Technical University of Denmark

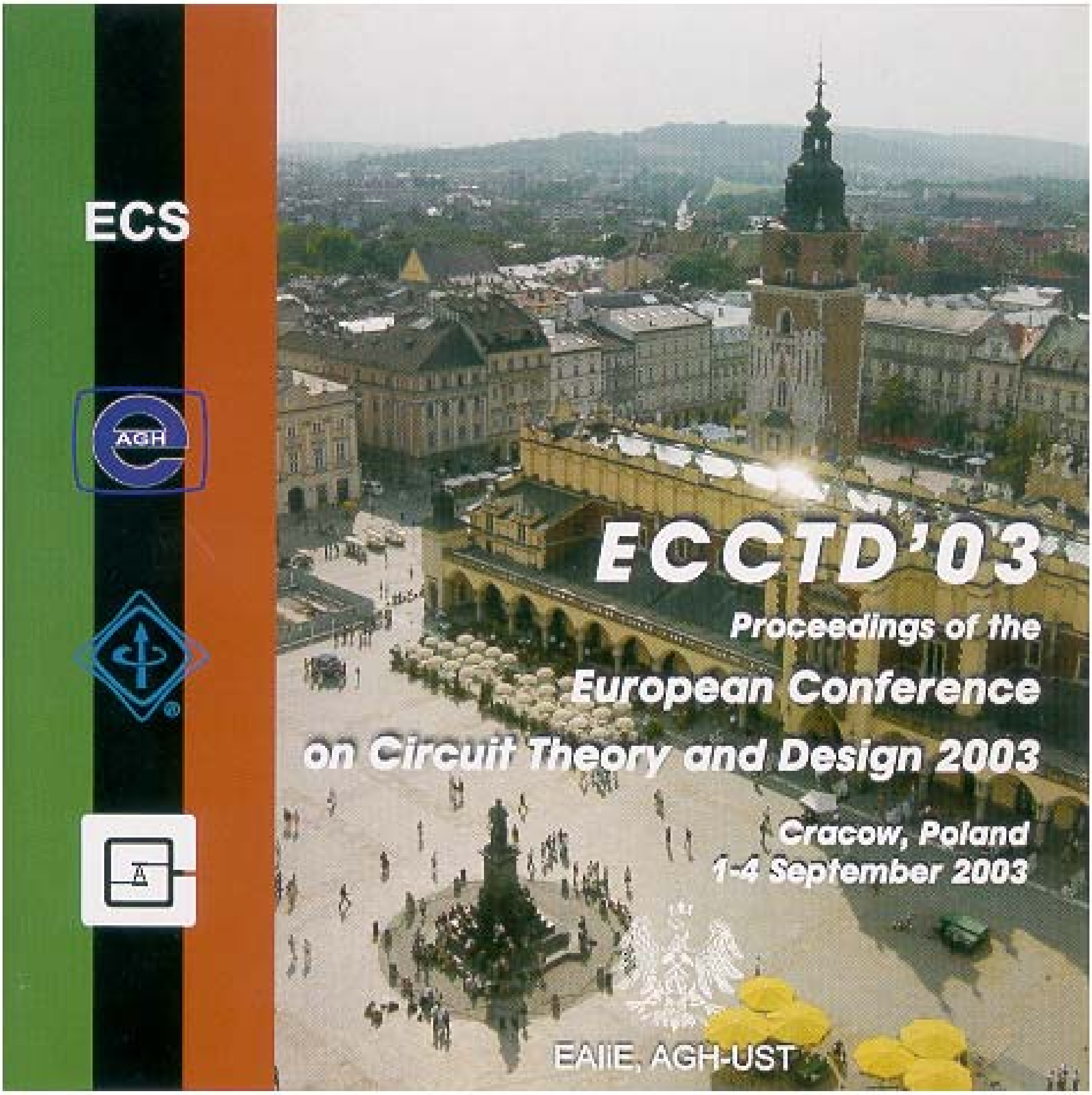
Oscillators - an approach for a better understanding

Lindberg, Erik

Published in:
Proceedings ECCTD'03

Publication date:
2003

Document Version
Publisher's PDF, also known as Version of record



"Oscillators - an approach for a better understanding"

Invited Lecture, Tuesday 2nd September 2003, 9:00-9:45

Oscillators - *an approach for a better understanding* (tutorial presented at ECCTD'03)

Erik Lindberg, IEEE Lifemember *

Abstract — **The aim of this tutorial is to provide an electronic engineer knowledge and insight for a better understanding of the mechanisms behind the behaviour of electronic oscillators. A linear oscillator is a mathematical fiction which can only be used as a starting point for the design of a real oscillator based on the Barkhausen criteria. Statements in textbooks and papers saying that the nonlinearities are bringing back the poles to the imaginary axis are wrong. The concept of "frozen eigenvalues" is introduced by means of piece-wise-linear modelling of the nonlinear components which are necessary for steady state oscillations. A number of oscillator designs are investigated and the mechanisms behind the control of frequency and amplitude are discussed.**

1 Introduction and General Remarks

An oscillator is a physical system for which you may observe some kind of periodic behaviour. Nature is filled with oscillators all the way from the superstrings of size 1e-33 to the galaxies in the universe of size 1e+33. Possibly the first human oscillator experiments were done many thousands years ago with mechanical oscillators (swings), acoustical oscillators (tubes, flutes) or mixed mechacustic oscillators (drums, wind harps or small pieces of wood or bone rotating in the end of a string). Later when the wheel was invented water mills were examples of human made oscillators. The first mechanical precision oscillator was the pendulum clock invented by C. Huygens (1629-1695) in 1656. Please see later where an electronic analogue of the pendulum clock is discussed.

1.1 Amplifiers

Electronics was born in 1883 when T. A. Edison installed a small metal plate near the filament in one of his incandescent lampbulbs. He applied a current to heat the filament and noticed that a galvanometer connected between the filament and the plate showed a current that flowed when the plate was at a positive potential with respect to the filament. At that time nothing was known of electrons and the phenomena was referred to as the Edison effect. J. A. Fleming conducted experiments on the Edison effect from 1896 to 1901. He used the unidirectional flow of current between the cold plate and the hot filament in his "Fleming Valve" or the vacuum tube diode as it is known today. In 1906 Lee de Forest introduced the grid between the filament and the plate so that the flow of electrons could be controlled. This type of tube became known as the triode. In its semiconductor version, the transistor (J. Bardeen and W. H. Brattain 1948), it is the basic element of electronic amplifiers. Also the tube diode has been replaced or supplemented with the semiconductor diode dating back to the point-contact diodes of the "crystal" radio.

Today we are able to produce large integrated circuits (SoC, Systems on Chips) which are able to perform almost any kind of signal processing. The operational amplifier is a simple integrated circuit which may be used as an almost ideal device for signal amplification.

During the last 100 years electronic engineers have succeeded enormously by means of the assumption that the dc bias point of an amplifier is time invariant so that it may be used as reference for the time varying signals, the linear small signal approach or *the linear blinkers approach.* By means of negative or degenerative feed back the amplifier performance is improved by sacrifying gain.

1.2 Sinusoidal Oscillators

Concerning oscillators they are divided into two groups according to the type of signal. Oscillators generating sinusoidal signals are termed "linear" oscillators. All other oscillators are termed relaxation or switching oscillators. *"Linear" oscillators* are normally considered second order systems. Many topologies have been proposed for sinusoidal oscillators (Colpitts, Clapp, Hartley, Pierce etc.).

