Wake decay constant for the infinite wind turbine array

Application of asymptotic speed deficit concept to existing engineering wake model

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Published in:
EWEC 2010 Proceedings online

Publication date:
2010

Document Version
Publisher's PDF, also known as Version of record

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DTU

Application of asymptotic speed deficit concept to existing engineering wake model

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Acknowledments:
Dong, Vattenfall for providing data
EU Upwind, Danish Strategic Research Council, Energinet.dk (PSO) for sponsoring the work

WAKE DECAY FOR THE INFINITE WIND TURBINE ARRAY

Application of asymptotic speed deficit concept to existing engineering wake model

Outline

- Background
- Asymptotic speed deficit from boundary layer considerations
- “WAsP Park” model details
- Asymptotic speed deficit of the “WAsP Park” model
- Adjustment of WAsP Park model
- Comparative wind farm predictions
- Conclusions

Background

Very large wind farms:

- Standard wake models seems to underpredict wake effects.

Recent investigations by Sten Frandsen [1, 2]:

- The reason is the lack of accounting for the effect a large wind farm may have on the atmospheric boundary layer, e.g. by modifying the vertical wind profile.

- In some way the effect of an extended wind farm resembles that of a change in surface roughness: increased equivalent roughness length.

Idea:

- While more detailed models are underway [3],
  modify the existing WAsP Park engineering wind farm wake model to take this boundary-layer effect into account.

[1] Frandsen, S.T., Barthelmie, R.J., Pryor, S.C., Rathmann, O., Larsen, S., Højstrup, J. and M. Thøgersen , Analytical modelling of wind speed deficit in large offshore wind farms. Wind Energ. 2006, 9: 39-54.
[2] Frandsen, S: The wake-decay constant for the infinite row of wind turbine rotors. Draft paper (2009).
[3] Rathmann O., Frandsen S, Barthelmie R., Wake Modelling for intermediate and large wind farms, Paper BL3.199, EWEC 2007

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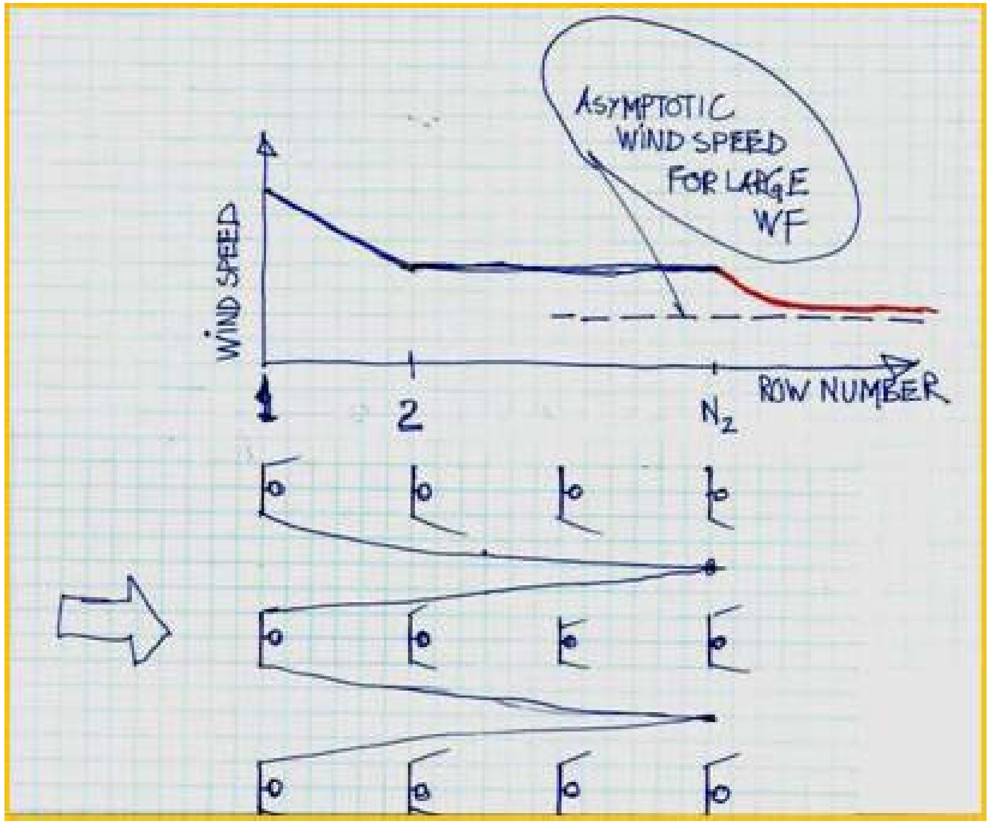
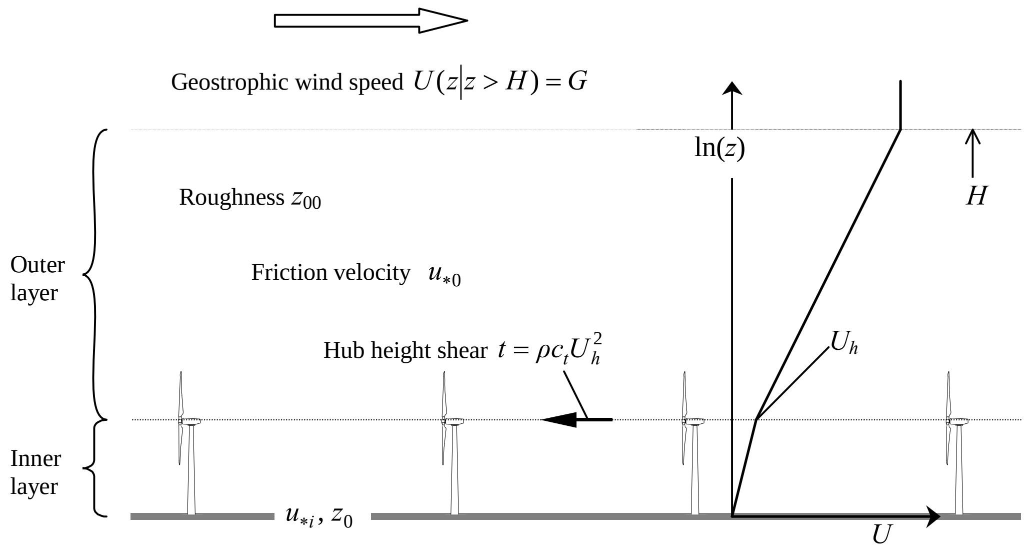
When should a wind farm be considered as large/infinite?

