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The coherence of reverberant sound fields

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A new method of measuring spatial correlation functions in reverberant sound fields is presented. It is shown that coherence functions determined with appropriate spectral resolution contain the same information as the corresponding correlation functions, and that measuring such coherence functions is a far more efficient way of obtaining this information. The technique is then used to verify theoretical predictions of the spatial correlation between various components of the particle velocity in a diffuse sound field. Other possible applications of the technique are discussed and illustrated with experimental results obtained in an ordinary room. © 2000 Acoustical Society of America. [S0001-4966(00)00807-9]

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INTRODUCTION

Reverberation rooms are used in a variety of standardized measurements, e.g., in measuring the absorption of materials, the sound power of noise sources, and the transmission loss of partitions. They are also used for testing satellite structures at high sound-pressure levels. All measurements in reverberation rooms are based on the assumption that the sound field is diffuse; therefore, it is of interest to validate the diffuse field theory and to examine the agreement between theoretical predictions and the behavior of sound fields in real rooms.

The diffuse sound field is an idealized concept, and the sound field in any real room differs in fundamental respects from a diffuse field. One possible way of testing the diffuseness of the sound field in a given room might be to compare theoretically predicted spatial correlation functions with measurements. This idea goes back to the middle of the 20th century; the spatial autocorrelation of the sound pressure in a perfectly diffuse sound field was derived by Cook *et al.* as early as in 1955.¹ Cook and his coauthors also made an experimental investigation, using warble tones in a reverberation room equipped with a rotating vane.¹ Since then, many authors have studied pressure correlation functions in diffuse fields, experimentally²⁻⁸ and theoretically.^{3,6-13} Spatial correlation functions of other quantities than the sound pressure in a diffuse sound field were derived more than 20 years ago,^{10,12} but have never been verified experimentally, although they have found application in various areas, e.g., in active noise control^{14,15} and in prediction of the response of a plate to excitation by a diffuse sound field.¹⁶

The purpose of this paper is (i) to discuss the concept of a diffuse sound field, (ii) to present a new, efficient method of determining spatial correlation functions in a room, (iii) to present experimental results that validate the theoretical predictions in a real room, and (iv) to discuss possible applications.

I. CORRELATION AND COHERENCE IN A DIFFUSE SOUND FIELD

A. A brief review of diffuse-field theory

Many different models of diffuse fields have been described in the literature, including models that make use of concepts such as ergodicity, entropy, and “mixing.”^{1,12,17-20} However, most acousticians would agree on a definition that involves sound coming from all directions. This leads to the concept of a sound field in an unbounded medium generated by distant, uncorrelated sources of random noise evenly distributed over all directions. Since the sources are uncorrelated there would be no interference phenomena in such a sound field, and the field would therefore be completely homogeneous and isotropic. For example, the sound pressure level would be the same at all positions, and temporal correlation functions between linear quantities measured at two points would depend only on the distance between the two points. The time-averaged sound intensity would be zero at all positions. An approximation to this “perfectly diffuse sound field” might be generated by a number of loudspeakers driven with uncorrelated noise in a large anechoic room, as in the experimental investigations described in Refs. 3 and 7. The sound field in a reverberation room driven with noise from one source is quite different, of course.

A more realistic model of the sound field in a reverberation room above the Schroeder frequency²¹ describes the sound field as composed of plane waves with random phases arriving from all directions.^{12,22} This is a pure-tone model, and therefore the various plane waves interfere. The result is a sound field in which the sound-pressure level depends on the position, although the probability of the level being in a certain interval is the same at all positions. Temporal correlation functions between linear quantities measured at two positions depend on the positions, although the probability of a given correlation function being in a certain interval depends only on the distance between the two points. The time-averaged sound intensity assumes a finite value at all positions.^{23,24} Since infinitely many plane waves with completely random phases are assumed, this model is also idealized, but it gives a good approximation to the sound field in

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a reverberation room driven with a pure tone with a frequency above the Schroeder frequency. With averaging over an ensemble of realizations, the perfectly diffuse field described above is obtained. An approximation to ensemble averaging is obtained if the room is equipped with a rotating diffuser.^{12,17}