The design of a "linear" oscillator is normally based on *the Barkhausen criteria* [1] according to which an oscillator is looked upon as an ideal finite gain amplifier with a linear frequency determining feed-back circuit (Fig. 1).

In order to startup the oscillations some parameters of the circuit are adjusted so that the poles (eigenvalues) of the linearized circuit are in the right-half plane, RHP, i.e. in the dc bias point the circuit is unstable and signals will start to increase

*Ørsted•DTU Department, 348 Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark, e-mail: el@oersted.dtu.dk, tel.: +45 4525 3650, fax: +45 4588 0117.

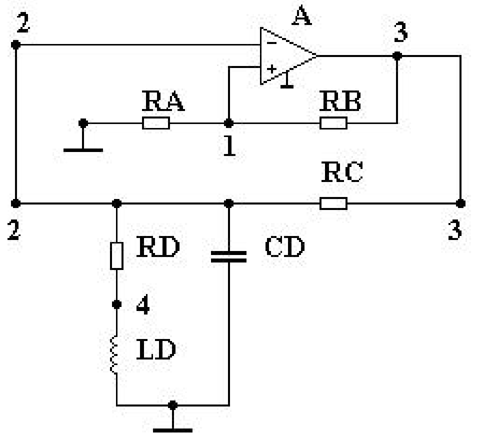


Figure 1: A negative resistance oscillator, “Barkhausen topology”.

when the power supply is connected. It is obvious that if the poles initially are very close to the imaginary axis (high Q) the time constant is very large (days, months, years ?) and the transient time to steady state behaviour is very large, i.e. *apparently* steady state sinusoidal oscillations take place. The placement of the poles on the imaginary axis is an impossible act of balance. The crucial point is whether there will be oscillations or just a transition to a new *stable* dc bias point. Very little is reported about how far out in the RHP the poles should be placed initially.

In textbooks and papers you may find statements like:

”A ”linear” oscillators is an amplifier with positive or regenerative feed back for which the Barkhausen condition is satisfied at a particular frequency”

or

”A ”linear” oscillator is an unstable amplifier for which the nonlinearities are bringing back the initial poles in the right half plane of the complex frequency plane, RHP, to the imaginary axis”

or

”There is always electrical noise in a circuit. The frequency component of the noise corresponding to the Barkhausen criteria will be amplified by the feed back circuit so that the proper sine signal builds up. The nonlinearities will limit the extent to which the oscillations builds up”.

Apparently it is assumed that a ”linear” real oscillator in the steady state behave like an ideal linear mathematical harmonic lossless LC oscillator with a complex pole pair on the imaginary axis of the complex frequency plane. This is of course not the case.

The aim of this tutorial is to investigate the behaviour of ”linear” oscillators by means of a study of the movements of the poles of the ”linear” oscillator as function of time. This approach is called *the frozen eigenvalues approach.* The poles are the eigenvalues of the linearized Jacobian of the nonlinear differential equations used as a mathematical model for the ”linear” oscillator. By means of piece-wise-linear modelling of the amplifier gain, i.e. large gain for small signals and gain zero for large signals, the ”linear” oscillator becomes linear in time slots so it make sense to calculate the poles. Please see later (Fig. 8) where a quadrature oscillator [8] is discussed.

2 Oscillators

An oscillator is a circuit which for constant input signal (dc battery) produce an oscillating output signal (a steady state time varying signal). An old rule of thumb says that if you want to design an oscillator try to design an amplifier instead and if you want to design an amplifier try to design an oscillator (violation of Murphys Law).

The ideal mathematical linear harmonic oscillator may be realized in principle as an electronic circuit by means of a coil L and a capacitor C coupled in parallel (or in series). With the initial conditions: a charge on the capacitor and no current in the coil, the voltage of the capacitor will be a cosine and the current of the coil a sine function of time with constant amplitudes.

The eigenvalues of the Jacobian of the two coupled first order differential equations of the coil and the capacitor are on the imaginary axis (no losses) and the oscillator may start up with any initial condition with no transition to the steady state. This is of course mathematical fiction. It is impossible to realize in the real world an oscillator having the poles on the imaginary axis for all times. It is an impossible act of balance to fix the poles on the imaginary axis. *Real oscillators must rely on nonlinearities.*

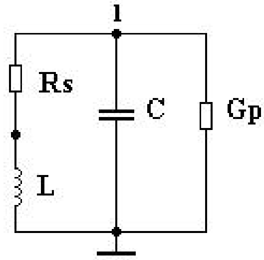


Figure 2: A damped linear oscillator.

In the real world coils and capacitors are always connected with loss mechanisms which as a first simple model may be inserted as a resistor R_s in series with the coil and a conductor G_p in parallel with the capacitor as shown in Fig. 2.

If you introduce a negative conductance in parallel with G_p you may compensate the losses and make the coefficient to s zero or negative so that the poles are on the imaginary axis or in RHP.

The characteristic polynomial of the linear differential equations describing the circuit becomes

$$s^2 + 2\,\alpha\,s + \omega_0^2 \;=\; 0\;,$$

where

$$2\,\alpha = \left(\frac{R_s}{L} + \frac{G_p}{C}\right)$$

and

$$\omega_0^2 \;=\; \frac{1 + R_s G_p}{L\,C}\;.$$

The poles or the natural frequencies of the circuit - the eigenvalues of the Jacobian of the differential equations - are the roots of the characteristic polynomial.

$$p_{1,2} \;=\; -\,\alpha \;\pm\; j\,\sqrt{\omega_0^2 - \alpha^2} \;=\; -\,\alpha \;\pm\; j\,\omega\;.$$

With certain initial conditions: flux in connection with the coil and charge in connection with the capacitor, damped voltages and currents may be observed. The damping of the signals is given by the factor $e^{-\alpha t}$. If $\omega_0^2 > \alpha^2$ the poles are complex and the signals become damped sine and cosine functions of the time $A \times\; e^{-\alpha\,t} \times\;\; \sin(\omega t + \varphi)$.

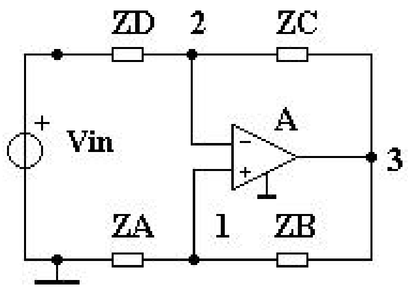


Figure 3: An amplifier with positive and negative feed-back.

In order to obtain undamped signals - steady state signals - we must introduce an energy source (battery) and some kind of electronic component (transistor, operational amplifier) which may amplify signals.

Figure 3 shows a general linear amplifier with positive and negative feed-back. The input impedance of the amplifier is assumed infinite and the gain of the amplifier is assumed constant, i.e. $V_3 = A\;(V_1 - V_2)$.

If we observe time varying signals for zero input signal $V_{in} = 0$ we have an oscillator. If the poles of the linearized circuit are in the left half plane (LHP) of the complex frequency plane the signals are damped. If the poles are in RHP the signals are undamped. Only if the poles are on the imaginary axis the signals are steady state signals. This is of course impossible in a real world circuit. The ideal harmonic oscillator may be started with any initial condition and keep its amplitude and frequency constant with no initial transient.

