; (r, φ) are the polar coordinates of the point x; ν is the normal vector. We consider the radian frequency ω as a positive parameter and the propagation constant β as a complex parameter which for each fixed ω belongs to Reimann surface Λ of the function $\ln \chi(\beta)$. By $\Lambda_0^{(1)}$ denote the principal ("proper") sheet of Λ, which is specified by the following conditions: $-\pi/2 < \arg \chi(\beta) < 3\pi/2$, $\mathrm{Im}\,(\chi(\beta)) \geq 0$.

A nonzero function $u \in U$ is referred to as an eigenfunction (generalized mode) of the problem (1)-(3) corresponding to some eigenvalues $\omega > 0$ and $\beta \in \Lambda$ if the relations of problem (1)–(3) are valid. The original problem is spectrally equivalent to the following spectral problem with integral operator:

$$v(x) = \lambda^2(\omega) \int_\Omega K(\omega, \beta; x, y) v(y) dy, \quad x \in \Omega, \tag{4}$$

where $K(\omega, \beta; x, y) = \frac{i}{4} H_0^{(1)} (\chi\,|x - y|)\, p(x)p(y)$, $v = up$, $p^2 = (n^2 - n_\infty^2)/(n_+^2 - n_\infty^2)$, $\lambda^2 = k^2/(n_+^2 - n_\infty^2)$. For all $\omega > 0$ and $\beta \in \Lambda$ the operator $B(\beta) : L_2(\Omega) \to L_2(\Omega)$ defined by the right side of equation (4) is compact. If ω has a fixed positive value, then the spectrum of the operator-valued function $A(\beta) = I - \lambda^2 B(\beta)$ can be only a set of isolated points on Λ, moreover on the principal sheet $\Lambda_0^{(1)}$ it can belong only the set $G = \left\{\beta \in \Lambda_0^{(1)} : kn_\infty < |\beta| < kn_+, \ \mathrm{Im}\beta = 0\right\}$. Each eigenvalue β of the operator-valued function $A(\beta)$ depends continuously on $\omega > 0$ and can appear and disappear only at the boundary of Λ, i.e., at $\beta = \pm kn_\infty$ and at infinity on Λ. Increasing at infinity leaky modes satisfy to propagation constants β which belong to "improper" sheet of Λ. This sheet is specified by the following conditions: $-\pi/2 < \arg \chi(\beta) < 3\pi/2$, $\mathrm{Im}\,(\chi(\beta)) < 0$. The well known surface modes satisfy to propagation constants $\beta \in G$. In this case $\chi(\beta) = i\sigma(\beta)$, where $\sigma(\beta) = \sqrt{\beta^2 - k^2 n_\infty^2} > 0$.

Let transverse wavenumber σ has a fixed positive value. Rewrite problem (4) in the form of usual linear spectral problem with selfadjoint compact operator

$$v = \lambda^2 B(\sigma) v, \quad B: \ L_2\,(\Omega) \to L_2\,(\Omega)\,. \tag{5}$$

There exist the denumerable set of positive characteristic values λ_l^2, $l = 1, 2, \ldots$, with only cumulative point at infinity. The set of all eigenfunctions v_l, $l = 1, 2, \ldots$, can be choose as the orthonormal set. The smallest characteristic value λ_1^2 is positive and simple, corresponding eigenfunction v_1 is positive. Each eigenvalue λ_l^2, $l = 1, 2, \ldots$, depends continuously on $\sigma > 0$, and $\lambda_1^2 \to 0$, if $\sigma \to 0$. The well known fundamental mode satisfies to smallest characteristic value λ_1^2. If some values of the parameters λ^2 and σ are known, then β and ω can be calculated by evidence formulas.

The piecewise-constant collocation method for numerical solution of the spectral problem with integral operator (4) is proposed. The convergence of this method is proved and practically investigated. Some results of the numerical experiments for surface and leaky modes will be presented at the conference.

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Interaction between Relativistic Beam and Scattered Field in Relativistic Diffraction Generator

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The numerically studies of the multiwave mechanisms of interaction between tubular relativistic electron beam and fields of super-dimensional axisymmetric periodical slow-wave structures (SWS) of two types diffractive profile — sinusoidally corrugated waveguide and open structure on tory sequence — in relativistic diffractional generator in the frequency range of 2π-type oscillations are presented at this report.

We developed Integral Equation Method as a numeric realisation of rigorous solution of a diffraction problem of proper radiation of relativistic tubular electron beam modulated on frequency ω on periodic obstales of axisymmetric SWS at both continuous (sinusoidally corrugated) and non-continuous (as on tory sequence) profile surfaces. Maxwell equations with boundary conditions for perfectly conducting surface are reduced to surface Fredholm integral equation of second kind in H_ϕ considering the axisymmetric mode. Its fundamental solution (Green function for free space) is represented as the azimuthal integral depending on the distance between the point of observation and the integating point and then solved numerically using conjugate operator formalism [1]. The surface currents derived this way as a result of solving of Toeplitz matrix equation are used for pointing out the fields inside the whole bulk of periodic structure as the second step of the problem of Smith-Purcell radiation (or diffraction radiation) detection solution. For consideration of nonstationary nonlinear selfconsistent problem and processes in time iteration we applied Matrix Multiwave Method [1] which is based on eigenmodes of axially symmetric periodic structures.

In open structures on tori sequence the surface current distributions and EM fields for different length of periodic structure were investigated. The length of the open type structure should be long enough (12–16 periods) in order to satisfy the condition of Smith-Purcell radiation harmonic component formation with amplitude compared to one of EM fields' membrane functions of every separate torus. For quasi-plate case we found out the regimes of mode selection. These features distinguish the structures of significant distributed power losses from the close ones with continuous profile, which manifests the resonance response in frequency range of 2π-type oscillations even for much shorter length (4–5 periods) and for moderate diameters.

We compared the electromagnetic field profiles, mode structure and radiation frequency spectrum for two types of slow-wave structure with continuous profile: sinusoidally corrugated and half-torus on a pedestal. For two-sectional structure (two periodic section are separated with drift tube) we matched section and drift tube length for the stable long-pulse generation with wide beam current value range.

The conditions of the RDG generating frequency stabilisation were found to be related to the longitudinal resonances of the SWS surface wave. The resonance bulk field excitation at the frequency close to the critical one for the mode of equivalent smooth waveguide is shown to be an additional agent for the generation power boost.

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Reversed Cherenkov-transition radiation of a charge passing from vacuum into anisotropic medium

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Reversed Cherenkov-transition radiation (RCTR) in the case of a charge intersecting the interface between vacuum and isotropic left-handed medium (LHM) was investigated in detail recently [1, 2]. This radiation occurs due to the fact that Vavilov–Cherenkov radiation (VCR) in LHM is reversed, i.e. the Poyting vector of VCR forms an obtuse angle with the direction of charge motion (this feature mentioned in Veselago's paper [3] was proved by recent experiments [4]). As a result, VCR incidents the interface and produces reflected and transmitted waves. We have called these waves RCTR [1, 2]. One of the most attractive properties of RCTR in the vacuum area is the presence of lower and upper thresholds in both frequency and charge velocity domains. Moreover, RCTR can dominate transition radiation at certain parameters. Estimation of RCTR decay due to the losses in LHM gives hope that this radiation can be detected in experiments. However, the most significant problem with this attractive phenomenon consists in practical realization of isotropic LHM. Although metamaterials with left-handed properties do exist [5] they cannot be characterized by isotropic effective parameters (in other words, these materials are substantially anisotropic). Therefore, it is expedient to analyze fields generated by a moving charge in the case of anisotropic medium, which nevertheless can support reversed VCR and therefore RCTR.

In the present paper, we investigate electromagnetic field generated by a point charge passing from vacuum into a nonmagnetic electrically anisotropic uniaxial medium with the plasma-type frequency dispersion of the dielectric permittivity tensor components. The charge moves along the main crystal axis with the constant velocity. As it is known [6], at certain relation between plasma frequencies of the model VCR in the anisotropic medium under consideration is reversed, thus RCTR effect should occur. This specific situation is analyzed in detail in the present report.

General expressions for the electromagnetic field in the problem under consideration is given, for example, in monograph [7]. We investigated these expressions in two ways. In the first one the asymptotic representations were obtained for the electromagnetic field components in the far field zone with the use of steepest descend technique. This asymptotic takes into account the main terms of spatial radiation: spherical wave of transition radiation (contribution of the saddle point neighborhood) and cylindrical wave of RCTR (pole contribution). As was shown, RCTR is generated in vacuum given that the charge velocity exceeds certain threshold value connected with the total internal reflection of the reversed VCR at the interface. In the frequency domain RCTR effect in vacuum possesses both lower and upper thresholds. The lower one coincides with the lower plasma frequency, while the upper one corresponds to the total internal reflection. At the mentioned conditions, vacuum RCTR exists in certain angular interval near the interface. It is interesting that losses in medium result in damping of RCTR both in medium and in vacuum. However, performed estimation shows that RCTR is essential in vacuum up to the distance of the order of thousand wavelengths. This gives hope that this radiation can be detected in experiments.

In the second way, the effective algorithm of the field components computation was produced. With this method both spatial and frequency distribution of the Fourier harmonics of the field, as well as time evolution of the field were investigated. The obtained results show that, similarly to the case of isotropic LHM, RCTR can dominate transition radiation at certain parameters. Moreover, comparison of the exact numerical results with the asymptotic ones demonstrates their good agreement in the domain of validity of asymptotic.

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Diffraction by a subwavelength concaved perfectly conducting wedge

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The fields and current densities behaviors in the vicinity of an isolated infinitely sharp edge or vertex are well known. Many analytic results are available to date for various diffraction problems. Just to name some of the most important ones, among others : Sommerfeld solved the half plane illuminated by a plane wave [1], Mac Donald generalized this result to the wedge [2]. Radlow [3] solved the the quarter plane sector case and Satterwhite and Kouyoumjian [4][5] found the general solution for any sector and incident field source.

To ensure unicity of the solution in presence of an edge, it has been recognized that a condition was necessary : the so called "edge condition". It states that induced current densities must be integrable to produce finite scattered fields. The edge condition actually selects the solution having the lowest order of singularity at an edge, ruling out any singularities of order greater than $\rho^{-1/2}$, where ρ is the distance to the edge.

By expanding the fields as a power serie of ρ, Meixner characterized their behavior close to an edge surrounded by a combination of metal and dielectric sectors [6][7]. After the pioneering work of Meixner, several authors analyzed the edge fields and current densities for several more complex configurations [8] [9] [10]. The general conclusion of all these works is that the singular fields grow like $\rho^{-\nu}$ very close to the edge, the exponent ν depending both on the geometry and the electromagnetic properties of the wedge-shaped sectors surrounding the edge. Two cases worth remembering are the metallic half plane ($\nu = -1/2$) and the metallic 90° wedge ($\nu = -1/3$) in free space.

More analytically challenging problems involving two or more edges have been tackled by various authors. Exact solutions have been obtained for an infinity [11] [12] or for only two [13] parallel stacked half planes. The so called "thick" half plane has been addressed first in an approximate way by Hanson [14], and later solved exactly by Jones [15] who obtained tractable expressions and numerical results for the far field only for subwavelength thicknesses.

In most of these examples it is found again, or simply assumed based on Meixner's work [16], that infinite current densities may arise in regions of the scattering surface where the local radius of curvature is zero, but much less effort is made to obtain the exact behavior of these current densities in the vicinity of the more or less coupled edges. The purpose of this work is to derive explicit laws for the growth to infinity of the current density for the case of a subwavelength perfectly conducting concaved wedge [17].

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Analysis of Electromagnetic Waves Diffraction on Metal-Dielectric Nanodipole Over Substrate in Optic Range

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One of the possible design of OA is a nanocrystal coated with metal film. Efficient method for treatment of submillimeter antenna made of carbon nanotubes is described in [1]. Suggested method is based on numerical-analytically approach of solving paired integral as well as IDE equations for dipole with surface impedance. Integral representation of Green's function is lies in the bases of the method.

The main goal of the presented work is to develop efficient numeric-analytical method for IE solving and implementation of obtained method to problem of diffraction electromagnetic waves on nanodipole (ND) on substrate.

In the work we investigate a structure consisting of nanodipole standing on a substrate. Nanodipole is a crystal (ZnO, $\varepsilon = 4{,}11$) with radius a covered with metal film (Au, Ag or Cu) with thickness of h. L is a length of the nanodipole. Nanodipole is located along z axes at the origin of Cartesian coordinate system. We consider cases when substrate is sapphire Al2O3 with $\varepsilon = 3{,}136$ or ZnO.

We use assumption, that field doesn't depend on azimuth angle and also we assume there is no radiation at the ends of the dipole. In this case boundary problem reduces to IE. Solution of such IE is described in [1]. Of cause in our case kernel as well as matrix element are different from [1]. The main difficulty of calculation of matrix elements is concerned with presence of logarithmic singularity in IE kernel.

One of possible approach is to use integral representation (IR) of the kernel in [1].

Function $g(x, x', y, y', z, z')$ is a Green's function (GF) in a case when current is perpendicular to substrate. When $z \geq 0$ and , $z' \geq 0$ FG may be presented in form of:

$$g(x, x', y, y', z, z') = \frac{1}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[e^{-\gamma_1 |z-z'|} + Q\left(\rho\right) \, e^{-\gamma_1 (z+z')} \right] \frac{1}{\gamma_1} e^{i\alpha \bar{x} + i\beta \bar{y}} d\alpha d\beta.$$

where: $\bar{x} = x - x'$, $\bar{y} = y - y'$, $Q = \frac{\gamma_1 - \gamma_2 t}{\gamma_1 + \gamma_2 t}$, $t = \frac{\varepsilon_1}{\varepsilon_2}$, $\gamma_{1,2} = \sqrt{\rho^2 - k_{1,2}^2}$, $\rho = \sqrt{\alpha^2 + \beta^2}$, k_2 — wave number in substrate.

Obtained IDE is solved by method combining collocation method and method of moment. Matrix elements of derived SLAE are presented in Fourier integrals. Such representation of kernels and matrix elements enables to avoid difficulties connected with singularity of the IDE kernels. It is shown that derived method has fast internal convergence.

Algorithm described here is implemented in our computer program. Numerical results show when thickness of metal layer of nanodipole (ZnO) increases resonant wavelength shifts to short-length range. Nanodipole is placed on sapphire or ZnO substrate. Such dependence takes place for Ag, Au and Cu. Material of substrate (ZnO or sapphire) has no strong effect on resonant wavelengths because there permittivity are not very different. Permittivity of substrate has no strong effect on resonant wavelength but has effect on magnitude of scattered field.

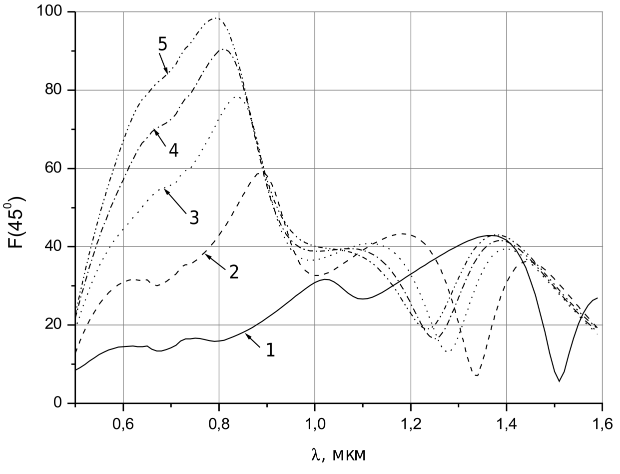

Fig. 1. Resonant characteristics of nanodipole. Material of dipole and substrate - ZnO, a=50nm, L=1um; curves 1 to 5 designate thickness of metal cover 10, 20, 30, 40, 50 nm.

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Diffraction of Optic Electromagnetic Waves on Metal Nanodipole

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Nanocrystal coated with metal foil is one of the possible structure of optical nanoantenna. Such antenna can be stimulated with both plain wave and surface wave propagating along a substrate. In it well-known that in optical range metal possesses complex dielectric conductivity (http://www.luxpop.com).

Metal can be presented as solid-state plasma generated by free electrons with plasma frequency of ultraviolet range. All this obviously leads to changes in properties of analyzed devices comparing to the structures with perfect metal. Electromagnetic methods of analysis are also changes. It caused analysis of diffraction on anisotropic dielectric nanodipole. Metal nanodipole in optical range is a particular case of such structure.

The aim of the research is to develop effective numerical analytical solution method of one-dimensional integral equation and its applying in theoretical study of electromagnetic wave diffraction on nanodipole in optical range.

The problem of diffraction electromagnetic wave on dielectric dipole can be reduced to solution of integral equation (IE) in case when field does not depend on the angle and radiation on the ends can be neglected:

$E(r,z) = E_0(r,z) + \left(\frac{d^2}{dz^2} + k^2\right) \int\limits_0^L \int\limits_0^a \tau E(r',z') g(r,r',z,z') r' dr' dz'$, where $g(r,r',z,z') = \frac{1}{4\pi} \int\limits_0^{2\pi} \frac{e^{-ikR}}{R} d\varphi$ – Green function (GF); $R = \sqrt{r^2 + r'^2 - 2rr'\cos(\varphi - \varphi') + (z-z')^2}$, $\tau = \varepsilon(r') - 1$, $\varepsilon(r')$ – dielectric conductivity of dipole, external field is defined as: $E_0(r,z) = \frac{1}{2\pi} \int\limits_0^{2\pi} E^{ext}(x,y,z) d\varphi$.

The kernels of the IDE are presented in the form of Fourier integral. Obtained IDE is solved by method combining collocation method and method of moment. Matrix elements of derived SLAE are presented in Fourier integrals. Such representation of kernels and matrix elements enables to avoid difficulties connected with singularity of the IDE kernels. It is shown that derived method has fast internal convergence.

Software was developed based on obtained results to compute diffraction of electromagnetic waves on metal nanodipole (ND). Performed analysis has shown that dependence of scattered field from wave length has resonant behavior.

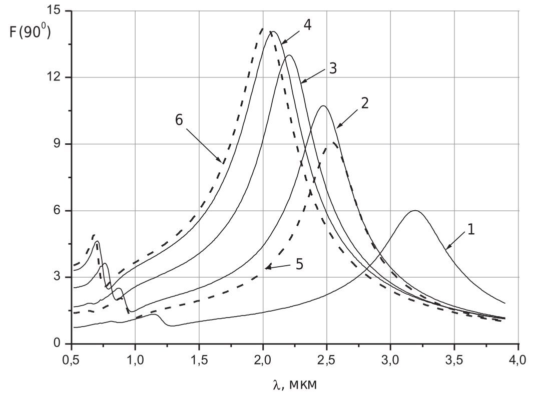

Fig. 1 Resonant characteristics of nanodipole made of ND made of ZnO (radius 0.01 μm, L=0.7 μm) in case of different thickness copper layer. Copper layer 5, 10, 15, 20 nm – curves 1–4, accordingly. Curves 5, 6 – copper NV with radius 15 and 30 nm.

In case of diffraction on nanocrystal ZnO covered with metal film (Fig. 1) resonance occurs even in case of film thickness t of few nanometers. Resonant wave length depends much on film thickness. On the same diagram there are also characteristics for totally copper dipoles for comparison. In case of $t = 20$ nm (curve 4) nanocrystal and copper nanodipole (curve 6) characteristics are really close. In case of smaller t values field penetrates in nancrystal and that why its influence on scattering pattern increases.

Thus in the work effective solution method of parameters of antenna-nanodipole as well as numerical analytical solution of boundary problem is obtained. Software was developed allowing compute characteristics of antenna-nanodipoles in optical range. Electrodynamic characteristics of antenna-nanodipoles were calculated. Behavior of its amplitude-frequency response was analyzed in case of different system parameters. Characteristics of copper, silver and gold nanoantennas were teoretically analyzed in optical range. Dependence of scattering field from frequency is of resonant behavior and also nanodipole resonant wave length values are higher than resonant wave length values of perfectly conducting dipole of the same size.

Integral equation based inverse diffraction problem solving for low-dimensional periodically-arranged nanocrystals

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Carrier confinement in low-dimensional periodically-arranged nanocrystals (LDPAN) leads to the dependence of the operating wavelength in LDPAN-based optoelectronic devices on the average size, shape, and material properties of heterostructures. Scatterometry as a non diffraction-limited optical method is applied to LDPAN (i.e. quantum dots, nanowhiskers, their combinations, etc) which are arranged on periodical masks. We propose numerical algorithms for the morphology determination of periodic relief nanostructures from light diffraction patterns measured in the UV–IR wavelength range. The solution of the direct 3D diffraction problem is reduced for some symmetrical LDPAN to the solution of boundary integral equations [1] for the 2D or even 1D Helmholtz equation. The ratio wavelength-to-period is small for typical inverse scattering problems and so the modified boundary integral equation method [2] has to be applied to obtain accurate results at a fast convergence rate for the direct problem [3]. The inverse problem is formulated as a non-linear operator equation in the Euclidean space with an assumed set of unknown structural parameters of nanocrystals (height, width, slope angles, and refractive indices) and a given set of determined efficiency and phase shift values. The operator maps the sought structural parameters to the amplitudes of diffraction orders. The present approach employs the gradient Levenberg-Marquardt method to solve the operator equation. This type of iterations is close to the Gauss-Newton method, but more efficient and stable for poor or improper input data [4] due to an additional regularization parameter. The inverse problem, mathematically, severely is ill-posed [5] and regularization techniques have to be used to improve the solution. Unfortunately, such a regularization is not enough even for simplest geometry considerations and serious restrictions to the number and values of reconstructed parameters must be applied. Here we consider problems with a few polygonal-type boundary profiles having several edge points only. The method can be applied to LDPAN grown in different material systems (group III–V, IV, II–VI, their combinations, etc) by various techniques (MBE, MOCVD, MOVPE, etc).

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Nonlinear effects by propagating of elastic waves in the samples of Outokumpu drill hole

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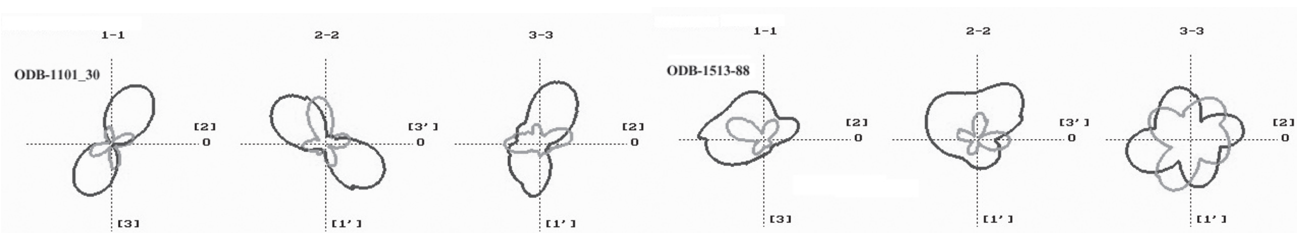
The investigation drill hole in Finland, Outokumpu (OKU), reached a final depth of 2516 m. The drill hole is located in SE Finland near the worked-out deposit bearing the same name. The real section showed that the upper drill hole has passed through mica schist (0–1310 m), black schist, biotite gneiss, serpentinite and diopside-tremolite skarn (1310–1515 m). Below 1515 m mica schist alternates mainly with bodies of pegmatite granite and biotite gneiss [1]. At the first stage of investigations we identified the types of rocks, texture, structure, composition and density of 43 core samples brought from Finland. The nearest to the surface sample was taken from a depth of 94 m and the deepest one from the 2297.85 m depth. At the subsequent stages we determined the rock density and performed acousopolariscopy of the whole sample collection [2]. Acoustopolarization measurements were made by the polarization planes of transducers brought in line (VP) and at the crossed (VC) position.

According to the acoustopolarization measurements, rocks in the upper section are highly homogenous (down to a depth of about 1300 m) and strongly anisotropic. Their homogeneity is explained by the preferred linear orientation of mineral (biotite, plagioclase, etc.) grains stretched in one direction. The acoustopolarigrams for the lower part (down to a depth of 2516 m) show that its structure is less ordered and heterogenic. The samples from the deep Outokumpu drill hole display the effect of linear acoustic anisotropic absorption (LAAA). The medium in which LAAA is observed is acting on the propagating beam of shear waves as a polarizer. Moreover, the directions of "the largest" (LA) and "the smallest" (SA) amplitudes of the beam passing through the media are connected, as a rule, with the linear or planar elements, stretching along one direction, such as crystalline grain borders, aligned microcracks and others. An example of the sample with strong manifestation of the LAAA effect is the sample ODB-1101_30 Bt-Ms schist with veinlets of gypsum-carbonaceous composition. All the sides of the sample show the presence of two elastic symmetry element projections (VP - dark lines, VC - light lines). The angles between the projections of symmetry elements are about 900. The acoustopolarigrams VP of all three sides have an elongated form. Such form is observed in the presence of strong LAAA. The degree of LAAA measured on sides 1, 2 and 3 is equal to D1 = 0.90, D2 = 0.81 and D3 = 0.51 accordingly. The 1-st and 2-nd sides show strong homogeneity and very strong LAAA.



Another effect observed in propagating seismic waves through geological media is associated with a change in the degree of polarization of shear waves during their propagation through the sample. Such a change is accompanied by an increase in the ellipticity of shear waves or by their depolarization. The shear wave depolarization (SWD) effect is appreciably manifested as shear waves propagate in a medium consisting, for example, of differently oriented elastic anisotropic layers. The SWD effect is also rather frequently observed in anisotropic crystalline rocks composed of grains whose elastic symmetry is sufficiently maintained, for example, in two directions [2]. The SWD effect is manifested in enormous dimension of the VC acoustopolarigram minimums. This effect is demonstrated in the acoustopolarigrams of the black schist sample ODB-1513-88. The differently oriented anisotropic layers are normal to direction 3-3' and parallel to 1-1' and 2-2' directions. The LAAA and SWD effects are common for the samples from the Outokumpu drill hole.

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The Poincaré wavelet transform: implementation and interpretation

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We present here examples of calculations of the Poincaré wavelet transform for model space-time signals.

Let analyzed signal f be square integrable function of time t and coordinate x, $f(ct, x) \in \mathcal{L}_2(\mathbb{R}^2)$. We construct a family of wavelets $\psi_{a,\gamma,c\tau,b}$ using mother wavelet $\psi(ct, x) \in \mathcal{L}_2(\mathbb{R}^2)$ in the following way:

$$\psi_{a,\gamma,c\tau,b}(ct,x) = \frac{1}{a}\psi(\tilde{ct},\tilde{x}), \quad \begin{pmatrix}\tilde{ct}\\ \tilde{x}\end{pmatrix} = \frac{1}{a}\Lambda_{-\gamma}\begin{pmatrix}c(t-\tau)\\ x-b\end{pmatrix},$$

$$\Lambda_\gamma = \begin{pmatrix}\cosh\gamma & -\sinh\gamma\\ -\sinh\gamma & \cosh\gamma\end{pmatrix}, \quad \tanh\gamma = \frac{v}{c},$$

where c is a constant which usually has a meaning of light speed, γ is rapidity, v is speed, a is scale and $c\tau, b$ are shifts in time and space, Λ_γ is a matrix of hyperbolic rotations which describes the Lorentz transform. Inner product of f and $\psi_{a,\gamma,c\tau,b}$ is called the Poincaré wavelet transform:

$$F(a,\gamma,c\tau,b) = \iint\limits_{\mathbb{R}^2} dtdx f(ct,x)\overline{\psi_{a,\gamma,c\tau,b}(ct,x)},$$

where overline stands for complex conjugation. If mother wavelet ψ satisfies some admissibility conditions reconstruction formula is valid:

$$f(ct,x) = C_\psi \int\limits_{\mathbb{R}} d\gamma \int\limits_0^\infty \frac{da}{a^3} \int\limits_{\mathbb{R}^2} d\tau db\; F(a,\gamma,c\tau,b)\psi_{a,\gamma,c\tau,b}(ct,x),$$

where C_ψ is a constant depending on ψ.

One can interpret the reconstruction formula as the superposition of localized functions living in different inertial reference systems which move with speeds v. Here we discuss the Poincaré mother wavelet construction and wavelet coefficients interpretation. It is demonstrated that the speeds of moving reference systems can be estimated employing wavelet coefficients. Numerical examples of the Poincaré wavelet transform are given. The Poincaré wavelet transform may be applied for integral representation of solutions of wave equation [2].

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Small perturbations in the spherical prestressed state in a nonlinear isotropic elastic reduced Cosserat medium: waves and instabilities

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We suggest a nonlinear elastic theory of a reduced Cosserat continuum for isothermal or adiabatic processes. Point-bodies of the reduced Cosserat continuum can rotate and move in general ways. However, the reduced Cosserat medium does not react on the gradient of rotation. Therefore, though the stress tensor in the medium is not symmetric, the couple stress tensor is zero. This follows from the law of balance of energy. Such a theory can be used for the description of granular materials to take into account rotational degrees of freedom of particles, which become important, in particular, in shear processes. Wave propagation in soils also may be influenced by proper rotational dynamics of heterogeneities contained in soil. Rotational degrees of freedom can be important on different scale levels. In some earthquakes there are large rotations of objects observed in the near field. Recently, a new branch of science has appeared, called "rotational seismology", where this model also can be used (see `http://www.rotational-seismology.org/`). We think that this model is more appropriate for the descriptions of granular materials, rocks and soils, than the full Cosserat continuum with the strain energy providing the stability of the material (e.g. positively defined quadratic form of nonlinear strain tensors), since in these media we do not observe the ordered structure of rotations: the interaction between neigbouring particles or other heterogeneities with the medium around them usually does not try to keep their rotations close, and often works vice versa.

The law of balance of energy gives the constitutive equation for the stress tensor in terms of the derivative of the strain energy with respect to the nonlinear Cosserat deformation tensor, depending on the gradient of the radius-vector and on the turn tensor. We consider a strain energy of almost general kind (sufficiently smooth) and write down the dynamic laws for such a material. Then we consider a nonlinear equilibrium stress state and small perturbations (translational and rotational) near this equilibrium. We write down the equations for infinitesimal perturbations for a material with sufficiently smooth strain energy. We consider a particular case, a prestressed spherical state for an isotropic material, and show that by their structure the equations for perturbations coincide with the equations for a linear reduced isotropic Cosserat continuum. Effective constants and the weight of the rotational strain in the equations depend on the prestressed state. We show that for a large class of materials under sufficiently large compression the prestressed state becomes unstable, and the loss of stability is due to shear perturbations. If we impose an additional condition for the strain energy, the prestressed spherical state is also unstable under large tension, and the loss of stability is due to rotational perturbations with a certain frequency. In the stable domain, equations of perturbations are analogous to the wave equations for the linear reduced Cosserat continuum and inherit their properties: coupling of the shear-rotational wave, strong dispersion in some domain of frequencies, existence of a "forbidden zone" of frequencies for the shear-rotational wave, shear-rotational localization phenomena in this zone etc.[1]. Similar localization effects are observed by seismologists near bassin edges [2].

In literature on fracture mechanics and phase transitions we can find qualitatively similar mechanisms of the instability of materials, however, we have not considered yet the corresponding prestressed states. In granular materials, their failure under large compression and large tension is described by the Roscoe diagram [3] and possibly may be interpreted in terms of our model.

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Diffraction in nanostructures: does scattering go beyond the Born approximation?

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The neutron scattering on a two-dimensional ordered nanostructure with the third nonperiodic dimension can go beyond the Born approximation. In our model supported by the exact theoretical solution a well-correlated hexagonal porous structure of anodic aluminum oxide films acts as a peculiar two-dimensional grating for the coherent neutron wave. The thickness of the film L (length of pores) plays important role in the transition from the weak to the strong scattering regimes. It is shown that the coherency of the standard small-angle neutron scattering setups suits to the geometry of the studied objects and often affects the intensity of scattering. The proposed theoretical solution can be applied in the small-angle neutron diffraction experiments with flux lines (Abrikosov's flux lines) in superconductors, periodic arrays of magnetic or superconducting nanowires, as well as in small-angle diffraction experiments on synchrotron radiation.

Propagation of elastic waves through a lattice of cylindrical cavities

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We consider the wave propagation through a doubly-periodic array of cavity cylinders in an isotropic elastic medium. The array contains a set of infinitely long circular cylindrical cavities of radius a and the axes of the scatterers are parallel to the z-axis and regularly arranged in two dimensions. It is assumed that the radius of the cylinders a is much smaller than the wavelength $(1/k)$ and the array periodicity L. Therefore $\epsilon = ka$ is a small parameter and the method of matched asymptotic expansions is used to obtain approximation solutions. Here kL is allowed to be an order one quantity so that phenomena associated with the periodicity of the array, such as band gaps, may be described.

All solutions considered satisfy a Bloch condition that, for a specified Bloch wave vector $\boldsymbol{\beta} = \boldsymbol{\beta}_0$, relates the solutions at corresponding points in different cells of the lattice. In the absence of the scatterers, and for the given Bloch wave vector $\boldsymbol{\beta}_0$, plane wave solutions are possible for discrete values $\omega_1, \omega_2, \ldots$ of the frequency ω. For each ω_i, there are $M \geq 1$ plane waves corresponding to a particular pair $(\boldsymbol{\beta}_0, \omega_i)$ — these solutions may be shear or dilatational waves, or a mixture of the two. With the scatterers present, the asymptotic analysis yields an algebraic system to determine ω for the M perturbed modes that exist for each $\boldsymbol{\beta}$ within a neighbourhood of $(\boldsymbol{\beta}_0, \omega_i)$ in the $(\boldsymbol{\beta}, \omega)$-space. Explicit expressions for the frequencies are readily obtained that show how the mode frequencies depend on $\boldsymbol{\beta}$, the geometry, and the Lamé constants for the medium. Results are given to illustrate the appearance of local band gaps, the splitting and crossing of double modes, and the switching between dilatational and shear modes.

Analytical solutions for diffraction problem of nonlinear acoustic wave beam in the stratified atmosphere

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The nonlinear wave equation and modified Khokhlov–Zabolotskaya type equation for high intensive acoustics wave beams propagating in stratified atmosphere with inhomogeneous of sound speed is set up. Some approaches to find analytical solutions of this equation are developed. The geometrical acoustics approximation and modified Raley integral for this problem is suggested. The profile distortion of broadband waves and waves with discontinuities is investigated. Time profiles of the single impulse, dependencies of its peak amplitude and duration are obtained. The asymptotical procedure is developed for describing of wave profile near the axis of wave beam. This method allows to take into account effect of shock front formation and nonlinear refraction of the shock front along the wave beam axis.

How accurate is molecular dynamics?

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In this talk we show that Born–Oppenheimer molecular dynamics accurately approximate observables based on the time-independent Schrödinger equation, in the limit of large ratio of nuclei and electron masses. The derivation, based on a Hamiltonian system interpretation of the time-independent Schrödinger equation and stability of the corresponding Hamilton–Jacobi equation, bypasses the usual separation of nuclei and electron wave functions, includes caustic states and crossing electron eigenvalues, and gives a different perspective on Schrödinger Hamiltonian systems and numerical simulation in molecular dynamics modeling at constant energy microcanonical ensembles.

Reflection of magnetoelastic bound plane waves from the boundary of elastic half-space

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Requests of modern technology require a more in-depth study of the wave problems of theory of elasticity. Many technical problems re associated with the theory of propagation of elastic waves. In this paper the reflection of magnetoelastic bound plane waves from the boundary of elastic half-space is considered. In the beginning the incidence of quasi-longitudinal wave, and then the quasi-transverse wave on the boundary $x = 0$ is considered. In this case, both the quasi-longitudinal wave and the quasi-transverse wave will be reflected from the boundary.

The system of equations defining the motion of the bound waves had been considered [1]. he characteristic equation for this system is received. With the first boundary conditions, when displacements U and V along the axes x and respectively at the boundary $x = 0$ are equal to 0, a condition has been found between the quasi-longitudinal, quasi-transverse and Alfvén speeds, at which the following result is received: the falling quasi-longitudinal wave is transforming to the reflected quasi-transverse wave and the falling quasi-transverse wave is transforming to the reflected quasi-longitudinal wave.

It is shown that the effect of the magnetic field changes the characteristics of the wave reflection. Numerical results are received, that characterize the relationship between the quasi-longitudinal, quasi-transverse and Alfvén speeds. It was found out, that quasi-longitudinal and quasi-transverse waves will be transformed into the reflected quasi-transverse and quasi-longitudinal waves respectively, which need to have a much more greater magnitude of the vector of magnetic-field strength.

Similar results are received at the three other boundary conditions: when displacement $U = 0$ at the boundary $x = 0$ and moves freely along the axes $x = 0$, when displacement $V = 0$ at the boundary $x = 0$ and moves freely along the axes x, and finally, when the boundary $x = 0$ is free. Unlike the rigid boundary condition for the above boundary conditions it turns out that at quite a low value of the vector of magnetic-field strength can be received transformation of quasi-longitudinal and quasi-transverse waves can be received to the reflected quasi-transverse and quasi-longitudinal waves respectively.

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Modelling photonic crystal waveguides using finite volume method

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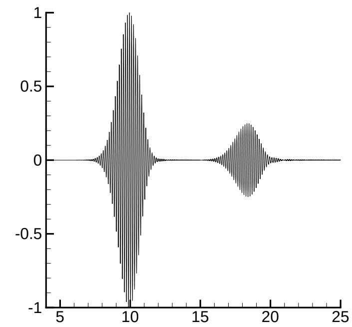
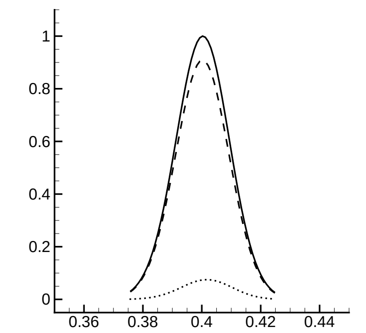
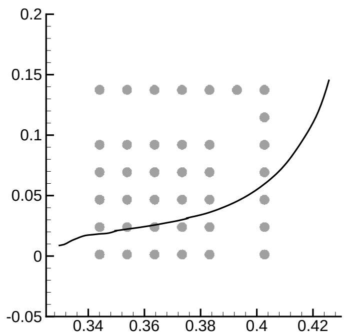
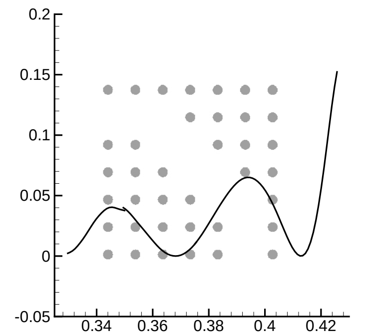
We applied parallel finite volume algorithm to evaluation of optical properties of photonic crystal waveguides with a bend. Reflection and transmission coefficients were studied for a range of frequencies inside the photonic crystal bandgap for different bend configurations. A series of pulses with a Gaussian envelope in time and different central frequencies were sent down the straight section of the waveguide before the bend. Electric field was recorded at two points one before the bend and another one after the bend. Spectra of incident, reflected and transmitted pulses were used to obtain transmission and reflection coefficients.Photonic crystal had square lattice with a period of $a = 0.059$. Structural elements were cylinders with a radius of 0.01062. Dielectric constant of cylinders ε was 11.56. An example incident and reflected pulses recorded before the bend for a signal with central frequency $0.4 \times 2\pi/a$ are shown on Figure 1. Corresponding spectra of incident (solid), reflected (dotted) and transmitted (dashed) pulses are shown on Figure 2. Reflection coefficients for a range of frequencies for two examples of bend configurations are shown on Figure 3 and Figure 4. Our results show that for a bend composed of two straight sections at 90° no frequency exhibits 100 % transmission. For configurations in which between two straight sections there is a section at 45° the number of frequencies that exhibit 100 % transmission is equal to the number of periods in this section. Our results compare well with simplified analytic models from previous researchers.









Spatially modulated optical vortex solitons in nematic liquid crystals

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A link between fundamental optical spatial solitons [1] and optical vortices [2] is provided by the existence of the dynamic bound states of solitons in the form of azimuthally modulated vortex solitons, or *azimuthons* [3]. For the first time to our knowledge, we observe in experiment and describe theoretically the formation of dipole azimuthons in nematic liquid crystals (NLCs) which undergo nontrivial charge-flipping of on-axis phase dislocation.

Our experiments are carried out in a planar cell, filled with 6CHBT NLC. A single-charged vortex beam is generated with a fork-type amplitude diffraction hologram from extraordinarily polarized cw laser beam. At low input power, $P < 0.9$ mW, the vortex beam uniformly diffracts without any noticeable self-action. As the power increases, $1 < P[\text{mW}] < 2.2$, the beam experiences the self-focusing and the output size is visibly reduced. In this regime, the initial radially symmetric vortex intensity undergoes drastic transformation: the vortex "doughnut" breaks up into two beams with a dark core transformed into a tilted stripe. Such symmetry-breaking can be understood in the context of vortex astigmatic transformations [4]. Further increasing the input vortex power

(P >2.3 mW) leads to the formation of self-trapped rotating *dipole azimuthon* [Fig.1]. For different input powers the output dipole has strongly varying elliptic shape tilted at different angles with respect to cell boundaries [Fig. 1(d–f)]. We showed the two-lobe beam is spatially twisted inside the cell, the direction of the twist is defined by topological charge of the input vortex beam, and the rate of twist depends on the excitation level.

Next, we investigate the singular phase structure of dipole azimuthons. At small powers, such as P=2.3 mW in Fig. 1(a), the input phase dislocation with topological charge m_0=+1 is preserved. However, with further increase of power, we observe *a triplet of vortices*, with the central (on-axis) vortex of the opposite charge m_0 =-1 and two satellite vortices with $m_{1,2}$=+1, so that the total charge remains identical to the input [Fig. 1(h)]. Following the outputs for increasing power P we observe spatial separation of the triplet, then attraction of vortices. Then, the central vortex flips its charge from m_0=-1 to m_0=+1, see Figs. 1(h,i). With further increase of power the charge flipping process repeats again. The apparent reason for such complex dynamics is the anisotropic deformation of a soliton-induced waveguide, similar to the astigmatic mode-converter [4] which can lead to nontrivial topological reactions of vortex.

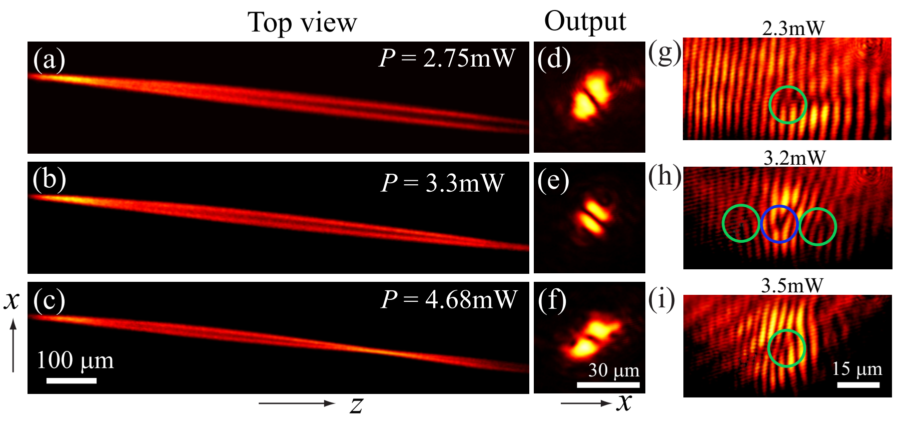

Fig. 1. Generation of a dipole azimuthon demonstrating the power-controlled twist and breathing of the beam. (g-i) Experimental interferograms with topological reactions of non-local dipole azimuthon. The circles indicate positions of phase singularities with charges +1 (green) and -1 (blue).

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Towards x-ray diffraction by objective structures: A Bravais-like classification of isometry groups

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The starting point for structure analysis of crystals by x-ray diffraction is the translation invariance of both plane waves and crystal lattices. The resonant frequencies reveal the translational structure of the crystal given by its Bravais lattice type. This classification identifies lattices that have the same symmetries up to conjugacy in the Euclidean group [1].

Recently [2, 3], R. D. James introduced a more general concept of invariant arrangements of atoms or molecules — the so-called objective structures — that include crystal lattices as special case. The translational structure of the latter is replaced by the uniformly discrete orbit of an isometry group. Many structures in nanotechnology and biology, like graphene, carbon nanotubes and the buckyball, tails and capsids of certain viruses as well as some secondary and quaternary protein structures are realizations of objective structures.

As crystal lattices are finite superpositions of Bravais lattices, objective structures are finite unions of isometry group orbits. So, the natural question arises, what different symmetries are produced by the same generating isometry group.

A general setting is provided by interpreting the proper isometries that fix some subset of the $\mathbb{R}^3$ as geodesic translations on certain manifolds. It is shown that every objective structure can be generated by a group of these geodesic translations starting with a finite molecule. A Bravais-like classification of these isometry groups is discussed.

We believe that these symmetry groups are essential in interpreting the response of objective structures to x-rays, as will be illustrated by examples.

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Monodromy of the deformed hypergeometric equation

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Euler integral symmetry for the hypergeometric system of linear differential equations is described. Reduction of the hypergeometric system leads to integral symmetry for the deformed hypergeometric equation. Analytic continuation of the corresponding contour integral gives the possibility to calculate the monodromy group for this equation in explicit terms. Solutions of the deformed hypergeometric equation in integral form are obtained too.

Schrödinger operator on axis with potentials depending on two parameters

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We study the eigenvalues of an operator

$$\mathcal{H}^{\mu,h} := -\frac{d^2}{dx^2} + \mu^{-1}\left(\sum_{j=1}^{n} V_j\left(\frac{x-x_j}{h}\right) + hW(x)\right), \quad x \in \mathbf{R}, \qquad 0 < h \ll 1, \tag{1}$$

assuming the existence of a number $\gamma > 0$ such that

$$\mu^{-1}h^{1/2} = o(h^{\gamma}). \tag{2}$$

Here x_j are arbitrary distinct numbers, $V_1(x), \ldots, V_n(x), W(x)$ are complex-valued functions in $C_0^{\infty}(\mathbf{R})$, and at least two of these functions are non-zero.

We establish the sufficient conditions for the absence and existence of the eigenvalues of such operator, including in the critical case

$$\int_{-\infty}^{\infty}\left(\sum_{j=1}^{n} V_j\left(\frac{x-x_j}{h}\right) + hW(x)\right)dx = 0. \tag{3}$$

In the case the eigenvalues occur, we construct their asymptotics.

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Structured Elastic Waves

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A short review of exact solutions for surface and body elastic waves is presented. We discuss surface and interfacial waves with plane [1] and non-plane wavefronts [2]. Also, we mention body waves with plane wavefronts but having polynomial amplitudes [3].

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Diffraction tomography of inhomogeneous plasma with double weighted Fourier transform

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The method of processing the results of tomographic measurements, based on the representation of the field by double-weighted Fourier transform is proposed. The numerical simulation of the proposed processing is performed taking into account the finite sizes of the antenna systems. Additionally our numerical simulation takes into account a finite number of elements in the receiving and transmitting antenna systems.

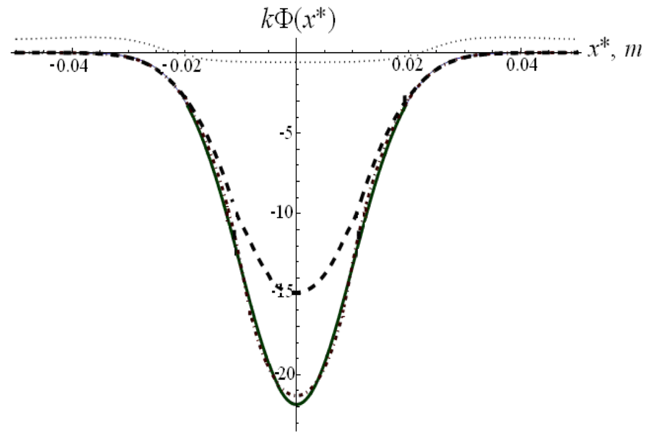

Figure 1. Behavior of the phase $k\Phi(x^*) = k\Phi(x^*, x^*, 0, 0)$ for the case without treatment (dotted line), for $D = D_0 = 28cm$ (dashed line), for $D = D_0 = 35cm$, (dash-dotted line) and for $D = D_0 \sim Z$ (solid line).

Figure 1 presents the values of the phase $k\Phi(x^*) = k\Phi(x^*, x^*, 0, 0)$, caused by Gaussian inhomogeneity $\tilde{\varepsilon}(x, z) = \varepsilon_m \exp\left\{-\left[(x - x_m)^2 + (z - z_m)^2\right] / (2l^2)\right\}$, on distance x^* in cross section $y^* = y_0^* = 0$ under following parameter $x_m = z_m = 0$, $\varepsilon_m = -0.55$, $l = 1cm$, $z_t = -3m$, $z_0 = 3m$, $\lambda = 2mm$, $z_s = 0m$. Fresnel radius under these parameters is $a_F = 5,4cm$. The dotted line corresponds to the case without treatment, dashed line corresponds to $D = D_0 = 28cm$, dash-dotted line – to $D = D_0 = 35cm$ and solid line corresponds to very long antennas with $D = D_0 \sim Z$. Where D and D_0 are sizes of the transmitting and receiving antennas.