The second model can be extended to excitation with a band of noise.^{25,26} If a reverberation room is driven with a pure tone whose frequency is shifted slightly, the phases and amplitudes of the plane waves that compose the sound field are changed, which means that the entire interference pattern is changed. The longer the reverberation time of the room, the faster the sound field will change as a function of the frequency.²⁷ It now follows that excitation with a band of noise corresponds to averaging over the band. As a result the sound field becomes more uniform,^{25,26} temporal correlation functions between linear quantities measured at pairs of positions tend to depend less on the particular positions,⁹ and the sound intensity is reduced.²⁴ The effect of this spectral averaging depends not only on the bandwidth of the excitation (or the analysis) but also on the damping of the room; the longer the reverberation time, the more efficient the averaging. It is apparent that there are many similarities between the sound field in a room driven with noise and the perfectly diffuse sound field, but there are also important differences. It can also be concluded that diffuseness at low frequencies requires a large room, and that long reverbera-

tion times (within the limits determined by the requirement of sufficient modal overlap²¹) are favorable.

B. Space-time correlation in a pure-tone “diffuse” sound field

The theory has been described elsewhere,^{10,12,28} so only the results will be presented here. Using the second model described above (the sum of plane waves with random phases and amplitudes), it can be shown that the normalized space-time correlation of two pressure signals separated in distance by r and in time by τ is

$$\rho_{pp}(r, \tau) = \frac{\sin(\omega_o r/c)}{\omega_o r/c} \cos(\omega_o \tau), \quad (1)$$

where ω_o is the radian frequency and c is the speed of sound, as already shown in the middle of the 1950s by Cook *et al.*¹ The corresponding correlation function between the pressure and a component of the particle velocity is

$$\rho_{pu_r}(r, \tau) = \sqrt{3} \frac{\sin(\omega_o r/c) - (\omega_o r/c) \cos(\omega_o r/c)}{(\omega_o r/c)^2} \sin(\omega_o \tau). \quad (2)$$

The space-time correlation between two particle velocity components in the direction of the line that joins the two points is

$$\rho_{u_r u_r}(r, \tau) = 3 \frac{(\omega_o r/c)^2 \sin(\omega_o r/c) + (2\omega_o r/c) \cos(\omega_o r/c) - 2 \sin(\omega_o r/c)}{(\omega_o r/c)^3} \cos(\omega_o \tau), \quad (3)$$

and the space-time correlation of two parallel particle velocity components perpendicular to the line that joins the two points is

$$\rho_{u_y u_y}(x, \tau) = 3 \frac{\sin(\omega_o x/c) - (\omega_o x/c) \cos(\omega_o x/c)}{(\omega_o x/c)^3} \cos(\omega_o \tau). \quad (4)$$

Equation (1) has been verified experimentally by many authors.^{1-5,7,8}

C. Space-time correlation in a perfectly diffuse sound field

The perfectly diffuse sound field is not a pure-tone field. In the perfectly diffuse sound field, the normalized temporal correlation functions between pressure and particle velocity signals measured at any pair of positions equal the expressions given by Eqs. (1), (2), (3), and (4) multiplied by a factor of $\sin(\tau\Delta\omega/2)/(\tau\Delta\omega/2)$, provided that $\Delta\omega \ll \omega_o$, where $\Delta\omega$ is the bandwidth of the analysis. (If the bandwidth is wider a correction described in Ref. 13 can be applied.) Conversely, at a

given fixed time the corresponding spatial correlation functions equal the expressions given by Eqs. (1), (2), (3), and (4) with $\tau=0$.

D. Coherence in a perfectly diffuse sound field

By definition, the coherence of two signals is the squared modulus of the cross spectrum normalized with the two autospectra,²⁹

$$\gamma_{xy}^2(\omega) = \frac{|S_{xy}(\omega)|^2}{S_{xx}(\omega)S_{yy}(\omega)}. \quad (5)$$

