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An Eigenvalue Study of a Double Integrator Oscillator

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Abstract—A tutorial study of an oscillator built from a loop of two active RC integrators and an ideal inverter. As a reference the linear harmonic oscillator is modeled as a LC circuit and as an ideal two integrator loop. The nonlinear amplifiers of the active RC integrator circuits are assumed to be linear with time varying gain and the eigenvalues are found as function of time. A design strategy based on the time-constants of the integrators is presented.

I. Introduction

In the recent 20 years the classic circuit theory with analysis and synthesis of linear electrical circuits has more or less disappeared from the curriculum of the electrical and electronic engineering students. The digital systems dominate over the analog systems. But in the future possibly nonlinear analog systems will be used instead of or as a supplement to digital systems due to lower power consumption and higher speed [1]. The aim of this tutorial is to demonstrate that the classic circuit theory with poles and zeros (eigenvalues) may be used to gain insight in the behavior of nonlinear circuits.
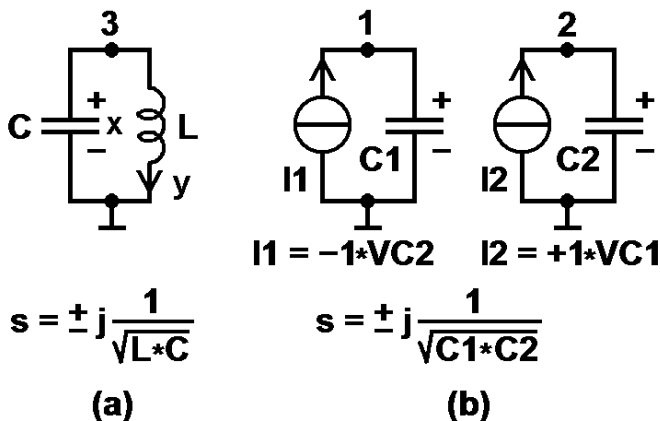

Fig. 1. Harmonic Oscillator Models.
x = V(3) = V(1) = cos(t),   y = I(L) = V(2) = sin(t)
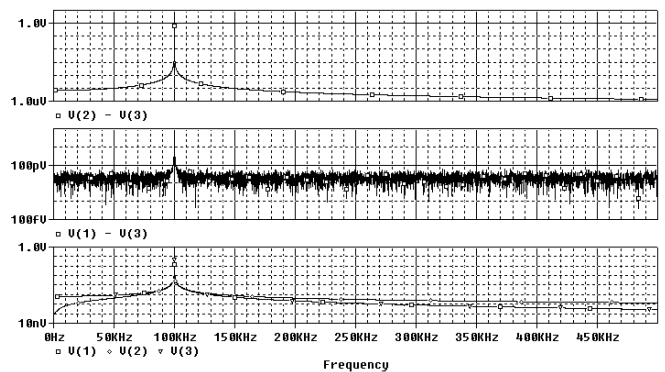

Fig. 2. Comparison of Harmonic Oscillator Models. FFT analysis.   V(1) = V(3) = cos(t),   V(2) = sin(t)

II. Harmonic Oscillator Models

The harmonic oscillator may be modeled as a capacitor coupled in parallel to a coil, Fig. 1(a). If the initial condition is a voltage across the capacitor and a current equal to zero through the coil the voltage becomes a cosine of time and the current becomes a sine of time. The capacitor may be modeled as a current source controlled by the time derivative of its voltage. The coil may be modeled as a voltage source controlled by the time derivative of its current. As variables we choose the voltage across the capacitor: x, and the current through the coil: y, (state variables). We choose voltage V and current I as variables because they are easy to measure (signals). We should use charge $q = C * V$ and flux $\phi = L * I$ as variables because they represent the energy in the system. Unfortunately they are difficult to measure. In the real world charge and flux are nonlinear functions of voltage and current. Note that current is time derivative of charge $I = dq/dt$ and voltage is time derivative of flux $V = d\phi/dt$. Charge provide the coupling between the chemical world and the electrical world (electrolysis). Flux provide the coupling between the mechanical world and the electrical world (generators, motors). Capacitors and coils are memory elements. The resistor

with memory - the memristor $\phi = M * q$ - has recently been implemented in the real world [2].
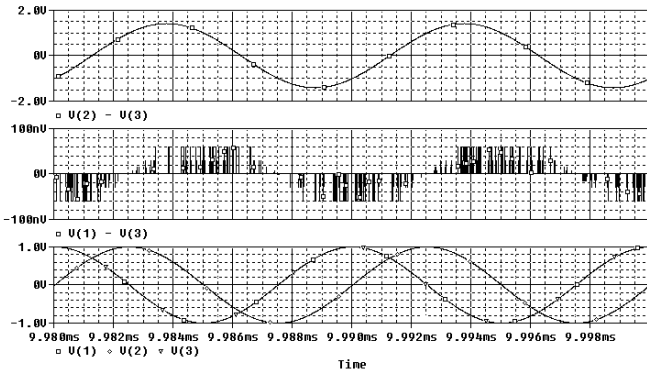

Fig. 3. Comparison of Harmonic Oscillator Models.
Time analysis. $V(1) = V(3) = cos(t)$, $V(2) = sin(t)$

Now the harmonic oscillator may be modeled (defined) by means of two first order differential equations

$$L \frac{dy}{dt} = +x \qquad (1)$$

$$C \frac{dx}{dt} = -y \qquad (2)$$

The two equations may be combined into a second order differential equation.

$$\frac{d^2x}{dt^2} + \frac{x}{LC} = 0 \qquad (3)$$

The roots of the characteristic polynomial - the eigenvalues - are a complex pole pair $s = \pm j * \omega$ on the imaginary axis where $\omega = 2\pi f$. With $\omega^2 = 1/(LC)$ the frequency becomes $f = 1/[2\pi\sqrt{(LC)}]$. The amplitude is given by the initial condition.

An ideal integrator may be modeled as a capacitor loaded current source. The voltage of the capacitor is the time integral of the current source. By means of two ideal integrators the harmonic oscillator may be modeled as shown in Fig. 1(b).

The figures Fig. 2 and Fig. 3 show a PSpice comparison of two 100kHz oscillators with the following component values: $L3 = 1.591549431\mu H$, $C3 = 1.591549431\mu F$ and $C1 = C2 = 1.591549431\mu F$. With an accuracy of $RELTOL = 1\mu = 1e - 6$ in PSpice a very close agreement between the models is observed.

Linear steady state oscillators are mathematical fiction. In the real world linear systems are always damped due to losses. Some kind of non-linearity must be introduced in order to obtain steady state oscillations. Oscillators are non-linear systems. It is to be expected that oscillators based on a double integrator will behave very close to sinusoidal so they have been reported frequently in the literature [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. In the following a double integrator oscillator based on operational amplifiers is investigated.
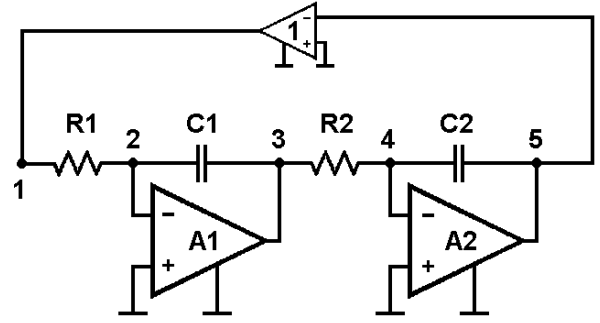

Fig. 4. Double integrator oscillator. $V(1) = -V(5)$

III. Double Integrator Oscillator

The circuit is a dedicated analogue computer circuit based on the definition of the sine and cosine functions: $d(sin(t))/dt = cos(t)$ and $d(cos(t))/dt = -sin(t)$. Input to the first integrator is $-sin(t)$ which give rise to input $cos(t)$ of the second integrator with output $sin(t)$ to be inverted for feed-back to the first integrator (see Fig. 4). Assuming ideal operational amplifiers ($A_1 = A_2 = \infty$) the relation between the components and the frequency becomes $\omega_0^2 = (2\pi f_0)^2 = 1/(\tau_1 * \tau_2)$ where $\tau_1 = R_1 * C_1$ and $\tau_2 = R_2 * C_2$ are the time constants of the integrators. If we choose $R_1 = R_2 = R = 10k\Omega$ and $C_1 = C_2 = C = 15.91549431nF$ the nominal oscillating frequency becomes $f_o = 1$ kHz, $\tau_1 = \tau_2 = \tau = 159.1549431e - 6$, $\omega_o = 1/(RC) = 6.283185308e + 3 = 2\pi f_o$.

