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Published in:
Proceedings of the 19. Nordic Seminar on Computational Mechanics

Publication date:
2006

Document Version
Early version, also known as pre-print

Citation (APA):
Krenk, S., & Høgsberg, J. R. (2006). Design of Multiple Tuned Mass Dampers on Flexible Structures. In Proceedings of the 19. Nordic Seminar on Computational Mechanics (pp. 103-106). Lund, Sweden: Lund Institute of Technology.

Design of Multiple Tuned Mass Dampers on Flexible Structures

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Introduction

Tuned mass dampers are increasingly used for structures such as bridges, towers, buildings and structural parts such as e.g. staircases. The classic design basis described e.g. by Den Hartog [1], refers to a single damper mounted on a single structural mass. In the case of flexible structures, such as e.g. the Millennium Bridge in London [2] and the newly completed Langelinie Pedestrian Bridge in Copenhagen [3], several modes must be damped. This may introduce two new effects into the design, namely a change in modal frequencies due to the damper masses associated with the other modes, and a possible change in the mode shapes.

This paper describes a simple procedure for design of a number of tuned mass dampers used to introduce controlled damping in several modes of a flexible structure. The following section gives a brief summary of the design basis for a single damper, and this procedure is then extended to flexible structures via a two-step procedure consisting of an initial estimate and a correction based on modal vibrations including the damper masses, but excluding the damping effect to provide a real-valued vibration problem. The procedure is illustrated by an example concerned with damping of the four lowest modes of a four-span bridge.

Single tuned mass damper

The classic design problem for a single tuned mass damper is illustrated in Fig. 1. The structure is represented by a mass m_0 supported by a spring k_0. The tuned mass damper consists of a damper mass m_d mounted on the structural mass by a spring with stiffness k_d and a viscous damper with damper constant c_d. The motion is described by the motion of the structural mass x_0 and the relative motion x_d of the damper.

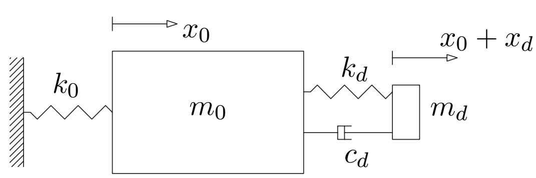

Figure 1: Single degree of freedom with mass damper

The design procedure for a single tuned mass damper mounted on a well defined structural mass makes use of the mass ratio μ, the structural angular frequency ω_0, the angular frequency of the rigidly mounted damper mass ω_d and the damping ratio of the rigidly mounted damper mass ζ_d,

$$\mu = \frac{m_d}{m_0} \quad , \quad \omega_0^2 = \frac{k_0}{m_0} \quad , \quad \omega_d^2 = \frac{k_d}{m_d} \quad , \quad \zeta_d = \frac{c_d}{2\omega_d m_d} \qquad (1)$$

The usual design procedure consists in selecting a sufficiently large mass ratio - usually in the order of 3-5%. The optimal frequency tuning and damping ratio are then determined by

$$\omega_d \,=\, \frac{\omega_0}{1+\mu} \qquad , \qquad \zeta_d \,=\, \sqrt{\frac{1}{2}\frac{\mu}{1+\mu}} \tag{2}$$

The classic damping value of Den Hartog [1] has a factor $\frac{3}{8}$, but it has recently been demonstrated that the factor $\frac{1}{2}$ leads to better damping of the structure and minimizes the relative motion of the damper mass, [4]. Use of the optimal tuning parameters in Eq. 2 leads to two coupled structural modes with identical damping ratio $\zeta_s \,\simeq\, \frac{1}{2}\zeta_d$, [4]. Thus, a mass ratio of $\mu \,=\, 0.05$ leads to a structural damping ratio of $\zeta_s \simeq 0.077$, sufficient to eliminate most vibration problems.