If we want to build an oscillator we must introduce an amplifier with *nonlinear* gain so that for small signals the poles of the linearized circuit are in RHP and for large signals the poles are in LHP. In this case we may obtain steady state behaviour based on balance between energy we obtain from the battery when the poles are in RHP and energy we loose when the poles are in LHP.

In other words *an oscillator is a feed-back amplifier with an unstable dc bias point.* Due to the nonlinear components the linearized small signal model corresponding to the instant bias point will vary with time. The dominating behaviour of the circuit is based on the instant placement of the poles of the linearized model. If the poles are in RHP the signals will increase in amplitude. If the poles are in LHP the signals will diminish in amplitude.

Seen from the source V_{in} the load is

$$Z \;=\; Z_D \;+\; Z_C\;\left(\frac{Z_B \;+\; Z_A\;(1 - A)}{Z_A \;+\; Z_B\;(1 + A)}\right)\;.$$

If we introduce memory elements - capacitors, coils, hysteresis - in the four impedances various types of oscillators may be obtained [2].

If we replace the impedances Z_A, Z_B and Z_C with resistors R_A, R_B and R_C and introduce an operational amplifier with large gain A for small input signals and zero gain for large input signals the admittance Y in parallel with Z_D becomes

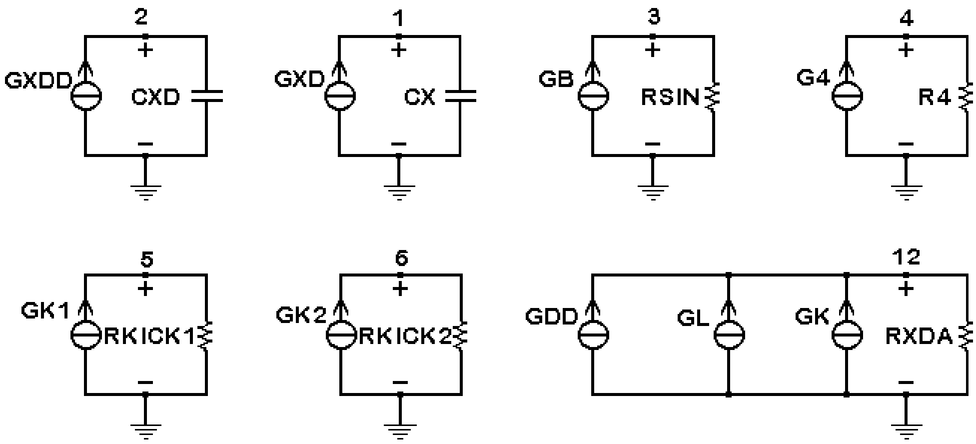
$$Y \;=\; \left(\frac{1}{R_C}\right)\left(\frac{R_A + R_B\;(1 + A)}{R_B + R_A\;(1 - A)}\right)\;,$$

which for $A = 0$ gives

$$Y \;=\; \frac{1}{R_C}\;,$$

and for A very large (positive or negative) gives

$$Y \;=\; -\,\frac{R_B}{R_A R_C}\;.$$




```
title pend-clock.cir,  pendulum clock modelling and simulation
* integrating capacitors
  cx    1 0       1      ic=0.0982   ; v(cx) = v(1) = x(t), x(0)=pi/32
* cx    1 0       1      ic=0.3927   ; v(cx) = v(1) = x(t), x(0)=pi/8  = 22.50 deg
  rcx   1 0       1e+20
  cxd   2 0       1      ic=0   ; v(cxd) = v(2) = dx/dt
  rcxd  2 0       1e+20
  gxd   0 1   2 0  1            ; gxd = v(2) = dx/dt
* sin function, v(3) = rsin*gb = b*sin(x)
  rsin  3  0        1
  gb    0  3        value={1*sin(V(1,0))}  ; b = (G/L) = +1
*                                           G = gravity, L = length
* losses assumed proportional with dx/dt = v(2)
  r4       4  0    1
  g4       0  4    2  0    0.03    ; a=0.03
* kick mechanism c*f(x)
  rkick1   5  0    1
  gk1      0  5    table {V(1,0)} = (-10 0), (-0.010 0), (-0.009 1),
+                                          (+0.009 1), (+0.010 0), (+10 0)
  rkick2   6  0    1
  gk2      0  6    value={1*sgn(v(2,0))}
* calculate xdd
  rxda   12  0       +1   ; v(12) = xdd
  gdd     0 12  3 0  -1   ; gdd   = -1*sin(x)
  gl      0 12  4 0  -1   ; gl    = -a*dx/dt   loss
  gk      0 12       value = {0.2*v(6,0)*v(5,0)}   ;  c = 0.002
* feed back
  gxdd   0  2  12  0  1
* .tran  1e-6   500  400   1  uic
.tran   1e-6    500    0   1  uic
.probe
.options opts nopage
+  RELTOL =    1.0000E-6 ; 03
+  ABSTOL =    1.0000E-12
+   VNTOL =    1.0000E-12 ; 06
+    GMIN =    1.0000E-12
+    ITL4 =   10
+  CHGTOL =   10.0000E-15
+  NUMDGT =    4
*
.end
```

Figure 4: PSpice model for a pendulum clock.

It is seen that for small signals we have a negative conductance in parallel with Z_D and for large signals we have a positive conductance.

Here the feed-back is simplified to voltage division. In the general voltage feed-back case a three terminal two-port may be used and the impedances used here become two of the three impedances in the Π-equivalent of the two-port. Similar investigations may be used for the three other kinds of feed-back and for other kinds of amplifiers.

3 EXAMPLES

The Pendulum Clock
Two Quadrature Oscillators
Vidal's Quadrature Oscillator
Mancini's Quadrature Oscillator
A Negative Resistance Oscillator
A Wien Bridge Oscillator
A Common Multi-vibrator

3.1 The Pendulum Clock

The pendulum clock with unit mass may be modeled with the following second order differential equation

$$\frac{d^2x}{dt^2} + a * \frac{dx}{dt} + b * sin(x) \;=\; c * f(x)$$

where x is the angle from vertical. The losses are modeled by the constant a. The constant $b = G/L$ is the ratio between gravity G and length L. For small values of x the pendulum is "kicked" by the function $c * f(x)$ by means of the escapement mechanism when the weights go down a step changing potential energy into a kinetic energy impulse. The model may easily be changed into two coupled first order differential equations with x and $\frac{dx}{dt}$ as the primary variables.

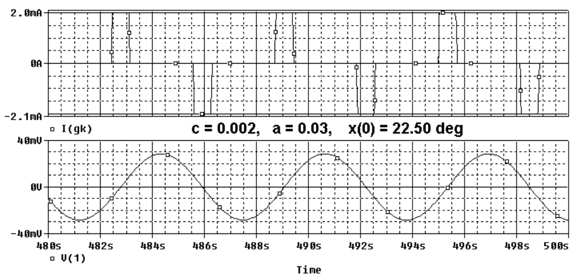


Figure 5: Pendulum Clock, angle $x(t) = V(1)$, pulse $I(gk)$.

A PSpice model (i.e. an ideal analogue computer model) may be set up as shown in Fig. 4. The constants are chosen as $a = 0.03$, $b = 1$, $c = 0.002$. Fig. 5 shows the angle $x = V(1)$ and the escapement impulse $I(gk)$ as functions of time.

