Technical University of Denmark

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**Munoz Queupumil, Eduardo Javier; Stolpe, Mathias; Bendsøe, Martin P.**

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# Benders Decomposition for Discrete material optimization in Laminate Design with Local Failure Criteria

*** E. Muñoz[1], M. Stolpe[2] and M.P. Bendsøe[3]**

[1] Dept. of Mathematics
Technical University
of Denmark (DTU)
Matematiktorvet, B. 303S
DK – 2800 Kgs. Lyngby
Denmark
E.Munoz@mat.dtu.dk

[2] Dept. of Mathematics
Technical University
of Denmark (DTU)
Matematiktorvet, B. 303S
DK – 2800 Kgs. Lyngby
Denmark
M.Stolpe@mat.dtu.dk

[3] Dept. of Mathematics
Technical University
of Denmark (DTU)
Matematiktorvet, B. 303S
DK – 2800 Kgs. Lyngby
Denmark
M.P.Bendsoe@mat.dtu.dk

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## ABSTRACT

We study the problem of designing a minimum compliance (or weight) composite laminate. The optimization is made by selecting, out of a set of candidate materials, the materials that will be used in the building process of a laminated structure. The material candidate set is given as data of the design problem, and are represented by the stress-strain relationship for each material. For example, consider an orthotropic material which can be oriented in four different angles (0, +/-45, and 90 degrees), then we must consider it as four different materials. This setting can be naturally applied in any discrete angle optimization design, or material selection problems. The mathematical modeling of this problem is more general than the one of standard topology optimization. When considering only two material candidates with a considerable difference in stiffness, it corresponds exactly to a topology optimization problem. The problem is modeled as a discrete design problem coming from a finite element discretization of the continuum problem. This discretization is made of shell or plate elements. For each element (selection domain), only one of the material candidates must be selected. The result is a mixed-integer formulation of the minimum compliance (weight) problem for the material selection problem of laminate design. The mixed-integer nature comes from the fact that each material candidate in each element is related to a single 0-1 variable. Each binary variable describes the selection condition of a material candidate in a selection domain. We study these problems, both from the theoretical and numerical point of view. The models include suitable failure criteria, as well as other necessary or convenient technical constraints. We propose to solve this class of problems by using a generalized Benders Decomposition technique, a global optimization technique for mixed-integer optimization. The method, under certain conditions, solves optimization problems to global optimum and allows us to solve larger problems. The technique solves the optimization problem iteratively, where at each iteration, a local linear approximation of the objective function and the feasible set are included in a mixed integer linear program, called the relaxed master problem. The approximations are obtained by solving one analysis problem and one adjoint problem. The process continues, until the optimal value of the relaxed master

problem and the current best compliance (weight) found get close enough with respect to certain tolerance. The method is investigated by computational means, using the finite element method to solve the analysis problems, and a commercial branch and cut method for solving the relaxed master problems. The theoretical results related to the single load minimum compliance and minimum weight problem will be presented. These results include convergence results, and conditions for global optimality. We will present standard benchmark examples of discrete angle optimization and isotropic material selection problems, and their comparison with standard methods of the discrete material optimization field. We aim to find global optimum designs, with respect to a certain convergence tolerance. We also aim to assess the possible impact in the design and manufacture process of laminated composite structures.

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