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Experimental Investigation of Some Effects of Multipath Propagation on a Line-of-Sight Path at 14 GHz

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TABLE 2
NUMERICAL COMPARISONS BETWEEN $Q(x)$, $R(x)$, $P(x)$ AND $Q_a(x)$, BEST UPPER BOUND, LOWER BOUND AND APPROXIMATION FOR $x \geqslant 0$. THE COLUMN SNR DENOTES THE SIGNAL-TO-NOISE RATIO IN A PSK (PHASE SHIFT KEYING) SYSTEM WITH THE BIT ERROR PROBABILITY $Q(x)$

x	SNR, dB	Q(x)	R(x) a = 0 b ≥ 0	P(x) a = 1 b = 1	$Q_a(x)$ a = 0.344 b = 5.334 upper bound for x ≥ 0	$Q_a(x)$ a = 1/π b = 2π lower bound for x ≥ 0	$Q_a(x)$ a = 0.339 b = 5.510 approximation for x ≥ 0
0	----	.5	----	.3989	.5021	.5000	.5013
.5	-6.02	.3085	.7041	.3149	.3086	.3050	.3077
1.0	0	.1587	.2420	.1711	.1590	.1570	.1586
1.5	3.52	$6.681\cdot10^{-2}$	$8.635\cdot10^{-2}$	$7.184\cdot10^{-2}$	$6.706\cdot10^{-2}$	$6.634\cdot10^{-2}$	$6.690\cdot10^{-2}$
2.0	6.02	$2.275\cdot10^{-2}$	$2.700\cdot10^{-2}$	$2.415\cdot10^{-2}$	$2.285\cdot10^{-2}$	$2.265\cdot10^{-2}$	$2.281\cdot10^{-2}$
2.5	7.96	$6.210\cdot10^{-3}$	$7.011\cdot10^{-3}$	$6.510\cdot10^{-3}$	$6.236\cdot10^{-3}$	$6.191\cdot10^{-3}$	$6.227\cdot10^{-3}$
3.0	9.54	$1.350\cdot10^{-3}$	$1.477\cdot10^{-3}$	$1.401\cdot10^{-3}$	$1.355\cdot10^{-3}$	$1.347\cdot10^{-3}$	$1.354\cdot10^{-3}$
3.5	10.88	$2.326\cdot10^{-4}$	$2.493\cdot10^{-4}$	$2.397\cdot10^{-4}$	$2.334\cdot10^{-4}$	$2.323\cdot10^{-4}$	$2.332\cdot10^{-4}$
4.0	12.04	$3.167\cdot10^{-5}$	$3.346\cdot10^{-5}$	$3.246\cdot10^{-5}$	$3.177\cdot10^{-5}$	$3.164\cdot10^{-5}$	$3.174\cdot10^{-5}$
4.5	13.06	$3.398\cdot10^{-6}$	$3.552\cdot10^{-6}$	$3.467\cdot10^{-6}$	$3.407\cdot10^{-6}$	$3.396\cdot10^{-6}$	$3.404\cdot10^{-6}$
5.0	13.98	$2.867\cdot10^{-7}$	$2.973\cdot10^{-7}$	$2.916\cdot10^{-7}$	$2.873\cdot10^{-7}$	$2.865\cdot10^{-7}$	$2.872\cdot10^{-7}$
6.0	15.56	$9.866\cdot10^{-10}$	$1.013\cdot10^{-9}$	$9.989\cdot10^{-10}$	$9.883\cdot10^{-10}$	$9.863\cdot10^{-10}$	$9.879\cdot10^{-10}$

imations. In the literature, several power series approximations of $Q(x)$ have been given, see for example [4]. We have found these approximations less useful for the programmable calculator. They are either only applicable in a restricted interval of x, require several numerically precise coefficients, or require programs with a loop which increases the computation time. For example, the rational approximations in [4, p. 299], are given with a maximum absolute error. For large x, these approximations have a larger relative error than the class $Q_a(x)$ considered in this paper.