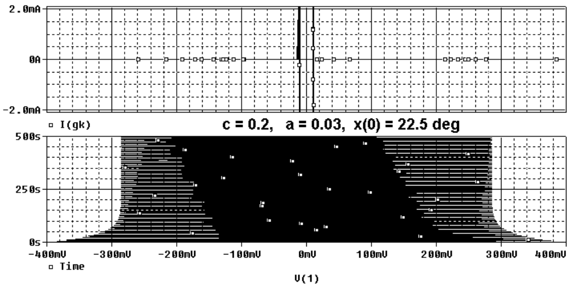


Figure 6: Pendulum Clock, angle $x(t) = V(1)$, pulse $I(gk)$.

The figures 6 and 7 show the escapement impulse $I(gk)$ and the time as functions of the angle $x = V(1)$ for $c = 0.2$ with different initial values of the angle. It is seen how the impulse determine the amplitude.

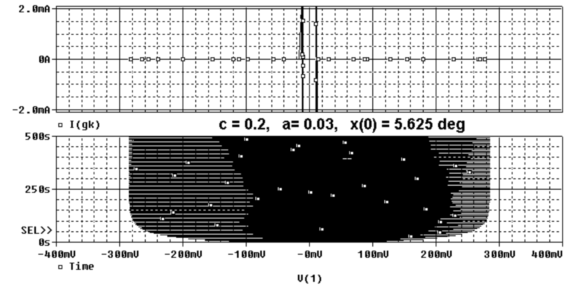


Figure 7: Pendulum Clock, angle $x(t) = V(1)$, pulse $I(gk)$.

The eigenvalues of the pendulum clock for very small angles ($sin(x) = x$, $cos(x) = 1$) are:

$$-0.01500000 \;\pm j \;*\; -0.99988749$$

For an angle $x = 16.5deg$ ($V(1) = 288mV$) the eigenvalues become:

$$-0.01500000 \;\pm j \;*\; -0.96880081$$

The pendulum clock is an oscillator with a stable dc bias point. It needs an amount of external energy in order to start up. This initial energy is potential energy if the initial condition is an angle x larger than zero. It is kinetic energy if the pendulum is given a push. The eigenvalues of the linearized Jacobian for the nonlinear differential equations used as a mathematical model for for the oscillator are moving around in the left half plane of the complex frequency plane as function of time. In the steady state the pendulum receives an energy pulse in each half period for $x \approx 0$

when the the weights go down controlled by the escapement. The size of this pulse determine the maximum swing. For small swing the movement is very close to sinusoidal.

3.2 Two Quadrature Oscillators

Quadrature oscillators are based on a feed-back loop with at least one almost ideal integrator for which input and output are two sinusoids with 90 degree phase difference. In [7] an active RC integrator and a passive RC integrator are combined with a negative resistance. The mechanism with a complex pole pair moving between RHP and LHP may be observed.

3.2.1 Vidal's Quadrature Oscillator

If two amplifiers are coupled as shown in Fig. 8 you may have a quadrature oscillator if the impedances are chosen as resistors according to [8]. In this case you make use of the operational amplifier poles as frequency determining memory ("integrating real poles") and the nonlinear saturation characteristic of the operational amplifier as the amplitude limiting mechanism.

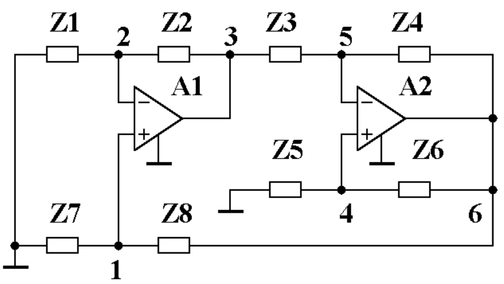


Figure 8: Quadrature Oscillator.

The quadrature oscillator is made from three circuits. For the first circuit (Z1, Z2 and A1) you have:

$$\frac{V_3}{V_1} = \frac{1+\frac{R_2}{R_1}}{1+\frac{1}{A_1}\left(1+\frac{R_2}{R_1}\right)}$$

.
For the second circuit (Z3, Z4, Z5, Z6 and A2) you have:

$$\frac{V_6}{V_3} = \frac{1}{\left(1+\frac{R_3}{R_4}\right)\left(\frac{R_5}{R_5+R_6}-\frac{1}{A_2}\right)-\frac{R_3}{R_4}}$$

.
The third circuit is the feed-back circuit (Z7 and Z8) for which:

$$\frac{V_1}{V_6} = \frac{R_7}{R_7+R_8}$$

. The loop gain is the product of the three network functions.

With reference to [8] you may now design the oscillator. With $R_1 = R_4 = R_7 = 10\text{k}\Omega$ you get $R_2 = 33.5\text{k}\Omega$, $R_3 = R_8 = 3\text{k}\Omega$, $R_5 = 1\Omega$ and $R_6 = 1130\Omega$. Figure 9 shows the result of a PSpice analysis with two μA741 Op Amps. The frequency is 530kHz and the output amplitude of any of the two Op Amps is 155mV.

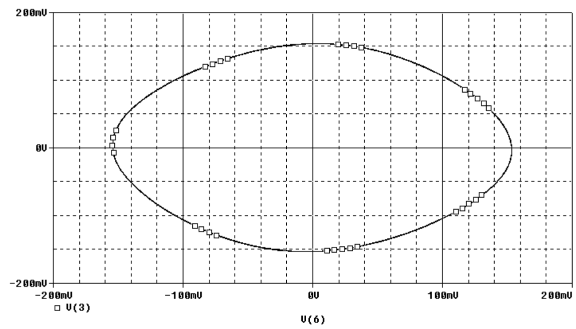


Figure 9: Quadrature Oscillator. V(3) as function of V(6).

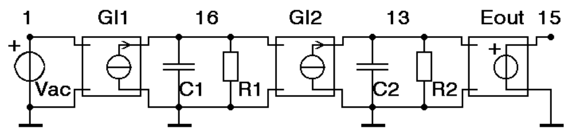


Figure 10: Op Amp macro model. Eout = A(s)Vac, $\omega_1 = 197.77$, $\omega_2 = 68.375\text{M}$.

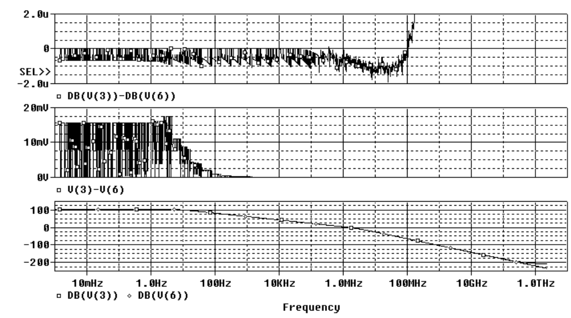


Figure 11: Comparison of the open loop ac-transfer characteristics of the Op Amp macro model of Fig. 10 and the PSpice μA741 library macro model. *Amplitude characteristic.* Logarithmic frequency scale.