R1 = R2 = 10e+3, C1 = C2 = 15.91549431E-9

V(3) = A1 output voltage, V(5) = A2 output voltage
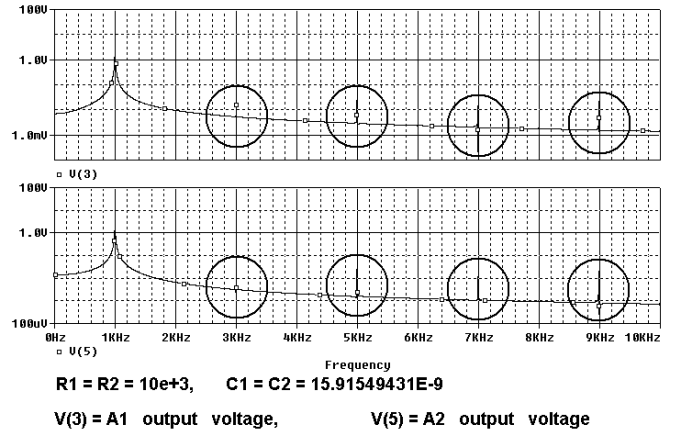
Fig. 5. Frequency spectrum. Logarithmic y-axis.

A. PSpice analysis

Now the two ideal operational amplifiers are replaced with PSpice op-amp macro-models, $\mu A741$ and an ideal PSpice inverter model (EVA3 1 0 5 0 -1) is introduced. The power supply is ± 22 volts.

Figure 5 shows the frequency spectrum of the oscillator in steady state (FFT analysis over 800ms). Higher harmonics may be observed. In this case - where the resistors R_1 and R_2 are chosen equal and the capacitors C_1 and C_2 are chosen equal

Fig. 6.   Amplifier output voltages $V(3), V(5)$ and Power supply currents.
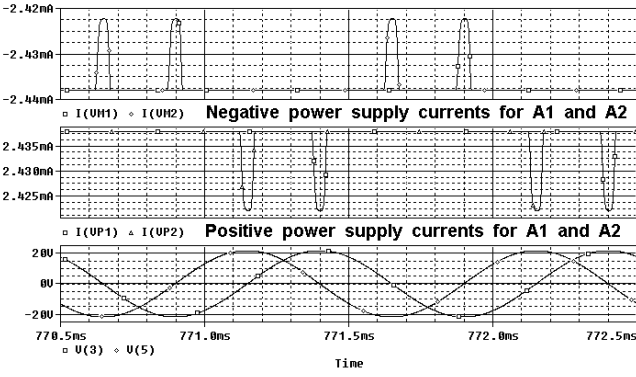
Fig. 8.   Amplifier output voltages $V(3), V(5)$ and Power supply currents.

- clipping of both amplifier output voltages $V(3)$ and $V(5)$ is observed, Fig.6.

Figure 6 shows that energy is transferred to the amplifiers as very narrow pulses at the maximums of the output voltages. This behavior is very similar to the behavior of the pendulum clock where the escape mechanism [13] delivers the energy as pulses when the angel from vertical is zero and the weights go down a step changing potential energy into a kinetic energy impulse. For small swing - i.e. when x and sin(x) are almost equal - the pendulum clock is very close to a damped linear oscillator with very high quality factor and no harmonics. A strategy for design of electronic oscillators with minimum distortion could be optimization of the energy impulses observed.

Fig. 9.   Amplifier voltage gains and output voltages.
V(3) = A1 output voltage, V(5) = A2 output voltage

$R_2 = 12.0\text{k}\Omega, C_2 = 25.33172448\text{nF}$, time constant amplifier A_1: $\tau_1 = R_1 * C_1 = 83.33333333e-6$, time constant amplifier A_2: $\tau_2 = R_2 * C_2 = 303.9806938e - 6$, $\omega^2 = 1/(\tau_1 * \tau_2) = 39.47619124e6 = (2\pi f), f = 0.9999718022\text{kHZ}$.

A comparison of Fig. 7 with Fig. 5 shows that the harmonics of the output voltage $V(5)$ of amplifier $A2$ have been reduced. Figure 8 shows that the power is supplied to amplifier A_1 with an almost constant current of 2.44mA and pulses of $15\mu A$ in the short clipping time intervals. The power is supplied to amplifier A_2 with an almost constant current of 2.438mA. A comparison of Fig. 8 with Fig. 6 shows that the pulses in the power supply currents of amplifier $A2$ have disappeared. This result depends of course on the op-amp macro-model used. Experiments with $LM741$ instead of $uA741$ did not show pulses but reduction of harmonics was obtained. Figure 9 shows the amplifier gains as functions of time over two periods. It is seen that the gain is varying slowly in the interval ± 5000 in the whole period except at the maximums of the amplifier output voltages where it vary between very large and very small values in very short time intervals.

B. Eigenvalue calculation

A study of the eigenvalues of the time-varying Jacobian of the linearized differential equations may give some insight in the behavior of the oscillator [14]. The linear time-varying approach (LTV) is a method to to calculate the time-varying

R1 = 8.333333333e+3,   C1 = 10.0e-9,   R2 = 12.0e+3,   C2=25.33172448e-9

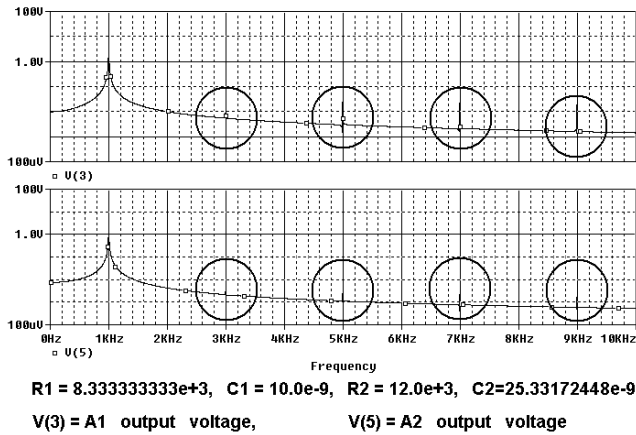V(3) = A1 output voltage,   V(5) = A2 output voltage

Fig. 7.   Frequency spectrum. Logarithmic y-axis.

A large number of PSpice simulations have been made with various combinations of the values of the two resistors and the two capacitors. Apparently it is possible to remove the energy impulses in connection with one of the amplifiers so that the harmonics in the output voltage becomes smaller. The following PSpice results demonstrates this assertion and gives more insight in the behavior of the oscillator.

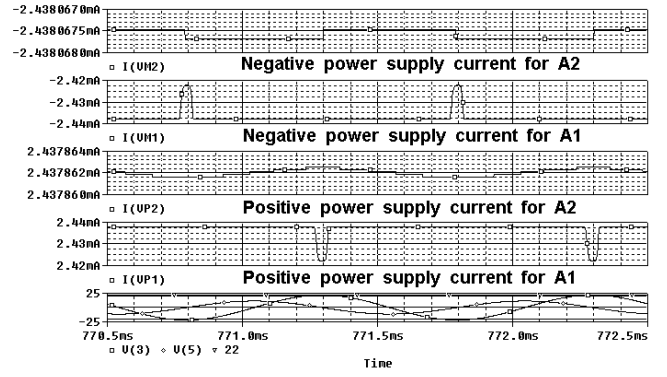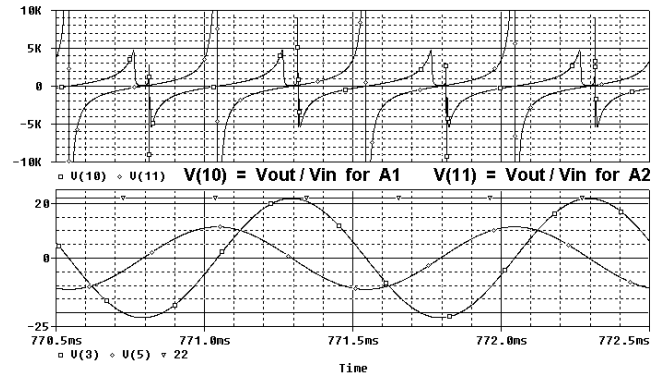By means of the following values a number of PSpice simulations have been made: $R_1 = 8.333333333\text{k}\Omega, C_1 = 10.0\text{nF}$,

eigenvalues (dynamic eigenvalues) of a nonlinear circuit [15], [16], [17], [18], [19]. To calculate the dynamic eigenvalues, the Riccati equation must be solved. In the following it is assumed that a nonlinear circuit can be treated as a time-varying linear circuit.

