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ABSTRACT

In this paper we propose a novel anisotropic smoothing scheme based on Markov Random Fields (MRF). Our scheme is formulated as two coupled processes. A vertex process is used to smooth the mesh by displacing the vertices according to a MRF smoothness prior, while an independent edge process labels mesh edges according to a feature detecting prior. Since we should not smooth across a sharp feature, we use edge labels to control the vertex process. In a Bayesian framework, MRF priors are combined with the likelihood function related to the mesh formation method. The output of our algorithm is a piecewise smooth mesh with explicit labelling of edges belonging to the sharp features.

Keywords: Mesh, smoothing, Markov Random Fields.

1 INTRODUCTION

Markov Random Fields (MRF) have been used extensively for solving Image Analysis problems at all levels. The local property of MRF makes them very convenient for modeling dependencies of image pixels, and the MRF-Gibbs equivalence theorem provides a joint probability in a simple form, making MRF theory useful for statistical Image Analysis. While some examples are mentioned below, MRF have rarely been used for mesh processing. One reason could be that MRF are usually defined on regular grids, but this is by no means required.

In this paper we demonstrate that feature preserving mesh smoothing may conveniently be cast in terms of MRF theory. Using this theory we can explicitly model our knowledge of properties of the surface (*prior knowledge*, e.g. how smooth the surface should be, which sharp features should it contain) and our knowledge of the noise (*likelihood*, e.g. how far do we believe the measured position of a vertex is likely to be from the true position). The central element of the MRF formulation is that we use Bayes rule to express the probability of any mesh configuration by defining its of prior and likelihood independently. This division of responsibilities often turns out to be a benefit.

For instance, a big advantage of the MRF formulation is that we can use the likelihood to keep the mesh fairly close to the input, avoiding the shrinkage associated with many other schemes. Unlike [Hildebrandt and Polthier, 2007] we do not obtain a hard constraint, but meshes far from the input can be made arbitrarily unlikely by choosing an appropriate likelihood function.

We investigate the use of MRF for formulating priors on 3D surfaces in a number of different ways. The smoothness prior encodes the belief that a smooth surface (according to some fairness criterion) is more probable than a noisy surface. In particular, we show how we can use one MRF to perform explicit labelling of edges according to how sharp they are, and another MRF to find optimal vertex positions according to the smoothness prior. Using our edge labelling from the first MRF to control the vertex smoothing, we are able to recapture very subtle sharp features on the noisy mesh.

2 RELATED WORK

Mesh-smoothing algorithms have a long history in the field of geometry processing since the early work of [Taubin, 1995], which demonstrated the connection between various explicit linear methods using the so called umbrella operator and low pass filtering. In [Desbrun *et al.*, 1999] a discrete Laplace Beltrami operator was introduced and the connection between smoothing and mean curvature flow was explained. Both techniques are efficient, but fail to distinguish between the noise and the features of the underlying object.

To address this problem, anisotropic diffusion [Desbrun *et al.*, 2000] and diffusion smoothing of the normal field [Tasdizen *et al.*, 2002] were proposed. The results are impressive, but the computation complexity puts a limit on the size of the model. More efficient methods were also developed, such as non-iterative feature-preserving smoothing [Jones *et al.*, 2003] based on robust statistics, and an adaptation of bilateral filtering to surface meshes [Fleishman *et al.*, 2003].

Another feature preserving smoothing method, fuzzy vector median smoothing [Shen and Barner, 2004], is a two-step smoothing procedure. In the first step face normals are smoothed using a robust method which employs distance to median normal as smoothing weight. In the next step vertex positions are updated accordingly. More recently, in [Diebel *et al.*, 2006] a Bayesian approach was proposed. This method uses a smoothness prior and the conjugate gradient method for optimization. It is feature-preserving, but without an explicit feature detection scheme. Similar to [Diebel *et al.*, 2006], we use a Bayesian approach, but unlike that method we obtain feature preservation by explicitly detecting the set of chosen features. Our method is also more flexible, allowing us to use a variety of priors and likelihood potentials.

The method for recovering feature edges proposed in [Attene *et al.*, 2005] is based on the dual process of sharpening and straightening feature edges. Vertex-based feature detection using an extension of the fundamental quadric is utilized in a smoothing method described by [Jiao and Alexander, 2005].