(Hand drawing illustrating the initial idea)



BL-Limited infinite wind farm



Jump in friction velocity at hub-height due to rotor thrust: $\rho(u_*^{eff})^2 = \rho(u_{*i})^2 + t$

Approximation: homogeneously distributed thrust c_t

$c_t = \pi/8\ C_t/(s_r\ s_f)$, $t = \rho C_t U_h^2$

s_r and s_f: dimensionless* WTG-distances (along- and across-wind) *by D_{rotor}

Z<h: profile according to ground surface friction velocity u_{*i} / roughness z_0.

Z>h: profile according to increased friction velocity u_*^{eff}(= u_{*0}) / roughness z_0^{eff} (=z_{00}).

Equivalent, effective surface roughness: $z_0^{eff} = h_H \cdot \exp\left(-\kappa / \sqrt{c_t + \left(\kappa / \ln(h_H / z_0)\right)^2}\right)$

Asymptotic speed deficit from boundary layer considerations (3)

Approximate geostrophic drag-law

$$G \approx \frac{u_*}{\kappa}\left(\ln\left(\frac{G}{fz_0}\right) - A_*\right)$$

General hub-height wind speed:

$$U(h) = \frac{G}{1 + \left(\ln \dfrac{G}{h \cdot f} - A_*\right) i}$$

Free flow: $i_0 = 1/\ln\dfrac{h}{z_0}$

Flow over wind farm: $i_{Tot} = \sqrt{{i_0}^2 + {i_{add}}^2}, \quad i_{add} = \dfrac{\sqrt{c_t}}{\kappa}$

Relative speed deficit ε:

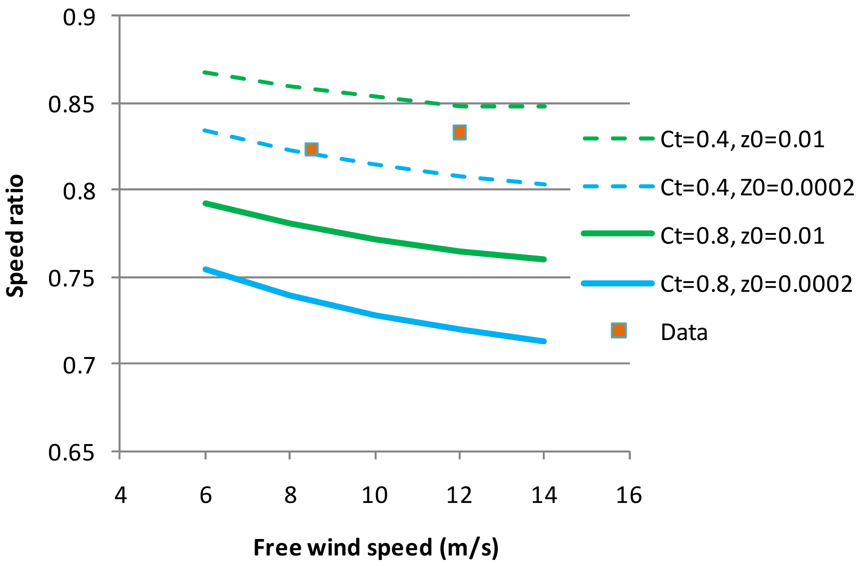
$$1 - \varepsilon = \frac{1 + \ln\left(\dfrac{G}{h \cdot f'}\right) i_0}{1 + \ln\left(\dfrac{G}{h \cdot f'}\right) i_{Tot}}$$

Asymptotic speed deficit from boundary layer considerations (3)

Comparison with wind farm (Horns Rev):

$s_r \approx s_f \approx 7$, h=80m, D_R = 60 m

Wake deficit about 50% of the BL-limiting value.
Horns Rev wind farm NOT "infinite".

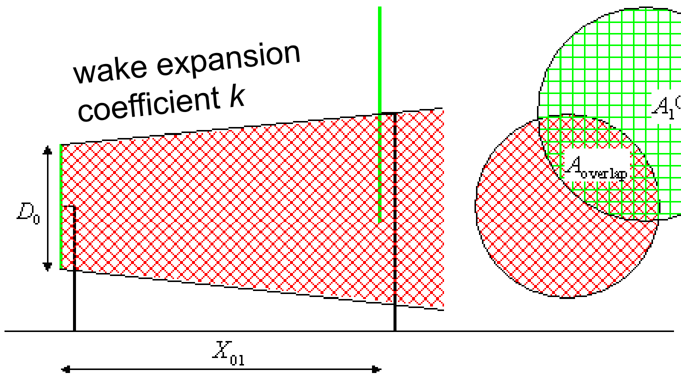


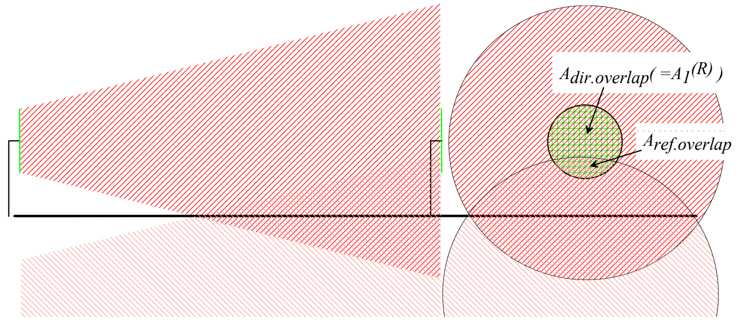
Horns Rev	
Distance for severe wake interference (k_{wake}=0.075)	Actual extension
7.5 km	5 km

Power density (W/m²) [4]	
Horns Rev 2MW turb's (observed)	Entire North Sea 5 MW turb's (Frandsen – BL-limited)
2.9	1.8

[4] Barthelmie,R.J., Frandsen,S.T., ., Pryor, S.C., Energy dynamics of an infinitely large offshore wind farm., Paper 124, European Offshore WInd 2009 Conference and Exhibition, Stockholm, Sweden (Sept. 2009).

Wake Evolution and speed deficit [5,6]





Speed deficit from single wake:

$$\delta V_{01}^{(type)} = U_0\left(1-\sqrt{1-C_t}\right)\left(\frac{D_0}{D_0+2kX_{01}}\right)^2 \frac{A_{(type)overlap}}{A_1^{(R)}}, \quad (type) = "dir.", \quad "ref."$$

Resulting speed deficit at a downwind turbine:

$$\delta V_{turb}^2 = \sum_{i \in upw.turb's} \left((\delta V_{i,turb}^{(dir.)})^2 + (\delta V_{i,turb}^{(ref.)})^2 \right)$$

[5] N.O.Jensen, A Note on Wind Generator Interaction, Risø-M-2411, Risø National Laboratory 1983.
[6] I.Katic, J.Højstrup, N.O.Jensen, A Simple Model for Cluster Effeciency, Paper C6, EWEC 2006, Rome, Italy, 1986.

Speed deficit for a turbine in an infinite wind farm

Speed deficit the same for all turbines, thus also the turbine thrusts.