It is seen from Figure 1 that chosen antennas parameters provide super Fresnel resolution even in conditions of large phase variations.

Transient Electromagnetic Fields in a Shielded Quasi-TEM Transmission Line

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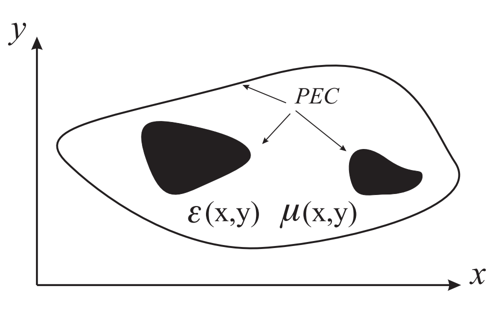
We consider a geometrically regular shielded quasi-TEM transmission line with a cross section of arbitrary shape. The cross section of such a line is shown in Fig. 1.



The cross section of shielded quasi-TEM transmission line.

A perfect electric conductor (PEC) screen covers finite number of conductors and thus forms the multiconductor line. The space between the conductors and screen is filled with transverse inhomogeneous magnetodielectric medium. It means that the relative permittivity and permeability depend on transverse coordinates only $\varepsilon = \varepsilon(x, y)$, $\mu = \mu(x, y)$. The constraints for permittivity and permeability are given as: $\varepsilon \geq 1$, $\mu \geq 1$. The structure is excited with some presented spatial distributions of transient electric and magnetic currents and charges. Electric field strength $\vec{\mathcal{E}}(\vec{r}, t)$ and magnetic field strength $\vec{\mathcal{H}}(\vec{r}, t)$ in the line under consideration is to be found.

In the present work a mathematical model of shielded quasi-TEM transmission line is developed in time domain on the base of the mode basis method (O.A. Tretyakov method) modification. Within the framework of the developed model the electric and magnetic fields can be presented as the following expansions:

$$\vec{\mathcal{E}}(\vec{r}, t) = \vec{E}(\vec{r}, t) + \vec{z}_0 E_z(\vec{r}, t), \quad \vec{\mathcal{H}}(\vec{r}, t) = \vec{H}(\vec{r}, t) + \vec{z}_0 H_z(\vec{r}, t),$$

$$\begin{aligned} \sqrt{\varepsilon_0}\vec{E}(\vec{r}, t) &= \sum_m e_m^H(z, t)\vec{E}_m^H(\vec{r}_\perp) + \sum_n e_n^E(z, t)\vec{E}_n^E(\vec{r}_\perp) + \sum_k e_k^T(z, t)\vec{E}_k^T(\vec{r}_\perp), \\ \sqrt{\mu_0}\vec{H}(\vec{r}, t) &= \sum_m h_m^H(z, t)\vec{H}_m^H(\vec{r}_\perp) + \sum_n h_n^E(z, t)\vec{H}_n^E(\vec{r}_\perp) + \sum_k h_k^T(z, t)\vec{H}_k^T(\vec{r}_\perp), \\ \sqrt{\varepsilon_0}\vec{E}_z(\vec{r}, t) &= \sum_n e_n^z(z, t) q_n \Phi_n^E(\vec{r}_\perp), \quad \sqrt{\mu_0}\vec{H}_z(\vec{r}, t) = \sum_m h_m^z(z, t) p_m \Phi_m^H(\vec{r}_\perp). \end{aligned} \tag{1}$$

Here $\vec{r} = \{x, y, z\}$, $\vec{r}_\perp = \{x, y\}$, ε_0 and μ_0 are the permittivity and permeability of free space correspondingly, $\vec{z}_0$ is unite vector of longitudinal coordinate z. The vector and scalar functions depending on the transverse coordinate $\vec{r}_\perp$ only are the basis functions in expansions (1). Scalar functions depending on longitudinal coordinate z and time t are the coefficients of expansions (1). Superscript indicates the wave mode type. Indexes H, E and T correspond to transverse electric (TE), transverse magnetic (TM) and transverse electromagnetic (TEM) waves correspondingly. The particularity of expansions (1) is that each basis function does not depend on frequency. In order to find the basis functions for the line with arbitrary cross section and filling the elliptic eigenvalue boundary problems are formulated and solved. The coefficients of expansions (1) satisfy the inhomogeneous system of hyperbolic linear partial differential equations of second order with constant coefficients.

Numerical experiments show that the process of excitation and propagation of any impulse signal with finite spectrum in arbitrary shielded quasi-TEM transmission lines is described with high accuracy by using a finite number of terms in the expansions (1). The relationship between the required number of terms and the upper frequency of the signal spectrum is found. The conditions of impulse wave propagation without essential distortion of its waveform are found for arbitrary transversely inhomogeneous multiconnected cylindrical transmission line.

On the base of the developed model the problems of transient electromagnetic waves excitation and propagation in various practical quasi-TEM transmission lines are solved.

Bistable Wave Transmission through a Grating of Nonlinear Dielectric Bars

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We consider a normal incidence of a plane x-polarized monochromatic electromagnetic wave on the periodic grating which consists of dielectric bars of rectangular cross section (Fig. 1). The structure is infinite in the x and y directions, and is placed in free space. The structure thickness and period are a and d, respectively. Each periodic layer of grating is assumed to be dielectric material with permittivity ε_{gi}, $i = 1, 2$. Suppose that the each second layer is a Kerr-nonlinear dielectric which permittivity ε_{g2} depends on electric field as follow $\varepsilon_{g2} = \varepsilon'_{g2} + \varepsilon''_{g2}|E|^2$.

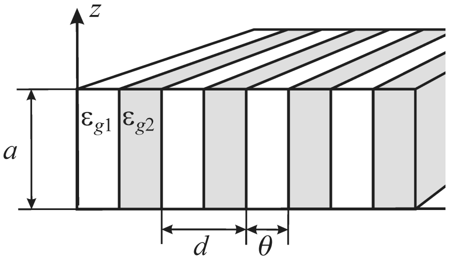

Fig. 1. The problem geometry.

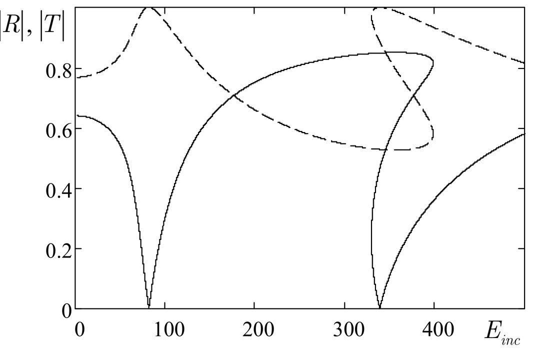

Fig. 2. Dependences of the reflection $|R|$ and transmission $|T|$ coefficients versus the incident field magnitude.

The problem under consideration is solved in two stages. At the first stage the long-wave approximation ($d << \lambda$) based on the anisotropic effective medium theory is used. Under this approximation 1-D periodic dielectric bars are viewed as a uniaxial anisotropic medium with the optical axis along the periodic direction. The relationship between the effective constitutive parameters and the geometrical and electrical parameters of the periodic grating is obtained. Thus the original structure is reduced to the anisotropic layer with an effective permittivity ε_{eff}. The optical response of anisotropic layer is determined analytically using the traditional electric and magnetic field boundary conditions. As a result the reflection and transmission coefficients are obtained in the form $R = R(\varepsilon_{eff})$, $T = T(\varepsilon_{eff})$.

At the second stage, on the basis of the linear problem solution, the nonlinear equation on unknown field intensity inside the anisotropic layer is used to solve the nonlinear problem. At a fixed frequency, the numerical solution of this equation give us the reflection and transmission coefficients as the functions of the incident field magnitude $R = R(\varepsilon_{eff}(E_{inc}))$, $T = T(\varepsilon_{eff}(E_{inc}))$.

In the case of the nonlinear effective permittivity, dependences of the reflection and transmission coefficients magnitude versus the incident field magnitude are typical and have the form of hysteresis dependence. At a certain intensity of the incident field, the coefficients stepwise changes theirs value from small to large level or vice versa (Fig. 2). The obtained solution shows clearly that the reflection and transmission coefficients are multiple-valued functions of the incident field intensity. Thus the bistable operation conditions are achieved.

To conclude, in the present report, the optical properties of the periodic grating of dielectric bars with a nonlinear dielectric are studied. It is shown that the structure can support a bistable operating mode similarly to the devices based on the interferometric electromagnetic cavities.

Iterative computation of compactly supported solutions of functional-differential equations with linearly transformed argument

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Thanks to their unique properties atomic functions (AF) [1–2] are widely applied in interpolation, digital signal and image processing and boundary-value problems. Then construction of new AF and improvement of computational algorithms for known ones is an actual problem.

In this work a new iterative method for computation of compactly supported solutions of functional-differential equations (FDE) with linearly transformed argument is presented. We consider FDEs in such form:

$$f'(x) = \sum_{k=1}^{N} c_k f(a_k x + b_k), \quad a_k > 1. \tag{1}$$

Compactly supported solutions of FDEs (1) with condition $a_k = a$, $k = 1 \dots N$ are atomic functions.

Algorithm consists of two steps: iterative construction of self similar sequence with structure based on the form of right-hand side of FDE (1) and its summation. Modified algorithm representing alternation of operations of continuation of sequence and summation requires considerably smaller computing expenses [3]. In [3] presented the fast algorithm for computation of AF $\mathrm{up}(x)$, and $\mathrm{h}_a(x)$ satisfying to the following equations:

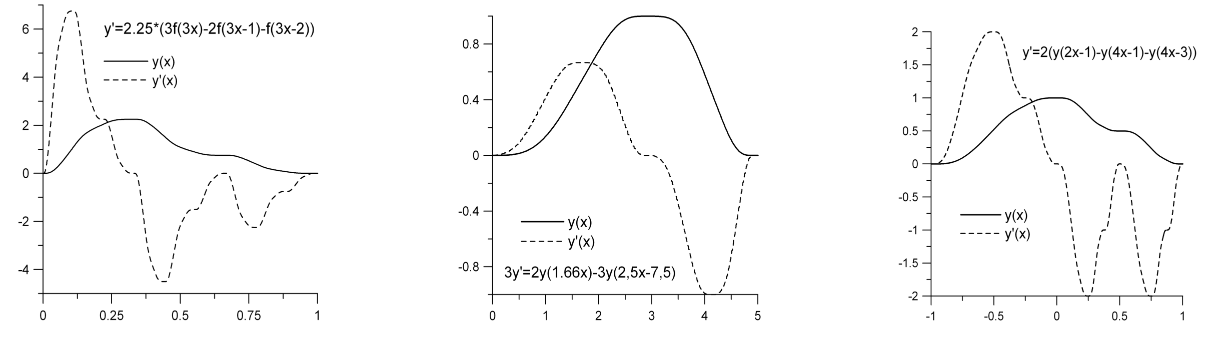
$$\mathrm{up}'(x) = 2\mathrm{up}(2x+1) - 2\mathrm{up}(2x-1), \qquad \mathrm{h}_a'(x) = \frac{a^2}{2}\left(\mathrm{h}_a(ax+1) - \mathrm{h}_a(ax-1)\right). \tag{2}$$

Note that atomic functions are even and symmetric and their increase and decrease intervals are in same scale. These properties make possible to use Fourier series for their computation.

In the case of more complicated structure of FDE usage of Fourier method may be more difficult or impossible. However iterative algorithm allows to find compactly supported solutions of such FDEs. In the report demonstrated examples of application of algorithm to the following cases:

- Function is not symmetric (increase and decrease intervals alternates in any arbitrary order). Example: $y' = \frac{9}{4}(3f(3x) - 2f(3x-1) - f(3x-2))$.
- Increase and decrease intervals are in different scales (a_k in equation (1) are different). Example: $f' = \frac{1}{3}\left(2f\left(\frac{5}{3}x\right) - 3f\left(\frac{5}{2}x - \frac{15}{2}\right)\right)$ or $f' = 2\left(f(2x-1) - f(4x-1) - f(4x-3)\right)$.
- Supports of terms $f(a_k x + b_k)$ intersects.

On the Figure 1 examples of the work of algorithm are presented.



Solutions of FDEs.

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Separation of Variables in Acoustically Concordant Fluid Media

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Acoustically concordant we name such fluid media in which dependences of density and of sound speed on co-ordinates have the following form:

$$\rho(x,y,z) = \rho(x)\cdot\rho(y)\cdot\rho(z),$$

$$c^{-2}(x,y,z) = c^{-2}(x) + c^{-2}(y) + c^{-2}(z).$$

In these media the wave equation

$$\Delta p - \nabla \ln\rho \nabla p + \omega^{-2}c^{-2}p = 0$$

by the search of solution in the form of

$$p(x,y,z) = p(x)\cdot p(y)\cdot p(z)$$

breaks up to three ordinary differential equations

$$d^2p(\xi)/d\varsigma^2 - d\ln\rho(\xi)/d\xi\cdot dp(\xi)/d\xi + \omega^2c^{-2}\left[c^{-2}(\xi) - m_\xi\right]p(\xi) = 0,$$

where $\xi = x, y, z$, m_ξ- separation constants, $\sum\limits_{\xi} m_\xi = 0$.

Thus, the solution of a three-dimensional problem is reduced to the solution of three one-dimensional problems. For example, being based on "solvable" [1] one-dimensionally non-uniform (flat-layered) media, we design a set of two-dimensionally and three-dimensionally non-uniform media, at once obtaining for them solutions by multiplication of already known one-dimensional solutions. Non-uniformity can be both smooth and stepwise. In sectionally-uniform media turn out simple plane-wave solutions. Studying of periodic structures — phononic crystals — is being carried out so, as studying of photonic crystals [2].

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Diffraction of nonstationary linearly inhomogeneous wave, which slides off a semi-infinite screen

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Exact solution for non-stationary diffraction by a semi-infinite soft screen is presented. Incident field has a plane wavefront, with the amplitude linearly varying along the front. The incident field runs from infinity along the screen and gives birth to a wave diffracted by the edge.

Rayleigh wave's transmission through a surface crack in elastic half-space

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Propagation of a Rayleigh wave in homogeneous elastic half-space that contains a surface vertical crack of finite depth is considered. Rayleigh wave's transmission through a crack is studied. Approximate approach proposed in [1] is used. According to this approach transmission coefficient is found by means of comparison of the energy flow of the incident Rayleigh wave's lower part (which is not cut off by the crack) with its whole flow of energy. Explicit analytical expression for transmission coefficient of a plane Rayleigh wave is obtained. The theoretical results calculated under the formula and the experimental results received in [2] are compared. It is shown that the results are close to each other for sufficiently long (in comparison with the crack's depth) Rayleigh waves.

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Planar waveguides: 3 kinds of dispersion equations and the number of optical modes

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We consider the propagation of monochromatic polarized electromagnetic waves trough a multilayer planar dielectric waveguide. An equation with roots equal to eigenvalues of effective refractive index of the waveguide is called a dispersion equation for this waveguide. Two kinds of dispersion equations were already known for planar waveguides. The first one is traditional dispersion equation, obtained by the well known method of characteristic matrices [1]. The second is equation, derived by equating to zero the determinant of the system of linear equations, expressing conditions on the bounds of layers. A new form of dispersion equation appeared in [2].

The author has carried out [3] the comparison and analysis of this 3 types of dispersion equations. Moreover, analyzing a new equation, the author has established formulas for the total number of optical TE an TM-modes possible in an arbitrary planar waveguide with a finite number of layers. This formula is effective. Using personal computer one can rapidly calculate the number of modes for waveguides consisting of hundreds of layers. The result is obtained by the development of the analitic method previously applied for calculating the number of TE and TM-modes in waveguides, consisting of layers of only two types [4], and the number of energy levels of a quantum particle in a comblike potential field (MQW structure)[5].

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Exact solutions for wave resonances in rectangular pyramidal cavity

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In spite of the great interest in pyramids as archeological and mysterious objects, only numerical solutions for wave resonances of pyramids are available in the literature till now [1,2]. The exact analytical solutions to the wave equation in a resonant pyramidal cavity are obtained for the first time in the present study. The results obtained correspond to the right pyramid with a square base and orthogonal opposite lateral faces. The shape of such a "rectangular" pyramid is characterized by four rectangular angles between the adjacent sides of the base and two rectangular angles between the opposite oblique faces. This particular shape allows one to search for the solution as a superposition of degenerate modes of a composite cube. The cube is formed by junction of six identical pyramids of the selected shape. The Neumann boundary conditions are assumed to be valid both at the pyramid boundaries and the cube walls. A Cartesian coordinate system with the origin at the apex of the pyramid is used to search for the solution. The detailed analysis of the solution is performed for three particular types of the pyramid modes. These are modes corresponding to the orientation of the wavevectors of partial waves along (i) the cube edges, (ii) the cube face diagonals, and (iii) the cube space diagonals. The analysis includes consideration of the following questions: (a) the coplanarity condition for the wavevectors of incident and reflected waves, and the normal to the reflecting boundaries, (b) the space arrangement of the nodes and loops of the wave field, (c) the relation of ray paths and resonant frequencies. For all mentioned modes beside antisymmetric ones, the wave field amplitude reaches a maximum at the apex and angles of the pyramid. From these modes, for the third order ones the geometric center of the pyramid, lying at the distance of one third of the height from the base of the pyramid, appears to coincide with one of the periodic maximums of the resonant wave field. The eigenfrequencies found from the exact wave solutions are in full agreement with the known resonance condition for the length of closed ray paths to be a multiple of the wavelength. It is remarkable that the derived formula for the resonant frequencies of modes of the first type coincides with the empirical approximate relation suggested in Ref. [1] for the numerical solution in a pyramid of similar (but not identical) shape.

The general wave solution for the pyramid resonances is found in the case when the corresponding wavevectors are oblique to the mentioned specific directions in the composite cube. According to the ray analysis of the coplanarity condition all less symmetric degenerate cube modes, corresponding to the oblique ray propagation, are potentially suitable as partial waves for constructing the pyramid modes. However the wave theory restricts the number of these modes by only those for which the quantum numbers defining the allowed coordinate projections of the wavevectors are of equal parity. Namely, the permitted pyramid modes are to be simultaneously symmetric or anti-symmetric with respect to all coordinate axes. To study the completeness of the solution the asymptotic number of pyramid eigenmodes is calculated in the high-frequency limit. This number happens to be four times less than that determined by the Weyl's law. It means that the mentioned above relation between the eigenmodes of the pyramid and of the composite cube can be used to calculate only one fourth of all modes of the pyramid.

The found solutions provide a better physical insight into the problem under study. They could be applied as initial approximations in solving more complicated problems by perturbation theory. In particular, the limitation of Q-factor of the pyramidal resonators due to small bulk dissipation or cavity-wall scattering and perturbation of the resonant frequencies due to regular shape changes, such as compression or elongation of the pyramid along a given axis, presence of facets or rounded edges etc, can be estimated in this way. It should be pointed out, that the mathematical formulation of the problem under study corresponds to the problems of acoustics. For acoustic waves propagating in a cavity filled by inviscid fluid (gas or liquid), grazing propagation along the cavity walls is permitted and the wave function has the meaning of the particle velocity potential. Besides, for plane harmonic waves involved in the found solution this potential is proportional to acoustic pressure. The excitation of acoustic resonances in the pyramidal cavity can be done by different methods, in particular, by the use of radiation of leaky Rayleigh waves propagating along the cavity walls. The value of the radiation angle should correspond in this case to the inclination of the wavevectors of partial waves of resonant modes of the pyramid. Note that the phase correction of the reflection coefficient at the Rayleigh incident angle should be included in this case into the found solution. The pyramidal cavity excited at the resonance can be used in microfluidic applications as ultrasonic concentrator in droplet ejectors. The presented acoustic solution for pyramidal cavity may be also of interest for its generalization to the case of waves of different nature.

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About problem of interaction of field and substance in classical electrodynamics

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It is considered the problem of interaction of the electromagnetic field and substance in form of the solid bodies fixed in the space. This problem may be solved on base of consideration of such integrals of Maxwell's equations as the momentums of the electrodynamic field with the help of the conception of the sourses of the electromagnetic field.

Four correct versions of the momentum are considered:

$$\mathbf{Q}_A = \mu_0\varepsilon_0 \int [\mathbf{E},\mathbf{H}]dv, \mathbf{Q}_M = \int [\mathbf{D},\mathbf{B}]dv, \mathbf{Q}_S = \varepsilon_0 \int [\mathbf{E},\mathbf{B}]dv, \mathbf{Q}_L = \mu_0 \int [\mathbf{D},\mathbf{B}]dv, \tag{1}$$

accordingly, to Abragam's, Minkovsky's, Stretton's and Lorentz's forms.

If the integration in (1) is carred out in V region, which contains one or several solid bodies, than the following general equation is obtained for momentum of any form:

$$\frac{\partial}{\partial t}\mathbf{Q}^V_{()} + \mathbf{F}_{()} + \mathbf{I}_S = 0, \tag{2}$$

where $\mathbf{F}_{()}$ is the force over the body or bodies in V, which depends on the form of momentum; $\mathbf{I}_S$ is a quasi-force produced by the Maxwell's stress space tensor over the external side of the surface S of region V; the quasi-force doesn't depends on the form of the momentum.

In the static case (2) gives the equation:

$$\mathbf{F}_A = \mathbf{F}_M = \mathbf{F}_S = \mathbf{F}_L = -\mathbf{I}_S, \tag{3}$$

where $\mathbf{I}_S = 0$ if there are no bodies exterior to the surface S, otherwise $\mathbf{I}_S \neq 0$. The formulae (3) is accorded to the laws Coulomb and Ampere and, in magnetostatics, to the articles in IEEE Trans. Mag. (for example, 2008, v. 44, no. 9, p. 2132).

In the case of the periodic electromagnetic presses averaged forces for the period are

$$\langle\mathbf{F}_A\rangle = \langle\mathbf{F}_M\rangle = \langle\mathbf{F}_S\rangle = \langle\mathbf{F}_L\rangle = -\langle\mathbf{I}_S\rangle.$$

In the general case the law of the preservations of the momentum doesn't hold because of the fixing the bodies in the spase.

Atomic functions in nonparametric estimations of probability density functions and their derivatives

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Nonparametric estimations of probability density function (PDF) [1–3] and its derivatives are considered. On the basis of atomic functions (AF) [4–8] the new constructions of weight functions (WF) with the compact support are offered that allowing to build admissible estimations of PDF and its 1st and 2nd order derivatives. The carried out physical analysis confirms quality of new admissible WF.

Constructions of admissible weight functions. Let $X_1, X_2, ..., X_n$ is sequence of n samples of random variable independent observations with unknown PDF $f(x)$. Its nonparametric estimation is defined by $f_n(x) = \frac{1}{nh}\sum_{j=1}^{n} K\left(\frac{X_j - x}{h}\right)$, where $h = h(n)$ is some sequence of positive numbers, $\lim\limits_{n\to\infty} h(n) = 0$ and $K(x)$ is even function satisfying to normalization requirement $\int_{-\infty}^{\infty} K(x)dx = 1$ and $K(x) \in L_2$. Weight function is called 'admissible' if its Fourier transform (FT) is nonnegative and no more than 1 for all real frequencies. Quality criteria of estimation of $f_n(x)$ is an mean integrated square error (MISE). If WF is admissible then MISE of estimation received with its help can't be reduced simultaneously for all PDFs. Therefore will consider of admissible WF construction on example AF $h_a(x)$ [4]. Let's enter the function $ch_a(x) = h_a(x) * h_a(x)$. Repeating operation of convolution of $(l-1)$ times will make function $ch_{a,l}(x)$. It is obvious that

$$\widehat{ch_{a,l-1}}(\omega) = \left(\prod_{k=1}^{\infty} \frac{\sin\left(\omega/a^k\right)}{\omega/a^k}\right)^l, \quad \sup\left(ch_{a,l}(x)\right) = \left[-\frac{l}{(a-1)}, \frac{l}{(a-1)}\right],$$
$$\int_{-l(a-1)^{-1}}^{l(a-1)^{-1}} ch_{a,l}(x)dx = 1, \quad l = 1, 2, ... \quad (1)$$

Admissible WF will construct by means of spectrum expression $\Psi_r(\omega) = 1 - \left(1 - \hat{\phi}_a(\omega)\right)^{r/2}$, $r = 2, 4, ...$, which positively and also no more than 1. Thus expression for calculation admissible WF the following

$$K_{a,r}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{1 - \left(1 - \hat{\phi}_a^2(\omega)\right)^{r/2}\right\}\exp(i\omega x)\,d\omega, r = 2, 4, ... \quad (2)$$

For example $K_{a,2}(x) = ch_{a,1}(x)$, $K_{a,4}(x) = 2ch_{a,1}(x) - ch_{a,2}(x)$, $K_{a,6}(x) = 3ch_{a,1}(x) - 3ch_{a,2}(x) + ch_{a,3}(x)$. Admissible value estimation of $f'(x)$ will receive in form $Df_n(x) = \frac{1}{nh^2}\sum_{j=1}^{n} N\left(\frac{X_j - x}{h}\right)$, where $N_r(x) = -K'_{r-1}(x)$, $r = 3, 5, ...$ Similarly for the second derivative of PDF $D^2 f_n(x) = \frac{1}{nh^3}\sum_{j=1}^{n} M\left(\frac{X_j - x}{h}\right)$, $M_r(x) = -K''_{r-2}(x)$, $r = 3, 5, ...$ Constructed WF have compact support and they are infinitely differentiable.

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Two-dimensional analytical Kravchenko–Rvachev wavelets in digital signal and image processing

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Construction of multidimensional filters is an actual problem of digital signal and image processing [1–7]. For the first time on the basis of the theory of R-functions [2–5] are constructed two-dimensional analytical Kravchenko–Rvachev wavelets with complex shape basic area (RA). The theory of R-functions allows to describe areas with various geometry at analytical level. Thanks to it procedure of construction analytical wavelets doesn't depend on a choice of the concrete form of basic area. Construction of two-dimensional analytical wavelet consists of several stages. The locus of basic area in time or frequency space is determine. By means of full system of R-functions and atomic functions (AF) [2–7] form its equation $\Omega(x_1,x_2) \geq 0$ such that $\max\Omega(x_1,x_2) = \Omega(0,0) = 1$. In time area the surface $W(x_1,x_2) \equiv \Omega(x_1,x_2)$ or $W(x_1,x_2) = \mathbb{F}^{-1}\{\Omega(f_1,f_2)\}$ where $\mathbb{F}^{-1}$ designates inverse Fourier transform is constructing. Two-dimensional function $W(x_1,x_2)$ is modulated by two-dimensional complex exponent [5,8,9] with necessary parameters. Thus, we receive two-dimensional analytical Kravchenko–Rvachev wavelets with basic area in time or frequency space of complex shape. They can improve considerably quality of a filtration and recognition in problems of digital signal and image processing. Numerical experiment and the physical analysis of modeling results confirm their efficiency.

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New constructions of complex orthogonal Kravchenko wavelets

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On basis of atomic function (AF) [1,2] the new complex orthogonal Kravchenko wavelets [3-10] are constructed. They characteristics are analyzed and numerical experiments of image wavelet-decomposition are carried out.

Complex Orthogonal Kravchenko wavelets are based on AF $ha(t)$ [1] and have smooth Fourier transform. The quadrature mirror filters $\hat{m}_0(\omega)$ [3,4] is using for their construction. To similarly to [6,7] results will receive equations for computation of scaling $\hat{\phi}(\omega)=\frac{1}{\sqrt{2}}\hat{m}_0\left(\frac{\omega}{2}\right)\hat{\phi}\left(\frac{\omega}{2}\right)=\prod_{k=1}^{\infty}\frac{1}{\sqrt{2}}\hat{m}_0\left(\frac{\omega}{2^k}\right)$ and wavelet $\hat{\psi}(\omega)=\frac{1}{\sqrt{2}}\hat{g}\left(\frac{\omega}{2}\right)\hat{\phi}\left(\frac{\omega}{2}\right)$ functions Fourier transforms, where $\hat{g}(\omega)=\mathrm{e}^{-i\omega}\hat{m}_0(\omega+\pi)$. Then $\hat{\psi}(\omega)=\frac{1}{\sqrt{2}}\mathrm{e}^{-i\omega}\hat{m}_0(\omega+\pi)\prod_{k=1}^{\infty}\frac{1}{\sqrt{2}}\hat{m}_0\left(\frac{\omega}{2^k}\right)$. Let's choose the following quadrature filters $\hat{m}_0(\omega)=\sqrt{2}\cdot\sqrt{\frac{2}{a}h_a\left(\frac{2}{a\pi}\omega-2\nu\right)}$. Here ν is frequency shift. This function is satisfied to following conditions: $\sup\hat{m}_0(\omega)=\left[-\frac{2}{3}\pi+\frac{\nu}{2};\frac{2}{3}\pi+\frac{\nu}{2}\right]$, $\hat{m}_0(\omega)=\sqrt{2}$ for $\omega\in\left[-\frac{1}{3}\pi+\frac{\nu}{2};\frac{1}{3}\pi+\frac{\nu}{2}\right]$, $\hat{m}_0(0,5\pi)=1$. The constructed functions satisfy to all wavelets properties [6-8] basic of which are zero average $\hat{\psi}(0)=0$ and stability condition $A\le\sum_{k=-\infty}^{\infty}\left|\hat{\psi}\left(\frac{\omega}{2^k}\right)\right|^2\le B$ and also they are complex. Functional basis generated from compressions and shifts of $\psi(x)\in W_0$ ($V_{j+1}=V_j\oplus W_j$, $W_j\perp V_j$, $W_j\perp W_k$ for $j,k\in\mathbb{Z},\ k\neq j$) is orthogonal. Energy of scaling and wavelet functions of level N it is equal to energy of scaling function of level N+1.

Conclutions.

New class of complex orthogonal Kravchenko wavelets with various physical characteristics and time-and-frequency properties are received. In specific case, when $\nu=0$ wavelets are real. At $\nu=0$ and $a\to\infty$ receive Kotelnikov–Shannon wavelet. The numerical experiments of image wavelet-decomposition and also comparison with known wavelets have shown efficiency of new complex wavelets.

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Properties of a solution to the Dirichlet problem for the Helmholtz equation in a planar domain with cracks

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We discuss the Dirihlet problem for the Helmholtz equation in an interior or exterior multiply connected domain. The propagative Helmholtz equation is considered in an exterior domain, while the dissipative Helmholtz equation is considered in both interior and exterior domain. The boundary of the domain contains both closed curves and double-sided open arcs (cracks). More precisely, the boundary may contain finite number of smooth cracks of finite length as well as finite number of smooth closed curves. The Dirichlet boundary condition is specified on the whole boundary, and the different boundary data may be specified on the opposite sides of the cracks.

If the boundary data satisfies matching conditions at the ends of the cracks, and so we look for the classical solution, which is continuous at the ends of the cracks, then the well-posed formulation of the problem was given in [1–4]. The existence of the unique classical solution was proved in [1–4] under certain smoothness conditions. The integral representation of the problem has been obtained in the form of potentials. The problem has been reduced to the uniquely solvable integral equation for the densities in potentials. Singularities of a gradient of the solution at the ends of the cracks have been studied.

In the present talk we consider the case, when the boundary data does not satisfy matching conditions at the ends of the cracks, and so we look for the solution, which is not continuous at the ends. The well-posed formulation of the problem is given. The uniqueness of the classical solution is proved by integral equalities. The existence of the classical solution is proved by the boundary integral equation method. The integral representation for a classical solution of the problem is obtained in the form of potentials. The densities in the potentials satisfy the uniquely solvable integral equations. So, we reduce the original Dirichlet problem to the uniquely solvable integral equations. The singularities of the gradient of the solution at the ends of the cracks is studied using integral representation for a classical solution. It is shown that the weak solution of the problem does not exists typically unlike the classical solution. We formulate the conditions under which a classical solution exists, while a weak solution does not exist. This result is very important for construction of numerical methods for solving problems in cracked domains, since all finite element and finite difference methods imply existence of a weak solution.

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Electrodynamic characteristics of a strip loop antenna located on the surface of a gyrotropic cylinder

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Much previous work on the current distribution and input impedance of a loop antenna in anisotropic and gyrotropic media applies to the case where the medium parameters are independent of the spatial coordinates (see, e.g., [1, 2] and references therein). In recent years, a substantial degree of interest has been shown in the characteristics of loop antennas in inhomogeneous gyrotropic media. Of special interest is the case where the antenna is operated in the presence of a gyrotropic cylinder [3]. Such a situation is frequently encountered in many practical applications and represents a geometry for which the antenna problem can be solved using the integral equation method. It is the purpose of the present paper to find the current distribution and input impedance of a strip loop antenna located on the surface of a gyrotropic cylinder in free space.

We consider an antenna having the form of an infinitesimally thin, perfectly conducting, narrow strip of half width d coiled into a circular loop of radius a ($d \ll a$). The antenna is located coaxially on the surface of a uniform circular gyrotropic cylinder placed in free space and aligned with the gyrotropic axis taken as the z axis of a cylindrical coordinate system (ρ, ϕ, z). The medium inside the cylinder is described by a general dielectric tensor with nonzero off-diagonal elements. The antenna is excited by a time-harmonic voltage which creates an external electric field with only nonzero azimuthal component $E_\phi^{\rm ext}$ on the surface of the strip in a narrow angular interval.

Using the boundary conditions on the surface of the cylinder ($\rho = a$) as well as on the antenna surface ($E_\phi = -E_\phi^{\rm ext}$ and $E_z = 0$ at $\rho = a$ and $|z| < d$) for the field excited by the antenna current, we derive the integral equations

$$\int_{-d}^{d} K_{\phi,m}(z - z')I_m(z')dz' = -A_m, \quad \int_{-d}^{d} K_{z,m}(z - z')I_m(z')dz' = 0$$

for the complex amplitudes of the angular harmonics $I_m(z)$ of the surface current density $I(\phi, z)$. Here, m is the azimuthal index ($m = 0, \pm 1, \pm 2, \ldots$), $|z| < d$, A_m are the coefficients of a Fourier series expansion with respect to the azimuthal angle for the voltage supplied to the antenna and creating the field $E_\phi^{\rm ext}$, and $K_{\phi,m}(\zeta)$ and $K_{z,m}(\zeta)$ are the kernels of the integral equations. The kernels can be represented as the sums of singular and regular parts. For a sufficiently narrow strip (i.e., in the limit $\zeta \to 0$), the kernels $K_{\phi,m}(\zeta)$ have logarithmic singularity, while the kernels $K_{z,m}(\zeta)$ have Cauchy singularity. Upon replacing the kernels by their small-argument approximations and taking into account the fact that the regular parts of the kernels $K_{z,m}(\zeta)$ are zero for $\zeta = 0$, it can be shown that for each m, a solution to the equation with the kernel $K_{\phi,m}(\zeta)$ satisfies the corresponding equation with the kernel $K_{z,m}(\zeta)$. These properties of the integral equations make it possible to obtain solutions for $I_m(z)$ in the form $I_m(z) = B_m(d^2 - z^2)^{-1/2}$, where B_m are coefficients depending on the medium parameters and proportional to A_m. Note that the singularities of the solutions for the surface-current angular harmonics at $z = \pm d$ are integrable. Hence, the total current along the strip, yielded (upon summation over m) by integration of the surface current density $I(\phi, z)$ with respect to z from $-d$ to d, is finite. Using the above-described approach, we calculated the current distribution and input impedance of the antenna. Detailed results of the performed numerical calculations will be reported for some cases of interest.

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Oscillations passing through a layer of nonlinear noncompressible elastic medium

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The problem for periodic (or nearly periodic) transverse one-dimensional waves to pass through a layer of weakly anisotropic elastic medium is considered. It is assumed, that the layer separates two half-space consisting of a medium that is much more rigid, as compared to the medium in the layer. This enables us to consider in the zero approximation the velocity components of one boundary of the layer as known ones and to determine the strains at the other fixed boundary. It is assumed, that the period of the incoming waves is close to the period of self-oscillations of the layer.

A system of differential equations which is obtained early for describing the slow variations of layer nonlinear oscillations is employed to investigate the zero approximation solution. The qualitative analysis of the strains behavior in time at the fixed boundary is performed. It is shown, that the functions describing the time evolution of strains can be discontinuous. The discontinuity conditions are obtained and it is noticed, that they coincides with shock conditions for the shocks propagating in a homogeneous anisotropic elastic medium.

The investigation of the first approximation is performed on the base of the above mentioned solution for zero approximation. The solution demonstrates the complicate dependence of outgoing wave on an incoming one.

On the coupled time-harmonic motion of water and a freely floating body

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In the present work we study (with a particular attention to the uniqueness property) a problem, that describes the time-harmonic small-amplitude motion of the mechanical system that consists of a three-dimensional water layer and a body (either surface-piercing or totally submerged), freely floating in it. The latter means that there are no external forces acting on the body, for example, due to constraints on the body motion. Water extends to infinity in horizontal directions, but has a finite depth being bounded from below by a horizontal rigid bottom and from above by a free surface. Thus our model describes a vessel freely floating in an open sea of a constant depth. The coupled boundary value problem under consideration contains a spectral parameter — the frequency of oscillations — in the boundary conditions as well as in the equations governing the body motion. We prove that the total energy of the water motion is finite and establish the equipartition of energy of the whole system. Under certain restrictions on body's geometry the problem is proved to have only a trivial solution for sufficiently large values of the frequency. The uniqueness frequencies are estimated from below.

About one new method of solving diffraction problems

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Pattern equation method (PEM) is widely used for solving diffraction problems. One of the Pattern Equation Method (PEM) [1] advantages is a weak dependence of the speed and accuracy of computational algorithm on the distance between single scatterers. This fact leads us to the idea of modeling scattering characteristics of complex geometry bodies by solving wave diffraction problems on the group of more simple geometry bodies, which in the aggregate reproduce examined complex object. Calculations have showed the efficiency of such approach [2]. Natural generalization of this idea is to substitute a scatterer with complex geometry and structure, for example a non homogeneous magneto-dielectric scatterer, by the group of simple homogeneous bodies, for example spheres (or circles in two dimensional case), which sizes are small in comparison to the wave length.

As an example let's consider two dimensional diffraction problem with Dirichlet boundary condition.

Using typical for PEM technique we can reduce this problem to the infinite algebraic system relatively to Fourier expansion coefficients c_{jn} of body j scattering pattern $g_j(\phi)$ [1].

Let's perform mentioned above modeling of complex geometry bodies scattering characteristics by solving this system for elementary scatterers in the form of circular cylinders, using one mode approximation, at which scattering pattern Fourier expansion reduced to the one summand. In this case above system takes the following form:

$$c_{j0}H_0^{(2)}(ka_j) + J_0(ka_j)\sum_{l=1}^{N}(1-\delta_{jl})H_0^{(2)}(kr_{lj})c_{l0} = -e^{-i\vec{k}\vec{r}_{0j}}J_0(ka_j), \quad j=\overline{1,N}.$$

Here a_j – cylinder j radius, $\vec{k}\vec{r}_{0j} = kr_{0j}\cos(\phi_0-\phi_{0j})$, and r_{0j}, ϕ_{0j} – coordinates of the body j center from the general origin, ϕ_0 – incident angle of the primary plane wave, r_{lj} – distance between body j and body l centers.

As an example let's consider a plane wave diffraction problem on a strip, which has a width $kL = 5$. Let's substitute this strip by the array of 25 circular cylinders with radius $ka = 0,1$, positioned close by next to each other. Performing the comparison of strip and circular cylinder array scattering patterns (see figure 1), we obtain good coincidence of these two patterns, although we substituted infinitely thin strip by the scatterer with finite width δ ($k\delta = 0,2$).This fact gives us foundation to expect more accurate results by using similar "approximations" for scatterers, which are not infinitely thin.

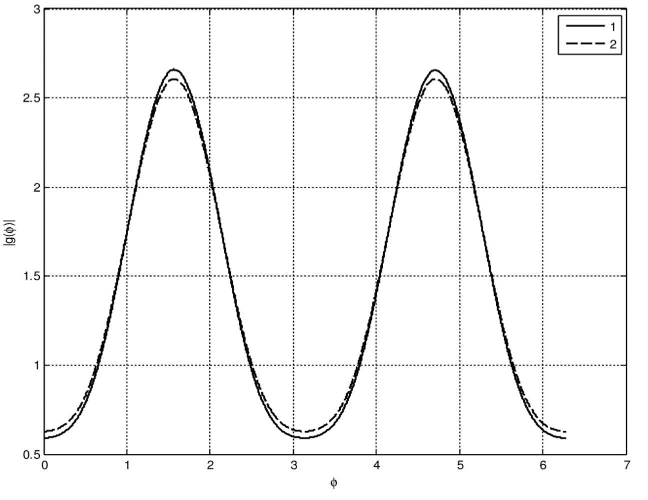

Figure 1. Scattering patterns of strip (curve 1) and circular cylinder array (curve 2)

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Edge Diffraction Phenomena in Formation of Acoustic Images

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The output signal of ultrasonic vision systems is formed by secondary radiation coming from the focal region of a probe beam. For monostatic (one-lens) configuration the main source of the received signal is back scattering. In flat and homogeneous areas backscattering is reduced to reflection - distinctions in reflectivity, interference of directly reflected radiation and leaky surface waves cause variations of the output signal value and, finally, contrast in acoustic images. But principal components of images are linear and point elements– phase boundaries, object contour and inclusion borders, edges and so on. Just sharpness of these elements determines quality of images. The work is aimed at analysis and experimental investigation of imaging features caused by contribution of diffracted waves. The study has been focused at short-pulse acoustic imaging with low-aperture probe beams. This variant is the main for bulk ultrasonic imaging – incident radiation effectively penetrates into the specimen bulk when the half-aperture angle of the incident beam is smaller than critical angles at the solid specimen interface with a fluid immersion; for such probe beams excitation leaky waves at the interface is lacking.

According to general concept of radiation interaction with objects, the output signal is a superposition of echo signals resulted from the incident beam reflection at a plane obstacle and beam diffraction at its linear or point elements. When probe beam is focused at the obstacle face, arrival times of the both signals coincide. While defocusing, the signals are separated in time. For the reflected signal its arrival time and a position of its abrupt variation retain unchangeable. The diffracted wave is received within a rather wide range of lens positions; its arrival time varies when shifting the probe beam axis about the edge position. In B-scans trajectories of diffracted impulses are displayed as parabolic whiskers bounded by angle aperture of the probe beam. The whiskers are observed at outer contour of specimens, at numerous small-sized irregularities of rough boundaries and interfaces, at elements of internal microstructure. elements originating from edge diffracted waves are indispensable details of all acoustic images – echo patterns, B- and C-scans. It has been shown edge diffracted waves are the main cause of line blurring in C-scans under defocusing.

Field Singularities in Focused Beam Interaction with Anisotropic Specimens

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Specific features of focused beam interaction with anisotropic specimens are considered to deduce basic principles of bulk acoustic imaging and local measuring in anisotropic objects.

Main features of ultrasound propagation in anisotropic media are caused by flat areas at the slowness surface. Waves with wave vectors directed to such a flat region possess the same group velocity. This velocity invariability provides specific structure of the acoustic field. At big distances r the dominant result of elastic anisotropy is energy concentration along preferential directions given by null-curvature points of the slowness surface. Just this type of radiation patterns is observed in phonon pulse propagation through crystals, when the rate of amplitude decay because of diffraction divergence $r^{-\alpha}$ is reduced ($\alpha < 1$) along the acoustic concentration direction comparing the ordinary value $\alpha = 1$ (*phonon focusing* phenomena). But in acoustical imaging of primary importance are near-field effects – phase relations between wave components of a received beam play a crucial role in output signal formation. Field singularities – focuses and ray convergence points; dictate positions of a focusing receiver for effective registration of reflected or transmitted signals in single- and double-lens configurations.

The dominant phenomenon governing structure of the secondary radiation from focused beam interaction with an anisotropic specimen is equal time of specimen traverling along each of parallel rays corresponding to a flat part of the slowness surface. It results in transfer of the sonic spot from the specimen face onto its backside along the direction given by the normal to the corresponding flat region of the slowness surface. After refracting transmitted radiation forms a new focus behind the specimen. Transferred focal points arise for each flat domain at the slowness surface. Positions of secondary focuses are determined by the group velocity direction for each flat area. The same transfer and multiplication of partial focuses can be observed in the reflected radiation, if wave vectors of radiation reflected from the specimen bottom are brought into another flat region of the slowness surface. For reflected radiation partial focuses are formed in front of the specimen. Partial focuses are formed not only for null-curvature elements, but also for small-curved domains of the slowness surface sheets. Probable specimen arrangements in single- and double-lens systems to observe displacement and multiplication of focus points are discussed; results of experimental studying structure of singularities in the reflected field are presented.

Wigner–von Neumann perturbations of the Kronig–Penney model

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It is known since 1929 due to J. von Neumann and E. P. Wigner that the half-line Schrödinger operator

$$-\frac{d^2}{dx^2}+\frac{c\sin(2\omega x)}{x}$$

has $\sigma_{\mathrm{ac}}=[0,+\infty)$ and it may have an eigenvalue at the point ω^2 embedded into the absolutely continuous spectrum, and in fact it has in the case $|c|>2|\omega|>0$ for one and only one self-adjoint boundary condition at the origin.

At the same period of time in 1931 R. de L. Kronig and W. G. Penney considered the whole line Schrödinger operator formally given by an expression

$$-\frac{d^2}{dx^2}+\sum_{n\in\mathbb{Z}}\alpha\delta(\cdot-dn)$$

with some $\alpha>0$ and $d>0$. This operator describes the behavior of a non-relativistic particle in a one-dimensional lattice, where α corresponds to the charge of repulsive interaction centers in the lattice and d corresponds to the period of the lattice. The spectrum of the Kronig–Penney operator has band-gap structure with bands of the purely absolutely continuous spectrum.

In the talk perturbations of the Kronig–Penney model by changing strengths of the interactions or changing the distances between centers of the interaction in a Wigner–von Neumann way will be discussed.

Using the modification of the Levinson theorem for discrete linear systems obtained by Benzaid and Lutz and the subordinacy theory developed by Gilbert and Pearson we show that the absolutely continuous spectrum is stable, but in each band two "critical" points appear. The conditions for the existence of embedded eigenvalues in these "critical" points will be given. Mathematical results will be compared with known results of numerical experiments.

On a behaviour of localized solution of the 2-D wave equation with variable velocity for small time

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We study the solution to the Cauchy problem for the wave equation with variable velocity and localized initial data before a creation of a front. The final formula for the solution is presented in the form of the Maslov canonical operator. For the localized sourses of special type we got an effective formula. Our results can be used for modeling of the initial phase of the creation of tsunami waves.

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Lateral relativistic electron beam synergetic creation and transport by petawatt laser radiation

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In experiments on interaction of superintense p-polarized laser radiation ($I \sim 10^{18}$–10^{20} W/cm^2) of near IR spectrum range with metal foils the occurrence of powerful electron beam transfer along the metal target out of the irradiated zone has been observed and investigated. The lateral relativistic electron beam has a speed $v \sim 0.9$ c, average electron energy $\geq$1MeV, and propagates up to few millimeters from the irradiated zone [1]. Here c is the speed of light in vacuum. Up to 30% of incident laser energy was transformed into the lateral electron beam energy. Simultaneously, high strength magnetostatic fields were recorded near surface [2]. The propagation of high energy electron beam causes high energy positive ions acceleration by Coulomb force in the direction of metal foil surface normal.

Theoretical models have been developed to explain the observed phenomenon. At present time, the model developed by Nakamura et al [3] is accepted.

We propose the physical model to explain the lateral electron beam creation and transport based on the conception of powerful surface plasmon polariton resonant excitation by incident beam of p-polarized laser radiation at large or moderate angles of incidence. The surface plasmon polariton wave is excited at and propagates along *the layer of emitted from metal foil surface electrons – vacuum* boundary. The wave is excited by means of self-organized electron density resonant grating according to the universal polariton model of laser-induced periodic structures formation [4] which involves the positive feedback through the grating modulation depth. The powerful amplified in limits of irradiated zone propagating wave of surface plasmon polariton drags the electrons of charged layer and transports the relativistic lateral electron beam. The spatial charges layer of electrons is balanced in the direction of surface normal by the attractive action of Coulomb force and repulsive one of ponderomotive force of surface plasmon polariton electric field exponentially decaying into vacuum. The existence of powerful relativistic lateral electron beam due to the Coulomb force action produces energetic positive ions from the metal surface in the direction of surface normal.

In conclusion, the considered phenomenon is explained self-consistently without any suggestion about the causes of magnetostatic field origination.