If it is assumed that the amplifiers are perfect - i.e. the input impedance is infinite and the output voltage is equal to the input voltage times a time varying constant A i.e. $V_{out} = (V_+ - V_-) * A(time)$ - the characteristic polynomial of the circuit becomes

$$s^2 + 2\alpha \, s + \omega^2 = 0 \tag{4}$$

where

$$2\alpha = \frac{C_1 R_1 (1 + A_1) + C_2 R_2 (1 + A_2)}{C_1 R_1 C_2 R_2 (1 + A_1)(1 + A_2)}$$

and

$$\omega^2 = \frac{1 + A_1 A_2}{C_1 R_1 C_2 R_2 (1 + A_1)(1 + A_2)}$$

The roots are

$$p_{1,2} = -\alpha \pm j\sqrt{(\omega^2 - \alpha^2)}$$

The roots of the characteristic polynomial (the poles) are found as function of time by means of a table of the gains as function of time found by means of PSpice. The result is shown in Fig. 10. It is seen that the imaginary part is close
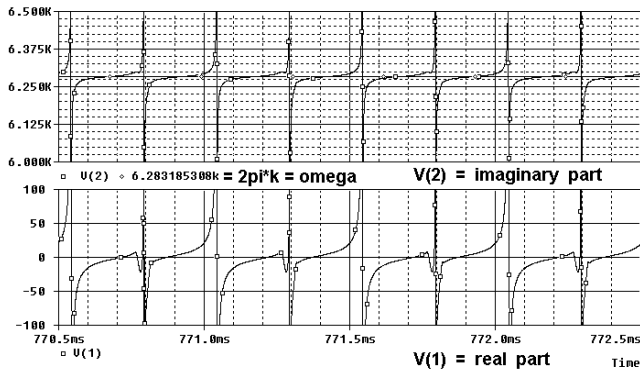

Fig. 10. Complex pole pair as function of time. $\omega_0 = 2\pi * 1k$.

to $\omega_o = 1/\sqrt{(R_1 C_1 R_2 C_2)} = 6.283185308e + 3 = 2\pi f_o$ and the real part is close to zero most of the time. Only in the short intervals - where the energy pulses come from the power supply - the complex pole pair goes to the real axis in the right half plane and split-up into two real roots which via zero and infinity comes back and unite into a complex pole pair as expected. The result is in agreement with the very little phase noise observed.

IV. CONCLUSION

It is demonstrated that the classic circuit theory with poles and zeros (eigenvalues) may be used in connection with the design of a double integrator oscillator. The non-linear circuit is treated as a linear time-varying circuit. The parameters are chosen in such a way that the imaginary part of the poles

of the linear time-varying circuit is as close as possible to ω_0 over a period. The time constants of the integrators are chosen different in order to obtain minimum distortion of the output voltage of one of the amplifiers. The circuit is used mainly for low-frequency oscillators. The structure may be a candidate for IC implementation because it is based on resistors and capacitors only.

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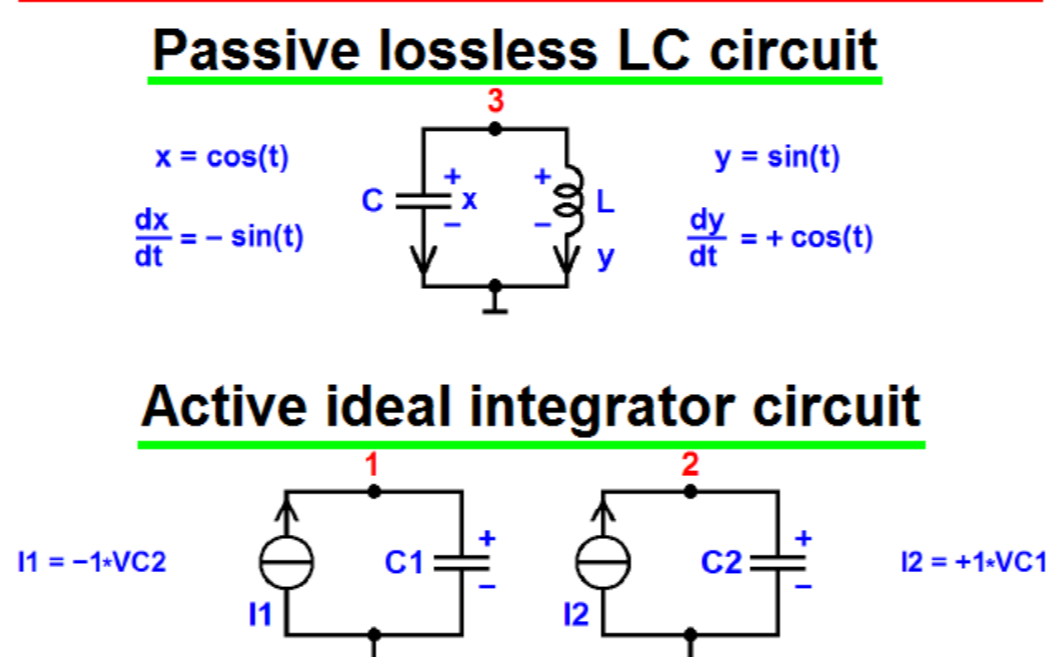
By means of **MEASUREMENTS** you may investigate the behaviour of the **REAL WORLD**

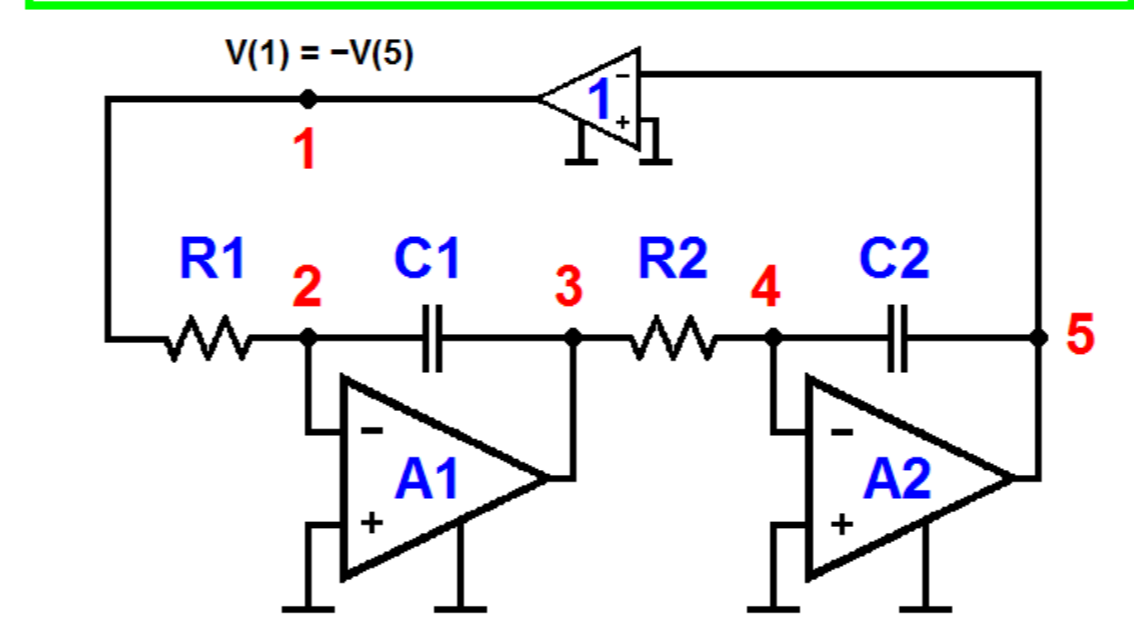By means of **SIMULATION** you may investigate the behaviour of your **MODEL**

If you observe **AGREEMENT** between measurements and simulation **YOU BELIEVE IN YOUR MODEL**

