Modelling Cow Behaviour Using Stochastic Automata

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Report

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Abstract

This report covers an initial study on the modelling of cow behaviour using stochastic automata with the aim of detecting lameness. Lameness in cows is a serious problem that needs to be dealt with because it results in less profitable production units and in reduced quality of life for the affected livestock. By featuring training data consisting of measurements of cow activity, three different models are obtained, namely an autonomous stochastic automaton, a stochastic automaton with coinciding state and output and an autonomous stochastic automaton with coinciding state and output, all of which describe the cows' activity in the two regarded behavioural scenarios, non-lame and lame. Using the experimental measurement data the different behavioural relations for the two regarded behavioural scenarios are assessed. The three models comprise activity within last hour, activity within last hour suplying with information on which hour of the day it is and lastly modelling the general activity level. Diagnosis algorithms for the three approaches are implemented and tested using the real data measurements and show that the diagnosis algorithm can distinguish between data belonging to nominal behaviour and data belonging to lame behaviour.

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1 Introduction

To every responsible farmer the health and well-being of his livestock is importand both in respect to ethics and economics. Ethically it has become more important to treat livestock in a humane way and economically there is a connection between well-being of the livestock and the production. A dairy cow that is feeling unwell will probably eat less and therefore produce less milk[10]. This has the effect that automatic detection of deviant behaviour amongst dairy cows has become a task of growing interest in the farming field. The perspective is to detect deviant behaviour caused by oestrus or some kind of disease. An early detection of a cow in oestrus or a cow suffering from a disease can save the farmer from a loss in production and the animal from a prolonged period of pain/uneasiness.

Lameness is a serious health and wellfare problem in dairy cows and is yet not sufficiently dealt with. It is a term including different illnesses that all have in common that the patients ability to move around is affected. It causes discomfort and is normally some pain as well.

In this report the possibility of using discrete-event models that describe the cows' behaviour for detecting deviant behaviour amongst dairy cows is investigated. The model should describe the cows' behaviour in terms of parameters which change in the presence of lameness. The parameters describing the cow's behaviour are observations of the cow's actions and activity. The observations available for this study include measurements of activity, feeding behaviour and milking behaviour. The available data are described in more detail in chapter 2. The act of assessing the change in behaviour due to lameness involves selecting parameters to analyse and if advantageous to perform data preprocessing and quantisation and to model the behaviour in terms of the pre-processed and quantised observations. An overview of the data processing and modelling is shown in Figure 1.1.

In this study the modelling of cow behaviour is using discrete-event models, in particular the automata, [5, 3]. The automata are means of describing a system where the system inputs, states and reactions can be described by discrete values. By modelling such a system in faulty and faultless mode respectively a fault or a change in the system behaviour can be detected by checking the consistency between the model and the observed behaviour. Lack of consistency is a sign of a fault or a change in the behaviour [3] (see Figure 1.2).

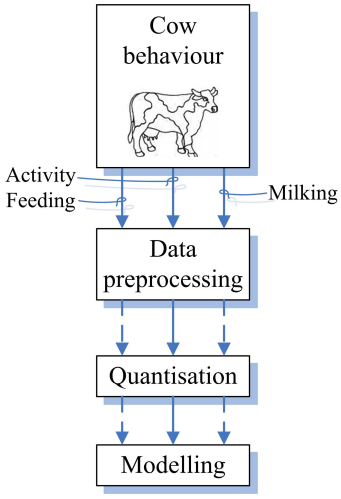


Figure 1.1: Overview of data processing and modelling.

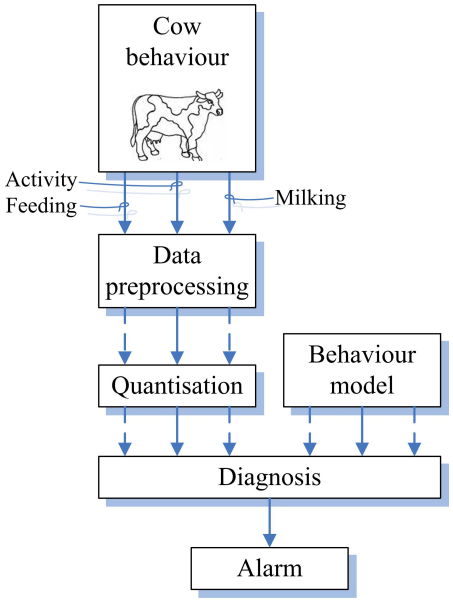


Figure 1.2: Overview of data processing and diagnosis.

1.1 Problem

The task at hand is to detect lameness in dairy cows. A primary goal is to be able to detect whether a cow is lame or not. An excellent result would be a lameness detector that could score the lameness into the lameness levels 1-5 (see section 2.1). This initial study will focus on distinguishing between the two behavioural scenarios *healthy* and

lame. In the literature there exist vast amount of material describing which changes in behaviour are to be expected when a cow becomes lame ([32, 12, 9, 33, 4]). Table 1.1 provides a list of some relevant changes in behaviour as symptoms of lameness.

Table 1.1: Hypotheses about changes in behaviour as symptoms of lameness.

Behaviour	Trait	Change
Activity	walking	−
	lying time	+
Feeding	Duration	−
	time between	+
Milking	milk yield	−
Other	ovarian cycle	delayed cyclicity

1.2 Results on automatic lameness detection in dairy cattle

Automatic lameness detection has been addressed in several studies. There are mainly two approaches that have been used for the detection of lameness namely behaviour assessment and gait assessment. In the first one the focus is on general behaviour in terms of e.g. activity, feeding, milking and so forth. In the latter one attempts are made to assess the cows' gait, i.e. the pattern of movement of the limbs.

1.2.1 Behaviour assessment

In [16] a simple activity detector was set to detect lameness. The authors used pedometer data from a leg attached sensor that reports average number of steps pr. hour since last milking. The authors set the system to daily identify cows that had average number of steps pr. hour during the last day that was 5% less than the average number of steps pr. hour during the preceding 10 days. The authors found that of cows with recorded clinical lameness 55.3% had at least 5% reduction in average number of steps pr. hour. Another alarming result is that 54.3% of the cows that had at least a 5% reduction in average number of steps pr. hour did not develop clinical lameness and are therefore false positives. The system can therefore not be assumed to work sufficiently.

Changes in short-term feeding behaviour of dairy cows in connection with lameness were investigated in [8]. The changes in short-term feeding behaviour were investigated with respect to the applicability as early indicators of the disease. The authors found that daily feeding time was the parameter that changed most consistently with respect to the

different types of lameness studied. A detection algorithm that was set to identify cows with a daily feeding time that was shorter than the average of the daily feeding time during the past seven days minus 2.5 standard deviations was able to detect more than 80% of cows at least one day before manual detection by farm employees.

In [13] a fuzzy logic model for classification of lameness and mastitis in dairy cows was developed. The authors included the traits milk yield dry matter intake, dry matter intake behaviour, water intake, activity and information about preliminary diseases, in their investigations. The best results for lameness detection were obtained using the traits, dry matter intake, feeding time, number of feeding visits activity and preliminary cases of lameness in the actual lactation. The authors reported that the algorithm was able to detect 75% of the lameness cases with an error ratio of 98.3%. Although the sensitivity or error ratio is acceptable the number of false alarms or error ratio is much to high for the algorithm to be considered applicable in real applications.

1.2.2 Gait assessment

When assessing lameness using gate assessment the focus is on detecting deviations in movement pattern, e.g. deviation in step length, attempts to reduce weight on legs and swinging legs while walking.

In [24] and [23] two parallel force plates were used to measure limbs ground reaction forces when cows walked over the plates. The authors utilised different models and came to a conclusion that the system was sufficiently accurate to use in a commercial application. In [24] the authors stated that the system was able to recognise lame cows and identify limbs affected by lameness. The test was performed using only three lame cows and three healthy cows. In [23] the logistic regression models were developed for the detection of lameness using measurement limbs ground reaction forces. The system showed promissing results and the authors claim that the methods could result in automated methods for lameness detection by further development.

In [20] four strain gauge balances installed into a milking robot were used to measure the load of each leg, number of kicks and total time in the milking robot. The authors observed the changes in data and concluded that limb and hoof disorders can be detected using the system. In [19] the authors presented data acquisition and algorithms for detecting leg problems, but stated that there were to many false alarms ([21]). M. E. Pastell and M. Kujala in [21] improved the algorithm by introducing a neural network model for classification of cows into groups of lame and sound cows respectively.

In [6] and [7] cows wearing reflective markers on each leg walked along a 40 m test alley after morning milking for 7 consecutive days and recorded with a video camera. The video recordings were analysed with image processing software and the authors

stated that the method showed distinct differences between cows with no visible hoof pathologies and those with painful injuries but more detailed analysis was needed to decide whether the method was usable for early detection of lameness.

In [26] the authors also used vision techniques to detect and predict lameness in dairy cows. The equipment extracted hoof location from images of cows freely passing a video recording device in a narrow pathway of 9 m length. The method's validity was shown by calculating correlation between the automatically calculated hoof trackway and visual locomation scores. The authors claim that the method has great potential for use in detection and prediction of lameness in dairy cattle.

1.3 Report structure

The report is organised as follows. The second chapter provides an overview of data available for the study. The third chapter contains a description of the methodologies that form the basis of the diagnosis. The fourth chapter addresses the application of the methods on the task of the detection of lameness in dairy cows. The fifth chapter demonstraits some initial tests of the applied algorithms. Lastly the report is finalised with a discussion of future development and a conclusion on the work that has been carried out.

2 Data

This chapter describes the data available in this study for the analysis of the cows' behaviour in connection with the lameness.

The data consist of measurements of activity, feeding behaviour, data from milking robot and manually performed lameness scoring. In addition there is access to all relevant logs on each cow. The logs contain information on diseases, medication, calving, insemination and so on. All data are recorded the research facility at the Danish Cattle Research Centre in Foulum Denmark. Figure 2.1 shows a picture of the stable.

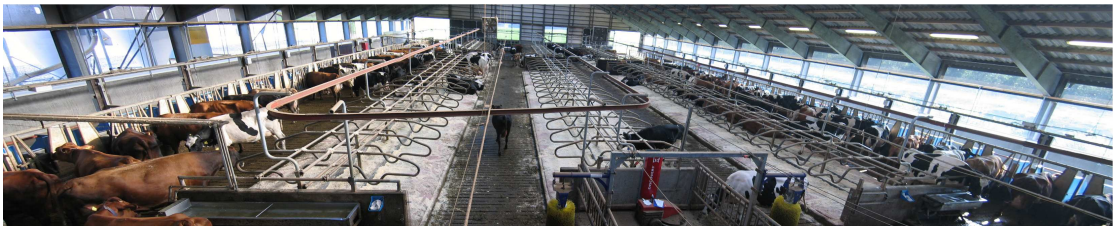


Figure 2.1: The research stable at the Danish Cattle Research Centre.

The two subsections below describe the data used in this particular study, the lameness scoring and the activity data.

2.1 Lameness scoring

The lameness scoring was performed by veterinarians and specially trained personnel at the Danish Cattle Research Centre, with around 2 weeks interval, i.e. sample time $T \approx 2\ weeks$. The scoring is done by visually inspecting each cow for signs of lameness. The cow gets a "score" that describes the cow's physical condition with respect to lameness. The lameness scoring system is described in [28] and a rough description of the system is showed in Table 2.1.

It can be seen in Table 2.1 that having a lameness score ≤ 2 means that the cow is considered not to be affected by lameness while having lameness score > 2 means that the cow is considered to be suffering from lameness.

Table 2.1: Lameness scoring system. An abridged version of that of [28].

Score	Term	Rough description
1	Normal	The cow walks normally. No signs of lameness
2	Uneven gait	The cow walks (almost) normally. No evident signs of lameness
3	Mild lameness	Some signs of lameness. In most cases, an observer is not able to tell which leg is affected
4	Lameness	Obviously lame on 1 or more legs. In most cases, an observer is able to tell which leg is affected
5	Severe lameness	Obviously lame on 1 or more legs. Cow is unwilling to bear weight on the affected leg.

In this study the methods suggested for detecting lameness are methods that are typically used for fault detection, the terms *nominal* and *faulty* will often be used in order to distinguish between data belonging to *non lame* behaviour and data belonging to *lame* behaviour respectively.

2.2 Activity

The activity data consist of measurements of activity on cows in the research stable which is a loose housing with cubicles. The activity is measured by means of commercial activity tags placed on the cows collar. The activity sensors ALPRO® by DeLaval return an activity measurement which consists of an activity index for each hour. Figure 2.2 shows a picture of one activity tag.

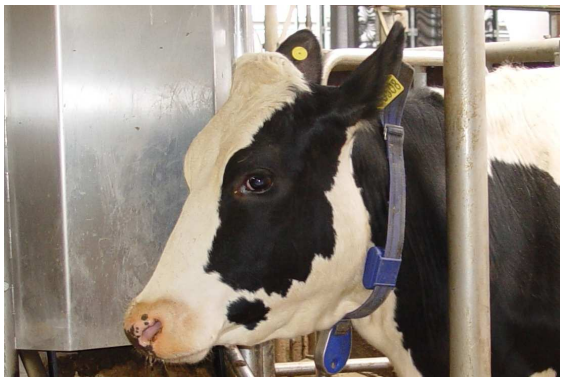


Figure 2.2: The ALPRO® activity tag.

2.3 Feeding

Observations of the cows feeding behaviour are recorded by a number special feeding boxes that identify each cow that puts its head into a feeding box and register time for arrival and departure as well as consumed weight for each visit. Figure 2.3 shows a picture of a feeding box.

Figure 2.3: A feeding box.

2.4 Data from milking robot

The milking robot identifies each cow entering and registers time for arrival and departure as well as amount of milk and a vast number of other parameters related to the milking that are not relevant for a behaviour study.Figure 2.4 shows a picture of a milking robot.

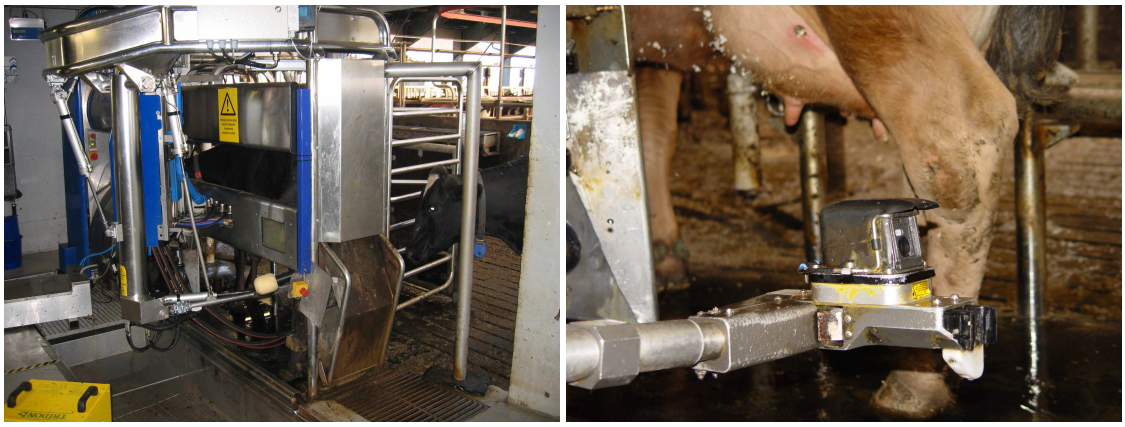


Figure 2.4: A milking robot.

2.5 Selection/generation of test data

The *training dataset* should be as representative for the cows' behaviour at each of the behavioural scenarios, as possible. The aim for the selection of test data for this study is to find the cow, that has the longest consecutive sequence of lameness scoring ≤ 2 *(nominal)* and of lameness scoring > 2 *(faulty)* respectively. That way a reasonable amount of data for training models that reflect the behaviour of each scenario can be obtained. The cow selected for this purpose is cow no. 842. The lameness scoring for cow no. 842 and the interpolated values of that are shown in Figure 2.5.

It can be seen in the data for cow no. 842, in Figure 2.5, that in the beginning there is a period of ~ 8 *months* with observations of lameness scoring ≤ 2 *(nominal)* followed by a period of ~ 2 *months* of changing lameness scoring that again is followed by a period of ~ 2 *months* with lameness scoring > 2 *(faulty)*.

Periods of lameness scoring fluctuating between *nominal* and *faulty* state were removed from the dataset. A plot of the extracted activity data along with interpolated lameness values shown in Figure 2.6.

When looking at the observations of activity index in Figure 2.6 the activity index seems generally higher in the period on nominal behaviour than in the period of faulty behaviour, which is in accordance with the hypotheses presented in Table 1.1. The model to be set up should reflect this property.

