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THE SHORT-CIRCUIT CONCEPT USED IN FIELD
EQUIVALENCE PRINCIPLES

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Introduction. In field equivalence principles electric and magnetic surface currents are specified and considered as impressed currents [1-3]. Often the currents are placed on perfect conductors. These currents may be treated by using two points of view. The first point of view is decomposition of the total field into partial fields caused by the individual impressed currents. Using this partial field point of view it is stated that on a perfect electric (magnetic) conductor, impressed electric (magnetic) surface currents are short circuited.

The second point of view is to note that since Maxwell's equations and the boundary conditions are satisfied, none of the impressed currents are short circuited and no currents are induced on the perfect conductors. Since all currents and field quantities are considered at the same time, this point of view is referred to as the total field point of view. The partial field point of view leads to alternative formulations for computations of the total field. This is not the case for the total field point of view. However, it increases understanding to be aware of the two points of view. Therefore, both points of view are discussed in some detail in the following section. Furthermore, this leads to the following correction.

In [3] the author has stated that an impressed electric current is short circuited on a perfect electric conductor. In relation to the problem discussed this is only correct in case the partial field point of view is used. Therefore, this condition should have been pointed out in [3]. The association of the short-circuit concept with the partial field point of view is mentioned in [4].

Discussion. A perfect electric conductor is defined as an object for which the electric conductivity σ_e is infinite and the magnetic conductivity σ_m is finite. For a perfect magnetic conductor σ_e is finite and σ_m is infinite. Inside and on the interior side of the conducting surface of such objects the electric field intensity $\bar{E}$ and the magnetic field intensity $\bar{H}$ are zero. Then, the boundary conditions are

$$\bar{J}_{ms} = -\hat{n} \times \bar{E}_1$$

$$\bar{J}_{es} = \hat{n} \times \bar{H}_1$$

where $\bar{E}_1$ and $\bar{H}_1$ are the electric and magnetic field intensities, respectively, on the exterior side of the conducting surface and $\bar{J}_{es}$ and $\bar{J}_{ms}$ are electric and magnetic surface currents, respectively. The unit vector $\hat{n}$ is normal to the surface and pointing in the direction from the interior side to the exterior side of the surface.

Let us first consider a perfect electric conductor. On it a $\bar{J}_{es}$ can be induced due to the fact that σ_e is infinite. But, a $\bar{J}_{ms}$ cannot be induced because σ_m is finite. In case an impressed $\bar{J}_{es}$ is placed on the conductor image theory shows that in the limit of zero distance, an oppositely directed $\bar{J}_{es}$ is induced so that the resultant field is zero. It is stated that the impressed $\bar{J}_{es}$ is short circuited. However, this consideration is only correct in case the impressed $\bar{J}_{es}$ is acting alone. In case the impressed $\bar{J}_{es}$ is acting together with other appropriately chosen impressed magnetic surface currents, the impressed $\bar{J}_{es}$ may from one point of view be considered as short circuited and from another point of view as not short circuited. This is illustrated by a discussion of Love's equivalence principle.

Love's principle is illustrated in Fig. 1. In Fig. 1a, original sources in a volume V generate a field $\bar{E}_1$, $\bar{H}_1$ exterior to V. The surface of the volume is S. Now, in Fig. 1b we have removed the original sources from V and specified a zero field inside V and a field exterior to V identical to that in Fig. 1a. This field can be generated by impressed currents given by Eqs. (1 and 2). Since the field inside V is zero we may fill out V with a perfect electric conductor as shown in Fig. 1c. Decomposing the total field into fields caused by the individual impressed
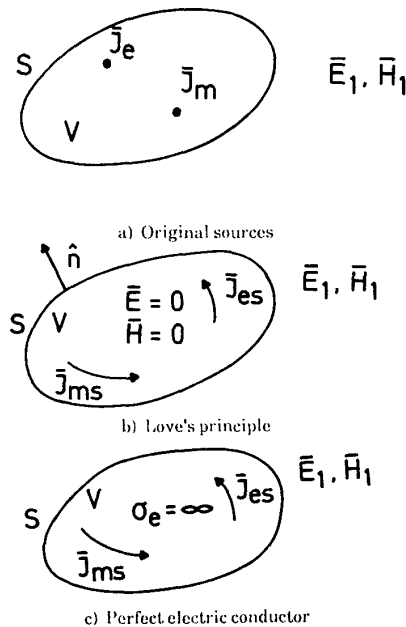

a) Original sources

b) Love's principle

c) Perfect electric conductor

Fig. 1. Steps in equivalence principles.

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currents leads to the result that the impressed $\overline{J}_{es}$ is short circuited and the impressed $\overline{J}_{ms}$ induces a current distribution equal to that of the impressed $\overline{J}_{es}$. It is noted that the short circuiting requires that the impressed currents flow on a surface coincident with that of the conductor, i.e., the surface S, and not as often stated just outside S [3].

Thus, the short circuiting is a result of the partial field point of view. However, from a total field point of view it may be argued that no currents are induced since Maxwell's equations and the boundary conditions are satisfied without the need of induced currents in addition to the impressed currents. Therefore, from a total field point of view the impressed $\overline{J}_{es}$ is not short circuited. Thus, only impressed currents exist on the perfect conductor. Since the total currents are the same under both viewpoints, the total field is the same. As another example, which is a usual scattering problem, let $\overline{J}_{es}$ be induced by currents impressed by sources exterior to the conductor. Now suppose that the induced $\overline{J}_{es}$ is substituted by an impressed $\overline{J}_{es}$ of the same value. Furthermore, suppose that this impressed $\overline{J}_{es}$ exists together with the original sources exterior to the conductor Then, again from a partial field point of view the impressed $\overline{J}_{es}$ is short circuited; but from a total field point of view no additional induced currents are required, i.e., the impressed $\overline{J}_{es}$ is not short circuited.

Consideration of an impressed $\overline{J}_{ms}$ on a perfect magnetic conductor can be made in a manner analogous to that given above for the perfect electric conductor.

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