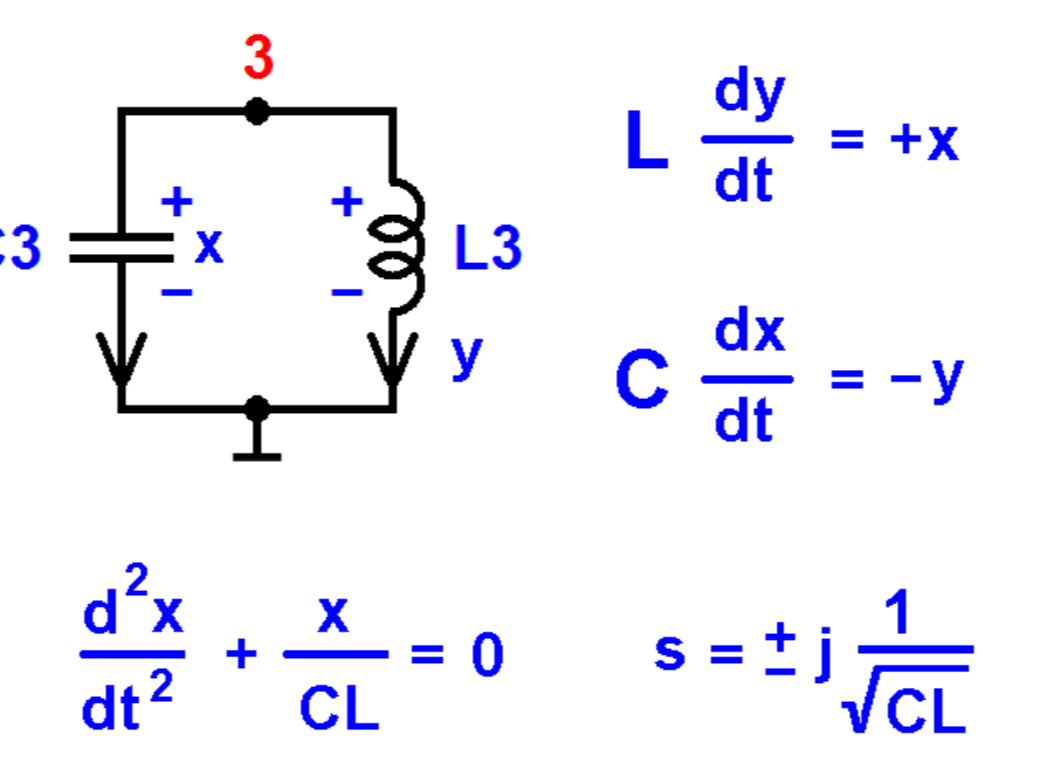By means of a **POSTER** you may present the slides of a **LECTURE** of several hours

Abstract

A tutorial study of an oscillator built from a loop of two active RC integrators and an ideal inverter. As a reference the linear harmonic oscillator is modeled as a LC circuit and as an ideal two integrator loop. The nonlinear amplifiers of the active RC integrator circuits are assumed to be linear with time varying gain and the eigenvalues are found as function of time. A design strategy based on the time-constants of the integrators is presented.
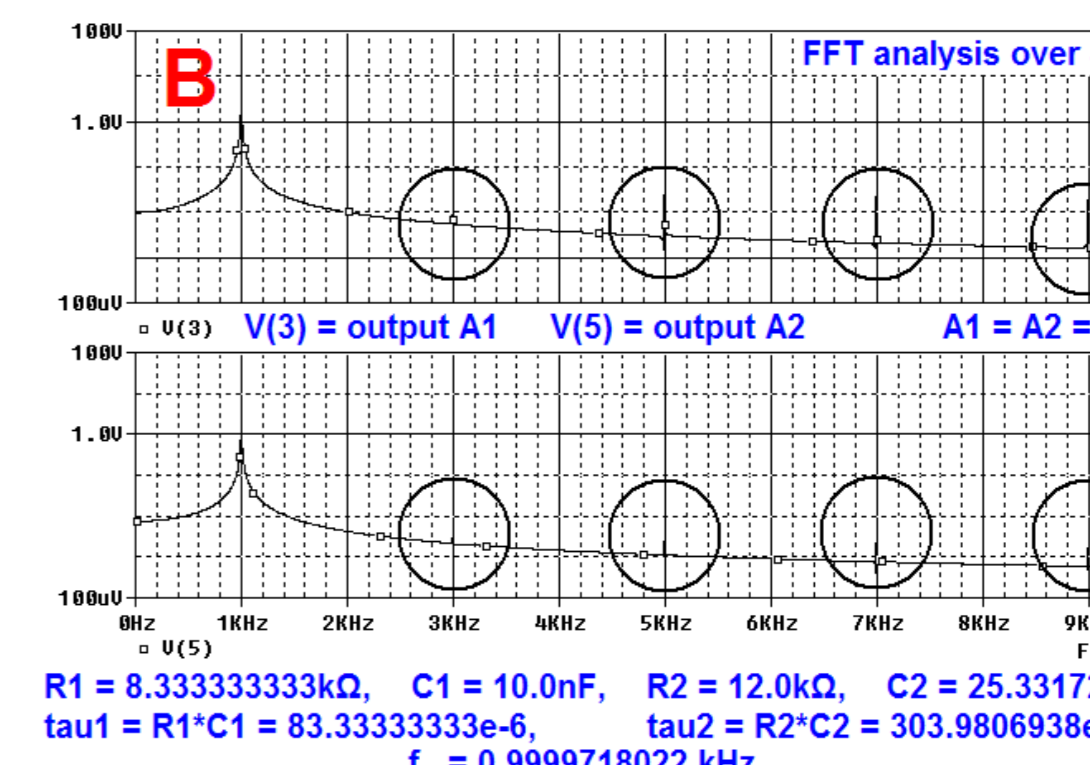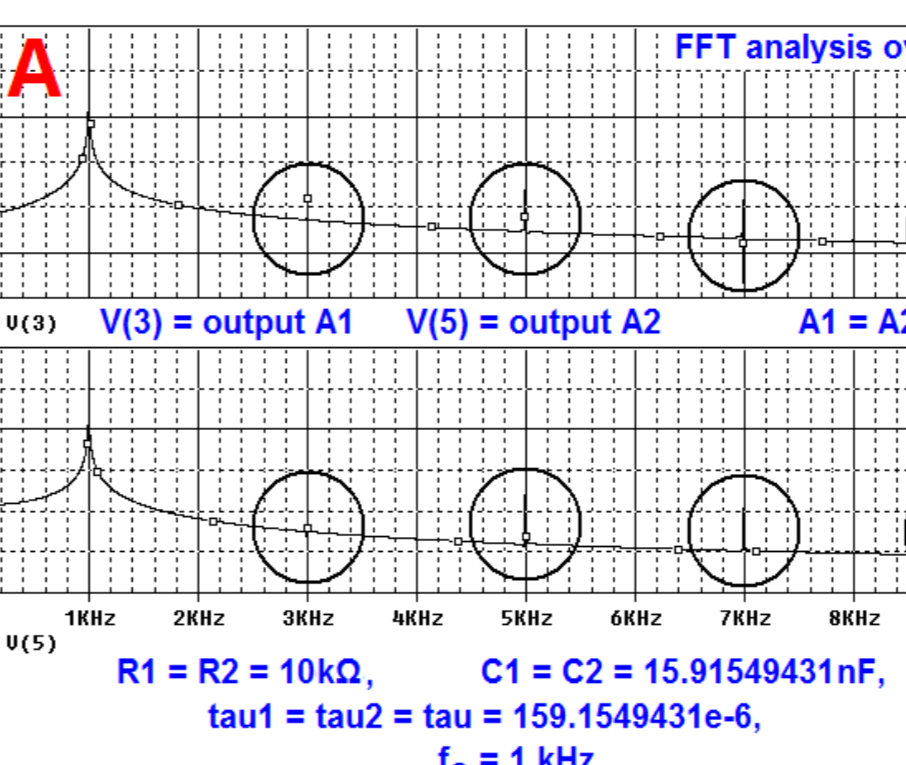
The aim of this tutorial is to demonstrate

that the classic circuit theory with

poles and zeros (eigenvalues)

may be used to **gain insight in**

the behavior of nonlinear circuits

COMPUTER AIDED CIRCUIT ANALYSIS

- The kernel of analyzing nonlinear circuits is the solution of a linear circuit
- All elements may be modelled by means of controlled sources
- During the iterations the elements may be approximated with either a dynamic value or a static value