Box plots of the activity for lameness score ≤ 2 and lameness score > 2 respectively are shown in Figure 2.7.

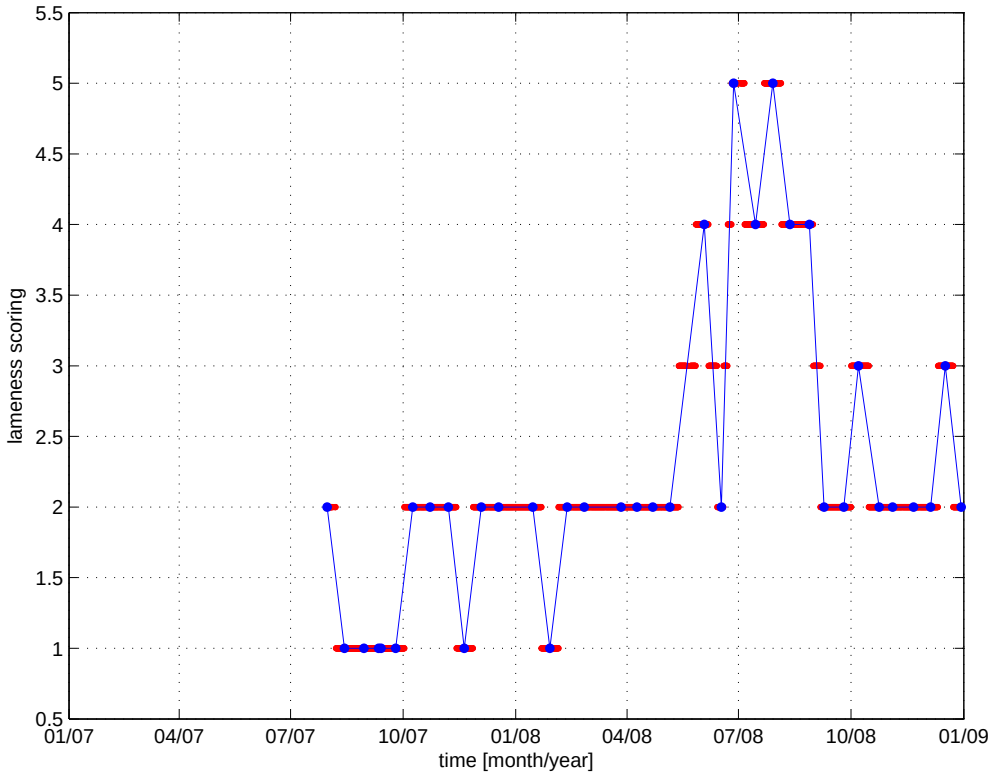


Figure 2.5: Assigned lameness scoring, cow no. 842. The lameness observations are shown with blue dots, the interpolation is shown with blue lines and the assigned lameness scoring is shown in red.

When observing Figure 2.7 one can see that the median values of the faulty behaviour are generally lower than the median values of the nominal case. This is a confirmation that the activity is actually reduced when suffering from lameness seen with respect to the activity under normal circumstances. Another conclusion that can be drawn from the box plots is that there is some sort of a diurnal rhythm in the activity as the medians are generally lower in the time interval $1-7$ *hours* than in the rest of the day. Thus the cow is more active during day than night. These diurnal variations in the activity have been observed earlier in [11].

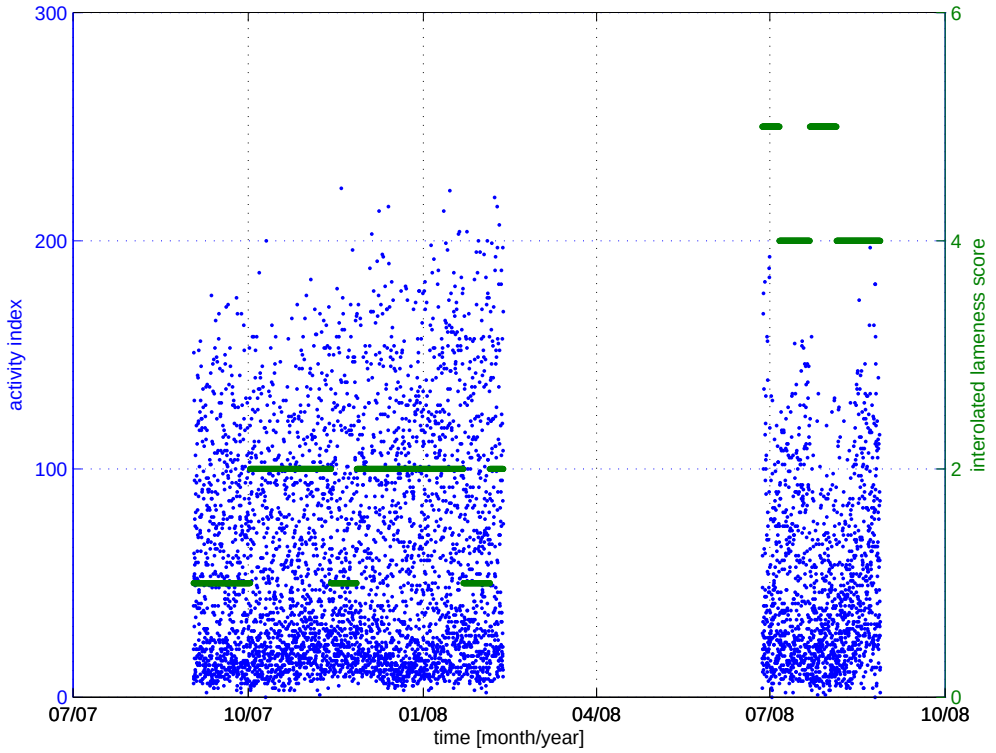
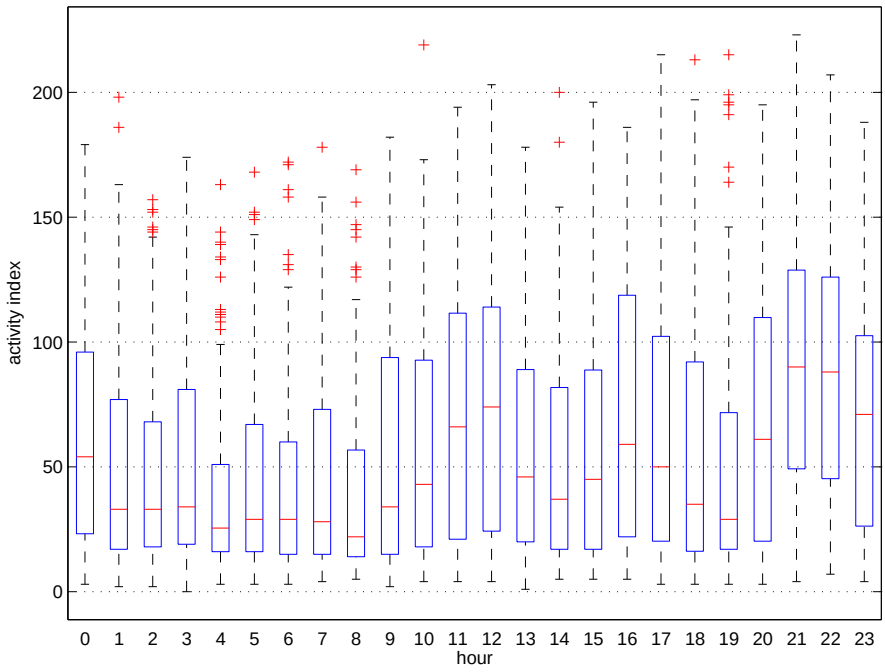
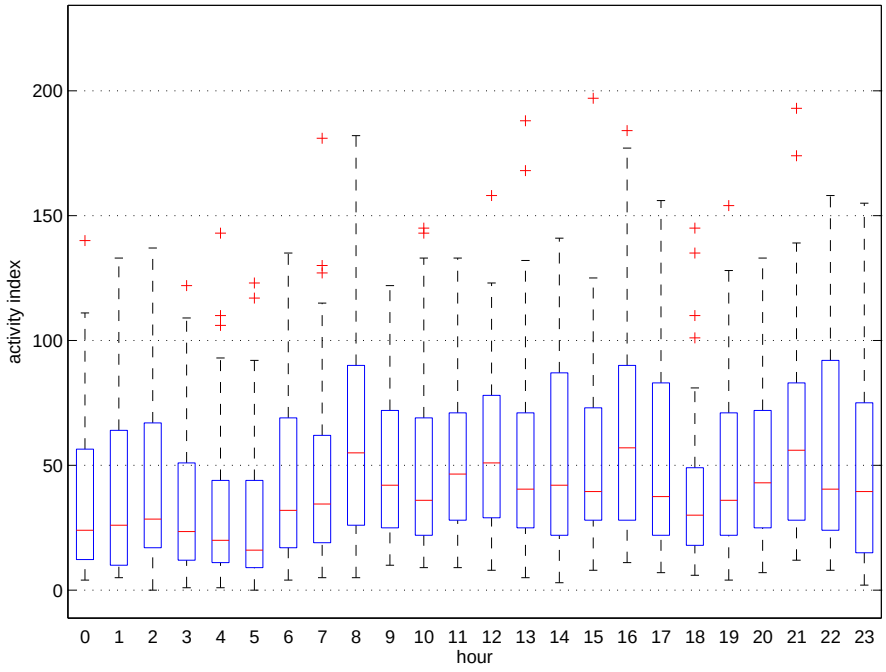


Figure 2.6: The test data; observations of activity index for cow 842 plotted together with interpolated lameness scoring of cow 842.



(a) nominal behaviour (lameness score ≤ 2)



(b) faulty behaviour (lameness score > 2)

Figure 2.7: Box plots of the observed activity for each hour in the day. The central line in each box is the median value of the observed activity index for each hour, the edges of the box are the 25th and 75th percentiles and the stapled lines show the interval where data are not considered as outliers. The individual plus-signs are the outliers.

3 Modelling and diagnosis methods

This chapter describes the methods used for modelling and diagnosis of the cows' activity behaviour with the aim of detecting lameness.

3.1 Model-based diagnosis

Model based diagnosis deals with defining a model of a system containing system components and connections between the components [17]. The task is to compare observations of the system modelled with that of the model. If the two deviate from each other a deviant behaviour is detected. The model can describe a nominal behaviour as well as the faulty behaviour that is supposed to be detected. The approach described in the following comprises model based diagnosis.

3.2 Modelling methods

Ideas for modelling the cows' behaviour include e.g. discrete-event models such as automata, stochastic automata, timed automata, Markov chains, hidden Markov models and etc. Each modelling method has it's degree of relevance with respect to applying to the problem of modelling cow behaviour with the goal of detecting lameness. A range of methods can be utilised for modelling a system behaviour [31, 29, 30]. If dealing with a system obeying known laws of physics an analytical quantitative model might possibly be set up. In total lack of information about behavioural structure one might fall back to solely relying on history. An in-between approach it to set up a qualitative model for describing the behaviour where the focus is on relevant changes instead of simply any change. Therefore among the decisions that have to be addressed is whether to model the cows' behaviour in a qualitative or quantitative way.

In general quantitative models describe a system in a more specific and precise manner than qualitative models. At the same time modelling a system with a quantitative approach can require a large effort. When attempting to model the cows' behaviour with the aim of detecting deviant behaviour and judging whether to use quantitative or

qualitative modelling one should bare in mind what is expected of the model and which are the desired results. In the case of modelling the cows' behaviour in order to detect lameness the model is really supposed to be able to indicate whether the behaviour is following an expected pattern or not. If the behaviour is following the expected pattern the cow is considered healthy. In this case it is not consider important to assess the exact state but merely if everything is within reasonable limits or not. As the lameness is expected to result in considerable changes in the behaviour a qualitative modelling approach is justifyable.

The qualitative models used in this study for modelling the cow behaviour are the automata. In the following subsections the automata and the diagnosis of automata are described.

3.3 Automata

The automata are a class of models suitable for describing discrete-event systems. Various versions of the automata are described in the literature, which includes e.g. standard automata [5], input/output automata (I/O automata) [15], learning automata [22] and so on. The version of automata applied in this study is the I/O automata.

In general the automaton describes the system's state and takes into consideration events that affect or are affected by the system.

Modelling a system using I/O automata can be done using deterministic automata, non-deterministic automata, stochastic automata or even timed automata (timed automata are addressed in e.g. [27] and [1]). The methods for the usage of of the deterministic automata, the non-deterministic automata and the stochastic automata are well established and are described in e.g. [3], [14] and [25]. Sections 3.3.1−3.3.3 describe briefly the deterministic automata, the non-deterministic automata and the stochastic automata. The timed automata are not addressed in this study.

3.3.1 Deterministic automata

Introducing a system described by inputs v outputs w and states z described by the sets

$$\begin{aligned} v \in &\quad \mathcal{N}_v = \{1, 2, \ldots, M\} \\ z \in &\quad \mathcal{N}_z = \{1, 2, \ldots, N\} \\ w \in &\quad \mathcal{N}_w = \{1, 2, \ldots, R\} \end{aligned} \tag{3.1}$$

where M, N and R are finite values a deterministic automata for such a system has the form

$$\mathcal{A} = (\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, G, H, z_0) \tag{3.2}$$

where G and H are the state transition function and the output function ([3]). The state transition function G and the output function H are given as

$$\begin{aligned} G &: \mathcal{N}_z \times \mathcal{N}_v \to \mathcal{N}_z, \qquad z' = G(z, v) \\ H &: \mathcal{N}_z \times \mathcal{N}_v \to \mathcal{N}_w, \qquad w = H(z, v) \end{aligned} \tag{3.3}$$

where z is the present state and z' is the successor state ([3]). The deterministic automaton has the properties that given an input v and state z the successor state z' is always known. Thus the state transition function describes the next state z' given the present state z and the input v. Similarly the output function describes the output generated during a transition.

3.3.2 Non-deterministic automata

A non-deterministic automaton for the system in (3.1) has the form

$$\mathcal{N} = (\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, L_n, z_0) \tag{3.4}$$

where L_n describes the behavioural relation. The behavioural relation L_n is given as

$$L_n : \mathcal{N}_z \times \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_v, \quad L_n(z', w, z, v) \to \{0, 1\} \tag{3.5}$$

and describes whether the state can change from state z to z' while producing the output w for the input v ([3]). The non-deterministic automaton has the properties that given an input v and state z the successor state z' belongs to a set of possible states that the automaton can move towards while producing the output w.

3.3.3 Stochastic automata

The stochastic automaton is for the system in (3.1) described by the 5-tuple

$$\mathcal{S} = \langle \mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, L, \mathrm{Prob}(z(0)) \rangle \tag{3.6}$$

where $\mathrm{Prob}(z(0))$ is the initial state probability distribution.

The stochastic automaton behaviour is described by its behavioural relation L.

$$L : \mathcal{N}_z \times \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_v \rightarrow [0, 1] \tag{3.7}$$

$$L\left(z', w, z, v\right) = \text{Prob}(z_p(k+1) = z', w_p(k) = w | z_p(k) = z, v_p(k) = v) \tag{3.8}$$

where z_p, w_p and v_p denote the stochastic variables of the state, output and input respectively and k is the sample instance. The stochastic automaton has the properties that given an input v and state z the successor state z' belongs to a set of possible states that the automaton can move towards with a certain probability, described by L, while producing the output w. For every state z and input v the sum of probabilities for moving towards any successor state $z' \in \mathcal{N}_z$ while producing the an output $w \in \mathcal{N}_w$ is equal to one. Thus

$$\sum_{z' \in \mathcal{N}_z} \sum_{w \in \mathcal{N}_w} L\left(z', w | z, v\right) = 1. \tag{3.9}$$

The state transition relation is given as,

$$\begin{aligned} &G : \mathcal{N}_z \times \mathcal{N}_z \times \mathcal{N}_v \rightarrow [0, 1], \\ &G(z'|z, v) = \text{Prob}\left(z_p(1) = z' | z_p(0) = z, v_p(0) = v\right), \end{aligned} \tag{3.10}$$

and the output relation

$$\begin{aligned} &H : \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_v \rightarrow [0, 1], \\ &H(w|z, v) = \text{Prob}\left(w_p(0) = w | z_p(0) = z, v_p(0) = v\right). \end{aligned} \tag{3.11}$$

The relationship between the behavioural relation and the state transition relation and the output relation can be seen from the boundary distributions,

$$\begin{aligned} G(z'|z, v) &= \sum_{w \in \mathcal{N}_w} L\left(z', w | z, v\right), \\ H(w|z, v) &= \sum_{z' \in \mathcal{N}_z} L\left(z', w | z, v\right). \end{aligned} \tag{3.12}$$

It should be mentioned that *only in special cases* does the relation

$$L\left(z', w | z, v\right) = G(z'|z, v) \cdot H(w|z, v) \tag{3.13}$$

hold.