Figure 10 shows a simple macro model for the Op Amp with two negative real poles. The figures 11 and 12 show the result of an optimization of the simple model with only one nonlinear element: the piece wise linear voltage controlled current source GI1. The component values obtained are: $C1 = 71.173000\mu$F, $R1 = 446.36969\Omega$, $C2 = 205.87424p$F, $R2 = 446.35600\Omega$ corresponding to a dominant pole at 31.48HZ and a HF pole at 10.88MHz.

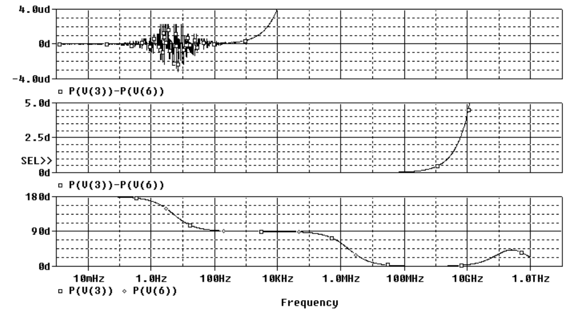


Figure 12: Comparison of the open loop ac-transfer characteristics of the Op Amp macro model of Fig. 15 and the PSpice μA741 library macro model. *Phase characteristic.* Logarithmic frequency scale.

The figures 11 and 12 show the close agreement between the output voltage V(5) of the PSpice library macro model and the output voltage V(15) of the macro model in Fig. 10 in the frequency range 1mHz to 1THz. The figures 13 and 14 show that the hysteresis of the PSpice library macro model is *NOT* modelled in Fig. 10.

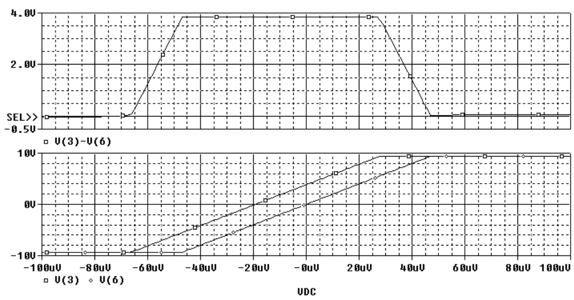


Figure 13: Comparison of the dc-transfer characteristics of the Op Amp macro model of Fig. 10 and the PSpice library macro model.

There are only two nonlinear components GI1A1 and GI1A2 in the quadrature oscillator model Fig. 8 when the simple Op Amp model of Fig. 10 is used. These nonlinearities are piece-wise-linear. There are four combinations of large gain and zero gain so you may calculate the poles as follows: For small signals i.e. when V(1,2) and V(4,5) both are in the range from -47μV to $+47\mu$V, there are two complex pole pairs, one in RHP and one in LHP:

real	$\pm j$	imag	real	$\pm j$	imag
+400k		4.49M	−11.3M		4.49M

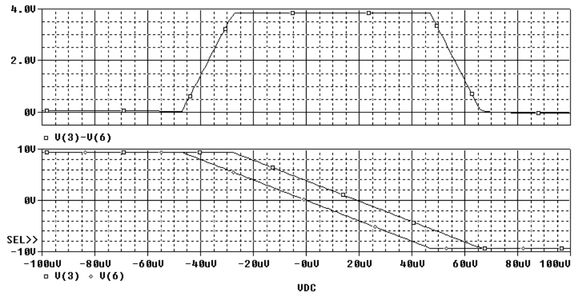


Figure 14: Comparison of the dc-transfer characteristics of the Op Amp macro model of Fig. 10 and the PSpice library macro model.

When V(1,2) is outside the range from -47μV to $+47\mu$V and V(4,5) is in the range from -47μV to $+47\mu$V, there are four negative real poles:

real	real	real	real
−31.5	−1.71M	−9.17M	−10.9M

With V(1,2) in the range from -47μV to $+47\mu$V and V(4,5) outside the range from -47μV to $+47\mu$V, there are four negative real poles:

real	real	real	real
−31.5	−1.71M	−9.17M	−10.9M

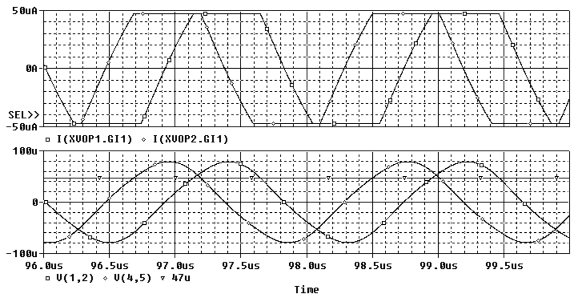


Figure 15: Input voltages of Op Amps (V(1,2), V(4,5)) and nonlinear currents of Op Amps (GI1, GI2) as functions of time (steady state). Op Amp macro model of Fig. 10 used.

When V(1,2) and V(4,5) both are outside the range from -47μV to $+47\mu$V, there are two real double poles in LHP:

real	real	real	real
−10.9M	−10.9M	−31.5	−31.5

Only in the case where the input voltages of A1 and A2 *both* are small signals, a complex pole pair in RHP is present corresponding to the unstable dc bias point. In the other cases the poles are real and negative and the circuit is damped. This is in agreement with fig. 15. It is seen how the two op amps synchronize so that only one Op Amp is active at a time. A certain amount of energy is moving between the memory elements corresponding to the input level of 47μV above which the gain is zero.

Vidal's quadrature oscillator is an oscillator with an unstable dc bias point (complex pole pair in RHP). In the steady state the poles apparently are real and negative all the time. The system is damped but for small signals energy is delivered from the power supply as pulses the same way the pendulum clock receives the energy from gravity.

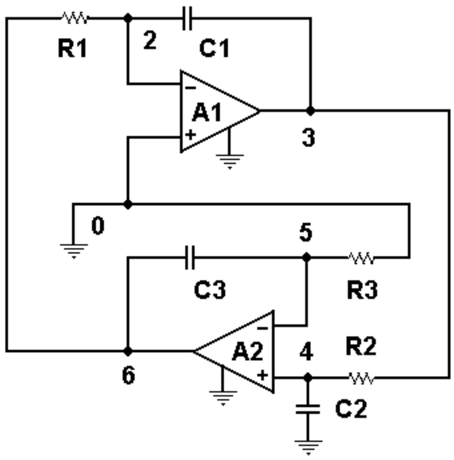


Figure 16: Quadrature Oscillator (16).

3.2.2 Mancini's Quadrature Oscillator

Figure 16 shows a quadrature oscillator made from two active and one passive integrator [3].

The characteristic polynomial for the linearized differential equations becomes:

$$s^3 \ +$$

$$s^2 \left(\frac{G_2}{C_2} + \frac{G_1}{C_1} \frac{1}{(1 + A_1)} + \frac{G_3}{C_3} \frac{1}{(1 + A_2)} \right) \ +$$

$$s \left(\frac{G_1 G_2}{C_1 C_2} \frac{A_1 A_2}{(1 + A_1)(1 + A_2)} \right) \ +$$

$$s \left(\frac{G_1 G_2}{C_1 C_2} \frac{1}{(1 + A_1)} \ + \ \frac{G_2 G_3}{C_2 C_3} \frac{1}{(1 + A_2)} \right) \ +$$

$$s \left(\frac{G_1 G_3}{C_1 C_3} \frac{1}{(1 + A_1)(1 + A_2)} \right) \ +$$

$$\frac{G_1 G_2 G_3}{C_1 C_2 C_3} \frac{(1 + A_1 A_2)}{(1 + A_1)(1 + A_2)}$$

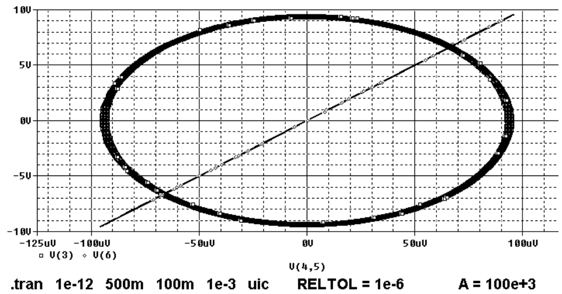


Figure 17: Linear Op Amp quadrature oscillator ($A = 100$k). Output V(3) and V(6) as function of Op Amp A2 input V(4,5).