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Overcoming diffraction limit in optics by nonlinear laser-matter interaction

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Limits of the usual far-field optics resolution is well known and is fundamental in nature. Physics of electromagnetic waves propagation obstacles the possibility of pattern resolution lower than so called Abbe–Rayleigh limit. Information about subwavelength patterns of irradiated object is transferred by evanescent waves which exponentially decreases from the surface distance. The known approaches to overcome the diffraction limit utilize as condensed matter nonlinear properties as the properties of surface plasmon polaritons. The proposed by us method unites both above approaches.

The theory of the approach is based on the nonlinear mathematical model – one-dimensional unimodal logistical map which models the process of spatial periodic structures formation under the action of linear polarized laser radiation [1]:

$$f\left(\mu,\, x\right) = 1 + \mu x \left(1 - x\right), \qquad x \in [0,\ 1]; \qquad \mu \in [\mu_1,\ \mu_2].$$

Here $x \sim \left(\frac{I_s}{I}\right)^{1/2} \sim \frac{E_s}{E}$, I ($I_s$) is the absorbed intensity of incident laser radiation (surface plasmon polariton wave), μ is a controlling parameter.

The example of experimental realization of periodic structures under the action of series of 125 fs linear polarized radiation (λ=800 nm) on the HOPG graphite surface [2] showed the formation the structures with periods $\frac{\lambda}{8\eta} \approx 70$ nm and $\frac{\lambda}{4\eta} \approx 180$ nm which is lower than diffraction limit value $\frac{\lambda}{2\eta}$. One more example is the production of the structures with period $\frac{\lambda}{8n\eta} \approx 130$ nm for ceramic TiN for 40 fs laser pulses [3] (n(λ=800 nm)=0,7). Here η is the real part of refractive index of the boundary TiN-nonequilibrium metallic plasma TiN for surface plasmon polariton, $\eta \geq 1$. The produced nanostructures orientation in experiments was $\vec{g}//\vec{E}$.

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New aspects in the search of exact analytical wave solutions of different equations on the base of the relatively undistorted wave anzats

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In the famous book (R. Courant and D. Hilbert, Methods of Mathematical Physics, v.2, 1937) the concept of "relatively undistorted wave" was introduced. The corresponding anzats is

$$u = g(x_1, x_2, x_3, t) \cdot W(S(x_1, x_2, x_3, t)) \tag{1}$$

that are the generalization of the traditional solution in the form of invariable (the case of $g \equiv 1$) progressive wave for the classical linear wave equation. In expression (1) S is a phase function; the function of one variable $W(.)$ presents the waveform and "amplitude" function g describes the scale change of waveform under the conservation of typical shape of this waveform.

Much later (beginning from nineties of past century) this concept plays the big role in the search of new analytic solutions of the wave equation. But there is not any variety in the work with this object: the main method is the complication and transformation of the known now solution.

The aim of the report is the presentation of any new ideas concerning the possibilities of anzats (1):

1. Traditionally (starting with Courant R. and Hilbert D.) the search of new solution of wave equation with help of the relatively undistorted wave anzats is realized under the condition of arbitrary function $W(.)$. Such solutions are relative to the class of nondispersive wave solutions. But there are many important possibilities for solutions in the use of connection of dispersive condition with the realization of considered anzats. This case and its details will be object of consideration in report.

2. There is good physical validity for successful use of the relatively undistorted wave anzats (with little modification in its form and application method) in the finding of new exact analytical solutions for all basic model nonlinear acoustic equations in addition to the active traditional "exploitation" of this anzats for classical linear wave equation. Indeed, in nonlinear acoustics under the absence of dispersion the propagation of one-dimensional (in space) wave or a spatially localized wave structure (e.g., a beam) is accompanied by transformation of the wave profile (time profile). In the general case, this transformation has a complex character (e.g., transformation of a sinusoid into a saw-tooth profile), but there can be certain types of profiles that transform according to a simpler law reflecting only their variations in scale in the case when they retain their specific forms that describe the desired solution in the form of a relatively undistorted wave. The simplest and very well known example of this situation is a periodic saw-tooth solution to the equation of a simple wave with a profile that retains its form during propagation. However, in this case, their peak values decrease; i.e., this profile is relatively undistorted. In report this idea is evolved and some exact solutions of all types of nonlinear acoustics equations are found.

Seismological implications of impedance-like boundary conditions

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Impedance-like boundary conditions are successfully applied in different fields of physics and material science. However, they are not very well-known in seismology. The purpose of this paper is to discuss potential applications of these boundary conditions in the framework of theory of elastic wave propagation with emphasis on seismology. Generally, such boundary conditions are always interesting in those cases, where the influence of any body on a wave field in its interior is to be described by an integral effect on its surface (keyword: equivalent boundary conditions). It is well-known that we encounter a great variety of different boundaries in the Earth's interior. Basically, the contact between two solids is a complicated phenomenon. However, for the interaction of seismic waves with discontinuities very often an ideally welded contact is assumed which implicates continuity of the relevant displacement and stress components. This is only a first approximation which can be and must be extended. It is not unusual to assume contact planes as very thin layers which results in the formulation of novel boundary conditions.

We pick in this paper from the big family of impedance-like boundary conditions only two ones: (1) the boundary conditions after Tiersten (1969) and its improvement by Bövik (1996) and (2) the boundary conditions after Pod"yapol'sky (1963), which are important for the description of different phenomena. Tiersten's boundary conditions simulate the influence of a very thin layer on half-space for low frequencies. By applying these boundary conditions we study the influence on the dispersion of Love and Rayleigh waves and compare with the new results of Tran Thanh Tuan et al. (2008).

The conditions of Pod"yapol'sky (1963) are characterized by a continuous normal component of displacement but a discontinuous tangential component on the plane of contact. The jump is proportional to the tangential stress. It can be shown that these conditions simulate an embedded layer with vanishing S-wave velocity. They were successfully applied by Its and Malischewsky (1987) for the propagation of surface waves in laterally disturbed media.

Finally it is demonstrated that such conditions significantly influence the propagation of usual Stoneley waves and allow, at the very least mathematically, the existence of Stoneley waves with SH-polarization.

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Tilted-front laser pulses for efficient terahertz generation: Effects of diffraction and dispersion

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Optical rectification of femtosecond laser pulses with tilted intensity front in electro-optic crystals provides record optical-to-terahertz conversion efficiencies nowadays [1]. In this technique, a pump optical pulse with the intensity front tilted at a certain angle α with respect to the phase fronts propagates with the optical group velocity V in the direction normal to the phase fronts (Fig. 1). The projection of this velocity on the direction perpendicular to the intensity front is $V\cos\alpha$ and can be made equal to the phase velocity of terahertz wave with a given frequency by proper choice of the tilt angle α. Thus, a phase matching with the quasiplane terahertz wave propagating in the direction normal to the intensity front can be achieved. Using the tilted-front excitation technique is particularly a way to circumvent the large velocity mismatch between optical pulses and terahertz waves in the materials with high optical nonlinearity, such as $LiNbO_3$.

Tilting the intensity front of the laser pulse is typically achieved by reflection from or transmission through a diffraction grating. A drawback of the tilted-front laser pulses is that the pulses suffer from significant diffraction distortion. Additionally, in an electro-optic crystal the pulses experience dispersion broadening. These two factors restrict the interaction length between the pump laser pulse and generated terahertz waves, thus, limiting the optical-to-terahertz conversion efficiency. In this paper, we study evolution of the tilted-front laser pulse in $LiNbO_3$ crystal typically used in experiments.

We consider the geometry with the diffraction grating (of a period p) placed on the entrance boundary of the $LiNbO_3$ crystal [2] (Fig. 1). The laser pulse with Gaussian temporal (of a 100 fs duration) and transverse (of a 2 mm width) profiles is incident from vacuum on the grating at the angle θ_i. We consider the pulse transmitted to the crystal in the first order of diffraction, i.e., at the angle θ_d.

Figure 2 shows the incidence angle θ_i that should be used for a given period p to provide the tilt angle $\alpha = 62.7°$ necessary for phase-matching in $LiNbO_3$ [3]. Figure 3 shows the evolution of the pulse in the crystal for $\theta_i = 28°$ and $p^{-1} = 2860\,\mathrm{mm}^{-1}$.

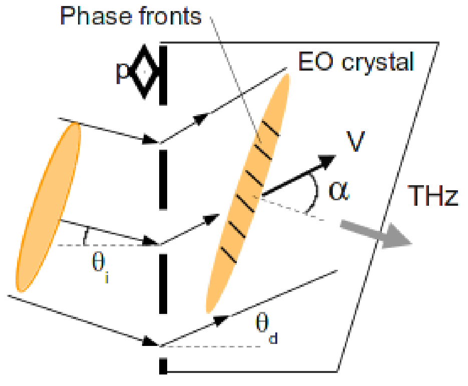

Fig. 1. Schematics of the tilted-front pulse excitation in the EO crystal.

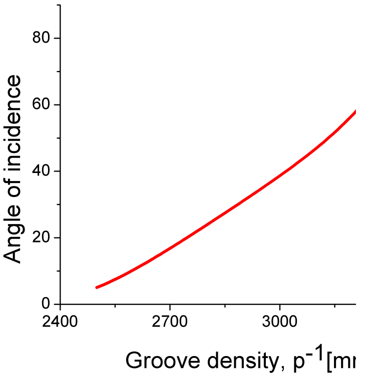

Fig. 2. The incidence angle as a function of the grating period for a fixed tilt angle $\alpha = 62.7°$.

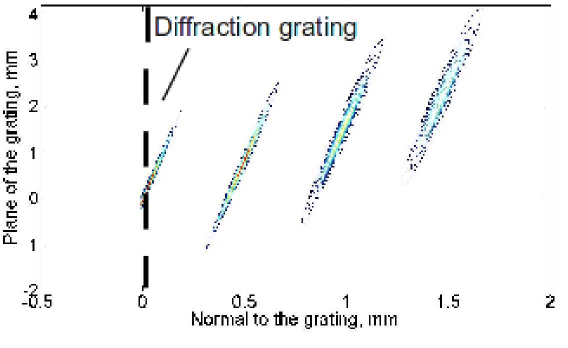

Fig. 3. Snapshots of the tilted-front laser pulse for several moments of time.

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Reconstruction of Complex Permittivity of a Nonhomogeneous Body in a Rectangular Waveguide Using the Method of Volume Singular Integral Equation

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We develop a method of solution to the inverse problem of reconstructing the media parameters (complex permittivity) of a body in a waveguide [1–3]. The analysis is based on the use of a volume singular integral equation and its reduction to an equation which can be solved, using iterations, both numerically and analytically. This enables us to determine, for a given single mode rectangular waveguide, the complex permittivity from the transmission coefficient. The approach also yields the proof of uniqueness of reconstruction of complex permittivity in a rectangular waveguide from the transmission characteristics. The results of test computations performed for the inclusion in the form of a parallelepiped justify the proposed approach.

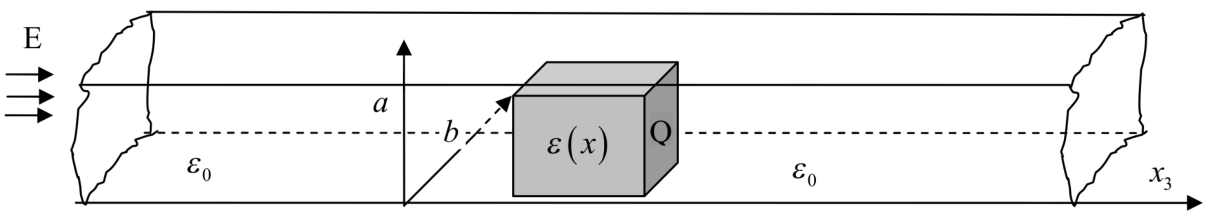

Fig. 1

Using a VSIE technique elaborated earlier for (forward) diffraction problems in waveguides, we have developed a method aimed at reconstruction of complex permittivity of a lossy arbitrarily-shaped body inside a waveguide of rectangular cross-section from the complex transmission coefficient. The approach can be implemented for a cylindrical or rectangular sample and generalized for a waveguide of arbitrary cross-section. The latter implies that the methods presented in this work can be used with data obtained from a variety of laboratory equipment and in a wide range of practical applications.

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Amplification of the sound waves in phonon crystals by subsonic electric current

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Sound propagation in medium with periodic lattice (the phonon crystal) in the presence of charge carriers drift is considered. The periodic lattice provides existence in sound field scattered components with wave numbers, equal to the sum and difference of the wave numbers of the initial sound wave k and wave number of the periodic lattice q. The case of $q >> k$ is considered. A phase velocities of scattered components are much smaller as compared with phase velocity of the sound. Interacting with current of the charge moving with subsonic velocity, scattered component can take up the energy of moving charges, in accordance with well known mechanism Landau for plasma physics and with mechanism Miles for wind generated waves. Analytical estimations for one dimension model of the effect — wave vectors of the wave and the lattice and velocity of the charge drift are collinear — are given.

Propagation of the normal waves in Biot rods with opened and closed pores on free boundaries

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For the both porous rods the axially symmetric wave fields are constructed and the dispersion equations are derived. Roots of these dispersion equations are velocities of the normal waves and be considered as functions of frequency. In small frequencies one normal wave propagates in the porous rod with opened pores on boundaries. In the case of closed pores two normal waves propagate in the rod for small frequencies. For the velocities of these waves and for the volume velocities in the Biot medium, the estimating inequalities are derived. Besides the indicated waves, in the both rods there is a set of waves with infinite velocities for zero wave number. With increasing frequency the velocities of these waves are decreased and tend to the smallest volume velocity. But one peculiar velocity can approach the velocities of the Rayleigh wave on boundary of the rods. In the case of closed pores on boundary, the Rayleigh wave exists always [1]. In the case of opened pores on boundaries, the Rayleigh wave can be absent [2] and then velocities of all normal waves tend to the smallest volume velocity.

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On trapping of time-harmonic water waves by an axisymmetric, motionless, freely floating body

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We consider a spectral problem describing the time-harmonic small-amplitude motion of the mechanical system that consists of water, occupying a half-space, and a freely floating body (either surface-piercing or totally submerged). A spectral parameter — the frequency of oscillations — appears in the boundary conditions of this coupled problem as well as in the equations governing the body motion. For the problem under examination we investigate three questions.

First, we prove that the total energy of the water motion is finite and establish the equipartition of energy for the whole system. Second, we show that any value of frequency is an eigenvalue of the problem for a certain body. In our proof, such a body is surface-piercing and has an axisymmetric submerged part that belongs to some family of geometries obtained with the help of the inverse procedure. Infinitely many such families are constructed, and for each of them the existence of trapped modes (non-trivial solutions) is proved. The characteristic feature of the obtained solutions is that the body remains motionless despite the fact that it floats freely in the presence of the trapped time-harmonic waves. Finally, we prove that under certain restrictions on body's geometry the problem has only a trivial solution for sufficiently large values of the frequency.

A new method for the calculation of diffraction loss due to building

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In this paper an advanced model for radio wave propagation in urban and suburban areas is proposed. The proposed model is based on geometry theory of diffraction (GUTD) as well as uniform theory of diffraction (UTD) and so it is appropriate for numerical calculation. Recently we have proposed a general UTD-based model for multiple diffractions by buildings therefore in this paper we have endeavored to extend the new model to involve trees and vegetable effects. In our model we supposed buildings are the same height and spacing and are organized by street systems into rows. Moreover we assumed that, trees and vegetable are situated adjacent to the buildings. The canopies of trees are modeled as discrete ensembles of leaves and branches. After comparing our results with the measurements it is found good agreement between them which justify our model. In addition the model is capable for considering the spherical wave instead of plan wave, since this point is important in situation where the transmitter is near the building.

The enforced stability of eigenvalues embedded into the continuous spectrum

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Two facts are well-known: Eigenvalues in the discrete spectrum are always stable but an eigenvalue in the continuous spectrum is not stable and a small perturbation of the problem data leads it out from the spectrum and turn into a point of complex resonance. Regarding a quantum waveguide with curved walls as an example, a new notion of the enforced stability is introduced and discussed together with concrete asymptotic formulas. The approach is based on asymptotic analysis of the so-called augmented scattering which gives a sufficient condition for the existence of an eigenvalue embedded into the continuous spectrum. The techniques developed also provides a new approach for justification of asymptotics of (stable) eigenvalues in the discrete spectrum as well as an asymptotic explanation of Wood's anomalies in the vicinity of the spectrum thresholds.

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Electromagnetic bound states in radiation continuum and applications to optical non-linear effects in periodic dielectric arrays

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Electromagnetic bound states in the radiation continuum are studied for periodic double arrays of subwavelength dielectric cylinders in TM polarization. They are similar to localized waveguide mode solutions of Maxwell's equations for metal cavities or defects of photonic crystals, but, in contrast to the latter, their spectrum lies in the radiation continuum. The phenomenon is identical

to the existence of bound sates in the radiation continuum in quantum mechanics, discovered by von Neumann and Wigner. In the formal scattering theory, these states appear as resonances with the vanishing width. For the system studied, the bound states are shown to exist at specific distances between the arrays in the spectral region where one or two diffraction channels are open. Analytic solutions are obtained for all bound states (below the radiation continuum and in it) in the limit of thin cylinders (the cylinder radius is much smaller than the wavelength). The existence of bound states is also established in the spectral region where three and more diffraction channels are open, provided the dielectric constant and geometrical parameters of the system are fine-tuned. The near field and scattering resonances of the structure are investigated when the distance between the arrays varies in the neighborhood of its critical values at which the bound states are formed. In particular, it is shown that the near field in the scattering process becomes significantly amplified in specific regions of the array as the distance approaches its critical values. This effect is then used to demonstrate that optical non-linear effects can be significantly enhanced and controlled by varying the distance between the arrays near its critical values. An example of the second harmonic generation is investigated in detail by means of perturbation theory for non-linear Maxwell's equations. The range of validity of the perturbation theory is also established.

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Periodic problems of diffraction of an elastic wave in space

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Investigate a problem of diffraction of an elastic wave the Flock on border of elastic semispace with bi-periodic system of defects. We will show that the problem can be reduced to two pair of the summatorial equations which can be reduced to the integrated equations.

Let on a plane Oxy the bi-periodic system of defects is located, and l_x and l_y – the periods of system of defects on axes x and y accordingly. Over a plane at $z > 0$ the elastic environment, and at $z < 0$ the rigid not deformable basis, therefore we will consider indignations of environment only in the top semispace. Let parameters of environment are set by constants ρ, λ and μ, and dependence on time in a kind $e^{i\omega t}$. Conditions of interaction of the elastic environment with the rigid basis we will set as follows: the area $[0, l_x) \times [0, l_y)$ at $z = 0$ is divided on two areas $\mathcal{M}$ and $\mathcal{N}$ with various conditions of contact. Let's define two types of conditions: 1) the elastic semispace is in full contact to the rigid basis; 2) the elastic semispace slides without a friction on the basis.

Consider the following problem: from the top semispace on border of section of environments, with located on it periodic system of defects, the elastic wave $w^{(0)}$ runs. Thus on $\mathcal{N}$ conditions of full contact are satisfied, and on $\mathcal{M}$ conditions of sliding without a friction are satisfied. It is required to find difraction wave w. In works [1] and [2] the similar class of problems in a two-dimensional case is investigated. Leaning against results from these works it is possible to prove following theorems:

Theorem 1. *The problem of diffraction of an elastic wave in semispace on bi-periodic system of the defects located on border of interface of the elastic environment with the rigid basis, is equivalent to system of two pair summatorial functional equations*

$$\left[2B_{sr}L_{xs} + D_{sr}\left(\beta_{2sr} - \frac{L_{xs}^2}{\beta_{1sr}}\right) - H_{sr}\frac{L_{xs}L_{yr}}{\beta_{1sr}}\right] e^{i\widetilde{L}_{xs}x} e^{i\widetilde{L}_{yr}y} +$$
$$+ \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\left[C_{nm}\left(\beta_{2nm} + \frac{L_{xn}^2}{\beta_{1nm}}\right) + E_{nm}\frac{L_{xn}L_{ym}}{\beta_{1nm}}\right] e^{i\widetilde{L}_{xn}x} e^{i\widetilde{L}_{ym}y} = 0, \quad (x,y) \in \mathcal{N},$$
$$\left[2B_{sr}L_{yr} + H_{sr}\left(\beta_{2sr} - \frac{L_{yr}^2}{\beta_{1sr}}\right) - D_{sr}\frac{L_{xs}L_{yr}}{\beta_{1sr}}\right] e^{i\widetilde{L}_{xs}x} e^{i\widetilde{L}_{yr}y} +$$
$$+ \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\left[E_{nm}\left(\beta_{2nm} + \frac{L_{ym}^2}{\beta_{1nm}}\right) + C_{nm}\frac{L_{xn}L_{ym}}{\beta_{1nm}}\right] e^{i\widetilde{L}_{xn}x} e^{i\widetilde{L}_{ym}y} = 0, \quad (x,y) \in \mathcal{N},$$
$$\left[-D_{sr}\left(L_{xs}^2 + \beta_{2sr}^2\right) - H_{sr}L_{xs}L_{yr}\right] e^{i\widetilde{L}_{xs}x} e^{i\widetilde{L}_{yr}y} +$$
$$+ \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\left[C_{nm}\left(L_{xn}^2 + \beta_{2nm}^2\right) + E_{nm}L_{xn}L_{ym}\right] e^{i\widetilde{L}_{xn}x} e^{i\widetilde{L}_{ym}y} = 0, \quad (x,y) \in \mathcal{M},$$
$$\left[-H_{sr}\left(L_{yr}^2 + \beta_{2sr}^2\right) - D_{sr}L_{xs}L_{yr}\right] e^{i\widetilde{L}_{xs}x} e^{i\widetilde{L}_{yr}y} +$$
$$+ \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\left[E_{nm}\left(L_{ym}^2 + \beta_{2nm}^2\right) + C_{nm}L_{xn}L_{ym}\right] e^{i\widetilde{L}_{xn}x} e^{i\widetilde{L}_{ym}y} = 0, \quad (x,y) \in \mathcal{M}.$$

Theorem 2. *The problem of diffraction of an elastic wave in semispace on bi-periodic system of the defects located on border of interface of the elastic environment with the rigid basis, is reduced to pair the integrated equations.*

Proofs of these theorems and a substantiation of search of the decision for periodic problems in the form of quasiperiodic decisions will be presented at conference.

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Effects associated with a saddle point of the dispersion surface of a photonic crystal

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We consider the propagation of a monochromatic electromagnetic wave in semi-inifinite photonic crystal. Crystal consists of alternating layers of two types of dielectric. The surface of the crystal is parallel to the layers. If the frequency of the wave coincides with the frequency of the saddle point, then the main properties of wave propagation are described by the hyperbolic wave equation, where the longitudinal coordinate plays the role of time. The solution of the hyperbolic equation propagates at a certain angle to the normal. We establish the relation between the parameters of the photonic crystal and the angle of the propagation, which varies from 0 to 45 degrees and does not depend of the angle of the incident wave. We propose the distribution of the field on the boundary of the crystal which provides the focusing beam of the light in the body of the crystal. The results of the numerical simulation are also presented.

Asymptotics of surface plasmons near curved interfaces

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Fabrication of nanoscale devices, problems of near-field optics, progress in metamaterials stimulate investigations of surface plasmons polaritons (SPP) and surface magnetic plasmons (SMP). The known results in surface modes deal with simple geometries (spherical or cylindrical cases) or with numerical calculations. An analytical methods, if they are applied, are aimed to development of more effective numerical methods.

In the present paper a new simple explicit formula demonstrating the dependence of the field of surface modes on the curvature of the surface, dielectric permittivities and magnetic permeabilities is obtained. It works in the boundary layer of the wavelength order near the surface. Qualitative consequences are discussed.

arXiv:1104.3651

Stability Conditions and Explicit Formulae for Basic Mode of Two Classes of Rotating Two-Mirror Optical Cavities

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Two-mirror astigmatic optical cavity rotating around its optical axis with extremely high frequency was recently considered by Steven J.M. Habraken and Gerard Nienhuis [1],[2]. Using the paraxial approximation and its generalization to the time-dependant case, they obtained round-trip time-independent 4×4 ray matrix in corotating frame. For resonator to be stable, absolute values of eigenvalues of this matrix must be equal to unity. Using approach, which is similar to traditional Babich–Popov method [3]–[7], they expressed basic mode and ladder operator in terms of eigenvectors of the round-trip matrix (monodromy matrix).

In this paper, an alternative technique [8]–[10] is applied to obtain formulae for the leading term of fundamental mode's asymptotics in explicit form; we don't need with eigenvectors mentioned to do it. Such formulae for two classes of astigmatic resonators are presented. Additionally, rotating modes of axially symmetric cavities are described.

As rotating resonator has the same monodromy matrix as some ring resonator with tetrahedral contour [6], [9], [11], [12], both stability conditions and formulae expressing field distribution coincide for these two types of cavities.

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Invisible bodies with specular surface

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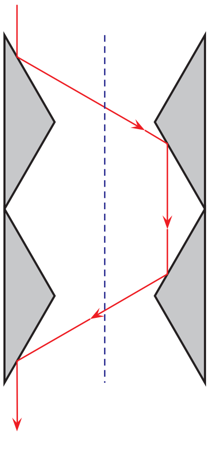
We consider the problem of invisibility for bodies with specular surface within the scope of geometrical optics. Examples of bodies invisible in one and two directions and a body invisible from one point are provided in the talk. In particular, a two-dimensional body invisible in the vertical direction is shown in the figure below. It is the union of 4 isosceles triangles with the angle 30^0 at the base. The three-dimensional invisible body is obtained by rotating it around the vertical axis.



It is proved that bodies invisible in *all* directions do not exist.

The question of maximal number of directions or points of invisibility remains open.

If time allows, I will tell about retro-reflecting bodies with specular surface. A retroreflector is an optical device reversing the direction of any incident beam of light. We provide several examples of asymptotically retro-reflecting sequences of bodies: the fraction of retro-reflected beams from a body B_n of such a sequence goes to 1 as $n \to \infty$.

Resonant scattering of electromagnetic waves at a two-dimensional electron system with lateral metal gates

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We have solved a problem of the scattering of electromagnetic waves at a two-dimensional electron system (2DES) with lateral metal gates. A plane electromagnetic wave is incident normally from vacuum onto a perfectly conductive plane (gate plane) with a slot of width w in it. An infinite sheet of 2DES located on the surface of a dielectric substrate is separated from the gate plane by the barrier layer of thickness d. Areal conductivity of 2DES is discribed by the Drude model as

$$\sigma(\omega) = i\frac{Ne^2}{m^*(\omega + i\nu)},$$

where ν is the electron momentum scattering rate, N is the sheet electron density, e and m^* are the charge and effective mass of electron, respectively. The electric field of the incident electromagnetic wave is polarised across the gate-plane slot.

The problem is solved in a first principle of electromagnetic approach similar to that elaborated by us earlier in [1] for the transistor structure with a topside gate. Using Ohm's law in the 2DES and the condition of vanishing in-plane electric field in the perfectly conductive lateral gates, we reduce the Maxwell equations to an integral equation for the electric field in the slot between perfectly conductive lateral gates. The integral equation is solved numerically by the Galerkin method through its projection onto an orthogonal set of the Chebyshev polynomials within the interval $[-w/2; w/2]$. As a result, we can find the induced electric fields in the ambient medium and the total electric field in the substrate below the 2DES as well as oscillating currents and charges in 2DES. Finally, we can calculate the scattering and absorption spectra of such a structure, which reveal the fundamental and higher plasmon resonances excited in 2DES under the slot between the lateral gates. Varying the width of the slot between the metal lateral gates we can calculate the dispersion of plasmons in the structure under consideration. We show that the screening of plasma oscillations in 2DES by lateral gates increases with increasing the plasmon wavevector as opposed to conventional screening of the plasmons in 2DES by a topside gate. Recent experimental observations of the plasmon resonances in 2DES discs of sub-mm diameter with lateral gate electrodes in gigahertz frequency range reported in [2] can be explained based on the results of this study by enhancement of the plasmon screening by lateral gates in the structures with 2DES discs of smaller diameter.

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Spectral analysis of the connected semicrystals model

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Spectral problem for two connected one-dimensional semicrystals is considered in the framework of Kronig–Penney model. The aim of our work is studing of the discrete spectrum of operator

$$H = -\frac{d^2}{dx^2} \tag{1}$$

with domain

$$\begin{aligned} &dom\ H = \left\{ f \in H^{2,1}(R) \cap H^{2,2}(R - \Lambda) \middle| f'(nT+) - f'(nT-) = \alpha f(nT), n \in Z \right\}, \\ &\Lambda = \{nT, n \in Z\}, \end{aligned} \tag{2}$$

where T is a period of potential in the Kronig–Penney model

$$p(x) = \sum_{n \in Z} \alpha \delta(x - nT), \tag{3}$$

.

We consider two semicrystals with different periods T_1, T_2 for negative and positive half-axes. We study two variants of this connections (presence or absence of point-like potential at the connection point). Using the the monodromy matrix technique, we obtain the dispersion equations and investigate the problem of bound states existence.

Another model we considered is the model of the branching graphene nanoribbons (Fig. 1).

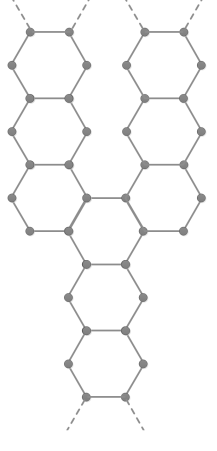

Fig. 1: The branching graphene nanoribbons

Many recent experiments on graphenes show remarkable properties of this material. The spectral analysis of this problem is made.

Flexural edge waves with general time dependence

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The paper is concerned with construction of solutions for flexural edge waves [1] in terms of harmonic functions. A generally similar approach is well-known for surface and interfacial waves, see [2]–[3]. An intuitive knowledge of underlying links between surface and edge waves leads to a guess that something similar is possible for the Konenkov's wave. However, this is not an easy problem. In case of the Rayleigh wave it is natural to consider a travelling wave solution of the form of $f(x - ct)$ at the surface. As it comes to the flexural edge wave, it is no longer the case.

The analyzed domain is $-\infty < x < \infty$, $0 \le y < \infty$, and the governing equation of motion is given by

$$\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4} + \frac{3\rho}{E}\frac{1-\nu^2}{h^2}\frac{\partial^2 W}{\partial t^2} = 0,$$

where E, ν, ρ and $2h$ denote the Young's modulus, the Poisson ratio, volume density and plate thickness, respectively, and $W(x, y, t)$ is the normal deflection. The key point of the problem is a vague idea that the analogue of the string equation for the Rayleigh wave in case of the Konenkov's wave is the beam equation. Assuming that

$$\frac{\partial^4 W}{\partial \xi^4} + c\frac{\partial^2 W}{\partial \tau^2} = 0,$$

where $\xi = x/h$ and $\tau = tc_2/h$ is the dimensionless scaling, one arrives at an elliptic equation indicating the presence of harmonic functions as the flexural edge eigenmodes. Substituting those into the free edge boundary conditions yields

$$c = \frac{3}{2\left[3\nu - 1 + 2\sqrt{2\nu^2 - 2\nu + 1}\right]},$$

coinciding with the results of Konenkov [1]. An example illustrating the existence of localized solution for certain initial conditions is also considered. Finally, the development of the approach leading to the asymptotic model for the flexural edge wave similar to that for Rayleigh surface wave [4] is discussed.

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Radial Airy function and related light beams

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The transformation of the radial Airy function with an additional cubic phase under propagation in Fresnel diffraction zone is investigated theoretically and numerically. The initial field depends on a number of real parameters. It is shown that for certain values of these parameters, the resulting field in the Fourier zone is real-valued. The found formulae are used to investigate the diffraction integral for the elliptic umbilic and propagation of the Gaussian function with an additional cubic phase.

Nonlinear dynamics of the Bragg soliton propagation in the 2D grating structure

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The nonlinear waveguides and periodical nonlinear structures exhibit attractive properties that make them suitable for utilizing in all-optical ultrafast application and novel devices development with wavelength tunable characteristics. The Bragg gratings are widely used in optical communications systems, as notch filters, in dispersion compensation modules, in pulse compressors, as Bragg grating biosensors, in the all-optical switching devices, all-optical buffers, storing devices and in fiber-optical accelerometers [1, 2, 3].

In this work the nonlinear dynamics in the 2D Kerr-nonlinear Bragg grating was investigated and the gap soliton generation was studied. As opposed to the conventional solitons, which can be formed due to the dispersion of the glass material [4], the solitons in the Bragg grating exist because of the balance between the nonlinearity and the dispersion of the grating [5].

The numerical simulations of the nonstationary dynamics in the periodical nonlinear structures are of great importance. In our work the Finite-Difference-Time-Domain (FDTD) method is applied because it is straightforward solution of the six-coupled field components of the Maxwell's equations. The free electromagnetic code from the MIT MEEP package [6] was used to perform the FDTD computational method. The size of periodic dielectric structure was choosen to be 200×1 μm, the period of structure was 1 μm, the linear part of layers' refractive indices was $n_1 = 1.45$ and $n_2 = 2.0$. The layers consist of the media with positive Kerr-like nonlinearity $n_{\mathrm{I,II}} = n_{1,2} + n_{nl}|E|^2$, where $n_{nl} = 3{\cdot}10^{-8}$ $\mu\mathrm{m}^2/\mathrm{W}$ – the nonlinear additive to the linear part of refraction index of silica glass.

The dispersion of the structure was studied and the values of cut-off frequencies were approximately estimated by the modification of the effective refractive index method (MERI). These results obtained by the MERI method were subsequently compared with the results of the transmission spectrum computation by FDTD method and the relatively good correspondence between these two methods was obtained.

The spatio-temporal dynamics of CW (continuous wave) signal propagation are studied in detail near the upper branch of the photonic band gap, where the dispersion is anomalous and the criterion of modulation instability [4] is satisfied. A series of simulations was performed with the different value of the amplitude of the input signal. When the power of source and therefore amplitude is small, the electromagnetic wave exponentially decays as it propagates along the structure. It was demonstrated that increasing the input signal amplitude one can obtain the signal propagation along the periodical structure. This can be explained as the transformation of the band structures of the periodic nonlinear system and the nonlinear frequency shift. Moreover the signal profile inside the Bragg grating is specifically modulated and the formation of multiple gap solitons is obtained. These gap solitons propagate stably through the grating and because of modulation instability in the grating the periodic train of pulses was observed. As the intensity is increased more gap solitons are generated and when the amplitude gets larger, the interaction between gap solitons was demonstrated in numerical simulations. The maximum value of the nonlinear additive $(n_{nl}|E|^2)$ to the linear part of refraction index in the numerical simulation was approximately 5 %. At this intensities of total launched power of light the structure of the real Bragg grating doesn't suffer permanent damage. The common method of reduction the threshold intensities for the nonlinear effects near the band gap region is the fabrication of Bragg gratings of increased length and using the chalcogenide glasses [7].

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Wave propagation in elastic waveguides with a finite length cracks

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In the presented work the propagation of SH-wave in elastic waveguides with finite length cracks in the case of free boundaries is considered. The complete analysis of diffraction of elastic waves on cracks of finite length is performed. This problem was solved by the method of partial regions. Matching procedure reduce to the infinite system of algebraic equations for unknown amplitudes. This system is solved by the use of method of residues of analytical functions. Residues method is based on calculating of integral as the sums of residues of analytical function $f(w)$ in the complex plane. Properties of function $f(w)$ determined by state of poles and zeros which are chosen so that the residue series coincide with the consideration system of equations. It is possible identify the unknowns in the system of equations with the residues of function. A finite crack in elastic waveguides makes it necessary to solve additional infinite system of algebraic equations caused by shift of zeroes of functions. Shift zeroes of function are solutions of an additional system. A displacement components of diffraction fields is obtained. The exact analytical solution on the base of the analytical functions methods is built. The reflection coefficient (the ratio of power flux incident wave to the power flux reflected wave) as well as transmission coefficient (the ratio of power flux of incident wave to the power flux of transmission wave) are calculated. All of that was obtained for different wavelengths of incident wave. The graphs of reflection and transmission coefficients depending of dimensionless frequency were built. Number of members in the infinity products, which are includes in defined amplitudes, to ensure energy conversation law is defined.

Diffraction series on a unit sphere and multiply diffracted waves for a quarter-plane problem

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The problem of diffraction by a quarter-plane (i.e by a flat cone) in the 3D space is studied. It is known that the total field (the solution of the problem) is composed of the following components: the incident plane wave, the reflected plane wave, the spherical wave diffracted by the tip, cylindrical waves diffracted by the edges of the quarter-plane, and the waves multiply diffracted by the edges. In this case the word "multiply" means "two times", however for a slightly more complicated problem, namely for a flat cone of an arbitrary opening there can be bigger amount of scattering acts. Up to the knowledge of the author, the multiply diffracted waves are not described in the literature. The aim of the work is to provide an expression for these waves.

We are starting from the expression for the field provided by the Smyshlyaev's theory. Namely, we perform the separation of variables, and then the Watson's transform, replacing the series over the eigenfunctions of the spherical problem by a contour integral. Instead of the eigenfunctions, one should use the Green's function for the Laplace–Beltrami equation on the unit sphere with a Dirichlet cut.

This well-known field representation is then transformed as follows. We propose an asymptotics for the Green's function on the unit sphere in the form of a diffraction series. Then we demonstrate that some terms of the series correspond to wave components of the initial 3D diffraction problem. As the result, asymptotic expressions for the multiply diffracted waves are obtained.

Generalized source method for light scattering and diffraction calculations

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There is a growing need in diffractive optics for an exact vectorial modeling of diffractive optical elements (DOEs) of smaller size and larger aperture as well as of microelectronic reticles towards the 22 nanometer technology node. DOEs have so far been analyzed and optimized by means of scalar modeling techniques. As the need for larger aperture grows, scalar techniques reach their limit and must be completed or even if possible superseded by an exact vectorial approach. An exact modeling technique then faces the deterring challenge of dealing with 2D structures of very large area and variable local pitch. The challenge encompasses both issues of calculation time and computer memory.

A number of techniques are capable of solving 2D dielectric diffractive elements exactly and in a short time. The C-method for instance [1] is one of them as far as periodic structures are concerned; however it is limited to smooth corrugations and is not adequate to solve planar digital profiles for which the Fourier methods [2] seems to be better adapted. The most popular among them is the Fourier Modal Method (FMM) [3, 4] which is the first one would think of to treat large diffractive systems. The state of the art methods however face the fact that the computation time increases with the cube of the number of diffraction orders M considered in the calculation of the scattering matrix of the element or of each slice of its profile. This property implies that even the calculation of a simple short period 2D grating like those used in microelectronics for the monitoring of the technological processes requires significant computation resources to solve the inverse problem in an acceptable time.

The present work is devoted to a methodology which exactly solves large systems and breaks through the M^3 limit and also decreases the needed memory size. It does so and calculates large 3D systems in a time proportional to M by means of a volume integral equation which the generalized source method (GSM) provides [5, 6]. The volume integral equation constructing the matrix of the usual system of linear equations of the diffraction problem in the form of a product of block-diagonal and block-Toeplitz matrices which is processed then by the known calculation algorithms of the fast Fourier transform (FFT) and generalized minimal residual method (GMRES).

In the presentation we will give a description of the method togeteher with a discussion of its' possible applications. Numerical examples will demonstrate consistency with reference methods and means of controling and improving the results' accuracy.

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Systems of equations admitting delta-shocks in modeling of physical processes

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We consider some systems of conservation laws which admit δ-shock wave type solutions. Components of such solutions *contain Dirac delta-functions.* The systems under consideration are used in modeling of some physical processes.

(1) We study a new class of systems of conservation laws admitting δ-shocks:

$$(u_j)_t + \big(u_j\, f_j(\mu_1 u_1 + \cdots + \mu_n u_n)\big)_x = 0, \quad x \in \mathbf{R}, \quad t \geq 0, \tag{1}$$

where $f_j(\cdot)$ is a smooth function, μ_j is a constant, $j = 1, 2, \ldots, n$. If $f_j(w) = 1 + \frac{a_j}{1+w}$, then (1) is the system of *nonlinear chromatography*, where $\mu_j = 1$, $u_j \geq 0$ and $\frac{a_j u_j}{1+\sum_{s=1}^{n} u_s}$ are the jth components of fluid and adsorbed (solid) phase concentrations, respectively, a_j is Henry's constant, $j = 1, 2, \ldots, n$ (see [2]).

(2) We study the system of zero-pressure gas dynamics with the energy conservation law (for modeling media which can be considered as *having no internal pressure*).

To study the above systems we use the technique developed in [1], [3].

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Simulation of the vortex dynamics in the mesoscopic superconducting films

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We discuss some aspects of numerical solution of 2D Ginzburg-Landau equations, describing the dynamics of the order parameter in mesoscopic superconducting films in external fields. We consider a discrete model of the GL system which preserves some important features of initial (continuous) equations such as gauge invariance and the absence of electric field in the steady state. We discuss also the important notion of the (discrete) vortex, which is an analogue of the corresponding continuous one and present the numerical method and algorithm for the "catching" of discrete vortices. We present some results of our numerical experiments to demonstrate how all this work.

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Oscillations of a fluid layer sandwiched between different elastic half-spaces

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Guided waves propagation in a fluid layer sandwiched between homogeneous elastic half-spaces have been analytically studied by several authors. As shown by Krauklis (1962) the most important feature of such a model is the fundamental mode, which exist for arbitrary elastic parameters and attenuates very slowly with the distance. Such a wave (known as a slow wave in a fluid layer) exhibits extraordinary behaviour, both phase and group velocities approach zero at low frequency. Krauklis has constructed the exact solution of the problem in terms of the contour integral method, but no seismograms and dispersion curves are shown. Paillet and White (1982) mentioned the plane-geometry analog to the cylindrical borehole and presented the dispersion equation for this case in the appendix of their work. Ferrazzini and Aki (1987) considered normal modes in a fluid layer and calculated dispersion curves for case, when two half-spaces are the same. Krauklis et al. (1996) analytically analyzed the dispersion equation and calculated the dispersion curves for a slow wave. In this paper we construct the exact solution of the problem by the Lamb's method. Then we calculate dispersion curves for the different geometries and point to some features of the case, when half-spaces are different. In addition, we use the axisymmetric finite-difference method to calculate seismograms. We show that a generalized Rayleigh wave analogue has more intensive vertical component of amplitude than a slow wave for some media parameters.

Let us consider in a cylindrical coordinate system the model consisted of two homogeneous half-spaces (1) ($z<0$) and (2) ($z>h$) separated by a fluid layer (f) ($0<z<h$). Each medium is characterized by the compression v_j ($j=p1,p2,f$) and shear v_j ($j=s1,s2$) velocities and densities ρ_j ($j=1,2,f$). The wave field $\vec{u}$, excited by the center of dilatation, satisfies the equation of motion

$$\mathrm{v}_{pj}^2 \text{ grad div } \vec{u} - \mathrm{v}_{sj}^2 \text{ rot rot } \vec{u} - \frac{\partial^2 \vec{u}}{\partial t^2} = 0,\ j=1,2,f,$$

zero initial conditions $\vec{u}\,(t\le 0) = \frac{\partial \vec{u}}{\partial t}\,(t\le 0) = 0$, and the boundary conditions

$$z=0,h:\ [u_z]=0,\ [t_{zz}]=0,\ [t_{rz}]=0,$$

$$z=d:\ [u_r]=0,\ [u_z]=\frac{1}{(2\pi)^2}\int\limits_{-\infty}^{\infty} e^{i\omega t}d\omega \int\limits_{0}^{\infty} kJ_0\,(kr)dk,$$

where t_{rz} – tangential stresses, t_{zz} – normal stresses and the square brackets denote the jump of function. We construct solutions in terms of Fourier and Fourier-Bessel integrals. Application of the boundary conditions at all interfaces including $(z = d)$ leads to the homogeneous system of algebraic equations for the unknown coefficients. If the source is located in half-space (2), the wave field in a fluid layer looks as

$$\vec{u}(r,z,t) = \frac{1}{(2\pi)^2}\int\limits_{-\infty}^{\infty} e^{i\omega t}d\omega \int\limits_{0}^{\infty} \begin{pmatrix} XJ_1(kr) \\ YJ_0(kr) \end{pmatrix} dk,$$

where

$$X = \frac{\rho_2 \mathrm{v}_{s2}^2 k^2 \left(\alpha_{s2}^2 + k^2\right)}{\rho_f \omega^2 \alpha_{p2} \Delta(k,\omega)} \left[\sinh(\alpha_f z) - Q_1 \cosh(\alpha_f z)\right] e^{-\alpha_{p2}(d-h)},$$

$$Y = \frac{\rho_2 \mathrm{v}_{s2}^2 k \alpha_f \left(\alpha_{s2}^2 + k^2\right)}{\rho_f \omega^2 \alpha_{p2} \Delta(k,\omega)} \left[Q_1 \sinh(\alpha_f z) - \cosh(\alpha_f z)\right] e^{-\alpha_{p2}(d-h)},$$

$$\Delta(k,\omega) = (Q_1 - Q_2)\cosh(\alpha_f h) + (Q_1 Q_2 - 1)\sinh(\alpha_f h), \quad Q_n = (-1)^n \frac{\rho_n \alpha_f \mathrm{v}_{sn}^4}{\rho_f \alpha_{pn} \omega^4} R_n(k), \ n = 1,2,$$

$$R_n(k) = \left(\alpha_{sn}^2 + k^2\right)^2 - 4k^2 \alpha_{sn}\alpha_{pn}, \ n = 1,2, \alpha_j = \sqrt{k^2 - \frac{\omega^2}{\mathrm{v}_j^2}}, \ \arg \alpha_j(k=0) = \frac{\pi}{2}, \ j = f, p1, s1, p2, s2.$$

We can simplify the dispersion equation $\Delta(k,\omega) = 0$ by substituting $k = \omega\varsigma = \omega \mathrm{v}^{-1}$. Then we reduce the equation to the following form

$$e^{2\omega h \widehat{\alpha}_f} = \frac{\left(\widehat{Q}_1 + 1\right)\left(\widehat{Q}_2 - 1\right)}{\left(\widehat{Q}_2 + 1\right)\left(\widehat{Q}_1 - 1\right)},$$

where the values with hat are not depending on frequency. In the case of different half-spaces there are no roots of the dispersion equation over the range $(\mathrm{v}_{1f}, \mathrm{v}_{2f})$, where v_{nf} $(n = 1,2)$ – velocity of Stoneley-Scholte wave propagating along interface between solid (1 or 2) and fluid (f) half-spaces. In the case of equal half-spaces $\mathrm{v}_{1f} = \mathrm{v}_{2f}$ and the range mentioned above is absent. The first mode possible at low frequencies is the slow wave, whose velocity falls to zero at zero frequency and rises to v_{1f} at high frequencies in both cases.

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Semiclassical localized solutions of the the two-component Hartree equation with the periodic Hermit matrix Hamiltonian

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We construct a new class of localized asymptotic solutions of the 2-component Hartree type equation with the small parameter at private derivatives nonlocal cubic nonlinearity and hermit matrix Hamiltonian of the Hartree equation. We show that, the constructed solutions are the nondiffusing wave packets. In case of quadric periodic matrix Hamiltonian, we present exact solutions of the Floke problem for the nonlinear equation.

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Method of dispersion curves calculation for waves with noninteger azimuthal wavenumbers in radially inhomogeneous cylindrical elastic waveguides

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Investigation of elastic waves propagating in media with cylindrical symmetry holds significance for non-destructive evaluation of materials and constructions (e.g., pipes) and for other applications. Often the model of the medium can be represented as radially layered inhomogeneous waveguide. The traditional way to describe waves propagating in such a medium is to expand them in series of eigenmodes possesing integer azimuthal wavenumbers ν. However, often for the sake of applicability waves with noninteger wavenumbers is considered. Such a solution can be treated as a helical wave propagating on a cylindrical surface, where wavefront can be presented as a spiral line. In this work a semi-analytical method is presented to calculate dispersion curves corresponding to these waves in the case where elastic parameters $\lambda(r)$, $\mu(r)$ and density $\rho(r)$ are piecewise continuous functions of the radius.

To justify the existence of helical waves with a noninteger ν in cylindrical waveguides, one can extend the reasoning of paper [1] for circular waves in the following way. A wave excited in an arbitrary direction along a cylindrical surface can be presented as a series of normal modes with integer ν_i. Assuming short wavelength asymptotic one can transform this series into integral over real ν numbers and derive the main contribution term, which corresponds to a helical wave propagating with a noninteger ν. This wave itself and its properties such as phase or group velocity can be experimentally measured. However, because this is not the normal mode solution, this wave cannot propagate in a waveguide for infinite time.