- The CAPACITOR is a voltage controlled current source (VCCS) controlled by the time derivative of its voltage:

$$IC = sC*VC = C\frac{dVC}{dt}$$

- The INDUCTOR is a current controlled voltage source (CCVS) controlled by the time derivative of its current:

$$VL = sL*IL = L\frac{dIL}{dt}$$

EXTENDED NODE EQUATIONS

- First order differential equations
- Variables: Node Potentials and Impedance Branch Currents
- Current sources are Admittance Branches
- Voltage sources are Impedance Branches

NUMERICAL INTEGRATION with

Variable Integration Step

and

Variable Order Polynomial Approximation

Newton-Raphson Iteration

Replace with dynamic value $g = di/dv$
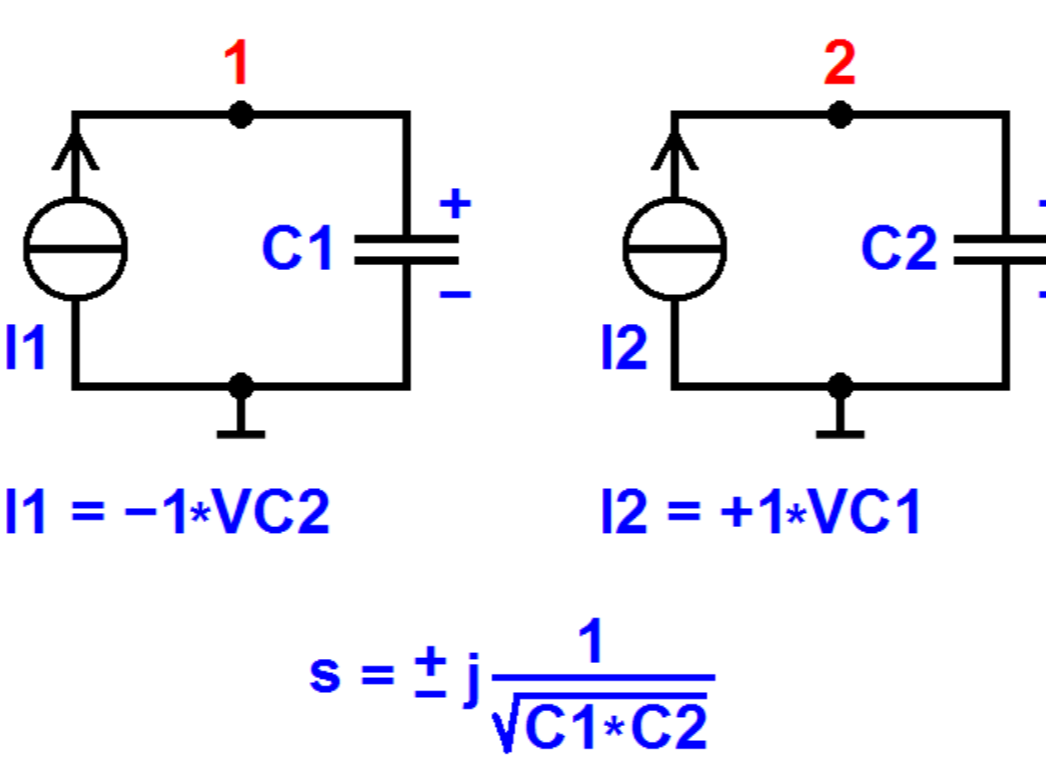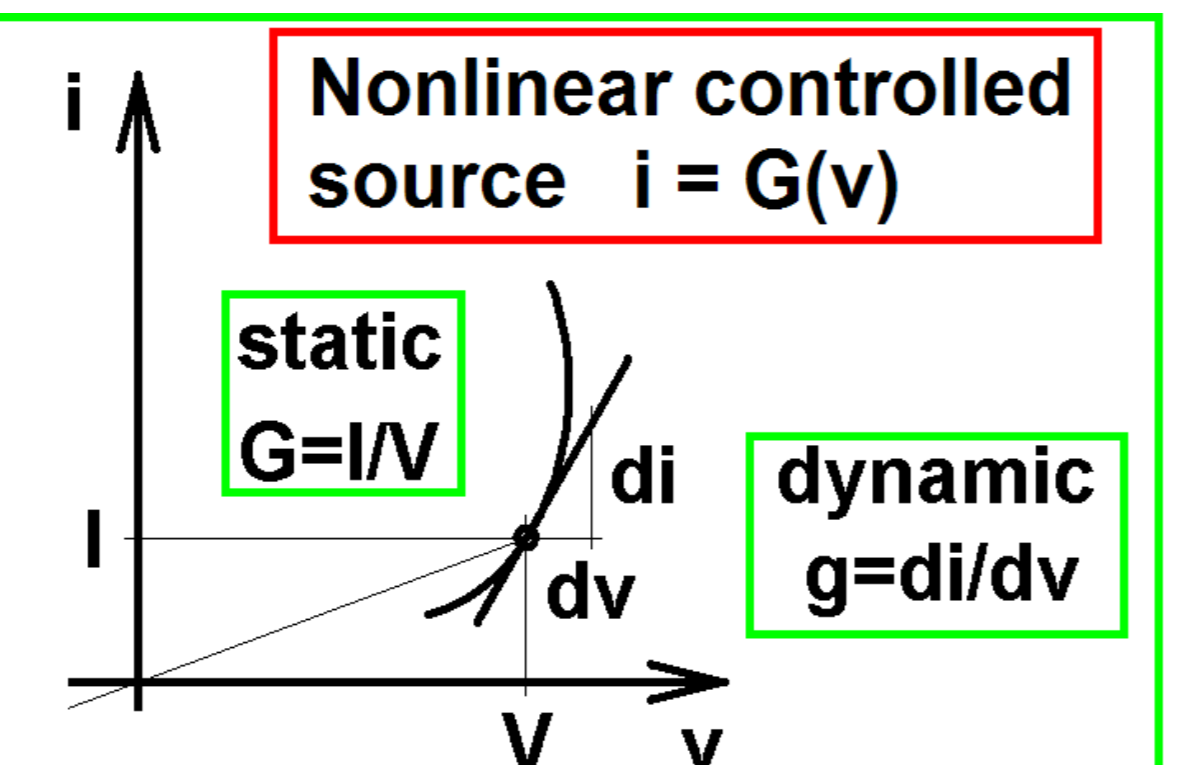
Picard Iteration

Replace with static value $G = I/V$

Nonlinear controlled source $i = G(v)$

static $G = I/V$

dynamic $g = di/dv$

Operational Amplifiers are made from transistors, resistors and capacitors - i.e. they are high order nonlinear circuits

Models are based on either **lumped element modeling** (e.g. BJT transistors, resistors and capacitors) or **macromodeling** (controlled sources, diodes, resistors, capacitors)

It make sense to study the **eigenvalues** of the linear small signal models used in each integration step of the numerical simulation process

- It is assumed that a nonlinear circuit can be treated as a time-varying linear circuit.
- Two approaches "freeze" time and replace nonlinearity with dynamic ratio (JACOBIAN) or with static ratio (LTV).
- It is assumed that the amplifiers are perfect - i.e. the input impedance is infinite and the output voltage is equal to the input voltage times a time varying constant A - i.e. $Vout = (V_+ - V_-) * A(time)$

"FREEZE" TIME

and

replace nonlinearity

with

dynamic ratio (JACOBIAN)

or with

static ratio (LTV)

HARMONIC OSCILLATOR MODELS

Passive lossless LC circuit

$x = \cos(t)$ $y = \sin(t)$

$\frac{dx}{dt} = -\sin(t)$ $\frac{dy}{dt} = +\cos(t)$

Active ideal integrator circuit

$I1 = -1*VC2$ $I2 = +1*VC1$

$$L\frac{dy}{dt} = +x$$

$$C\frac{dx}{dt} = -y$$

$$\frac{d^2x}{dt^2} + \frac{x}{CL} = 0 \qquad s = \pm j\frac{1}{\sqrt{CL}}$$

$I1 = -1*VC2$ $I2 = +1*VC1$

$$s = \pm j\frac{1}{\sqrt{C1*C2}}$$

- PSpice comparison of two 100kHz oscillators with the following component values:
  L3 = 1.591549431µH,   C3 = 1.591549431µF
  and
  C1 = C2 = 1.591549431µF
- With an accuracy of RELTOL = 1µ = 1e − 6 in PSpice a very close agreement between the models is observed
- The initial condition is a voltage across the capacitor and a current equal to zero through the coil
- The voltage becomes a cosine of time and the current becomes a sine of time

