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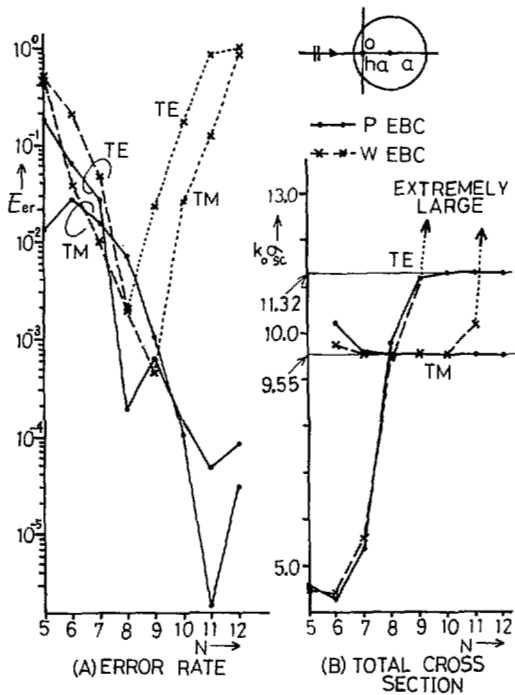Technical Information Center of Denmark

Fig. 3. Comparison between the proposed EBC and Waterman's EBC for an origin-shifted circular cylinder ($k_0 a = 2.0$, $h = 0.6$, $\kappa = 3.0$). ·

trophe in an earlier stage for the cases where the degree of convergence of the series solution is not so good. It is to be mentioned that the proposed method does not always give results superior to Waterman's and on the contrary in some cases of near-circular cross section shape Waterman's method happens to give better results. As for the computing time, the proposed method consumes more time than Waterman's method, but not so much as expected from a simple consideration of the number of multiplications and divisions. For example, it is only increased 30 percent at most in the case of $N = 16$ of Fig. 2. (The total computing time for this example is about 23 s on the NEAC 2200-700 computer.)

The EBC method is simpler and more useful than the method of solving directly the integral equations [6, eqs. (43)–(46) or (53)–(56)] when the boundary shape of the dielectric cylinder is smooth and convex. For the dielectric cylinder of relatively complex boundary shape, however, the limitation of applicability of the EBC method seems to exist.

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On Determining if a Specular Point Exists

W. V. T. RUSCH, FELLOW, IEEE AND O. SØRENSON

Abstract—A technique is presented whereby the existence of a specular point on a convex surface of revolution can be determined without actually finding it. Only the evaluation of two simple algebraic expressions is involved. Should a specular point be found not to exist, a search procedure has been thereby eliminated.

I. INTRODUCTION

All geometrical theories of diffraction require superposition of a classical specular or geometrical-optics ray to determine the total field. Geometrical laws relate the reflected and incident fields at a specular point in a straightforward manner [1], yet the location of a specular point on a complex surface can be a difficult, or at least tedious, procedure. Depending on the reflector geometry, and the location of the source and field points, one- or two-dimensional iteration schemes can be used. These techniques waste time and yield ambiguous results, however, if the source/field point orientation is such that a specular point does not exist.

A procedure based on Fermat's principle will be described whereby the existence of a specular point can be determined for a general convex surface of revolution. The following two cases are considered.

1) Source and field point are coplanar with the symmetry axis.
2) Source and field point location is unrestricted.

II. ANALYTIC TREATMENT

Let O be the source point, P be the observation point, R be a point on the surface, $\overline{OR}$ be the vector from O to R, and $\overline{RP}$ be the vector from R to P (Fig. 1). Let R_0 be a specular point on the surface. Define

$$d \equiv |\overline{OR}| + |\overline{RP}|. \tag{1}$$

For a convex monotonic surface d will be a global minimum at R_0 so that, at most, one specular point may exist on the surface. The gradient of d with respect to the coordinates of R is

$$\nabla_R d = \frac{\overline{OR}}{|\overline{OR}|} - \frac{\overline{RP}}{|\overline{RP}|} \tag{2}$$
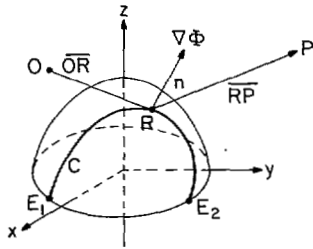
Fig. 1.   Geometry of general problem.