Waves with both noninteger and integer values of ν are solutions of a waveguide dispersion equation. To obtain this equation in inhomogeneous waveguides, a transfer matrix method [2, 3] was applied for piecewise constant elastic parameters $\lambda(r)$, $\mu(r)$ and density $\rho(r)$ and Riccati equation method [4] was used for piecewise continuous ones. Their combination provides efficient calculation of dispersion curves for helical waves in radially inhomogeneous cylindrical elastic waveguides. Both methods were formulated in terms of impedance matrix $Z(r)$, which is convenient to introduce in tasks for which the knowledge of stress and displacement fields is not required [5]. The methods under discussion were used to calculate the impedance matrix at one boundary of the waveguide by knowing its value at the other boundary. It is noteworthy that, in most cases these matrices can be shown to be Hermitian. To avoid treatment of matrix Riccati equation in singular points the medium in the vicinity of the axis and at $r \to \infty$ is considered as homogeneous. Matrix impedances of such homogeneous rod and cavity of radii r_0 and r_n, respectively, can be easily found. Treating them as initial value $Z(r_0)$ for matrix Riccati equation and as impedance load $Z^{(L)}(r_n)$ one can calculate impedance matrix $Z(r_n)$ at the boundary $r = r_n$ and formulate the dispersion equation as follows:

$$\det\left[Z(r_n) - Z^{(L)}(r_n)\right] = 0 \tag{1}$$

Calculation of dispersion curves satisfying this equation consists in computation and separation of the roots of implicit complex-valued function $f(\nu, k, \omega)$, where k means axial wavenumber and ω stands for wave's circular frequency. This task was fulfilled be means of predictor-corrector type of parametric continuation method, where correction was done with Davidenko's method [6].

Use of the presented technique enabled calculation of dispersion curves for helical waves propagating in cylindrical waveguides with different values of elastic and dimensional parameters. The study also assessed the dependence of the curves' behavior on the different types of inhomogeneity found in the medium. In the framework of this work linear, quadratic and square root dependence of compressional and shear waves velocities on the radial coordinate was considered. It was also shown that curves for waves with noninteger ν locate between curves calculated for neighboring integer wavenumbers and possess similar dispersion properties.

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Electromagnetic waves produced by abruptly starting luminal source

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We consider electromagnetic field produced by abruptly starting luminal source moving along a straight line with the velocity of light. This problem is solved in [1,2] by a non-trivial calculation based on Riemann function. We obtain the result by an easier way, based on convolution of source with Green function for the wave equation.

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Causal model of launching an X-wave by a superluminal rectilinear current pulse

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The early 1970s witnessed the start of the intensive study of radiation emanated by superluminal sources [1]. The space-time structure of charged-tachyon radiation was discussed and some alternative physical models, in which the superluminal pattern is created by the coordinated subluminal motion of aggregates of "normal" charged particles, were proposed [2]. The research received new impetus by discovering that the superluminal sources may generate localized waves, the most promising of which being the X-shaped waves. In a more resent paper, Recami et al. [3] proposed a "toy model" of localized X-shaped field generation by an axisymmetric superluminal four-current $J^{(\alpha)} = (c\varrho, 0, 0, j_z)$, where c is the speed of light, ϱ the electric charge density, j_z is the only non-zero component of the current density vector $\mathbf{j}$ in the cylindrical coordinates ρ, ϕ, z. Considering the inhomogeneous problem for the electromagnetic four-potential $A^{(\alpha)}$ in the form

$$A^{(\alpha)} = (\varphi/c, \mathbf{A}) = (\varphi/c, 0, 0, A_z)\,, \quad A^{(\alpha)}(\rho, \phi, z, t) = A^{(\alpha)}(\rho, \zeta)\,, \quad \alpha = 0, 1, 2, 3,$$

where $\zeta = z - Vt$ is a V-cone variable introduced for the constant superluminal velocity V, the authors of [3] inevitably constrained the investigation to the cases that (i) describe a steady-state process that lasts all times from $-\infty$ to ∞; (ii) require four-current dependence $J^{(\alpha)}(\rho, \phi, z, t) = J^{(\alpha)}(\rho, \zeta)$, and (iii) bound the nonzero components of the four-vectors by relations $j_z(\rho, \zeta) = V\varrho(\rho, \zeta)$ and $A_z(\rho, \zeta) = V\varphi(\rho, \zeta)/c^2$ due to, respectively, the continuity equation and the Lorentz gauge. The problem was solved for the case of a pointlike charge, that is, $j_z = eV\delta(\rho)\,\delta(\zeta)/\rho$ using the traditional spectral method, yielding a "true X wave" localized solution

$$A_z = const \times \frac{V}{\sqrt{\zeta^2 - (V^2/c^2 - 1)\,\rho^2}} \tag{1}$$

In the present work we consider a far more challenging modification of the above model, corresponding to much more physically admissible source in the form of a current pulse

$$j_z = eV\frac{\delta(\rho)}{2\pi\rho}h(z)\,h(t)\,h(Vt - z)\,f(Vt - z) \equiv eV\frac{\delta(\rho)}{2\pi\rho}h(z)\,h(Vt - z)\,f(Vt - z)\,, \quad h(\tau) = \begin{cases} 0 & \tau < 0 \\ 1 & \tau > 0 \end{cases}$$

of an arbitrary shape $f(\xi) = f(-\zeta)$ launched with a constant velocity $V > c$ at a fixed moment of time, taken as $t = 0$, along the positive z direction. The potential is sought in the form $A^{(0,3)}(\rho, \phi, z, t) = A^{(0,3)}(\rho, z, t)$ as a solution of the initial-value problem, $A^{(\alpha)} = 0$ for $t < 0$, invoking the additional condition $J^{(\alpha)} = 0$ for $t < 0$, which in the absence of external charge sources immediately implies

$$\begin{aligned} \varrho &= e\frac{\delta(\rho)}{2\pi\rho}\left[h(z)\,h(Vt - z)\,f(Vt - z) - Vh(t)\,\delta(z)\int_0^t f(Vt' - z)\,\mathrm{d}t'\right] \\ &= \frac{1}{V}j_z + (-e)\,h(t)\,\frac{\delta(\rho)\,\delta(z)}{2\pi\rho}\int_0^{Vt} f(\xi)\,\mathrm{d}\xi \end{aligned}$$

due to the continuity equation. The second term describes the accumulation of the opposite charge at the pulse-generation point $\rho = 0, z = 0$ due to current outflow (can be compensated in the long term by consideration of either tachyon creation in particle-antiparticle pairs or transient bipolar current pulses for which $\int_0^\infty f(\xi)\,d\xi = 0$). The general-type solution is constructed via incomplete separation of variables, yielding a causal description of X-wave generation in the space-time domain comprising the transient and steady-state process. The latter is shown to exist within a certain spatiotemporal area in its pure form, whose particular representation for $f(\xi) = \delta(\xi)$ is given by Recami's solution (1).

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Embedding formulae for Laplace–Beltrami problems on the sphere with a cut

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We consider the problem of diffraction of a plane by a quarter-plane with Dirichlet boundary conditions. For this problem exist various expressions for diffraction coefficient. These expressions have the form of contour integrals over separation parameter. Integrands are constructed from the solutions of Laplace–Beltrami problems on the unit sphere with a cut produced by the quarter-plane. In this paper we derive embedding formulae which connect these solutions and show the possibility to derive expressions for diffraction coefficient from one another.

Ferromagnetic nanodots as metamaterials for a superconducting current of Josephson junctions and magneto optic

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The prospective usage of patterned ferromagnetic particles of submicron sizes is accounted for by the well-defined local magnetic field. In the first part of report it will be shown possibility of e-beam lithography combined with dry etching techniques for fabrication different magnetic nanostructures and manipulation of magnetic states of such structures.

Next part of report will be about of ferromagnetic nanostructures formed on Josephson junction. In the case of periodic lattice of ferromagnetic particles we observed commensurability effects between the periodic phase-difference distribution created by the magnetic particles and the spatial wave of the Josephson current formed by the magnetic field H. The dependence of the critical current through the junction on a uniform applied magnetic field H is shown to differ strongly from the conventional Fraunhofer diffraction pattern in the case of ferromagnetic order of the particles magnetic layers. In this case, ferromagnetic nanodots become as meta material for supercurrent, it is possible to suppress critical current of the junction.

Last part of discussion will show magnetic dots with vortex magnetic states. It will be shown that using special form of particles it is possible to control vorticity of the magnetic state and form lattice of nanomagnets with one direction of vorticity (for example, clockwise direction). The interest to such periodic lattice of magnets for magnetooptics and possibility of measurement of in the non-zero diffraction maxima will be discussed.

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Numerical method for the inverse problem of near field microscopy of layered media

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We present the numerical method for retrieving of the profile of layered media permittivity from the near field microscopy data. The method is based on the minimization of the discrepancy function describing the difference between the experimental measured of the near field antenna impedance and the calculated one for given permittivity profile. Such calculation is based on the numerical algorithm which developed on the base of rigorous solution of corresponding direct problem. We suppose that the profile to be retrieved is described by the function with finite number of unknown parameters and the discrepancy is minimized with respect of these parameters. We present some results of numerical experiments with method suggested.

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Diffraction focusing of the radiation by acoustic (phononic) crystal

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The purpose of the present work — an experimental research of the phenomena of focusing of radiation, generated by practically dot source of radiation (piezotransduser), by a two-dimensional acoustic crystal in the form of a parallelepiped (with dimensions... $220\times167\times93\text{mm}^3$), formed by a hexagonal lattice (with a lattice constant $a \sim 18,6$mm) and the steel cylinders of circular cross section (diameter $d \sim 16$mm) located in the air. Radiation source with dimensions smaller than the wavelength placed at the center of the crystal (on a place of a withdrawn core) and measured focal length to the generated image of a source out of a crystal. Proceeding from received values in accordance with Snell's law calculated the effective refractive index depending on the frequency of radiation (Table).

Table. Frequency dependences of wave parameters of phononic crystal

Frequency ν, kHz	Focal distance F, mm	Refraction index n	Normalized frequency $\omega a/2\pi c$	Normalized wave vector $ka/2\pi$
12	78	-0,906	0,656	0,594
12,5	84	-0,858	0,683	0,586
13	90	-0,814	0,711	0,579
13,5	98	-0,762	0,738	0,563
14	108	-0,705	0,766	0,540
14,5	126	-0,620	0,793	0,492

The observed negative dispersion of acoustic waves in the crystal leads to negative values of its effective refractive index. The calculation of the difference between the phase shifts of axial and off axis (at angle $\alpha = 45°$ in the crystal) rays gives differences lengths paths less than 1mm (at wavelength about $\lambda \sim 3$cm) within a range of frequencies corresponding to observable area of a negative dispersion of a crystal. On the one hand and should be in case of usual lenses when geometrical parameters on some orders of size exceed length of a wave of radiation and the beam theory of formation of the focused image of a dot source is fair. On another hand in our case all geometrical parameters do not exceed length of a wave of radiation even on an order value of its size.

Observed phenomenon can be explained in terms of the quantum uncertainty principle. If we have a stream of phonons with a certain impulse at attempt to filter out through the slit photons with a specific coordinate we will receive the big uncertainty on an impulse, that is on a wave vector. In our case, it seems, we have an reverse process when phonons from well-localized source pass through a spatial filter with diffraction at cores of acoustic crystal, varies the distribution on an impulse — the wave vector at first is directed to a source (back wave) in the environment with negative refraction — then, leaving the environment (in air), it changes the direction for the opposite. Therefore, an uncertainty in the impulse can collect phonons with very good accuracy in the focus (there where phases of all phonons are equal). In other words, the source excites the maximum number of phonon modes of the crystal, each of which occupies the entire crystal, and therefore long-range order interaction of the elements of the lattice leads to a very good homogenization of the phases of many of the elements of the lattice. We also observed at any condition subwave resolution about quarter of wavelength of radiation. It may be useful for some applications.

Thus, justice of the classical beam theory in the conditions of strong diffraction on elements of a lattice and field quantization in two-dimensional periodic structure of phononic crystals is found experimentally.

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New solutions to the electrostatic problem for highly excentric axysimmetric particles

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The electrostatic model of a particle in the uniform electric field is often used for approximating the light scattering by small particles in different fields of applications including nano-scale optics [1, 2].

The distortion of the uniform field by particles of the canonical shapes (spheres, ellipsoids) has been found analytically in the classical monograph [3]. The electrostatic problem has also been solved for confocal and non-confocal multilayered ellipsoids, clusters of spheres, and 2 spheres near a plane. Besides these geometries other axisymmetric particles are also widely applied. However, only the numerically intensive methods, e.g. the finite elements methods or the discrete dipole approximation, are available for their modelling.

We solve the electrostatic problem for axisymmetric particles of analytical shapes by using the generalized separation of variables (gSVM) and the extended boundary condition (EBCM) methods. These methods were originally developed for the light scattering problem and are known to be highly efficient for such shapes [4]. We extend our previous work [5] by providing solutions in the spheroidal coordinates, that allows obtaining numerically stable results for highly oblate and prolate particles.

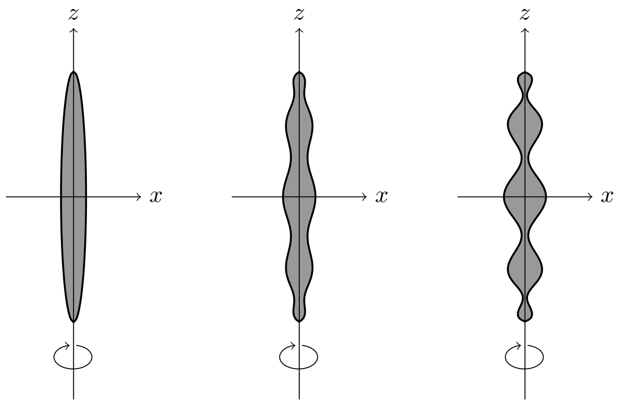

Fig. 1: Sample particles for which accurate numerical results were obtained with the proposed methods: a prolate spheroid with $a/b = 10$ and spheroidal Chebyshev particles with the waviness parameter $n = 10$ and the deformation parameter $\varepsilon = 0.03$ and 0.07.

We present some results of numerical tests including comparison with the discrete dipole approximation (DDA) for prolate and oblate spheroids and spheroidal Chebyshev particles (see fig. 1).

The gSVM and EBCM methods in the electrostatic limit were implemented within the ScattPy python package [6] and are available for public use. For more details see `http://scattpy.github.com`.

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Electromagnetic surface waves guided by the boundary between bi-anisotropic composite and dielectric media

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Over the past decade the impressing success in the development and creation of artificial media described by both dielectric and magnetic permeability tensors which separate elements simultaneously may be negative values in a certain frequency ranges (from microwaves up to the optical wavelengths) has been achieved. The electrodynamics properties of surface waves guided by interface of anisotropic metamaterials have been the subject of many studies.

This report is devoted to the propagation of surface waves guided by the boundary between dielectric and bi-anisotropic artificial media. The composite medium is described by the permittivity and permeability tensors $\hat{\varepsilon}$ and $\hat{\mu}$ with zero off-diagonal elements

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_2 \end{pmatrix}, \qquad \hat{\mu} = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_2 \end{pmatrix}.$$

The anisotropic axis is perpendicular to the media interface. As it is known both direct and backward waves of TE and TM types may propagate along such interface in certain frequency range if even one of the elements (ε_2 or μ_2) of tensors is negative [1]. In particular we analysed the case of isotropic composite medium. The permittivity and permeability constants in this case of the monochromatic signal with frequency ω can be written in the form $\varepsilon_1 = \varepsilon_2 = 1 - (\omega_p/\omega)^2$ and $\mu_1 = \mu_2 = 1 - F\omega^2/(\omega^2 - \omega_m^2)$ where parameters F, ω_p, ω_m are determined by the technological properties of elementary cells of composite materials [2]. The main attention is given to the nonlinear interaction of the surface waves in the presence of a high frequency (2ω) external magnetic field which is perpendicular to interface of the media. Such intense external magnetic field may effect on the medium properties, and permeability of the composite medium depends on the amplitude of magnetic field. The parametric instability has been developed if the space-time conditions between the external magnetic field and surface waves take place. The increments of the parametric instability have been obtained in analytical form for the case of isotropic composite medium.

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Semiclassical analysis of electron dynamics in grathene structures in magnetic field

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Application of semiclassical analysis in studying quantum mechanical behaviour of electron has been demonstrated in various fields of modern physics such as nano-structures, electronic transport in mesoscopic systems [1], quantum chaotic dynamics of electronic resonators [2], and many others. One of the examples of application of semiclassical analysis are quantum electronic waveguides and resonators.

In this talk an application of semiclassical analysis to graphene quantum electron dynamics is demonstrated for two important problems. The first problem is construction of semiclassical series of eigen-states of an electron-hole motion inside a strip graphene resonator with zigzag boundary conditions. Constructions of semiclassical approximation of Green's function in electronic waveguide inside graphene structure is the second example. Both problems can be described by the following 2D Dirac system with magnetic and electrostatic potentials in axial gauge $\mathbf{A} = 1/2B(-x_2, x_1, 0)$

$$v_F < -i\hbar\nabla + \frac{e}{c}\mathbf{A}, \bar{\sigma} > \psi + U(\boldsymbol{x})\psi(\boldsymbol{x}) = E\psi(\boldsymbol{x}), \qquad \psi(\boldsymbol{x}) = \left[\begin{array}{c} u \\ v \end{array} \right],$$

where u, v are the components of spinor wave function describing electron localisation on sites of sublattice A or B of honeycomb graphene structure, v_F being Fermi velocity, and $\bar{\sigma} = (\sigma_1, \sigma_2)$ with Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

In the first problem, similar to [3], an asymptotic method of construction of the energy spectrum and eigenfunctions, localized in the small neighborhood of a periodic orbit, is described. The isolated periodic orbit is confined between two interfaces of graphene strip. Such a system represents a quantum electronic resonator or Fabry–Perot interferometer, an analog of the well-known high-frequency optical or acoustic resonator with eigen-modes called "bouncing ball vibrations".

The first step in the asymptotic analysis involves constructing a solitary localized asymptotic solution to the Dirac equations (electronic Gaussian beam – wavepackage). Then, the stability of a closed continuous family of periodic trajectories confined between two reflecting surfaces of the resonator boundary is studied. The asymptotics of the eigenfunctions were constructed as a superposition of two electronic Gaussian beams propagating in opposite directions between two reflecting points of the periodic orbits. The asymptotics of the energy spectrum are obtained by the generalized Bohr–Sommerfeld quantization condition derived as a requirement for the eigenfunction asymptotics to be periodic. For one class of periodic orbits, localised eigen-states were computed numerically by the finite element method using COMSOL, and proved to be in a very good agreement with the ones computed semiclassically.

Application of method of summation of Gaussian beams (integral over Gaussian beams), which was developed in [4], [5], for description of electron-hole motion inside graphene waveguide is given in the second part of the talk. Gaussian beam as a localised asymptotic solution is always regular near caustics or focal point. Realization of the method does not require any knowledge about geometrical properties of caustics. The method of summation of Gaussian beams gives universal semiclassical uniform approximation for solutions to various problems of wave propagation and quantum mechanics. This approximation is valid near caustics or focal points of arbitrary geometric structure.

This method is convenient to construct a semiclassical uniform approximation for Green's function for interior of waveguides and resonators quantum problems. Application of this method to problems of electron motion in magnetic field in graphene required a generalization of the approach originally developed for acoustic wave propagation problems. The first step in this direction has been done in ([6]).

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Non-diffracting beams resistant to attenuation in absorbing media

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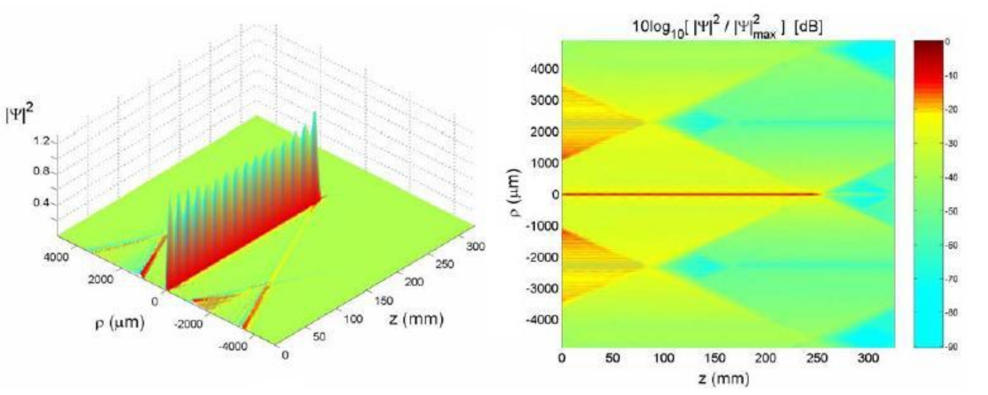
In this work we show how it is possible to model the longitudinal intensity pattern of vectorial optical beams in absorbing media. As particular cases, we obtain non-diffracting beams capable of resisting, for long distances, the attenuation effects due to energy losses in the medium. These beams are constructed by suitable superpositions of equal-frequency Bessel beams, exploiting in the highest degree the known self-reconstructing properties of non-diffracting waves. The results here presented can have important applications in free space optics, medical apparatus, remote sensing, etc..



(a) (b)

(a) 3D field-intensity of a diffraction-attenuation resistant beam in an absorbing medium. This beam, with $\lambda = 308$nm, possesses a spot radius $\Delta\rho = 5.6\,\mu$m, and is capable of maintaining the size and the intensity of its central spot until a distance of 25 cm, whereas in this same medium a plane wave or an ideal Bessel beam (with the same spot size) possess a penetration depth of 5 cm, and an ordinary Gaussian beam (with the same initial spot size), besides suffering attenuation, would also be affected by a strong diffraction, having a diffraction length of only 0.6 mm. **(b)** Orthogonal projection of the resulting beam intensity (normalized with respect to its maximum value, $|\Psi|^2_{max}$) in **logaritmic** scale.

Numerical analysis of elastic waves in a multilayered solid sphere excited by surface pulse-vibration force

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Global seismic waves induced by shallow-focus and deep-focus earthquakes in the Earth space are simulated by applying the ANSYS software. Earth models in the form of 3- and 4-layered elastic spheres are used. In a relative measurement, the spherical interface diameters correspond to the diameters of the real interfaces of the Earth's crust, mantle, molten core, and solid core. In the 3-layered model, the crust is not taken into account; in 4-layered model, the core is simulated by the 4th layer. The 4-layered model parameters are given in Table 1.

Table 1

4th type medium	3rd type medium	2nd type medium	1st type medium
$0 \le D_4 \le 2.19\,\mathrm{m}$	$2.19\,\mathrm{m} \le D_3 \le 5.1\,\mathrm{m}$	$5.1\,\mathrm{m} \le D_2 \le 9.5\,\mathrm{m}$	$9.5\,\mathrm{m} \le D_1 \le 10\,\mathrm{m}$
$\rho_4 = 2 \cdot 10^4\,\mathrm{kg/m^3}$	$\rho_3 = 2 \cdot 10^4\,\mathrm{kg/m^3}$	$\rho_2 = 1 \cdot 10^4\,\mathrm{kg/m^3}$	$\rho_1 = 3 \cdot 10^3\,\mathrm{kg/m^3}$
$\nu_4 = 0.24$	$\nu_3 = 0.45$	$\nu_2 = 0.3$	$\nu_1 = 0.38$
$E_4 = 1.2 \cdot 10^{11}\,\mathrm{N/m^2}$	$E_3 = 6 \cdot 10^{10}\,\mathrm{N/m^2}$	$E_2 = 2 \cdot 10^{11}\,\mathrm{N/m^2}$	$E_1 = 2 \cdot 10^{11}\,\mathrm{N/m^2}$

The sphere outer diameter in both models is chosen to be equal to 10 m; the 3-layered model is obtained by substituting the 1st type medium by the 2nd type medium in the outer layer 0.25 m thick simulating the crust layer. The disturbance source is the concentrated force having harmonic or pulse time dependence; this force is applied normally to the crust-mantle interface. The similarity principle is used to estimate oscillation-wave periods (or frequencies) in the real Earth by the model data:

$$R^E/\lambda^E_{l,t} = R^M/\lambda^M_{l,t} \to f^E = f^M(C^E_{l,t}R^M/C^M_{l,t}R^E), \tag{1}$$

where R^E, R^M, f^E, f^M, $C^E_{l,t}$, $C^M_{l,t}$ are radii, compression and shear wavelengths, oscillation frequencies, and propagation velocities of compression and shear waves in the Earth and in the model.

Global seismic waves in the Earth are simulated for wave excitation by a variable power source affecting the outer boundary or the crust-mantle interface. In this case, analogs of shallow-focus and deep-focus earthquakes are respectively realized. In particular, seismic field distributions in the meridian section of the planet are obtained in the harmonic oscillation mode of the source (at frequencies of about 8 kHz… 12 kHz). The patterns indicate a strong influence of the 4th layer on the spatial distributions of wave intensities. The frequencies corresponding to resonances of surface waves excited by the source that affects the above-mentioned sphere boundaries are determined. The effect of amplitude increase due to convergence of R-waves on the opposite pole of the sphere was previously studied for the 3-layered model. This effect is also observed for the 4-layered model, moreover, it becomes even more pronounced. At resonances, sphere plays the role of an acoustic concentrator of R-waves; the wave dispersion markedly selects definite frequencies.

It is found out that in spite of the negligibly small relative volume of the thin crust layer in the planet body, the density and the Poisson coefficient of the crust medium influence the pulse-excited surface R-wave velocity. The estimated surface R-wave velocity in the 4-layered model is close to 5 km/s; variation in the above-mentioned parameters leads to an increase in the surface R-wave velocity up to 5.5 km/s (cf. fig. 1 a and fig. 1 b).

The spatial amplitude distributions of the global field of body seismic waves and surface R-waves are shown in fig. 1 c, d. They indicate convergence of R-waves on the opposite pole of the sphere when the source acts either on the free surface of the sphere or on the crust-mantle interface. It is seen that the effect of the wave response in the first case is ~ 3.5 times higher than the similar one in the second case. This peculiarity is of interest for differentiation of shallow-focus and deep-focus earthquakes and can be used as an informative sign for classification of seismic events.

a b c d

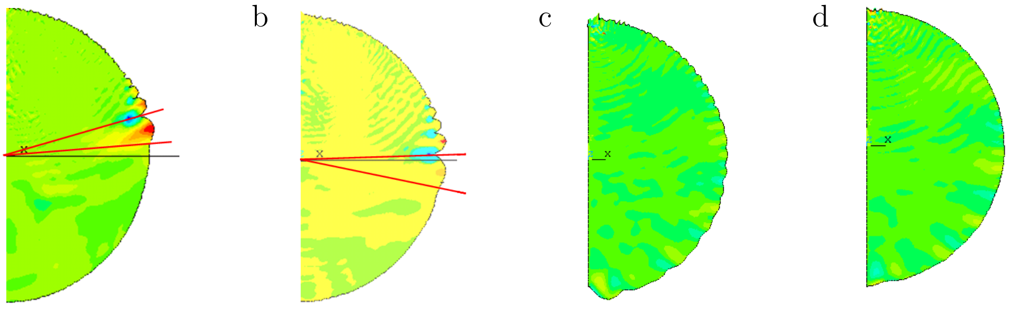

Fig. 1. Spatial-amplitude distributions of the global seismic field in the meridian section of the Earth: a – $\rho = 3 \cdot 10^3\,\mathrm{kg/m^3}$, $\nu = 0.38$, b – $\rho = 2.1 \cdot 10^3\,\mathrm{kg/m^3}$, $\nu = 0.45$ (time delay after a shock $\tau_1 = 2.5\,\mathrm{ms}$), c – a shock on the free boundary, d – a shock on the crust-mantle interface ($\tau_2 = 5.4\,\mathrm{ms}$).

Therefore, the peculiarities of excitation and propagation of global seismic waves in the planet scales can be simulated and clearly illustrated applying the ANSYS software. Quantitative comparison of the simulation results shows satisfactory agreement with the data known from classical papers [1, 2].

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Wave field from a point source on the boundary of half plane Biot

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Initial boundary value problem of wave propagation in half plane filled with fluid-saturated porous solid is considered. Explicit formulae for components of displacement vectors in elastic and fluid phases are obtained.

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Meta-session

Emphasizing the metamaterial behaviour of the Mie scattering coefficients and Debye series for negative refractive index spherical particles

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When an arbitrary wave impinges a spherical particle, the scattering fields can be completely described once the Mie scattering coefficients are known. Their associated Debye series are extremely useful in identifying the various contributions to these fields owed to the infinite series of reflections and refractions that the wave suffers once it encounters the surface of the scatterer. The matched condition (*viz.*, equal characteristic impedances) dictates the magnitude of the reflection (transmission) coefficients, which are expected to be zero (one) whenever the characteristic impedance η_p of the particle equals that of the surrounding medium (η_0), regardless of the metamaterial nature of the sphere. However, a closer analysis reveals that the associated reflection coefficients in the Debye series does not goes to zero when the matched condition is imposed on a negative refractive index (NRI) particle surrounded by a positive refractive index (PRI) host medium, thus rising the question of how physical these parameters are in the double-negative scattering problem. The non-zero reflection coefficients, on the other hand, are indispensable to account for the new external field profile due to the spatial redistribution of such fields provided by the NRI particle, thus demanding a redefinition of their physical meaning, or a new interpretation for them. Here, we propose to explore some physical insights into this problem, rewriting both the Mie scattering coefficients and the Debye series for a loss-less NRI spherical particle impinged by an arbitrary wave, emphasizing the metamaterial behaviour of the scattered fields by writing them as a sum of two terms, one corresponding to the PRI analogous and another one that solely appears for a metamaterial particle. This may help to clarify some previous results reported by the authors concerning the application of such particles in biomedical optics, especially in optical trapping [1–3].

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The Dynamics of Non-Radiative Plasmon Exitation and Rabi Oscillations in Spaser

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The dynamics of non-radiative plasmon excitation in Spaser (Surface Plasmon Amplifier by Stimulated Emission of Radiation) is studied theoretically. Spaser consists of two-level quantum dot placed in the vicinity of metallic nanoparticle [1]. The transition frequency of quantum dot is supposed to be close to the nanoparticle plasmonic resonance. If the quantum dot pumping exceeds the threshold value, the non-zero dipole moment oscillates at a certain spaser frequency [2–4]. It means that spaser generates surface plasmons in the metallic nanoparticle.

In the present communication we show that the steady-state lasing of spaser (spasing) is preceded by the regime of Rabi oscillations. During this stage the phase shift between dipole moments of quantum dot and metallic nanopaticle oscillates with terahertz frequency (Fig. 1). Thus, the energy flow from the quantum dot to the metallic nanoparticle periodically changes the direction. The direct numerical simulation of the spaser dynamics shows that as in the case of the interaction of two classical dipoles, the excitation of surface plasmon in the metallic nanoparticle by the near-field of quantum dot with population inversion is possible only if the phase shift between quantum dot and metallic nanoparticle dipole moments is present. The quantum dot, excited by the pump, transfers its energy to the surface plasmons and finally to the heat due to absorption in the bulk metal of the nanoparticle. It is shown that for nanoparticles of the size of several nanometers the metallic losses dominates the radiaive ones by some orders of magnitude. The influence of the external optical wave on the spasing threshold and losses is also discussed [5,6].

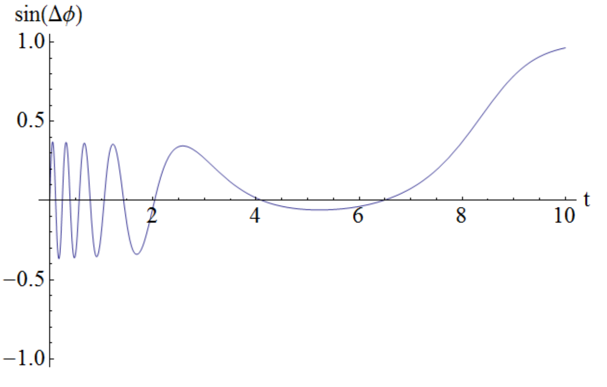

Fig. 1. The Rabi oscillations of the phase shift between dipole moments of quantum dot and metallic nanoparticle.

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The Non-Radiative Plasmon Exitation in Spaser under External Optical Field

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The dynamics of non-radiative plasmon excitation in Spaser (Surface Plasmon Amplifier by Stimulated Emission of Radiation) is studied theoretically. Spaser consists of two-level quantum dot placed in the vicinity of metallic nanoparticle [1]. The transition frequency of quantum dot is supposed to be close to the nanoparticle plasmonic resonance. If the quantum dot pumping exceeds the threshold value, the non-zero dipole moment oscillates at a certain spaser frequency [2–4]. It means that spaser generates surface plasmons in the metallic nanoparticle.

In the present communication we show that the spaser affected by an external optical wave can generate surface plasmons at the frequency of external wave [5, 6]. Such behavior corresponds to regime of synchronization and occurs on certain conditions. The region of values of the amplitude and the detuning of external wave in which synchronization exists is called Arnold tongue (Fig. 1) [7]. Outside of this region the chaotic regime is realized. The detailed investigation of the dependence of the plasmon dipole moment on the amplitude of the external filed for different values of the frequency mismatch shows that there are four different regimes. For $E < E_{Synch}(\Delta)$ the point (Δ, E) is outside of the Arnold tongue and the specter is in the stochastic regime. For $E_{Synch}(\Delta) < E < E_{QL}(\Delta)$ the spaser oscillations are close to stationary and their amplitude is mainly determined by pumping and the frequency mismatch. For $E_{QL} < E < E_L$ the dependence of the dipole moment amplitude on the external field is linear, $d = d_0 + \alpha_1 E$, where d_0 is negative. Finally, for $E > E_L$the influence of the QD on the NP dipole moment is negligible and $d = \alpha_2 E$. In this regime the metamaterial can be described by an effective permittivity. This regime is the most interesting for many applications.

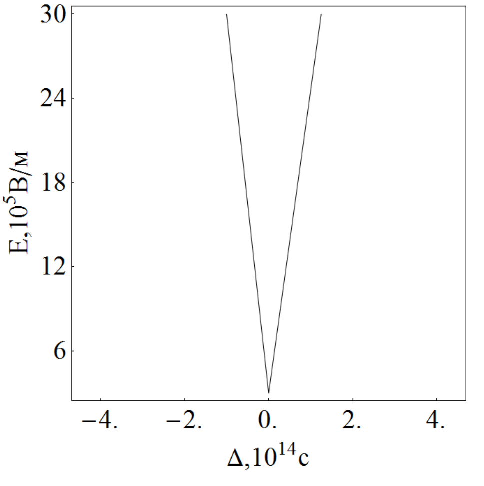

Fig. 1. Arnold tongue.

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Wave propagation phenomena in metamaterials for retrieving of effective parameters

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In the talk we give an overview of the developed restoration procedures and discuss their pros and cons in connection of assigning effective parameters (EP) to metamaterials (MMs). There are plenty of notorious physical phenomena preserving the unambiguous retrieving of EP, like strong coupling between constitutive elements, multipoles resonances, multimode or photonic crystal (diffraction type) regimes. There are also technical limitations of the retrieval methods connected with very strong losses, branching ambiguity, convergence to bulk parameters, etc. Moreover, most of the simple methods reveal so-called wave effective parameters, assigned for particular ligh propagation direction in numerical or real experiments. Therefore, finding the EP is a tricky problem, which still requires a lot of contribution to get deeper insight in it.

We report on our advances in restoration MMs EP taking into account propagation of eigenwaves in multilayered structures (thicknesses 10–100 unit cells). Thus, the question of parameters convergence is naturally resolved in our approach. The method has been tested on complex three-dimensional structures like a split-cube-in-carcass and with circular polarized waves on chiral MMs [1, 2].

Elaborating our approach the new method has been established, where the unit-cell volume and face field averaging procedures define wave and input (Bloch) impedances correspondingly. The first part of the method involves the extraction of the dominating (fundamental) Bloch modes from the simulation data of the field distribution in several unit cells [3]. Then, we explicitly perform either volume or surface averaging of the electric and magnetic fields of the dominant forward-propagating Bloch mode over the unit cell. The ratio of the surface averaged fields provides the value of the Bloch impedance and, respectively, enables the retrieval of wave EPs. The volume averaging of fields of forward propagating fundamental Bloch wave provides the wave impedance, which is needed for the retrieval of the materials parameters. The method is illustrated with several examples.

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Plasmonic nanoantenna based coupler for telecom range

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Scaling down of the electronic integrated circuits faces the limitations of the signal transmission through the metal wires. Nanophotonic waveguides are regarded as an alternative, since they provide higher bandwidth and lower power dissipation. A special attention has been given to the plasmonic waveguides [1–2] which use surface plasmon polaritons (confined surface modes on the interface between dielectric and metal). Among different types of them, the plasmonic slot waveguides [3] are prominent, as they provide small modal area and reasonable optical losses. However, small size of the slot waveguide mode makes difficult to couple light to it from thicker waveguide, for example, single mode optical fiber. Conventionally used tapered fibers have the large length of hundreds of micrometers and therefore cannot be used in the hybrid optical integrated circuits. Wen et al. [4] and Huang et al. [5] proposed to use a dipole nanoantenna for coupling the radiation from the free space. However, it is well known from the microwave antenna theory [6], that a single dipole antenna is not directive, so it does not provide the maximal coupling efficiency (CE) in the receiving regime.

In our presentation we show the substantial improvement of the nanoantenna coupler for the telecom range around $\lambda_0 = 1.55\mu$m with the broadband coupling efficiency close to theoretical maximum if one uses tapered connectors between the nanoantenna and waveguide and additional bottom and side reflectors. The design of the coupler is shown in the Fig. 1(a). Nanoantenna and plasmonic waveguide consist of gold, and they are embedded in silica.

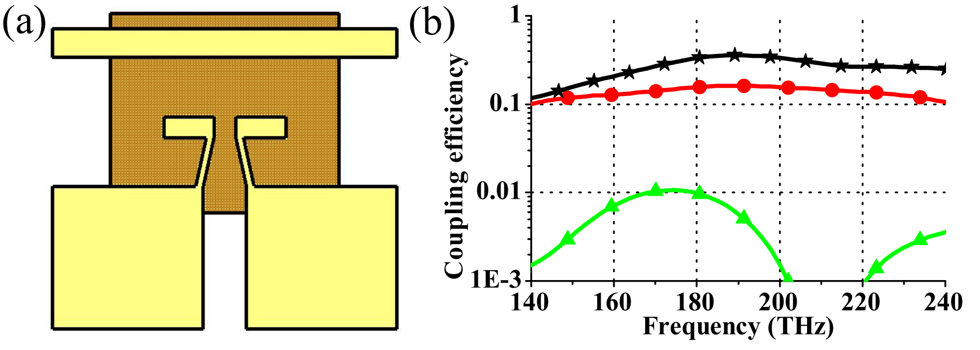

Figure 1. (a) Nanoantenna coupler and plasmonic waveguide: view from top. Metallic parts are embedded in silica (not shown). Dark area represents the bottom reflector which is separated from the nanoantenna with a silica layer. Excitation with an optical fiber is assumed to be from top. (b) Coupling efficiency around the telecom frequency $f = 193$THz (wavelength $\lambda_0 = 1.55\mu$m) without a nanoantenna (triangles), with just a nanoantenna (circles) and with a nanoantenna, side and top reflectors (stars).

We show the results of the nanocoupler numerical simulation and optimization in the Fig. 1(b). Coupling efficiency of the direct coupling from fiber to plasmonic waveguide is below 1% at the telecom frequency 193 THz ($\lambda_0 = 1.55\mu$m). Using the optimized nanoantenna increases CE to 17%. Adding to nanoantenna the side and bottom reflectors brings efficiency to 35%. Coupling efficiency full width at half-maximum is 107 THz. We also show that serial connection of several nanoantennas with the period equal to the wavelength of the surface plasmon polariton mode increase the directivity of the antenna array and therefore the effective area of the nanocoupler.

The nanoantenna based coupler is feasible for fabrication with the modern nanolithography methods, and it has a significant advantage of a small size. It can be used to couple light from the free space to the nanosize plasmonic waveguide as well as to provide the wireless interconnects in the optical integrated circuits between different functional layers.

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Microwave heating of composite metallic powders

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Theoretical and experimental investigation of composite metallic powders microwave heating is carried out in this work. Experimental part of the work is executed using multimode microwave resonator with frequency 2.45 GHz with no sintering atmosphere and nihrosil-nisil thermocouple for temperature measurement. Direct microwave heating of composite powders is realized at various compositions, densities of powders and volume fractions of metal.

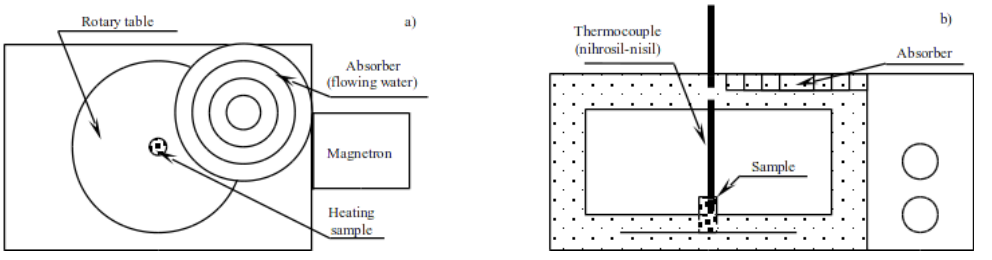

Fig. 1. The scheme of the experimental setup: a) top view; b) front view.

Approximate theoretical model of composite layer with effective electrical, magnetic [1] and thermal [2] properties for calculation of microwave heating of metallic powders is suggested in this work. Metal particles are covered with thin oxide shells. So, we use effective medium approximation to take into account the impact of this shell on the effective permittivity and permeability:

$$p\zeta\frac{\varepsilon_2\left[3\varepsilon_1+(\zeta-1)(\varepsilon_1+2\varepsilon_2)\right]-\varepsilon_{eff}\left[3\varepsilon_2+(\zeta-1)(\varepsilon_1+2\varepsilon_2)\right]}{2\alpha\varepsilon_{eff}+\beta\varepsilon_2}+(1-p\zeta)\frac{\varepsilon_g-\varepsilon_{eff}}{\varepsilon_g+2\varepsilon_{eff}}=0, \qquad (1)$$

where ε_1, μ_1 are permittivity and permeability of copper core; ε_2, μ_2 are ones of oxide shell; ε_g, μ_g are ones of gas; p is the volume fraction of metal in effective medium; $\zeta = (R_2/R_1)^3 = (1+l)^3$, $l = (R_2 - R_1)/R_1$, $\alpha = (\zeta-1)\varepsilon_1 + (2\zeta+1)\varepsilon_2$, $\beta = (2+\zeta)\varepsilon_1 + 2(\zeta-1)\varepsilon_2$; R_2 is radius of metal core with oxide shell; R_2 — without shell.

We use Mie theory to take into account the impact of skin-effect [3] on the permeability and permittivity of the small spherical conductive core. We numerically calculate a heat conduction equation with heat sources in the form of penetrated into layer electromagnetic energy to obtain the heating curves of investigated samples. Numerical calculations of penetrated electromagnetic fields and heating curves are performed using COMSOL Multiphysics 3.2 (License No: 1045581).

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Absorption of the electromagnetic wave in structure of nonmagnetic conductor - magnet

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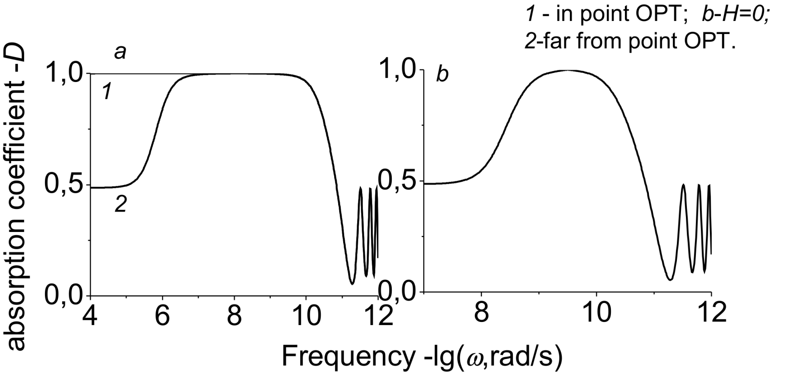
One of the main difficulties encountered in the development of electromagnetic protection, due to the fact that the widely used today kinds of materials even if they are combinations of protective properties in a limited range of frequencies. Establish effective systems to the screening or absorption as a low-frequency magnetic and high-frequency and microwave electromagnetic fields, is extremely challenging. Therefore, the current study is the material to absorb electromagnetic waves in a wide frequency range and having a small thickness.

In this paper we propose an approximate model to calculate the absorption coefficient of electromagnetic waves in a nonmagnetic conductor (NMC) - Magnet. The study conducted by a method based on solving the coupled system of Maxwell's equations and the Landau – Lifshitz equation with the boundary conditions on the electromagnetic field and the magnetization [1]. At normal and near normal incidence of electromagnetic waves due to the formation of a standing wave in a layer of NRM with a thickness less than the skin layer, the intensity of quasi-uniform magnetic field inside the NMC may be close to zero. In this case, a certain ratio of thickness and conductivity of the layer the NRM can be observed strong absorption of electromagnetic waves in a layer of NMC. Frequency range of strong absorption of electromagnetic waves is determined by the area in which the impedance at the lower boundary layer of NMC is greatly increased. Consequently, the frequency range of strong absorption depends strongly on the parameters of the magnetic layer (conductivity, thickness, magnetic anisotropy, saturation magnetization), its width can be controlled either changing the magnetic field, or due to changes in temperature and under certain conditions, this region can be quite large, including and important from a practical point of view of the microwave range. As the magnetic layer can use artificial magnets.



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Composite materials based on electromagnetic crystal

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The problem of development of system of protection, shielding and absorption of broadband electromagnetic radiation becomes more and more important and still remains difficult. There are different ways to solve such problems, for example, with the help of various metal shields, grids, special thin-film coverings. However this solution is expensive.

Composite materials consisting of dielectric matrix with various conductive inclusions (pieces of wire, metal powders, and powders of flake graphite) are cheaper and thus effective. Reflection and absorption of electromagnetic radiation by such materials are mainly determined by filling material and its geometry. It is possible to vary effective complex dielectric permeability by changing the contents of filler, allowing to adjust reflectance and absorptions of the material.

Multilayered absorbers with various dynamic characteristics of separate layers are also promising. In this case it is much easier to receive a material with the prescribed functional properties. For example, the dielectric matrix containing layers with partly oriented graphite flakes demonstrated the strong anisotropy of dielectric permeability [1]. Hence, depending on the polarization of the falling wave the anisotropic composite material will have various reflecting and absorbing properties.

This work presents the results of researches of an easy-to-manufacture anisotropic material. Samples of the material consisting of dielectric matrix with "the electromagnetic crystal" [2] were investigated. The electromagnetic crystal was composed of graphite conducting rods 0.7 mm in diameter arranged in the square lattice. The lattice distance a varied from 5 to 15 mm, which is comparable to the wavelength. That crystal is characterized by energy-band structure in a certain frequency range. Depending on the frequency of the incident radiation the crystal either transmits or forbids propagation of electromagnetic waves.

The researches were carried out on the angular spectrometer based on the measuring instrument KCBH P2-61 in the range of 8–12 GHz. The measurements showed, that the composite material (a=10 mm) has the low-loss transmission window (9–10 GHz) and the clear forbidden gaps (8–9 and 10–12 GHz) where the coefficient of reflection is equal to 1. Similar dependences were observed on the samples with the lattice distances of 8 and 9 mm. At reduction of the lattice distances down to a=8 mm, the shift of the low-loss transmission window of transparency and the forbidden gap to the higher frequencies is observed.

The results demonstrated the opportunity of essential change of reflection coefficient of electromagnetic waves in the range from 8 to 12 GHz.