$V(1) = V(3) = \cos(t)$ $V(2) = \sin(t)$

FFT analysis

$V(1) = V(3) = \cos(t)$ $V(2) = \sin(t)$

Double Integrator Oscillator

$V(1) = -V(5)$

R1   C1   R2   C2   A1   A2

- Linear steady state oscillators are mathematical fiction
- In the real world linear systems are always damped due to losses
- Some kind of non-linearity must be introduced in order to obtain steady state oscillations
- Oscillators are non-linear systems
- It is to be expected that oscillators based on a double integrator will behave very close to sinusoidal
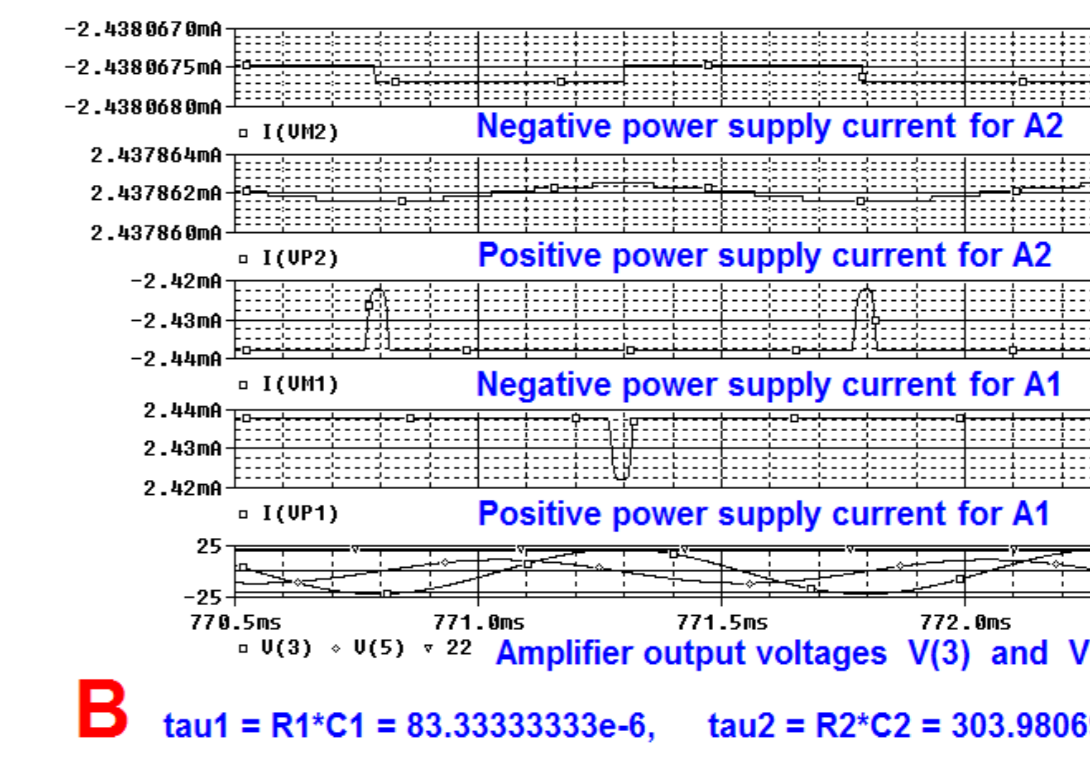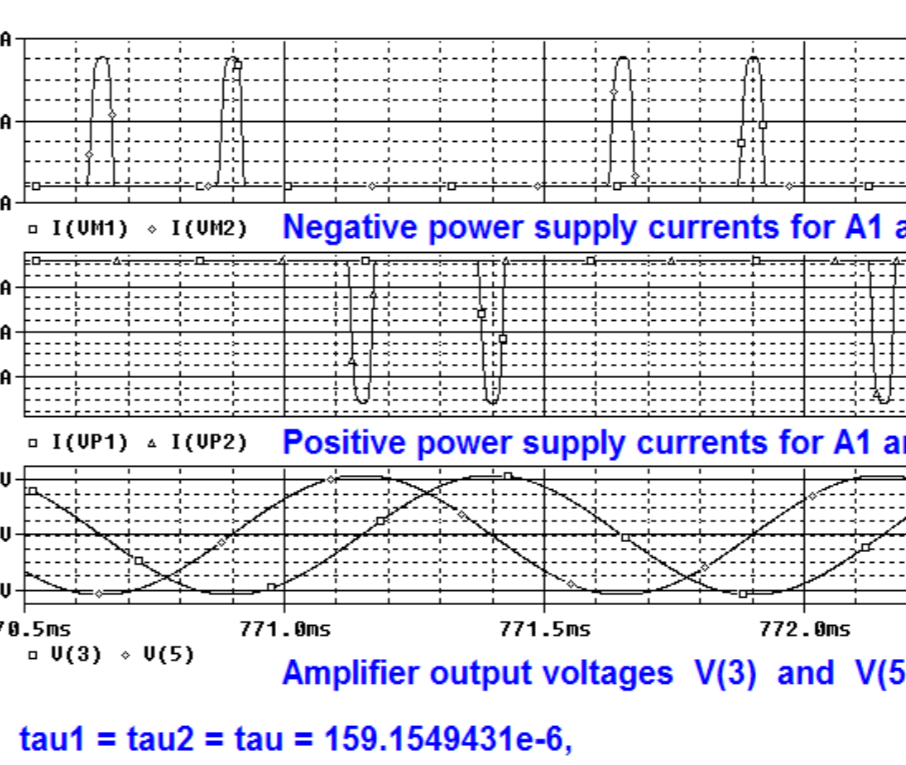
It is assumed that the amplifiers are perfect - i.e. the input impedance is infinite and the output voltage is equal to the input voltage times a time varying constant A - i.e. $Vout = (V_+ - V_-) * A(time)$

By means of Kirchhoffs current law for the nodes 2 and 4 we obtain

$$V(5) \times \left(\frac{\frac{1}{R1} \times \frac{1}{R2}}{\frac{1}{A_1}(\frac{1}{R1} + sC_1) + sC_1} + \frac{1}{A_2}(\frac{1}{R_2} + sC_2) + sC_2 \right) = 0$$

If V(5) is different from zero the expression within the brackets must be zero and the characteristic polynomial may be derived

For infinite gains the characteristic polynomial becomes

$$s^2 + \frac{1}{R_1C_1R_2C_2} = 0$$

The characteristic polynomial of the circuit becomes

$$s^2 + 2\alpha\ s + \omega^2 = 0$$

where $\quad 2\alpha = \dfrac{C_1R_1(1+A_1) + C_2R_2(1+A_2)}{C_1R_1C_2R_2(1+A_1)(1+A_2)}$

and $\quad \omega^2 = \dfrac{1 + A_1A_2}{C_1R_1C_2R_2(1+A_1)(1+A_2)}$

The roots are $\quad p_{1,2} = -\alpha \pm j\sqrt{(\omega^2 - \alpha^2)}$

Energy is transferred to the amplifiers as very narrow pulses at the maximums of the output voltages.

This behavior is very similar to the behavior of the pendulum clock where the escape mechanism delivers the energy as pulses when the angel from vertical is zero and the weights go down a step changing potential energy into a kinetic energy impulse. For small swing - i.e. when x and sin(x) are almost equal - the pendulum clock is very close to a damped linear oscillator with very high quality factor and no harmonics.

A strategy for design of electronic oscillators with minimum distortion could be optimization of the energy impulses observed.