Fig. 2.   Geometry of planar problem.

and it is easily shown that

$$\nabla_R d \cdot (\overline{OR} \times \overline{RP}) = 0 . \tag{3}$$

Thus $\nabla_R d$ lies in the plane containing O, R, and P. Next, define

$$\overline{v} \equiv \nabla_R d - (\nabla_R d \cdot \hat{n})\hat{n} \tag{4}$$

where $\hat{n}$ is the outward normal at R. Thus $\overline{v}$ is the projection of $\nabla_R d$ on a plane tangent to the surface at R, and $\overline{v}$ also lies in the plane containing O, R, and P. For R in the vicinity of R_0, $\overline{v}$ will be directed *away from* R_0 since d is a minimum at R_0.

A. Axially Symmetric Convex Reflector with Source and Observation Point in the Same Aximuthal Plane

The planar cross section containing O, P, and the reflector axis is shown in Fig. 2. The specular point R_0 will also lie in this plane. Consequently, this special case reduces to a two-dimensional problem.

As shown in the previous section, for all points R in the planar cross section, $\overline{v}$ also lies in this plane and can only be zero at a specular point which may or may not exist. Furthermore, if a specular point exists, as shown in the figure, $\overline{v}$ will point to the right for all R on the right side of R_0 and to the left for all R on the left of R_0. Thus, at both edges of the surface in the cross section (E_1 and E_2), $\overline{v}$ will point *away* from the surface. However, if no specular point exists, v cannot reverse direction (because it will be necessary to become zero in the process). Consequently, $\overline{v}$ will point in the same direction along the entire surface and will point *toward* the surface at one edge and away from it at the other.

Thus, for this two-dimensional special case, in order to determine if a specular point exists, $\overline{v}$ need only be evaluated at the two edges. The specular point will exist if and only if $\overline{v}$ points away from the surface at both edges. If a specular point is found to exist, many different algorithms are available to locate it, one of which may be based on the principles of this communication [2].

B. Axially Symmetric Convex Reflector with Source and Observation Point Not Restricted to the Same Azimuthal Plane

The geometry of this configuration is shown in Fig. 1. Define the surface by

$$\rho^2 = f(z) \tag{5}$$

$$\Phi(\rho, z) = \rho^2 - f(z) = 0 \tag{6}$$

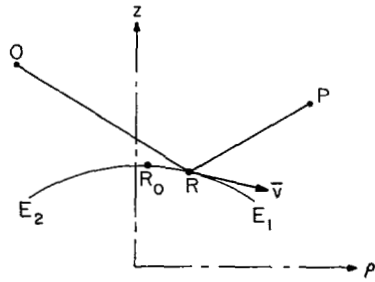$$\nabla\Phi = 2x\hat{a}_x + 2y\hat{a}_y - f'(z)\hat{a}_z . \tag{7}$$

In this case O and P do not necessarily lie in the same azimuthal plane. However, according to Fermat's Principle, at the specular point, O, R, P, and $\nabla_R\Phi$ (or $\hat{n}$) all lie in the same plane. Hence

$$\nabla_R\Phi \cdot (\overline{OR} \times \overline{RP}) = 0 . \tag{8}$$

Expanding (8) yields

$$Ax_R + By_R + C = 0 \tag{9}$$

where

$$A = -2[(y_O - y_P)(z_R - z_O) + y_O(z_O - z_P)]$$
$$\quad - f'(z_R)(y_O - y_P) \tag{10}$$

$$B = +2[(x_O - x_P)(z_R - z_O) + x_O(z_O - z_P)]$$
$$\quad + f'(z_R)(x_O - x_P) \tag{11}$$

$$C = f'(z_R)(x_P y_O - y_P x_O) . \tag{12}$$

Equations (9)–(12) represent a three-dimensional surface S, which reduces to the cross-sectional plane of the previous section when $x_O/y_O = x_P/y_P$.