However, the coherence is also related to the normalized cross-correlation function. It can be shown that, provided that $\Delta\omega \ll \omega_o$, the coherence estimated with finite spectral resolution is identical with the squared envelope of the normalized cross-correlation at zero time delay, that is,

$$\gamma_{xy}^2(\omega_o, \Delta\omega) = \rho_{xy}^2(0, \omega_o, \Delta\omega) + \hat{\rho}_{xy}^2(0, \omega_o, \Delta\omega), \quad (6)$$

where $\rho_{xy}(\tau, \omega_o, \Delta\omega)$ is the normalized cross-correlation of the bandpass-filtered signals $x(t, \omega_o, \Delta\omega)$ and $y(t, \omega_o, \Delta\omega)$, and $\hat{\rho}_{xy}(\tau, \omega_o, \Delta\omega)$ is the Hilbert transform of $\rho_{xy}(\tau, \omega_o, \Delta\omega)$.³⁰ From this equation, in combination

$$\gamma_{pp}^2(\omega, r) = \left(\frac{\sin(\omega r/c)}{\omega r/c} \right)^2, \quad (7)$$

$$\gamma_{pu_r}^2(\omega, r) = 3 \left(\frac{\sin(\omega r/c) - (\omega r/c) \cos(\omega r/c)}{(\omega r/c)^2} \right)^2, \quad (8)$$

$$\gamma_{u_r u_r}^2(\omega, r) = 9 \left(\frac{(\omega r/c)^2 \sin(\omega r/c) + (2\omega r/c) \cos(\omega r/c) - 2 \sin(\omega r/c)}{(\omega r/c)^3} \right)^2, \quad (9)$$

$$\gamma_{u_y u_y}^2(\omega, x) = 9 \left(\frac{\sin(\omega x/c) - (\omega x/c) \cos(\omega x/c)}{(\omega x/c)^3} \right)^2. \quad (10)$$

Equation (7) was shown by Piersol in 1978.³¹

E. Coherence in a reverberation room

The sound field in a real room excited by a source of noise is not composed of infinitely many incoherent sound waves arriving from all direction. In a room without devices such as rotating diffusers, driven by a single source, it is much more realistic to describe the sound field as composed of a large but finite number of coherent waves. Such a system is linear and time invariant, from which it follows that all linear quantities are fully coherent, that is, all coherence functions between pressure and particle velocity signals recorded in the room equal unity.³² In practice, however, coherence functions are estimated using fast Fourier transform (FFT) analyzers with a finite spectral resolution, and under certain conditions the estimated functions approach the functions given by Eqs. (7), (8), (9), and (10). The experimental technique described in the next section is based on this observation.

It takes a *very* fine spectral resolution (a bandwidth of less than $1/T_{60}$, where T_{60} is the reverberation time of the room) to show that the entire sound field is actually fully coherent. By contrast, if the room is driven with pseudorandom noise synchronized to the FFT analysis, the measured coherence is unity, irrespective of the spectral resolution.³²

II. COHERENCE VERSUS CORRELATION

The experimental techniques described in the literature, e.g., in Refs. 5 and 7, are based on measurement of correlation functions in different frequency bands. Alternatively, one might measure the corresponding coherence functions, using an appropriate spectral resolution. It follows from Eq. (6) that a coherence function determined with finite spectral resolution contains the same information as many correlation functions. That the spectral resolution is important can be explained as follows. Since the autocorrelation function of a random signal of a finite bandwidth decays to insignificance

with the foregoing, it now follows that the coherence functions between pressure and particle velocity signals measured at any pair of positions in an ideal diffuse sound field are

for a time shift that is inversely proportional to the bandwidth, it follows that the sound waves in a reverberation room *seem* to be practically uncorrelated, i.e., their correlation coefficient is much less than unity, if the time delay between the various waves exceeds the decay time of the autocorrelation, and this will be the case if the bandwidth of the analysis exceeds c/l , where l is a characteristic dimension of the room. Because the sound waves in a reverberation room are reflected many times before they die out, the sound waves that compose the field are essentially uncorrelated even with a somewhat finer spectral resolution. The spectral resolution must not be too fine, though, because in the limit of $\Delta\omega \rightarrow 0$ the estimated coherence will approach the true value, unity, just as estimated normalized correlation functions will approach sinusoids with an amplitude of unity, irrespective of the properties of the sound field. Expressed in simple terms, the waves that compose the sound field *seem* to be generated by independent sources since the analysis is based on short signal segments, and therefore the measured coherence functions will approach the expressions given by Eqs. (7), (8), (9), and (10), provided that the number of waves is sufficient and their directions are sufficiently random. The purpose of the test is to examine whether this is the case.