Comprehensive study on the use of MRF theory for solving Image Analysis problems can be found in books [Li, 2001; Winkler, 2003]. MRF theory is convenient for addressing the problem of piecewise smooth structures. In [Geman and Geman, 1984] a foundation for the use of MRF in Image Analysis problems is presented in an algorithm for restoration of piecewise smooth images, where gray-level process and line processes are used. Another application of MRF for problems involving reconstruction of piecewise smooth structures is [Diebel and Thrun, 2005], where high-resolution range-sensing images are reconstructed using weights obtained from a regular image.

There are some previous examples of using MRF theory to 3D meshes, but the applications are somewhat different. In [Willis *et al.*, 2004] MRF are used in the context of surface sculpting with the deformation of the surface controlled by MRF potentials modelling elasticity and plasticity. MRF was also used for mesh analysis and segmentation in [Lavoué and Wolf, 2008].

Our work investigates the possibility of formulating surface priors in terms of MRF, and using those priors for reconstructing the surface from the noisy date. Unlike most other mesh smoothing algorithms, our approach does not only preserve sharp ridge features, but also explicitly detects the ridges.

The method described here is not automatic and requires an estimation of a considerable set of parameters. However, this allows a great control over the performance of the priors.

3 MESH SMOOTHING USING MRF

Markov Random Fields is a powerful framework for expressing statistical models originating in computational physics, and it has proven highly successful in Image Analysis [Li, 2001; Winkler, 2003]. A MRF is, essentially, a set of sites with associated labels and edges connecting every site to its neighbors. The labels are the values which we wish to assign (e.g. pixel color, vertex position or edge label), and it is a central idea in MRF theory that the label at a given site must only depend on the labels of its neighbors. This framework lends itself well to mesh based surfaces, where the neighborhood of a vertex can be naturally defined via its connecting edges.

Apart from a well developed mathematical framework one of the main advantages of MRF is that its Markovianity (local property) makes is quite clear what the objective function is and what a MRF based algorithm aims at achieving. Exponential distributions are often used, and the joint probability distribution function of given configuration f (e.g. combined vertex location) is given by

$$P(f) \propto e^{-\sum U(f)} \quad ,$$

where the $U(f)$ can be seen as energy terms or potentials defined on neighborhoods. In order to find the most likely configuration f, we need to obtain

$$\min_f \sum U(f) \quad . \qquad (1)$$

In our proposed framework, we wish to smooth a given mesh. Some of the $U(f)$ in (1) are thus data (likelihood) terms penalizing the displacement of the vertices in the smoothed mesh relative to the original mesh. Other terms would be prior terms which express how likely a surface is *a priori*, i.e. without making reference to how far removed it is from the data.

3.1 Likelihood

We want the output of the smoothing to relate to the input mesh, which has an underlying true surface cor-

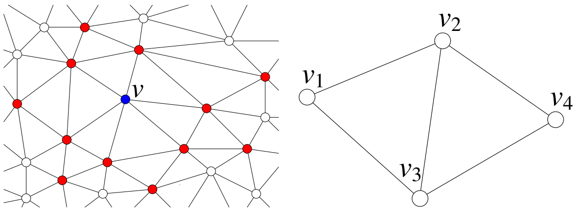

Figure 1: ***Left*: A neighborhood structure for the smoothness prior. The neighbors of the vertex v are marked red. When we move vertex v, we only need to look at its neighboring vertices to calculate the change in the joint smoothness potential. *Right*: A collection of 4 vertices, expressing two adjacent faces.**

rupted by the noise of the data-acquisition device. Assuming isotropic and Gaussian measurement noise we choose quadratic function for the likelihood energy

$$U_L(v) = \alpha \|\mathbf{v}^0 - \mathbf{v}\|^2$$

where $\mathbf{v}^0$ and $\mathbf{v}$ denote the initial and the current position of the vertex v. The constant α is used as the weight determining how much faith one has in the data.

There is always a possibility of plugging in a different likelihood function in our model, e.g. a volume preserving likelihood function or likelihood utilizing some specific knowledge about data acquisition process.

