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From Microwave Filter to Digital Filter and Back Again

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FROM MICROWAVE FILTER TO DIGITAL FILTER AND BACK AGAIN.

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ABSTRACT.

A new very simple state variable flow graph representation for interdigital transmission line bandpass filters is presented, which has led to two important results: 1) A new type of digital filter with properties, that surpass the properties of most other (all pole) digital filtertypes. 2) The study of the new digital filtertype has led to design formulas for interdigital transmission line filters that are very simple compared to the hitherto known formulas. The accuracy is the same or better.

INTRODUCTION.

In [1] a transmission wave (or propagation mode) flow graph for coupled transmission line filters, based on a matrix equation description was introduced, a flow graph which also may be used for a digital filter. The flow graph may be used for e.g. interdigital SCTL (Short Circuited Terminating Line) or OCTL (Open Circuit Terminating Line) filters etc.. For the interdigital filters, coupling between nonadjacent lines is ignored, and homogeneous dielectric is supposed. However, the flow graph may also be used for microstrip interdigital filters if the analysis and synthesis methods described in [2] to [5] are employed in a somewhat modified form.

In this paper it is shown how the analysis and synthesis procedures for the so called Digital Interdigital Filters (DIF) [1] lead to design formulas for interdigital transmission line filters. These formulas are very simple compared to the formulas presented by Matthei in [6]. They are, however, just as accurate.
Design formulas for both OCTL- and SCTL-interdigital filters are presented.
The author has shown, that the DIF has properties, that on almost all accounts surpass the properties of most other digital filtertypes. Although the DIF in its raw form is a bandpass filter, simple changes in the flow graph make it into e.g. a bandstop filter. The DIF is not in itself the subject of this paper, but the work, which has led to design formulas for the different types of DIF's, also led to design formulas for interdigital filters. It should be mentioned that the new digital filter is a State Space Wave Digital Filter, a filter type which hitherto has been deemed impractical [7].

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THE MATRIX EQUATION DESCRIPTION.

The terminating impedances (impedance matrices) (4) for a coupled transmission line filter may be chosen to make the filter into e.g. a SCTL- or a OCTL-interdigital filter.

In [1] it is shown that, if the transmission lines all have the same electrical length (homogeneous dielectric), then the same matrix equation describe the behavior of the filters, losses not taken into account. This matrix equation is presented in (1) for a filter with output at the opposit end of the input. The reflexion coefficient matrices $\underline{\rho}_1$ and $\underline{\rho}_2$ (2), (3) are expressed in terms of the capacitance matrix $\underline{C}$ for the coupled strips and the termination impedance matrices $\underline{Z}_g$ and $\underline{Z}_1$ (4).

The vectors $\underline{V}_g$ and $\underline{V}_2$ represent the generator voltages (normally only one) and the voltages across the load impedances. Normally only one output voltage is of interest.

$$2 \cdot \underline{V}_2 = (\underline{1} + \underline{\rho}_2) \cdot (\underline{1} - \underline{\rho}_1 \cdot \underline{\rho}_2 \cdot z^{-2})^{-1} \cdot z^{-1} \cdot (\underline{1} - \underline{\rho}_1) \cdot \underline{V}_g \quad , \quad z = e^{sT} \quad (1)$$

$$\underline{\rho}_1 = (\underline{Z}_1 \cdot v \cdot \underline{C} + \underline{1})^{-1} \cdot (\underline{Z}_1 \cdot v \cdot \underline{C} - \underline{1}) \quad (2)$$

$$\underline{\rho}_2 = (\underline{Z}_2 \cdot v \cdot \underline{C} + 1)^{-1} \cdot (\underline{Z}_2 \cdot v \cdot \underline{C} - \underline{1}) \quad (3)$$

v = the phase velocity.