OPAMP MACROMODEL

Gin   I4   GA   Rout   VOP

Gin = 0   GA = 1/A   Rout = 0

I4 = +1*V(Gin)   VOP = +1*V(GA)

ideal opamp: A = infinite for GA = 0

```
.subckt popamp   1   2  31 32  3
                 +   -   V+  V-   out
* Perfect OPerational AMPlifier
*
RPos   31   0   1e+20  ; dummy resistors
RNeg   32   0   1e+20  ; for connection to power
* input
Gin    1    2   1   2   0   ; infinite input resistance Rin = 1/Gin
Gia    0    4   1   2  +1   ; VCCS, controlled current source - Via
Ga     4    0   4   0  10e-6 ; gain = 1/Ga = 10e+3, perfect op amp
EVO    4    4   0   4  1e+20  ; infinite gain, ideal op amp
EGpi   4    0              ; SPICE resistor
EVOP   5    4   4   0  +1   ; Vout
VRout  5    3              ; zero output resistance
*
.ends
```

TWO EXPERIMENTS

A   Same time constants of the integrators

R1 = R2 = 10kΩ,   C1 = C2 = 15.91549431nF, tau1 = tau2 = tau = 159.1549431e-6, f_O = 1 kHz,

A FFT analysis over 800ms

V(3) = output A1   V(5) = output A2   A1 = A2 = µA741

R1 = R2 = 10kΩ,   C1 = C2 = 15.91549431nF, tau1 = tau2 = tau = 159.1549431e-6, f_O = 1 kHz,

Negative power supply currents for A1 and A2

Positive power supply currents for A1 and A2

A Amplifier output voltages V(3) and V(5)

tau1 = tau2 = tau = 159.1549431e-6,

V(10) = Vout/Vin for A1   V(11) = Vout/Vin for A2

Amplifier output voltages V(3) and V(5)

B V(2) = 6.283185308s = 2pi*k = omega   V(2) = imaginary part

V(1) = real part

Complex pole pair as function of time   ω0 = 2π*1k

TWO EXPERIMENTS

B   Different time constants of the integrators

R1 = 8.333333333kΩ,   C1 = 10.0nF,   R2 = 12.0kΩ,   C2 = 25.33172448nF, tau1 = R1*C1 = 83.33333333e-6,   tau2 = R2*C2 = 303.9806938e-6, f_O = 0.9999718022 kHz,

B FFT analysis over 800ms

V(3) = output A1   V(5) = output A2   A1 = A2 = µA741

R1 = 8.333333333kΩ,   C1 = 10.0nF,   R2 = 12.0kΩ,   C2 = 25.33172448nF, tau1 = R1*C1 = 83.33333333e-6,   tau2 = R2*C2 = 303.9806938e-6, f_O = 0.9999718022 kHz,

Negative power supply current for A2

Positive power supply current for A2

Negative power supply current for A1

Positive power supply current for A1

B Amplifier output voltages V(3) and V(5)

tau1 = R1*C1 = 83.33333333e-6,   tau2 = R2*C2 = 303.9806938e-6,

Comparing case **A** and case **B** the harmonics of the output voltage V(5) of amplifier A2 have been reduced in case **B** compared to case **A**
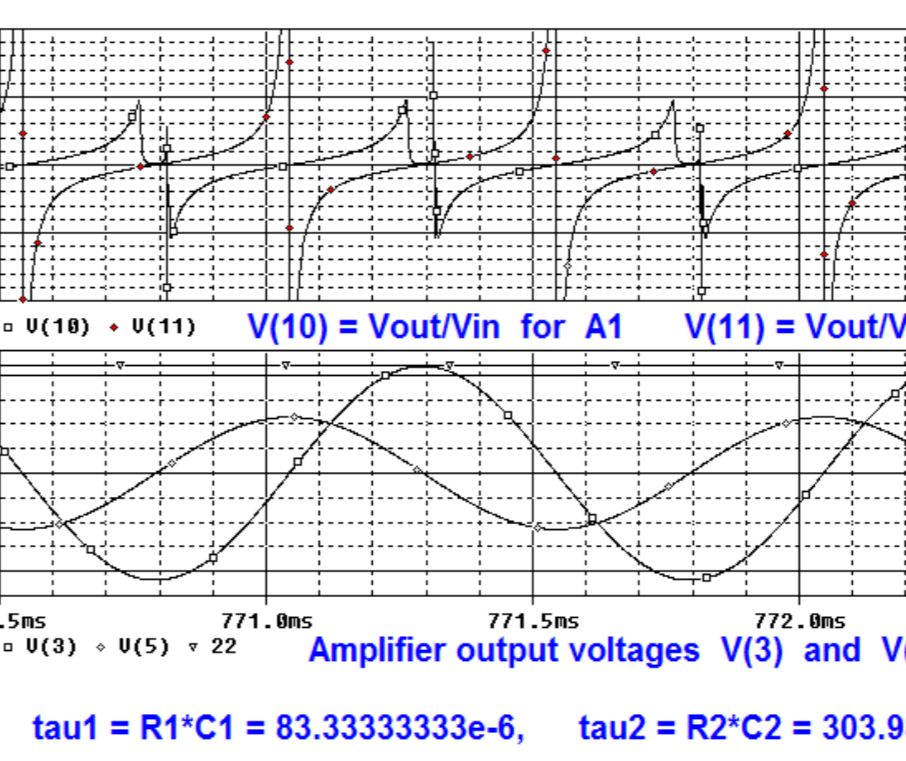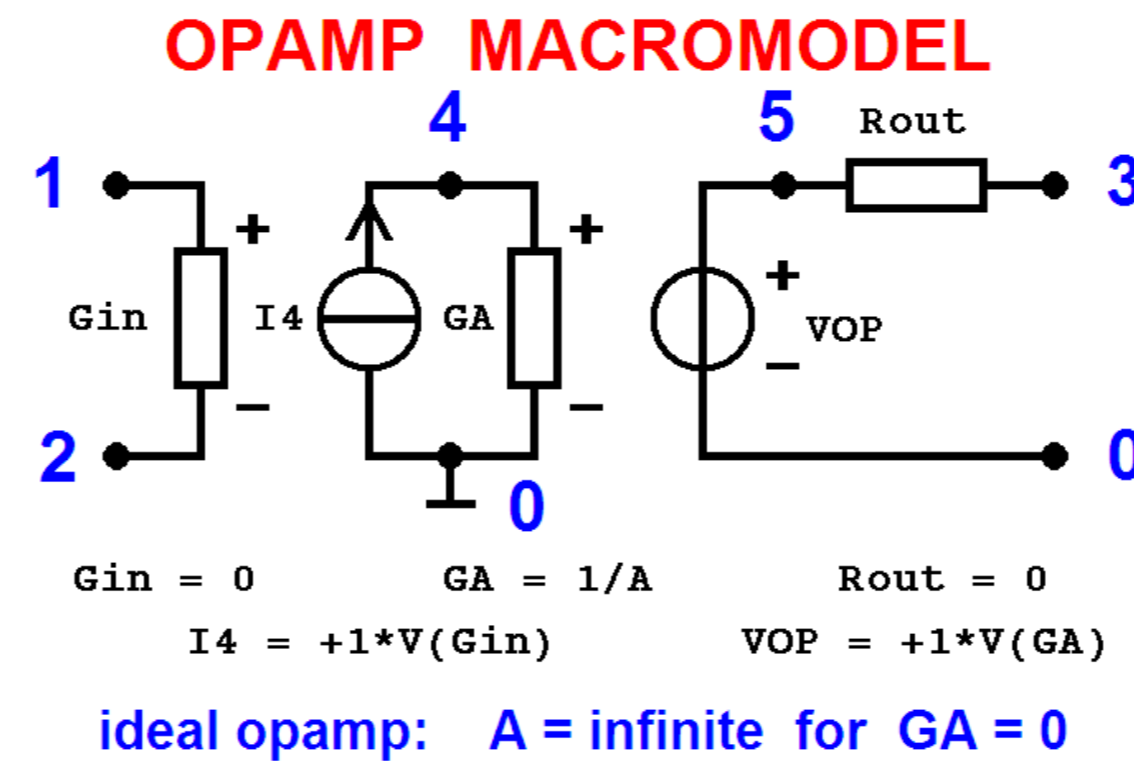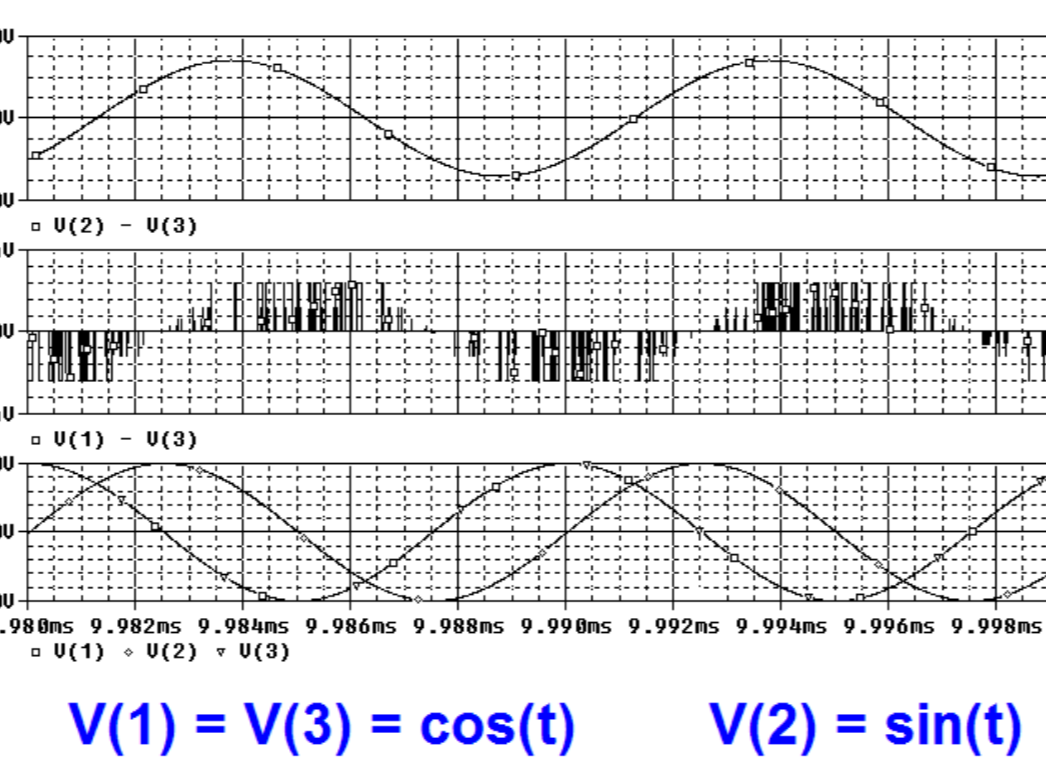
In case **B** the power is supplied to amplifier A1 with an almost constant current of 2.44mA and pulses of 15µA in the short clipping time intervals. The power is supplied to amplifier A2 with an almost constant current of 2.438mA.