The advantage of the proposed technique is that many correlation functions are, in effect, determined at the same time.

III. EXPERIMENTAL RESULTS IN A REVERBERATION ROOM

To test the validity of the foregoing considerations and Eqs. (7), (8), (9), and (10), some experiments have been carried out in a large (245 m^3) reverberation room with a reverberation time of about 5 s. Two sound-intensity probes of type Brüel & Kjær 3545 (“*p-p* probes” based on the finite difference principle) were used to measure the sound pressure and components of the particle velocity. It is prob-

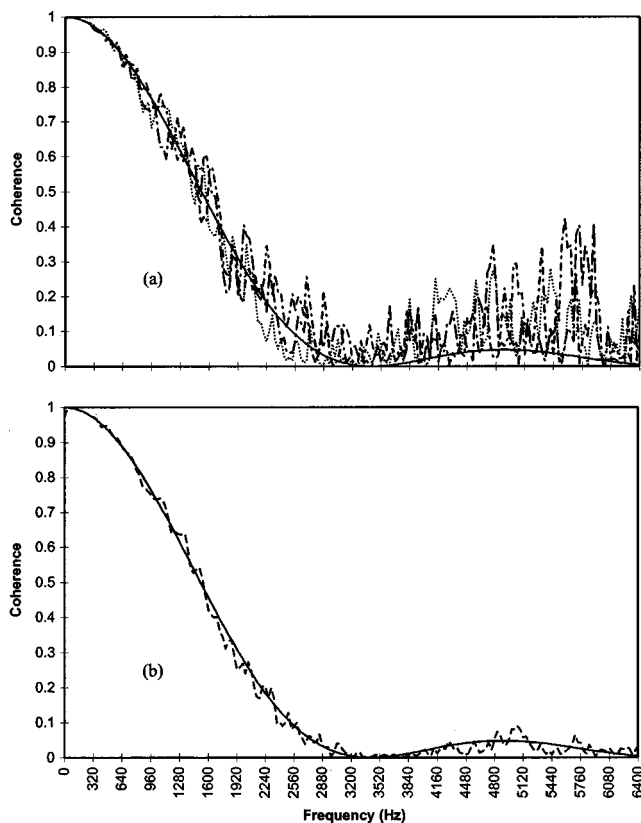


FIG. 1. Coherence between two pressure signals at positions 5 cm apart in a reverberation room, measured with a spectral resolution of 32 Hz. —, theoretical prediction [Eq. (7)]. (a) Measurements at three pairs of positions; (b) ---, measurement in which the cross and auto spectra have been averaged over ten pairs of positions.

ably advantageous to use a p - p sound intensity probe with the microphones in the “face-to-face” configuration separated by a solid spacer also in measuring pressure coherence functions, because this ensures that the distance between the acoustic centers of the microphones is almost independent of the frequency and the nature of the sound field.^{33,34} A single pressure microphone of type Brüel & Kjær 4192 was used in combination with one of the intensity probes in measuring the pressure-particle velocity coherence. The coherence functions were determined with a dual-channel FFT analyzer of type Brüel & Kjær 2035. The room was driven with random noise generated by the analyzer.

Figure 1(a) shows the result of measuring the coherence between two pressure signals recorded a distance of 5 cm from each other in the room with a spectral resolution of 32 Hz. Measurements at three different pairs of positions are shown. There are similarities between the measured coherence functions and the theoretical prediction [Eq. (7)], but it is also apparent that the measured functions deviate erratically from the prediction and from each other. However, with averaging of the cross and auto spectra over ten pairs of positions chosen at random,

$$\gamma_{xy}^2(\omega, r) = \frac{|\langle S_{xy}(\omega, r) \rangle|^2}{\langle S_{xx}(\omega) \rangle \langle S_{yy}(\omega) \rangle}, \quad (11)$$

where $\langle \rangle$ indicates the averaging, the agreement between predicted and measured coherence becomes very good, as