For $A_1 = A_2 = 100$k, $R_1 = R_2 = R_3 = R = 10\text{k}\Omega$ and $C_1 = C_2 = C_3 = C = 10$nF the characteristic polynomial becomes:

$$s^3 \ + \ s^2 \left(\frac{1}{RC} \right) (1.00002000) \ +$$

$$s \left(\frac{1}{RC} \right)^2 (1.00000000) + \left(\frac{1}{RC} \right)^3 (0.9999800004)$$

where $\frac{1}{RC} = 10$k so oscillation occurs at $\omega = 2\pi f = 10$k, $f = 1.59$kHz, $T = 0.628$ms.

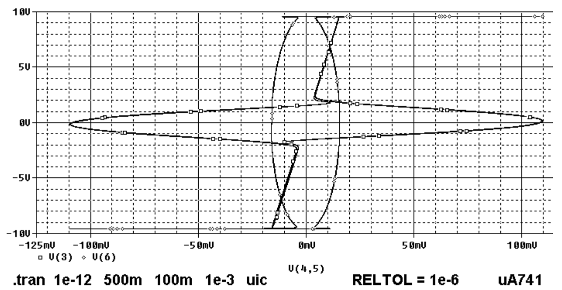


Figure 18: Nonlinear Op Amp quadrature oscillator ($A = \mu A741$). Output V(3) and V(6) as function of Op Amp A2 input V(4,5).

The figures 17 and 18 show the outputs V(3) and V(6) as function of the Op Amp A2 input V(4,5) for the linear and the nonlinear model of the quadrature oscillator. It is seen that the linear model is slightly unstable.

The upper part of Fig. 19 shows that stable oscillations take place. The lower part of Fig. 19 shows that heavy pulses occur at the op am inputs.

Mancini's quadrature oscillator is an oscillator for which a complex pole pair apparently are close to the imaginary axis all the time. It is reported by Mancini [3] that the outputs V(3) and V(6) have relatively *high distortion* which means that a gain

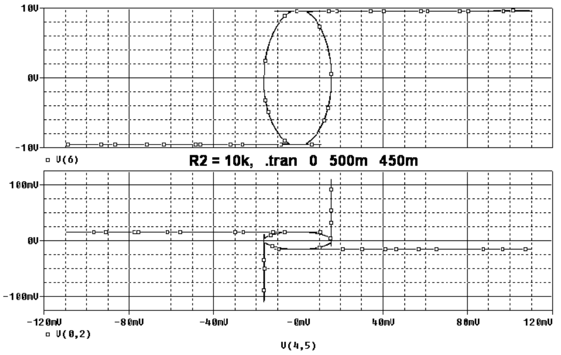


Figure 19: Nonlinear Op Amp quadrature oscillator ($A = \mu A741$). Output V(6) of Op Amp A2 and input V(0,2) of Op Amp A1 as function of Op Amp A2 input V(4,5).

stabilizing circuit is needed. This seems to be in agreement with Fig. 19.

R2 Ω	alpha $\pm j$	omega
10.00000k	-99.99900m	9.99990k
10.00000k	-10.00000k	0.00000
10.0004000k	-2.99985μ	9.99980k
10.0004000k	-9.99980k	0.00000
10.0004001k	$+21.99815\mu$	9.99980k
10.0004001k	$-9.99980k$k	0.00000
15k	$+0.81907$k	8.92190k
15k	-8.30500k	0.00000

.

Table 1: The poles of the linear quadrature oscillator as function of the passive integrator resistor R_2 for gain $A = 100k$.

.

3.3 A Negative Resistance Oscillator

If we replace Z_D in Fig. 3 with a passive LC circuit with losses we have a *negative resistance oscillator*, Fig. 20. If the circuit is redrawn as shown in Fig. 1 it is obvious that the placement of the complex pole pair on the imaginary axis by means of a negative resistor is equivalent to the *Barkhausen* criteria: loop gain = 1 and loop phase shift = 0 (or a multiple of 2π).

From the expression $2\,\alpha = \left(\frac{R_s}{L} + \frac{G_p}{C}\right) = 0$, we may calculate the value of the negative resistor needed for placing the poles on the imaginary axis.

$$R_p = \frac{1}{G_p} = -\left(\frac{1}{R_s}\right)\left(\frac{L}{C}\right).$$

For a 10kHz oscillator we may choose $L_D = 256$mH from which $C_D = \frac{1}{L_D(2\pi f)^2} = 989.4646837$pF. If we choose $R_D = 14.8\Omega$ (the measured series resistance of the coil) the value of the negative resistor becomes $R_p = -17.48147011\text{M}\Omega$ and the complex pole pair: $+7.3$n $\pm\ j$ 62.832k is on the imaginary axis. For $R_p = -100\text{M}\Omega$ the poles become $-23.853 \pm j\,62.832$k i.e. LHP. For $R_p = -100\text{k}\Omega$ the poles become $+5.024$k $\pm\ j$ 62.626k i.e. RHP ($f = 9.97$kHz, $Q = -6.3$). If we now set $R_p = -100\text{k}\Omega$ and choose $R_C = R_B$ we may calculate $R_A = +100\text{k}\Omega$ assuming infinite gain of the Op Amp. C_D is set to 1nF and the poles become $+4.971$k $\pm\ j$ 62.297k.

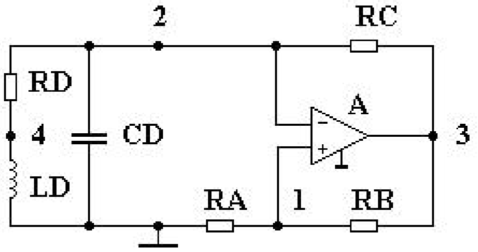


Figure 20: A negative resistance oscillator, $R_p = -R_A R_C / R_B$.

We may now vary the gain A from plus infinite via zero to minus infinite and calculate the trajectory of the complex pole pair ([4] and [5] page 434). Please note that for infinite gain the concept of positive or negative feed-back has no meaning due to the virtual short-circuit of the input terminals of the Op Amp.

Gain A	alpha $\pm j$	omega
$+10^{+12}$	$+4.971093$k	62.29735k
$+10.000000$	$+7.471093$k	62.04486k
$+2.3481$	$+62.42584$k	1.429039k

.

For large positive gain A the complex pole pair is $+4.971$k $\pm\ j$ 62.297k. For $A = +2.3480$ the complex pole pair goes to the real axis and split up into two real poles. One pole goes direction plus infinite and one pole goes direction zero.