Since the "system" at hand, the cow, is biological, a deterministic or a non-deterministic automaton alone will not provide the necessary level of detail to distinguish between quite similar stochastic behavioural patterns of live creatures with a wide variation between cows. A stochastic automaton extends the concept of the non-deterministic discrete-

event systems in such a way that the frequency of the occurrence of the different events can be addressed by assigning probabilities to transitions. Probabilities are a means to describe the cows in more details. Therefore the stochastic automaton yields more knowledge about the system and makes diagnosis more likely to be successful. It is therefore the stochastic automaton that is used in the following modelling efforts. In the study there are three types of the stochastic I/O automata used; an autonomous stochastic automaton, a stochastic automaton where output coincides with states and an autonomous stochastic automaton where output coincides with states. All three are described in the following subsections.

Autonomous stochastic automaton

The autonomous stochastic automaton does not use any input v but is otherwise no different from the automaton in eq. (3.6), hence the autonomous stochastic automaton is described by the 4-tuple

$$\mathcal{S} = \langle \mathcal{N}_z, \mathcal{N}_w, L, \text{Prob}(z(0)) \rangle \tag{3.14}$$

where like before L is the behavioural relation. The behavioural relation of the autonomous stochastic automaton is given as

$$L\left(z', w|z\right) = \text{Prob}\left(z_p(k+1) = z', w_p(k) = w|z_p(k) = z\right). \tag{3.15}$$

Stochastic automaton with coinciding states and outputs

The stochastic automaton with coinciding states and outputs and is described by the 4-tuple

$$\mathcal{S} = \langle \mathcal{N}_z, \mathcal{N}_v, G, \text{Prob}(z(0)) \rangle . \tag{3.16}$$

Omitting the output from the case described in section 3.3.3 leaves only the state transition relation in eq (3.10) as the behavioural relation for this automaton. For the state transition relation the relation

$$\sum_{z' \in \mathcal{N}_z} G\left(z'|z, v\right) = 1 \; for \; all \; z \in \mathcal{N}_z, v \in \mathcal{N}_v \tag{3.17}$$

holds ([3]).

Autonomous stochastic automaton with coinciding output and state

The autonomous stochastic automaton with coinciding states and outputs is described by the triple

$$\mathcal{S} = \langle \mathcal{N}_z, G, \text{Prob}(z(0)) \rangle \,. \tag{3.18}$$

Omitting the input the state transition relation becomes

$$G(z'|z) = \text{Prob}\left(z_p(1) = z'|z_p(0) = z\right) \tag{3.19}$$

and describes the probability of moving from one state to another. The transition relation has the property

$$\sum_{z' \in \mathcal{N}_z} G\left(z'|z\right) = 1 \; for \; all \; z \in \mathcal{N}_z. \tag{3.20}$$

3.3.4 Prediction

A stochastic automaton can be used to imitate the behaviour. According to page 344 in [3] the behaviour of the stochastic automaton is given as

$$\text{Prob}\left(Z(0 \ldots k_h)|V(0 \ldots k_h - 1)\right) = \prod_{k=0}^{k_h-1} G\left(z(k+1)|z(k), v(k)\right) \cdot \text{Prob}\left(z(0)\right) \tag{3.21}$$

where $\text{Prob}\left(Z(0 \ldots k_h)|V(0 \ldots k_h - 1)\right)$ is the probability distribution over all state sequences $Z(0 \ldots k) = (z(0), z(1), \ldots, z(k))$ for a given input sequence $V(0 \ldots k) = (v(0), v(1), \ldots, v(k))$.

The stochastic automaton can be simulated using the recursive algorithm that is on on page 385 in [3] given as

$$\begin{aligned} k_h > 1: \;& \text{Prob}\left(z(k_h)|V(0 \ldots k_h - 1)\right) \\ & = \sum_{z(k_h-1)} G\left(z(k_h)|z(k_h - 1), v(k_h - 1)\right) \cdot \text{Prob}\left(z(k_h - 1)|V(0 \ldots k_h - 2)\right) \end{aligned}$$

$$k_h = 0: \; \text{Prob}\left(z(1)|v(0)\right) = \sum_{z(0)} G\left(z(1)|z(0), v(0)\right) \cdot \text{Prob}\left(z(0)\right) \tag{3.22}$$

and can be used to calculate the probability of the automaton moving towards state z at sample k_h given the input sequence $V\left(0 \ldots k_h - 1\right)$.

For simulating the stochastic automaton with coinciding states and outputs the recursive algorithm in eq. (3.22) can be used directly.

For simulating the autonomous stochastic automaton and the autonomous stochastic automaton with coinciding states and outputs the recursive algorithm in eq. (3.22) can be used by omitting the input. It therefore changes to

$$\begin{aligned} k_h > 1: \quad & \mathrm{Prob}\left(z(k_h)\right) = \sum_{z(k_h-1)} G\left(z(k_h)|z(k_h-1)\right) \cdot \mathrm{Prob}\left(z(k_h-1)\right), \\ k_h = 0: \quad & \mathrm{Prob}\left(z(1)\right) = \sum_{z(0)} G\left(z(1)|z(0)\right) \cdot \mathrm{Prob}\left(z(0)\right). \end{aligned} \tag{3.23}$$

3.3.5 State observation

For obtaining a probability distribution for the present state the automaton can be observed. The observation problems deals with estimating which state the model is in at a given time instance, given measurement sequences of inputs and outputs and the automaton. This becomes particularly relevant in the diagnosis which is described in section 3.3.6. Assuming a consistent I/O pair (see page 391 in [3]) the state observation is given as

$$\begin{aligned} &\mathrm{Prob}\left(Z(0\ldots k_h)|V(0\ldots k_h-1), W(0\ldots k_h-1)\right) \\ &= \frac{\sum\limits_{z(k_h+1)} L(z(k_h+1), w(k_h)|z(k_h)) \cdot \ldots \cdot (z(2), w(1)|z(1)) \cdot (z(1), w(0)|z(0)) \cdot \mathrm{Prob}\left(z(0)\right)}{\sum\limits_{Z(0\ldots k_h+1)} L(z(k_h+1), w(k_h)|z(k_h)) \cdot \ldots \cdot (z(2), w(1)|z(1)) \cdot (z(1), w(0)|z(0)) \cdot \mathrm{Prob}\left(z(0)\right)}, \end{aligned} \tag{3.24}$$

which only deviates from the solution on page 392 in [3] due to the omission of the input v. For practical computational purposes the recursive solution to the state observation

is important. The recursive solution to the state observation problem is given as

$$k_h \geq 0: \ \text{Prob}\left(z(k_h)|k_h\right) = \frac{\sum\limits_{z(k_h+1)} L(z(k_h+1), w(k_h)|z(k_h)) \cdot \text{Prob}\left(z(k_h)|k_h-1\right))}{\sum\limits_{z(k_h),z(k_h+1)} L(z(k_h+1), w(k_h)|z(k_h)) \cdot \text{Prob}\left(z(k_h)|k_h-1\right)},$$

$$k_h > 0: \ \text{Prob}\left(z(k_h)|k_h-1\right) = \frac{\sum\limits_{z(k_h-1)} L(z(k_h), w(k_h-1)|z(k_h-1)) \cdot \text{Prob}\left(z(k_h-1)|k_h-2\right)}{\sum\limits_{z(k_h),z(k_h-1)} L(z(k_h), w(k_h-1)|z(k_h-1)) \cdot \text{Prob}\left(z(k_h-1)|k_h-2\right)}, \tag{3.25}$$

$$k_h = 0: \ \text{Prob}\left(z(0)|-1\right) ::= \text{Prob}\left(z(0)\right).$$

As before this is almost identical to the recursive solution on page 396 in [3]. The only difference lies in the omission of the input v.

3.3.6 Diagnosis

For identifying faults or abnormal behaviour in a system modelled by an automaton the consistency between the automaton and the actual behaviour is checked[3]. Lack of consistency implies that the observed behaviour is not described by the automaton. By modelling a specific behaviour to be detected and checking the consistency between the model describing the deviant behaviour and the actual behaviour a specific deviant behaviour can be detected. If the observed behaviour is not consistent with the normal case and at the same time there exists consistency with the model of the specific deviant behaviour the specific behaviour is isolated.

Stochastic automata for diagnosis

As mentioned above deviant behaviour can be detected by checking the consistency between the measured behaviour and models of the faulty behaviour. To be able to include different behavioural scenarios of faults in the stochastic automaton it is extended with a new input $f(k)$ which symbols the behavioural scenario at each sample k. The stochastic automaton for diagnosis is described by the 6-tuple

$$\mathcal{S} = \langle \mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_f, \mathcal{N}_w, L, \text{Prob}(z(0)) \rangle \tag{3.26}$$

with $\mathcal{N}_f$ denoting the set of possible behavioural scenarios. Correspondingly the behavioural relation now becomes

$$\begin{aligned} &L : \mathcal{N}_z \times \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_f \times \mathcal{N}_v \to [0,1]\,, \\ &L\left(z', w|z, f, v\right) = \text{Prob}(z_p(k+1) = z', w_p(k) = w|z_p(k) = z, f_p(k) = f, v_p(k) = v). \end{aligned} \tag{3.27}$$

The dynamical behaviour for the fault is introduced by the *fault model* which is a stochastic automaton described by the triple

$$\mathcal{S}_f = \langle \mathcal{N}_f, \mathcal{G}_f, \text{Prob}(f(0)) \rangle \tag{3.28}$$

where G_f is the state transition relation of the fault ([3]). The fault state transition relation G_f describes the conditional probability of the fault changing from f to f' in the time between two consecutive samples. The fault state transition relation is given as

$$\begin{aligned} &G_f : \mathcal{N}_f \times \mathcal{N}_f \to [0,1] \\ &G_f(f'|f) = \text{Prob}\left(f_p(k+1) = f'|f_p(k) = f\right). \end{aligned} \tag{3.29}$$

Combining the fault model with the extended automaton in eq. (3.27) gives the stochastic automaton

$$\tilde{\mathcal{S}} = \left\langle \mathcal{N}_{\tilde{z}}, \mathcal{N}_v, \mathcal{N}_f, \mathcal{N}_w, \tilde{L}, \text{Prob}(\tilde{z}(0)) \right\rangle \tag{3.30}$$

where the state set is (page 388 in[3])

$$\mathcal{N}_{\tilde{z}} = \mathcal{N}_z \times \mathcal{N}_f. \tag{3.31}$$

The behavioural relation of the combined automaton becomes

$$\tilde{L}(z', f', w|z, f, v) = L(z', w|z, f, v) \cdot G_f(f'|f). \tag{3.32}$$

Diagnosis of the stochastic automata

The task of diagnosing this combined automaton becomes an observation problem where the task is to observe the present *state* z together with the present *fault state* f, given the combined automaton and measurements of input v and output w. A solution to the

diagnostic problem is on page 418 in [3] given as

$$\begin{aligned}
k_h \geq 0: \;& \operatorname{Prob}\left(f(k_h)|k_h\right)) \\
&= \frac{\sum\limits_{\substack{f(k_h+1) \\ z(k_h+1),\, z(k_h)}} L(k_h) \cdot G_f(k_h) \cdot \operatorname{Prob}(f(k_h), z(k_h)|k_h - 1))}{\sum\limits_{\substack{f(k_h),\, f(k_h+1) \\ z(k_h),\, z(k_h+1)}} L(k_h) \cdot G_f(k_h) \cdot \operatorname{Prob}(f(k_h), z(k_h)|k_h - 1))}, \\
k_h > 0: \;& \operatorname{Prob}\left(f(k_h), z(k_h)|k_h - 1)\right) \\
&= \frac{\sum\limits_{f(k_h-1), z(k_h-1)} L(k_h - 1) \cdot G_f(k_h - 1) \cdot \operatorname{Prob}(f(k_h - 1), z(k_h - 1)|k_h - 2))}{\sum\limits_{\substack{f(k_h),\, f(k_h-1) \\ z(k_h),\, z(k_h-1)}} L(k_h - 1) \cdot G_f(k_h - 1) \cdot \operatorname{Prob}(f(k_h - 1), z(k_h - 1)|k_h - 2))}, \\
k_h = 0: \;& \operatorname{Prob}\left(f(0), z(0)| - 1\right) := \operatorname{Prob}\left(f(0)\right) \cdot \operatorname{Prob}\left(z(0)\right).
\end{aligned} \tag{3.33}$$

given that the relation

$$\sum_{f(k_h-1), z(k_h-1)} L(k_h - 1) \cdot G_f(k_h - 1) \cdot \operatorname{Prob}(f(k_h - 1), z(k_h - 1)|k_h - 2)) > 0 \tag{3.34}$$

holds and using the abbreviations $\operatorname{Prob}(f|k_h) = \operatorname{Prob}(f|V(0 \ldots k_h), W(0 \ldots k_h))$, $L(k_h) = L(z(k_h + 1), w(k_h)|z(k_h), v(k_h))$ and $G_f(k_h) = G_f(f(k_h + 1)|f(k_h))$.

Diagnosis of the autonomous stochastic automaton

For diagnosing the autonomous stochastic automata relations described in eq. (3.26) to (3.34) are valid by omitting the input $v(k)$. The same way, the diagnosis algorithm on page 420 in [3] can be used directly by simply omitting the input v.

Diagnosis of the stochastic automaton with coinciding states and outputs

The procedure for diagnosing the stochastic automaton with coinciding states and outputs is similar to what is described above. As done in the case of the stochastic automaton the stochastic automaton with coinciding states and outputs is extended with a new input $f(k)$. The automaton is therefore now described by the 4-tuple

$$\mathcal{S} = \langle \mathcal{N}_z, \mathcal{N}_f, \mathcal{N}_v, G, \operatorname{Prob}(z(0)) \rangle . \tag{3.35}$$

This automaton has the transition relation

$$G(z'|z,f,v) = \text{Prob}\left(z_p(1) = z'|z_p(0) = z, f_p(0) = f, v_p(0) = f\right). \tag{3.36}$$

The *fault model* is the same as presented before in eq. (3.28) Combining the automaton in eq. (3.35) with the fault model in eq. (3.28) results in the 4-tuple

$$\tilde{\mathcal{S}} = \left\langle \mathcal{N}_{\tilde{z}}, \mathcal{N}_v, \tilde{G}, \text{Prob}(\tilde{z}(0)) \right\rangle, \tag{3.37}$$

with the state set given in (3.31). The state transition relation of this combined automaton becomes

$$\tilde{G}(z', f'|z, f, v) = G(z'|z, f, v) \cdot G_f(f'|f). \tag{3.38}$$

Diagnosis for a model like the one in eq. (3.37) involves an observation of the state and the fault. In the case of the automaton with coinciding states and outputs and measurable state the diagnosis task becomes a task of observing the fault, but not the state. Applying theorem 8.3 on page 396 in [3] and keeping in mind that $z(k_h)$ is measured, the diagnosis algorithm becomes.

$$\begin{aligned}
&k_h \geq 0: \quad \text{Prob}\left(f(k_h)|k_h\right)) \\
&= \frac{\sum\limits_{\substack{f(k_h+1)\\ z(k_h+1)}} G(z(k_h+1)|z(k_h), f(k_h), v(k_h)) \cdot G_f(f(k_h+1)|f(k_h)) \cdot \text{Prob}(f(k_h), z(k_h)|k_h-1))}{\sum\limits_{\substack{f(k_h)\\ f(k_h+1)\\ z(k_h+1)}} G(z(k_h+1)|z(k_h), f(k_h), v(k_h)) \cdot G_f(f(k_h+1)|f(k_h)) \cdot \text{Prob}(f(k_h), z(k_h)|k_h-1))},
\end{aligned}$$

$$\begin{aligned}
&k_h > 0: \quad \text{Prob}\left(f(k_h), z(k_h)|k_h-1)\right) \\
&= \frac{\sum\limits_{f(k_h-1)} G(z(k_h)|z(k_h-1), f(k_h-1), v(k_h-1)) \cdot G_f(f(k_h)|f(k_h-1)) \cdot \text{Prob}(f(k_h-1), z(k_h-1)|k_h-2))}{\sum\limits_{\substack{f(k_h)\\ f(k_h-1)\\ z(k_h)}} G(z(k_h)|z(k_h-1), f(k_h-1), v(k_h-1)) \cdot G_f(f(k_h)|f(k_h-1)) \cdot \text{Prob}(f(k_h-1), z(k_h-1)|k_h-2))},
\end{aligned}$$

$$k_h = 0: \quad \text{Prob}\left(f(0), z(0)|-1\right) ::= \text{Prob}\left(f(0)\right) \cdot \text{Prob}\left(z(0)\right). \tag{3.39}$$

For implementation purposes, Algorithm 3.1 was derived. Algorithm 3.1 uses the same notation and setup as Algorithm 8.3 on page 420 in [3].