Multiple dampers on flexible structures

Let the flexible structure be represented by a discreized model with the displacement vector $\mathbf{u}$ and equation of motion

$$\mathbf{M}\ddot{\mathbf{u}}(t) \,+\, \mathbf{C}\dot{\mathbf{u}}(t) \,+\, \mathbf{K}\mathbf{u}(t) \,=\, \mathbf{Q}(t) \tag{3}$$

$\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrix of the structure, and $\mathbf{Q}(t)$ is the time varying external load vector. Damping is typically introduced into the individual vibration modes $\mathbf{u}_j$, determined from the generalized eigenvalue problem

$$(\,\mathbf{K} \,-\, \omega_j^2\,\mathbf{M}\,)\,\mathbf{u}_j \,=\, \mathbf{0} \quad , \quad j = 1, \cdots, n \tag{4}$$

where ω_j is the natural frequency of mode j. The modal mass is defined as

$$m_j \,=\, \mathbf{u}_j^T\,\mathbf{M}\,\mathbf{u}_j \tag{5}$$

and represents the part of the structural mass that participates for the particular mode. When one or more mass dampers are mounted on the flexible structure to introduce damping into mode j the effective mass ratio for this group of dampers is

$$m_j^d \,=\, \mathbf{u}_j^T\,\mathbf{M}_j^d\,\mathbf{u}_j \tag{6}$$

where $\mathbf{M}_j^d$ contains the masses of the dampers in the diagonal at the degrees of freedom corresponding to the locations of the dampers on the structure. Thus, the effective mass ratio of mode j is given as

$$\mu_j \,=\, \frac{m_j^d}{m_j} \tag{7}$$

The optimal parameters for the tuned mass dampers associated with mode j can be found by Eq. 2.

For the flexible structure with dampers the vibration modes and natural frequencies become complex valued due to phase differences. The complex modes and frequencies are found from the expanded symmetric eigenvalue problem

$$\left\{ \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & -\mathbf{M} \end{bmatrix} + i\omega \begin{bmatrix} \mathbf{C} & \mathbf{M} \\ \mathbf{M} & -\mathbf{0} \end{bmatrix} \right\} \begin{bmatrix} \mathbf{u} \\ i\omega\mathbf{u} \end{bmatrix} \,=\, \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{8}$$

where the system matrices $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and the displacement vector $\mathbf{u}$ include both structure and dampers. The damping ratio is extracted as the relative imaginary part of the natural frequency,

$$\zeta_j \,=\, \frac{\mathrm{Im}[\omega_\mathrm{j}]}{|\omega_j|} \tag{9}$$

Design procedure

The present design procedure for multiple tuned mass dampers relies on the optimal expressions in Eq. 2 for a single tuned mass damper, where the mass ratio is defined in Eq. 7. However, to take the effects from the other dampers into account, the procedure consists of two steps: 1) A preliminary design based on the undamped vibration modes and 2) a correction based on the mode shape for the structure including mass and stiffness (from step 1) of all dampers for *the other modes*. Thus, in step 2 the stiffness k_d and damper parameter c_d from the preliminary design are recalculated based on the modified vibration form due to the tuned mass damper associated with the other modes that are being damped.

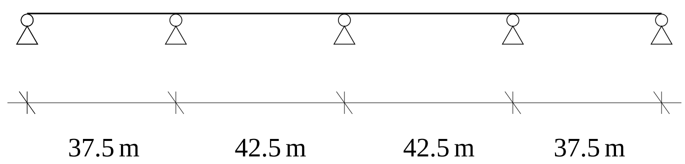

Figure 2: Four-span bridge.

The design procedure is illustrated in terms of the four-span bridge shown in Fig. 2. The elastic modulus E, cross section area A, moment of inertia I, mass per unit length ρ and length L are

$$E = 300\,\mathrm{GPa} \quad , \quad A = 0.5\,\mathrm{m}^2 \quad , \quad I = 0.08\,\mathrm{m}^4 \quad , \quad \rho = 6400\,\mathrm{kg/m} \quad , \quad L = 160\,\mathrm{m}$$

These properties represent typical values for e.g. pedestrian bridges with span lengths as shown in Fig. 2. The aim is to introduce damping into the lowest four vibration modes with mode shapes shown in Fig. 3. Two tuned mass dampers with equal properties are introduced for each mode and placed according to the maximum of the associated vibration modes as indicated in Fig. 3.

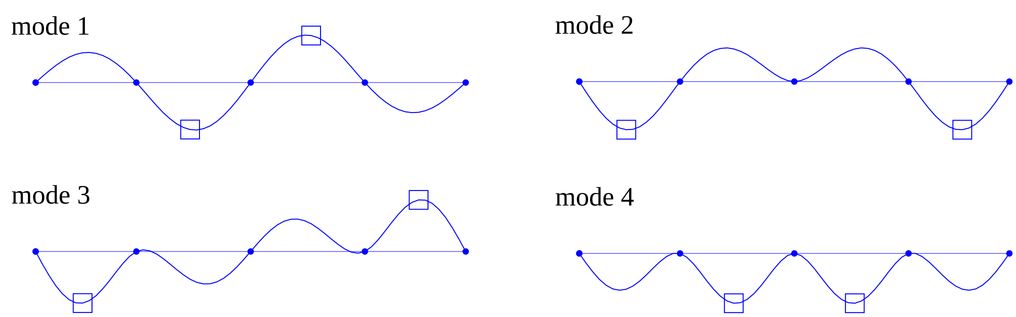

Figure 3: Undamped vibration modes and damper location.