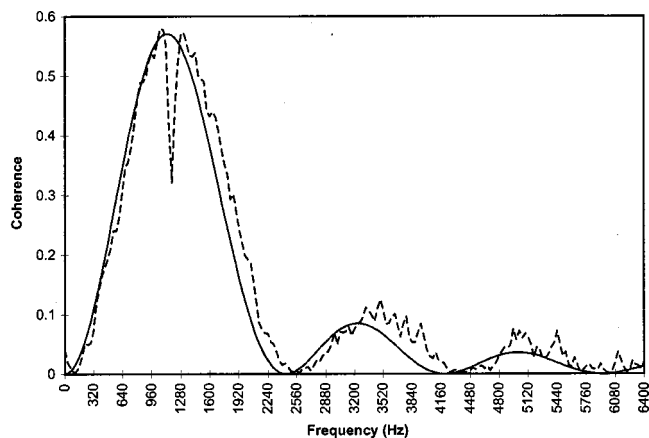


FIG. 2. Coherence between the pressure and a component of the particle velocity at a position 10 cm away in a reverberation room, measured with a spectral resolution of 32 Hz. —, theoretical prediction [Eq. (8)]; ---, measurement in which the cross and auto spectra have been averaged over ten pairs of positions.

can be seen in Fig. 1(b). In practice the spectral averaging process was simply stopped after a suitable number of spectra had been averaged, the probe was moved to another position, and the averaging was continued.

To measure the corresponding coherence functions between pressure and particle velocity components and between various components of the particle velocity directly with the analyzer would require a multichannel analyzer. Such a device was not available. Instead, signals proportional to the pressure gradients were determined from the two pressure signals from each intensity probe using analog subtracting circuits.

In Fig. 2 a comparison of the coherence between sound pressure and particle velocity and the theoretical prediction is shown. It is worth noting that the acoustic center of the half-inch microphone used in measuring the pressure is about 5 mm in front of the microphone.³⁵ If this is taken into account the agreement is almost perfect, except for the inexplicable trough in the experimental curve at 1.1 kHz.

Figures 3(a) and 3(b) show the results of similar measurements of the coherence between two components of the particle velocity in the direction of the line that joins the two points and perpendicular to the line, respectively. Again, there is very good agreement with the diffuse field prediction, except at very low frequencies where phase and amplitude mismatch may have affected the estimates of the particle velocity.

Finally, a few additional measurements with the reverberation room driven with pseudorandom noise synchronized to the FFT records were carried out. In the absence of transducer noise and nonlinearities, all coherence functions should equal unity,³² and without spatial averaging the measured coherence function between two pressure signals recorded a distance of 5 cm from each other was indeed close to unity, as shown in Fig. 4. Since driving the room with pseudorandom noise in effect corresponds to driving it with many pure tones that are analyzed separately, the pure-tone “diffuse”-field theory applies. Therefore we can expect Eqs. (1), (2), (3), and (4) to describe the spatial correlation, and