The functions in the family $Q_a(x)$ have some significant advantages for the communication engineer, namely: simply by choosing two parameters in Table 1, the best upper bound, lower bound or approximation for the interval $x \geqslant x_0$ ($x_0 = 0$, $Q^{-1}(0.1)$, $Q^{-1}(0.01)$, $Q^{-1}(0.001)$) is obtained. With a simple pocket calculator program, very good approximations for $Q(x)$ are obtained.

Different analytical expressions for the approximation of $Q(x)$ can be considered. We have tried several other configurations, but none of them is as simple a formulation as (17). However, we do not claim that $Q_a(x)$ in the form (17) is the optimum solution to the approximation problem in terms of the minimum number of programming steps required. Better approximations with fewer programming steps might be possible. For the communication engineer, the solutions in Table 1 are quite satisfactory in terms of simplicity, precision and feasibility.

A slightly more complex structure which gives more exact approximations can be derived from $Q_a(x)$ in the form (13). By letting the two parameters depend on x, i.e., replacing a and b by simple functions of x, a new structure is obtained with new parameters in $a(x)$ and $b(x)$ which can be optimized by the same techniques discussed above.

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Experimental Investigation of Some Effects of Multipath Propagation on a Line-of-Sight Path at 14 GHz

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***Abstract*—A microwave line-of-sight propagation experiment is carried out in Denmark at frequencies around 14 GHz. Results from long**

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term measurements of multipath propagation are presented. The multipath fade durations are shown to be log-normally distributed. The level dependence of the probability of fading, P, the average duration of fades, $\bar{t}$, and the number of fades, N_0, is investigated. Results show that $P \propto L^2$, $\bar{t} \propto L^{2/3}$ and $N_0 \propto L^{4/3}$. This differs from previously published results, where proportionalities of L^2, L, and L, respectively, have been proposed. Statistical results on enhancements above free space level are also presented. The results presented are believed to enable an improved description of the effects of multipath propagation.

1. INTRODUCTION

The quality of microwave line-of-sight communication systems may be severely degraded by the effect of multipath propagation. The main effect is a generation of fades, i.e., periods with decreased signal level. Generally, this is of importance for all path lengths in excess of 10-20 km. A commonly less recognized effect is an occurrence of enhancements of the signal level received.

Statistics on the probability of multipath fading at different frequencies have been reported by several authors. Theoretical and experimental results on characteristics of fades, i.e., number of fades and distributions of fade durations, have been published for frequencies of 4, 6 and 11 GHz,[1]-[4]. Statistical information on enhancements has not been published so far.

This paper presents comprehensive experimental results on characteristics of the effect of multipath propagation in the frequency band 13.5-15.0 GHz. Results on the probability distribution of fading, the probability distribution of fade duration, the average duration of fades, and the number of fades are described. Similar data on the distributions of enhancements are included for signal levels up to +11 dB above free space value.

2. DESCRIPTION OF THE EXPERIMENT

The test setup utilizes the towers of an existing microwave trunk system on a 44.7 km overland path southwest of Copenhagen; see Fig. 1 for a path profile. The area is partly urban. The terrain roughness is 21 m as measured by the standard deviation of terrain elevations at 1 km intervals.

The test signal is a carrier which is frequency modulated with a 10 MHz test tone. The carrier is swept sinusoidally from 13.5 to 15.0 GHz at a rate of 1 Hz. The polarization is linear, horizontal. The amplitude, the differential gain and the differential phase are measured at 2.5 MHz intervals during a sweep, supsequently digitized and stored on magnetic tape. A minicomputer supervises the whole measuring system.

The data recorded are analyzed off-line. The validity of the data is tested very carefully to assure that the different kinds of analyses do not give misleading results due to erroneous data. The fading events are automatically classified as due to precipitation or due to multipath propagation, [5].

In order to be able to generate distributions of the various parameters it is necessary to divide the dynamic range of the parameters into a number of intervals of finite widths. However, since the parameters at a fixed frequency may change significantly from sweep to sweep, it is very likely that the parameters cross at least one interval boundary between successive measurements. The use of interpolation will thus improve the quality of the prepared statistics considerably. Therefore automatic procedures with interpolation functions have been included in the calculations.