3.2 Smoothing Potential

Alongside the data term we also have some a priori terms expressing our assumptions about how a smoothed mesh should look. Firstly, we have a smoothing potential, which is basically a penalty function, ρ, based on the difference between the normals of adjacent faces, see Figure 1

$$U_s(v_1, v_2, v_3, v_4) = \rho(\mathbf{n}_{123} - \mathbf{n}_{243}) \quad , \qquad (2)$$

where $\mathbf{n}_{123}$ and $\mathbf{n}_{243}$ are the normals of the two adjacent faces. The suitable MRF neighborhood for above formulation is defined as follows: two different vertices are neighbors if they belong to the adjacent faces. In this smoothing scheme the label of each mesh vertex is its spatial position, which is adjusted to minimize the chosen energy function.

The choice of the smoothness potential can greatly influence the feature preserving property of the smoothing. On the one side, there is a over-smoothing quadratic potential developed by [Szeliski and Tonnesen, 1992]

$$\rho(\mathbf{x}) = \|\mathbf{x}\|^2 \quad ,$$

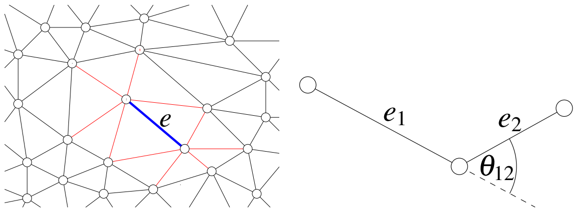

Figure 2: ***Left*: A neighborhood structure for the edge support prior. The neighbors of the edge e are marked red. The neighboring edges support each other if they lie along the same line. *Right*: A pair of edges. The support for the edges e_1 and e_2 depends on the size of the angle θ_{12}.**

on the other side, there is a feature preserving square root potential developed by [Diebel *et al.*, 2006]

$$\rho(\mathbf{x}) = \|\mathbf{x}\| \quad .$$

In our case, feature preservation will be handled by the explicit edge labelling, which allows us to use the aggressive quadratic potential for smooth regions, without thinking about its feature preservation properties.

3.3 Edge Labelling

In many mesh smoothing tasks the presence of clear ridge features in the result is part of our a priori expectation. This is included in our MRF model where we, as an integral part of the smoothing process, label mesh edges as being ridge edges or not. Edge label ε is a number from the interval $[0,1]$ which indicates how probable it is that the given edge is a part of a sharp ridge feature. Those labels will later be used to introduce discontinuities in the smoothing process.

Edge labelling is in itself based on a MRF model consisting of two terms, edge sharpness term U_{E1}, and the neighborhood support term U_{E2}.

The larger the dihedral angle ϕ_e, of a mesh edge is, the more probable it is that the edge lies along the surface ridge. The first term is thus given by

$$U_{E1}(e) = (\phi_0 - \phi_e)\varepsilon \quad , \qquad (3)$$

where ϕ_0 is a ridge sharpness threshold, and ε is the label assigned to the edge e.

The second term of the edge labelling is the neighborhood support, i.e. the presence of other ridge edges along the same ridge line. We assign a support energy to all pairs of edges, see Figure 2. A measure of parallelism between the edges is used in the formulation of the support potential

$$U_{E2}(e_1, e_2) = -\cos(\theta_{12})\varepsilon_1\varepsilon_2 \quad , \qquad (4)$$

where θ_{12} is the angle between the edges e_1 and e_2, and ε_1 and ε_2 are the labels assigned to e_1 and e_2. Feature edges lying on a straight line will have a maximum support, the orthogonal edges do not support each other, and feature edges meeting at a sharp angle are discouraged.

There are additional constrains one can use to define ridge edges, like e.g. dihedral angle changing slowly along the ridge line, or the expectation that the ridge edge itself is smooth.

3.4 The Coupled Model

The smoothing potential and the edge labelling are coupled in a feature preserving scheme, which smoothes the mesh, but not over the edges labelled as sharp. This is obtained by using edge labels as weights for the smoothing potential, which is now, for the setting as in Figure 1

$$U_s(v_1, v_2, v_3, v_4) = (1 - \varepsilon_{23})\rho(\mathbf{n}_{123} - \mathbf{n}_{243}) \ .$$

The edges labelled as sharp with will not contribute to the smoothness potential, and the smoothed surface will be allowed to form a ridge along those edges.

In total, we are minimizing the sum of three terms: the likelihood term, (weighted) smoothing potential, and the edge labelling potential, which in turn consists of the edge sharpness term and neighborhood support term.

3.5 Optimization

At present we use the Metropolis sampler [Winkler, 2003] with simulated annealing for the optimization, i.e. computing a solution to (1). This is a somewhat cumbersome but flexible method, allowing for widespread experimentation with different objective functions. The clear advantage of this approach is that we do not make any assumptions about the potentials.