$$\underline{Z}_1 = \begin{bmatrix} Z_{g1} & Z_{g2} & & \\ & & \ddots & \\ & & & Z_{gN} \end{bmatrix} \quad , \quad \underline{Z}_2 = \begin{bmatrix} Z_{11} & Z_{12} & & \\ & & \ddots & \\ & & & Z_{1N} \end{bmatrix} \quad (4)$$

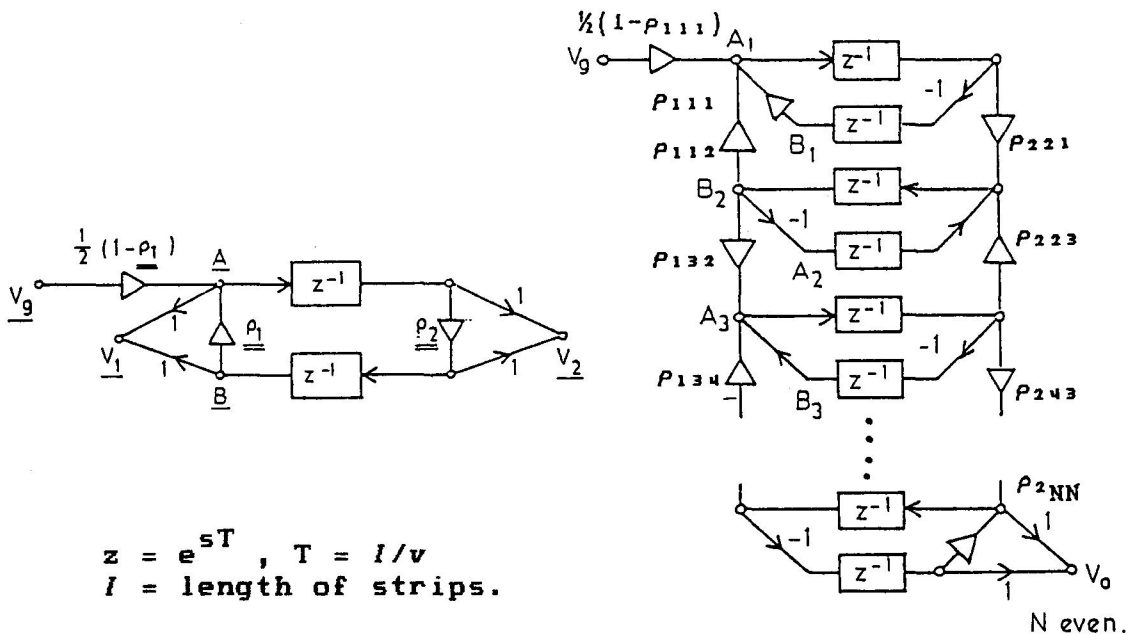$$z = e^{sT} \quad , \quad T = l/v$$
l = length of strips.

Fig.1 Flow graphs for an even order SCTL interdigital filter or a DIF. a) The matrix (state space) flow graph. b) The "unfolded" flow graph.

The terminating impedances may be chosen to make the filter into a SCTL- or a OCTL-interdigital filter or a combline filter etc.

The flow graph representation of the matrix equation (1) for a SCTL interdigital filter is shown in fig.1, where also the "unfolded" version of the flow graph is shown. This flow graph is simple, because the reflexion coefficient matrices $\underline{\varrho}_1$ and $\underline{\varrho}_2$ (2), (3) each only have 2N-1 elements different from 0, and N-1 of these are equal to +1 or -1.

In the case of a DIF, the flow graph may be simplified drastically, so that the corresponding digital filter only has N+1 multipliers. The digital filter is very simple and has exellent properties, and it may easily be changed into e.g. a lowpass filter.

DESIGN FORMULAS FOR A SCTL-INTERDIGITAL FILTER:

The flow graph in fig.1b represents a N+2 strip Nth order filter. The two outer strips transforms the impedance level. The condition under which this is true is that the elements $P_{1,11}$ and $P_{2,NN}$ (N even) in the refexion coefficient matrices are 0. This corresponds to utilizing one of the well known Kuroda identities. With $P_{1,11} = 0$ and $P_{2,NN} = 0$ (for N even), the flowgraph may be reduced to the flowgraph shown in fig.2.