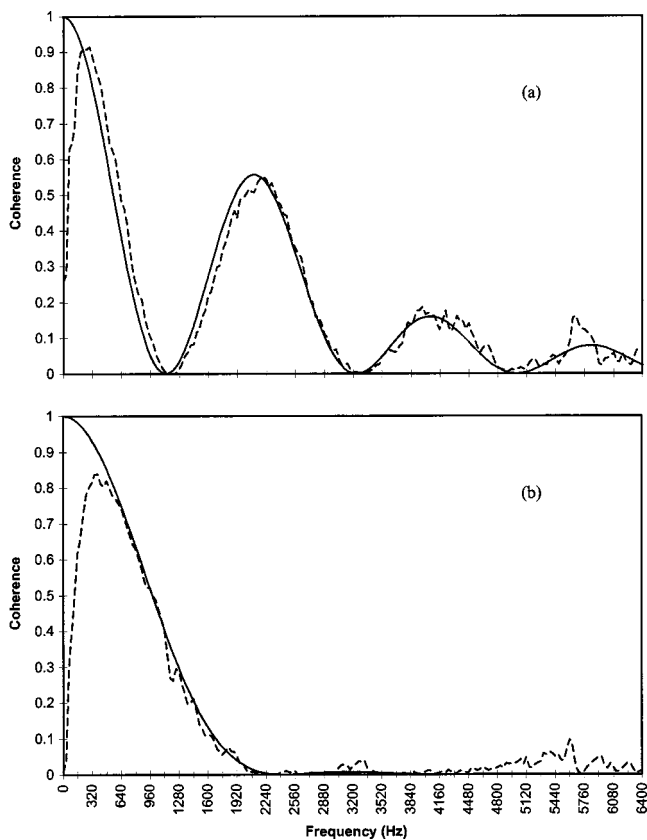


FIG. 3. Coherence between two components of the particle velocity at positions 10 cm from each other in a reverberation room, measured with a spectral resolution of 32 Hz. (a) Components in the same direction. —, theoretical prediction [Eq. (9)]; ---, measurement in which the cross and auto spectra have been averaged over ten pairs of positions. (b) Perpendicular components. —, theoretical prediction [Eq. (10)]; ---, measurement in which the cross and auto spectra have been averaged over ten pairs of positions.

Eqs. (7), (8), (9), and (10) to describe the spatial coherence. However, without the inherent frequency averaging of a normal FFT analysis, much more spatial averaging is needed, which can be seen by comparing the erratically fluctuating curve in Fig. 4 with Fig. 1(b). In both cases averaging over ten pairs of positions was carried out. With continuous averaging over many positions and directions, carried out by moving the intensity probe slowly around far from the source, the agreement with the theoretical prediction becomes very good.

IV. EXPERIMENTAL RESULTS IN AN ORDINARY ROOM

Further experimental results were determined in an ordinary room of about 180 m^3 with a reverberation time of about 0.5 s. One would not expect the sound field in this room to be particularly diffuse, and the lack of diffuseness is confirmed by the relatively large deviations between the measured and theoretical pressure coherence functions shown in Fig. 5(a). However, with averaging over many positions and directions the agreement between prediction and measurement becomes almost perfect, as can be seen in Fig. 5(b).

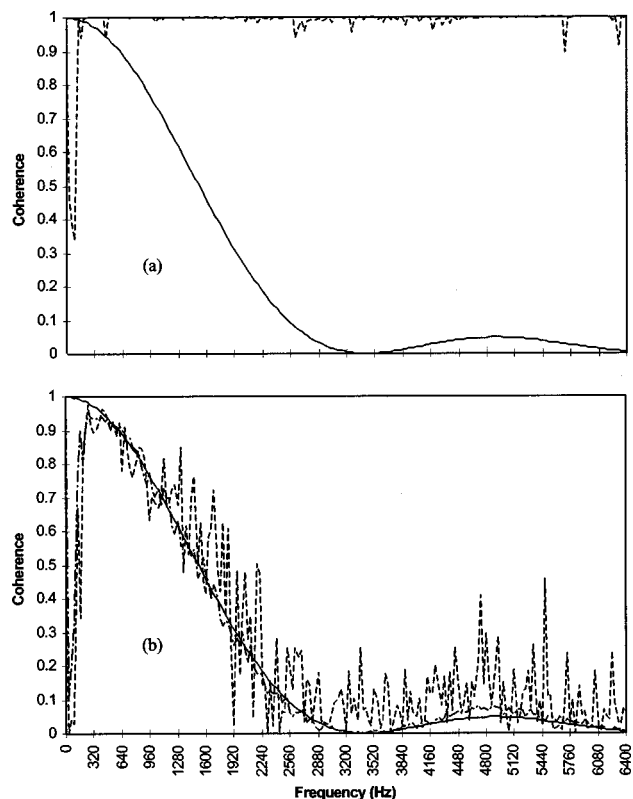


FIG. 4. Coherence between two pressure signals at positions 5 cm apart in a reverberation room driven with synchronized pseudorandom noise generated by the FFT analyzer, measured with a spectral resolution of 32 Hz. —, theoretical prediction [Eq. (7)]. (a) ---, measurement at one position. (b) ---, measurement with averaging of the cross and auto spectra over ten pairs of positions; —, measurement with continuous averaging over many positions.

Instead of moving the intensity probe around in the room, one might move the source that generates the sound field, and with sufficient averaging one should expect the measured pressure coherence to approach Eq. (7). This is confirmed by the result presented in Fig. 6.