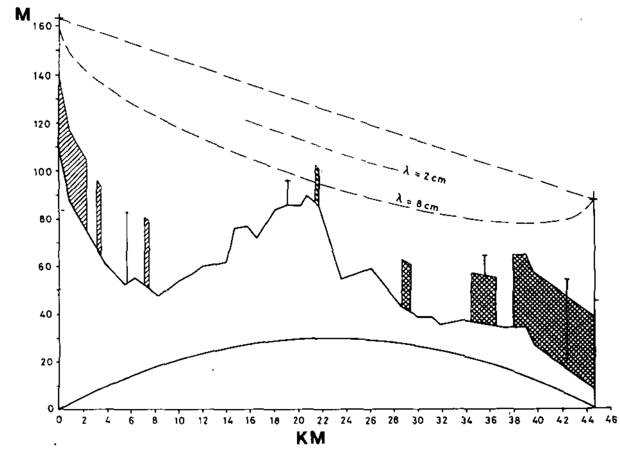

Figure 1. Path profile, $k = 4/3$. First Fresnel zone indicated for frequencies 15 GHz (λ = 2 cm) and 3.75 GHz (λ = 8 cm).

The dynamic range of the system is from +12 dB to −60 dB relative to nominal, with some nonlinearities above +6 dB and particularly above +10 dB. A linearization and calibration procedure is used to compensate for nonlinearities and long term variations in the reference level. A more detailed description of the system may be found in [6] or [7].

The data presented are based on results from a total measuring time of 91 weeks, including the late summer periods of 1974, 1975 and 1977. All statistics presented in the paper represent average distributions which have been obtained by averaging distributions for 125 equally spaced frequencies in the frequency band 13.5-15.0 GHz. Averaging is applied to improve the statistical basis of the data. It is believed to be a valid approach, as no pronounced frequency variation has been observed in the frequency band.

3. AMPLITUDE PROBABILITY DISTRIBUTIONS

The variation of the signal level received during multipath activity results from a combination of signals with different amplitudes and randomly distributed phases. It has been shown that cumulative distributions of amplitude variations can very often be approximated by simple analytical expressions, e.g., described by power laws of the received signal level, [3].

Measured amplitude probability distributions are shown in Fig. 2. The distribution for the total period represents 91 weeks measuring time with approved data from 4.63×10^7 s. The average worst month data are the average of data from months with the highest multipath fading probability each year, August 1974 and 1975, and July 1977.

3.1 Probability of Fading

The measured cumulative fading distributions can be approximated by straight lines when plotted on a log-log scale. The level dependence of the measured data can be approximated by a square law in the deep-fade region, i.e.,

$$P(V \leqslant L) \propto L^2, \qquad \text{for } L \leqslant 0.1$$

where

P is the probability function

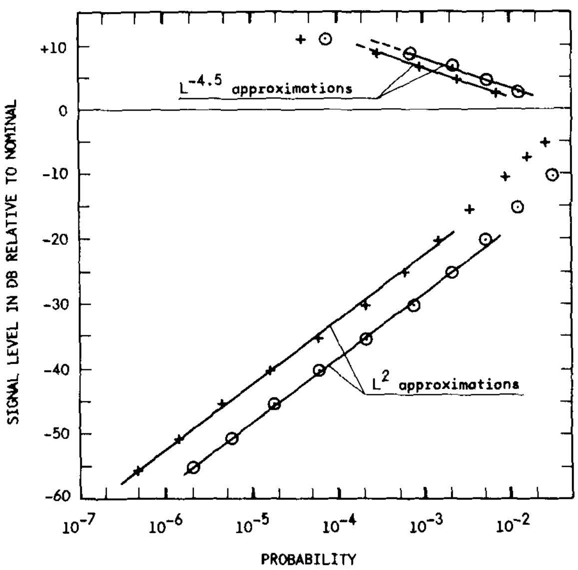

Figure 2. Measured cumulative probability distributions of multipath fades and enhancements. Frequency: 13.5-15.0 GHz, path length: 44.7 km, + : total period, ⊙ : average worst month.

V is the received amplitude in linear measure
L is the specified amplitude in linear measure.

This relation is valid for data from the total period as well as for data from the average worst month. The L^2 dependence for deep fades agrees well with the theory of Rayleigh fading often used to describe fading due to multipath propagation. At higher levels the measured distributions approaches a proportionality to L.