The flow graph now represents a N strip filter and thus a Nth order bandpass filter (2·Nth order in z). The remaining problem is to find expressions for the elements in the capacitance matrix for the N+2 strips. Using the design formulas for a Nth order DIF presented in the full paper [1] and utilising some signal flow graph theory together with the theory presented in [2] to [5], it is found that the reflexion coefficient elements can be expressed as shown in the equations (5) to (8). Here the constants g_1, g_2, g_3 etc. are the normalized all pole lowpass filter components. Observe that the element C_{11} in the capacitance matrix is deter mined by the equation (5) $P_{1,11} = 0$.
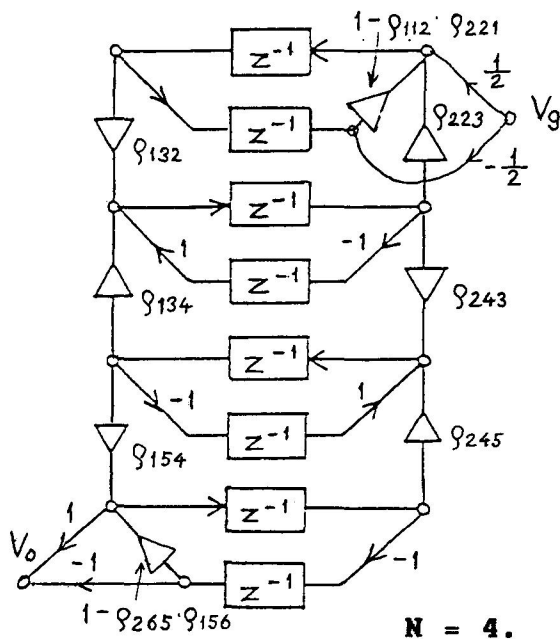

N = 4.

Fig.2 Simplified flow graph for N+2 strip SCTL interd. filter.

$$P_{1,11} = (Z_0 \cdot v \cdot C_{11} - 1)/(Z_0 \cdot v \cdot C_{11} + 1) = 0 \tag{5}$$

$$P_{1,12} \cdot P_{2,21} = 2 \cdot n \cdot B_{rel}/(4 \cdot g_1 + n \cdot B_{rel}) \tag{6}$$

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$$P_{1,32} \cdot P_{2,23} = (n \cdot B_{rel})^2 / ((4 \cdot g_1 + n \cdot B_{rel}) \cdot g_2) \tag{7}$$

$$P_{1,34} \cdot P_{2,43} = (n \cdot B_{rel})^2 / (4 \cdot g_2 \cdot g_3) \tag{8}$$

etc.

$$B_{rel} = 2 \cdot n \cdot B / \omega_0 \quad . \quad B = \text{the bandwidth.}$$

Observe that except for $P_{1,11}$ and $P_{2,NN}$ it is the product of reflexion coefficient elements that determine the filter response.
The expressions (2) and (3) give the elements of the reflexion coefficient matrices in terms of the capacitance matrix elements. The resulting very simple design formulas (9) to (15) for SCTL interdigital filters are presented below.

$$Z_0 \cdot v \cdot C_{11} = 1 \quad , \quad v = c / \sqrt{\varepsilon_r} \quad , \quad Z_0 = 50\Omega \text{ normally} \tag{9}$$

$$Z_0 \cdot v \cdot C_{12} = - \sqrt{\frac{k_1}{g_1 \cdot T + 1}} \tag{10}$$

$$Z_0 \cdot v \cdot C_{22} = k_1 \tag{11}$$

$$Z_0 \cdot v \cdot C_{23} = - (1/T) \cdot \sqrt{\frac{k_1 \cdot k_2}{(g_1 + 1/T) \cdot g_2}} \tag{12}$$

$$Z_0 \cdot v \cdot C_{33} = k_2 \tag{13}$$

$$Z_0 \cdot v \cdot C_{34} = - (1/T) \cdot \sqrt{\frac{k_2 \cdot k_3}{g_2 \cdot g_3}} \tag{14}$$

$$Z_0 \cdot v \cdot C_{44} = k_3 \tag{15}$$

etc.