Even in the direct field close to the source where the sound field is far from being diffuse, one can, with appropriate spatial averaging, measure pressure coherence functions that approach the theoretical functions of a perfectly diffuse sound field (although some systematic deviations can be seen), as demonstrated by Fig. 7(b). However, as can be seen in Fig. 7(a), without spatial averaging the measured pressure coherence at any position and in any direction close to the source is completely different from the theoretical value for a diffuse sound field.

V. DISCUSSION

There are strong similarities between the sound field in a reverberation room and a perfectly diffuse field, but there is also a significant difference that reveals itself by the influence of the spectral resolution on the estimated coherence functions: In the limit of very fine spectral resolution, it becomes apparent that the sound field in a room unlike the diffuse field is perfectly coherent. However, estimated with a

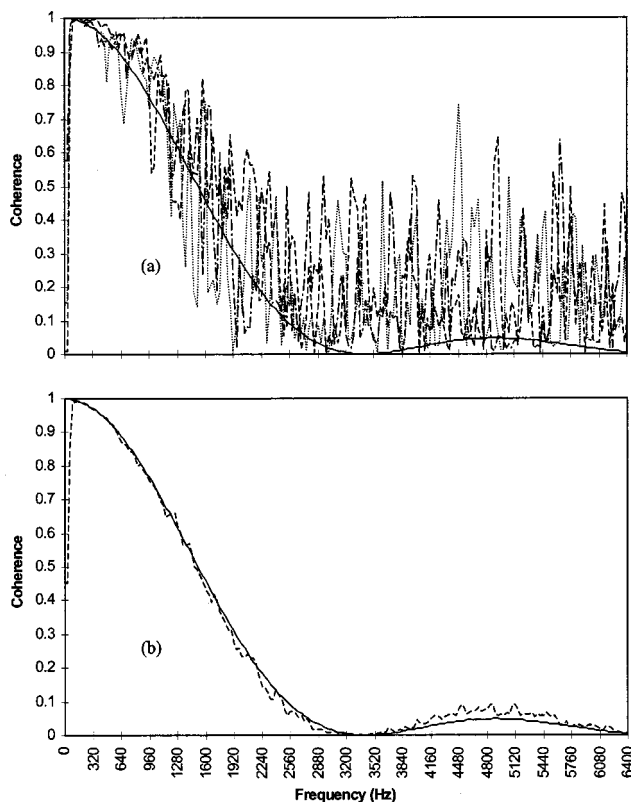


FIG. 5. Coherence between two pressure signals at positions 5 cm apart in an ordinary room, measured with a spectral resolution of 32 Hz. —, theoretical prediction [Eq. (7)]. (a) Measurements at three pairs of positions; (b) ---, measurement in which the cross and auto spectra have been averaged continuously over many positions.

less fine resolution the coherence functions approach those predicted by the diffuse-field theory. The longer the reverberation time of the room the less averaging over frequency is needed, and the finer the spectral resolution can be. Another difference is the influence of spatial averaging. Without spatial averaging, the agreement between measurements and predictions for the perfectly diffuse sound field is no more than tolerable even in a large reverberation room with a long

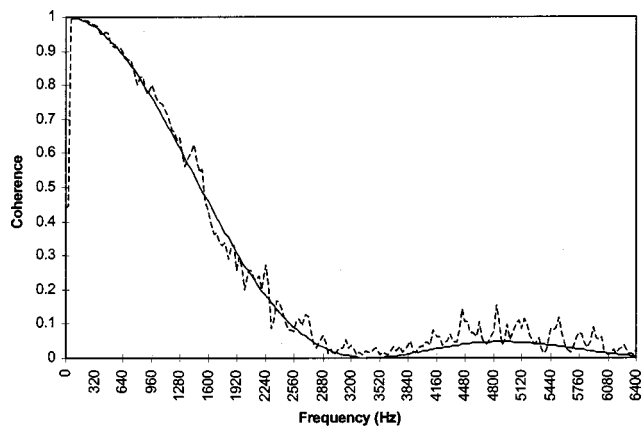


FIG. 6. Coherence between two pressure signals at positions 5 cm apart in an ordinary room, measured with a spectral resolution of 32 Hz. —, theoretical prediction [Eq. (7)]; ---, measurement in which the source has been moved continuously over many positions.