Comparing case **A** and case **B** it is seen that the pulses in the power supply currents of amplifier A2 have disappeared in case **B**. This result depends of course on the op-amp macro-model used. Experiments with LM741 instead of uA741 did not show pulses but reduction of harmonics was obtained

In case **B** the amplifier gains as functions of time over two periods is shown. It is seen that the gain is varying slowly in the interval ±5000 in the whole period except at the maximums of the amplifier output voltages where it vary between very large and very small values in very short time intervals

In case **B** the roots of the characteristic polynomial (the poles) are found as function of time by means of a table of the gains A1 and A2 as function of time found by means of PSpice

In case **B** the complex pole pair as function of time is shown. $\omega_O = 1/\sqrt{(R1C1R2C2)}$

It is seen that the imaginary part is close to $\omega_O = 1/\sqrt{(R1C1R2C2)} = 6.283185308e+3 = 2\pi f_O$ and the real part is close to zero most of the time
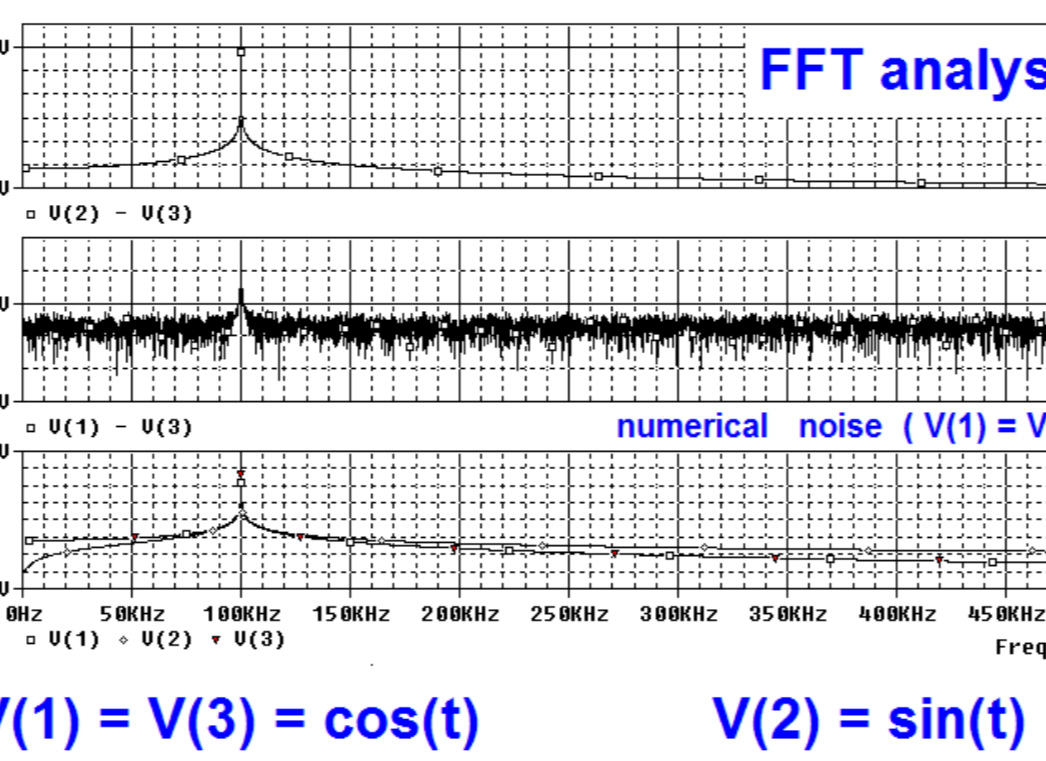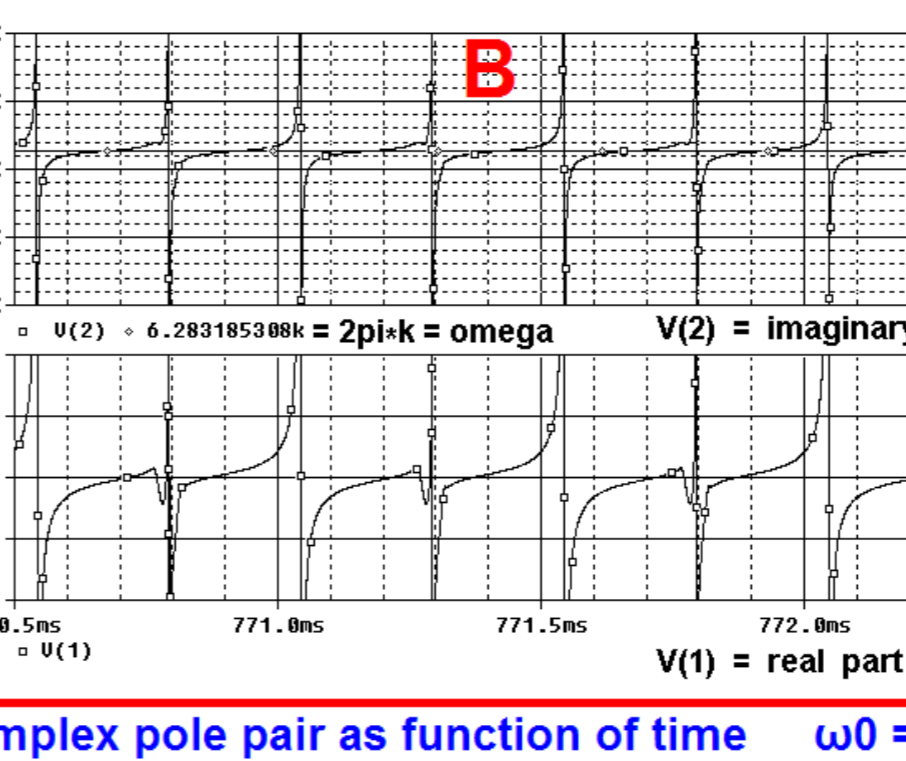
Only in the short intervals - where the energy pulses come from the power supply - the complex pole pair goes to the real axis in the right half plane and split-up into two real roots which via zero and infinity comes back and unite into a complex pole pair as expected

The result is in agreement with the very little phase noise observed

CONCLUSION

It is demonstrated that the classic circuit theory with poles and zeros (eigenvalues) may be used in connection with the design of a double integrator oscillator.

The non-linear circuit is treated as a linear time-varying circuit. The parameters are chosen in such a way that the imaginary part of the poles of the linear time-varying circuit is as close as possible to ω_O over a period.

The time constants of the integrators are chosen different in order to obtain minimum distortion of the output voltage of one of the amplifiers.

The circuit is used mainly for low-frequency oscillators. The structure may be a candidate for IC implementation because it is based on resistors and capacitors only.