A formula for estimating the probability of deep fading for the worst month has been proposed by [8] and adopted by the CCIR [9]:

$$P(V \leqslant L) = KQL^2 f^B d^C$$

where

P is the probability
V and L are amplitudes in linear measure
K is a factor for climatic conditions
Q is a factor for terrain conditions
f is the frequency in GHz
d is the path length in km
B and C are constants.

This formula, with the CCIR figures for the N.W. Europe applied for this path at these frequencies, underestimates the probability by a factor of 5.5 compared with the measured data:

$$\text{CCIR formula: } P(V \leqslant L) = 0.119L^2$$

$$\text{measured: } P(V \leqslant L) = 0.659L^2.$$

The discrepancy may be due to the uncertainty in the determination of the constants K, Q, B and C for specific regions and the year to year variability in worst month data.

3.2 Probability of Enhancement

The distributions of probability of enhancement above free space level show a distinct level dependence for enhancements between +2 and +9 dB. No difference is observed between the level dependence for the worst month data and that for the total period. The probability function for the worst month can be approximated to:

$$P(V \geqslant L) = 0.059L^{-4.5}, \qquad \text{for } 1.25 \leqslant L \leqslant 3.0$$

The measured distributions tend to flatten out above a level of +9 dB/nominal. This may be due to the limited dynamic range of the system or an effect of a small sample size.

4. FADE DURATION DISTRIBUTIONS

The duration of a single fade is defined in Fig. 3. The analysis of the data recorded includes preparation of frequency distributions of the duration of fades at different amplitude levels. Only fades for which data have been fully approved from the time of the onset of the fade to the time of recovery are included in these distributions. No difference was observed between the worst month distributions and distributions based on data from the total period. The greater time basis is therefore applied in this presentation.

4.1 Duration Probability Distributions

Measurements made in United States in a late summer period, [1] and [2], have shown that the fade durations tend to be log-normally distributed.

The fade duration distributions obtained from the Danish measurements are shown in Fig. 4 for different signal levels. The measured distributions are approximated by straight lines, i.e., log-normal approximations. Straight lines are close approximations to the measured values for durations of more than one second, hence strongly supporting the assumption that fade durations are log-normally distributed. However, the measured distributions tend to bend downwards for durations shorter than one second. This behavior may be due to an effect of a small sample size (a subdivision of the total amount of data), the one-second sampling rate or an inadequacy of the interpolation functions applied in the analysis.

The standard deviation of the distributions decreases linearly with decreasing signal level, from 6.9 dB at a level of −10 dB/nominal to 4.3 dB at −50 dB/nominal. This implies that the fade duration distributions with durations normalized to the average duration at the levels cannot be independent of the level as suggested by [4] for instance.

The probability distributions of the duration of enhancements (not shown) can also be approximated by log-normal distributions. In this case the standard deviation was found to decrease slightly with increasing signal level, from 8.0 dB at a level of +3 dB to 6.7 dB at +11 dB/nominal.

4.2 Average Duration Distributions

Average durations have been calculated from the log-normal approximations to the measured duration probability distributions. The approximations have been extended beyond the median values for the low signal levels. The average duration versus signal level is shown in Fig. 5.

The average duration of fades shows a distinct linear dependence of the level when plotted on a log-log scale. This

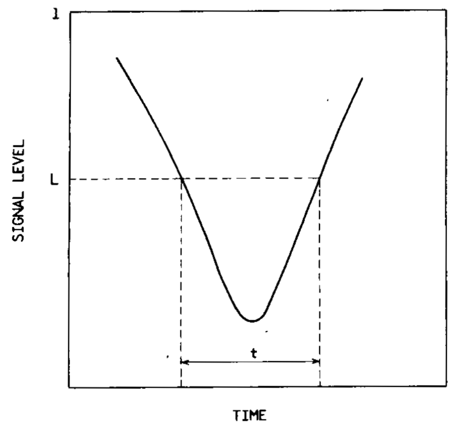

Figure 3. Definition of fade duration t at level L.