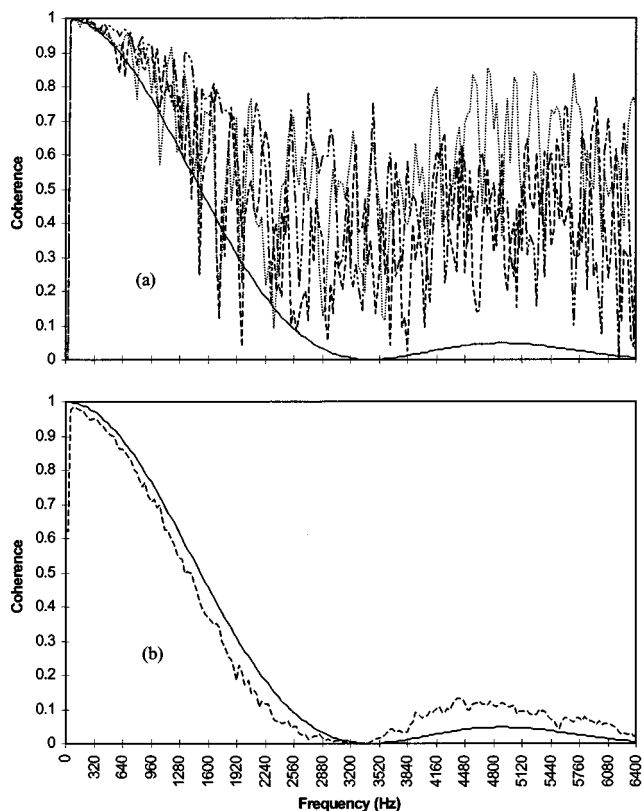


FIG. 7. Coherence between two pressure signals at positions 5 cm apart in an ordinary room, measured with a spectral resolution of 32 Hz. —, theoretical prediction [Eq. (7)]. (a) Measurements at three pairs of positions one meter from the source; (b) ---, measurement in which the cross and auto spectra have been averaged continuously over many directions at positions 1 m from the source.

reverberation time, but with averaging over just a few points the agreement becomes almost perfect. However, the amount of averaging that is required depends on the quality and size of the room. It is apparent from the results obtained in the ordinary room that with sufficient spatial averaging one can “prove” that almost any sound field is diffuse. A rotating diffuser (as used in Refs. 1, 2, and 8) would reduce the amount of spatial averaging required. Exciting the room with several loudspeakers driven with independent noise signals (as in Ref. 7) or moving the source around during the measurement would have a similar effect. The better the agreement between theory and measurement *with the same amount of spatial averaging*, the more diffuse is the sound field, and the more one would expect other predictions from the diffuse-field theory to be valid.

It is more difficult to measure particle velocity than sound pressure or even sound intensity, because not only phase but also amplitude mismatch between the two pressure microphones gives rise to errors, in particular at low frequencies. This cautions against using measures based on particle velocity, and since Eqs. (2) and (3) have been derived by differentiating Eq. (1) once and twice,¹² there is no reason to expect these functions or the corresponding coherence functions to be more sensitive to deviations from diffuseness.

Applications of the theoretical correlation functions in a diffuse sound field were mentioned in the Introduction. Pos-

sible applications of the experimental technique include evaluating the quality of reverberation rooms. However, it is clear from the results presented here that a comparison of results obtained in different rooms would not make sense unless the same averaging procedure was used.

VI. CONCLUSIONS

Spatial coherence functions in a reverberant sound field have been examined experimentally. The theoretical coherence functions between pressure and particle velocity components in a perfectly diffuse sound field have been shown to give very good predictions of the corresponding functions in a large reverberation room, provided that the analysis is carried out with a spectral resolution of the order of c/l , where l is a characteristic dimension, and provided that a certain amount of spatial averaging is carried out. Since these coherence functions are closely related to the corresponding spatial correlation function, the results verify the spatial correlation functions of reverberant fields derived more than 20 years ago. However, it is important to note that a measured coherence function in perfect agreement with predictions for a diffuse sound field by no means proves that the sound field is diffuse.

The measurement technique described in this paper is far more efficient than measuring correlation coefficients at different distances in a number of frequency bands. Measuring coherence has the advantage that many correlation functions, in effect, are determined at the same time.

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