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Dichroism, chirality and optical activity in planar metamaterials

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Metamaterials have attracted strong scientific interest in recent years for their unusual physical properties, which are rare or absent in nature. One example of such properties is giant optical activity [1] in composite materials build from spiral-like or otherwise twisted elements ("meta-atoms"). Planar chiral metamaterials (PCMs) is another prominent example [2]. In PCMs, the meta-atoms possess two-dimensional (2D) rather than three-dimensional (3D) enantiomeric asymmetry (Fig. a-b). That makes PCMs distinct from both 3D chiral and Faraday media. Polarization eigenstates of PCMs are co-rotating elliptical rather than counter-rotating elliptical or circular [2]. This leads to exotic polarization properties, e.g., asymmetry in transmission for left-handed (LH) vs. right-handed (RH) circularly polarized incident wave without nonreciprocity present in Faraday media. The same asymmetry is observed when the structure is replaced with its enantiomeric counterpart or when the direction of incidence is reversed. Such exotic transmission and polarization properties, combined with the small dimensions of planar structures, make PCMs promising candidate for polarization sensitive integrated optics applications. Several different PCM designs have been proposed recently, the most notable example being the asymmetric chiral split rings [3]. In the same time, this geometry is complicated enough to make microscopic theoretical analysis difficult. It can be shown that simpler geometries may suffice to achieve the desired chiral properties. Numerical simulations reveal (Fig. c) that straightening the split-ring segments into two rods results in similar enantiomeric asymmetry in transmission. Such double-rod dimer meta-atoms, unlike more complicated particles [4–5], lend themselves to a quite straightforward theoretical analysis and more importantly are much easier to fabricate than split-ring or gammadion structures, especially in the optical domain. In this presentation we demonstrate that double-rod plasmonic dimers can function as planar chiral meta-atoms. We derived effective material tensors of PMC base on double-rod dimers analytically in terms of multipole expansion. We find that the resulting effective medium supports elliptical dicroism necessary for PCM effects [6]. We confirm both analytically and numerically that any dimer with distinct 2D enantiomers intrinsically exhibits planar chiral behavior, and systematically investigate the relations between the strength of chiral properties, the dimer's geometrical parameters and contributions of different multipole orders. By doing so, we propose a simple PCM design that lends itself to easy fabrication and simple analytical treatment.

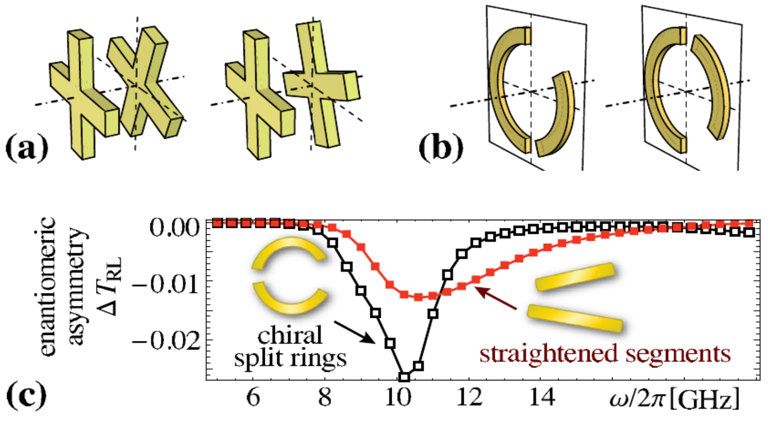

Figure: Example of (a) 3D and (b) 2D enantiomeric meta-atoms; (c) difference in transmission for left-handed vs. right-handed circularly polarized wave for split-ring vs. "straighened split-ring" (rod dimer) structure.

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Coupling classic and quantum objects: from nano-laser to quantum metamaterials

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We present an analytical model for describing complex dynamics of a hybrid system consisting of interacting classical and quantum resonant structures. Classical structures in our model correspond to plasmonic nano-resonators of different geometries, as well as other types of nano- and micro-structures optical response of which can be described without invoking quantum-mechanical treatment. Quantum structures are represented by atoms or molecules, or their aggregates (for example, quantum dots and carbon nanotubes), which can be accurately modelled only with the use of quantum approach. Our model is based on the set of equations that combines well-established density matrix formalism appropriate for quantum systems, coupled with harmonic-oscillator equations ideal for modelling sub-wavelength plasmonic and optical resonators. This model can also be straightforwardly adopted for describing electromagnetic dynamics of various hybrid systems outside the photonics realm, such as Josephson-junction metamaterials, or SQUID elements coupled with an RF strip resonator.

The study of inverse Co-based photonic crystal by microradian X-ray diffraction

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Artificial opals consisting of submicron monodisperse microspheres packed in a face-centered cubic structure and materials on their basis are good candidates for the creation of high quality Photonic Crystals (PhC). They have recently attracted great attention due to their unusual optical properties and promising applications in optical devices. Inverse opals can be synthesized by filling the voids of opal templates and subsequent removing the initial microspheres. Three-dimensionally ordered porous materials are thus obtained. One of the most promising methods of filling the voids is electrodeposition of metal, which enables almost 100% filling. The conducting PhC are of great interest from the viewpoint of multi-functionality and interplay between the optical, magnetic, and electronic transport properties.

The present study is aimed to investigate the structure of a film of the inverse metallic PhCs deposited onto a conductive substrate. We characterized the film of the Co-filled inverse crystal by microradian X-ray diffraction of synchrotron radiation.

Analysis of the diffraction images agrees with an face-centred cubic (FCC) structure with the lattice constant $a_0 = 770 \pm 10$ nm and indicates two types of stacking sequences coexisting in the crystal (twins of ABCABC and ACBACB ordering motifs). The spatial coherence of the periodic order is derived from the full width at half maximum ΔQ of the diffraction peaks: either from a single diffraction pattern for the in-plane coherence of the film or from the rocking curves for the transverse-to-film coherence. Thus, the degree of disorder has been described by three parameters: in-plane correlation length, transverse correlation length, and mosaic spread of the structure.

We showed that the electrochemical method of synthesis, indeed, allows one to duplicate the structure of artificial opals and to obtain the inverse photonic crystal. Among many methods to characterize photonic crystals, the microradian X-ray diffraction of synchrotron radiation provides the most detailed information on average structure, disorder phenomena, and structural coherence.

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Electrodynamic analysis of biisotropic composite materials

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In recent years one can notice the activation of investigations related to designing and analyzing artificial composite materials and structures created on the basis of resistive fibers located in dielectric matrix. Creation of such materials is realized by including randomly-oriented conductive particles into a matrix of epoxy, silicon caoutchouc or rubber [1]. In a series of papers the data is obtained that demonstrates the obvious benefits of using such materials, but the results, as a rule, are presented for the case of arbitrary electrodynamic parameters of a composite, such as dielectric permittivity and magnetic permeability, chirality factor [2]. The composites that contain conductive inclusions (few-coil helices, omega-particles, combinations of helix coil with attached rectilinear conductors) have resonant properties in a frequency range. In this connection it is important to reliably estimate their electrodynamic parameters while designing different appliances on the basis of similar materials.

To describe such media the electric and magnetic dipole moment of a conductive element are defined using, as a rule, its equivalent radiotechnic scheme [3], and after that the averaging is made using some distribution function of these elements in the media. Similar approach does not consider the influence of both geometry peculiarities of conductive particles and the peculiarities of the matrix, in which they are located.

The paper presents a self-consistent approach to analyzing chiral metamaterials and devices on their basis. The approach is founded on analyzing the interaction of electromagnetic field with separate inclusions without limitations on their geometry and considering the possible magneto-dielectric coating of inclusions or impedance properties of the conductor, which they are made of.

The method of integral equations in thin-wire approximation is used as a basis of the calculation. It was suggested to use the corresponding modifications of the known Pocklington integral equation to calculate current distribution in a separate conductive inclusion considering either surface impedance or possible magnetodielectric coating [4]. The influence of the coating is taken into account by introducing electric and magnetic polarization currents that are considered as foreign currents in the free space within the volume of magnetodielectric. Within the frames of quasistatic thin-wire approximation electric and magnetic polarization currents are replaced with equivalent currents, flowing along the conductor axis.

After that, using the known current distribution, the values of electric and magnetic dipole moments of a wire element are calculated. The information about dipole moments of inclusions gives an opportunity to define their polarizability factors (PF). And by the known PF of a single scatterer, using Maxwell-Garnett method for biisotropic chiral media, the effective dielectric permittivity, magnetic permeability and chirality parameter of a composite are determined [5]. The knowledge of material parameters allows to solve the tasks of practical use of similar composites.

Electrodynamic analysis is carried out for composites with inclusions in the form of few-coil helices, omega-structures, particles in the form of a helix coil with attached rectilinear conductors, oriented perpendicular to coil plane. The dependence of material parameters on particle geometry, coating parameters, inclusions concentration in the composite volume is investigated. Numerical experiments are carried out in the frequency range, corresponding to the first resonant response, when conductor length of a particle is close to one half of a wavelength of the electromagnetic field interacting with the particle. Some calculation results were compared to the data obtained via other numerical methods and analytical approximations [3,5].

The advantage of the proposed method for analyzing biisotropic composites is the combination of calculation speed with versatility relative to inclusions geometry, the possibility of considering magnetodielectric fibers.

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Tunable Optical Antennas Based on Metallic Nanoshells with Nanoknobs

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Near-field optical probes for tip-enhanced spectroscopy can be based on metallic nanostructures due to their unique optical properties, which are underpinned by surface plasmon resonances. One of the possibilities to create such probes (known as optical antennas) is using spherical metallic nanoparticles. However the plasmon resonance frequency of metallic spheres has just faint dependence on particle diameter so it is hardly possible to tune the resonance to the desired part of the spectrum. In this paper optical properties of a complex metallic nanostructure are investigated. The structure consists of a spherical metallic nanoshell (dielectric sphere covered with a metallic layer) and a smaller metallic nanoparticle ("nanoknob") on its surface. It was found that the plasmon resonant frequency of the entire structure illuminated with light is guided by the geometrical and material properties of the metallic nanoshell, while the local field enhancement is observed near the metallic "nanoknob". The idea is supported with electromagnetic modeling: the resonant wavelength changes from 530 nm to 630 nm upon the decreasing of gold layer thickness of the nanoshell. The designed structures were created by means of a novel method based on precise manipulation under electron beam: a single 120 nm dielectric sphere was obtained at the tip of a tungsten needle in an electron microscope. It was then covered with a 20 nm thick gold layer to create a shell and a 40 nm platinum particle was formed on its surface afterwards. The fabricated structures were investigated by means of scanning near-field optical microscopy.

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A novel optical component: compact nonreciprocal divider based on 2D photonic crystals

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We suggest and analyze a new nonreciprocal optical device based on 2D photonic crystals which fulfills simultaneously two functions: division of the input signal and isolation of the input port from two output ones. This allows one to reduce the dimensions of optical subsystems.

The minimum number of ports of the projected two-way divider is 4: input port 1; two output ports where the input signal is divided (numbered for example by 2 and 3); and an additional port 4 where the signals due to undesirable reflections in ports 2 and 3 are collected so that the input port 1 is isolated from output ports 2 and 3. The ideal matching, the 3dB division and isolation conditions, symmetry and unitary property allow us to write the desired scattering matrix in terms of power as follows:

$$[P] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1/2 & 0 & 1/2 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \end{pmatrix} \qquad (1)$$

We choose the direction of uniform magnetization by a dc magnetic field $\mathbf{H}_0$ along the axis z which is perpendicular to the plane xoy of a photonic crystal. Using magnetic group theory, it can be shown that for 2D structure of the four-port, the possible elements of symmetry are two-fold (C_2) and four-fold (C_4) rotations around the z-axis and planes of symmetry σ combined with the Time reversal operator T, i.e. $T\sigma$. It can be also shown that the symmetries C_4 and C_2 do not serve for our purposes. Thus, the only symmetry which can give the desired scattering matrix is the antiplane of symmetry $T\sigma$. Two orientations of the ports with respect to the antiplane can exist. Using the commutation relations $[R][S] = [S]^t[R]$ for the antiuntary element $T\sigma$ we calculate the scattering matrices in terms of the voltages.

In electrodynamical analysis, we consider transverse electric (TE) mode of the connected waveguides with the components H_z, E_x, E_y. Parameters of 2D crystal are as follows: hexagonal lattice of air holes of radius 0.3a, $a = 0.48\mu m$ is the period of lattice. Magnetic material is bismuth iron garnet (BIG) material with refractive index n=2.5. The magnetic media permittivity tensor of the BIG magnetized in z-direction has the diagonal element $\epsilon = 6.25$ and the nondiagonal one $k = 0.3$, the Voigt parameter is $k/\epsilon = 0.048$. As a starting point for the divider simulation, one can choose for a given central frequency a resonator without magnetization which provides the resonance of lowest (dipole) rotating modes [1-3]. Then, connecting four waveguides to the magnetized resonator in the way discussed above we realize the nonreciprocal divider. We will show the calculated characteristics of the divider obtained by software COMSOL.

Finally, using the theory of magnetic groups, we have shown that in 2D geometry with homogeneous magnetization the only element admissible for the nonreciprocal divider is antiplane of symmetry. By numerical simulations we have shown realizability of the nonreciprocal divider based on 2D photonic crystal. The frequency band of the 3dB divider is about 100 GHz at the level of isolation 20dB for the wavelength $\lambda = 1.55\mu m$.

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Determination of invisible inhomogeneities of refractive index from eikonal equation

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Basing on eikonal equation we develop a theoretical approach for evaluation of the parameters of refractive index inhomogeneities which do not disturb ray pattern of the scalar wave field outside the inhomogeneities. As an example, calculation of the characteristics of isotropic dielectric covering which shield spherical cavity from radiation of a given source is presented. It is demonstrated that 2D eikonal equation and 2D wave equation lead to the same expressions for invisible inhomogeneities of refractive index. Finally, we discuss the possibility of designing 3D refractive index inhomogeneities that are invisible to radiation which eikonal is independent on one of Cartesian coordinates.

Lasing of surface plasmons in metallic groove

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Recently, a new branch of the quantum optics, quantum nanoplasmonics, has been developed. The advantage of the surface plasmons over the photons consists in a smallness of their wavelength and in the possibility of the field localization in the small (subwavelength) regions, which opens the perspective of construction of subwavelength optical devices, such as waveguides, resonators, antennae, etc. Of particular interest are plasmons, travelling along 1D objects like wires, wedges and grooves [1].

However, plasmons have an essential shortcoming: their existence requires presence of media with $\varepsilon < 0$, i.e., metals, possessing substantial losses. In order to increase the plasmon propagation length, the use of gain media was suggested [2]. If the gain exceeds the threshold value, the system generates plasmons of large amplitude without any incident wave. It is the case in spaser [3], in which a quantum dot excites plasmons at a metallic nanoparticle in a nonradiative way. This process is $(kd)^3$ times more effective than the radiative one ($k = \omega/c$ is a free-space wave number and d is a distance between the quantum dot and nanoparticle). As a result, the energy is mostly radiated into a plasmon instead of photon.

We suggest a new kind of spaser based on plasmons excited at a groove in a metal surface. The plasmons are amplified by quantum dots places at the groove bottom. It is shown that at the plasmon propagation along the groove the gain can exceed loss at realistic parameters of the system, which is enough for lasing in the circular groove. Employment of 1D plasmons instead of a nanoparticle plasmonic oscillations gives additional advantages to the spaser. First, the radiation is directional. Second, use of the proper geometry (form of the groove) can eliminate radiation into continuous spectrum (leaky modes).

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Magnetooptical Switching of the Tamm State in VC Laser

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The lasing regimes of the Faraday laser using the Tamm state as a working mode are studied. The Tamm state is excited at the interface of a magneto-photonic crystal and a photonic crystal [1,2] with gain. The magnetooptical effects are shown to switch the lasing on and off. In our research we use modal approach. In addition, we use the slow amplitudes approach:

$$\frac{\partial E}{\partial t} + \frac{\omega}{2Q}E = 2\pi i P$$
$$\frac{\partial P}{\partial t} + \frac{1}{\tau_p}P = -i\left|d\right|^2 nE$$
$$\frac{\partial n}{\partial t} + \frac{1}{\tau_n}(n - n_0) = Re\left[-iE^* P\right].$$

These equations determine the amplitude of electric field, the polarization and the inverted population, respectively.

Presence of the layer wedging causes anisotropy of two linear polarizations, leading to different values of the Q factors, Q_x and Q_y[3]. Furthermore, the anisotropy can be introduces intentionally [2].

It is shown that two kinds of transition, stationary lasing/beating and stationary lasing/absence of lasing, can be observed when varying amplitude of the external magnetic field and Q factors of both the linear polarizations. The condition of switch between the operating modes is obtained.

Employing both the linear stability analysis and the numerical methods we have obtained the values of relaxation times for switching the lasing on and off.

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Enhanced radiative emission of dyes coupled to long range surface plasmons in asymmetric structures

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Studies of the interactions between metallic structures and fluorescent dyes has a long and rich history dating back almost a century. Over the last decade it has been receiving renewed interest as a result of the boom in nanophotonics and metameterials research, and applications for e.g. loss compensating/spasing. A distinct and also potentially useful application would involve optimizing these interactions for higher contrast fluorescent imaging near surfaces. It is well known that a strong enhancement in the radiative emission into free space may be observed for fluorophores near (less than $\sim \lambda$) nano-structured/roughened metallic surfaces. The effect can be attributed to both enhancement of the incident excitation field, and coupling of the fluorophore emission, to surface plasmons, which in the latter case are scattered into radiation by the subwavelength structural features. On the other hand it is also well understood that a decrease in the radiative decay will be observed when distances between the fluorophore and the metallic interface are $< \lambda$ and the metallic interfaces are very smooth. This is the result of excited state flourophores efficiently coupling to different bound surface and bulk modes in the metal, which due to the phase matching conditions can not radiate into the lower refractive index space away from the interface above the fluorophore. A means to enhance the free-space radiative decay rate of a fluorescent dye near a *smooth* interface could among other things be of interest for single molecule/photon detection applications as well as of possible technological relevance due to the reduced large-scale fabrication costs and improved homogenuity that can be achievable between samples.

In the presented study we consider the near-field interaction of a typical fluorescent dye with planar asymmetric multilayer metal/dielectric films that supports a bound symmetric long range surface plasmon polariton mode (s_b), above a certain cut-off energy (E_c). It is found that if the structures are designed such that E_c lies above, but close to, the lowest vibration/rotational excited state of the fluorophore a measurable increase in the far-field radiative emission from the fluorophore should be observable in the direction away from the structure. This is the result of the coupling of higher excited energy states of the fluorophore to the s_b mode in the structure, which due to the finite spectral width of the mode, cause an increased near-field intensity at $E \sim E_c$ which scales with the population of the s_b near E_c. The probability of decay via radiative spontaneous emission (as compared to other lossy, non-radiative and dissipative channels) is significantly larger for $E < E_c$ than for $E > E_c$, and one therefore obtains an increase in the free-space radiative emission compared to the case of the same fluorescent dye in a homogenous environment. The enhancement factor will depend in detail on the material and design parameters of the multilayer structure, the fluorophore-structure spacing, and the Franck–Condon coefficients of the fluorophore.

We consider the case of an arbitrary generic fluorophore and propose several realistic structures that would exhibit radiative emission enhancements by more than an order of magnitude. We derive the criterion for the various coefficients and transition rates for the proposed structures. We find that the effect is robust in that the enhancement factor is not too sensitive to perturbations in the geometric and material properties of the structure and the fluorophore location above the structure. As expected, in all cases the enhancement is found to be much larger for perpendicular dipole orientations which can couple more efficiently to the plasmonic modes of the structure. This effect is likely to be useful for a wide range of applications ranging from improved solid state light emitting devices to high resolution and contrast fluorescence bioimaging near surfaces (e.g. for DNA arrays).

Metatronics

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In recent years, in my group we have been working on various aspects of metamaterials and plasmonic nano-optics. We have introduced and been developing the concept of "metatronics", i.e. metamaterial-inspired optical nanocircuitry, in which the three fields of "electronics", "photonics" and "magnetics" can be brought together seamlessly under one umbrella – a paradigm which I call the "Unified Paradigm of Metatronics" [see e.g., 1–8]. In this novel optical circuitry, the nanostructures with specific values of permittivity and permeability may act as the lumped circuit elements such as nanocapacitors, nanoinductors and nanoresistors. Nonlinearity in metatronics can also provide us with novel optical nonlinear lumped elements. We have investigated the concept of metatronics through extensive analytical and numerical studies, computer simulations, and recently in a set of experiments at the IR wavelengths. We have shown that nanorods made of low-stressed Si_3N_4 with properly designed cross sectional dimensions indeed function as lumped circuit elements at the IR wavelengths between 8 to 14 microns [7]. We have been exploring how metamaterials can be exploited to control the flow of photons, analogous to what semiconductors do for electrons, providing the possibility of one-way flow of photons. We are now extending the concept of metatronics to other platforms such as graphene, which is a single atomically thin layer of carbon atoms, with unusual conductivity functions. We study the graphene as a new paradigm for metatronic circuitry and also as a "flatland" platform for IR metamaterials and transformation optics, leading to the concepts of one-atom-thick metamaterials, and one-atom-thick circuit elements and optical devices [8]. I will give an overview of our most recent results in these fields.

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Broadband Compensation of Losses in Layered Optical Metamaterials with Dispersive Gain

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One of the main problems preventing a rapid growth of applications of metamaterials in the optical spectral range is their practically unavoidable losses. The loss compensation is possible using media with optical amplification (gain), e.g., in layered structures [1]. Important are also applications of active photonic crystals where high values of gain are achievable near the spectral band edges where the group velocity is small [2, 3]. However, in layered structures with large transverse sizes, amplified spontaneous emission can develop, thus affecting substantially the light propagation. Here we consider the light propagation in the composite system for the layers with absorption and gain. For weak amplitude, the problem is linear, and it can be solved in terms of the transfer matrix; the main goal here is the analysis of the conditions and efficiency of the losses compensation with an optical gain in a periodic system of layers with absorption and gain. We demonstrate that introduction of layers with engineered frequency dependence of gain would allow to compensate the fixed losses in such a way that the transmittance through an arbitrary large number of layers in the system will remain the same as that for the system of transparent layers. As a result of this compensation, the transmittance increases in a fairly wide angular range, but it is accompanied by oscillations. The competing process of amplified spontaneous emission for radiation propagating mainly in the direction orthogonal to the system axis develops, and it can have a higher threshold.

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New interference effects in spectra of opal-like photonic crystals under multiple diffraction regime

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Photonic crystal (PhC) structures (spatially periodic solid-state structures whose permittivity is modulated with spacing comparable to the wavelength of light) attract much attention in up-to-date studies. Considerable interest to PhCs is stimulated by currently emerging applications in laser technology and optoelectronics utilizing nanostructured materials [1] and by the fundamental scientific problems which come about in attempting to explain PhC mediated optical phenomena [2]. One of the striking phenomena is the multiple Bragg diffraction appearing when two or more families of crystal planes with different plane indices are involved in Bragg reflection at the same frequency [3].

We report here on theoretical studies of the Bragg reflection and transmission spectra formed by resonant diffraction of light from a three-dimensional (3D) opal-like PhC slab for the case when the multiple diffraction effects are of importance. We focus primarily on new interference effects which are expected to be observed for thin PhC films. Particular attention is paid to the analysis of the complex spectral structure governing the Bragg reflection contour which is shaped due to the interference of light on two plane boundaries of the PhC slab and strongly depends on structural perfection of PhC.

The eigenmode dispersion curves and zero-order diffraction (reflection and transmission) spectra are calculated using the dynamical multiple diffraction theory generalized to the case of high dielectric contrast of 3D spatially periodic medium, within the frameworks of two- and three-band mixing approximations [4]. The incidence angle $\theta = 57^\circ$ for s-polarized light was taken in our numerical calculations. The case of the three-band mixing model corresponds to the 3D situation and takes into account simultaneous diffraction from two systems of crystal planes (111) and $(11\bar{1})$, with $(11\bar{1})$ being inclined to the reflecting surface of the slab. The Bragg reflection spectra are depicted for this case in Figure (a) by solid lines. Dashed lines in the Figure (a) are referred to the two-band mixing model (one-dimensional (1D) situation) and describe the light diffraction from the only one system of crystal planes, (111), parallel to the lateral surface plane. We see, comparing the two models, that the calculated spectra differ qualitatively in the spectral range of the photonic band gap (PBG) where maximum (minimum) values of reflectance (transmittance) take place. Outside the PBG resonant region, classic Fabry-Pérot interference fringes are observed.

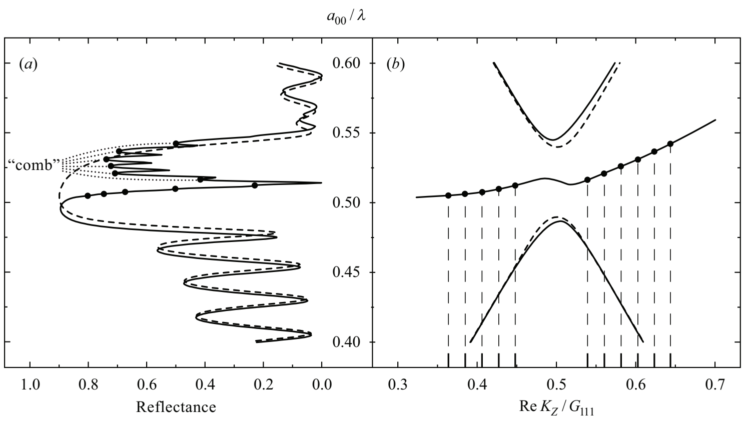

Figure: Calculated Bragg reflection spectra (a) and corresponding dispersion curves for the electromagnetic eigenmodes (b) of an opal-like photonic crystal film (assembled from polystyrene spheres of 300 nm in diameter) with the 20 monolayers thickness. The dashed curve describes the light diffraction from the only one system of crystal planes (111). The solid curve corresponds to the simultaneous diffraction from two systems of crystal planes (111) and (11$\bar{1}$).

Calculations performed in the framework of 3D model show that an additional short-periodic Fabry-Pérot-like interference structure (interference "comb") appears in the resonant range. It should be noted that such a "comb" disappears with increasing the imaginary part of permittivity, which corresponds usually to the deterioration of the PhC structural quality. Thus, in order to register the interference "comb" one needs to use PhC specimens of enough high perfection.

The physical cause of the short-period oscillations in the resonant region of the reflection spectrum becomes clear after comparison of this spectrum with the electromagnetic eigenmodes spectrum in the PBG region. Figure shows Bragg reflection spectra (a) and corresponding dispersion curves for the PhC eigenmodes (b). Finite thickness of the PhC film leads to a spatial quantization ΔK_Z of additional low-group-velocity modes, which determines the corresponding oscillation period in the resonant region of the spectrum.

Comparison of the results of the full-wave electrodynamics simulation (using discretization of Maxwell's equations for the exact formulation of the problem) with the results obtained in the three-band mixing approximation is performed. It is shown that the used approximation correctly describes the complex shape of the Bragg reflection contour associated with the 3D nature of diffraction.

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Limit analysis of time-harmonic Maxwell equations with fast oscillating coefficients and dispersion

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The scattering of electromagnetic waves by a periodic medium with dielectric or metallic inclusions is considered. In the situation where the wavelength λ is larger than the period d, a small parameter $\eta = d/\lambda$ is introduced and the limit of the electromagnetic field when η tends to 0 is investigated. Various regimes are shown to exist: the usual quasi-static approach, artificial magnetism near resonances, spatial dispersion. It is stressed that the approach deals directly with a finite-size medium and is NOT based on a Bloch wave analysis, that it DOES provide boundary conditions for the electromagnetic field and also that it gives a VERY PRECISE MEANING to the notion of limit field, by providing the topology for which fields converge when η tends to 0. The extension to upper orders will be discussed as well as the phenomenon of boundary layer. Finally, we will discuss the connection with other, non rigorous, approaches.

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Second harmonic generation in spatially inhomogeneous metamaterials

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We studied the process of second harmonic generation in metamaterials responding with a different sign of the refraction index to a pump field and to its second harmonic. Characteristics of a second harmonic generation in homogeneous metamaterials depend on a phase mismatch. In conventional dielectrics, optical fields are periodically varying along the sample if the phase mismatch is nonzero. The fields exhibit monotonic behavior in the ideal case of a zero mismatch. In the last case the energy flows unidirectionally from the pump to the second harmonic wave which leads to high conversion efficiency. The direction of energy flow varies along the sample if phase mismatch is nonzero. We demonstrated that in contrast to conventional dielectrics, unidirectional energy transfer takes place not only for a single zero value of a phase mismatch but also for an interval of phase mismatch values. Boundaries of this interval determine critical values of a phase mismatch which separate periodic behavior from monotonic behavior. We studied second harmonic generation in inhomogeneous metamaterials when the value of the phase mismatch changes gradually along a sample and crosses its critical value. In this case both regimes of second harmonic generation are present in the same sample. We analyzed transient effects, the distributions of the fundamental and second harmonic fields, conversion efficiency, and the influence of losses.

Stability of nonlinear pulse in waveguide structure with metamaterial slab

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In this paper the nonlinear pulse propagation in a two-layered waveguide structure is considered. One layer is a double negative metamaterial, the other one is an usual dielectric. This layers are divided by thin film consisted of two thin layers: thin type II superconductor and nonlinear insulator with Kerr nonlinearity. In this structure can propagate the nonlinear soliton-like pulses. The combination of double negative metamaterial and usual dielectric acts in considered waveguide as a slow-wave structure. The presence of high temperature superconducting film in the mixed state allows to change the parameters of nonlinear pulses by changing the external magnetic field. We have studied the regions of stability of nonlinear pulses on the basis of the Lighthill criterion. It is shown that the boundary separating regions of stability and instability, changes with the pulse amplitude and external magnetic field. It is revealed that nonlinear pulses can be amplified in the region of instability due to the energy of the moving Abrikosov vortex lattice in superconducting film. The dependence of velocity and pulse duration on the magnitude of magnetic field allows to control the parameters of propagating in the region of stability of nonlinear pulses.

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Structural and magnetic properties of inverse opal photonic crystals studied by small-angle x-ray and polarized neutron diffraction

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The structural and magnetic properties of inverse opal photonic crystals are studied by complementary experimental techniques, including wide-angle and small-angle diffraction of synchrotron radiation, and polarized neutrons. The samples are fabricated by electrochemical deposition of nickel (cobalt) in voids in a colloidal crystal film made of polystyrene microspheres followed by their dissolving in toluene. It is established from the microradian XRD that inverse opal photonic crystals repeat completely the structure of a matrix of a photonic crystal with minor alteration of a lattice constant. The wide-angle x-ray powder diffraction shows that nanosize fcc nickel (or hcp cobalt) crystallites, which form an inverse opal framework, have some texture prescribed by principal directions in inverse opals on a macroscale, thus showing that the atomic and macroscopic structures are correlated. The polarized small-angle neutron scattering is used to detect the transformation of the magnetic structure under applied field. Different contributions to the neutron scattering are analyzed: the nonmagnetic nuclear one, the pure magnetic one, and the nuclear-magnetic interference. The latter in the diffraction pattern shows the degree of the spatial correlation between the magnetic and nuclear reflecting planes and gives the pattern behavior of the reversal magnetization process for these planes. The field dependence of pure magnetic contribution shows that the three-dimensional geometrical shape of the structure presumably leads to a complex distribution of the magnetization in the sample.

Nonlinear Surface Plasmons and Giant Second Harmonic Generation Under Temporal Resonance of Main and Second Harmonic

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Applications of principles of plasmonics and metamaterials are now the most actual areas of nanophotonics, as routs to subwavelength devices [1]. In the present paper, we study the propagation of nonlinear surface waves in a semi-bounded electron plasma and find the conditions of giant second harmonic generation. We consider nonlinearities connected with both volume and surface plasma charges. Contributions of both Lorenz and "kinematics" plasma nonlinearities are taken into account. We are looking, first of all, for the conditions of giant second harmonic generation and therefore consider the neighborhood of "temporal resonance" with the second harmonic, in other words plasma resonance at the second harmonic of the fundamental frequency of a surface wave. It is interesting to note that under these conditions, spatial resonance of the second harmonic is absent, and the second harmonic is not eigenmode of the surface plasma waves. The system of equations for slowly varying amplitudes of coupled main and second harmonics is obtained. For this purpose, the method, similar to that used in the course of derivation of plasma version of Umov-Pointing equations was used and bilinear relationships in differential [2] and then in integral form were obtained. Note that self-interaction of the main harmonic, connected with the contribution from zero harmonic, is accounted for. Zero harmonic is found as a solution of corresponding problem with "nonlinear driving force" and proper boundary conditions. Numerical computations were done under the condition of "constant pumping" for the main harmonic at the "input of the system" ($Z = 0$, where Z in coordinate along the surface of plasma). The main result of the present paper is a possibility of giant generation of the second harmonic, but only in the close vicinity of the input ($Z = 0$) of the system (because the second harmonic is only temporally resonant). Also, conditions of "modulation-like" instability of the main harmonic are investigated. It was shown that the effect of surface nonlinearity may lead to a change of sign of total cubic nonlinear coefficient in the equation for the main harmonic. This is important for the developing the process of modulation instability. Nonlinear dynamics of coupling harmonics is searched for different signs of above mentioned nonlinear coefficient (either in the absence or presence of surface nonlinearity) and deviation of a frequency from the resonant one. It is shown that an instability of the main harmonic is possible, but it demonstrates unusual properties under the conditions of (near) resonant coupling with the second harmonic. Namely, the modulation-like instability is present and even the most pronounced in the range of parameters, where Lighthill criteria [3] of standard modulation instability is not fulfilled. Obtained results, in particular giant generation of the second harmonic on the plasma surface could be useful, for example, for sensing applications.

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Homogenization of metallic metamaterials and electrostatic resonances

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It has been noted that the usual approaches for homogenization (e.g. Bruggeman) could lead to singularities when considering composites comprising materials with positive and negative permittivities. This situation is specifically encountered in the field of metamaterials, in particular in the visible range. The question then arises to unerstand if these resonances are just artifacts or possess, on the contrary, a clear physical meaning.

In this work, we study the homogeneous properties of a bidimensional structure that is made of a periodic set of metallic wires embedded in a dielectric host medium. The structure is considered in a region of wavelengths that are much larger than the period of the structure. The work comprises a theoretical part where we develop a two-scale approach to the homogenization of the structure. As it is the case for the common physical approach, it leads to an effective permittivity with strong (electrostatic) resonances as well. The theoretical results, and the presence of resonances, are confirmed by numerical computations based on a rigorous modal approach. In the numerical results we consider specifically the case of silver and gold nanowires, described by a dispersive negative permittivity. We show that the main parameters for the onset of resonances is the optical filling ratio of the structure. Keeping in mind the possibility of performing experiments, it is far easier to keep the geometrical filling ratio constant and to consider strongly dispersive materials in the range of wavelengths considered. Here, besides the crucial fact that they are widely used in nanotechnology, silver and gold nanowires comply with our needs in the visible region of the spectrum.

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Surface states at the interface of metall-dielectric and magnetic periodic nanostructures.

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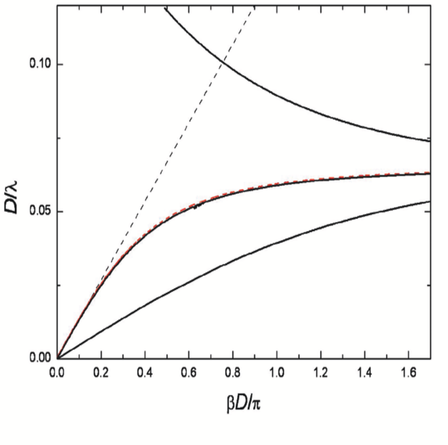
The properties of the surface states which may exist at the boundary of 2 semi-infinite one-dimensional periodic structures are studied. Such periodic structures can exhibit left handed material (LHM) behaviour which means that they can simultaneously have negative effective dielectric permittivity and negative effective magnetic permeability. Such periodic structures are commonly treated as an effective media, i.e. permittivities and permeabilities are averaged over the period which seems to be justified while the period of the structure is far less than the characterstic wavelength. Thus, the surface states on the interface of such structures are commonly treated in the effective media approach. The dispersion of these surface states however can also be obtained in a fully rigorous way through transfer matrix method approach and is defined by the solution of following transcendent equation:

$$\frac{\lambda^L - T_{11}^L}{T_{12}^L} = \frac{\lambda^R - T_{11}^R}{T_{12}^R}$$

Where λ^L and λ^R are the eigenvalues of the transfer matrices for left and right periodic structures and $T_{11}^L, T_{12}^L, T_{11}^R, T_{12}^R$ are the components of these matrices. In this work the dispersion curves for the surface states at the interface of different periodic structures is obtained, particularly metal-dielectric nanostructures and magnetic periodic structures, using rigorous transfer matrix method approach and compared to the results provided by the effective media approach. It is shown that although effective media approach is in good agreement with rigorous results, there are several features which it does not predict. For example, if we are looking for dispersion relations of the surface states at the interface of 2 metal-dielectric nanostructures, we can see that together with a conventional surface plasmon predicted by the effective media approach (depicted with the dashed red line) there exist 2 more surface states: one with positive and one with negative group velocity. It is notable that effective media approach fails to completely describe the dispersions of the surface states even in the case when the characteristic wavelength is much larger than the period of the structure which is due to the nonlocal plasmonic effects at the metall-dielectric interfaces of periodic structure. In the work applicability bounds of effective media approach for the periodic structures with plasmonic and magnetic resonanses is also studied.



The diffraction of single cycle terahertz waves

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In this paper the spectral approach [1] is used for the analysis of the diffraction of single cycle terahertz waves. We derive the analytical expressions for one period THz waves of Gaussian transverse profile propagating in Fresnel's and Fraunhofer's diffraction zones and demonstrate the diffraction caused changes in both temporal and spatial profiles of pulses. We also estimate the typical diffraction distances for one period THz waves of Gaussian transverse profile.

Assuming that the radiation is linearly polarized along x and its spectrum at $z = 0$ has the form

$$C_x(\omega, k_x, k_y) = \pi\rho^2 \exp\Big(-\frac{\rho^2(k_x^2 + k_y^2)}{4}\Big)G_0(\omega), \tag{1}$$

where ρ is the spatial size of pulse, kx and ky are frequencies of temporal and spatial spectrum respectively, $G_0(\omega)$ is the initial temporal spectrum of the pulse near the axis.

The spatiotemporal spectrum of a wave at an arbitrary distance z can be described by the following relation

$$g(\omega, k_x, k_y, z) = \pi\rho^2 \exp\Big(-\frac{\rho^2(k_x^2 + k_y^2)}{4}\Big(1 - i\frac{2cz}{\rho^2 n(\omega)\omega}\Big)\Big)\exp\Big(-i\frac{n(\omega)\omega z}{c}\Big)G_0(\omega) \tag{2}$$

Using the Fourier transform formulae

$$E_{x,y,z}(t, x, y, z) = \frac{1}{(2\pi)^3}\iiint\limits_{-\infty}^{+\infty} g_{x,y,z}(\omega, k_x, k_y, z)\exp(i(\omega t + k_x x + k_y y))\,d\omega dk_x dk_y \tag{3}$$

and the relation (2) we can present the field evolution in the form

$$G(\omega, x, y, z) = \frac{1 + i\dfrac{2cz}{\rho^2 n(\omega)\omega}}{1 + (\dfrac{2cz}{\rho^2 n(\omega)\omega})^2}\exp(-\frac{x^2 + y^2}{\rho^2}\cdot\frac{1 + i\dfrac{2cz}{\rho^2 n(\omega)\omega}}{1 + (\dfrac{2cz}{\rho^2 n(\omega)\omega})^2})\exp(-i\frac{n(\omega)\omega z}{c})G_0(\omega) \tag{4}$$

From (2) and (4), we can introduce the following characteristic distances for Fresnel and Fraunhofer diffraction zones respectively:

$$z_1 = \frac{\rho^2}{2c}\{n(\omega)\omega\}_{\min}, \tag{5}$$

$$z_2 = \frac{\rho^2}{2c}\{n(\omega)\omega\}_{\max} \tag{6}$$

For $z >> z_2$ in case of the dispersionless media (like atmospheric air) with refractive index we have simpler expression for temporal spectrum (4):

$$G(\omega, x, y, z) = T(z)\exp(-T^2(z)\omega^2\cdot\frac{x^2 + y^2}{\rho^2})\cdot i\omega G_0(\omega) \tag{7}$$

where $T(z) = \rho^2 n_0/2cz$.

At the center of the wave packet (at $x = 0$), the spectrum (7) takes a simple form

$$G(\omega, 0, 0, z) = T(z)\cdot i\omega G_0(\omega), \tag{8}$$

from which it follows that for any pulse shape the temporal structure of the field at the medium edge $E_0(t)$ in the far field diffraction region is defined by its derivative

$$E(t^{'},0,0,z) = T(z)\frac{\partial E_0(t^{'})}{\partial t'} \tag{9}$$

As follows from (7) and (8), in the Fraunhofer diffraction region for small x and y, the temporal spectrum of the radiation field is shifted towards higher frequencies in comparison with the spectrum at the input $G_0(\omega)$; however for larger x and y it is shifted downwards, to the low frequencies.

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Trapped mode resonances in optic planar structures with asymmetry metal or dieletric elements

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Not long ago, the possibility of high-quality factor "trapped mode" resonances in microwave planar periodic structures was shown theoretically [1], then studied in [2], and confirmed experimentally [3]. These trapped mode resonances are excited in planar double-periodic structures with at least two metal strips in a periodic cell. A small asymmetry of the metal elements of such structures results in excitation of the strong anti-phased current modes through a weak free space coupling, which provides low radiation losses and therefore high Q-factor resonances in microwaves.

Now such type resonances for optical and infrared range attract a big attention. It is explained by their main feature which lay in that the eigenfrequency of trapped mode resonance may be real for infinitesimal dissipation of structure, i.e. radiation losses may be zero. It was shown in [4] that planar arrays of two weakly asymmetric metal elements reveals trapped mode resonances. The quality factor of such resonances is higher than the ordinary plasmon-polariton resonances in planar arrays of optical range.

Thus using of the planar structure with the "trapped mode" resonance in infrared and optical ranges is very perspective. For example, due to greatly increasing electric fields near the structure in trapped mode resonance regime, the structure can be effectively used as a biosensor. Another perspective application is a "lasing spacer", which was proposed in [5]. The lasing spacer is a planar source of spatially and temporally coherent radiation which consists of a planar double-periodic structure in trapped mode resonance regime and a thin substrate of a gain medium.

The trapped mode resonances are accompanied with relatively high value of the electric field in and around metal strips. It is well known that the high level of dissipation inherent to the metal in infrared and optical ranges. It results in increasing of Joule losses for the structures in regime of trapped mode resonances. Thus, a Joule loss in metal elements is a main factor limiting of the trapped mode resonance quality factor in infrared range. One way of the compensation of the Joule losses consists in using of active medium layer between array of metal elements and substrate. A recent papers [6, 7] experimentally demonstrated evidence of such compensation by using a layer of semiconductor quantum dots.

Another way to increasing the quality factor of trapped mode resonances consist in using the array of dielectric elements. Such type elements have not Joule losses, but the nature of resonances in arrays of dielectric elements is different from nature of resonances in metal ones. Thus, the existence of trapped mode resonances in arrays of dielectric elements requires an additional study.

In presentation we want to discuss condition of the trapped mode resonance forming in periodic array of dielectric or metal elements and theirs properties. For this aim the light diffraction on periodic array with two almost symmetry metal or dielectric elements was considered. The period of metal arrays includes two concentric metal rings or two C-elements which have equal or different length. In such structures can exist a high order trapped mode resonance, which has quality factor much higher than first order one.

The array of dielectric elements consists of two dipole-like elements of different length. In our presentation we will analyze the properties of most long-wave trapped mode resonance. Such type resonance is very promising for different applications.

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Novel Horn Antennas Based on Woodpile Structures

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The scientific and technological interest towards electromagnetic band gap (EBG) structures has been growing rapidly since their discovery. The essence of any EBG structure, a periodical dielectric structure with certain geometry and dimension, is that it is a resonance structure. Due to this fundamental property different interesting well-known effects, such as band gaps, controllable dispersion, defect-based waveguiding, field localization or resonant transmission are observed [1, 2]. In recent years EBGs have been widely exploited for the purpose of shaping and improving the radiation characteristics of antennas of different types. In particular, EBG resonator cavities and defects [1–3] have been used to create antennas with large directivities and high efficiencies. Finally, following the analogy with classical metallic horn antennas, introducing a horn-shaped hollow defect can make the EBG-based system work as a horn antenna [4–6]. EBG antennas can substitute metallic horns in certain applications, which is especially valuable for millimeter and THz devices.

This paper demonstrates a new concept of evanescent feeding for EBG horn antennas (Fig.1*a*). It is shown that such horn antennas, realized as hollow defects in a three-dimensional (3D) periodic structure can be coupled to an EBG waveguide via evanescent fields existing in the embedding medium. This concept allows one to design compact arrays of EBG antennas, where horns can share feeds. A woodpile-based evanescently fed horn antenna operating in the F band (at around 110GHz) was designed and its scaled-up Ku band prototype was fabricated and measured.

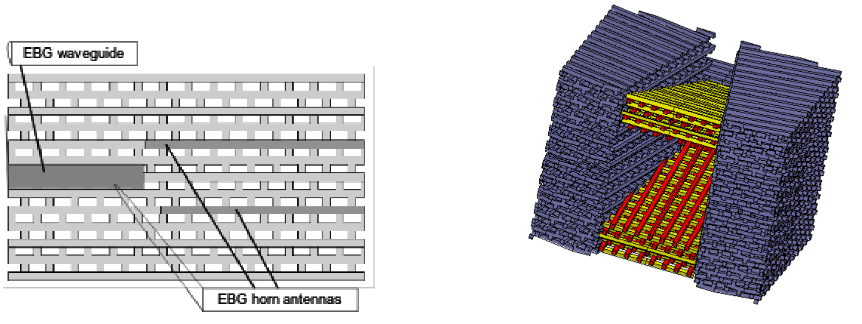

Fig.1. a) Side-view of the evanescently-fed EBG horn antenna, b) 3D view of the pyramidal symmetric EBG horn antenna.

The EBG-based horn antennas in their present form despite offering a wide spectrum of applications still have several fundamental gaps and drawbacks. Up to now only sectoral horns embedded in one layer of rods in a woodpile structure have been designed. The reason for this is the lack of understanding of electromagnetic processes inside such structures. The other principal drawback of the woodpile horn antennas is the absence of mirror symmetry in the stacking direction of a woodpile structure, which results in non-symmetrical radiation patterns of the antennas.

This paper presents the design of a novel symmetric EH-horn antenna based on EBG structures (Fig.1*b*). The main concept of the design lies in turning the EBG structure at the antenna flare angle and thus forming the EBG "walls" of the horn.The main difficulty in creating an EH-antenna or even an H-antenna embedded in various adjacent layers of a woodpile structure, lies in the fundamental difference between the processes of mode formation in metallic and EBG waveguides and horns. This research is aimed at better understanding of the phenomena, observed in 3D EBG defects. Basing on careful modal analysis, this paper explains the fundamental difference between a gradual transition in metallic and EBG waveguides. It is shown that unlike the analogous situation in metallic horns, forming a horn antenna by cutting out a pyramidal shaped defect results in creating abrupt transition between effective EBG waveguides (the local waveguides in each cross-section of the horn antenna). Using the proposed concept of rotating the woodpile structure and thus shifting the EBG lattice nodes provides the real adiabatic transition between en EBG waveguide and the free space.

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Transformation Optics with Metamaterials

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The analysis of propagation of light in nanostructured dispersive metal-dielectric composites is a subject of fundamental as well as practical importance. In recent years attention has focused in particular on artificial metal-dielectric composites (optical metamaterials, OMMs) in which for example either a negative magnetic permeability (metamagnetics) or a negative refractive index (negative index materials) can be obtained in the visible range.

OMMs offer the opportunity for compressing optical and energy in space, dramatically reducing the device footprint and enhancing light–matter interactions. Thus, the interaction of light with optical gain materials is of great importance especially in metal-dielectric nanostructures, where the compensation of losses is essential for practical engineering — so that lasers, amplifiers, detectors, absorption modulators and wavelength converters could be miniaturized, and the cost of integrated optical circuits and devices could be reduced. In order to design active nanoplasmonic devices, accurate models of this light-matter interaction are required.

A loss-free and active OMM in the visible has been recently experimentally demonstrated at our Birck Nanotechnology Center at Purdue [1]; this experiment was supported by finite-element simulations in frequency domain and the appropriate retrieval of the effective bianisotropic properties. Our recent progress in this field will be reviewed followed by a brief discussion of the challenges of the bianisotropic characterization of thin OMM samples.

Along with the development of OMMs, the potential of using the invariant transformations of Maxwell's equations for solving complex electromagnetic problems became clear after the early studies of Weyl, Tamm, Dolin, Post and Lax–Nelson followed by Chew–Weedon, Ward–Pendry, Milton, and Leonhard–Philbin. But in contrast to the early works dealing with simulations, the *transformation optics* (TO) approach solves a fundamental problem of designing a continuous material space for arranging a desired flow of electromagnetic energy [2]. This true revolution started with the development and publication on transformation-designed optical elements by Shurig-Pendry-Smith. Although some theoretical aspects of their approach were known, it was the publication of their method confirmed by critical experiments that made it so important.

Evidently, the TO tactics require a new breed of artificial optical composites (optical metamaterials) that feature extraordinary electromagnetic properties. OMMs are for example capable of providing optical mimicry with invisibility 'carpets' that conceal objects underneath and blend with the environment.

Following a semi-classical branch of TO theory a novel approach to broad-band omnidirectional light absorption has been recently developed at Purdue [3, 4]. The proposed system is based on trapping light with a metamaterial structure forming an effective 'black hole' that can be used in photovoltaics, solar energy harvesting, and optoelectronics. On top of photonics applications similar 'black holes' and other specially designed metamaterials proposed by the Berkeley group can serve as 'table-top universe' for modeling the motion of massive celestial bodies in gravitational potentials under a controlled laboratory environment.