$$T = (n \cdot B_{rel} / 4)^{-1}$$

The constants k_1, k_2 $\cdots$ may in theory be chosen arbitrarily, but practical considerations limit the choice.
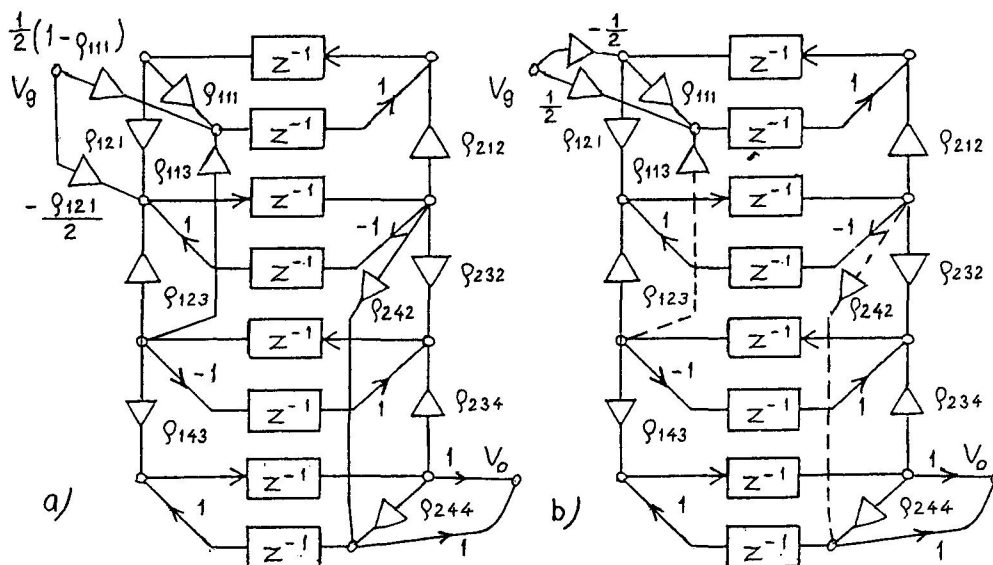Compare these formulas with the corresponding formulas in [6].

Fig.3. a) The OCTL interdigital filter flow graph.
       b) The reduced flow graph.   N = 4.

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THE OCTL INTERDIGITAL FILTER.

The unfolded flow graph for a OCTL filter is shown in fig.3a. It is somewhat more complicated than the SCTL flow graph. However, by again utilizing some signal flow graph theory, the flow graph may be reduced to the flow graph presented in fig.3b. Except for the elements $P_{1,13}$ and $P_{2,42}$, $(N = 4)$, the flowgraph is almost identical to the flow graph for the SCTL filter. These two elements are very small but do have some influence on the transfer function. The design equations for an Nth order DIF [1] may be used for the Nth order OCTL filter, with a little modification. The resulting design equations are presented below (16) to (21).