In the idealized form with a single structural mass the use of a single damper splits the original undamped mode into two modes with equal damping ratio $\zeta_s \simeq \frac{1}{2}\zeta_d$, the use of two dampers introduces a third mode in which the dampers act in opposite phase and thereby retain the full damping ratio $\zeta_s \simeq \zeta_d$. For flexible structures the behavior of the dampers may be more complicated as illustrated for mode 1 in Fig. 4 showing the real part of the three vibration forms.

The desirable mass ratio of the design procedure is $\mu = 0.05$ for all four modes. The results of the two-step procedure is summarized in Table 1, where the damper parameters and the damping ratio for the structural modes are given for both the preliminary tuning and the correction. It is seen how the correction step leads to a larger critical damping ratio for each mode.

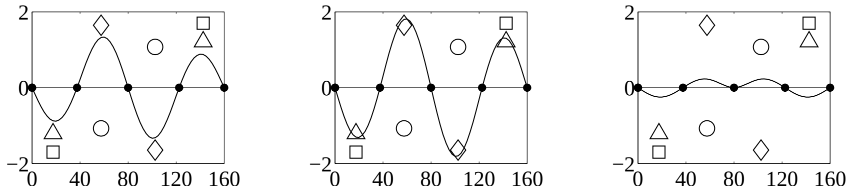

Figure 4: Mode shapes with dampers for mode 1. Dampers tuned to: mode 1 (circle), mode 2 (square), mode 3 (triangle) and mode 4 (diamond).

	mode j	1	2	3	4
	ω_0	16.51	20.97	27.17	32.31
	Damper mass	4700 / 4700	4450 / 4500	3900 / 3900	4150 / 4150
	ω_d	15.72	19.97	25.87	30.77
Step 1	ζ_d	0.155	0.154	0.155	0.154
	ζ_j	0.055 / 0.107	0.069 / 0.110	0.096 / 0.103	0.116 / 0.083
	ω_d	14.91	17.17	26.26	32.29
Step 2	ζ_d	0.163	0.161	0.153	0.147
	ζ_j	0.071 / 0.074	0.075 / 0.081	0.083 / 0.099	0.091 / 0.091

Table 1: Modal properties of bridge with $\mu = 0.05$.

The effect of the correction step is illustrated in Fig. 5, showing the complex natural frequencies for modes 1 and 2. The lines represent two half circles, representing the root loci for a single tuned mass damper, see [4]. The loci initiate from the three natural frequencies when $c_d \to 0$. The dashed line represent the expected damping ratio of $\zeta_j = 0.077$, whereby the two intersections (circles) between the dashed line and the small locus represent the expected optimal tuning. The three natural frequencies from step 1) of the procedure (asterisk) are seen to be relatively far away from the the expected locus. However, following step 2) of the procedure the natural frequencies (crosses) move very close to the expected locus and to the intersections (circle), representing the desired efficiency of 7.7% of critical damping.

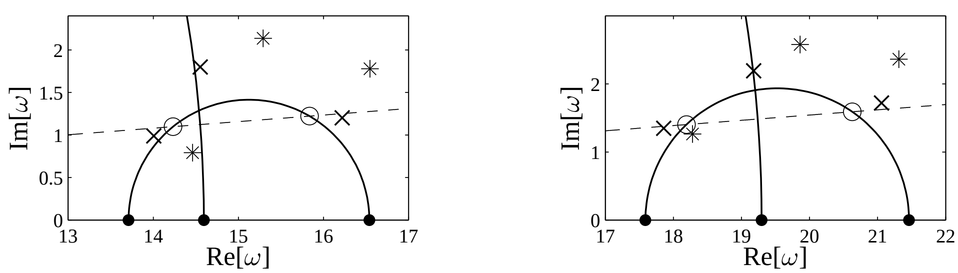

Figure 5: Complex roots for mode 1 (left) and 2 (right).

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