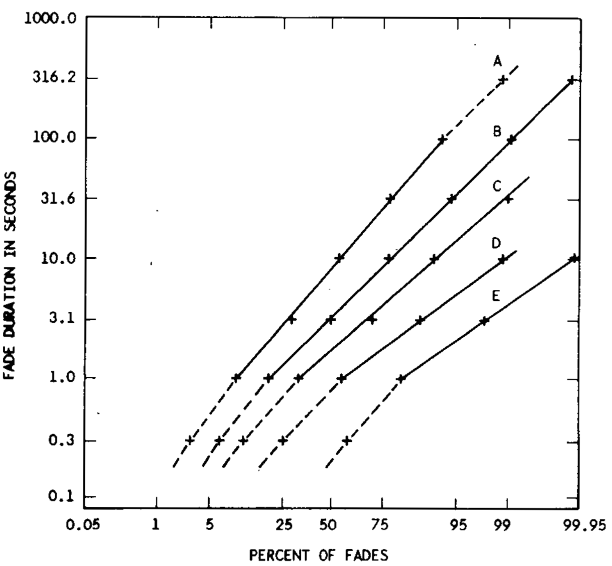

Figure 4. Measured fade duration probability distributions, total period. Full lines indicate log-normal approximations. 13.5-15.0 GHz, 44.7 km. Level $A, B, \cdots, E$: $-10, -20, \cdots, -50$ dB/nominal.

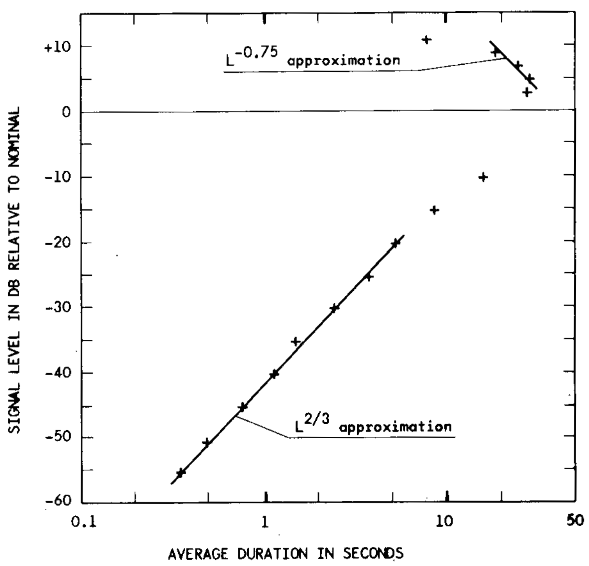

Figure 5. Average fade duration versus level for multipath fades. 13.5-15.0 GHz, 44.7 km.

dependence can be expressed as

$$\bar{t} = 24.5L^{2/3}, \qquad \text{for } L \leqslant 0.1$$

where

$\bar{t}$ is in seconds
L is the amplitude in linear measure.

This equation yields estimated durations which are in close agreement with the experimental results.

The results reported in [1], [2] and [4] differ from the results obtained by this experiment in two ways. In the first place, the average durations have been approximated to be proportional to L in [1], [2] and [4]. In the second place, the average durations measured at 4, 6 and 11 GHz on paths of 45-50 km in length are considerably longer than the durations shown in Fig. 5. Table 1 illustrates the results obtained by the different experiments. The apparent discrepancy in the level dependence cannot easily be explained. Regarding the absolute values, the difference may be partly explained by a frequency dependence. Reference [2] indicates a frequency dependence of approximately $1/\sqrt{f}$, while [4] concludes that the average durations are frequency independent. The frequency dependence thus has to be further investigated. Another source of deviation is the difference in the level dependence of the approximations applied, but this can only partly account for the discrepancy.

The average durations of enhancements do not exhibit a similar clear level dependence. The measured values can, for levels between +3 and +10 dB/nominal, be approximated to, see Fig. 5:

$$\bar{t} = 41.4L^{-0.75}, \qquad \text{for } 1.4 \leqslant L \leqslant 3.2$$

where

$\bar{t}$ is in seconds
L is the amplitude in linear measure.

The average duration observed at a level of +3 dB/nominal is slightly shorter than the average duration observed at the +5 dB level. This fact is at present unexplained.