$$\text{Define } g_1' = g_1 - 1/(T^2 \cdot g_2) \text{ and } g_2' = g_2 \cdot g_1/g_1' \tag{16}$$

$$Z_0 \cdot v \cdot C_{11} = 1/(T \cdot g_1') \tag{17}$$

$$Z_0 \cdot v \cdot C_{12} = -(1/(T \cdot g_1')) \cdot \sqrt{\frac{k_1}{T \cdot g_2'}} \tag{18}$$

$$Z_0 \cdot v \cdot C_{22} = k_1 \tag{19}$$

$$Z_0 \cdot v \cdot C_{23} = -(1/T) \cdot \sqrt{\frac{k_1 \cdot k_2}{g_2' \cdot g_3}} \tag{20}$$

$$Z_0 \cdot v \cdot C_{33} = k_2 \tag{21}$$

$$Z_0 \cdot v \cdot C_{34} = -(1/T) \cdot \sqrt{\frac{k_2 \cdot k_3}{g_3 \cdot g_4}} \tag{22}$$

$$Z_0 \cdot v \cdot C_{44} = k_3 \tag{23}$$

etc. Here presented for $N > 4$.

A DESIGN EXAMPLE.

A 7th order (14th. order in z) 0.5 dB ripple Chebyshev bandpass DIF or a 9 strip SCTL interdigital stripline filter with 50% relative bandwidth was designed using the design formulas presented above. A 50% bandwidth is rather large. A SCTL interdigital transmission line filter will in fact be impractical with this bandwidth.
Fig 4 shows the output from a digital filter analysis program developed by the author. This is also the theoretical response for an ideal stripline interdigital filter. The theoretical response for a microstrip SCTL interdigital filter ($\varepsilon_r = 2.37$) is also shown. The actual design method for the microstrip filter is a combination of the design method presented in [2] to [5] and the method used for a DIF.

CONCLUSION.

The introduction of a state variable description for interdigital transmission line filters led to the development of a simple State Space Wave Digital Filter with exellent properties, a filter which hitherto has been deemed impractical. A simple design procedure for this digital filter was developed, which again led to simple design formulas for interdigital transmission line filters.
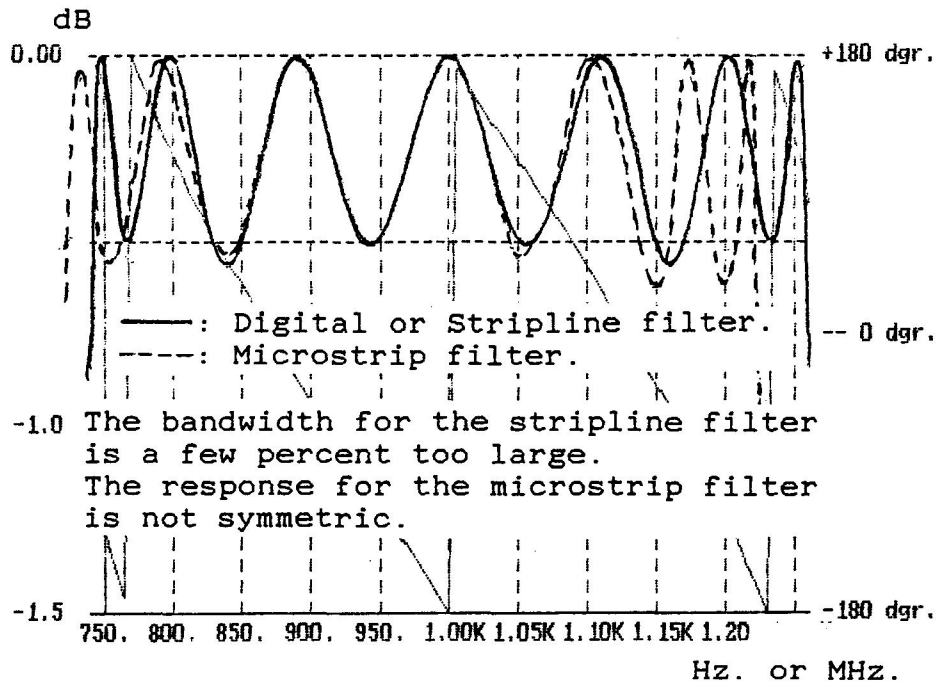
Fig.4. The filter response for a digital filter, a stripline filter and a microstrip SCTL interdigital filter.

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