Infinite (convergent!!) sum:

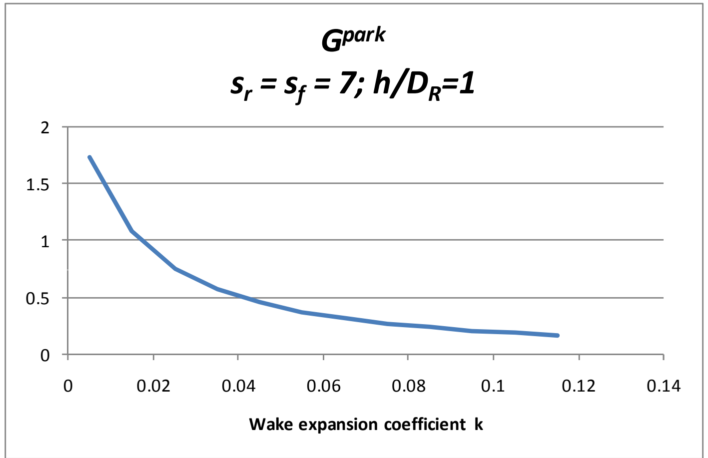
$$(\delta V)^2 = (U_{upwind}\varepsilon_0)^2 \sum_{j=1}^{\infty} N(s_j)\varepsilon_w(x_j)^2; \quad \varepsilon_w(x) = \left(\frac{D_R}{D_R + 2kx}\right)^2; \quad \varepsilon_0 = \left(1 - \sqrt{1 - C_t}\right)$$

x_j: Distance to upwind turbine row j. $N(x_j)$: number of turbines row j throwing wake on the rotor in focus.

U_{upwind}: Wind speed immediately upwind of a turbine

The infinite sum may be approximated by an infinite integral - a simple function *G*:

$$\frac{\delta V}{U_{upwind}\varepsilon_0} = G^{Park}\left(k; s_r, s_f, h/D_R, C_t\right)$$



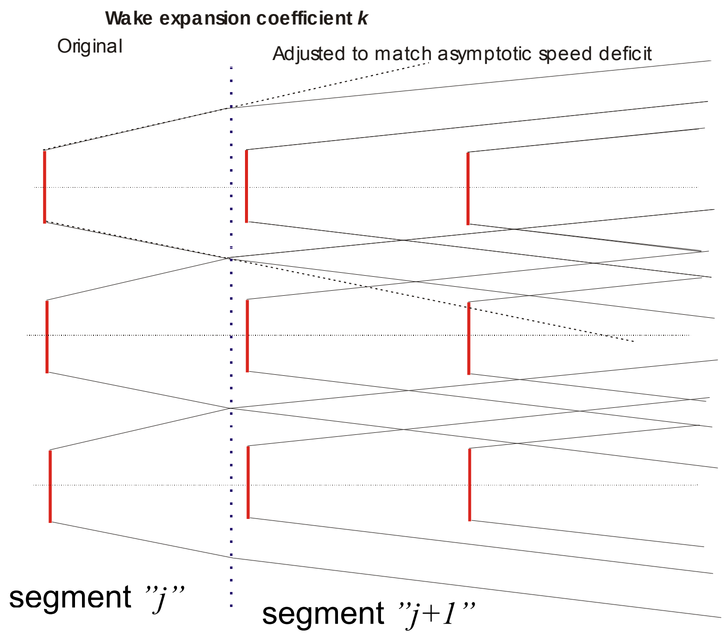
Since $U_{upwind} = U_w = U_{free} - \delta V$:

$$\frac{\delta V}{U_{Free}} = \varepsilon_w = \frac{\varepsilon_w^{app}}{1 + \varepsilon_w^{app}}; \quad \varepsilon_w^{app} = \varepsilon_0 G^{park}(layout; k)$$

Adjustment to match the BL-based asymptotic speed deficit

For “deep” positions the wake expansion coefficient *k* of the Park Model is modified to approach the BL-based asymptotic speed deficit value k_{inf}:

$$\delta V_{infin.park}(k_{inf};[s_r, s_f, h, C_t]) = \delta V_{BL-based}(s_r, s_f, h, C_t)$$



The *k*-change applies when a wake overlaps with a downwind rotor (to both wakes involved), using a relaxation factor F_{relax}:

$$k_{j+1}^{adj} = k_j^{adj} + (k_{inf} - k_j^{adj}) \frac{A_{overlap}}{A_w} F_{relax}$$

The change of the wake expansion cofficient is indicated.