Gain A	alpha	alpha
$+2.3480$	$+62.58839$k	$+62.29631$k
$+2.05$	$+805.148$k	$+4.793427$k
$+2.0005921$	$+67.56615$M	$+0.0012077$
$+2.0005920$	$+67.57758$M	-0.0085549
$+2.000001$	$+40.00001$G	-57.714843
$+2.000000$	∞	-57.812500
$+1.999999$	-39.99999G	-57.910156
$+1.75$	-0.116434M	-33.62341k
$+1.703831$	-62.67297k	-62.44286k

.

For $A = 2$ we have a real pole at -57.8125

and the other real pole is infinite. Please note that "infinite" and "zero" in a sense is the same "number". The two poles now go together for $A = 1.703831$ and leaves the real axis as a complex pole pair with decreasing real part and increasing imaginary part.

Gain A	alpha $\pm j$	omega
$+1.703830$	-62.55769k	0.123544k
$+1.5$	-35.02890k	51.80031k
$+1.0000000$	-15.02890k	60.68044k
$+0.5$	-8.362239k	61.94583k
$+1\mu$	-5.028911k	62.30199k
$+10^{-24}$	-5.028906k	62.30199k
$+0$	-1.695572k	62.47853k

.

For $A = 0$ the complex pole pair is

$$-1.695572\text{k} \pm j\, 62.47853\text{k}.$$

It is seen that for positive gain ($A > 0$) the mechanism behind the negative resistance oscillator is relaxation for the component values chosen.

For negative values of the gain the complex pole pair moves further from LHP to RHP. For $A = -2.0232555$ the complex pole pair passes the imaginary axis. For negative gain ($A < 0$) the poles moves smoothly across the imaginary axis as it e.g. is seen for the Colpitts oscillator [6].

Gain A	alpha $\pm j$	omega
-10^{-24}	-5.028906k	62.30199k
-1μ	-5.028901k	62.30199k
-0.5000000	-3.028906k	62.42934k
-1.0000000	-1.695572k	62.47853k
-2.0000000	-28.906250	62.49999k
-2.0230000	-0.3206173	62.49997k
-2.0232550	-5.521610m	62.49997k
-2.0232600	$+0.656338$m	62.49997k
-2.10	$+93.044969$	62.49981k
-2.5	$+526.64930$	62.49726k
-10.0	$+3.304427$k	62.40949k
-1k	$+4.951133$k	62.29896k
-10^{+12}	$+4.971093$k	62.29735k

.

The behaviour for positive and negative values of the gain A has been verified by means of PSpice simulation with RC4136 Op Amp by monitoring the current in the negative resistance.

By means of experiments with PSpice for variation of the resistors R_B and R_C the distortion of the amplifier may be minimized.

3.4 A Wien Bridge Oscillator

If we in Fig. 3 replace Z_A with R_A in parallel with C_A, Z_B with R_B in series with C_B, Z_C with R_C and Z_D with R_D we have a *Wien Bridge Oscillator*. Please note that the frequency determining components are placed in the positive feed-back path. The coefficients of the characteristic polynomial become

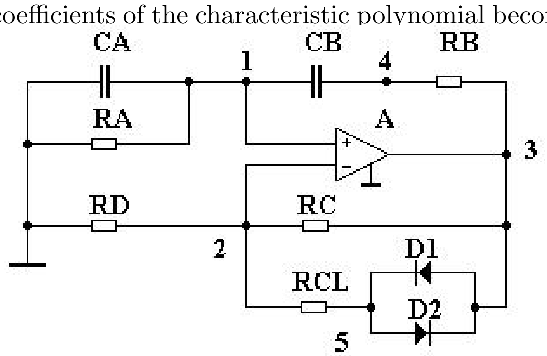


Figure 21: Wien Bridge Oscillator.

.

$$2\,\alpha = \left(\frac{1}{C_A R_A} + \frac{1}{C_B R_B} - \frac{R_C}{C_A R_B R_D}\right)$$

and

$$\omega_0^2 = \frac{1}{C_A C_B R_A R_B}\,.$$

The resistor R_C is crucial for the sign of the loss coefficient $2\,\alpha$. If R_C is amended with a large resistor in series with a nonlinear element made from two diodes in antiparallel as shown in Fig. 21 you have a mechanism for controlling the movement of the poles between RHP and LHP so you can avoid making use of the nonlinear gain. In this way you may *control both frequency and amplitude of the oscillator.*

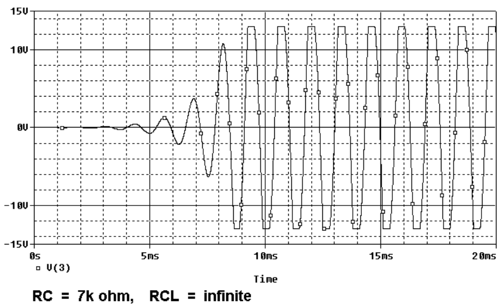


Figure 22: Wien Bridge Oscillator, $RC = 7k\Omega$, $RCL = \infty$.

In order to study the mechanism behind the amplitude control an oscillator is designed as follows: The operational amplifier is assumed a perfect linear amplifier with gain 100k. Choose the capacitors as $C_A = C_B = C = 10$nF. Choose the resistors as $R_A = R_B = R = 20\text{k}\Omega$. From $\omega_0^2 = \frac{1}{C_A C_B R_A R_B}$ the frequency becomes 795.7747151 Hz and $\omega_0 = 5$k rad/sec. Choose $R_C = 2R_D$ so that 2α becomes zero e.g. $R_D = 3\text{k}\Omega$ and $R_C = 6\text{k}\Omega$.

RC Ω	alpha ± j	omega
12k	+4.99937k	79.0524
10k	+3.33286k	3.72719k
9k	+2.49960k	4.33035k
8k	+1.66633k	4.71416k
→ 7k	+833.055	4.93011k
6100.00	+83.1033	4.99930k
6010.00	+8.10783	4.99999k
6003.00	+2.27485	4.99999k
6002.00	+1.44157	4.99999k
6001.00	+0.60829	4.99999k
6000.270008101	$+6.31994e-10$	5.00000k
6000.270008100	$-1.99284e-10$	5.00000k
5999.00	−1.05827	4.99999k
5998.00	−1.89155	4.99999k
5990.00	−8.55782	4.99999k
5900.00	−83.5533	4.99930k
→ 5k	−833.511	4.93003k
4k	−1.66680k	4.71399k
3k	−2.50009k	4.33006k
2k	−3.33340k	3.72671k
1k	−4.16671k	2.76378k
800.00	−4.33337k	2.49436k
600.00	−4.50003k	2.17937k
400.00	−4.66669k	1.79497k
200.00	−4.83336k	1.28008k
100.00	−4.91669k	908.914
50.00	−4.95835k	643.952
25.00	−4.97919k	455.682
10.00	−4.99169k	288.119
5.00	−4.99585k	203.466
1.00	−4.99919k	89.9028

.

Table 2: The poles of the linear Wien bridge oscillator as function of resistor R_C.