Algorithm 3.1 Diagnosis of stochastic automaton with coinciding states and inputs *(Based on Algorithm 8.3 on page 420 in [3])*

Given: autonomous stochastic automaton with coinciding states and inputs $\mathcal{S}$ and fault model $\mathcal{S}_f$
Initial state probability distribution $\text{Prob}\left(\hat{z}(0)\right)$
Initial fault probability distribution $\text{Prob}\left(\hat{f}(0)\right)$

Initialisation: $p_r(f,z) = \text{Prob}\left(\hat{f}_p(0) = f\right) \cdot \text{Prob}\left(\hat{z}_p(0) = z\right)$ for all $f \in \mathcal{N}_f$ and $z \in \mathcal{N}_z$
$k_h = 0$

Loop:

1. Measure the current state z_m and input v.
2. For all $f \in \mathcal{N}_f$ determine $h(f, z_m) = \sum_{\bar{f},\bar{z}} G(\bar{z}|z_m, f, v) \cdot G_f(\bar{f}|f) \cdot p_r(f, z_m)$
3. If $\sum_f h(f, z_m) = 0$ holds, stop the algorithm as the measured state sequence is inconsistent with the automaton behaviour.
4. For all $f \in \mathcal{N}_f$ determine $p_k(f, z_m) = \frac{h(f,z_m)}{\sum_f h(f,z_m)}$.
5. For all $f \in \mathcal{N}_f$ and $z \in \mathcal{N}_z$ determine $p_r(f, z) = \frac{\sum_{\bar{f}} G(z|z_m,f,v) \cdot G_f(f|\bar{f}) \cdot p_r(f,z_m)}{\sum_f h(f,z_m)}$.
6. Determine $\text{Prob}\left(f(k_h) = f|k_h\right) = p_k(f, z_m)$
7. $k_h = k_h + 1$
   Continue with step 1.

Result: $\text{Prob}\left(f(k_h) = f|k_h\right)$

Diagnosis of the autonomous stochastic automaton with coinciding states and outputs

The autonomous stochastic automaton with coinciding states and outputs is identical with the automaton shown in eq. (3.35) apart from omission of the input v. The automaton is therefore now described by the triple

$$\mathcal{S} = \langle \mathcal{N}_z, \mathcal{N}_f, G, \text{Prob}(z(0)) \rangle \,. \tag{3.40}$$

This automaton has the transition relation

$$G(z'|z, f) = \text{Prob}\left(z_p(1) = z'|z_p(0) = z, f_p(0) = f\right). \tag{3.41}$$

The diagnosis of the autonomous stochastic automaton with coinciding states and outputs therefore becomes almost identical to what is described in eq. (3.37) to (3.39) and in Algorithm 3.1. The only difference lies in the omission of the input. Algorithm 3.2 is used for diagnosing the autonomous stochastic automaton with coinciding states and outputs.

3.3.7 The relation between measured signals and finite value sets

The relation between the measured signals/quantities and the discrete value sets $\mathcal{N}_z$, $\mathcal{N}_v$ and $\mathcal{N}_w$ is done by partitioning the measured signals into the finite number sets. The partitioning of the signals is done by quantisers. If denoting a measured quantity by y the partition sets are denoted as $\mathcal{Q}_y(w)$ hence $y \in \mathcal{Q}_y(w)$ where $w \in \mathcal{N}_w$. The measured signals can thus be quantised into the finite number of sets using the quantisers.

An example of this can be seen in Figure 3.1.b where the measurements of activity index belonging to nominal behaviour are quantised into the discrete value set $\mathcal{N}_z \in \{1, 2, 3, 4\}$ using the partition intervals shown in Figure 3.1.a.

3.4 Identification of automata

An obvious challenge for modelling the cows' behaviour by means of e.g. automata is identifying the automata.

The precise behaviour is not known to follow a well established pattern other than assumptions based on basic needs and desires of the animal. A model of the behaviour could therefore only in a vague manner be based on a fundamental understanding of how

Algorithm 3.2 Diagnosis of autonomous stochastic automaton with coinciding states and inputs *(Based on Algorithm 8.3 on page 420 in [3])*

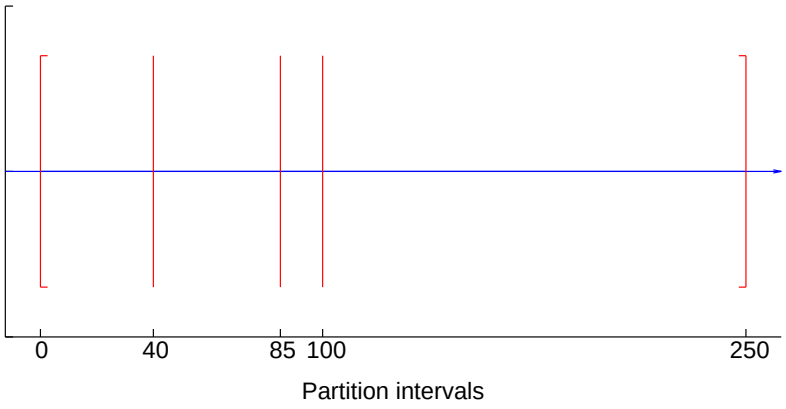
Given: autonomous stochastic automaton with coinciding states and inputs $\mathcal{S}$ and fault model $\mathcal{S}_f$
Initial state probability distribution $\text{Prob}\left(\hat{z}(0)\right)$
Initial fault probability distribution $\text{Prob}\left(\hat{f}(0)\right)$

Initialisation: $p_r(f,z) = \text{Prob}\left(\hat{f}_p(0) = f\right) \cdot \text{Prob}\left(\hat{z}_p(0) = z\right)$ for all $f \in \mathcal{N}_f$ and $z \in \mathcal{N}_z$
$k_h = 0$

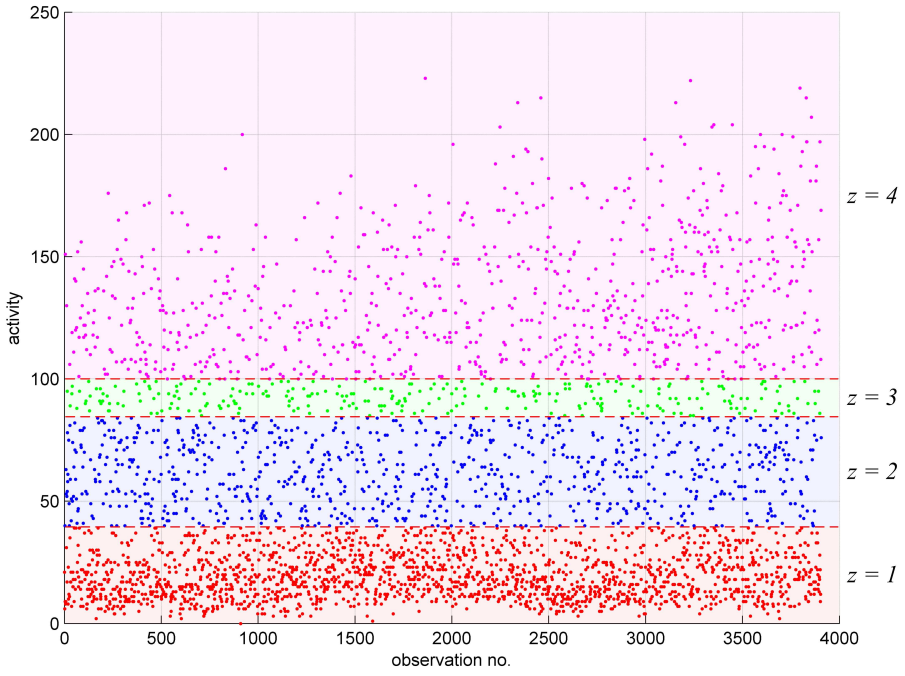
Loop:

1. Measure the current state z_m.
2. For all $f \in \mathcal{N}_f$ determine $h(f, z_m) = \sum_{\bar{f},\bar{z}} G(\bar{z}|z_m, f) \cdot G_f(\bar{f}|f) \cdot p_r(f, z_m)$
3. If $\sum_f h(f, z_m) = 0$ holds, stop the algorithm as the measured state sequence is inconsistent with the automaton.
4. For all $f \in \mathcal{N}_f$ determine $p_k(f, z_m) = \frac{h(f,z_m)}{\sum_f h(f,z_m)}$.
5. For all $f \in \mathcal{N}_f$ and $z \in \mathcal{N}_z$ determine $p_r(f,z) = \frac{\sum_{\bar{f}} G(z|z_m,f) \cdot G_f(f|\bar{f}) \cdot p_r(f,z_m)}{\sum_f h(f,z_m)}$.
6. Determine $\text{Prob}\left(f(k_h) = f|k_h\right) = p_k(f, z_m)$
7. $k_h = k_h + 1$
   Continue with step 1.

Result: $\text{Prob}\left(f(k_h) = f|k_h\right)$



(a) Partition intervals



(b) Quantisation of the activity level

Figure 3.1: Quantisation example

the system works. Building a model mainly from observing past observations is subsequently the approach and the a priori knowledge is based on qualitalive assumptions.

To the knowledge of the author there do not exist methods for identification of I/O

automata from data measurements. The existing literature on I/O automata mainly addresses the abstraction of systems already described by continuous models.

3.4.1 Beginning the modelling process

It is evident that the modelling process can involve many steps as it can involve all possible observed data regarding the cow behaviour, e.g. activity, feeding behaviour and milking behaviour.

The best approach is to try to identify the simplest task and complete that before moving on to the next model/trait.

3.4.2 Modelling steps

In the process of finding the most suitable model and diagnosis method the following steps are elaborated:

1. Model selection.
2. Definition of states, inputs and outputs.
3. Quantisation.
4. Calculating transition probabilities from data.

Each of the above listed steps is described briefly in sections 2.5−3.4.6, below. Figure 3.2 provides an overview of the modelling process.

3.4.3 Model selection

A suitable model construction is suggested. In this study there are three different versions of the stochastic automaton tested for modelling and diagnosing the activity.

3.4.4 Definition of states, inputs and outputs

When defining states one should bare in mind that the states should symbolise the state of the system in question, especially with respect to which changes in the system the

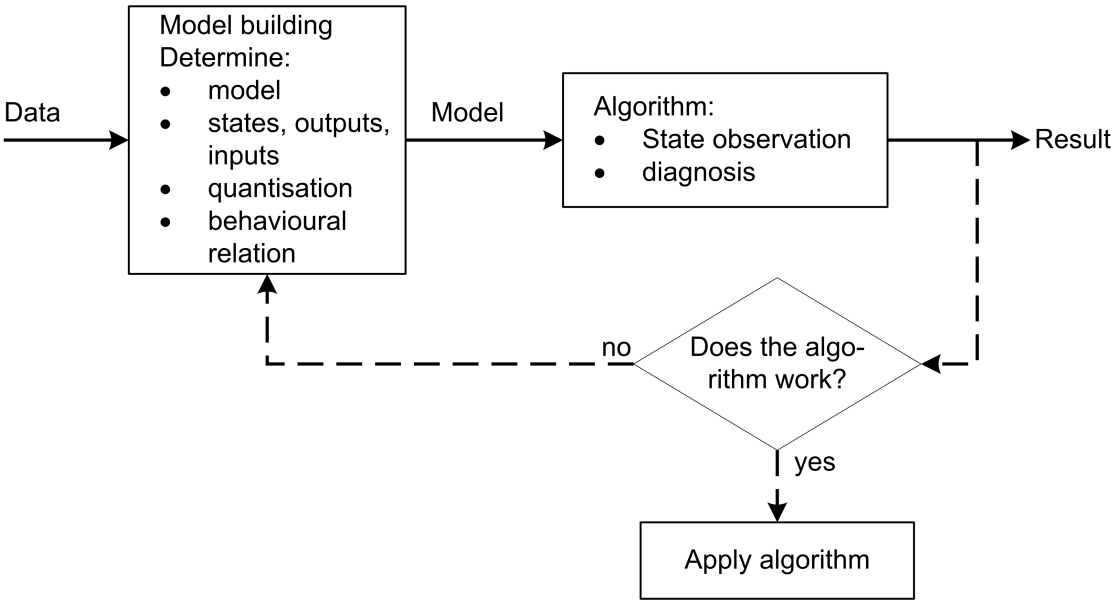


Figure 3.2: Overview of the modelling steps.

model is supposed to emphasise. The inputs should be chosen as those external circumstances that influence the state of the system. The output is preferably a parameter that is dependant on the system state.

3.4.5 Quantisation

The main aim of selecting the quantisation is to retain the difference in the data between behavioural scenarios the detection algorithm is supposed to distinguish between. The quantisation can be tuned to bring out the differences in the behaviour for each fault.

3.4.6 Calculating transition probabilities from data

The counting has to do with training the automata. At this stage the frequency of each transition for each value of input v and output w for each of the behavioural scenarios is assessed by observing a *training dataset*.

The training algorithm utilises the Markov property and has eq. (3.13) as the underlying basis. The sum in eq. (3.13) describes the relation that the sum of probabilities for all transitions from a state z towards a state z' given the input v while producing the output w under the behavioural scenario f should therefore indicate e.g. whether a

certain transition is more probable for input $v = 1$ or $v = 2$.

An evaluation of the approximated probability for each transition and output generation will compare the number of observed transitions from state z in the training dataset and assign the highest probability to the most frequent transition and output generation.

Thereby denoting by $N_{z',w,z,f,v}$ how many times in the training dataset the transition from state z towards state z' given the input v while producing the output w under the behavioural scenario f has been observed the approximated probability of the given transition and output generation is found as.

$$L\left(z', w|z, f, v\right) = \frac{N_{z',w,z,f,v}}{\sum\limits_{z' \in \mathcal{N}_z} \sum\limits_{w \in \mathcal{N}_w} N_{z',w,z,f,v}} \tag{3.42}$$

where $L\left(z', w|z, f, v\right)$ is the transition probability that complies with the relation given in eq. (3.13).

The quality of the approximation of the transition and output generation probabilities depends largely on how representative and extensive the training dataset is with respect to the modelled behaviour. If using a small dataset one could experience that some of the transitions do not occur at all resulting in the approximated transition probability equal to zero thus indicating that the transition is not possible although in reality it might very well be possible under some circumstances.

4 Application

This chapter describes the application of the methods described in the previous chapter on the task of modelling behaviour of cows with the goal to detect lameness.

The procedure will follow the design steps earlier described in section 3.4.2.

4.1 Considerations for model selection

The modelling aim is to be able to detect lameness by observing changes in behaviour due to lameness.

As described in section 3.4.2 the first steps in the modelling process involve model selection and definition of states, inputs and outputs. With respect to selection of the model it has already been stated in section 3.3.3 that the model is based on the stochastic automata.

It is mentioned in section section 2 and shown in Figure 1.1 that available traits in data include observations of the traits activity, feeding behaviour and milking behaviour. When selecting the model one can choose between modelling the behaviour with respect to one trait or one could choose to combine two or more traits in the modelling efforts. In this study the choice was to model one trait and to investigate its relevance with respect to lameness detection using discrete event models. If the modelling is shown to be relevant the model can be expanded with more traits at a later stage.

All three traits have been pointed out as relevant indicators of lameness ([2, 8, 16]). As shown in section 1.2.1 all three have been used in some way in earlier attempts to automatically detect lameness ([8, 13, 16]). If only considering earlier results of automatic lameness detection traits regarding feeding behaviour seem most likely to indicate lameness behaviour as they have given the best results with respect to number of successful detections and the number of false alarms. There therefore already exist methods that comprise feeding behaviour for the detection of lameness with promissing results. To the author of this report, the most intuitive beginning task is to model the activity. As the main symptoms of lameness involve feet illnesses it's intuitive to associate

these with changes in activity. Changes in activity in connection with lameness have been pointed out in e.g. [18] and [16] and the detection of lameness using observations of activity has been carried out in [16]. Although the authors of [16] did not succeed in developing a convincing lameness detector solely based on activity it is still a relevant parameter for further studies of lameness detection. The detection method used in [16] was quite plain and results should be possible to improve. The trait that is used in the following is therefore the activity.

As mentioned in section 3.3 the cows' behaviour follows a stochastic pattern, therefore the stochastic automata is selected for describing the cows' activity.

For the selection of model and the definition of states, inputs and outputs it should be kept in mind that the model is supposed to describe the cows' activity and that the diagnosis task is to detect changes in the activity behaviour due to feet illnesses (lameness). The modelling aim is therefore to construct a model that retains differences in activity between nominal and lame behaviour.