Here we review our recent progress in developing a new class of specially designed optical metamaterial spaces with functionalities that cannot be obtained with conventional optics and natural materials. We also outline our recent development and deployment of an easy-to-use, multifaceted, on-line research environment for the nanophotonics research community [5]. In particular, we show representative examples of our online software tools addressing a growing need in efficient numerical simulations in the area of TO.

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Shaping of light in metamaterial and plasmonic structures

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We will review recent theoretical and experimental results on tunability of periodic photonic structures and metamaterials. First, we will discuss several effects associated with tunability and control of negative index of refraction and specific properties of backward waves. Next, we will overview our recent theoretical and experimental results on the study of subwavelength nanofocusing and shaping of light in plasmonic structures. This includes:

(i) *Nonlinear transverse self-action of plasmon beams propagating in tapered metal-dielectric-metal waveguides* with the Kerr-type nonlinear dielectric [1]. We demonstrate that in contrast to light focusing in straight waveguides, an appropriate choice of the taper angle allows an effective compensation of attenuation with the propagation of a spatial plasmon soliton. For larger tapering angles, significant soliton narrowing is observed, it leads to three-dimensional spatial light nanofocusing of the soliton beams.

(ii) *Nanofocusing and spatial shaping of polychromatic plasmonic beams [2].* We introduce the concept of polychromatic plasmonics and demonstrate several functionalities of the so-called broadband plasmonic lens based on a metal-dielectric-metal curved structure. We utilize quadratic modulation of the dielectric layer thickness in transverse direction to produce a parabolic optical potential which is practically wavelength independent. We develop analytical descriptions and employ simulations to show capability of three-dimensional subwavelength manipulations and beam shaping, including nanofocusing, self-collimation, and optical pendulum effects. The nanofocusing of our lens is demonstrated over a bandwidth exceeding an optical octave (>500 nm) thus allowing for polychromatic plasmon nanofocusing.

(iii) *Generation and the experimental study of near-field evolution of Airy plasmons [3].* We demonstrate experimentally generation, shaping, and near-field mapping of the propagating Airy plasmons earlier discussed theoretically by D. Christodoulides et al. These self-accelerating Airy plasmons exhibit self-healing properties and enable novel applications of plasmonics, including selective surface manipulation of nanoparticles.

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Theory of wave propagation and dispersion in fluid-saturated porous media allowing for spatial dispersion

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Three years ago, we conjectured in this conference and elsewhere [1,2], equations having a "Maxwell" form [3]

$$\phi\partial_t B = -\nabla_i V_i \qquad \partial_t D_i = -\phi\nabla_i H \tag{I$_{1,2}$}$$

$$\begin{aligned}\rho_0^{-1} D_i(t,\mathbf{R}) &= \int_0^\infty d\tau \int d\mathbf{R}' \rho_{ij}(\tau;\mathbf{R},\mathbf{R}') V_j(t-\tau,\mathbf{R}') \\ \chi_0 H(t,\mathbf{R}) &= \int_0^\infty d\tau \int d\mathbf{R}' \chi^{-1}(\tau;\mathbf{R},\mathbf{R}') B(t-\tau,\mathbf{R}')\end{aligned} \qquad (\mathrm{II}_{1,2})$$

to describe, at the macroscopic level, the sound propagation in a fluid-saturated rigid-framed porous medium.

In these Eqs we denoted $\mathbf{V}$ the macroscopic velocity, B the field defined by (I_1), ρ_0 the fluid density, χ_0 the fluid adiabatic compressibility, ϕ the porosity, and H and $\mathbf{D}$, the abstract acoustic "Maxwell" fields. Given these Eqs the question was: how to define and compute from the microgeometry and fluid-solid physical parameters, the constitutive kernels $\rho_{ij}(\tau;\mathbf{R},\mathbf{R}')$and $\chi^{-1}(\tau;\mathbf{R},\mathbf{R}')$? We have now a better understanding of the above acoustic "Maxwell" Eqs and we may now answer this question.

Indeed, Eqs (I-II) apply both to the fluid-saturated rigid-framed porous medium and, in absence of the latter, to the unbounded viscothermal fluid itself. In the latter case the operators are difference kernels operators, *i.e.* $\rho_{ij}(\tau;\mathbf{R},\mathbf{R}') = \rho_{ij}(\tau;\mathbf{R}-\mathbf{R}')$and $\chi^{-1}(\tau;\mathbf{R},\mathbf{R}') = \chi^{-1}(\tau;\mathbf{R}-\mathbf{R}')$, and they may be explicitly constructed based on the fluid-mechanics equations. The formulation is not unique. Nevertheless, using Penfield and Haus [4] notion of an energy flux $S_i(t,\mathbf{R})$ brought "in acoustic form" (their so-called "power-flow in the hydrodynamic system"), and assuming that

$$S_i(t,\mathbf{R}) = H(t,\mathbf{R}) V_i(t,\mathbf{R}) \qquad (\mathrm{III})$$

we arrive at a simple understanding of the field H identified through (III) as the pressure field. In this formulation a nice disconnection principle is automatically satisfied, that is, the inertial and viscous effects determine the density kernel, and the elastic and thermal effects determine the compressibility kernel. For the more complicated case of a fluid-saturated porous material that appears homogeneous at the macroscopic scale (in a Gibbsian sense), this formulation (I-II-III) lends itself to natural generalization and yields definite recipes to compute, from the microgeometry and fluid-solid physical parameters, the nonlocal kernels $\rho_{ij}(\tau;\mathbf{R},\mathbf{R}')$and $\chi^{-1}(\tau;\mathbf{R},\mathbf{R}')$.

The resulting new general nonlocal theory is in principle amenable to direct numerical and experimental tests. In particular, it will predict the negative compressibilities described in Fang et al. [5].

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Conformational nonlinearity and tunability in metamaterials

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In this talk, we review our recent results on various phenomena in metamaterials achieved through the conformational changes in the metamaterial structure (Fig. 1).

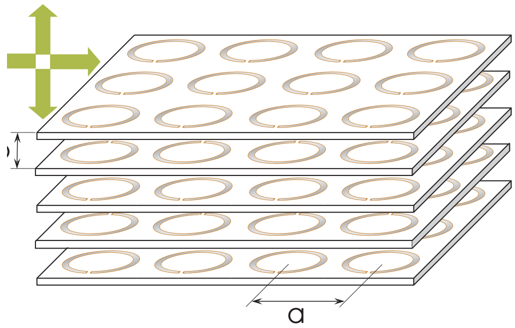

Fig. 1. General schematic of anisotropic metamaterial with conformational tunability and nonlinearity.

First, we revisit our efficient approach [1] for tuning metamaterial properties by a structural reconfiguration. In that setup, strong mutual interaction between the elements [2] enables a significant change in transmission properties which can be controlled by the lattice parameters [Fig. 2 (left)]. We discuss the tuning schemes based on axial and lateral lattice reconfiguration. It turns out that a complex near-field interaction is essential for understanding the tunability patterns [3]. We discuss the corresponding theory and validate it with numerical and experimental support.

Making the next step forward, we further exploit the collective nature of metamaterial response to develop a novel type of nonlinear metamaterials. In contrast to the initial designs [3, 4], where the nonlinearity was provided on the level of an individual element, we now allow an extra degree of freedom in the structure — flexible conformation. This provides an efficient self-action on the level of mutual interactions. The arising nonlinear response is characterized with a complex pattern of amplitude and frequency bistability [Fig. 2 (middle and right)] and is promising for unusual nonlinear effects.

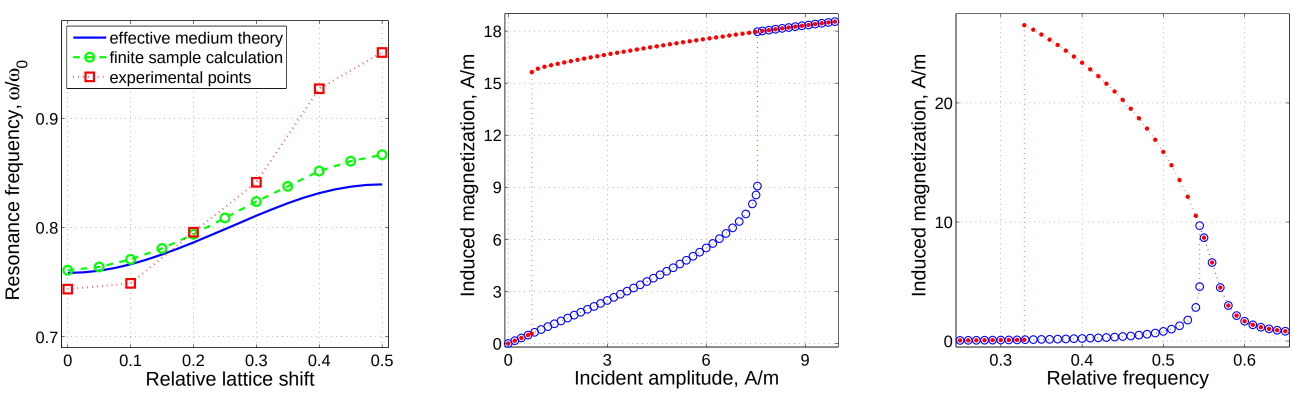

Fig. 2. Examples of the resonance frequency shift by conformational tuning (left), and of the bistability of the magnetization observed with conformational nonlinearity: amplitude (middle) and frequency (right) hysteresis.

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VLSI Photonics: Nano-Photonic Integration using Metamaterial, Plasmonic, Photonic Crystal Devices and Nano-Optics

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This lecture presents an overview on our frontier research on the science and engineering of nano-optical integration of diverse devices and structures that we have been pursuing at the OPERA National Research Center for VLSI Photonics. It discusses on the theory, design, fabrication, and integration of nano-photonic materials and devices of generic and application-specific nature. The nano-photonic devices include: metamaterial devices; plasmonic devices; photonic crystal devices; and nano-optical wires. First, we developed, for the first time, a theory that explains the physics of light propagation through dielectric nanowires that lie beyond the diffraction limit. It uses the model of plasmonic waves caused by free electrons generated by dangling bonds on the surfaces of dielectric wires. These free electrons then carry the lightwaves through the nanowires. We discuss this model using Maxwells equations and hydro-dynamic equations. Next, we analyzed the nano-optical and propagation characteristics of the plasmonic lightwaves, either fast or slow, through various forms of plasmonic, metamaterial, and photonic crsytal structures. We then examine the optical mismatches between the wires. We analyze, for example, the effective index of the silicon wire and the plasmonic wire and design a guided directional coupler based on the optical matching between two wires. We use silver or gold for plasmonic wave generation. We investigated three kinds of plasmonic coupled structures: Strip wires, slot wires, and stripe wires. We calculate the mode fields of the individual plasmonic nano-wires of these structures. We then calculate the coupled eigen-modes of even and odd eigen-modes. In the case of the strip wire coupling, for example, we find that, for the even mode, the magnetic field has the same direction in all the position while the odd mode has opposite field direction on the two nano-waveguides. In terms of energy transfer, we find that the lightwave coming into a nano-wire is transferred to the plasmonic wire due to the optical impedance matching. For a single plasmonic waveguide we find that the plasmonic modes are not excited by TE mode but by TM mode. In the slot palmonic waveguide, on the other hand, we find that the plasmonic modes are not excited by TM mode but by TE mode. Using the results of these optical mismatch characteristics, we designed functional plasmonic devices, such as micro-ring switches, splitters, and modulators, which we use as building blocks of VLSI photonics. We also examined, for the first time, a novel Si-SiO_2 dielectric metamaterial structure that can couple the light from an optical fiber of 9 um diameter to a nano-size optical waveguide of $400\times250\text{nm}^2$ size, which lies well beyond the diffraction limit. This matamaterial system consists of an array of $50\times250\text{nm}^2$ Si waveguides embedded in SiO_2 at a certain regular spacing. A Si waveguide having the size of $50\times250\text{nm}^2$ in the subwavelength regime has no TE mode and therefore cannot transmit light. However, when the Si waveguides are placed in an array, the light feels the whole array as one unit medium and generates a propagation mode. The effective index can be precisely controlled by the number of Si waveguide and the inter-waveguide spacing. We will discuss the physics of this phenomenon in more detail. We will also discuss the optical coupling of nano- photonic silicon wires and photonic crystal devices of high-Q nano-lasers and nano-wires. We will also discuss lightwave transmission through photonic crystal nano-wires that lie beyond the diffraction imit. VLSI photonic chips, incorporating these devices, are designed to perform the functions of sensing, processing, switching, modulating, routing, distributing optical signals. We discuss the scientific and technological issues, challenges, and progresses regarding the optical issues in terms of the optical matches and mismatches between various devices and structures. The issues include the mismatches in terms of their sizes, shapes, losses, dispersion, non- linearity, modes, polarization, birefringence, refractive index, and mechanical/thermal mismatches. We also consider nano-optical effects, micro-cavity effects, non-linear effects, and quantum optical effects in nano-scale devices and in their integrated structures. Needs for new physics and chemistry for new nano-materials, new nano-structures, and new nano-approaches will be discussed and examples of recent progresses will be presented along with the global and historical perspectives of the VLSI photonics technology for the 21st century.

Multilayer 3D photonic crystals for application as highly reflective thermal barrier coatings

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At temperatures in excess of 1200°C, the contribution of thermal radiation to the overall heat transfer inside of the gas turbines substantially increases. If the low thermal conductivity of conventional thermal barrier coatings (TBCs) would be combined with a high reflectivity in the infrared (IR) range, the thermal efficiency of the turbines could be improved and/or their lifetime could be substantially extended.

Three-dimensionally ordered structures can provide TBCs with functionality of a photonic crystal (PhC). Radiation with wavelengths corresponding to the photonic bandgap (determined by the periodicity of the structure) can be efficiently reflected. Coating consisting of several PhC-layers with gradually changing periodicity constant can reflect light in a wide wavelength range and could potentially be used as IR-reflecting TBCs. The conventional TBCs are made of Yttrium stabilized zirconia which predefines the material of choice and thus the index contrast of 1.9 to 1 of the air. The simulations were optimized for maximal reflection in the opal structures made of this material.

Test coatings with the inverse opal topology were obtained by self-assembly of monodisperse polystyrene (PS) particles (400–1000 nm diameter) followed by infiltration of the so-obtained templates with the ceramic phase (by atomic layer deposition) and removal of PS by calcination in air at 500°C. Titanium dioxide was used instead of Yttrium stabilized zirconia for first experiments. The samples demonstrated well-defined stopgaps in the IR-range. Multilayer inverse opals were produced by the repetitive self-assembly of PS particles of different sizes and ALD-infiltration steps.

Analysis for Metamaterials having Losses and Origins of Bandgaps

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This talk presents an overview of my recent work on the analysis of metameterials for nanophotonics as a part of diverse research at the OPERA National Research Center.

We first consider a few basic concepts of metamaterials in terms of (1) components and (2) systems. In plasmonics materials, we need to live with the unwanted material losses in exchange for the mode confinements. Normally, the relaxation rate employed in the conventional Drude model is small in comparison to the plasma frequency in the case of noble metals, and hence the analysis with linearization works fine for most of the time. In contrast, the relaxation rate is comparable to the plasma frequency in the case of gas-phase plasmas, for which we have analytically found resonance characteristics for isolated wires and nanoparticles.

The relaxation rate can be extended to take complex values in consideration for composite media such as thin films containing metallic nanoparticles. The well-established circuit theory of the metal optics has originated from the concept of kinetic inductance employed for superconductors. In fact, the complex relaxation rate has been introduced to deal with the time-dependent Ginzburg-Landau model for type-II superconductors. Along this line of reasoning, we will present subtleties encountered in modeling plasmonic materials by a resistance-inductance-capacitance circuit.

Getting to the possibility of superconducting metamaterials, we will present our preliminary results about the relationship between the split-ring resonators and the vortex dynamics. In this aspect, we try to find how rotating dynamics of magnetic fields arouse electric fields, and vice versa. In addition, we will examine the photonic-crystal aspect of the Abrikosov vortices, and the commensurability issue of the underlying pinning lattices. We will also consider the coupling between the electromagnetic waves and the electron dynamics in terms of two coupled second-order harmonic oscillators, which has a similarity to the two-Josephson-junction circuit.

With respect to the periodic structures of metal wires, the different origins of the low-frequency and intermediate-frequency bandgaps will be elucidated. In addition, a few numerical results will be presented for metal wires as well as the anti-metal wires (namely, periodic holes punched through metallic film).

Universal formula, describing angular width of diffractive beam in anisotropic media and structures for 2-d geometries

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As it is known, different kind of waves, propagating in various media and structures, are characterized by common physical laws. An examples of such well-known common laws for isotropic media are the laws of geometrical optics and the formula, describing angular width of diffractive beam (arising as a result of incidence of plane wave on the wide slit in opaque screen) as a ratio of incident wavelength λ_0 and slit length D. The question is appear evidently: is it possible to deduce similar universal formula, describing angular width of diffractive beam for anisotropic media (at least for 2-D geometries)? How it is seen from analysis of special literature, diffractive phenomena in anisotropic media were investigated mainly for electromagnetic wave in plasma, for light in optical crystals, for acoustic waves and for dipole spin waves, named usually magnetostatic waves (MSWs) [1]. However, it seems that such formula is absent for these types of waves at the moment (probably, because of mathematical difficulties). So an attempt is taken in the present work to obtain such universal formula through the study of MSW diffraction in ferrite slab.

MSWs, effectively excited and propagated in various ferrite structures, are enough convenient object both for theoretically and experimentally investigation of diffraction phenomena in anisotropic media. Due to small phase velocity MSWs have wavenumbers $k \sim 10$–$10^4\,\mathrm{cm}^{-1}$ (wavelengths $\sim$ 5 mm–50 mm), which is much higher than corresponding wavenumbers of electromagnetic waves k_{emw} in a vacuum at microwaves ($k >> k_{emw} \equiv \omega/c \sim 1\,\mathrm{cm}^{-1}$). So characteristics of MSW can be studied in magnetostatic approximation. In other words, one can neglect by the terms with

$\sim \partial/\partial t$ in Maxwell equations and can use equations of magnetostatics $\operatorname{rot}\boldsymbol{h} = 0$ and $\operatorname{div}\boldsymbol{b} = 0$, where $\boldsymbol{h}$ and $\boldsymbol{b}$ — microwave magnetic field and induction. Thus to describe MSWs characteristics it is usually introduce magnetostatic potential Ψ in agree with formula $\boldsymbol{h} = \operatorname{grad}\Psi$ (because of $\operatorname{rot}(\operatorname{grad}\Psi) \equiv 0$) [1].

Due to magnetostatic potential Ψ is scalar function, the study of MSW diffraction become much more simple and it is possible in general follow by the widely known analytical way, applied for isotropic media. However, this study is differ appreciably from the case of isotropic media because MSW has noncollinear character in general case (i.e., the wavevector $\boldsymbol{k}$ and group velocity vector $\boldsymbol{V}$ are not collinear). In particular, for the case of Fraunhofer MSW diffraction on the slit (when the plane MSW with non collinear vectors $\boldsymbol{k}$ and $\boldsymbol{V}$ is incident on the wide slit in opaque thin screen with arbitrary orientation) it was shown, that angular distribution of MSW's magnetic potential Ψ in the far-field region is described by the expression of type $\sim \sin\Phi/\Phi$, where the phase function Φ is more complex, than in isotropic media (it is considered the example of surface MSW in free ferrite slab). Analyzing the dependence $\sin\Phi/\Phi$ it is possible to obtain analytical formula, describing angular width $\Delta\psi_{0.5}$ of diffractive MSW beam in the far-field region:

$$\Delta\psi_{0.5} = \frac{\lambda_0}{D}\left|\frac{d\psi}{d\varphi}(\varphi_0)\right| F. \tag{1}$$

Here D is the length of slit, λ_0 — the wavelength of incident plane MSW, φ_0 — the angle, describing orientation of the wave vector of incident MSW, $d\psi/d\varphi$ and $\psi(\varphi)$ — respectively, derivative and dependence of MSW group velocity vector orientation angle ψ on MSW wavevector orientation angle φ, F — function, depending on φ_0, slit orientation θ and angular derivative of isofrequency dependence (ID) (i.e. the indicatrix or the section of wavevector surface) at $\varphi = \varphi_0$. All angles are counted in coordinate system, connected with collinear propagation axis (analog of optical axis in optic crystals), i.e. axis, along which the MSW has collinear character (its vector $\boldsymbol{k}$ and vector $\boldsymbol{V}$ are collinear). Thus, the angular width $\Delta\psi_{0.5}$ is defined substantially by mathematic properties of ID for the certain wave (as it is known, the dependence $\psi(\varphi)$ is completely defined by ID too). It was found, that function F is equal to unity for the case, where the wavevector $\boldsymbol{k}_0$ of incident MSW is perpendicular to the slit line. In this case for isotropic media (whose ID is circumference, dependence $\psi(\varphi)$ has the form $\psi = \varphi$ and $d\psi/d\varphi \equiv 1$) we find from the formula (1) well-known expression $\Delta\psi_{0.5} = \lambda_0/D$.

There is a hope, that formula (1), deduced for MSW, will be valid for any anisotropic media and structures for 2-D geometries, including metamaterial structures (whose IDs is differ from circumference too). As it is seen from the formula (1) an unusual phenomenon may be appear in anisotropic 2-D geometries: if incident wave is characterized by such value φ_0 that $d\psi/d\varphi = 0$ at $\varphi = \varphi_0$ then $\Delta\psi_{0.5} = 0$! It means, the diffractive beam conserve its wide during propagation! Mention must be made, that ID of not any medium contains the point(s), where $d\psi/d\varphi = 0$, but such points present on ID of surface MSW. Evidently, it is possible to design metamaterials, that will be characterized by ID with points, where $d\psi/d\varphi = 0$.

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The electrodynamic analysis of composites and metamaterials on the basis of the method of minimal autonomous blocks

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Metamaterials possess great potential possibilities for design on their basis of new types of optical and microwave devices. For the description of electromagnetic properties of composites and metamaterials effective material parameters (dielectric permittivity and magnetic permeability, chirality and nonreciprocity factors) are widely used. Use of this approach is related to a series of problems: a large dynamic range of effective material parameters near resonance frequencies; applicability complexities of modeling of the systems containing bianisotropic materials.

The complex approach to the electrodynamic analysis of composites, metamaterials and systems, based on the method of the minimal autonomous blocks (MAB) [1] is considered. The MAB method is based on decomposition of investigation object on a system of minimal autonomous blocks. For three-dimensional problems blocks have the shape of a rectangular parallelepiped, and for two-dimensional - a rectangle. The decomposition scheme includes system of MABs, transition blocks, metallization blocks etc. Electromagnetic properties of all blocks are described by scattering matrixes. Algorithms of realization of the MAB method are:

- the recomposition algorithm in which the multichannel scattering matrix is calculated by connecting of common channels of the neighboring blocks on the basis of the theory of multiport network;
- the iterative algorithm in which modeling process of multiple scattering of channel waves on system of MABs;
- the direct solution of the matrix equation with a sparse matrix;
- the hybrid algorithm combining recomposition and iterative approaches.

Various approaches to modelling of the systems containing composites and metamaterials are considered: direct modeling by the MAB method; use of multichannel scattering matrixes of macroblocks, containing structural inhomogeneities; use of average scattering matrixes of the nonuniform blocks.

Comparative results of use of the specified approaches for the analysis of electromagnetic properties of various types of the objects containing metamaterials are given.

The technique of the analysis of electromagnetic properties of the composites with the multiscale organization of interior structure is proposed. It is based on complex use of the MAB method and technique of average scattering matrix. At each scale level average scattering matrixes for heterogeneous blocks are calculated. These matrixes are used at a following scale level.

Results of the analysis of electromagnetic properties of composites with multiscale interior structure are presented.

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Virtual metamaterials — the analysis and synthesis

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Materials with new unique electromagnetic properties are object of intensive research and use for designing and optimization of devices and systems of optical and microwave ranges. Traditionally for the description of electromagnetic properties of composites and metamaterials effective electromagnetic parameters (dielectric permittivity and magnetic permeability, chirality and nonreciprocity factors) are used.

As the alternative approach to the description of electromagnetic properties of structurally nonuniform materials on the basis of the method of the minimal autonomous blocks (MAB) [1] the technique of average scattering matrixes has been developed [2] . On informativeness average scattering matrixes do not yield to the tensor description of the material parameters for any bianisotropic materials. Average scattering matrixes are used together with usual scattering matrixes of the homogeneous blocks at the solution of electrodynamic problems by the MAB method.

Traditionally synthesis of metamaterials is reduced to structure search for which effective material parameters have the given frequency dependences. Development of metamaterials is carried out as a rule for practical realization of devices and the systems which are based on already known physical effects.

The new approach to development of the metamaterials, based on investigation of wave processes in the nonuniform mediums which electromagnetic properties are described by arbitrarily average scattering matrixes is considered. Such metamaterials are termed as virtual, as not all from them can be realized in practice.

Designing of the virtual metamaterials includes following stages: generation of average scattering matrixes; modeling by the MAB method of amplitude-phase distribution of an electromagnetic field in a metamaterial for the given types of sources; the analysis of electromagnetic field distribution for the purpose of finding of new physical effects, having practical and theoretical interest.

The problem of synthesis of a metamaterial on the given average scattering matrix is considered. It is supposed generally that the interior structure and material parameters of blocks can be arbitrary. The block is decomposed on a system of homogeneous MAB.

The solution of a problem of synthesis is reduced to finding of space distribution of material parameters on blocks at which the average matrix of the macroblock is close to the given scattering matrix. The successful solution of a problem of synthesis translates the virtual metamaterial in a category of realized structurally inhomogeneous materials. The solution of a problem of synthesis is carried out with use of methods and algorithms of global optimization.

Results of synthesis of composites and metamaterials with known constructive and electromagnetic properties are presented. New types of the metamaterials which are of interest for practical use are proposed.

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Motion-induced repulsive Casimir–Lifshitz forces

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In 40's of the last century, Hendrik B. G. Casimir predicted [1] that two electrically neutral metal plates in vacuum attract when placed close enough one to another due to the quantum fluctuations of the electromagnetic field in the space between the plates. This result was later generalized to the slabs of arbitrary dispersive dielectrics by Lifshitz [2]. The Casimir–Lifshitz forces between nonmagnetic objects in vacuum are prevalently attractive. Casimir's attraction may be turned into a repulsion if objects of different permittivities are immersed in a fluid of an intermediate permittivity [3] or when one of the objects has a magnetic response.

The magnitude of the Casimir–Lifshitz force is determined by the electric properties of the interacting bodies, by their shape, and by their mutual disposition. For bodies of simple geometries, the force is typically monotonic with the separation of the bodies r and in the geometries involving planar slabs decays as $1/r^4$, i.e., this force is a short-range force observable at distances on the order of tens to hundreds of nanometers. However, recently it was shown that the Casimir–Lifshitz forces can be enhanced dramatically if the bodies are immersed in a uniaxial fluid. Moreover, it was found that the Casimir forces in uniaxial arrays of plasmonic nanowires decay with the distance as $1/r^2$ and, thus, are ultralong-range [4].

In this work we consider motion-induced Casimir–Lifshitz interactions in layered dielectrics. When the layers are at rest, the quantum fluctuations of the electromagnetic field in the layers, and the Casimir interaction between the layers may be described in the frames of the standard Lifshitz theory. Introducing relative motion along the interfaces between the layers perturbs this system and may result in nonconservative friction-like forces (which we do not consider in this talk) and, as well, in an additional velocity-dependent attractive or repulsive interaction.

The most interesting case is the case of moving layers of the same material (i.e., the layers of a nonuniformly moving fluid), for example, the case of a pair of sliding dielectrics separated by a stationary layer of the same dielectric in which both attractive and repulsive Casimir–Lifshitz forces may be observed depending on the direction of the relative movements of the outer layers. Without any movement the force vanishes. In contrast to the cases of moving layers separated by vacuum studied previously [5], the Lifshitz theory requires a generalization in the case of nonuniformly moving continuous media [6].

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Perfect lensing with phase-conjugating surfaces: approaching practical realization

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In 2003, it was shown [1] that two parallel sheets with phase-conjugation boundary conditions for tangential fields on the two sides of the sheets

$$\boldsymbol{E}_{\mathrm{t}+} = \boldsymbol{E}^*_{\mathrm{t}-}, \qquad \boldsymbol{H}_{\mathrm{t}+} = \boldsymbol{H}^*_{\mathrm{t}-} \tag{1}$$

have the property of *perfect lens.* In the above relations the indices $\pm$ indicate the field values on the two sides of the infinitely thin sheet. The boundary conditions are written for the complex amplitudes of the fields, and $*$ denotes complex conjugation operation. Perfect lens is a device which focuses the field of a point source into a point, that is, the perfect lens focuses both propagating and evanescent fields.

Obviously, boundary conditions (1) cannot be realized using linear materials, and in the same paper [1] a three-wave mixing phase-conjugating surface was proposed as a possibility to realize this effect. Phase conjugation and "time-reversal" devices were studied also earlier for other applications. It is interesting that in the same year (2003) an experimental microwave realization of a phase-conjugating surface was published [2] independently from the work of the authors of [1]. Later, the idea of perfect lensing based on phase conjugation sheets was developed theoretically in [3] and more experimental realizations of nonlinear negative refraction were published [4, 5].

However, in devices based on sheets of nonlinear materials or arrays of particles with nonlinear inclusions [5] or antenna arrays with mixers like in [2, 5], the phase-conjugated ("time-reversed") products create waves propagating symmetrically to the left and right sides of the sheet. While the perfect lens operation can be (theoretically) approached if the amplitudes of the nonlinear products tend to infinity [3], the ideal phase-conjugating boundary conditions (1) can be realized only approximately.

Considering the ideal phase-conjugating boundary conditions (1), one can realize that the induced electric and magnetic surface currents should form Huygens sources. These electric and magnetic surface currents form sets which radiate only towards one of the sides of the sheet, negating fields incident from one side of the sheet in the other half space and creating complex-conjugated fields in the corresponding half space. In the presentation we will show first conceptual designs of structures that can potentially realize such performance. The suggested structures contain electrically and magnetically polarizable particles with appropriate nonlinear loads. The necessary balance between electric and magnetic polarizations can be ensured by using combinations of complex-shaped resonant particles, such as helices or Ω-shape metal particles with symmetrically positioned nonlinear elements. For potential optical applications, where the realization of complex-shape nonlinear inclusions is probably not realistic, a possible realization route is the use of layered structures (the thickness cannot be negligible because both electric and magnetic currents are needed).

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Quarter-wave splitter based on heterogeneous composite with prolate silver nanoparticles

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The optical properties of the composite medium incorporating metallic nanoparticles can be controlled by careful selection of geometric parameters of nanoparticles [1,2]. In this work, we explore the possibility of realizing a sub-wave plasmonic polarizer integrated into a transparent dielectric. In our design, the high polarization contrast of composite slab is obtained by using uniformly oriented silver nanoparticles of ellipsoidal or parallelepiped shape. Parameters of metal-dielectric composite slab required to get high polarization contrast are calculated on the basis of the theory of effective dielectric function of a heterogeneous medium with nanoparticles of ellipsoidal shape. The applicability of the effective medium model for describing the optical properties of the composite slab is verified using full-wave finite-element simulations [3].

It is shown that anisotropic plasmonic absorption in uniformly oriented non-spherical nanoparticles leads to polarization-dependent reflectance and transmittance in the visible region. Composite slab with thickness $h \approx \lambda/4$ exhibits high absorption for light polarized parallel to the long axis of nanoparticles, and for the perpendicular polarized light such slab is nearly transparent. The reflectance and transmittance of composite slab with nanoparticles of ellipsoidal shape possess the values either more than 0.7 or less than 0.1 over a wide optical wavelength range $\Delta\lambda \approx 150$ nm. For the case of parallelepiped-shaped nanoparticles a high degree of polarization are observed over narrower wavelength range ($\Delta\lambda \approx 80$ nm). From the practical point of view it is interesting that high reflectance corresponds with low transmittance and vice versa. Such sub-wave structure work for normal-incidence light and can be used as a polarizing beam splitter with high performance in transmission and reflection. By selecting the proper shape of the nanoparticles or choosing matrices with different refractive indexes it is possible to achieve the polarization contrast at specified spectral regions.

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Negative effective permeability at optical frequencies produced by clusters of plasmonic particles

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Metamaterials (MTMs) with both negative permittivity and permeability attracted a great attention of scientific community in the last decade. The most challenging part of the problem is a realization of strong magnetic response, especially in the optical range, there are no natural magnetic exist. In this report we suggest several designs solutions, which theoretically allow us to obtain negative permeability at optical frequencies.

Our designs solutions are based on the idea presented in the seminal paper [1] by the group of N. Engheta. In this work authors suggested to utilize small silver spheres to form so-called plasmonic nanorings. MTM made of such nanorings possesses strong magnetic response near the frequency of plasmonic resonance of a single sphere. In the present talk we suggest further modifications of the nanoring design. The main idea of the modifications is utilization of particles with stronger plasmonic resonance than a single sphere to increase the amplitude of the magnetic resonance.

The first design solution is based on the use of plasmonic dimers instead of spheres, which allows us to provide a stronger robustness to Ohmic absorption in silver and, as a result, obtain the negative permeability at the edge between infrared and visible ranges [2]. Next modification of the nanoring design is based on utilization of plasmonic triangular nanoprisms [3,4]. Such particles possess very strong dipolar plasmonic resonance in the visible range, which we use to engineer isotropic magnetic response of the cluster leading to isotropic negative permeability of the composite in the visible range.

In the talk we also concern different characterization procedures, which we used to extract material parameters of the composite. One of them is based on the works of the group of C.L. Holloway [5,6,7], later developed in our study [8]. In this method material parameters of a bulk MTM are extracted from reflection and transmission coefficients of a single metasurface, consisting of the same clusters as an original MTM. We compare the results of different approaches of calculation of material parameters and discuss the domains of their applicability.

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Modelling the optical response of metal nanoparticles

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The current interest in the optical properties of metal nanoparticles is due to their ability to host localized surface plasmon (SP) modes. Localized SPs can be tailored by controlling the size and morphology of metal nanoparticles. This unique property enables a wide range of applications, such as switching, light guiding, light manipulation on the nanoscale, and bio-sensing. This has given rise to an active field of research aimed at finding new recipes for controlling the size and shape of metallic nanoparticles [1, 2] and developing advanced experimental techniques that give information on the spatial variation of the near fields associated to SPs with high energy and space resolutions.

The experimental study of the near field associated with particle plasmon modes by optical means is limited by the low resolution available in common techniques such as scanning near-field optical microscopy (around 50nm). In contrast, methods that are based on the interaction of fast electrons with nanostructures allow retrieving local information on plasmons with nanometer resolution. In particular, electron energy-loss spectroscopy (EELS) provides a powerful tool for studying plasmons in metal nanostructures. The maxima in the energy loss spectra of transmitted electrons reflect the energies of the particle-plasmon excitations to which they couple. Part of these energy losses results in the emission of light (cathodoluminescence, CL), which has also been used to probe localized plasmons in metallic nanoparticles [3].

In this work, we present an experimental and numerical study of the optical properties of noble-metal nanoparticles prepared via lithography or colloidal chemistry. In particular, the rich structure of SP modes in nanoparticles of different morphologies (rods, octahedra, decahedra, prisms, and split-ring resonators) is explored by optical spectroscopy [4, 5], spatially resolved EELS performed in a scanning transmission electron microscope (STEM) [6, 7, 8], and by electron-beam-induced radiation emission performed in a scanning electron microscope [9]. Spectral features and spatially resolved maps of SP modes collected for nanoparticles are compared with theoretical STEM-EELS and optical excitation calculations obtained by using the boundary element method that is based upon rigorous solution of Maxwell's equations in a frequency space [10].

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Electromagnetic wave properties of carbon nanotube films in the mid infrared range

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In this paper we discuss propagation of electromagnetic waves in finite-thickness slabs, filled with aligned carbon nanotubes (CNTs) with different orientations. We assume that arrays consist of metallic CNTs as possessing a high conductivity. Propagation in mid and far infrared ranges is considered since namely in these ranges CNTs exhibit lowest attenuation. Electromagnetic properties of individual carbon nanptubes are described in terms of a surface conductivity and effective boundary conditions [1]. Electromagnetic interaction between carbon nanotubes is taken into account using periodic Green's function [2]. For analysis of finite-thickness CNT slabs 2×2 transfer matrix method can applied since azimuthal currents on a CNT surface can be neglected compared to axial ones and Maxwell's equations are split into two subsystems corresponding to ordinary (TE) and extraordinary

(TM) waves. Here we illustrate remarkable wave properties of CNT arrays by two examples. In the first case (see Fig. 1) a slab of vertically standing CNTs is combined with a layer of isotropic dielectric. Opposite directions of the Poynting vector in the CNT and dielectric layers cause "stop light" situation when energy flows in both layers are compensated.

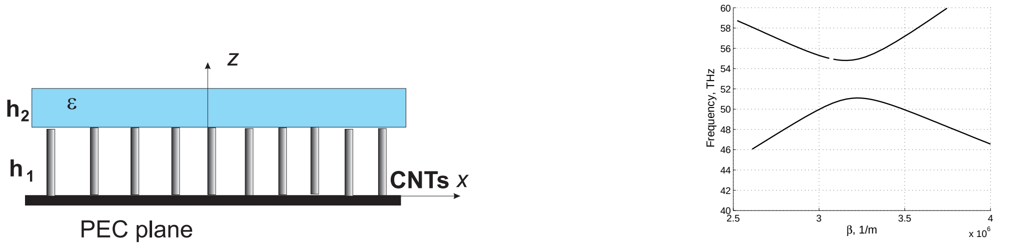

Fig. 1. Left: Schematic view of a vertically standing CNTs, adjusted to a dielectric slab. Right: Frequency versus the wave propagation constant β, calculated at $h_1 = 1.2\,\mu$m, $h_2 = 1.5\,\mu$m.

The second example (see Fig. 2) illustrates that a slab of CNTs, aligned in plane of the slab, is characterized by a hyperbolic-type dispersion and can be considered as a planar *indefinite medium*.

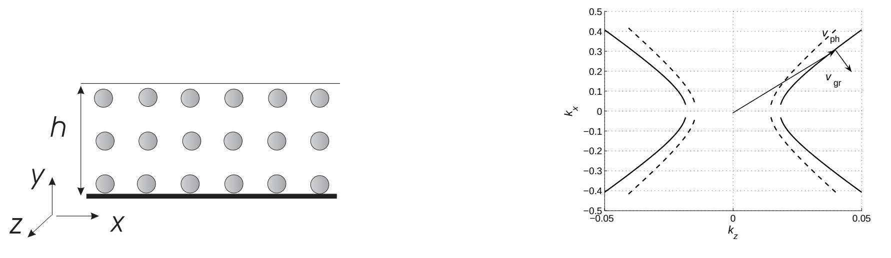

Fig. 2. Left: Cross-section of a planar waveguide consisting CNTs. Right: Isofrequencies, calculated at 10 THz (solid) and 8 THz (dashed line).

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Nanophotonics simulation tools in the cloud: PhotonicsSHA-2D and PhotonicsCL at nanoHUB.org

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In this paper we introduce a set of software tools for simulating the electromagnetic (EM) fields in some particular classes of nanostructured optical metamaterials. In these systems, the solution of Maxwell's equations can be found either analytically, e.g. using transfer matrix approach and the Mie solution to wave equations, or semi-analytically, e.g. using the spatial harmonic analysis (SHA) method.

Significantly, the tools take advantage of the cloud computing capabilities built on the Hub0 platform for scientific collaboration created by Purdue University and the host network infrastructure brought to users and developers by the Network for Computational Nanotechnology (NCN) located at the Birck Nanotechnology Center. First of all, the web-based tools do not require local software installation and can be freely accessed using any of the supported by nanoHUB.org browsers. Second, the virtual workspace of the simulation sessions running on the remote servers, the obtained data, and other available resources are smartly managed and organized as a personalized hub for each user. At the same time, the system provides services for group discussions and collaborations. This concept of easy-to-publish and easy-to-use resources has already attracted a large number of researchers and has helped to achieve a great knowledge base in nanotechnology.

Nanophotonics Tools at NanoHUB. So far, the developed set of nanophotonic tools consists of PhotonicsDB [1], Hyperlens Designer&Solver [2, 3], PhotonicsRT [4], PhotonicsSHA-2D [5], and PhotonicsCL [6]. The PhotonicsDB tool provides users with the interpolation and comparison of the frequency-dispersive optical properties of various materials; this material database is used to construct optical devices in other tools. The Hyperlens Designer&Solver has been developed to simulate a specific transformation optics device – the magnifying cylindrical hyperlens [7]. The PhotonicsRT tool is a simple engine, based on the transfer matrix approach, for calculating the reflection and transmission coefficients of a multilayered stack with a plane wave incident on the stack at an arbitrary angle. The most powerful and sophisticated solvers for simulating nanophotonics devices are PhotonicsSHA-2D and PhotonicsCL. The first tool employs the spatial harmonic analysis method to model EM fields for single-period, multilayered gratings. Importantly, besides the natively handled rectangular geometries of gratings, the tool also can be used for curvilinear objects if a staggered approximation of the initial objects is applied. The last tool, PhotonicsCL, is based on Mie theory and has been developed for simulating circular cylindrical, multilayered, and continuously graded index structures. The multilayered, cylindrical structures designed by the tool can serve as a piecewise-constant approximation for a variety of ideal optical devices with a continuous distribution of material constants, e.g., Eaton lenses, Luneburg lenses, and the recently proposed optical "black hole" devices[8]. The numerical and theoretical study of an omnidirectional light concentrator (a "black hole") and its absorption efficiency are discussed in detail.

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Plasmon-driven spontaneous symmetry breaking in clusters of nonlinear metallic nanoparticles

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Great interest to the investigation of light interaction with isolated metallic nanoparticles and nanoparticle arrays is caused, in particular, by plasmonic effects which bode a lot of promising applications in nanophotonics, near-field optics, nano-waveguiding, optical lithography, biosensorics, etc [1–3]. To date, sufficient progress in fabrication of well-ordered one-, two- and three-dimensional arrays of metallic nanoparticles has been achieved [4–9], that made possible experimental demonstration of such intriguing optical effects as negative refraction [10] and cloaking [11]. Special interest concerns nonlinear plasmon-driven phenomena in such structures. The strong nonlinear optical response of metallic nanoparticles arising due to intra- and interband electron transitions is well-known [12–14] and used for frequency conversion [15,16], determination of optical field enhancement [17] and high-resolution near-field imaging [18]. However, despite some advances in this realm, the real potential of nonlinear optical metamaterials for practical applications remains obscure.

In this report we present comprehensive consideration of *plasmon-driven spontaneous* symmetry breaking in clusters of nonlinear metallic nanoparticles. Contrary to some recent papers where effects induced by *structural* symmetry breaking in different metallic nanostructures were analyzed [19,20], we discuss symmetry breaking of completely other nature, namely we show that in *geometrically symmetric* clusters of two or three identical spherical/cylindrical particles *electrodynamical* symmetry can be broken due to intrinsic cubic nonlinearity of particles that leads to instability of fundamental symmetric dipole modes of the system. The instability growth, in turn, results in spontaneous excitation of fundamental asymmetric modes of the cluster. In our recent articles [21,22] we showed that when the cluster contains two spherical particles with a real cubic susceptibility, different scenarios of the instability growth allow spontaneous magnetization of the system and slowly time dependent energy swapping between fundamental modes with the possibility of modulation frequency tuning approximately from 1 to 60 THz by means of variation in intensity and frequency of incident light, that open perspectives for creation of ultra compact and extremely widely tunable source of THz radiation based on arrays of metallic nanodimers. In the present work we make a step further and consider the more realistic case of complex particle susceptibility. We also extend our previous investigations to three-particle clusters and clusters of cylindrical particles. Our results give exact recommendations on how to observe the predicted phenomenon of plasmon-driven spontaneous symmetry breaking in experimental conditions.

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Nonlocality in Multilayered Metal-Dielectric Optical Metamaterials

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Optical metamaterials formed by periodic layered metal-dielectric nanostructures (MDNs) provide great possibilities for near-field manipulations. This property is employed in numerous applications including subwavelength imaging [1–3], nanolithography [4], optical nanocircuitry [5], and even invisibility cloaks [6].

The effective medium model (EMM) is a conventional approach to description of MDNs. EMM describes the metamaterial under consideration as an uniaxial anisotropic medium with permittivity tensor of the form:

$$\varepsilon_{\text{eff}} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\parallel & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \ \varepsilon_\parallel = \frac{\varepsilon_1 d_1 + \varepsilon_2 d_2}{d_1 + d_2}, \varepsilon_\perp = \left(\frac{\varepsilon_1^{-1} d_1 + \varepsilon_2^{-1} d_2}{d_1 + d_2} \right)^{-1},$$

where $\varepsilon_1, \varepsilon_2$ and d_1, d_2 are dielectric permittivities and thicknesses of the layers, respectively.

The principal elements of the permittivity tensor can have nearly arbitrary values. For example, if $\varepsilon_\parallel$ and $\varepsilon_\perp$ have different signs then the MDN is a typical realization of indefinite medium [7]. Such medium features negative refraction effect since its isofrequency contours have hyperbolic form.

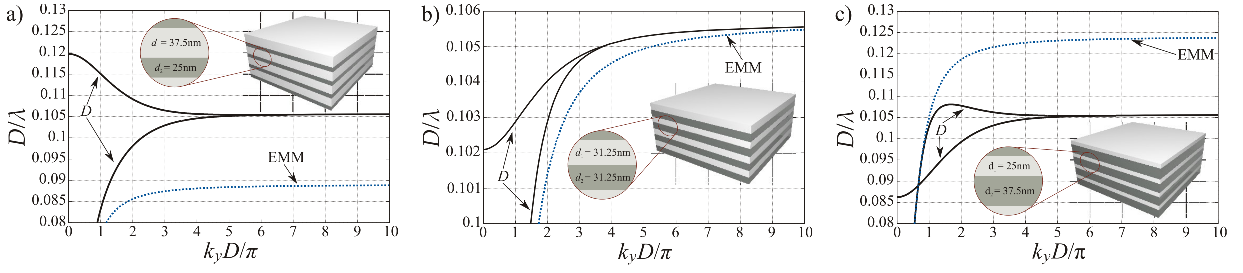

Fig. 1. The dispersion diagrams and geometry of the MDNs composed of alternating HfO_2 and Ag layers. The permittivity of HfO_2 is assumed to be equal to $\varepsilon_1 = 4.6$. The permittivity of silver is given by Drude model: $\varepsilon_2 = 1 - \omega_p^2/\omega^2 = 1 - \lambda^2/\lambda_p^2$, where $\lambda_p = 2\pi c/\omega_p$=250 nm. The ratio of layers thicknesses is varied as follows: a) $d_1 = 1.5 d_2$, b) $d_1 = d_2$, and c) $d_2 = 1.5 d_1$.

In this work we considered three MDNs formed by layers of metal and dielectric with various thickness ratios (3:2, 1:1, and 2:3), but fixed total period. Configurations and parameters of the structures are illustrated schematically in the insets of Fig. 1. The dispersion diagrams $\omega(k_y)$ for the three MDNs under consideration shown in Fig. 1 were computed using two approaches: the effective medium model (approximate approach) and the well-known classical dispersion relation for 1D photonic crystals (exact description).

Different ratios of layers thicknesses were chosen in order to demonstrate different behaviors of dispersion curves. In all cases the dispersion curves consists of two branches with joint surface plasmon polariton (SPP) resonance as asymptote if $k_y \to \infty$ for actual MDN and of one branch with $\epsilon_\perp = \infty$ resonance for effective medium model. The presence of two branches of dispersion curve (in contrary to just one brunch predicted by effective medium model) is a consequence of strong spatial dispersion in the structure.

In the first case (Fig. 1.a), the SPP resonance appears above he frequency where epsilon very large behavior is expected and the dispersion diagram features forward and backward wave branches with different frequency bands. In the second case (Fig. 1.b), the frequencies are chosen to be equal and both branches correspond to forward waves, but the waves exist at the same frequency band in contrary to the previous case. In the third case (Fig. 1.c), the branches cross each other at certain point and one of the branches has a maximum leading to existence of backward and forward waves simultaneously at the same range of frequencies. The effective medium model in all cases predicts only one forward propagating wave at all frequencies. The presence of two propagating waves is a consequence of nonlocality and strong spatial dispersion which are caused by SPPs at the interfaces of the layers.