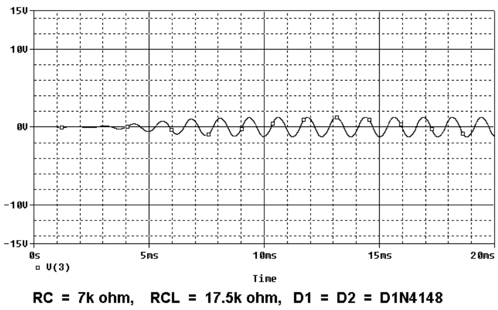


Figure 23: Wien Bridge Oscillator, $RC = 7k\Omega$, $RCL = 17.5\mathrm{k}\Omega$, D1 = D2 = D1N4148.

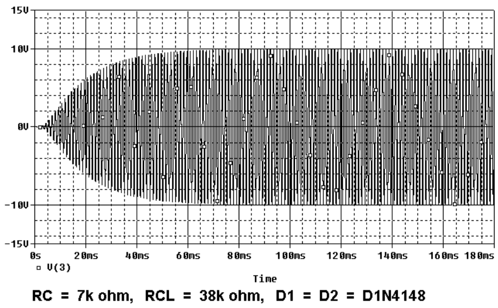


Figure 24: Wien Bridge Oscillator, $RC = 7k\Omega$, $RCL = 38\mathrm{k}\Omega$, D1 = D2 = D1N4148.

Table 2 shows the poles of the oscillator as function of resistor R_C. Let us choose $R_C = 7\mathrm{k}\Omega$ so that the initial poles are in the RHP 2. For large signals the poles are moved into LHP by means of R_{CL} in parallel with R_C when the diodes open. Let us choose $R_C//R_{CL} = 5\mathrm{k}\Omega$ which correspond to an almost symmetrical pole placement in LHP. Now R_{CL} may be calculated as $17.5\mathrm{k}\Omega$. The figures 22, 23 and 24 show how the amplitude is determined by means of R_{CL}.

3.5 A Common Multi-vibrator

If you replace the impedance Z_D in figure 3 with a capacitor $C_D = 0.5\mu\mathrm{F}$ and the impedances Z_A, Z_B *and* Z_C with resistors $R_A = 1\mathrm{k}\Omega$, $R_B = 1\mathrm{k}\Omega$ and $R_C = 2\mathrm{k}\Omega$ you have a *common multi-vibrator* with real poles moving back and forth between RHP and LHP. The hysteresis is the second "memory component" needed for oscillations.

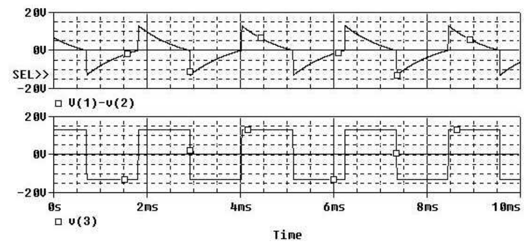


Figure 25: Common multi-vibrator, PSpice analysis, op. amp. input voltage V(1,2) and op. amp. output voltage V(3) as functions of time.

Figure 25 shows the voltages as functions of time using a RC4136 operational amplifier. The time constant becomes $\tau = RC = -((R_A R_C)/R_B)C_D = -1\mathrm{ms}$ for A very large and $\tau = R_C C_D = +1\mathrm{ms}$ for $A = 0$. This is in agreement with Fig. 25.

Assuming a perfect operational amplifier with

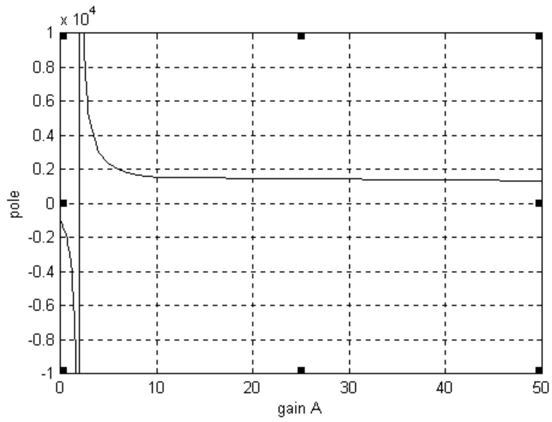


Figure 26: Multi-vibrator pole as function of amplifier gain A.

gain A the pole of the circuit is +1k for A very large and -1k for A very little. For $A = 2$ the pole pass from $+\infty$ to $-\infty$ as shown in Fig. 26. Please note that this is not a jump but a smooth transition.

4 CONCLUSIONS

"Linear" oscillators are <u>*nonlinear*</u> *electronic circuits.*

The nonlinearities do not bring back the poles to the imaginary axis.

The oscillator amplitude may be controlled by other means than the power supply.

If you place the poles as close as possible to the imaginary axis you may obtain a noisy oscillator

At a certain instant the linearized small signal model of an oscillator will try to oscillate according to the poles. The actual oscillator frequency is a kind of an average frequency.

In this tutorial it is demonstrated that it is possible to obtain insight in the mechanisms behind the behaviour of oscillators by means of <u>*the frozen eigenvalues approach*</u>. We must rewrite our textbooks and replace "obscure noise statements" and statements concerning "nonlinearities pulling the poles back to the imaginary axis" with proper statements concerning the mechanisms behind the steady state oscillations as e.g. *sinusoidal oscillation* where a complex pole pair is moving between RHP and LHP or *relaxation oscillation* where real poles are moving between RHP and LHP.

Apparently there are three basic types of oscillators. The first type has an unstable initial dc bias point. This type is selfstarting when the power supply is connected. The poles are moving between RHP and LHP so that a balance is obtained between the energy obtained from the power supply when the poles are in RHP and the energy lost when the poles are in LHP.

The second type has a stable initial dc bias point. This type needs some extra initial energy in order to start up. The poles are in LHP all the time and some special impulse mechanism is needed to provide energy from the power supply in the steady state.

The third type is a combination of the two types. It is unstable in the initial dc bias point and the poles are moving around in LHP only in the steady state.

Electromechanical crystal oscillators should be investigated with the presented technique in the future.

References

[1] H. Barkhausen, *Lehrbuch der Elektronen-Rohre*, 3.Band, "Rückkopplung" , Verlag S. Hirzel, 1935.

[2] J. R. Westra, C. J. M. Verhoeven and A. H. M. van Roermund, *Oscillators and Oscillator Systems - Classification, Analysis and Synthesis*, pp. 1-282, Kluwer 1999.

[3] Ron Mancini (edt. in Chief), *Op Amps For Everyone*, Design Reference, slod006b, Texas Instruments, August 2002, http://focus.ti.com/docs/apps/catalog/resources/appnoteabstract.jhtml?abstractName=slod006b.

[4] E. Lindberg, "Oscillators and eigenvalues", in *Proceedings ECCTD'97 - The 1997 European Conference on Circuit Theory and Design*, pp. 171-176, Budapest, September 1997.

[5] L. Strauss, *Wave Generation and Shaping*, pp. 1-520, McGraw-Hill, 1960.

[6] E. Lindberg, "Colpitts, eigenvalues and chaos", in *Proceedings NDES'97 - the 5'th International Specialist Workshop on Nonlinear Dynamics of Electronic Systems*, pp. 262-267, Moscow, June 1997.

[7] A.S. Sedra and K.C. Smith, *Microelectronic Circuits 4th Ed*, Oxford University Press, 1998.

[8] E. Vidal, A. Poveda and M. Ismail, "Describing Functions and Oscillators", *IEEE Circuits and Devices*, Vol. 17, No. 6, pp. 7-11, November 2001.