No date preprocessing other than a quantisation is performed in the following. If a detection is possible using observations of raw data observations this is likely to be faster than a detection performed on observations that have been pre-processed/filtered using e.g. an aggregation or a running mean and so on.

In subsections 4.2−4.4 three stochastic automata models are described, that all model the activity behaviour in the two behavioural scenarios namely nominal and lame. Diagnosis results using the three approaches are shown in sections 5.1−5.3 and a comparison of the results is done in section 5.4. Below is a brief description of the modelling aim of the three different approaches.

As the cow is a living being, an individual if you will, that can move freely within some boundaries in a loose housing system it is a natural assumption to consider the cow as an autonomous system with respect to its activity behaviour. This is what is comprised in the first approach described in section 4.2 where the cow is modelled as an autonomous system with raw measurements of activity index (see section 2.2) as its measurement output. In the first approach the states describe the activity during the last hour, i.e. a qualitative measure of the activity index described in section 2.2. This is realised by the autonomous stochastic automaton with coinciding states and outputs.

It is well known that the cows' activity follows a some sort of a diurnal rhythm (see e.g. [11]).One could therefore assume that it's justifyable to consider the time of day as some kind of an input or disturbance controlling the systems activity.This is what is done in the second approach by adding quantised time of day as an input to the stochastic automaton. In this case the system is as before considered to have raw activity index as measurement output. The modelling aim of the second approach is therefore to model the cow's activity as a function of the time of day.

In the third approach the modelling aim is to model a general activity level of the autonomous cow. The model should describe whether the cow is experiencing an active period rather than describing a more instantaneous activity as in the two earlier approaches. In this approach the state z is therefore defined as a measure of the general activity state of the cow in terms of a some sort of a mean value. Hence if the activity observations are generally high the activity state should also be high, but although a single low activity observation appears it doesn't necessarily mean that the state should change over to a low activity state, thus the activity state is a some kind of a moving average of the activity observations. The output w on the other hand expresses the direct measurement of the current activity index which means that the activity state is not directly observable.

Steps 2, 3 and 4 of the design steps (see. section 3.4.2) for the application each of the above mentioned models for the behavioural modelling are described in sections 4.2-4.4.

The two behavioural scenarios, i.e. *healthy* and *lame*, which are expressed by $f(k)$ (see section 3.3.6), are mapped into the discrete value set $\{healthy, lame\} \rightarrow \{1, 2\}$.

4.2 First approach: Autonomous cow

As mentioned in the previous section the cow is in the first approach considered as an autonomous system with respect to its activity behaviour. The modelling aim of this approach is to model the activity during the last hour in the autonomous cow for the two behavioural scenarios nominal ($f = 1$) and lame ($f = 2$) respectively.

4.2.1 Selection of inputs, states and outputs

As the cow is in this approach considered autonomous the input is neglected, which leads to the autonomous stochastic automaton. As the modelled behaviour is the activity during the last hour, which is the observed measure of the activity behaviour, it comes as a natural choice to also select the state z as a quantised measure of the activity during the last hour. As this is also considered to be the output of the system the model used is *the autonomous stochastic automaton with coinciding state and output*.

4.2.2 Quantisation and calculation of probabilities from data

The state is interpreted as the activity by quantising the activity into the finite number of states using a quantiser such as described in section 3.3.7.

In this first test the partition intervals were selected manually by observing the test data described in section 2.5 in Figure 2.6 and selecting quantisation intervals that would underline the difference in data between the two behavioural scenarios. The quantisation is thus chosen to retain the difference in activity, especially in the upper levels of the activity where the difference is more evident than otherwise. Hence in the first test the output was partitioned using

$$\mathcal{Q}_y(w) = \{0, 40, 85, 100, 250\} \,. \tag{4.1}$$

A graphical demonstration of the quantisation intervals is shown in Figure 4.1 and Figure 4.2.

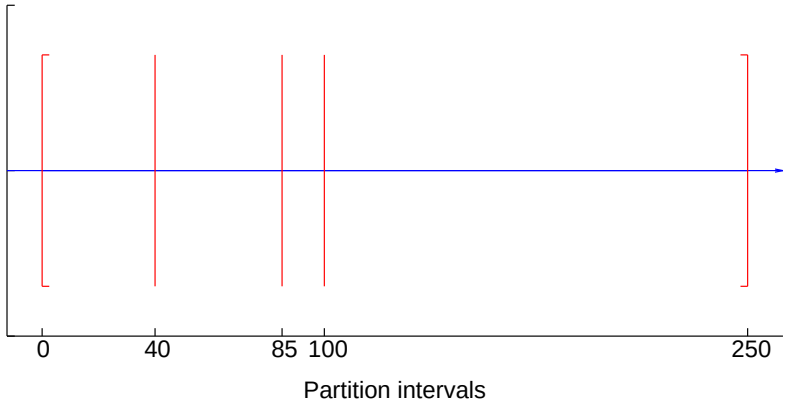
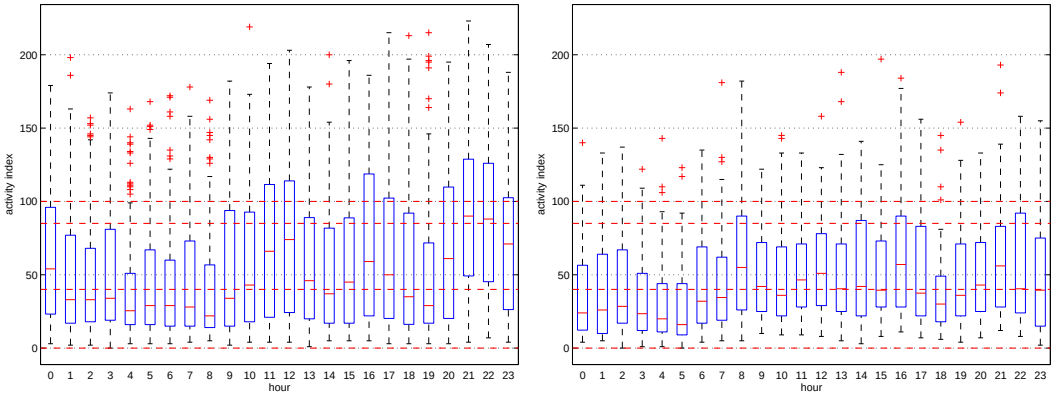


Figure 4.1: A plot of the quantisation intervals.

It can be seen in the box plots shown in Figure 4.1 that observations of activity index in the last two quantisation intervals is much more frequent for nominal behaviour than for faulty behaviour.

In order to assess the state transition probabilities of the model (automaton) the model was trained on the test dataset described in section 2.5 using the counting algorithm in eq. (3.42).

As the model is constructed to describe two different behavioural scenarios namely nominal and faulty, the automaton was trained on the nominal behaviour and the faulty behaviour.



(a) $f = 1$ (nominal) (b) $f = 2$ (faulty)

Figure 4.2: Quantisation of the activity level shown on box plots of the activity for each hour in the day

The model describing the nominal behaviour was trained using 162 days of data in the period between 02.09.2007 and 12.02.2008 while the model describing the faulty behaviour was trained using 62 days of data in the period between 27.06.2008 and 28.08.2008, setting $f = 1$ when training on the data belonging to the nominal behaviour and $f = 2$ when training on the data belonging to the faulty behaviour.

The test data was quantised using the quantisation given in eq. (4.1). Figure 4.3 shows how the observed values in the test data are quantised, using different colour for each quantisation level and the assigned state value is shown on the right side of the plot.

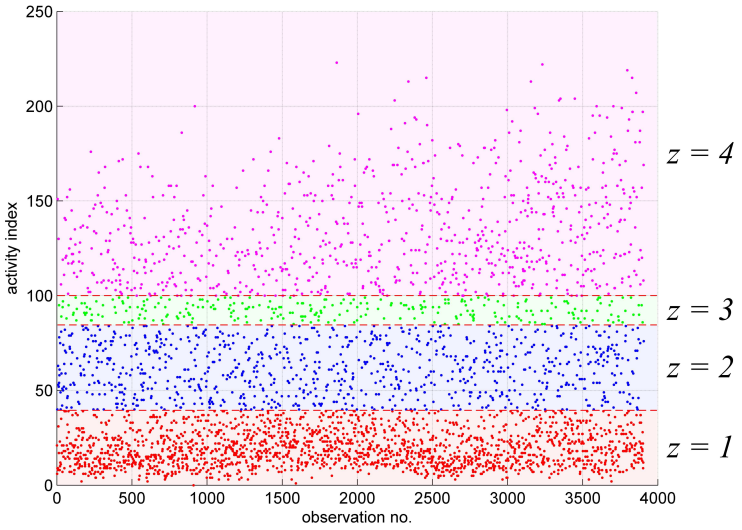
Bar plots showing the number of observations quantised into each of the partitions are plotted in Figure 4.4.

It becomes evident by looking at Figure 4.3 and especially the bar plot in Figure 4.4 that the number of observations in the highest quantisation interval is clearly higher for the nominal behaviour compared to the faulty (lame) behaviour.

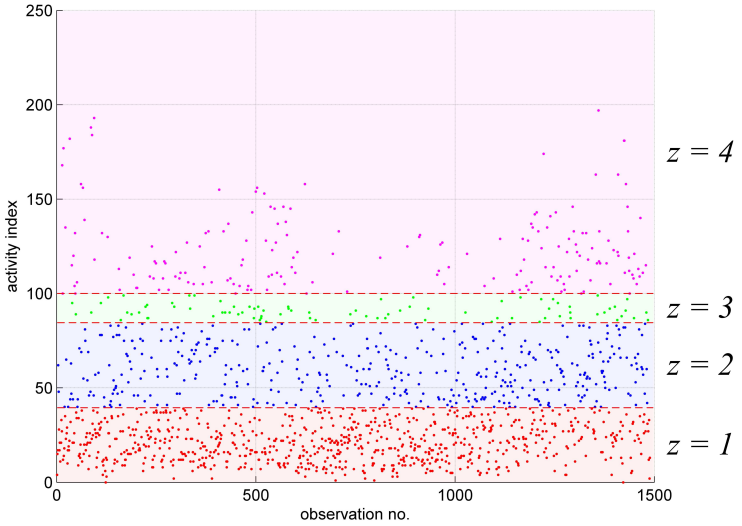
The automaton shown in eq. (3.40) was trained using the training algorithm in eq. (3.42). The state transition relation of the automaton was estimated as

$$G(z'|z,1) = \begin{pmatrix} 0.56 & 0.43 & 0.44 & 0.39 \\ 0.17 & 0.24 & 0.23 & 0.35 \\ 0.06 & 0.06 & 0.08 & 0.08 \\ 0.21 & 0.27 & 0.25 & 0.18 \end{pmatrix}, \quad G(z'|z,2) = \begin{pmatrix} 0.58 & 0.47 & 0.43 & 0.49 \\ 0.25 & 0.34 & 0.34 & 0.35 \\ 0.05 & 0.06 & 0.08 & 0.06 \\ 0.13 & 0.12 & 0.14 & 0.10 \end{pmatrix}, \tag{4.2}$$

where the row number represents the successor state z' and the column number represents



(a) $f = 1$ (nominal)

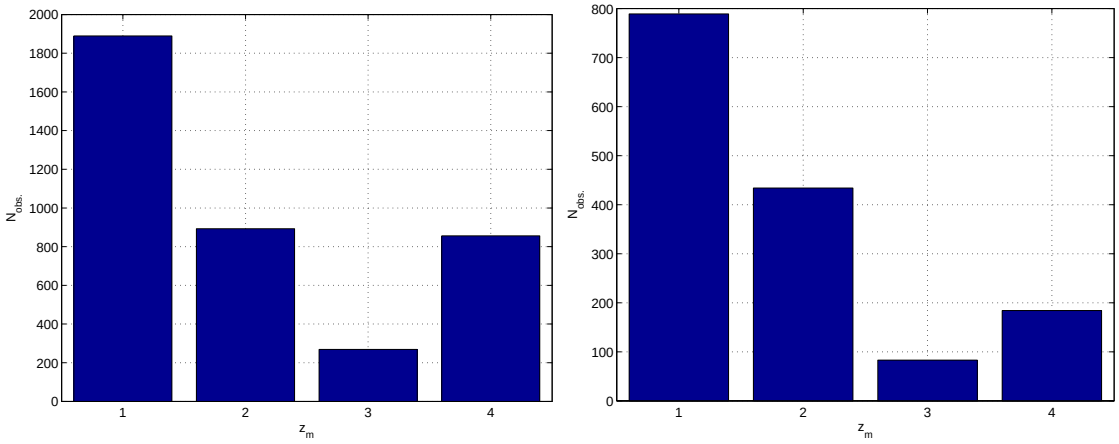


(b) $f = 2$ (faulty)

Figure 4.3: Quantisation of the activity level

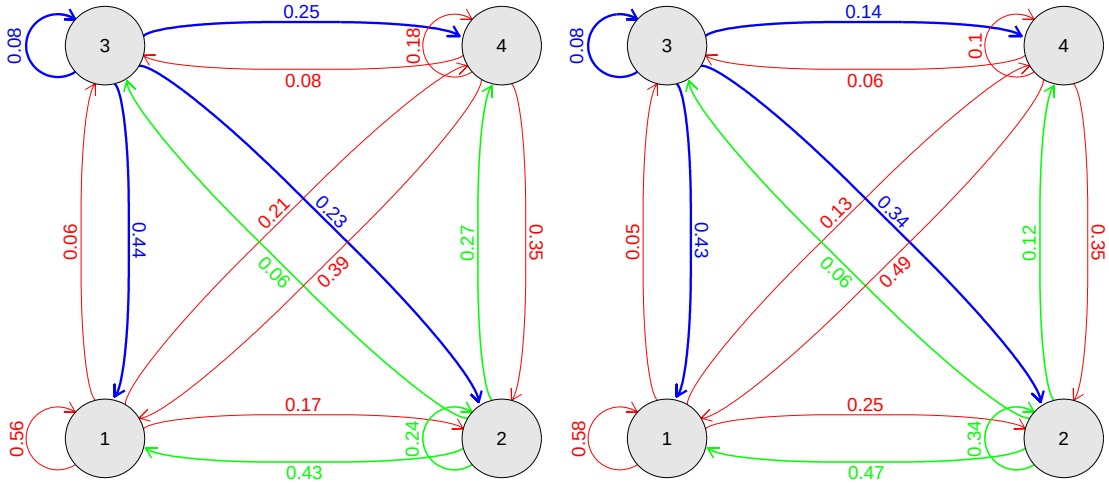
the present state z. A graphical interpretation of the automaton in eq. (4.2) are shown in Figure 4.5.

By looking at the state transition relation in eq. 4.2 and Figure 4.5 it is clear that transitions with especially successor state $z' = 4$ are much more probable for the nominal



(a) $f = 1$ (nominal) (b) $f = 2$ (faulty)

Figure 4.4: Bar plot showing the number of observations of the activity index quantised into each level.



(a) $f = 1$ (nominal) (b) $f = 2$ (faulty)

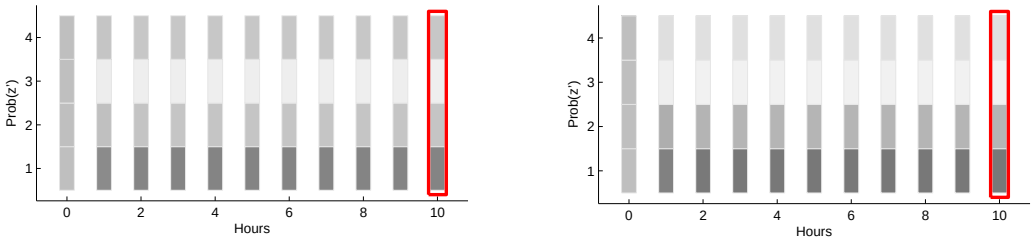
Figure 4.5: A graph plot of the autonomous automaton

case than for the faulty case, which is what was to be expected.

4.2.3 Model evolution

The autonomous stochastic automaton that is described by the transition probabilities given in eq. (4.2) was used to simulate the cows behaviour by using the recursive algorithm in eq. (3.23).

A simulation of the automaton which behavioural relations are expressed in eq. (4.2) and Figure 4.5 was performed and the result is shown in Figure 4.6 and Figure 4.7.



(a) $f = 1$ (nominal)

(b) $f = 2$ (faulty)

Figure 4.6: A grey scale plot of the simulation of the autonomous automaton with coinciding state and output over 10 *hours*. A black rectangle symbolises probability of one, white rectangle symbolises a probability of zero.

From the simulation in Figure 4.6 and Figure 4.7 it can bee seen that it is more probable that the automaton occupies state $z = 4$ under nominal behaviour while it is more likely to occupy state $z = 1$ and $z = 2$ under faulty behaviour.