Model-paramters used in the following (based on Horns Rev data):

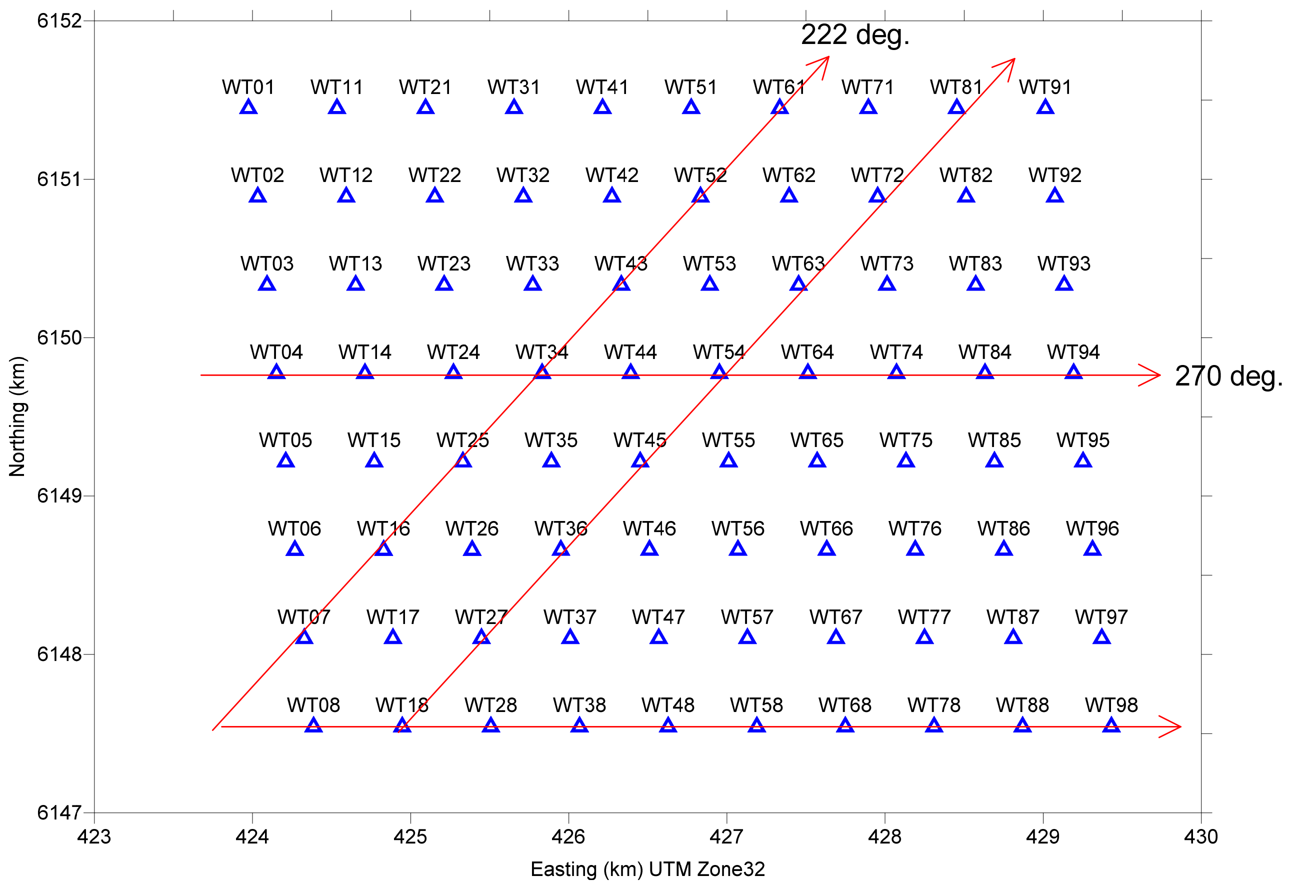
$k_{initial}$:= 0.075 (recommended value for onshore!)