5. NUMBER OF EVENTS

Separate distributions are prepared for the number of events. Measured distributions of the number of fades and the number of enhancements are shown in Fig. 6 for data from the total period and from the average worst month, respectively. Only events for which data are fully approved from the onset until the recovery are included in the distributions shown. The total number of fades may thus be underestimated since data are sometimes discarded. The real numbers may be up to 40% higher at some levels, estimated from a knowledge of the total sum of approved onsets and recoveries. Almost all data are fully approved for enhancements.

The measured numbers have been approximated to an $L^{4/3}$ dependence for the deep fades. This is in agreement with the approximations of the probability of fading and the average duration of fades. The approximation of the average worst month data yields

$$N_0 = 68\,000L^{4/3}, \qquad \text{for } L \leqslant 0.06$$

TABLE 1
AVERAGE FADE DURATIONS AS MEASURED BY DIFFERENT EXPERIMENTS

Reference	f GHz	D km	Fade duration (sec) at level L (-20 dB)	(-30 dB)	(-40 dB)
Bullington, [2]	4	48.3	40.0	12.6	4.0
Vigants, [1]	4 6	45.8§ - §	40.8 49.0	12.9 15.5	4.1 4.9
Barnett, [4]	11	45.8§	30.0	10.4	3.0
This experiment	14	44.7	5.3	2.5	1.1

§ : a common path, a common antenna

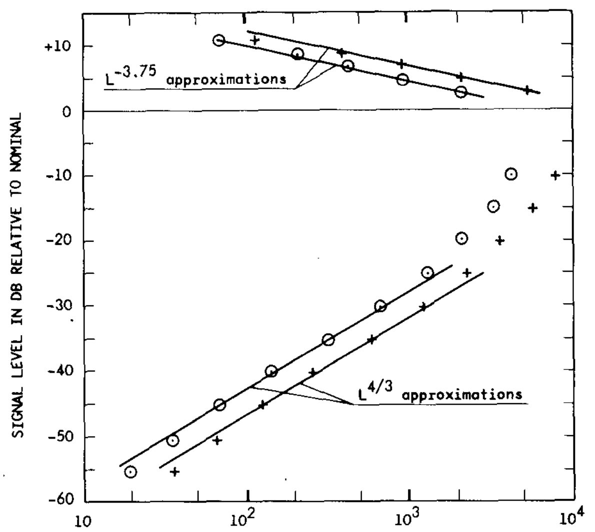

Figure 6. Measured number of entirely approved events versus level. 13.5-15.0 GHz, 44.7 km. + : total period, ⊙ : average worst month.

where

N_0 is the number of fades
L is the amplitude in linear measure

The number of enhancements has a more distinct level dependence. The measured numbers have been approximated to an $L^{-3.75}$ dependence. The approximation of the average worst month data yields

$$N_e = 7100L^{-3.75}, \qquad \text{for } 1.2 \leqslant L \leqslant 3.5$$

where

N_e is the number of enhancements
L is the amplitude in linear measure.

6. SUMMARY AND CONCLUSIONS

Extensive experimental data on multipath fading and enhancements have been presented. In some aspects the results are in accordance with the commonly adopted theory, e.g., the probability of fading $P(V \leqslant L)$ was shown to be proportional to L^2 and the measured probability distributions of fade durations to be log-normal. In other aspects the results differ from previously published results, e.g., the average fade duration at a level L was found to be proportional to $L^{2/3}$ and the number of fades to be proportional to $L^{4/3}$. Further, the average durations at 14 GHz were much shorter than the average durations reported elsewhere for 4, 6 and 11 GHz, [1]-[4], which may indicate that a reconsideration on the frequency dependence is needed.

Data on enhancements have been presented. The measured probability distribution was shown to be proportional to $L^{-4.5}$. The probability distributions of the durations were log-normal and the average durations could be approximated to an $L^{-0.75}$ dependence. The number of enhancements was found to be proportional to $L^{-3.75}$.

It is believed that the data presented will lead to an improved knowledge of the multipath propagation conditions which impair the quality of line-of-sight communication. A knowledge, which will be of practical value to the designers of microwave communication systems.

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