A diagnosis test of the automaton with transition relations shown in eq. 4.2 is addressed in section 5.1.

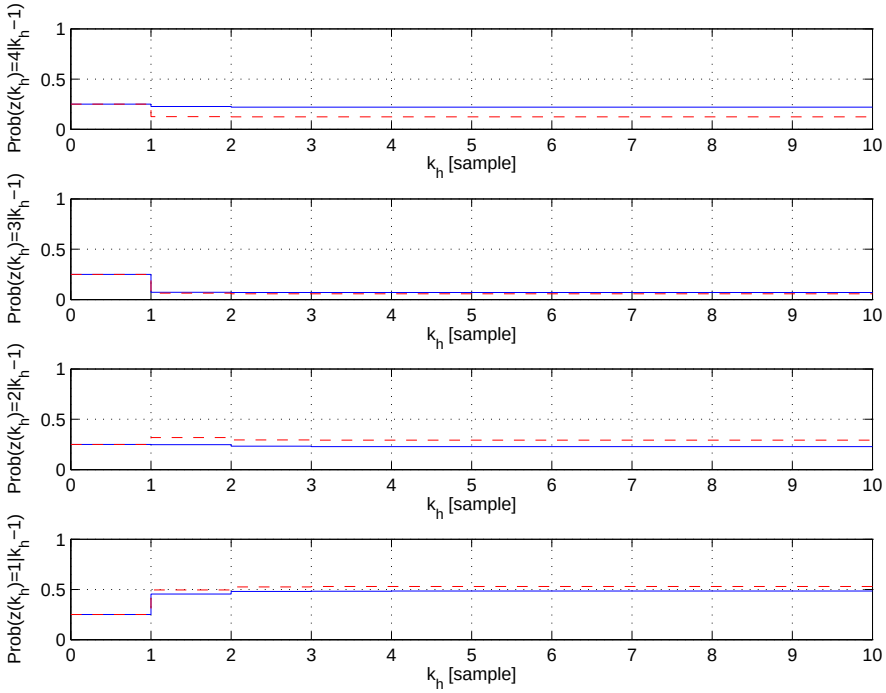


Figure 4.7: Simulation of the autonomous automaton with coinciding state and output over 10 *hours*. A simulation of the nominal behaviour is shown in blue and the faulty behaviour with a red dashed line.

4.3 Second approach: Diurnal activity

In this second approach the cow is considered to be affected by which time of day it is when it comes to its activity behaviour. The modelling aim of this approach is to model the cow's activity during the last hour as a function of the quantised time of dayfor the two behavioural scenarios nominal ($f = 1$) and lame ($f = 2$) respectively. The model should therefore not only express the difference in activity between the two behavioural scenarios but also include a diurnal rhythm.

4.3.1 Selection of inputs, states and outputs

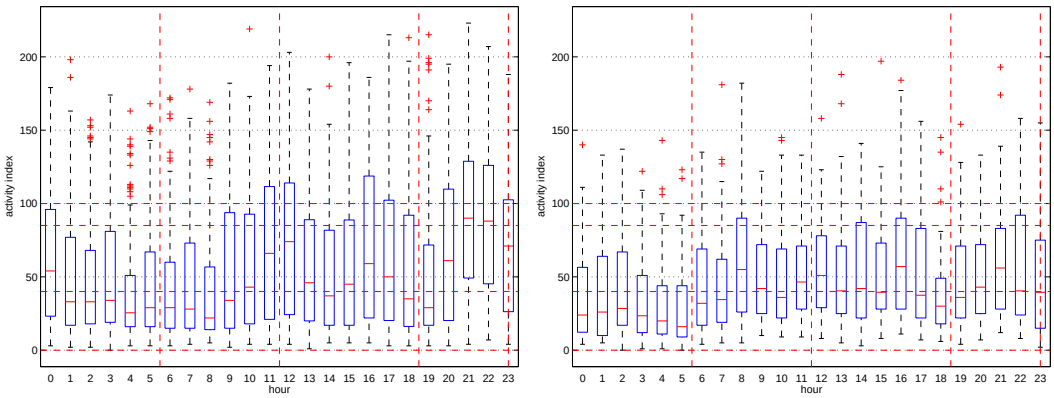
As the cow is in this approach considered to be affected by which time of day it is, a quantised time of day is selected as input to the stochastic automaton. As the modelled behaviour is still the activity during the last hour the state is a quantised measure of the activity during the last hour. The second test therefore comprises *a stochastic automaton with quantised time of day as input and coinciding state and output.*

4.3.2 Quantisation

The quantisation of the inputs and outputs in this example is done manually like in the previous example. The input quantisation is chosen by observing the mean value for each hour of the day (see Figure 4.9) and trying to select the quantisation levels where the activity level changes. The input was quantised using

$$\mathcal{Q}_u(v) = \{0, 6.5, 12.5, 19.5, 24\}\ , \tag{4.3}$$

while the output was quantised using the intervals shown in eq. (4.1). The quantisation intervals are shown in Figure 4.9.



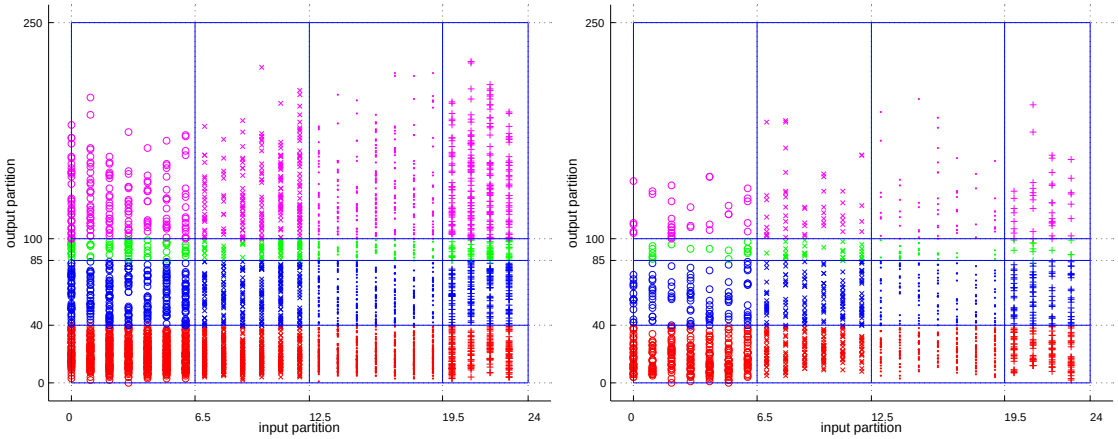
(a) $f = 1$ (nominal) (b) $f = 2$ (faulty)

Figure 4.8: Quantisation of the activity level and time of day shown on box plots of the activity for each hour in the day

From Figure 4.8 it can be seen that on the right figure there is a change in median value from before $05:30$ to the interval $5:30-11:30$. It is also evident that activity in the nominal case is especially high compared to the faulty case in the interval $18:30-24:00$. The differences in the activity that are visible only when looking at a certain time interval but disappear when looking at the whole time spectrum will aid the diagnosis when also including time of day as input. The quantisation is shown in Figure 4.9.

4.3.3 Behavioural relations

Estimating transition probabilities counting transitions in the dataset of nominal and faulty behaviour respectively gave the state transition relations shown in eq. (4.4) and eq. (4.5). The difference between the two transition relations is shown in eq. (4.6).



(a) $f = 1$ (nominal) (b) $f = 2$ (faulty)

Figure 4.9: Quantisation of the activity level and the time of day

State transition relation for the healthy cow becomes

$$G(z'|z,1,v) = \left(\begin{array}{cccc|cccc|cccc|cccc} 0.64 & 0.52 & 0.58 & 0.54 & 0.55 & 0.44 & 0.47 & 0.37 & 0.55 & 0.42 & 0.43 & 0.41 & 0.35 & 0.27 & 0.27 & 0.28 \\ 0.17 & 0.29 & 0.19 & 0.36 & 0.18 & 0.19 & 0.16 & 0.40 & 0.16 & 0.21 & 0.26 & 0.32 & 0.18 & 0.26 & 0.32 & 0.34 \\ 0.05 & 0.05 & 0.08 & 0.04 & 0.07 & 0.07 & 0.09 & 0.10 & 0.06 & 0.06 & 0.05 & 0.09 & 0.11 & 0.09 & 0.11 & 0.09 \\ 0.15 & 0.14 & 0.15 & 0.06 & 0.21 & 0.30 & 0.28 & 0.14 & 0.23 & 0.31 & 0.26 & 0.18 & 0.36 & 0.38 & 0.29 & 0.30 \end{array}\right), \tag{4.4}$$

where the 4 columns between each two vertical lines represent the present state z, the row number the successor state z' and the vertical lines mark a change in the input. Hence the values from the leftmost parentheses to the leftmost vertical line represent transitions for input $v = 1$ and so forth.

State transition relation for the lame cow is found as,

$$G(z'|z,2,v) = \left(\begin{array}{cccc|cccc|cccc|cccc} 0.68 & 0.59 & 0.65 & 0.53 & 0.47 & 0.43 & 0.34 & 0.42 & 0.58 & 0.42 & 0.35 & 0.50 & 0.49 & 0.50 & 0.55 & 0.53 \\ 0.18 & 0.27 & 0.24 & 0.44 & 0.31 & 0.41 & 0.34 & 0.38 & 0.25 & 0.35 & 0.38 & 0.29 & 0.32 & 0.30 & 0.36 & 0.33 \\ 0.04 & 0.05 & 0.06 & 0.00 & 0.08 & 0.06 & 0.10 & 0.10 & 0.04 & 0.08 & 0.12 & 0.06 & 0.04 & 0.04 & 0.00 & 0.05 \\ 0.10 & 0.08 & 0.06 & 0.03 & 0.14 & 0.10 & 0.21 & 0.10 & 0.14 & 0.15 & 0.15 & 0.15 & 0.16 & 0.17 & 0.09 & 0.10 \end{array}\right). \tag{4.5}$$

A relative difference in the transition relations between the two considered behavioural scenarios is found as

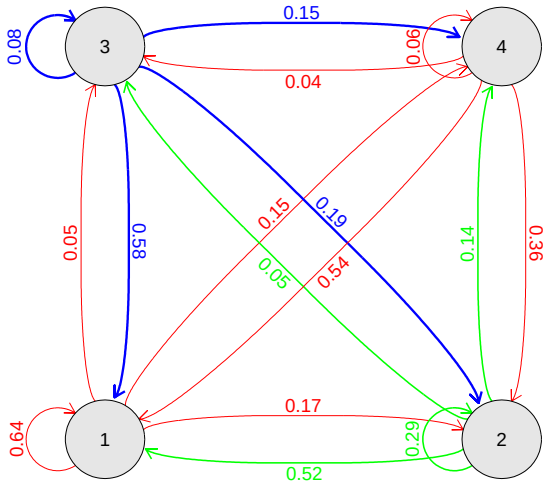
$$\frac{G(z'|z,1,v) - G(z'|z,2,v)}{G(z'|z,1,v)} \times 100 =$$

$$\left(\begin{array}{cccc|cccc|cccc|cccc} -6.3 & -13.2 & -11.4 & 2.5 & 13.9 & 4.3 & 26.7 & -13.5 & -4.9 & 1.0 & 19.2 & -21.4 & -40.2 & -87.5 & -98.9 & -90.3 \\ -9.7 & 5.0 & -21.6 & -23.2 & -73.3 & -121.1 & -113.2 & 5.3 & -51.7 & -66.5 & -48.1 & 8.6 & -75.7 & -14.2 & -12.7 & 3.3 \\ 15.3 & -5.4 & 27.1 & 100.0 & -23.0 & 9.3 & -17.2 & -2.7 & 30.3 & -40.0 & -122.1 & 25.2 & 68.9 & 60.7 & 100.0 & 44.8 \\ 33.5 & 40.8 & 59.5 & 52.4 & 34.0 & 66.0 & 26.0 & 23.0 & 40.5 & 51.6 & 40.8 & 21.2 & 55.4 & 56.3 & 68.7 & 66.4 \end{array}\right). \tag{4.6}$$

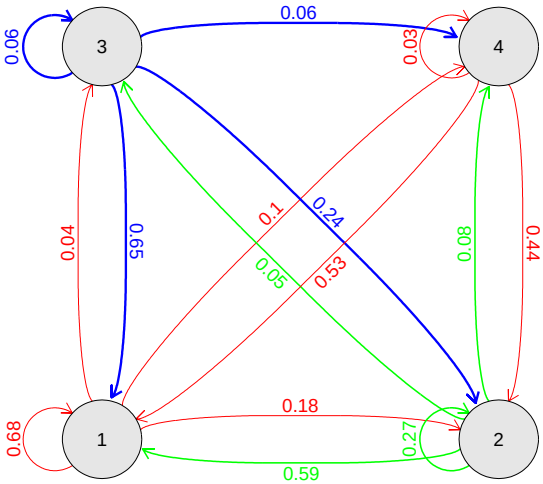
by observing the matrix in eq. (4.6) one can see that the largest relative difference in transition probability is when moving from $z = 3$ to $z' = 3$, i.e. it is more likely that the

activity remains at state $z = 3$, for input $v = 3$, between $11:30$ and $18:30$, where it is more likely to happen when the cow is lame than when it's not lame.

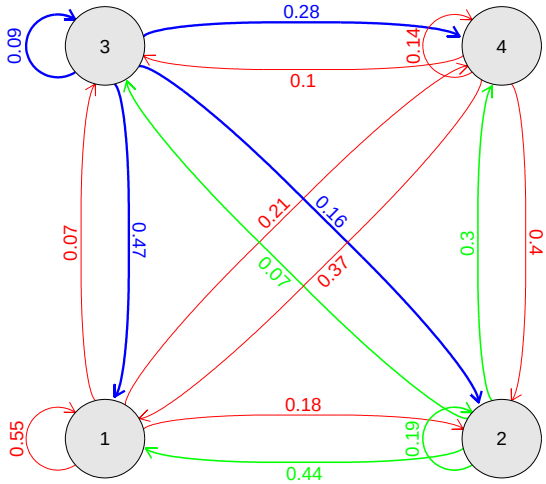
A graphical interpretation of the automaton in eq. (4.4) and eq. (4.5) are shown in Figure 4.10 and 4.11.