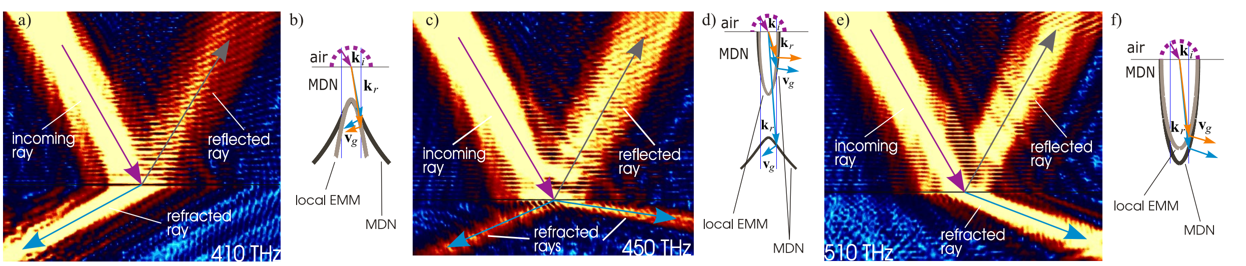

Fig. 2. Results of ray refraction simulations with corresponding refraction diagrams based on isofrequency contours both for local effective medium model and actual MDN.

Next, we have performed numerical experiments of ray refraction at the air-MDN interface at different frequencies (see Fig. 2). At 410 THz only one negatively refracted ray is observed (see Fig. 2.a,b) and effective medium model is applicable in such a situation. Appearance of an ellipse at the isofrequency contour at 450 THz shown in Fig. 2.d as compared to Fig. 2.b leads to birefringence phenomena in the MDN. The negatively refracted ray is still there while in addition to it a positively refracted ray corresponding to the ellipse appears. In this case the local effective medium model is not able to predict the presence of the two rays and describes only one of them. At the higher frequency (510 THz) only one positively refracted ray is observed and this fact is well described by effective medium model.

In conclusion, we compared dispersion characteristics of actual periodic structure with ones predicted by effective medium model and revealed significant differences between them. In particular, two dispersion branches of extraordinary waves are observed instead of one predicted by effective medium model. Our numerical simulations revealed the splitting of the TM-polarized wave at the interface between air and metal-dielectric-nanostructure into two refracted waves inside of the structure instead of one extraordinary wave predicted by effective medium model. All obtained results demonstrate presence of strong spatial dispersion in the structure and provide a proof that the metal-dielectric nanostructure is actually a nonlocal material. These conclusions are in a good agreement with results of precedent works where the guiding modes of metal-dielectric multilayered structures [8] and the surface waves at interfaces of metal-dielectric nanostructures [9] were investigated.

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Transformation Optics at Optical Frequencies

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"Invisible Cloaks & a Perfect Lens"

Electromagnetism encompasses much of modern technology. Its influence rests on our ability to deploy materials that can control the component electric and magnetic fields. A new class of materials has created some extraordinary possibilities such as a negative refractive index, and lenses whose resolution is limited only by the precision with which we can manufacture them. Cloaks have been designed and built that hide objects within them, but remain completely invisible to external observers. The new materials, named metamaterials, have properties determined as much by their internal physical structure as by their chemical composition and the radical new properties to which they give access promise to transform our ability to control much of the electromagnetic spectrum.

Slow-light waveguides in 2D triangular lattice line-defect waveguides

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A slab of a high index material periodically structured in two dimensions can demonstrate omnidirectional mirror properties for slab modes. A line-defect in the periodically structured slab can be used as a waveguide. The slow light properties in such waveguides were first demonstrated by Notomi et al. [1]. In the presentation we will give a review on the slow light phenomena in presented waveguides, including: dispersionless slow light [2], coupling into slow light regime, disorder induced losses [3]. Finally we will present the recent concept of dynamical effects in slow light waveguides for frequency conversion and light storage. The desired effects are achieved by performing an ultrafast change of the refractive index in the line-defect waveguide while the light signal is still confined to it (dynamic effect) [4].

The present approach for light storage uses a proper modification of the guided mode of a line-defect slow-light waveguide achieved by changing the refractive index of the photonic crystal in the hatched region shown in Fig.1 left. Fig. 1 left also shows the dispersion and group velocity curves of the guided mode in the initial state (solid line) and in the case of an applied refractive index change $\Delta n = 0.6$ (dashed). The grey shaded regions show the frequency and k-vector ranges occupied by the input pulse (FWHM approx. 60GHz). Once the whole pulse is in the waveguide, a refractive index change is applied, thus leading to a reduction of the signal's group velocity. The pulse returns to its initial state by switching off the refractive index change. Fig. 1 right shows the results of numerical simulations of the stopping process. A storing time of 70ps could be shown theoretically. For delays as large as 200ps the pulse distortion is strong due to the non-vanishing dispersion of the flattened mode.

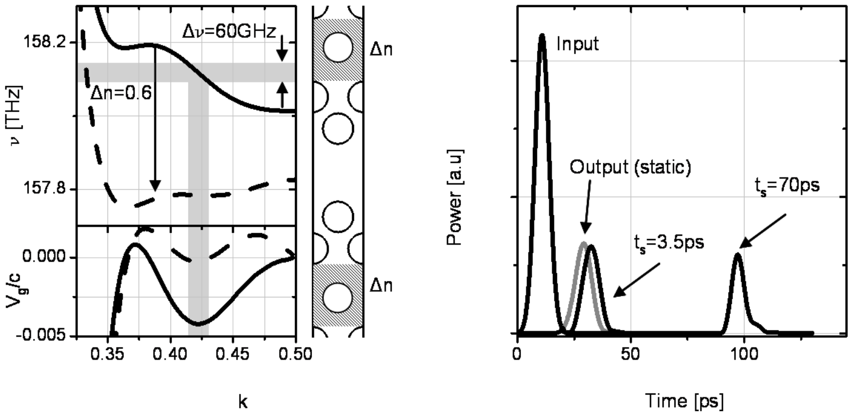

Figure 1. Left: Dispersion and group velocity of the guided mode in the initial state and after an applied refractive index change. The waveguide unit cell is also shown. Right: Numerical results for different storage times t_s.

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Purcell effect for distributed light source in hyperbolic medium

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Hyperbolic medium is an uniaxial medium where the transverse $\varepsilon_{xx} = \varepsilon_{yy} \equiv \varepsilon_\perp$ and longitudinal $\varepsilon_{zz} = \varepsilon_\parallel$ dielectric constants have opposite signs. It is characterized with hyperbolic isofrequency surface in wavevector space, see insets in Figure, and can be realized in metamaterials, artificial photonic structures. One of the multiple reasons why the metamaterials presently attract a large interest of researchers, is their applicability to control the radiative decay rates of the light sources.

Spontaneous emission enhancement of the light source, placed in complex electromagnetic environment, is generally termed as Purcell effect [1]. The huge Purcell effect for the point dipole embedded in the hyperbolic metamaterial has been reported in Refs. [2,3]. It has been shown the radiative rate is strongly enhanced when $\varepsilon_\parallel < 0$ and $\varepsilon_\perp > 0$ and diverges in the ideal case. The radiative rate remains finite only due to the inevitable losses or if one takes into account the inhomogeneous structure of the realistic metamaterial. This effect is related to the singular photonic density of states in such a structure.

In this work we consider the light source placed in the homogeneous uniaxial medium with $\varepsilon'_\parallel \varepsilon'_\perp < 0$ and take into account the finite spatial extent of the source a. We study the Purcell effect for arbitrary values of $\varepsilon_\perp$ and $\varepsilon_\|$ (see the Figure) and also analyze the role of the losses in the system. Explicit analytic results for the Purcell factor have been obtained. For such distributed source the radiative rate does not diverge but strongly depends on the source size instead. The maximum enhancement of the radiative rate is on the order of the ratio $(\lambda_0/a)^3$ of the light wavelength λ_0 and the source size a.

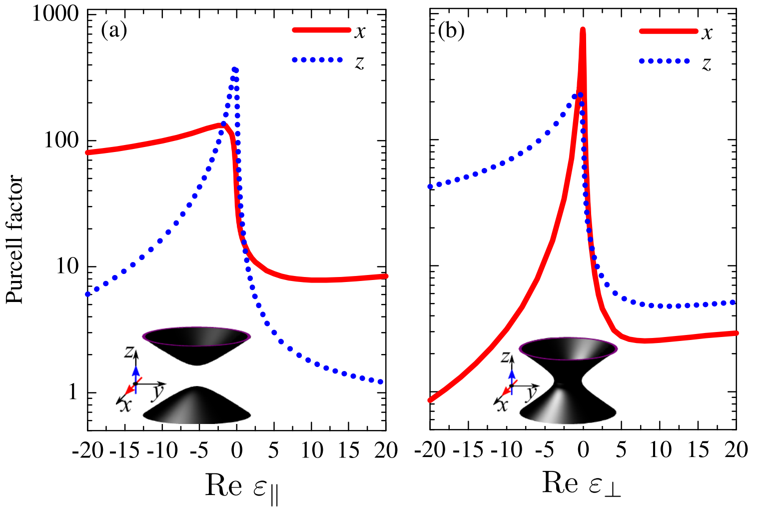

Fig. 1. Purcell factor relative to vacuum as a function of (a) $\varepsilon'_\|$ for $\varepsilon'_\perp = 1$ and (b) $\varepsilon'_\perp$ for $\varepsilon'_\| = 1$. Solid and dashed lines correspond to the dipole oriented along x and z axes, respectively. The insets schematically illustrate the isofrequency surfaces in wavevector space in the regimes when $\varepsilon'_\perp \varepsilon'_\| < 0$. The calculation was performed for the source size $2\pi a/\lambda_0 = 0.1$ and $\varepsilon''_\| = \varepsilon''_\perp = 0.1$, where a is the characteristic size of the emitter and λ_0 is the light wavelength in vacuum.

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Nonlinear electromagnetics in multi-domain negative index metamaterials

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Nanostuctured negative-index metamaterials (NIMs) form a novel class of artificial electromagnetic materials that promises revolutionary breakthroughs in photonics. Such metamaterials are expected to play a key role in the development of novel photonic microdevices and all-optical data processing chips. Significant progress has been achieved recently in the design of bulk, multilayered, negative-index, plasmonic slabs. The problem, however, is that these structures introduce strong losses inherent to metals that are difficult to avoid.

Unlike ordinary materials, the energy flow and wave vector (phase velocity) are counter-directed in NIMs. Negative-index properties and, therefore, backwardness of electromagnetic waves are usually achievable only within a certain wavelength band. Metamaterials remain ordinary, positive index, outside such interval. This opens the opportunities of unique schemes of nonlinear-optical coupling between the ordinary and backward electromagnetic waves, which meet the requirements of the phase matching. It is because all wave vectors remain parallel, whereas some of the energy flows inside the metamaterial appear counter-directed. Such unusual nonlinear propagation processes exhibit extraordinary properties not achievable in ordinary nonlinear optical materials and not described in the literature [1–7].

This paper is to propose several such coupling schemes and to present analysis of the operational properties for one of them in the context of their applications to compensating strong losses inherent to plasmonic metamaterials and to design novel photonic devices for to optical sensing and data processing. Each of the schemes provides different distribution of the coupled fields (hot zones) across the originally strongly absorbing metamaterial slab. The outlined possibilities to implement originally strongly absorbing microscopic samples of plasmonic metal-dielectric composites for the remote all-optically tailoring of their transparency and reflectivity as well as the options for creating of unique ultracompact photonic sensing devices is demonstrated through numerical simulations. Different schemes of coherent energy transfer from strong control field to the negative-index signals described here present alternative approaches to compensating losses in NIMs based on the population inversion (such as recent breakthrough reported in [8]).

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Coupling Effects in Rotated Split Ring Resonators

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We study the dynamics of two microwave split ring resonators broadside coupled to each other, varying twist angle θ between them, as shown in Fig. 1(a). A similar system operating at near infrared frequencies was presented in [1]. We show our experimental and numerical results in Fig. 1(b).

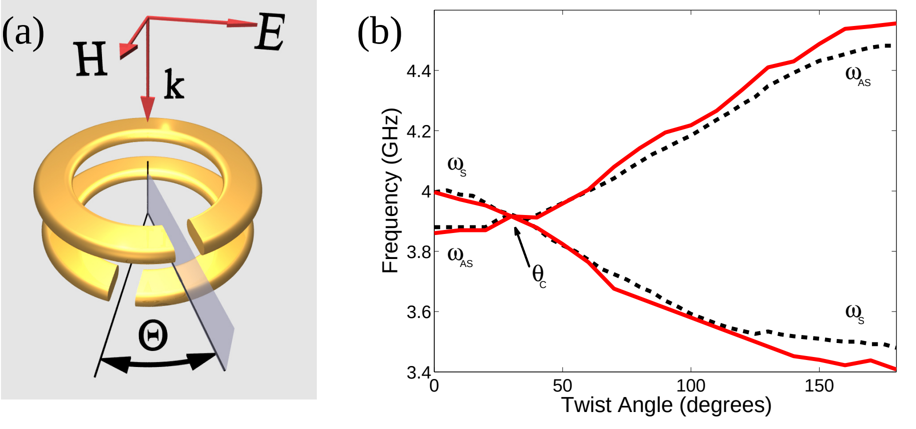

Fig. 1. (a) Schematic of interacting rings, (b) Resonant frequencies as a function of twist angle.

For $\theta = 0°$, there are two resonances ω_S and ω_{AS}, and by inspection of the currents in the rings we verify that these correspond to the expected symmetric and anti-symmetric modes. As θ increases, ω_{AS} increases and ω_S decreases, reaching their maximum and minimum values respectively at $\theta = 180°$. For our chosen parameters the resonances appear to cross at $\theta_c \approx 33°$, in contrast to [1], where an avoided crossing of resonances was reported.

The crossing of resonances usually only occurs in systems where the resonators are absolutely identical. First we describe this system as a pair of lossless resonators, coupled by electric and magnetic interaction constants, using a formulation derived in [2]. In this formulation, any difference between the rings, no matter how small, will cause an avoided crossing. However, using the theory of Morse critical points, it has previously been shown that once sufficient losses are introduced into a system of detuned resonators, the crossing can be restored [3]. This approach is based on mapping the determinant of the system equations over the whole (ω, θ) space, and performing a Taylor expansion about the Morse critical point. We add losses into our model, thus we are considering the determinant of the following matrix:

$$\begin{bmatrix} (\omega_0 + \delta\omega)^2 + j2\gamma\omega - \omega^2 & \beta\omega_0^2 - \alpha\omega^2 \\ \beta\omega_0^2 - \alpha\omega^2 & (\omega_0 - \delta\omega)^2 + j2\gamma\omega - \omega^2 \end{bmatrix} \quad (1)$$

where ω_0 represent the unperturbed resonant frequencies, $\delta\omega$ is the detuning, α and β are the magnetic and electric interaction constants (which depend on angle θ), and γ represents losses (both radiation and dissipation). The dispersion curve of the system is the set of points in (ω, θ) space where the determinant goes to zero. We show that for small values of losses these curves do not cross, however once realistic losses are included, the mode crossing can be restored. The different branches of the dispersion curves are easily distinguished since the imaginary parts of their complex frequencies do not cross.

These results help us understand the coupling mechanisms at work in metamaterials, which are of great importance for controlling their properties by adjustment of lattice properties.

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Time domain simulation of an active plasmonic metamaterial

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Employing metamaterials in optics provides tremendous opportunities for novel optical devices; however, one limitation is the inevitable loss of the optical signal in the metal structures. Recently, the experimental demonstration of a loss-free metamaterial was achieved by incorporating gain media into a metamaterial [1]. In that initial report, the experimental data was validated by a finite-element solver in the frequency domain (FD). However, FD modeling does not reveal the kinetic nature of gain media and therefore the multiphysics of light-matter interactions cannot be fully investigated. In contrast, time-domain simulations of the atomic systems and level transitions of a gain medium use a time-dependent system of rate equations (SRE). SRE is further coupled to Maxwell's equations, giving an accurate time-domain description and dynamics of interaction of light with an active metamaterial.

The metamaterial sample used for numerical demonstrations consists of a Rh800-epoxy composite sandwiched inside an Ag nanostrip-thin film structure. The gain medium is described by a four-level atomic system [2]

$$\begin{array}{rclcccc} {N_0}' & = & \tau_{10}^{-1}N_1 & & & - & f_{30} \\ {N_1}' & = & \tau_{21}^{-1}N_2 & - & \tau_{10}^{-1}N_1 & - & f_{21} \\ {N_2}' & = & \tau_{32}^{-1}N_3 & - & \tau_{21}^{-1}N_2 & + & f_{21} \\ {N_3}' & = & & - & \tau_{32}^{-1}N_3 & + & f_{30} \end{array} . \tag{1}$$

Here the driving terms for $ij \in \{30, 21\}$ are $f_{ij} = (\hbar\omega_{ij})^{-1}\mathbf{E}\cdot(\mathbf{P}'_{ij}+\mathbf{P}_{ij}\Delta\omega_{ij}/2)$, and the transition polarizations $\mathbf{P}_{ij}$ are used to couple SRE (1) with Maxwell's equations by the electric field $\mathbf{E}$

$$\begin{array}{c} \mathbf{P}''_{30} + \Delta\omega_{30}\mathbf{P}'_{30} + \omega_{30}^2\mathbf{P}_{30} = \kappa_{30}(N_0 - N_3)\mathbf{E} \\ \mathbf{P}''_{21} + \Delta\omega_{21}\mathbf{P}'_{21} + \omega_{21}^2\mathbf{P}_{21} = \kappa_{21}(N_1 - N_2)\mathbf{E} \end{array} , \tag{2}$$

where $\Delta\omega_{ij} = \tau_{ij}^{-1} + \tau_{jj-1}^{-1} + 2T_{2,ij}^{-1}$ are the transition line-widths, $\kappa_{ij} = 6\pi\varepsilon_0 c^3\gamma_{r,ij}/(n{\omega_{ij}}^2)$ are the coupling coefficients, and $\tau_{ij} = \gamma_{ij}^{-1} = (1/\tau_{r,ij} + 1/\tau_{nr,ij})^{-1}$ are the total lifetimes. The specific gain system parameters of Rh800 dye in a dielectric host have been retrieved by fitting the simulations to the pump-probe experiment in our previous work [3].

The frequency dispersion in the metal elements of the sample is described by the generalized dispersion model (GDM, [4]) that employs universal and accurate Pade approximations for the frequency-domain permittivity data

$$\varepsilon(\omega) = \varepsilon_\infty - \frac{\sigma}{\iota\omega\varepsilon_0} + \sum_{i\in I_1} \frac{a_{0,i}}{b_{0,i} - \iota\omega} + \sum_{i\in I_2} \frac{a_{0,i} - \iota\omega a_{1,i}}{b_{0,i} - \iota\omega b_{1,i} - \omega^2}\ . \tag{3}$$

For numerical demonstrations, we use the finite-difference time-domain (FDTD) method for solving Maxwell's equations, where the SRE is coupled using the auxiliary differential equations (ADE) method, while the dispersion of the permittivity (3) is implemented with a recursive convolution method following our recent work [4]. Modeling active metamaterials with optically dispersive plasmonic elements comes at a large additional computational cost as well as significant development cost, so the proposed optimized and universal numerical implementations play an essential role in reducing both the development and running times. The modeling results of the far-field optical responses and the population dynamics of the given active metamaterial system under various pumping and probing conditions are discussed. The stability analysis and the order of the accuracy for the proposed numerical schemes are reviewed.

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Magnetic properties of metamaterials based on asymmetric double-wire structures

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In this work we investigated the influence of a unit cell asymmetry on the metamaterials' magnetic properties. We considered a double-wire structure with the asymmetry introduced by the tuning of the wire lengths L_1 and L_2 (see Fig. 1(a)). The asymmetry of the structures was characterized by the parameter $\Delta L = L_1 - L_2$. Theoretical analysis of the system was done using a multipole model from Ref. [1] extended on the case of asymmetric wires. The theoretical model is based on the multipole expansion of the charge density in a unit cell of a metamaterial (MM). The macroscopic electromagnetic moments for MMs are introduced following the commonly accepted approach in the classical electrodynamics of continues medium.

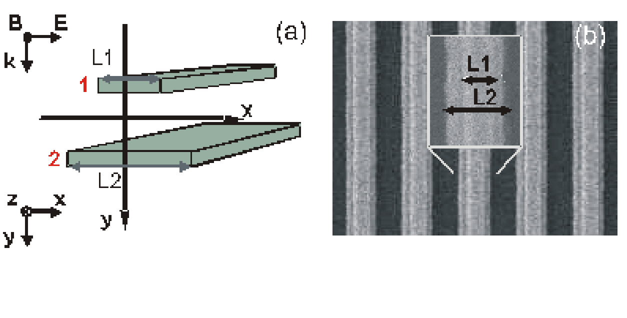

Fig. 1. (a) - Geometry of the double-wire structure considered in the theoretical model. (b) – SEM image of the experimental structure.

To simplify the description of the internal charge dynamics of a unit cell, the double-wire system was described as a pair of coupled plasmonic oscillators characterized with eigenfrequencies, amplitudes, damping constants, and a coupling constant. For each configuration of the system, these parameters were found by the fitting of the light dispersion of a MM from the theoretical model to the one found in numerical simulations. Such a simple model allowed us to connect the geometrical asymmetry of the structure with the magnetization of MMs and the effective permeability.

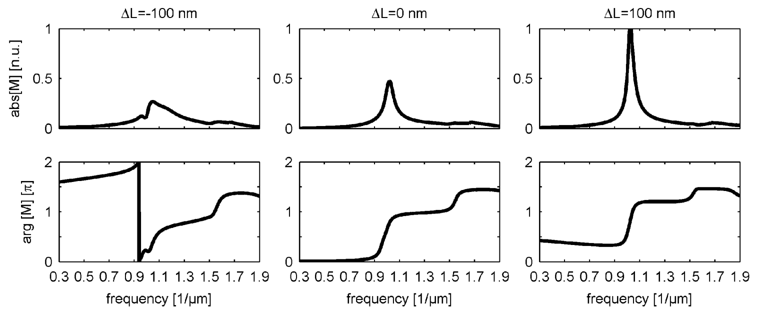

Fig. 2. Absolute values and phases of the magnetization of the MMs with $\Delta L = -100\,\text{nm}$, $\Delta L = 0\,\text{nm}$, and $\Delta L = 100$.

The investigation has shown that the magnitude of the magnetic moment and the phase shift relative to the magnetic field depends strongly on the configuration of the double-wire system. In general, the dynamics of the system is dominated by the larger wire, where the plasmon oscillations follow the external electric field. This defines the orientation of the effective current in the system of the coupled wires. In the system where the wire on the top is longer than the wire on the bottom ($\Delta L > 0$), magnetic moment is strong and has a phase shift of π with respect to the magnetic field of the illuminating wave (see Fig. 2). This leads to decreasing of the effective magnetic permeability. In the configuration where the wire on the top is shorter than one on the bottom, the magnetic moment decreases and oscillates in phase with the magnetic field; corresponding effective magnetic permeability grows. The obtained results correlate with ones presented in [2], where a set of asymmetric nano-disks was considered. However, implementation of the analytical model [1] allowed us to connect the geometrical asymmetry of the structure with the macroscopic effective parameters and facilitate understanding of the internal dynamics of the system.

Additionally, our experimental investigations Fig. 1(b) have shown that the results obtained in the model based on the double-wires structures in a symmetrical environment qualitatively describes dynamics of a double-wire system on a substrate.

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Transformational Optics, Complex Geometrical Optics and Nonlinear Electromagnetic Energy Concentrator

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The advantages of metamaterial-based devices are manifested in the possibility of providing predefined functions, and making full use of the new electrodynamics modes, including necessary inhomogeneities of the material parameters. One of the most promising ways of providing the given characteristics of new electromagnetic devices is using the principles of transformational optics (TO) [1, 2]. TO allows realizing, for example, "electromagnetic invisibility" of objects placed in an appropriate environment of inhomogeneous metamaterials [1], concentration of power [2–4], and other functions. In modeling (linear) "electromagnetic black hole" (energy concentrator, based on the using of inhomogeneous dielectric), intuitively clear method of (real) geometrical optics (GO) was applied [4].

In this paper we propose a new method suitable for simulation of advanced energy concentrators. This method is demonstrated by the example of a layered cylindrically symmetric media. Outer region of the media is homogeneous, and the inner is inhomogeneous, as in [4]. This method includes the "complex" geometrical optics (CGO) and full-wave numerical solution of Maxwell's equations. CGO is used in an external inhomogeneous region, while full-wave solution is applied in the inner cylinder with uniform electrodynamics parameters. For the media under consideration, the concentration of power occurs at the expense of a linear inhomogeneity, since as refractive index increases toward the centre of symmetry. Application of CGO, as opposed to "real" GO, allows to take into account the change in amplitude along the geometrical optics rays due to changes in distance to the center of the cylindrical symmetry (curvature of geometrical optic rays in inhomogeneous media) , and due to the presence of loss (of either sign) in a dissipative (active) media. Matching of "geometrical optics" and full-wave solutions at the interface of homogeneous and inhomogeneous media is carried out in the "local approximation". Calculations based on the CGO are therefore more accurate than those based on the standard "real" GO. As a result, CGO gives qualitatively new results, in particular, that (for the same structure) rays, which are "trapped" in "optical black hole "[4] in the GO approximation, in fact, are not captured and come out of the considered structure.

In addition to linear method, the algorithm of "nonlinear transformational optics" is proposed for modeling nonlinear energy concentrator, where inner homogeneous region is nonlinear. This method includes, again, matching of geometrical optic and full-wave solutions. In this case, "superfocusing" is used, which includes both linear focusing (due to linear inhomogeneity of the media) and nonlinear (in particular, saturating Kerr) focusing. Preliminary calculations show a possibility of effective energy focusing and even a tendency to an occurrence of "hot spots". Next, more detailed modeling will be dedicated to effects of "superfocusing" and nonlinear version of anisotropic layered concentrator [3].

Possible applications of the proposed method for energy harvesting and sensing are under consideration.

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Switching Waves and Solitons in Nonlinear Magnetic Metamaterials

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We study switching waves (fronts) and dissipative solitons in nonlinear magnetic metamaterials assembled either as 1D arrays or 2D lattices of interacting split-ring resonators driven by external coherent radiation. We analyze the governing discrete equations for the electric current amplitudes in different resonators [1] and find that, in the limit of strong coupling, these equations coincide with the equations for the driven wide-aperture interferometer with Kerr nonlinearity where switching waves and dissipative solitons have been studied earlier, both theoretically and experimentally [2]. Here we focus on the effect of the system discreetness when the coupling of resonators is not strong. We obtain both analytical and numerical solutions of the governing equations, and demonstrate hysteresis of discrete switching waves in the system, when the switching waves can be at rest or can move with a constant velocity depending on the initial conditions. We find stable localized structures, the so-called 1D and 2D discrete dissipative metamaterial solitons, including extremely narrow localized modes.

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Optical properties of 1D disordered photonic crystals

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The fundamental difference between crystals and photonic crystals is the uniformity of the crystal building elements (particles): atoms in crystals are absolutely identical, while the artificially produced or naturally grown particles (or other constituting elements) in PhCs inevitably exhibit a certain degree of non-uniformity in size and dielectric properties. This non-uniformity leads to additional disorder-induced scattering, the character of which is determined by the shape of the particles constituting PhC (Fabry-Perot scattering in the case of slabs, Mie scattering in the case of spheres and so on).

We present a new picture of optical spectra transformation in disordered 1D PhCs. The main emphasis is made on our recent results obtained by means of numerical calculations. We demonstrate that the optical phenomena in 1D PhCs dramatically depend on the character of the disorder. Variation in the dielectric constant of the constituents of the 1D PhCs gives rise to appearance of disorder-induced Fabry-Perot continuum. We reveal presence of two distinct types of scattering mechanisms, which results in their interaction and formation of a Fano-type resonance [1]. At the same time, one can observe a distinct transformation picture of the spectrally narrow specific for the Fano resonance, which manifests as a reversal of the Bragg band, i.e. its transformation from the stop-band to the pass-band. Note that the Fano resonance with an active involvement of the Bragg band was previously discovered in the opals [2, 3] – representatives of the 3D PhCs. This experimental result allows us to make a suggestion that the effects described here for the case of 1D PhCs can be also observed in disordered 2D and 3D PhCs.

It is worth of mentioning that here we limit our consideration to the case of lossless structures and the vast majority of the results are given for the case of small dielectric contrast. Moreover, we also consider a novel class of optical materials — structures composed of periodical arrangement of ordered components and components with fluctuating dielectric constant, i.e. 'disordered' components. For disordered components, the average dielectric constant is equal to that of ordered components, i.e. dielectric contrast on average vanishes. We show that such materials possess photonic band structure which can be considered as an inverted of that found in photonic crystals, i.e. instead of Bragg stop bands we observe Bragg pass bands. A detailed study shows that the main mechanism behind these extraordinary optical properties is localization of light.

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Two-dimensional optical diffraction from thin opal films

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Opal-based structures are photonic crystals (PhCs), which possess stop bands in the visible range due to the typical size of the constitutive a-SiO_2 spherical particles of some hundreds of nanometers. This provides a chance to study photonic properties not only by traditional methods like registering transmission or reflection with a spectrometer but also directly observe diffraction patterns on a screen.

The results of the experimental studies of light diffraction from bulk opal samples and opal films are presented in a number of works, e.g. [1–3]. With the incident beam normal to the film either three or six diffraction reflexes are observed depending on the specimen parameters, such as the sample thickness, the presence of disorder, the lattice spacing to wavelength ratio as well as the dielectric contrast. It is worth mentioning that six diffraction reflexes in the given geometry can be observed in two cases. The first one is 2D light diffraction from the hexagonal layer with the C_6 symmetry; the second is 3D Bragg diffraction from the twinned fcc lattice oriented so that [111] axis coincides with the incident beam direction. Each fcc twin has the C_3 symmetry in the direction [111] and defines a triple of diffraction reflexes. As the twins in the fcc lattice coincide with each other at a turn by 60° about axis [111], the result will bear the C_6 symmetry. The aim of this work is to identify the nature of diffraction from opal films. Up to now different authors gave different answers to this question.

In general, diffraction of electromagnetic waves from a periodic system of scatterers is defined by the Laue equation set [4]. Depending on dimensionality of the system under study one should take into account one, two or three equations. In case of the elastic scattering from 3D ordered structure the Laue equation set may be rewritten as an equivalent Bragg's law [4], which implies that diffraction follows the law of *specular reflection* from (hkl) crystallographic planes.

The condition of diffraction from 2D periodic structure which is defined by two out of three Laue equations may be written as [5]:

$$\lambda_{hk} = \frac{Dn_{eff}}{2}\left(\frac{3}{h^2+hk+k^2}\right)^{1/2}\left[\sin\left(\Theta-\theta\right)-\sin\theta\right], \tag{1}$$

where λ_{hk} is the radiation wavelength, h and k are integers, θ and $(\Theta-\theta)$ are the angles of incidence and scattering, counted off from normal to plane which contains the 2D structure, n_{eff} stands for the effective index of refraction, D is the lattice constant which for opal films can be equated to the SiO_2 particle diameter.

When comparing Bragg law for 3D diffraction with expression (1) written for 2D diffraction one can immediately notice substantial difference between angular dependences of 2D and 3D diffraction conditions. Therefore in terms of this work [5] it was suggested to investigate angular dependence of diffraction patterns experimentally.

We should emphasize two key features of our approach. The first is that instead of plain screen we use a narrow *cylindrical* one with a specimen fixed in its center. It allows us to avoid nonlinear distortions of diffraction reflexes. The white light diffraction patterns on the cylindrical screen are photographed from three different fixed points. While processing of the obtained images developed onto a flat stripe line, a full diffraction pattern is formed. The second key feature of these studies is the experimental data presentation. For flat screen case, the commonly used presentation is an (x, y) intensity diagram, i.e. dependence of diffraction reflexes intensity on coordinates along the axes parallel to the screen borders. But here we demonstrate an alternative approach: we plot the angle of light incidence θ and the angle of diffracted light registration Θ along the Cartesian axes, with the results being presented in the form of a color image obtained from an aggregate of a large number of the cylindrical screen photographs.

The advantage of such technique is that one can easily distinguish 2D and 3D diffraction, along with singling out a variety of effects induced by the disorder in the specimen. We have obtained such an aggregate diffraction pattern from thin opal film consisting of 6 hexagonal layers. It turns out to be in perfect agreement with the pattern calculated using formula (1) with particles diameter of D=720nm. This confirms the conclusion that diffraction from thin opal films possesses pure 2D character.

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2D to 3D diffraction transition in photonic structures

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The diffraction is a key optical phenomenon in photonic crystals (PhCs). Diffraction gives rise to energy stop-bands in the spectrum for the electromagnetic wave propagation in a periodic structure. The presence of photonic stop-bands along certain directions of electromagnetic wave propagation or the formation of a completely three-dimensional photonic band gap is the major feature of 3D PhCs. For a stop-band to investigate, the light wavelength should be comparable to the period of the spatial modulation of dielectric permittivity in PhCs.

The alteration of physical properties with the transition from two-dimensional to three-dimensional objects always attracts huge interest of the researchers, and the optical diffraction is not an exception. The aggregate diffraction patterns obtained by our team in experiments both on films [1] and bulk [2] opal samples were represented in the angle of light incidence θ and the angle of diffracted light registration Θ Cartesian axes. This kind of presentation allowed us to obtain new information about properties of light scattering depending on specimen parameters such as its thickness, the presence of disorder, lattice constant as well as the dielectric contrast. It is well known that diffraction on periodic systems can be described by Laue equations [3] in first approximation. Number of equations that should be taken into account coincides with the system dimensionality. It is well known that Bragg diffraction from 3D structures implies the law of *specular reflection* from (*hkl*) crystallographic planes. On the other hand according to the 2D diffraction grating equation [3] angular position of reflexes has more complicated behavior. At the same time there is no analytical solution for a specimen with limited thickness as well as for the one with interlayer disorder like twinning in opal-based PhCs. The aim of this work is interpretation of experimental data and modeling of influence of the factors mentioned above on light diffraction in such systems.

In order to solve this task we perform numerical modeling of the diffraction patterns, computing the scattering structure factor $S(\mathbf{b}_{is})$ given by expression [4]

$$S^2(\mathbf{b}_{is}) = \frac{1}{NLM} \cdot \frac{\sin^2(N\mathbf{b}_{is}\mathbf{a}_1/2)}{\sin^2(\mathbf{b}_{is}\mathbf{a}_1/2)} \cdot \frac{\sin^2(L\mathbf{b}_{is}\mathbf{a}_2/2)}{\sin^2(\mathbf{b}_{is}\mathbf{a}_2/2)} \cdot \frac{\sin^2(M\mathbf{b}_{is}\mathbf{a}_3/2)}{\sin^2(\mathbf{b}_{is}\mathbf{a}_3/2)} \quad , \tag{1}$$

where $\mathbf{b}_{is} = \mathbf{k}_s - \mathbf{k}_i$ is the difference of the wave vectors of the scattered and incident waves, $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ are the translation vectors of fcc opal lattice primitive cell. Equation (1) allows one to calculate $S(\mathbf{b}_{is})$ for different angles of scattering and specimen orientation determined by the $\mathbf{k}_s$ and $\mathbf{k}_i$ vector directions. The size of the specimen is the calculation variable and is determined by the number of scatterers N, L, M along the directions of $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$ vectors respectively. Thickness is determined by the varying number of layers N while the other geometrical parameters are fixed: $L = M = 3000$, $D = 320$ nm which corresponds to the linear dimensions of the specimen being about a millimeter.

Light scattering from a single hexagonal layer formed by close-packed SiO_2 spheres corresponds to the 2D diffraction case ($N = 1$). The calculated diffraction pattern presented in (θ, Θ) axes calculated for visible light in range of incident angles of $-90° \leq \theta \leq 90°$ consists of two ruptured ovals and a straight white line $\Theta = 2\theta$ separating them which corresponds to the specular reflection from the hexagonal layer.

The increasing of the specimen thickness leads to transition from 2D to 3D diffraction. In 2D case the diffraction is determined by two Laue equations while in 3D case the third equation is added. It applies additional limitations to the diffraction conditions and reduces the number of solutions.

With the increasing of the number of layers certain regions both of the semi-rings and of the diagonal fade out versus the initial 2D diffraction pattern. First, the closed semi-rings rupture, after that the regions between ruptures shrink and finally in case of the bulk specimen consisting of $3000 \times 3000 \times 3000$ unit cells these regions morph into a set of segments parallel to the straight line $\Theta = 2\theta$. As a result the expected Bragg diffraction pattern is indeed reached.

The calculated diffraction patterns are in good agreement with experimental data and allow one to follow the transition from 2D diffraction in thin opal films to 3D Bragg diffraction observed in bulk opal specimens.

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Parametric interactions in metamaterial structures

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Nonlinear parametric effects are substantially suppressed in natural materials, because of the so-called phase mismatch. Various techniques, such as quasi phase matching, are used in order to overcome this limitation for achieving, e.g., efficient second harmonic generation (SHG). The essence of phase matching is that the indices of refraction for the interacting waves should be equal. This is why it is hard to find in natural materials which should be dispersionless over the whole octave.

In this talk we overview the possibilities of achieving phase matching in metamaterial structures. In particular, we focus on backward-to-forward frequency conversion, as well as backward-to-backward wave interaction. We also demonstrate phase-matched interaction between the waves with zero phase velocity. Such waves appear, e.g. in plasma near the plasma frequency, or in the so-called epsilon-near zero metamaterials. Here, using the composite left-right handed (CRLH) transmission line we design the structure which supports zero phase velocity waves at both the fundamental frequency and at the second harmonic.

Firstly, we theoretically consider the problem of SHG during the scattering from a semi-infinite left-handed medium and demonstrate a phase-matching condition, specific for the harmonic generation by backward waves. With this condition, we demonstrate that exact phase matching between a backward-propagating wave of the fundamental frequency and the forward propagating wave at the second harmonics is indeed possible. This novel phase-matched process allows the creation of an effective "nonlinear quadratic mirror" that reflects the SH component generated by an incident wave [1].

Secondly, we study the possibility for the phase matching between two backward waves, as well as between two waves with zero phase velocity [2]. To demonstrate predicted effects experimentally, we design the transmission line with a unit cell structure made of a combination of two regular nonlinear CRLH transmission lines with different parameters in order to achieve the required properties. We analyze the dispersion properties of such a transmission line, and study nonlinear effects experimentally. We observe second harmonic generation, and demonstrate that it is enhanced near the frequency of zero phase velocity. We also measure the field distribution in the structure in order to experimentally determine its dispersion properties. We expect that similar regimes of the phase matching can be achieved in dual-band index near zero metamaterials.

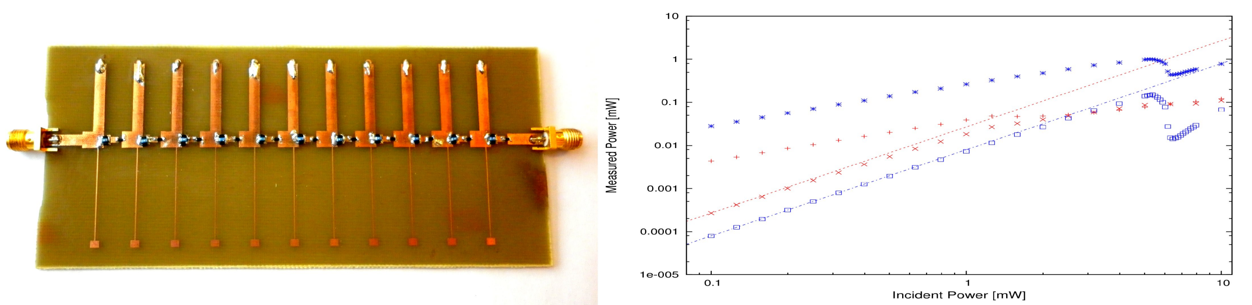

Fig. 1. Left: composite transmission line. Right: Transmitted power at fundamental frequency (stars and +'s) and at second harmonics (x's and squares) for two regimes of interaction.

Figure 1 shows structure of the designed nonlinear transmission line containing 11 unit cells. Right panel demonstrates second harmonic generation for two cases of interest: when both interacting waves have zero phase velocity (crosses and x's) and when both interacting waves are backward (stars and squares). We will discuss these results and present our experimental study for the dispersion of waves in the structure.

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Broadband optical blooming of a medium by a nanocrystalline layer

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At present, investigations directed at the creation of conditionally invisible materials and media having complete or almost complete transmission of incident radiation in a given spectral range are widely performed. These investigations are stimulated by the possibility of the controlled creation and use of nanoobjects whose application makes it possible to vary the optical properties of natural materials in a sufficiently wide range by modifying the existing antireflection coatings or appropriately tuning the refractive index of the medium as a whole. In particular, Xi et al. [1] demonstrated that the $Si0_2$ and $Ti0_2$ nanotube hairs deposited on the silicon surface can reduce the reflectivity of the substrate to 0.05% at certain wavelengths. A similar effect was also observed in arrays of carbon nanotubes [2] and is due to the entanglement of light in the diluted chaotic nanostructured system. The possibility of the achievement of the absolute transparency of the medium at a given wavelength owing to the deposition of an ordered layer of spherical nanocavities was theoretically predicted in [3]. However, this effect was obtained only for some exotic weakly refracting media. Yanagishita et al. [4] experimentally investigated the reduction of the reflection by an ordered nanocrystal on an example of the periodic layer of polymer nanocones on the surface of a lens.

In this work, the optical properties of a dielectric medium with the ordered distribution of nanopores introduced into the surface layer are investigated. It is shown that this system can be bloomed in a wide wavelength range. The blooming effect can be obtained both for weakly and strongly refracting materials.

In [3,5], the reflection of light from the monolayer of nanoparticles on the surface of the substrate was considered and a method was proposed to obtain an analytical solution having a high accuracy as compared to the *ab initio* numerical solution [5–7] when the radius (a) and refractive index (n) of the particles satisfy:

$$k_0 a = 1, \quad k_0 n a = 1, \tag{1}$$

where k_0 — wave number. It means that the field incident on the nanocluster varies weakly in its volume. Using this approach to describe optical properties of a system under investigation it is possible to obtain the following formula for reflection coefficient which is well-known from classical optics of antireflecting coatings [7]:

$$\hat{r} = \frac{\hat{R}_{12} + \hat{R}_l \exp\left\{2i\tilde{n}_m\left(\boldsymbol{k}_0\boldsymbol{\Delta}\right)\right\}}{1 - \hat{R}_{21}\hat{R}_l \exp\left\{2i\tilde{n}_m\left(\boldsymbol{k}_0\boldsymbol{\Delta}\right)\right\}}, \tag{2}$$

where $\hat{R}_l$ is the tensor of the non-Fresnel reflection coefficients of the monolayer, $\hat{R}_{12}$ is the tensor of the Fresnel reflection coefficients of a matrix medium in the absence of nanolayer, $\tilde{n}_m$ is refractive index of a matrix medium, and $\boldsymbol{\Delta}$ is the distance from the surface to the plane passing through the centers of the nanoparticles.

Relation (2) allows us to make the following conclusions:

1) The monolayer of nanoparticles is an imaginary interface between two media, which passes through the centers of the clusters and has complicated reflection and transmission coefficients. So the system under consideration can be represented as a film with the thickness Δ that is located on the surface of semi-infinite substrate, although both the film and substrate are made of the same material.

2) Phase shift $2i\tilde{n}_m\left(\boldsymbol{k}_0\boldsymbol{\Delta}\right)$ taking place in classical system "film on a substrate" due to wavelength changing does not allow us to satisfy interference minimum condition $2i\tilde{n}_m\left(\boldsymbol{k}_0\boldsymbol{\Delta}\right) = \pi$ in wide spectral range and is the reason for using multilayered coatings. In the present system this limitation has been partially excluded due to the phase shift $\rho\left(\lambda\right)$ (depending on a wavelength) appearing while reflecting from a nanolayer; it can be used to compensate phase shift $2i\tilde{n}_m\left(\boldsymbol{k}_0\boldsymbol{\Delta}\right)$.

3) It is necessary to use non-conductive particles with refractive index lower than the refractive index of matrix medium ($n < \tilde{n}_m$) for achieving compensation effect and a broadband medium blooming correspondingly.

In paper [7] we show that the semi-infinite medium can be bloomed in a wide wavelength range by the introduction of a single layer of nanocavities. This effect is inherent only in the nanostructured system and cannot be achieved in a bulk film. The results of this work can be significant for the development of materials with high transparency ("invisible" materials), the creation of blooming coatings with a quality higher than that of the existing materials, and an increase in the transmissivity of media whose blooming by the existing methods is difficult or impossible.

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Anisotropic left-handed metamaterials for imaging

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In this presentation we discuss the lensing properties of anisotropic metamaterials with negative permittivity and/or negative permeability tensor components. Such metamaterials, under certain conditions, can show perfect lensing capabilities, fulfilling all the perfect lensing conditions introduced by Pendry. Moreover, a new lens formula in such structures allows for more far-field imaging than in the isotropic metamaterial lenses, and gives the possibility for thinner lenses; such a possibility is extremely important in the optical regime where metamaterial losses are quite high. Finally, the structure parameter conditions required for the achievement of anisotropic perfect lenses are quite easy to be fulfilled using todays planar fabrication approaches. Specific examples of anisotropic perfect lenses will be demonstrated and discussed.

Edge waves and their use for the broadband field concentration

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In this paper we suggest and theoretically study a planar version of the metamaterial nanotip whose 3D variant was reported in the previous conference. This planar version is a tapered plasmonic nanostructure from silver or gold nanobars separated by dielectric nanoplates. It implements the light field concentration i.e. optically connects the region whose width is close to half-wavelength (the region of the incident wave beam) with a strongly subwavelength spatial region where the field is locally enhanced in a broad frequency range. This small spatial region (hot spot) has a frequency-independent location near the apex of the tapered structure. The broadband and spatially stable field concentration results from special waveguide modes whose field is strongly concentrated at the edges of the tapered structure. This field almost does not penetrate inside the metal nanobars. We called such waves the edge waves. The excitation of these waves at many frequencies is achieved because the size of the metal nanobars is gradually varied so that there are many plasmon resonances in the structure. These resonance overlap and cover a sufficient part of the visible range (nearly one half).

Over one quarter of this band the interference of edge waves which meet one another at the apex of the structure turns out to be constructive and this results in a hot spot.

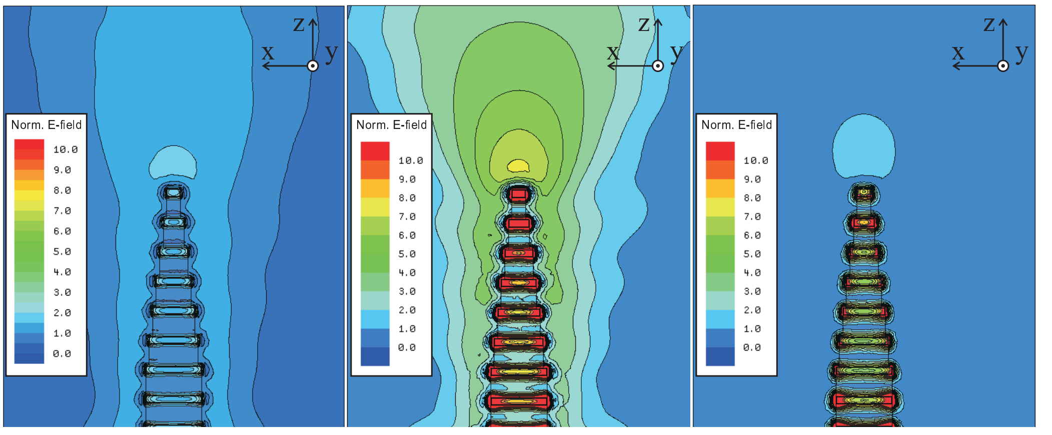

Fig. 1. The normalized amplitude of the electric field at (a) 530 THz, (b) 570 THz, and (c) 610 THz.

To obtain these results we needed to find proper design parameters of the nanotip. For it we numerically studied the dispersion properties of a planar periodic plasmonic waveguide where silver nanobars are sandwiched in between dielectric nanoplates. Since the dispersion in a periodic structure is the cell problem we repeated this study for many different sizes and studied different modes. Simulations were done using the HFSS and COMSOL simulators whereas the complex permittivity of the silver corresponded to the known experimental data (the permittivity of the dielectric was set equal to 2.2). We found that the edge waves possess strong localization but low attenuation. Due to their strong localization they propagate even if the nanobars are very optically short. Therefore the apex of the waveguide can be reduced to sub-wavelength dimensions without sacrificing too much in the matching between the wider waveguide sections. Briefly, the edge waves are 1D analogues of 2D waves already found and exploited in the previous work – surface waves propagating along the interface of the indefinite medium. These waves were used for the same purpose in our previous (3D variant) of the metamaterial nanotip.

We foresee many possible applications for our structure from prospective near-field scanning optical microscopes to interconnects between conventional optical waveguides and prospective optical nanocircuits.