$F_{relax} = 0.2$

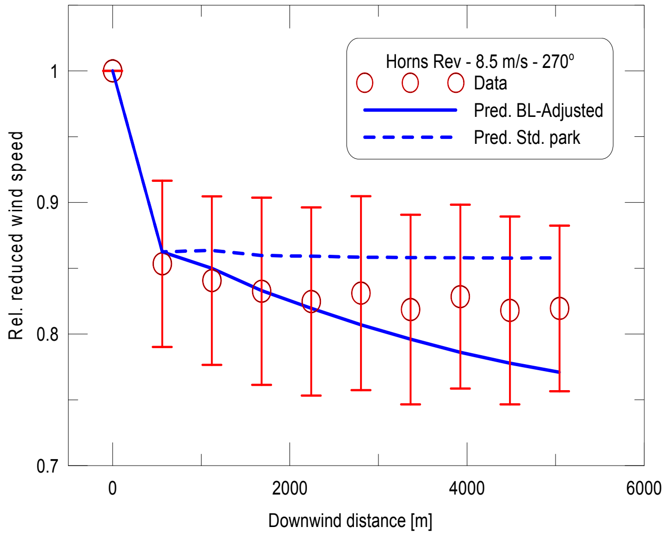
Comparative wind farm predictions: Horns Rev (1)

Turbines: 2MW, D_R = 80m, H_{hub} = 60m

Layout: $s_r = s_f = 7$

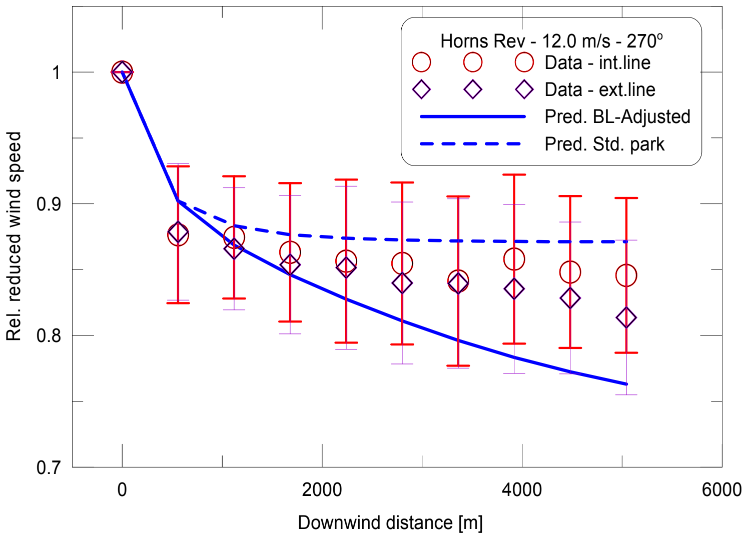


Comparative wind farm predictions: Horns Rev (2)