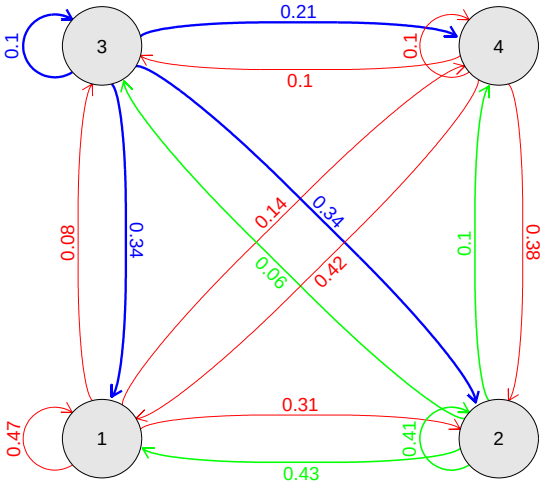
(a) $f = 1$ (nominal) $v = 1$



(b) $f = 2$ (faulty) $v = 1$

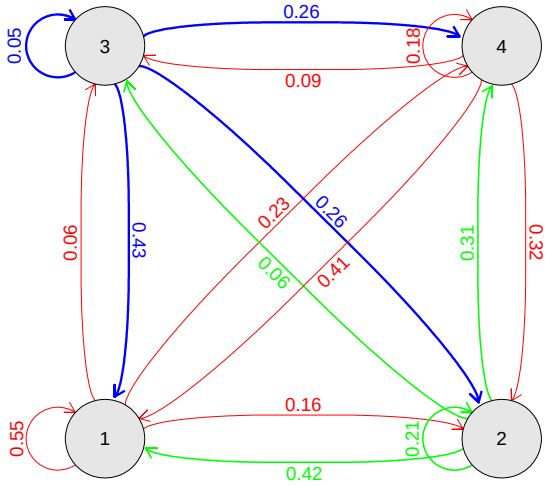


(c) $f = 1$ (nominal) $v = 2$

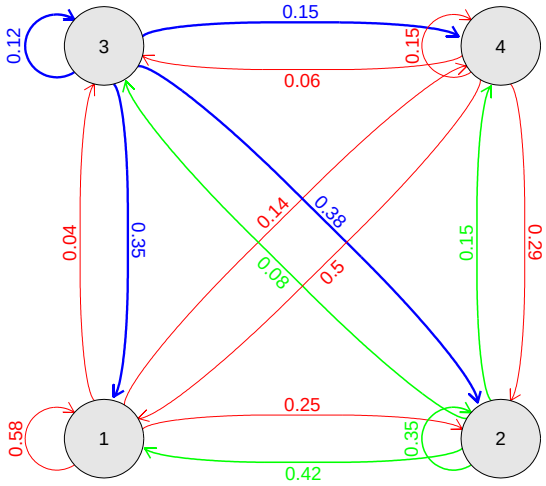


(d) $f = 2$ (faulty) $v = 2$

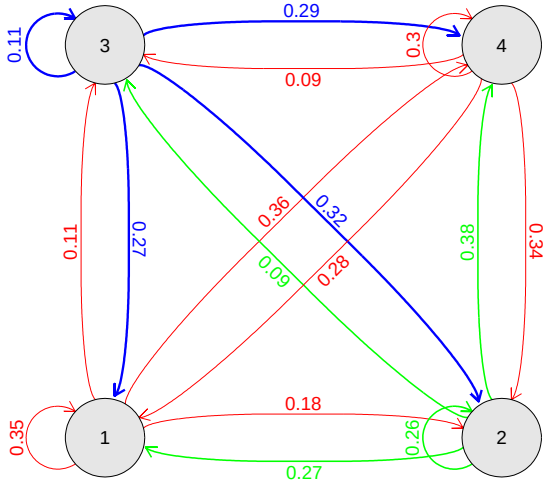
Figure 4.10: A graph plot of the autonomous automaton with time of day as input *(input $v = 1$ and $v = 2$)*



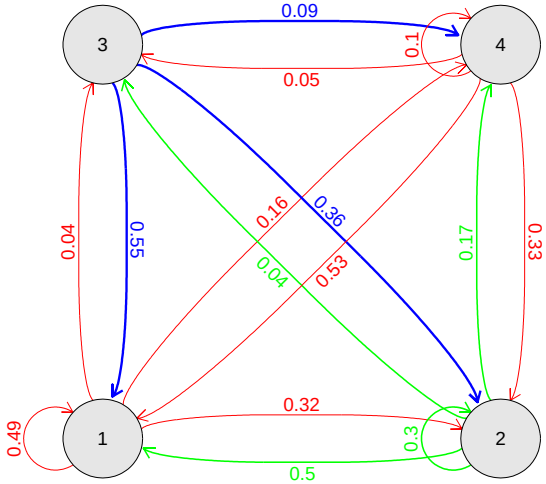
(a) $f = 1$ (nominal) $v = 3$



(b) $f = 2$ (faulty) $v = 3$



(c) $f = 1$ (nominal) $v = 4$

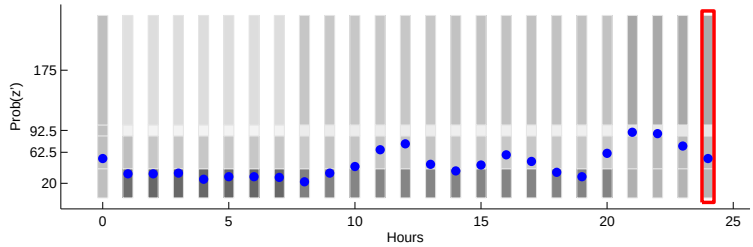


(d) $f = 2$ (faulty) $v = 4$

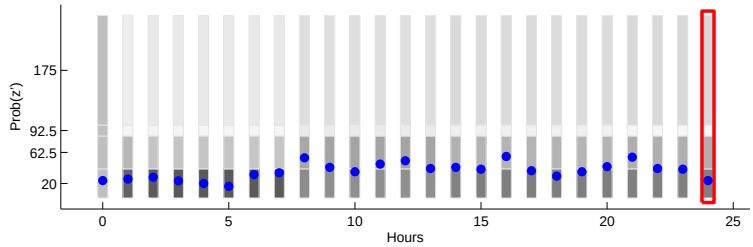
Figure 4.11: A graph plot of the autonomous automaton with time of day as input *(input $v = 3$ and $v = 4$)*

4.3.4 Simulation

For simulating the stochastic automaton with coinciding states and outputs and a *time of day* input the recursive algorithm in eq. (3.22) can be used directly. The normal behaviour was simulated using the behavioural relation given in eq. (4.4) and the behaviour belonging to lameness was simulated using the behavioural relation given in eq. (4.5). The simulation result is shown in Figure 4.12.



(a) $f = 1$ (nominal)



(b) $f = 2$ (faulty)

Figure 4.12: Simulation of stochastic automaton with time of day input plotted in greyscale and the median value of the activity for each hour of the day plotted with blue dots.

It can be seen by observing the simulations in Figure 4.12 and Figure 4.13 that the simulations now follow a diurnal rhythm and that the probability of moving towards a state of "high" activity is more probable for the cow when healthy than when lame.

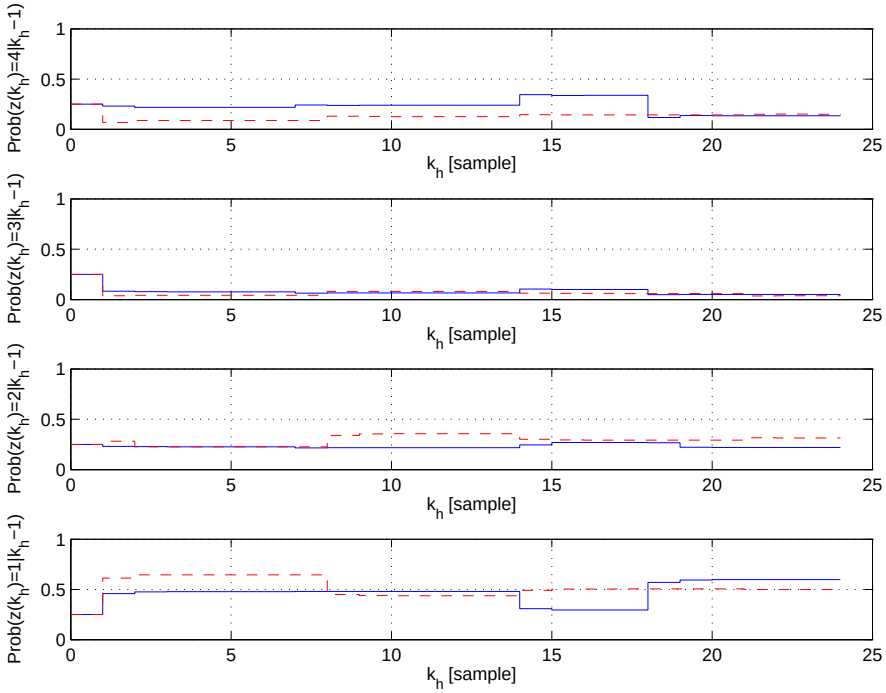


Figure 4.13: A plot of the simulation of the autonomous automaton with coinciding state and output over 10 *hours*. A simulation of the nominal behaviour is shown in blue and the faulty behaviour in red.

4.4 Third approach: General activity level of the autonomous cow

In the third approach the modelling aim is to model a general activity level of the autonomous cow.

4.4.1 Selection of inputs, states and outputs

As in the first approach (see section 4.2) the cow is considered autonomous and the input is neglected. In this approach the state z is defined as a measure of the general activity state of the cow in terms of a some sort of a mean value. The output w on the other hand expresses the direct measurement of the current activity index. The model therefore becomes *the autonomous stochastic automaton*.

4.4.2 Quantisation

For being able to come up with a reasonable estimate of the behavioural relation for the automaton one should built a dataset for training this automaton.

As a "truth" set for the states, and state transitions, median values of each hour in the day were used. As the cows' activity follows a diurnal rhythm it was considered reasonable to assume that all activity observations taken at e.g. $01:00$ occurred during a transition towards the activity state at $02:00$, i.e. under training the state was the same at $01:00$ every day and the same for at $02:00$ every day and so forth.

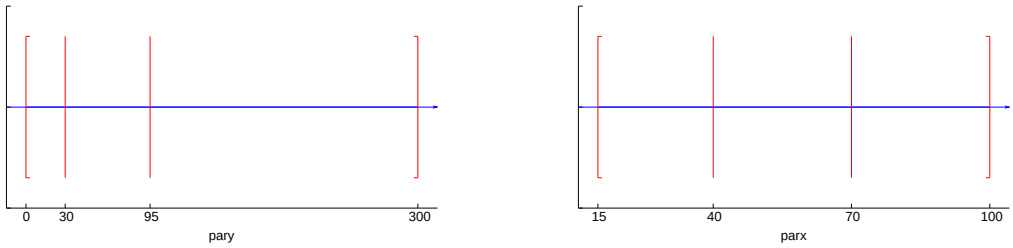
The median values seen in Figure 4.15 were therefore quantised into state sets and used as a truth set for the automaton training. The activity level was partitioned into a set of 3 states and the output into a set of 3 values, thus

$$\mathcal{N}_z = \{1, 2, 3\} \quad \text{and} \quad \mathcal{N}_w = \{1, 2, 3\} \tag{4.7}$$

As described above, both state set $\mathcal{N}_z$ and output set $\mathcal{N}_w$ have 3 values. The quantisation for states $\mathcal{Q}_x(z)$ and the quantisation of outputs $\mathcal{Q}_y(w)$ was selected as

$$\mathcal{Q}_y(w) = \{0, 30, 95, 300\} \quad \mathcal{Q}_x(z) = \{15, 40, 70, 100\}\,. \tag{4.8}$$

A graphical demonstration of the quantisation intervals is shown in Figure 4.14 and Figure 4.15.

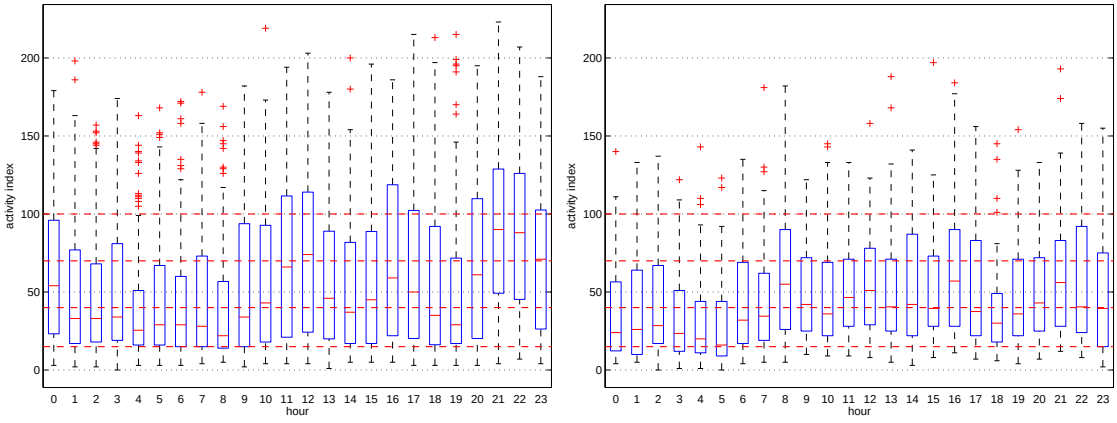


(a) quantisation of activity index

(b) quantisation activity medians

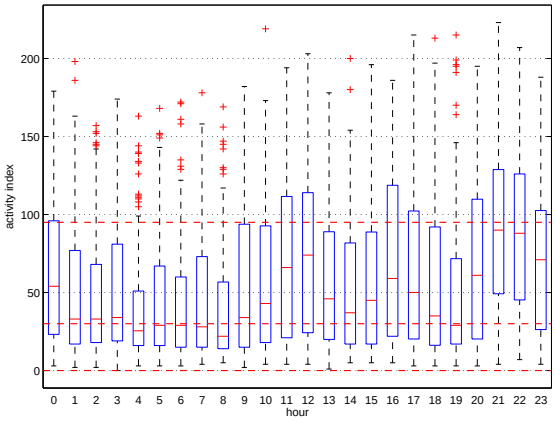
Figure 4.14: A plot of the quantisation intervals

From Figure 4.15 (a) and (b) it can be seen that for both behavioural scenarios the cow always stays in state $z = 1$ in the period $01:00 - 07:00$ and it can also be seen that the cow never enters state $z = 3$ when lame, which it does in the period $21:00 - 23:00$ in the nominal case.

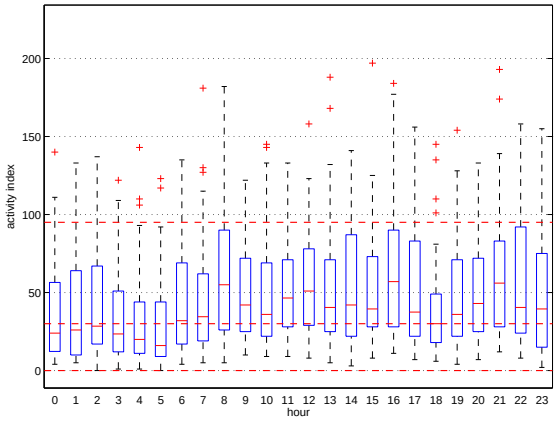


(a) State quantisation, $f = 1$ (nominal)

(b) State quantisation, $f = 2$ (faulty)



(c) Output quantisation, $f = 1$ (nominal)



(d) Output quantisation, $f = 2$ (faulty)

Figure 4.15: Quantisation of the activity level and time of day shown on box plots of the activity for each hour in the day

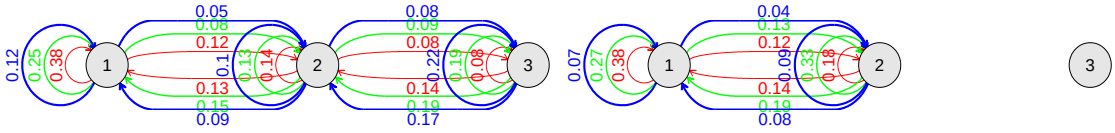
4.4.3 Behavioural relations

Estimating transition probabilities counting observations in the dataset of "normal" and "lame" behaviour respectively gave the state transition relations shown in eq. (4.9). The difference between the two transition relations is shown in eq. (**??**).

The behavioural relation of the normal activity

$$L(z', w|z, 1) = \left(\begin{array}{ccc} 0.38 & 0.13 & 0.00 \\ 0.12 & 0.14 & 0.14 \\ 0.00 & 0.08 & 0.08 \\ \hline 0.25 & 0.15 & 0.00 \\ 0.08 & 0.13 & 0.19 \\ 0.00 & 0.09 & 0.19 \\ \hline 0.12 & 0.09 & 0.00 \\ 0.05 & 0.10 & 0.17 \\ 0.00 & 0.08 & 0.22 \end{array}\right) \quad L(z', w|z, 2) = \left(\begin{array}{ccc} 0.38 & 0.14 & 0.00 \\ 0.12 & 0.18 & 0.00 \\ 0.00 & 0.00 & 0.00 \\ \hline 0.27 & 0.19 & 0.00 \\ 0.13 & 0.33 & 0.00 \\ 0.00 & 0.00 & 0.00 \\ \hline 0.07 & 0.08 & 0.00 \\ 0.04 & 0.09 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{array}\right) \tag{4.9}$$

A graphical interpretation of the automaton in eq. (4.9) are shown in Figure 4.16.



(a) $f = 1$ (nominal) (b) $f = 2$ (faulty)

Figure 4.16: A graph plot of the autonomous automaton

As shown in Figure 4.16 there are no transitions towards state $z' = 3$ in the model describing the activity during lameness. This should/could make the diagnosis more robust although the transition isn't very probable ($\text{Prob}(z_p(k+1) = 3, w_p(k) = w|z_p(k) = 2) \leq 0.09$) in the model describing the healthy behaviour either.

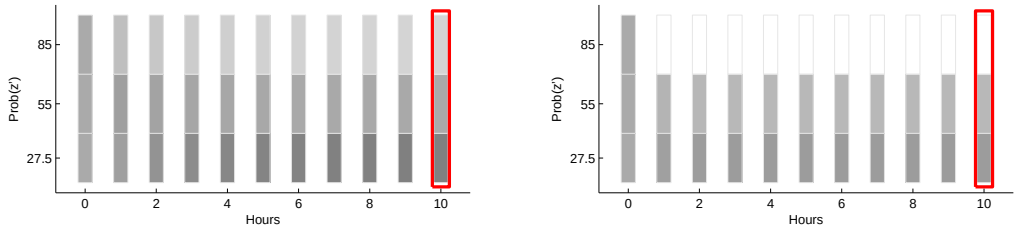
4.4.4 Simulation

For the autonomous stochastic automaton the simulation algorithm is the same as the one described in eq. (3.23).A simulation of the autonomous stochastic automaton which behavioural relations are expressed in eq. (4.9) and Figure 4.16 was performed and the result is shown in Figure 4.17.