The Metropolis sampler is a random sampling algorithm, which generates a sequence of configurations from a probability distribution using a Monte Carlo procedure. The sampling scheme consists of randomly choosing a new label for a single site, and replacing the old label with the probability which is controlled by the current temperature. For an initially high temperature, the new configuration can be accepted even if it has a smaller probability that the old one. This allows the algorithm to leave local energy minima. The temperature then gradually decreases and the system converges.

In our case, a new label is either a new vertex position (randomly sampled in the vicinity of the present position), or a new edge label for the ridge detection. Instead of optimizing simultaneously over all defined potentials, we have in each iteration of the optimization process first detected the feature edges (considering vertex positions to be fixed), and than displaced the vertices (considering edge labels to be fixed).

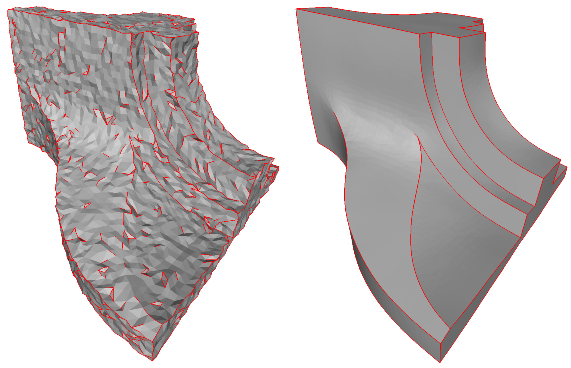

Figure 3: Smoothing fandisk model using our feature preserving method with explicit edge labelling. *Left:* Fandisk model corrupted with the Gaussian noise. Edges are initially labelled based only on the sharpness of the dihedral angle. *Right:* The resulting smooth mesh and the resulting edge labelling.

More specialized and efficient algorithms have been developed for many kind of MRF problems e.g. via filtering, belief propagation and graph cuts (in case of discrete labels). After showing that MRF is a good formulation of the mesh smoothing problem, the search for faster optimization method is part of our ongoing work. A conjugate gradient method would probably provide sufficiently good results in a more efficient way.

4 RESULTS

The results of our experiments prove the feasibility and versatility of using MRF on triangular meshes. Explicit edge labelling when smoothing models with sharp ridge features is shown it the Figure 3. In an initial noisy mesh it is impossible to detect feature edges based only on the local information. However, our algorithm converges to a configuration where all the ridges get correctly labelled and even the subtle feature edges get detected. Correct edge labelling allows us to choose aggressive smoothing prior and obtain results superior to using only a single feature preserving prior, as demonstrated in the Figure 4. Note that, unlike the fuzzy vector median smoothing (which is generally very successful in preserving edges and smooth regions), our method detects and preserves a subtle ridge in the front of the model, and is partly preserving a disappearing ridge close to models back. The most other smoothing methods will either miss those subtle ridges, or will not remove the low frequency noise.

5 DISCUSSION

There are many alternative ways of using MRF on triangle meshes. Instead of labelling vertices with spa-