The surface S intersects the axially symmetric reflector in a curve C as shown in the figure. The specular point R_0, if it exists, lies along C. At every point R along C, $\overline{v}$ is tangent to C. Consequently, as in the previous section, the problem reduces to finding the two points E_1 and E_2 where C intersects the reflector rim and to evaluating $\overline{v}$ at those points. A specular point will exist if and only if $\overline{v}$ points away from the surface at both E_1 and E_2.

The two edge points may be found by substituting

$$x_R{}^2 + y_R{}^2 = f(z_{\text{RIM}}) \tag{13}$$

into (9). The four possible types of solution are as follows.

1) If $B = O$:

$$x_{\text{RIM}} = -C/A \tag{14a}$$

$$y_{\text{RIM}} = \pm\sqrt{f(z_{\text{RIM}}) - (C/A)^2} . \tag{14b}$$

2) If $A = O$:

$$y_{\text{RIM}} = -C/B \tag{15a}$$

$$x_{\text{RIM}} = \pm\sqrt{f(z_{\text{RIM}}) - (C/A)^2} . \tag{15b}$$

3) If $A = O$ and $B = O$: O and P are coincident, causing (9) to become indeterminant. However, the techniques of the previous section can be used.

4) If $A \neq O, B \neq O$:

$$y_{\mathrm{RIM}} = -\frac{A}{B} x_{\mathrm{RIM}} - \frac{C}{B} \tag{16a}$$

$$x_{\mathrm{RIM}}{}^2 \left[1 + \left(\frac{A}{B} \right)^2 \right] + \left[\frac{2AC}{B^2} \right] x_{\mathrm{RIM}}$$

$$+ \left[\frac{C^2}{B^2} - f(z_{\mathrm{RIM}}) \right] = 0 . \tag{16b}$$

The descriminant of (16b) is never negative so that two real roots always exist.

As a specific example, let $x_O = x_P = O$ so that both O and P lie in the yz plane. Then, from (11) and (12), $B = C = O$ so that (14a) and (14b) yield $x_{RIM} = O$ and $y_{RIM} = \pm \rho_{RIM}$, as expected.

Having, again, as in the previous section, demonstrated the existence of a specular point for the nonplanar case, many different search techniques are available for locating it. However, should the simple evaluation of $\bar{v}$ at the two points on the reflector ring defined by (9) indicate that no specular point exists, a search procedure has been eliminated.

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Transient and Time-Harmonic Dyadic Green's Functions for a Perfectly Conducting Cone

K. K. CHAN AND L. B. FELSEN

Abstract—New representations for the time-dependent dyadic Green's functions for a perfectly conducting semi-infinite cone are presented. For the special case of small cone angles and an on-axis source, simplified expressions are given for both the time-dependent and time-harmonic regimes.

In a separate paper, new representations for the time-dependent scalar Dirichlet and Neumann Green's functions for a semi-infinite cone were developed [1]. It was shown that these formulations can be simplified substantially when the cone angle is small and the source is located on the cone axis. It was also shown that new closed-form time-harmonic solutions can be obtained by Fourier transformation of the transient fields.

The present communication extends these results to the dyadic Green's functions generated by vector dipole excitation.