4.4.5 Observation

As a test the observation properties for the system in eq. (3.14) are investigated and an observation of the model using a measured output sequence is performed.

An observation of the autonomous stochastic automaton which behavioural relations are expressed in eq. (4.9) and Figure 4.16 was performed and the result is shown in Figure



(a) $f = 1$ (nominal) (b) $f = 2$ (faulty)

Figure 4.17: A greyscale plot of the simulation of the autonomous stochastic automaton

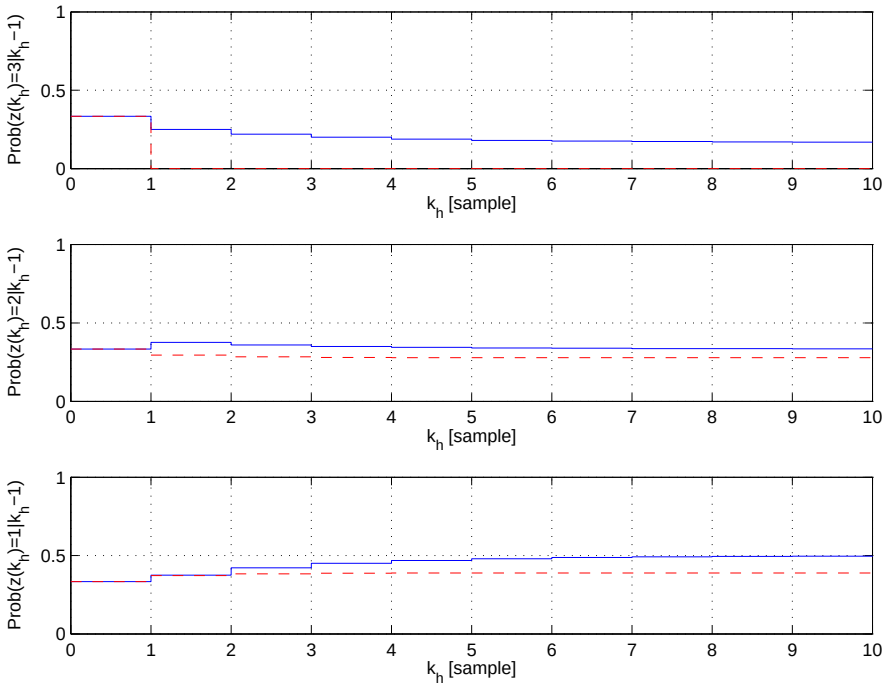
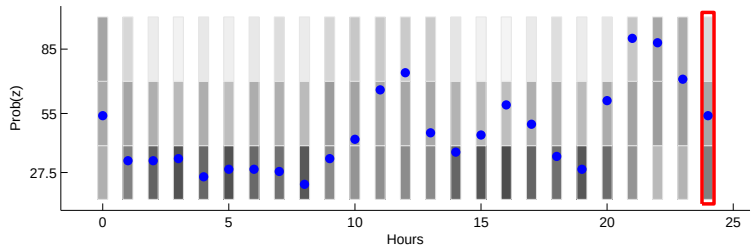


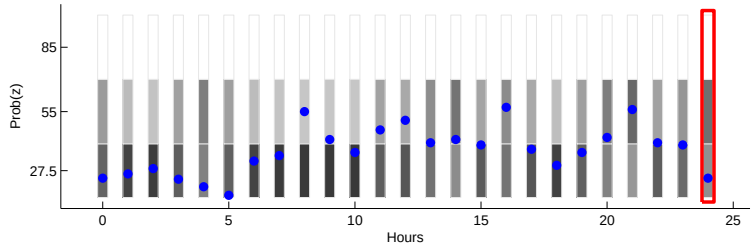
Figure 4.18: A plot of the simulation of the autonomous stochastic automaton. A simulation of the nominal behaviour is shown in blue and the faulty behaviour in red.

4.19.

From looking at Figure 4.19 it can be seen that the observed behaviour the first 24 *hours* in each dataset (nominal and faulty) follows reasonably well with the trajectory of the median values for each hour of the day which is what was to be expected.



(a) $f = 1$ (nominal)



(b) $f = 2$ (faulty)

Figure 4.19: A plot of the observation of the autonomous stochastic automaton plotted in greyscale and the median value of the activity for each hour of the day plotted with blue dots.

5 Application of the models for detecting deviant behaviour

5.1 Diagnosis results for the autonomous cow

For a relatively slowly changing phenomena like leg illness in cows the probability of changing behavioural model within one step should be very low (it is slowly changing seen in respect with the sampling time of $T = 1\ hour$). This is reflected in the selection of the fault transition probabilities in the fault model:

$$G_f(f'|f) = \begin{pmatrix} 0.99999 & 0.00001 \\ 0.00001 & 0.99999 \end{pmatrix} \tag{5.1}$$

The transition probabilities of the fault model reveal that the probability of the measured data sequence belonging to either of the fault models has to be very low before a transition becomes probable. The algorithm in 3.2 was applied on on the test data using the autonomous stochastic automaton with coinciding states and outputs and the transition relation given in eq. (4.2) and the fault model in eq. (5.1).

The diagnosis result is shown in Figure 5.1.

It can be seen from Figure 5.1 that the diagnosis algorithm is able to distinguish between nominal and faulty behaviour apart from a short period in the beginning and in the end and just after the cow becomes lame, at sample 3907.

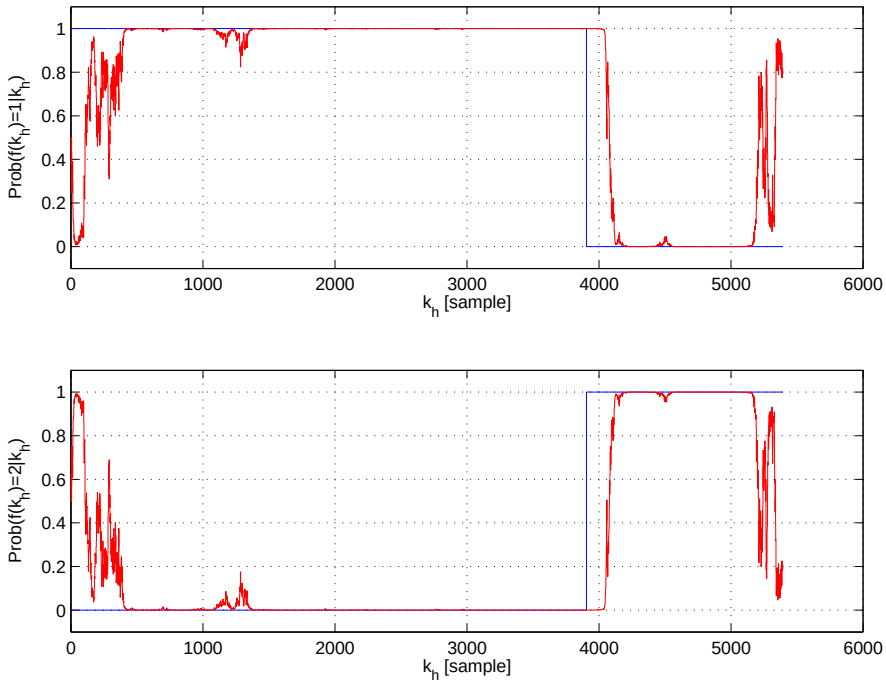


Figure 5.1: A plot of the diagnosis of the autonomous stochastic automaton with coinciding state and output. A behaviour reference is shown with a blue line and the diagnosis result is shown with a red line. Thus the closer the red line is to the blue line, the better the diagnosis result.

5.2 Diagnosis results for the diurnal activity

The algorithm in 3.1 was used for diagnosing on the test data using the behavioural relation given in eq. (4.4) and eq. (4.5) and the fault model in eq. (5.1). The diagnosis result is shown in Figure 5.2.

From Figure 5.3 it can be seen that like in section 5.1 the diagnosis algorithm is able to distinguish between the nominal and faulty behaviour. The result in Figure 5.2 is better than that of Figure 5.1 as the period of indecision in the beginning is shorter and the period with errornous diagnosis towards the end has disappeared. This improvement is a direct consequence of the adding of information in the time of day input v.

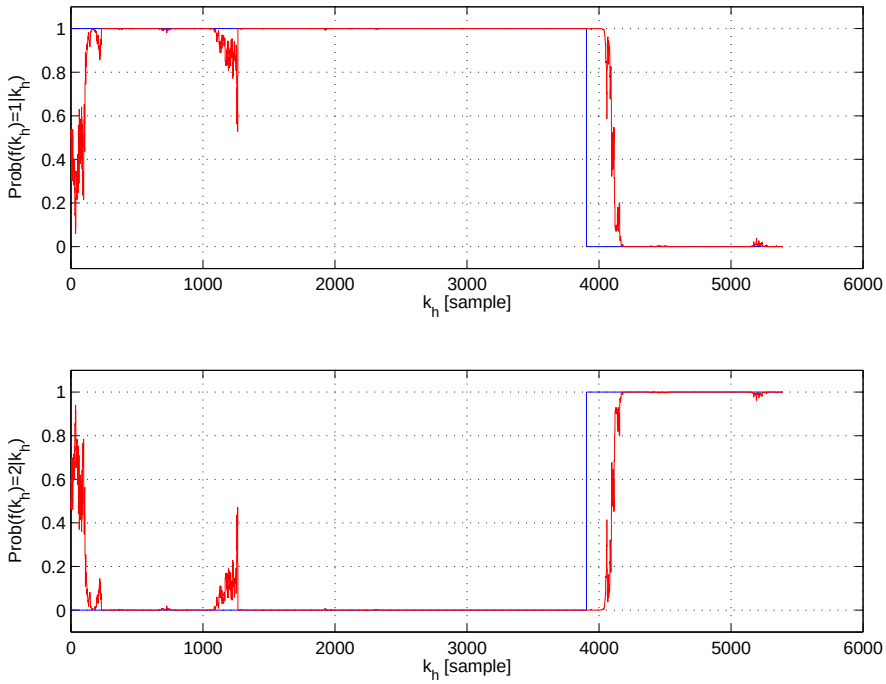


Figure 5.2: A plot of the diagnosis of the stochastic automaton with coinciding state and output

5.3 Diagnosis results for the general activity level of the autonomous cow

The model was diagnosed using the quantised output extracted from the data shown in Figure 2.6 and the fault model in eq. (5.1). The diagnosis result is shown in Figure 5.3.

From Figure 5.3 it can be seen that for the case of modelling the autonomous cows' general activity level the diagnosis algorithm is able to distinguish between the two behavioural scenarios nominal and lame. The results seem clearly better than when modelling the activity of the preceding hour for the autonomous cow. Further comparison is done in section 5.4.

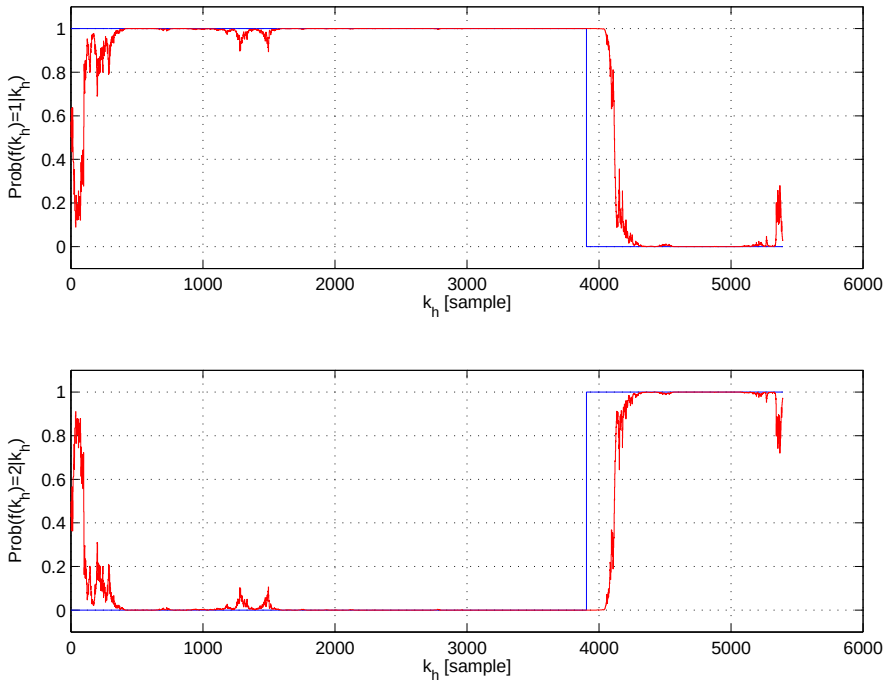


Figure 5.3: A plot of the diagnosis of the autonomous stochastic automaton

5.4 Comparison of the diagnosis results

In order to facilitate comparison of the diagnosis results of the three approaches a plot of the absolute difference between the probability of the data measurements belonging to the nominal behaviour for each of the tree approaches and a "truth" reference was drawn. The "truth" reference is meant to indicate whether a measurement sample belongs to nominal behaviour of faulty behaviour. Denoting the "truth" reference by q the truth reference is selected as

$$q(k) = \begin{cases} 1 & \text{if } k = 0 \dots 3906 \\ 0 & \text{if } k = 3907 \dots 5395 \end{cases} \tag{5.2}$$

and thus $q(k)$ is equal to one in the period belonging to the nominal behaviour and zero in the period belonging to the faulty behaviour. The absolute difference between the probability of the data measurements belonging to the nominal behaviour and the "truth" reference is therefore found as

$$\epsilon_f(k) = |q(k) - \text{Prob}(f(k) = 1|k)| \tag{5.3}$$

for each of the three approaches. $\epsilon_f(k_h)$ was calculated for the whole period for each of the three approaches and is shown in Figure 5.4.

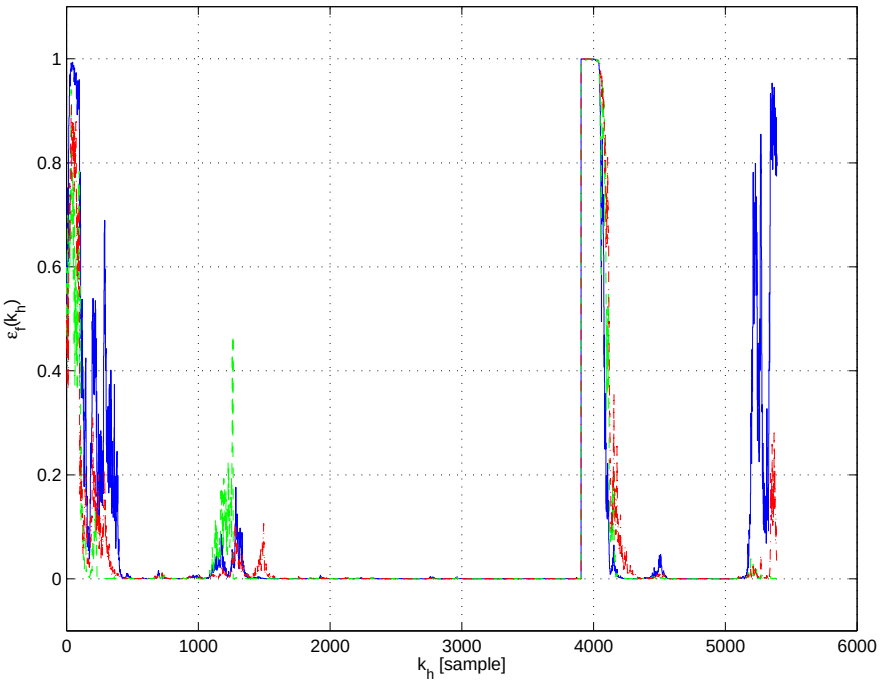


Figure 5.4: A plot of the diagnosis error $\epsilon_f(k_h)$ for the three modelling approaches. The blue line represents $\epsilon_f(k_h)$ for the diagnosis in the first approach, the green dashed line represents $\epsilon_f(k_h)$ for the diagnosis in the second approach and the red dash-dotted line represents $\epsilon_f(k_h)$ for the diagnosis in the third approach.

From Figure 5.4 it can bee seen that there is some difference between the three approaches and that approach two and three are clearly performing better that approach one.

6 Conclusion

This report describes a study on modelling cows activity using discrete event models with the aim of detecting lameness.

Three modelling approaches were comprised in order to investigate the possibility to detect reduced activity due to lameness. Tests of the diagnosis algorithm showed that detection was possible when modelling the activity during the last hour considering the cow as autonomous with respect to its activity behaviour using an autonomous stochastic automaton with coinciding state and output. Improved results were gained by adding an input with information on time of day and considering the cow as being affected by which time of day it is. Modelling general activity level instead of the activity during last hour also gave improved results.

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