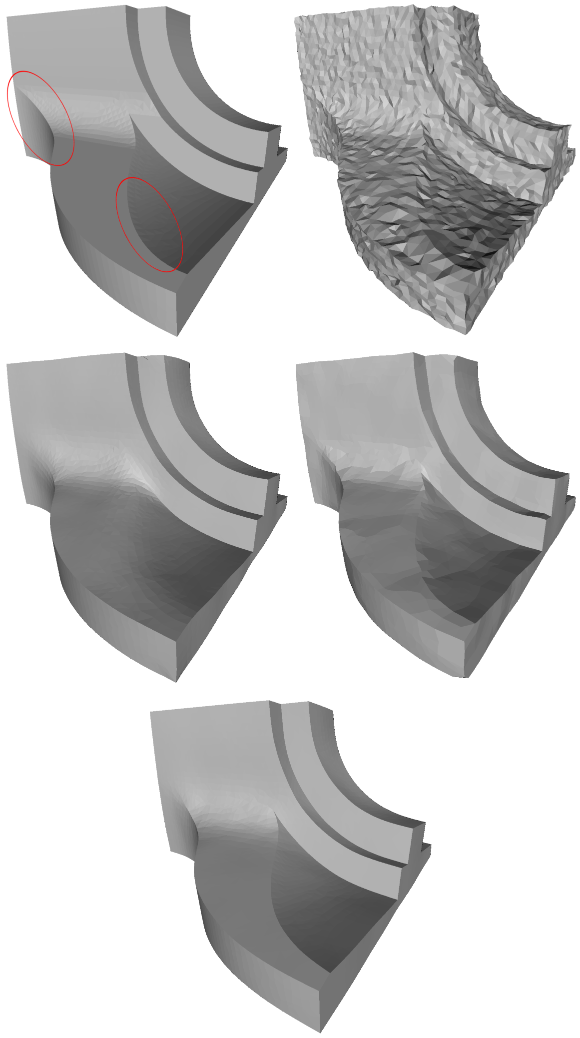

Figure 4: Smoothing fandisk model using the different feature preserving methods. *Top row:* Original model and the model corrupted with the Gaussian noise. The two subtle ridges are circled in the original model. *Middle row:* Results of fuzzy vector median smoothing and MRF smoothing using only the feature preserving square root potential. *Bottom row:* Results of MRF smoothing using the quadratic potential and the explicit edge labelling. Note the preserved subtle ridges.

tial positions, vertex labels can also be used to classify vertices into smooth segments. Furthermore, vertex labels could be used to detect features, classifying the vertices into those that are a part of the smooth surface, those that are on the ridge and vertices that are a corner, in a manner similar to [Lavoué and Wolf, 2008]. MRF can also be defined on mesh faces, either for segmentation or aligning face normals.

Having enough prior knowledge of the problem at hand, one can tailor the surface potentials to obtain the desired result. By including the curvature information

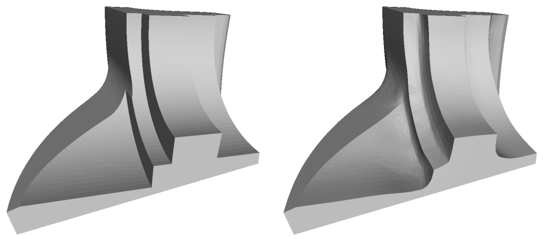

Figure 5: Obtaining curvature clamping by providing curvature information to edge detection process. *Left:* Initial mesh. *Right:* The result of clamping the curvature to discourage the concave sharp ridges.

in the edge labelling process we can detect only certain ridges, while skipping the others, obtaining curvature clamping behavior mentioned in [Botsch *et al.*, 2008] and being the focus of the recent article [Eigensatz *et al.*, 2008], see Figure 5. Extending the size of the vertex neighborhood it is possible to formulate the prior for piecewise quadratic surfaces and also model the ridge behavior more precisely.

To demonstrate the great flexibility and versatility of the MRF formulation we include another example of mesh smoothing. Inspired by a two-step smoothing method [Shen and Barner, 2004], we used MRF to obtain the smooth normal field, which is then used for reconstructing vertex positions. Now we have the mesh faces as the sites of the MRF, with the MRF labels being the normal direction of the faces. The vertex update step is taken directly from [Shen and Barner, 2004], which in turn uses a method developed by [Taubin, 2001] where the system of equations gets solved in a least squares sense to obtain the vertex positions update.

One of the important differences between the vertex based smoothing and face based smoothing is the possibility to preform smoothing of the normals without changing the geometry of the mesh, which makes this approach more effective. The disadvantage is that it is not so straightforward to include displacement-based likelihood function. The results of using this method can be seen on the Figure 6.

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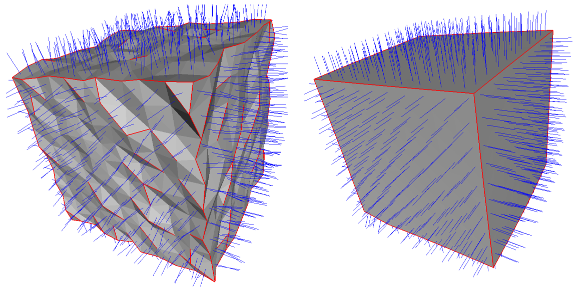

Figure 6: Smoothing a noisy cube using the face and the edge processes. *Left:* A synthetic cube corrupted with Gaussian noise with the initial normal field and the initial edge labelling. *Right:* The resulting mesh, with the smooth normal field and the resulting edge labelling.

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