Consider an electric dipole of vector strength $\mathbf{J}$ situated at the point $\mathbf{r} = \mathbf{r}'$ outside a perfectly conducting cone defined by the surface $\theta = \theta_0$. The components of $\mathbf{J}$ in the (r, θ, ϕ) directions are J_r, J_θ, and J_ϕ, respectively. The total field can be regarded as a superposition of the separate responses due to J_r, J_θ, and J_ϕ. The response to a radial electric dipole source may be obtained from the scalar Dirichlet Green's function discussed in [1]. The electric field due to a transverse dipole source of vector strength $\mathbf{J}_t$ can be represented in terms of functions S' and S'' as follows [2]:

$$\mathbf{E} = E_r \mathbf{r}_0 + E_\theta \theta_0 + E_\phi \phi_0 \tag{1}$$

$$E_r = \left(\frac{\partial^2}{\partial r^2} + k^2 \right) (\mathbf{J}_t \cdot {}_t \nabla') \frac{\partial}{\partial r'} \left(\frac{S'}{-i\omega\epsilon} \right) \tag{1a}$$

$$E_\theta = \frac{-1}{i\omega\epsilon r} \frac{\partial^2}{\partial r \partial \theta} (\mathbf{J}_t \cdot {}_t \nabla') \frac{\partial S'}{\partial r'}$$

$$- \frac{i\omega\mu}{r \sin \theta} \frac{\partial}{\partial \phi} (\mathbf{J}_t \times \mathbf{r}_0 \cdot {}_t \nabla' S'') \tag{1b}$$

$$E_\phi = - \frac{1}{i\omega\epsilon r \sin \theta} \frac{\partial^2}{\partial r \partial \phi} (\mathbf{J}_t \cdot {}_t \nabla') \frac{\partial S'}{\partial r'}$$

$$+ \frac{i\omega\mu}{r} \frac{\partial}{\partial \theta} (\mathbf{J}_t \times \mathbf{r}_0 \cdot {}_t \nabla' S'') \tag{1c}$$

where

$$\mathbf{J}_t \cdot {}_t \nabla' = \frac{J_\theta}{r'} \frac{\partial}{\partial \theta'} + \frac{J_\phi}{r' \sin \theta'} \frac{\partial}{\partial \phi'} \tag{1d}$$

$$\mathbf{J}_t \times \mathbf{r}_0 \cdot {}_t \nabla' = \frac{J_\phi}{r'} \frac{\partial}{\partial \theta'} - \frac{J_\theta}{r' \sin \theta'} \frac{\partial}{\partial \phi'} . \tag{1e}$$

The scalar functions S', S'' can be decomposed into a free-space portion and a perturbation term accounting for the effect of the cone. Since the free-space portion of the solution is known, only the evaluation of the perturbation fields (distinguished by subscript s) need concern us here. Suitable representations for the latter are [3]

$$S_s' = S_{s_1}' + S_{s_2}' \tag{2a}$$

$$S_{s_1}' = -\frac{i}{2\pi k} j_0(kr_<)h_0{}^{(1)}(kr_>) \sum_{m=1}^{\infty} \frac{\cos m(\phi - \phi')}{m}$$

$$\cdot \left[\tan \frac{\theta}{2} \tan \frac{\theta'}{2} \tan^2 \left(\frac{\pi - \theta_0}{2} \right) \right]^m \tag{2b}$$

$$S_{s_2}' = \frac{1}{8\pi k} \sum_{m=0}^{\infty} \epsilon_m \cos m(\phi - \phi') \int_{-1/2-i\infty}^{-1/2+i\infty} dv \, (2v + 1)$$

$$\cdot \frac{h_v{}^{(1)}(kr)h_v{}^{(1)}(kr')}{v(v+1)} \frac{P_v{}^{-m}(\cos \theta)P_v{}^{-m}(\cos \theta')}{\sin (v - m)\pi}$$

$$\cdot \frac{\Gamma(v + m + 1)}{\Gamma(v - m + 1)} \frac{P_v{}^{-m}(-\cos \theta_0)}{P_v{}^{-m}(\cos \theta_0)} \tag{2c}$$