3D optical diffraction and selective switching of diffraction reflexes in opal-based photonic crystals

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We report on a detailed diffraction study of the photonic band structure of 3D multi-component photonic crystals (McPhC). These include photonic crystals are comprised of either inhomogeneous or multiple (three or more) homogeneous components. Recently we discovered that opal-based McPhCs demonstrate quasiperiodic resonant behavior of their (*hkl*) photonic stop-bands as a function of the reciprocal lattice vector [1].

The experimental and calculation results indicate the selective principle of different $\{hkl\}$ stop bands 'off switching' in McPhCs while varying the permittivity of one of the components. In the work [1] there was demonstrated that the opal-based photonic crystals belong to the class of McPhCs. The multi-component nature of the opal structure is provided by the fact that the constitutive a-SiO_2 particles possess an inhomogeneous inner structure: the porous nucleus consisting of smaller amorphous silica dioxide particles covered by a denser thin coat [2].

In low-contrast PhC $\{hkl\}$ stop bands in transmission spectra and the $\{hkl\}$ reflexes in the diffraction patterns have common nature – the Bragg light reflection from the specific $\{hkl\}$ crystallographic plane sets. Taking into account the effects discovered in opal transmission spectra, one can expect some nontrivial transformation of the Bragg $\{hkl\}$ reflexes in the diffraction patterns depending on the dielectric contrast. Such effects are indeed observed during the experiments on light diffraction from bulk opal specimens while varying ε_f [3].

We should emphasize two key features of our experimental diffraction technique. The first is that instead of commonly used flat screen we use a narrow *cylindrical* one with a specimen fixed in its center. It allows us to avoid nonlinear distortions of diffraction reflexes. The experiments are performed using the immersion spectroscopy method, which implies consecutive acquisition of the diffraction patterns with change of the permittivity of the liquid filling opal matrix. The cylindrical flask was filled with the liquid being an opal filler as well as the ambient medium. During the experiment distilled water ($\varepsilon_{\mathrm{H_2O}}$= 1.78), propylene glycol (ε_{pr} = 2.05) and their mixes are used. It allows to vary the filler permittivity within the range $1.78 \leq \varepsilon_f^{\leq}$ 2.05.

The second key feature of these studies is the experimental data presentation. For flat screen case, the commonly used presentation is an (x, y) intensity diagram, i.e. dependence of diffraction reflexes intensity on coordinates along the axes parallel to the screen borders. But here we demonstrate an alternative approach: we plot the angle of light incidence θ and the angle of diffracted light registration Θ along the Cartesian axes, with the results being presented in the form of a color image obtained from an aggregate of a large number of the cylindrical screen photographs. This approach allows one to distinguish reflexes corresponding to specific (*hkl*) stop bands using both geometrical and spectral information.

As far as 3D Bragg diffraction implies that light follows the law of *specular reflection* from (*hkl*) crystallographic planes, angular position of specific (*hkl*) diffraction reflex has linear dependence on the angle of incidence and in $(\theta,\ \Theta)$ axes

$$\Theta = 2(\theta - \theta_{hkl}). \tag{1}$$

At the same time the color of diffraction reflex is determined by the Bragg's law [2]:

$$\lambda_{hkl}(\theta) = 2d_{111}\sqrt{\varepsilon_{av}}\left(\frac{3}{h^2+k^2+l^2}\right)^{1/2}\cos\left(\theta - \theta_{hkl}\right). \tag{2}$$

Therefore Bragg diffraction reflexes associated with different (*hkl*) plane sets appear in the (θ, Θ) diagram as a set of parallel straight colored segments lying on the lines shifted by θ_{hkl} along θ axis with respect to the $\Theta = 2\theta$ diagonal corresponding to the (111) reflex. These lines rupture in regions corresponding to diffraction in invisible (either UV or IR) radiation range.

Our diffraction studies have established that intensity of Bragg reflexes corresponding to the different $\{hkl\}$ families of crystal planes have different dependencies on the permittivity of the liquid filler ε_f [3]. The reflexes of the $\{111\}$ family significantly weaken when the filler permittivity approaches the value of $\varepsilon^0_{f\{111\}}$ ≈1.82, while the intensities of the dips corresponding to the $\{220\}$ family stop bands significantly decrease when ε_f approaches $\varepsilon^0_{f\{220\}} \approx 1.92$. These values coincide with the ones obtained from analysis of optical transmission spectra.

Therefore, one can conclude a possibility of the selective control over the intensity of Bragg diffraction reflexes in low-contrast McPhCs. Also it is of importance that the results of diffraction experiments show excellent agreement with results of transmission spectroscopy, analytical theory predictions and computational data.

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Influence of interface plane angle on transmission properties of superlens

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During last years, various types of lenses with super-resolution ("superlenses") using metamaterials are actively investigated. Sir J. Pendry proposed in [1] that a planar lens formed by left-handed material (LHM) [2] can be used to image source information with a spatial resolution below the diffraction limit. Dr. P. Belov proposed a flat wire medium (WM) lens with 'canalization' principle [3] to transport subwavelength source details to an image plane at a significant distance. In contrast to the case of an LHM, such devices are less sensitive to losses. Above mentioned lenses may find application in magneto resonance imaging (MRI) [4–5]. In this paper we use "canalization" principle for investigation and development of MRI WM lenses with sloped interfaces planes.

Variable parameter in superlens design is an angle between the wires direction and the interface planes of the superlens ("interface plane angle" = 30°, 60°, 90°). Same parameters for superlens design are the wire length of 250 cm, 20x20 array of brass wires with diameter of 2 mm and period of 10 mm.

The numerical simulation of three superlens designs were performed in the frequency range of 56–64 MHz using the CST Microwave Studio 2011 package. The operating frequency range was located near nuclear magnetic resonance frequency (63,8 MHz). The superlenses were excited by a source in the form of the flag. The transmission of image in the form of a flag with a deeply sub-wavelength resolution is shown on Fig.1. for superlens with "interface plane angle" = 30° In conclusion, we have numerically demonstrated the transmission of the image in the form of a flag with $\lambda/500$ resolution to a $\lambda/2$ distance in the very high frequency range (MRI frequencies) for three types of MRI superlens. It should be noted that the superlenses with 30°, 60° interface plane angle allow to transmit both s, p-polarizations.

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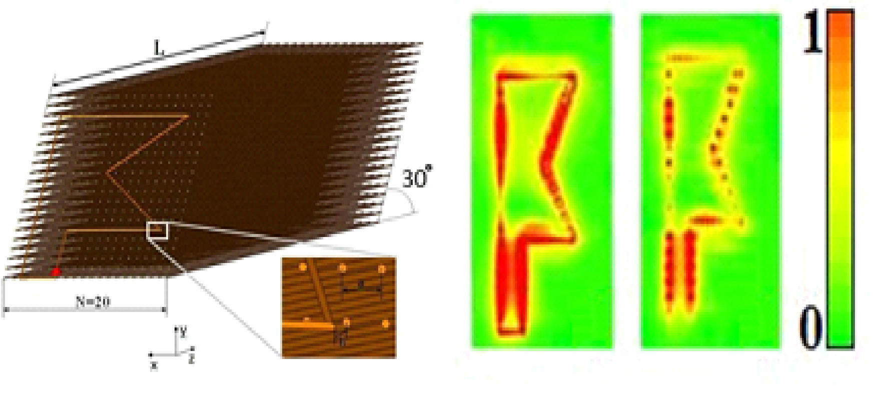

(a) (b) (c)

Fig. 1. a) Geometry of the sloped lens with "interface plane angle"=30^0 formed by the wire medium and the source in the form of the flag. Distribution of E_y electric field component at the vicinity of the source <4 mm from the front lens interface> (b) and in the image plane <4 mm from the back lens interface> (c). The results are presented for the frequency of the first Fabry-Pero resonance (60MHz) normalized by the maximum value of electric field intensity.

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Diffraction of Wave Beams in Inhomogeneous Metamaterials

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Metamaterials with negative refractive index, or left-handed materials (LHM), are in a marked contrast to any conventional materials that can be found in nature [1,2]. Such metamaterials were first engineered in microwave frequency range. Recently metamaterials possessing negative refractive index are constructed up to optical frequencies [3]. Metamaterials are characterized by rather high losses and small figures of merit (which is the ratio of real to imaginary part of the refractive index), but their further improvement opens new opportunities for optics and photonics. Among such new opportunities there is a construction of a subwavelength cavity [4]. The boundary between the positive-index material and the LHM acts like a lens, focusing divergent wave beam. It makes possible waveguide beam propagation in a periodic structure consisting of layers with alternating sign of the refractive index [5]. In our work is devoted to the theory of the novel types of open layered bulk and surface cavities containing layered structure with left-handed metamaterials (LHM). This structure contains layers of conventional materials (dielectrics) and layers of metamaterials with negative refractive index thus representing an artificial structure with alternating sign of the refractive index.

Inclusion of LHM inside a bulk cavity strongly affects its properties such as eigenmode profiles, eigenfrequencies and the stability conditions. We have investigated open cavity containing layered structure with. Our analysis of the propagation of the wave beams in the layered cavities based on the slowly varying amplitude method shows that cavity properties are determined by the values of the optical cavity length $L_{opt} = \sum\limits_i l_i n_i$ and so-called effective diffraction length $L_{dif} = \sum\limits_i l_i/n_i$. It is the remarkable for the considered cavity that in contrast to ordinary cavities both optical and effective diffraction lengths can have negative or zero as well as positive values.

The formation conditions for waveguide eigenmodes in flat- and convex-mirror cavities having Gaussian and Hermit-Gaussian profile were derived neglecting losses and assuming the layers to have the same wave impedance in order to avoid Fresnel reflection. Important feature of such cavity is the existence of eigenmodes in cavities with both concave and convex mirrors. Waveguide eigenmodes can also exist in the Fabri-Perrot cavities if the effective diffraction length is equal to zero. The most sufficient result in this case is that the transverse field distribution in eigenmode with zero diffraction length is arbitrary. In this case all eigenmodes have the same eigenfrequencies not depending on their cross-section profile. Conditions for the arbitrary waveguide eigenmodes in concave and convex-mirror cavities were derived. The case of different impedance values of the layers was also considered. It was shown that Fresnel reflection leads to the eigenfrequency shift and longitude spectrum is no more equidistant. We performed calculations basing on the dispersion of the existing metamaterials and found that these shifts sufficiently depends on the metamaterial type.

While the theory mentioned above can be used for the description of rather wide beams (with the width much larger than the wavelength) numerical approach can be used to describe the propagation of ultra-narrow beams with the widths about 1 wavelength. Finite-difference time-domain (FDTD) method is used to simulate narrow beam propagation in the layered cavity. Eigenmode profiles and eigenfrequncies of such cavity are analyzed.

Introduction of metamaterial layer inside of the cavity also changes its stability regions. It can be used to make stable initially unstable empty cavity. For example, empty cavity with both convex mirrors does not have eigenmodes. Introduction of metamaterial layer not only leads to appearance of eigenmodes but also can make such cavity stable.

Nonlinearity of dielectric or metamaterial layers leads to bistability of the cavity. We investigate the influence of nonlinearity on the stability conditions and eigenmode profiles of cavity with metamaterial. Stability conditions changes significantly in the cases of negative and zero diffraction, which are possible in cavity with metamaterial.

Previously we analyzed surface plasmon polariton beam diffraction and derived the diffraction equation for such beams [6]. We consider a surface planar cavity consisting of alternating interfaces of metal or metamaterial with dielectric. It can be shown that under certain conditions such layered structure is analogous to the bulk layered structure discussed in [5]. Thus basing on the theory of the bulk cavity we can develop analogous theory of planar Fabri-Perrot plasmon cavity with metamaterial layers. Properties of such cavities are determined by the values of optical and effective diffraction lengths where refractive indexes are defined as $n_i = \beta/k_0$ where β is the plasmon polariton propagation constant and k_0 is the wavevector value in vacuum.

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Mapping the plasmonic patterns in nanostructures with the resolution of AFM

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In the past ten years, a new class of materials has emerged and it has attracted a lot of attention due to its counterintuitive optical behavior and revolutionary potential applications — metamaterials.

Metamaterials are materials engineered for displaying unusual electromagnetic properties and they have been associated with negative refractive index,[1] invisibility,[2] light-based nanocircuits,[3–5] etc. All of these spectacular phenomena are based on surface plasmon resonances — the property whereby, in metallic nanostructures, light can collectively excite the electron population at surfaces. However, in mapping the plasmonic patterns on the surfaces of nanostructures, the diffraction limit of light remains an important obstacle. Here we demonstrate that this limit can essentially be removed. We have studied nanostructures made of nickel and palladium, deposited by electron beam lithography in different geometrical patters, including the G-shaped design that we proposed previously.[6–9] We show that, upon illuminating nanostructures, the resulting surface plasmon pattern is imprinted on the structures themselves allowing for subsequent imaging with Scanning Electron Microscopy and Atomic Force Microscopy (AFM), see Figure 1. The resulting resolution of plasmon pattern imaging can therefore, in principle, be brought down to that of the AFM. Our results open a new avenue for studying plasmonic patterns at the nanoscale.

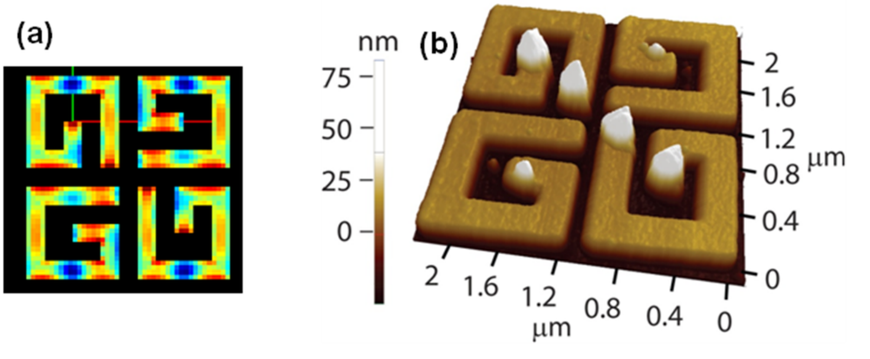

Figure 1 | A high resolution imprint of the plasmon hotspots. In (a), theoretical simulation of the electric currents in the nanostructures. In (b), after illumination of the sample, the localized field enhancements are imprinted on the nanostructures. Subsequently, an Atomic Force Microscopy image reveals the precise location of the hotspots.

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Electromagnetic TE wave propagation through a nonlinear metamaterial slab

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The last years, metamaterials are intensively investigated. Here we obtain and study a dispersion equation (DE) for TE-waves propagating through a nonlinear metamaterial layer [1, 2]. Some numerical results are shown. For the case of TM-waves see [3].

Let us consider electromagnetic waves propagating through a homogeneous isotropic nonmagnetic dielectric layer filled by media with general type of Kerr-law nonlinearity. The layer is located between two half-spaces $x < 0$ and $x > h$ in Cartesian coordinate frame $Oxyz$. Half-spaces are filled with isotropic nonmagnetic medium containing no sources and with constant permittivities ε_1 and ε_3, respectively (ε_1 and ε_3 are arbitrary real values). Everywhere below μ is the permeability of free space.

Electromagnetic field harmonically depends on time $\vec{\tilde{E}}(x,y,z,t) = \vec{E}_+(x,y,z)\cos\omega t + \vec{E}_-(x,y,z)\sin\omega t$; $\vec{\tilde{H}}(x,y,z,t) = \vec{H}_+(x,y,z)\cos\omega t + \vec{H}_-(x,y,z)\sin\omega t$, where ω is circular frequency; $\vec{E}_+$, $\vec{E}_-$, $\vec{H}_+$, $\vec{H}_-$ are real required functions. Form complex amplitudes: $\vec{E} = \vec{E}_+ + i\vec{E}_-$; $\vec{H} = \vec{H}_+ + i\vec{H}_-$. Below time multipliers are omitted.

Electromagnetic field $\vec{E}$, $\vec{H}$ satisfies Maxwell equations $rot\vec{H} = -i\omega\varepsilon\vec{E}$, $rot\vec{E} = i\omega\mu\vec{H}$, continuity condition of tangential components at the interfaces $x = 0$, $x = h$ and radiation condition at infinity: electromagnetic field exponentially decays at $|x| \to \infty$ in the regions $x < 0$ and $x > h$.

The permittivity inside the layer has the form $\varepsilon = \varepsilon_2 + \alpha|\vec{E}|^2 + \beta|\vec{E}|^4$, where ε_2, α, β are arbitrary real constants.

We seek solutions to the Maxwell equations in the entire space.

Consider TE-waves $\vec{E} = (0, E_y, 0)^T$, $\vec{H} = (H_x, 0, H_z)^T$. It is easy to show that components of electromagnetic field do not depend on y. Waves propagating along media interfaces z harmonically depend on z. It implies that components of electromagnetic field have the form: $H_y = H_y(x)e^{i\gamma z}$, $E_x = E_x(x)e^{i\gamma z}$, $E_z = E_z(x)e^{i\gamma z}$.

We perform the normalization the Maxwell equations in accordance with the following formulae $\tilde{x} = kx$, $\frac{d}{dx} = k\frac{d}{d\tilde{x}}$, $\tilde{\gamma} = \frac{\gamma}{k}$, $\tilde{\varepsilon}_j = \frac{\varepsilon_j}{\varepsilon_0}$ $(j = 1,\ 2,\ 3)$, $\tilde{\alpha} = \frac{\alpha}{\varepsilon_0}$, $\tilde{\beta} = \frac{\beta}{\varepsilon_0}$, where $k^2 = \omega^2\mu\varepsilon_0$ and ε_0 is the permittivity of free space. Introduce the notation $E_y(\tilde{x}) \equiv Y(\tilde{x})$ and omitting tilde from the Maxwell equations we get the equation which we are studying

$$Y''(x) = \gamma^2 Y(x) - \varepsilon Y(x), \quad \text{where } \varepsilon = \begin{cases} \varepsilon_1,\ x < 0; \\ \varepsilon_2 + \alpha Y^2 + \beta Y^4,\ 0 < x < h; \\ \varepsilon_3,\ x > h. \end{cases}$$

We look for real values of γ and $Y(x) \in C(-\infty;\ +\infty) \cap C^1(-\infty; +\infty) \cap C^2(-\infty;\ \ 0] \cap C^2[0;\ h] \cap C^2[h;\ \ +\infty)$.

It can be proved that DE has the form

$$h = -\wp^{-1}(\sigma_0) - \wp^{-1}(\sigma_h) + Tk, \quad k = 1, 2, ...,$$

where $\wp^{-1}$ is the inverse of Weierstrass elliptic function $\wp$; $\sigma_0 = \frac{3C_h + \left(\gamma^2 - \varepsilon_2\right)Y_0^2}{3Y_0^2}$, $\sigma_h = \frac{3C_h + \left(\gamma^2 - \varepsilon_2\right)Y_h^2}{3Y_h^2}$; value Y_h is assumed to be known (initial condition); $C_h = \left(\frac{\beta}{3}Y_h^4 + \frac{\alpha}{2}Y_h^2 + \varepsilon_2 - \varepsilon_3\right)Y_h^2$; and Y_0 is found from the equation $\frac{\beta}{3}Y_0^6 + \frac{\alpha}{2}Y_0^4 + (\varepsilon_2 - \varepsilon_1)Y_0^2 - C_h = 0$.

The invariants of $\wp$ are defined by formulae $g_2 = \frac{2}{3}\left(3\alpha\left(\frac{\beta}{3}Y_h^4 + \frac{\alpha}{2}Y_h^2 + \varepsilon_2 - \varepsilon_3\right)Y_h^2 + 2\left(\gamma^2 - \varepsilon_2\right)^2\right)$, and $g_3 = -\frac{2}{3}\alpha\left(\gamma^2 - \varepsilon_2\right)\left(\frac{\beta}{3}Y_h^4 + \frac{\alpha}{2}Y_h^2 + \varepsilon_2 - \varepsilon_3\right)Y_h^2 - \frac{8}{27}\left(\gamma^2 - \varepsilon_2\right)^3 + \frac{4}{3}\beta\left(\frac{\beta}{3}Y_h^4 + \frac{\alpha}{2}Y_h^2 + \varepsilon_2 - \varepsilon_3\right)^2 Y_h^4$.

The value T is connected with periods of a function $\wp(x)$ and defined depending on discriminant's sign $\Delta \equiv g_2^3 - 27g_3^2$ of the curve $f = 4t^3 - g_2 t - g_3$.

If $\Delta > 0$ then the equation $f = 0$ has three real roots $e_1 > e_2 > e_3$ and $T = 2\int\limits_{e_1}^{\infty} \frac{dt}{\sqrt{4t^3 - g_2 t - g_3}}$.

If $\Delta < 0$ then the equation $f = 0$ has one real root e_2 and $T = 2\int\limits_{e_2}^{\infty} \frac{dt}{\sqrt{4t^3 - g_2 t - g_3}}$.

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Electromagnetic TM wave propagation through a nonlinear metamaterial slab

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The last years, metamaterials are intensively investigated. Here we obtain and study a dispersion equation (DE) for TM-waves propagating through a nonlinear metamaterial layer [1, 2, 3]. Some numerical results are shown. For the case of TE-waves see [4].

Let us consider electromagnetic waves propagating through a homogeneous isotropic nonmagnetic dielectric layer filled by media with general type of Kerr-law nonlinearity. The layer is located between two half-spaces $x < 0$ and $x > h$ in Cartesian coordinate frame $Oxyz$. Half-spaces are filled with isotropic nonmagnetic medium containing no sources and with constant permittivities ε_1 and ε_3, respectively (ε_1 and ε_3 are arbitrary real values). Everywhere below μ is the permeability of free space.

Electromagnetic field harmonically depends on time $\vec{\tilde{E}}(x,y,z,t) = \vec{E}_+(x,y,z)\cos\omega t + \vec{E}_-(x,y,z)\sin\omega t$; $\vec{\tilde{H}}(x,y,z,t) = \vec{H}_+(x,y,z)\cos\omega t + \vec{H}_-(x,y,z)\sin\omega t$, where ω is circular frequency; $\vec{E}_+$, $\vec{E}_-$, $\vec{H}_+$, $\vec{H}_-$ are real required functions. Form complex amplitudes: $\vec{E} = \vec{E}_+ + i\vec{E}_-$; $\vec{H} = \vec{H}_+ + i\vec{H}_-$. Below time multipliers are omitted.

Electromagnetic field $\vec{E}$, $\vec{H}$ satisfies Maxwell equations $rot\vec{H} = -i\omega\varepsilon\vec{E}$, $rot\vec{E} = i\omega\mu\vec{H}$, continuity condition of tangential components at the interfaces $x = 0$, $x = h$ and radiation condition at infinity: electromagnetic field exponentially decays at $|x| \to \infty$ in the regions $x < 0$ and $x > h$.

The permittivity inside the layer has the form $\varepsilon = \varepsilon_2 + \alpha|\vec{E}|^2$, where ε_2, α are arbitrary real constants.

We seek solutions to the Maxwell equations in the entire space.

Consider TM-waves $\vec{E} = (E_x, 0, E_z)^T$, $\vec{H} = (0, H_y, 0)^T$. It is easy to show that components of electromagnetic field do not depend on y. Waves propagating along media interfaces z harmonically depend on z. It implies that components of electromagnetic field have the form: $E_x = E_x(x)e^{i\gamma z}$, $E_z = E_z(x)e^{i\gamma z}$, $H_y = H_y(x)e^{i\gamma z}$.

We perform the normalization the Maxwell equations in accordance with the following formulae $\tilde{x} = kx$, $\frac{d}{dx} = k\frac{d}{d\tilde{x}}$, $\tilde{\gamma} = \frac{\gamma}{k}$, $\tilde{\varepsilon}_j = \frac{\varepsilon_j}{\varepsilon_0}$ ($j = 1, 2, 3$), $\tilde{\alpha} = \frac{\alpha}{\varepsilon_0}$, where $k^2 = \omega^2\mu\varepsilon_0$ and ε_0 is the permittivity of free space. Introduce the notations $E_x(\tilde{x}) \equiv X(\tilde{x})$, $E_z(\tilde{x}) \equiv Z(\tilde{x})$ and omitting tilde from the

Maxwell equations we get the system of equations which we are studying

$$\begin{cases} -Z'' + \gamma X' = \varepsilon Z, \\ -Z' + \gamma X = \frac{1}{\gamma}\varepsilon X; \end{cases} \quad \text{where } \varepsilon = \begin{cases} \varepsilon_1, \ x < 0; \\ \varepsilon_2 + \alpha(X^2 + Z^2), \ 0 < x < h; \\ \varepsilon_3, \ x > h. \end{cases}$$

We look for real values of γ and $X(x) \in C\,(-\infty;\ 0] \cap C[0;h] \cap C\,[h;\ +\infty) \cap C^1\,(-\infty;\ 0] \cap C^1[0;h] \cap C^1\,[h;\ +\infty)$, $Z(x) \in C(-\infty;+\infty) \cap C^1\,(-\infty;\ 0] \cap C^1[0;h] \cap C^1\,[h;\ +\infty) \cap C^2\,(-\infty;\ 0] \cap C^2[0;h] \cap C^2\,[h;\ +\infty)$.

Introduce new variables $\tau(x) = \frac{\varepsilon_2 + \alpha(X^2(x)+Z^2(x))}{\gamma^2}$, $\eta(x) = \gamma\frac{X(x)}{Z(x)}\tau(x)$. It can be proved that DE has the form

$$h = -\int\limits_{-\frac{\varepsilon_3}{\sqrt{\gamma^2-\varepsilon_3}}}^{\frac{\varepsilon_1}{\sqrt{\gamma^2-\varepsilon_1}}} f(\eta)d\eta + N\int\limits_{-\infty}^{+\infty} f(\eta)d\eta, \quad N = \pm 1, \pm 2, ...,$$

where $f(\eta) = \frac{\tau}{\gamma^2\tau^2+\eta^2(\tau-1)}$; $\tau = \tau(\eta)$ is expressed from the equation $\eta^2 = \frac{\gamma^2\tau^2(\tau^2-C)}{C+3\tau^2-2\tau^3-2\tau(2-\tau)\tau_0}$; $\tau_0 = \varepsilon_2\gamma^{-2}$;
$C = \tau^2(h) - \frac{2\varepsilon_3^2\tau(h)(2-\tau(h))(\tau(h)-\tau_0)}{\varepsilon_3^2+\gamma^2(\gamma^2-\varepsilon_3)\tau^2(h)}$. Here $\tau(h) = -\frac{\varepsilon_3}{\gamma\sqrt{\gamma^2-\varepsilon_3}}\frac{Z(h)}{X(h)}$, where $Z(h)$ is supposed to be known (initial condition), $X(h)$ is the real root of the equation $X^3(h) + \frac{\varepsilon_2+\alpha Z^2(h)}{\alpha}X(h) + \frac{\gamma\varepsilon_3}{\alpha\sqrt{\gamma^2-\varepsilon_3}} = 0$.

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Tunable metamaterials for THz electromagnetic spectrum

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Nowadays terahertz (THz) region of the electromagnetic spectrum reveals huge possibilities for fabrication and a variety of different applications. Realization of novel devices for security, medicine, space and earth science became possible by using THz radiation [1]. THz rays are highly absorbed in water molecules and metal while most dielectrics are transparent for them. This property allows performing high resolution THz imaging. Important is that the THz radiation is non ionizing and consequently safe for biological objects (in contrast to x-ray radiation).

THz radiation is a very new science. For a long time this region was closed for practical application and was called "THz gap". From the beginning of 21$^{\text{th}}$ century a remarkable progress has been achieved in the development of terahertz science and technology. In 2008 a room temperature semiconductor source of coherent terahertz radiation has been introduced. In 2009 a new cheap method for producing THz waves at frequencies 2 THz and 18 THz has been introduced. Nevertheless there is still a lack of devices making it possible to control THz radiation due to the deficiency of materials having a suitable response in this frequency range. Functional THz devices such as filters, switches, modulators, and phase-shifting and beam-steering devices, largely are not available.

As compared to the microwave electronic and optical photonic technologies, the development of THz technology is lagging far behind. Natural materials do not exhibit strong magnetic or electric responses between 1 and 3 THz [2]. Conventional magnetic materials usually have resonances at frequencies below the gigahertz range, while metals possess resonances at frequencies beyond the mid-infrared owing to phonon modes. There are not any dielectric or magnetic materials with a strong response in the THz range.

In addition to active searching of novel materials, artificial structured materials have played an increasingly important role particularly in the construction of functional THz devices [3]. This new class of artificial composite materials, termed as electromagnetic metamaterials, is constructed to yield a designed resonant/nonresonant response to electromagnetic waves. Because of their customizable characteristics, metamaterials can complete the missing link, and become versatile tools for future terahertz research and development. Ultimately, these metamaterials are expected to develop into terahertz manipulating devices that outperform their predecessors, or moreover, which are not possible with conventional materials. The terahertz spectrum represents a fascinating arena for metamaterial research.

Especially interesting area of investigation is designing controllable metamaterials for THz tunable devices. Different methods of external influence can be implemented for changing metamaterial properties: electric or magnetic field, temperature and optical radiation. For different control methods different metamaterial structures can be used: split-ring resonators (SRR) metamaterials [4], layered metal-dielectric structures [5], wire medium [6], liquid crystal based metamaterials [7] and others. Also a combination of different structures can be used for designing tunable THz range metamaterial.

In this work we will consider some novel metamaterial structures for variable controllable response at terahertz frequencies.

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Surface plasmon propagation in a metalic nanocylinder array

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We consider propagation of surface plasmon (SP) modes in a array of metallic nanocylinders, assuming that the trasversal size of the cylinders is less than the skin layer depth. Maxwell equation was inverstigated in the quasi-static limit, when retarding effects are neglected.. Helmholtz wave equation is reduced to Laplace equaion in the limit. Our theory is based on conformal transformation technic [1] which allows us to take into account all orders of multipole expansion of electric field near the cylinders. Laplace equation is invariant relative to conformal transformations, and we choose the trasformation wich converts boundaries of the cylinders to lines, that allows us to write the boundary conditions in a simple form.

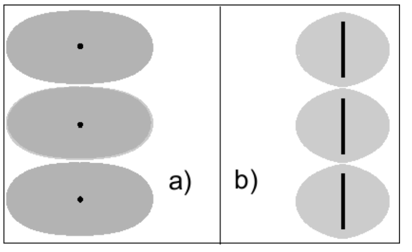

FIG. 1. Possible form of cylinders crossection. a) oblate form; b) near-circular form.

The SP modes of a chain of metal granules or grid of wires were considered earlier in the chains of metal granules or array of cylinders experimentally in [2], numerically in [3, 4] and analytically in [5, 6], where dipole-dipole approximation was used for cylinders which are apart from each other. We analyze close cylinders, see FIG. 1, where two cases a) and b) correspond to different values of the adjustment parameter in the conformal transformation. Dependence of SP zones positions and width in the system geometry are found analytically found. Also dispersion law and field distribution are found. In terms of dielectric permittivities ratio ε between the metal and surrounding dielectric the results are represented as

$$\varepsilon_{q=0} = -\frac{2/\pi}{2n-1}\frac{l}{\delta}, \qquad \varepsilon_{ql=\pi} = -\frac{2/\pi}{2n}, \qquad \varepsilon_{ql\ll\delta/l} \approx \frac{(2/\pi)l}{\delta}\left(1 - \frac{4}{\pi^2}\frac{l^2 q}{\delta}\right) \tag{1}$$

where $n = 1, 2, ...$ is the number of Brillouin zone, and the case FIG. 1a) is taken, for which the curvature of the cylinder's surface at the center of the gap between them is $a \approx (2/\pi^2)l^2/\delta$, where l is the period of the array and δ is the gap width. We discuss the qualitative relation (1) to the results [7, 8, 9] concerning SP resonance in systems of two close cylinders or spheres.

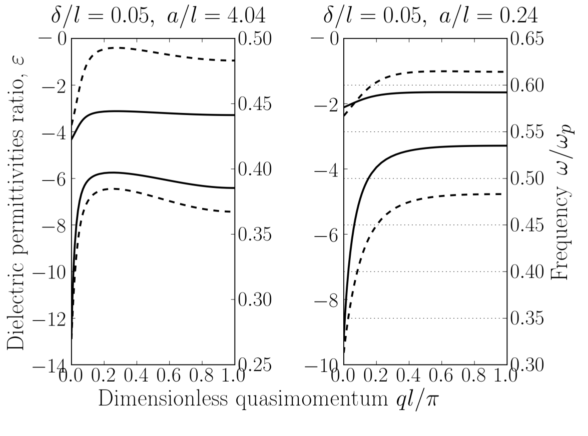

FIG. 2. Dispersion for 1st and 2nd Brillouin zones: solid lines correspond to FIG. 1a), dashed lines to FIG. 1b). We plot the curve in terms of frequency ω (right plot); ω_p is the plasma frequency.

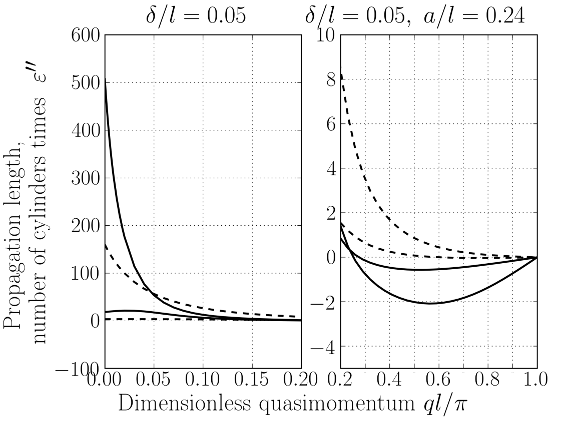

FIG. 3. Propagation length of surface plasmon mode times $\mathrm{Im}\,\varepsilon$.

The dispersion curve is not monotonic, see FIG. 2, thus the region with negative dispersion is presented. The resonance frequency has strong dependence in the region of small quasimomentums, $ql \lesssim \delta/l$, see .Eq. 1. This means, that the modes with relatively long wavelength primarily implement energy transfer. Due to small quasimomentum the modes can be easily excited by incident light. The surface plasmon modes are localized in tranverse direction on the scale of the cylinders diameter in the case of the array. For comparison, the modes in thin metallic layer with the same quasimomentum q are localized at the scale $\sim 1/q$. Another advantage of the modes in the array is large propagation length for modes at small quasimomentums, see FIG. 3.

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Metamaterials with negative refraction and some relativistic effects

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The appearance in the scientific circulation of a new class of objects — metamaterials — not only led to the emergence of new, very attractive applications, but also put the researchers several issues relating to some fundamental problems in physics, in particular relativistic physics. One important question is the direction of the field linear momentum P in the lefthanded material. If the linear momentum of the field in vacuum is determined by the well known relation

$$P = hk$$

and if assume that this relation is valid for the field propagating in the material, it should be recognized, in particular, that the light pressure in the absorption or reflection waves replaced by light attraction [1-2]. However, the situation is complicated by the fact that the equaqtion (1) is not universally recognized, even for the case when the light propagates in a medium with positive k. Indeed, there are two main viewpoints on the relationship between momentum and wave vector. One dependence proposed by Minkowski in his work [3], corresponds to (1) and can be written as

$$P = hk = hn\omega/c = nE/c = E/V_{ph}$$

n is refractive index, ω is the frequency of light, E is its energy, and V_{ph} is the phase velocity. Another form of expression offered by Abraham [4], is

$$P = h\omega V_{gr}/c^2 = EV_{gr}/c^2$$

and corresponds to the link between energy and momentum, which follows from the Lorentz transformations. Expressions (2) and (3), if we neglect the dispersion differs on n^2 times. Based on these expressions we can calculate the value of the mass, m, which is transferred from the transmitter to the receiver of light with energy E [2]. If we take for P the under Minkowski (2), m will be equal

$$m = P/V_{ph}V_{gr}$$

If, however, to recognize, following Abraham, the validity of (3), then we will have a standard expression

$$m = E/c^2$$

From relations (2) and (3) implies that in nature there are two classes of objects with significantly different link between the energy and momentum.

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Additional effective medium parameters for composite materials (excess surface currents)

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The possibility of designing structure and arrangement of the elements in electromagnetic artificial materials permits one to achieve electromagnetic properties different from the properties of constituents (e.g., [1]). Most fully this possibility realizes in metamaterials, in which the interaction of the electromagnetic fields with artificial structural elements is of the resonant nature or / and the solenoidal part of the electric field plays the dominant role in this interaction. As a consequence, metamaterials exhibit advantageous and unusual electromagnetic properties (see e.g. in [2,3]). Nevertheless, it is still desirable to describe the metamaterials as homogeneous ones introducing effective constitutive parameters. Assignment of conventional effective parameters (permittivity, permeability, chirality parameter) to metamaterial samples often produces the values of the parameters whose properties differ from those of any physically possible homogeneous medium. Firstly, the retrieved parameters may depend on the sample size and surrounding environment (see e.g. [4,5]). Secondly, this approach may result in the nonzero imaginary parts of ε and μ in the absence of real dissipation (see e.g. [6]). Thirdly, the sign of these imaginary parts may contradict to the passiveness of the system [7–9] and fourthly, the frequency dispersion of material parameters may violate the causality principle [7–9]. To fix the problems, additional constitutive material parameters are often introduced. This model extension may be determined by new physical phenomena (anisotropy, artificial permeability, chirality, etc.) or by the peculiarities of the homogenization scheme [10–12]. In the latter case the additional parameters not obviously have a clear physical meaning.

Below we show that the key moment of the problems is the boundary conditions. We show that modification of the boundary conditions by introduction of additional ('excess') surface currents can return the conventional permittivity and permeability of metamaterials their usual physical meaning, namely, the modified retrieval procedure yields values of effective impedance and refractive index through the reflection and transmission coefficients, which are independent of the system size and interface structure, whereas the susceptibilities of the excess surface currents depend on the local permittivity distribution near the interface. The accuracy of the latter procedure is the same as that of the quasi-static effective parameters.

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Enhancement of electric field between two metallic granules due to plasmonic resonance

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We study plasmonic modes of pair of metal granules and the response of this system to an external electromagnetic field. It is known that for a single sphere of subwavelength size plasmonic resonance occurs at frequencies corresponding to metal permittivity $\varepsilon \approx -2$ [1]. This condition is independent of sphere radius. Moreover, if a sphere is placed in external electric field (for example in the field of plane wave) the amplitude of the field inside the sphere is $E \sim E_0/\varepsilon''$ where E_0 is the amplitude of the external field and ε'' is the imaginary part of the permittivity. This field enhancement factor is independent of the sphere size and can only be altered by changing the permittivity.

Let us consider a pair of metallic granules separated by the distance δ which is much smaller than the granules size a (Fig. 1) and both a and δ are much smaller than the wavelength. In this system plasmonic modes have properties very different to those for single sphere due to a small separation between the granules. This also makes it impossible to write a perturbation theory, based on the multipole expansion. Nevertheless, the problem can be approached analytically in the quasistationary approximation which is acceptable due to the small size of the system compared to a wavelength. This allows one to find explicitly the resonance values of permittivity and the field distribution. In this system the electric field in the gap between the granules is much larger than the field of the incident wave and the enhancement factor is dependent on the system geometry.

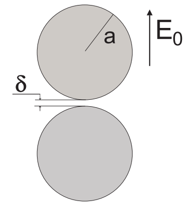

Fig. 1. Pair of metallic granules

The estimation for both resonance permittivity and field enhancement can be obtained from the general physical reasoning. The former one is the condition of the existence of standing wave in a gap of width δ and length $\sqrt{a\delta}$ between metallic surfaces. Thus we estimate:

$$\varepsilon_{res} \sim -\frac{1}{n}\sqrt{\frac{a}{\delta}} \tag{1}$$

where n is an integer number. This expression states that the permittivity is negative and large in value [2], which for good metals mean the shift to the lower frequencies compared to the single granule system [3].

The electric field enhancement factor can be estimated by comparing the energy losses in the system with the power supplied by the external field. This yields:

$$\frac{E_c}{E_0} \sim \frac{1}{\varepsilon''}\left(\frac{a}{\delta}\right)^{3/2} \tag{2}$$

where E_c is the field in the center of the gap. All the estimations are confirmed by a rigorous analytic solution [4, 5].

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Effect of Loss in Dispersion-Engineered Slow Light Waveguides

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Enhanced light-matter interactions in slow light (SL) waveguides present new opportunities for tunable delays, switching, and monitoring of optical pulses in compact photonic structures [1]. Dispersion engineered waveguides in photonic crystals (PhC) and plasmonic structures enable SL regimes with reduced group-velocity dispersion (GVD) away from photonic band edges. Light can be efficiently coupled into such SL waveguides [2], even in the extreme case approaching "frozen light" regime at zero group velocity [3]. Material loss is also enhanced by SL as was shown for strongly band-edge SL with strong GVD [4, 5], and in this work we reveal unique effects of loss in dispersion-engineered SL waveguides with zero GVD.

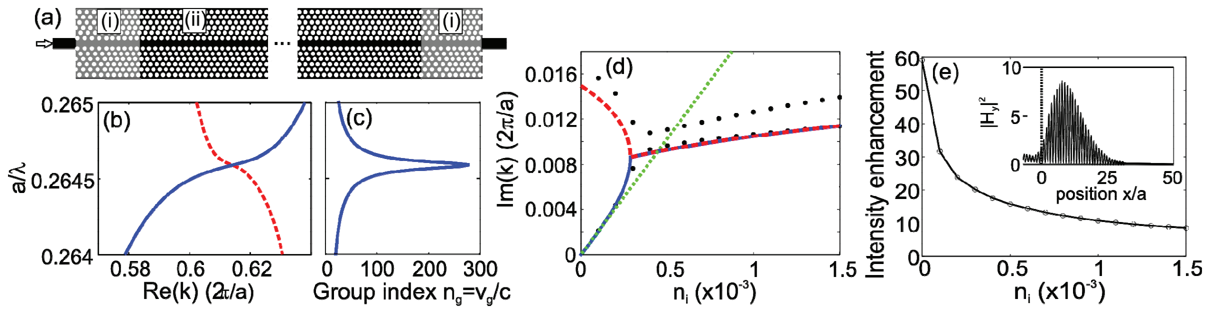

Fig. 1. (a) PhC waveguide geometry consisting of SL section (ii) and fast mode section (i). (b) Re(k) dispersion curves of the propagating (solid) and evanescent (dashed) modes of a lossless SL waveguide with a dispersion inflection point. (c) Group index of the propagating mode. (d) Im(k) vs. n_i at the inflection point frequency $\omega_0 a/2\pi c = 0.264585$. Lines – analytical perturbation theory, black dots – FDTD numerical results. The dotted green line is a linear fit to the propagating mode decay rate when n_i is small. (e) Maximum intensity enhancement in the SL waveguide section vs. n_i. Inset shows normalised magnetic field along the waveguide axis for $n_i = 1.5 \times 10^{-3}$.

We identify the generic effects of material loss for engineered SL waveguides, such as photonic-crystal waveguide shown in Fig. 1(a), with the help of analytical perturbation theory. We consider SL dispersion which, in the absence of losses, features an inflection point associated with the maximum reduction of the group velocity and zero GVD, see Figs. 1(b) and (c). Note that in addition to the propagating mode, such dispersion characteristics of SL waveguide always give rise to a presence of evanescent mode, which real wavenumber dispersion is shown with dashed line in Fig. 1(b). Material loss is introduced as a small imaginary part of the refractive index $n = n_r + in_i$ which perturbs the complex dispersion features of the lossless waveguide. We plot the dependence of the imaginary wavenumber parts Im(k) characterizing attenuation rate for the propagating and evanescent modes vs. material loss n_i in Fig. 1(d). For weak material loss, the decay rate of the original propagating mode scales linearly with n_i and it is enhanced proportionally to the slow-down factor as illustrated with the straight dotted line in Fig. 1(d). However, when material loss exceed a threshold value ($n_i > 2.2 \times 10^{-4}$), the decay rate saturates. This happens due to the hybridization of propagating and evanescent waves as their decay rates become equal to each other, which is a unique feature of dispersion-engineered SL waveguides not possible for the previously considered band-edge SL [4, 5]. We have confirmed the loss saturation behaviour with finite-difference time-domain (FDTD) simulations of a PhC waveguide with loss. The results are plotted as dots in Fig. 1(d), and show excellent qualitative and quantitative agreement with the analytic predictions. Most importantly, the FDTD simulations reveal that the loss saturation enables significant intensity enhancement in the waveguide, even in the high loss regime, as shown in Fig. 1(d).

We anticipate that these effects, predicted on the basis of general perturbation theory, should also occur in dispersion-engineered plasmonic SL waveguides [6] where losses appear due to metal absorption.

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Metamaterials as a platform for active plasmonics

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Guiding and manipulating light on length scales below the diffraction limit requires structural elements with dimensions much smaller than the wavelength. Recently, novel plasmonic metamaterial has been developed based on arrays of aligned gold nanorods grown in self-assembled anodic aluminium oxide templates [1, 2]. This metamaterial provides a flexible platform with tuneable resonant optical properties across the visible spectral range, that can be specifically designed by changing the length, diameter and separation between the nanorods [3]. Such metamaterials, with a controllable and engineered plasmonic response, can be used instead of conventional plasmonic metals for designing plasmonic waveguides [4, 5], plasmonic crystals, label-free bio- and chemo-sensors [6, 7] and for development nonlinear plasmonic structures with the enhanced nonlinearities. The high electromagnetic field confinement and hence enhancement makes these systems ideally suited to the creation of active plasmonic devices by hybridization with nonlinear optical materials [2, 3, 7, 8, 9] or sensing applications which benefit from the large effective surface area in such self-assembled, template based materials.

In this talk we will overview applications of plasmonic plasmonic nanorod metamaterial for designing new types of nanoscale waveguides, biosensing platforms and nonlinear optical devices with improved properties.

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Wave Propagation in Photonic Crystal with Gain

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Recently, a great interest to heterogeneous media, in which the interaction of the electromagnetic fields with constitutive elements is of resonant nature, has emerged [1]. However, in the most suggested applications the main limiting factor is the high level of losses [2]. For the reduction of losses it was proposed to introduce gain into such media [2,3]. The gain is often described as a negative loss, i.e., as a negative imaginary part of permittivity $\operatorname{Im}\varepsilon < 0$.

We discuss the applicability of the concept of negative losses to description of the light propagation in a system with gain at the example of 1D photonic crystal (PC) containing gain layers. It is shown that this approach is in agreement with the causality principle unless the lasing is present. Connecting the start of lasing with the appearance of transfer function pole in the upper half-plane of complex frequency we show that (i) if the pump frequency lies in a pass band then the increase of the number N of elementary cells sooner or later leads to the lasing; (ii) if the frequency of pump lies in the band gap, the lasing at the band-gap frequencies may occur in a sample with a small N while the band gap has not been formed. Nevertheless, the lasing will be necessarily suppressed by further increase of N. Anyway, at sufficiently large number of layers, because of the finite line width of permittivity dispersion, lasing appears in the pass band, even if the pump frequency belongs to the band gap.

Under the linear approximation, both the gain and effect of band gap result in an appearance of the imaginary part of the Bloch wave number k_B, which characterizes the propagation of the eigenwave in the PC. Intuitively, the band gap effect results in the positive value of $\operatorname{Im} k_B$ (Lyapunov factor), whereas the pump results in the negative value of $\operatorname{Im} k_B$. However, due to multiple scattering of waves in a finite slab of PC the competition between the effects is not trivial and it is not easy to find the resulting value of $\operatorname{Im} k_B$. Below the effect of gain and band gap is examined as the dependence of the poles' position on the number of layers and pump intensity.

The increase of pump shifts the poles upwards irrespective of whether they are in the transmission band or in the band gap. This shift is accompanied by the amplification of the transmission resonances, corresponding to the poles. At the threshold value of pump the pole reaches the real frequency axis, and the amplitude of transmission wave becomes infinitely high. When the pump exceeds the lasing threshold, the direct application of the T-matrix method gives such weak fields that linear approximation seems to be valid. Nevertheless, the presence of a pole in the upper half-plain of the complex frequency makes such calculations violating the causality principle. The exponentially growing eigenmode, corresponding to the pole, should be taken into account. It leads to the increase of field that makes the linear theory inapplicable.

The shift of the poles with the increase of the number of cells N in the PC sample depends on the poles' position: in the pass band they move upwards, whereas in the band gap they move downwards at the complex frequency plane. This shift is manifested as an amplification of the transmission resonances in the band and as a suppression of them in the band gap with the increase of N. Thus, the band gap suppresses the lasing.

If the pump frequency corresponds to the band gap, the competition between the wave amplification by the pump and its suppression by the band gap is displayed as follows. At small enough N, the threshold value of pump can produce lasing at the band gap frequencies, and the increase of N leads to the growth of the lasing threshold. With the further increase of N the lasing appears at the transmission band frequencies, and the lasing threshold becomes decreasing.

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