Wind direction: 270° +/- 3°

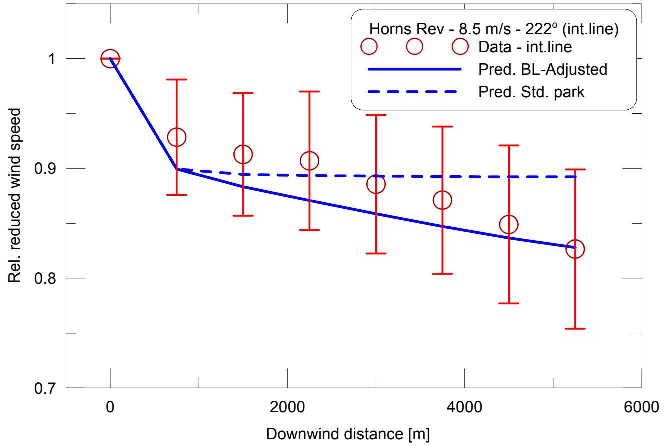
Wind speed: 8.5 m/s +/- 0.5 m/s

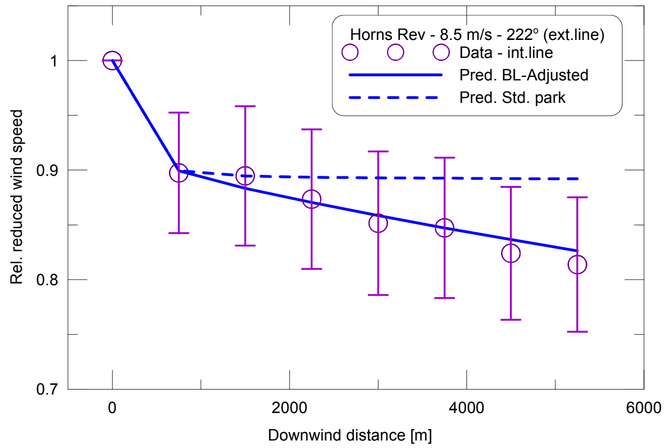


Wind direction: 270° +/- 3°

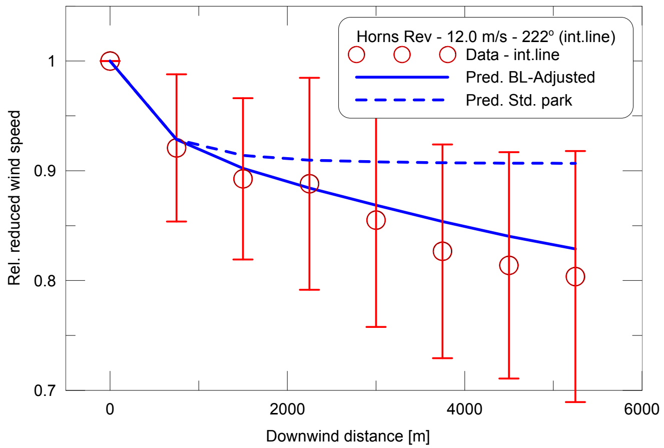
Wind speed: 12.0 m/s +/- 0.5 m/s

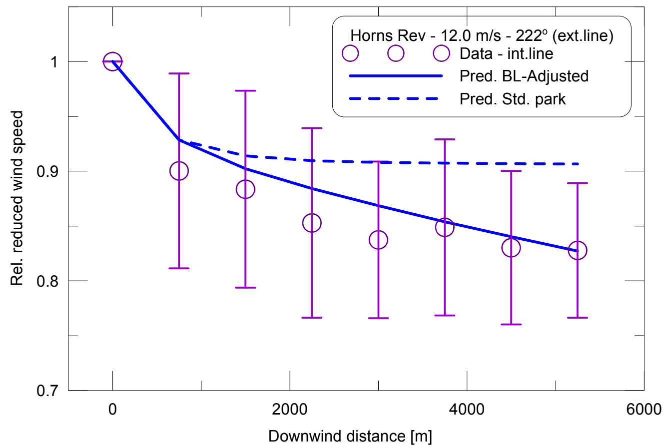
Comparative wind farm predictions: Horns Rev (3)





Wind direction:
222^{o} +/- 3^{o}
Wind speed:
8.5 m/s +/- 0.5 m/s



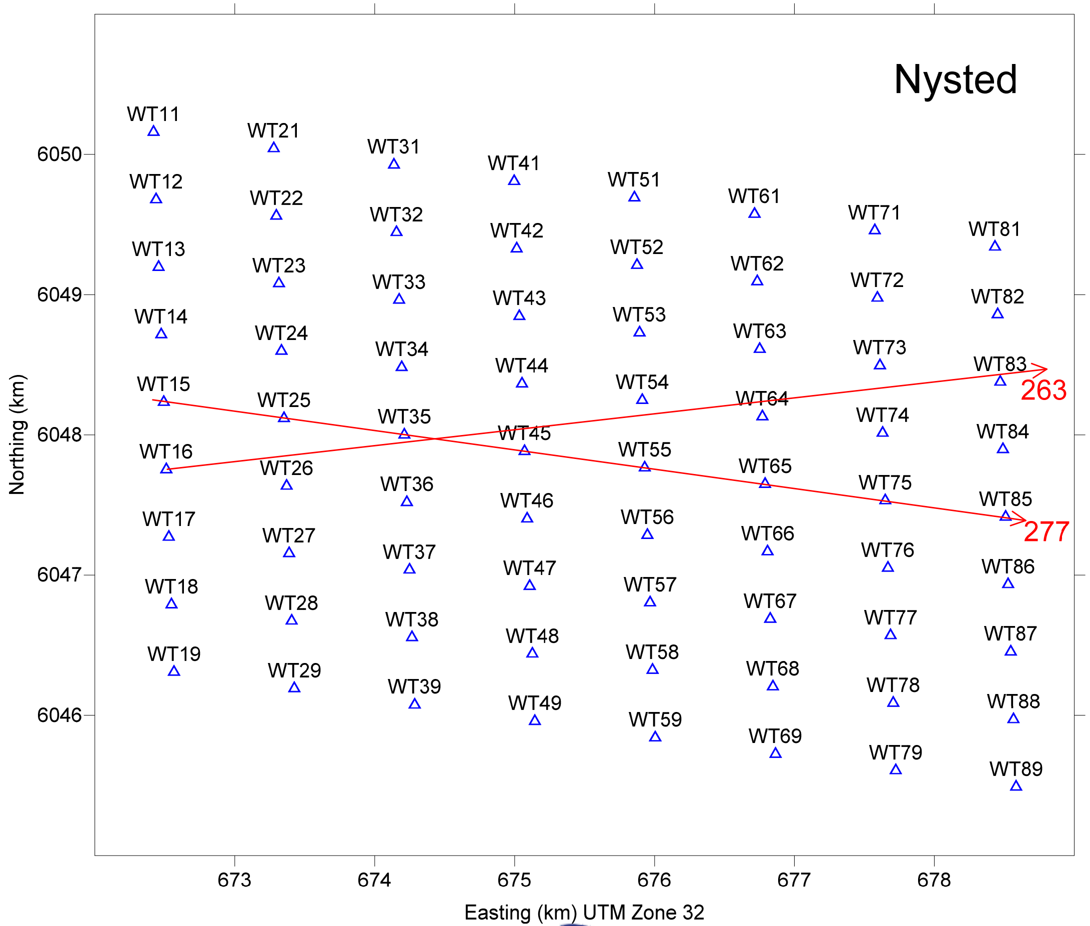


Wind direction:
222^{o} +/- 3^{o}
Wind speed:
12.0 m/s +/- 0.5 m/s

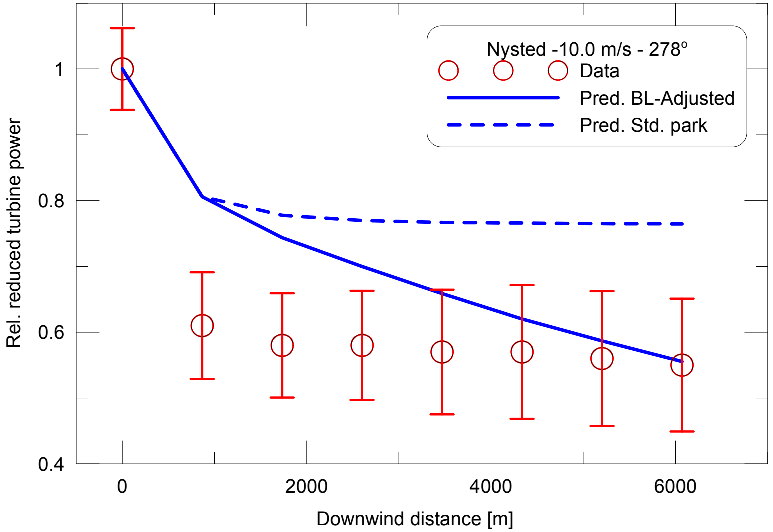
Comparative wind farm predictions: Nysted (1)

Turbines: 2.33 MW, D_R = 82m, H_{hub} = 69m

Layout: s_r = 10.6, s_f = 5.9

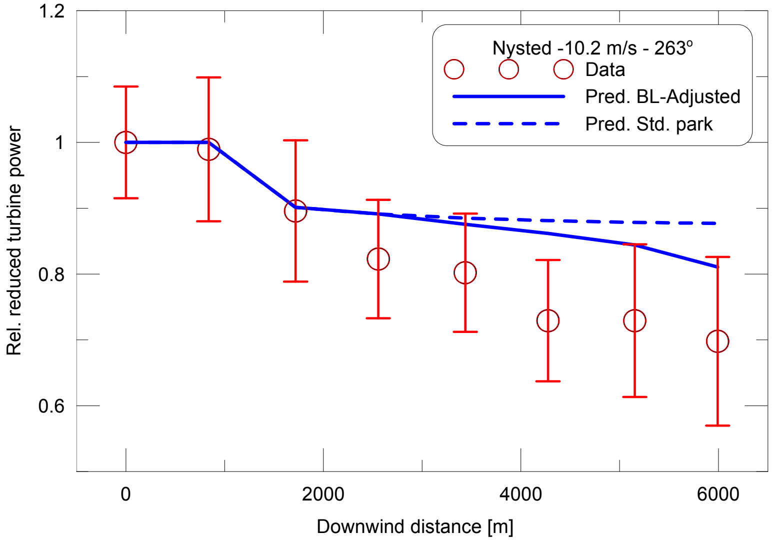


Comparative wind farm predictions: Nysted(2)



Wind direction: 278° +/- 2.5°

Wind speed: 10.0 m/s +/- 0.5 m/s



Wind direction: 263° +/- 2.5°

Wind speed: 10.2 m/s +/- 0.5 m/s

Conclusions

- The adjustment of the wake expansion coefficient towards a value matching the BL-limited asymptotic speed deficit seems a valuable engineering approach
- A value for the wake expansion coefficient close to that normally used for onshore – locations seems reasonable in this approach also for off-shore wind farms
- The model (relaxation factor) needs to be fine-tuned in order not to produce over estimations.
- The model needs to be tested on situations with wake effects between neighboring wind farms.