Technical University of Denmark

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Niemann, Hans Henrik; Søgaard-Andersen, Per

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NEW RESULTS IN DISCRETE-TIME LOOP TRANSFER RECOVERY

Hans Henrik Niemann and Per Søgaard-Andersen

Control Engineering Institute, Technical University of Denmark,
Building 424, DK-2800 Lyngby, Denmark.

ABSTRACT

For discrete-time compensators incorporating prediction observers asymptotic loop transfer recovery is not feasible. Instead loop transfer recovery objectives must be satisfied via exact recovery techniques. In this note the model-based compensators which achieves exact recovery are parametrized in terms of the system zeros and the corresponding zero-directions. Full-order as well as minimal-order observers are treated. Further it is shown how exact recovery is also applicable to non-minimum phase plants. In this case the achievable performance is parameterized explicitly.

1 INTRODUCTION

In recent years the LQG/LTR feedback design methodology for robust model-based compensation has received much attention [see e.g. 1-6]. This procedure works for continuous-time systems - and it is always effective for minimum-phase plants. Unfortunately a similar procedure is not generally feasible in discrete-time. If filtering observers are used asymptotic recovery (the LTR step) is often possible [11]. However, the application of filtering observers require that the processing time of computing the control signal is negligible in comparison to the sampling interval. Very often such an assumption cannot be satisfied in practice, and prediction observers must be used. For compensators based on prediction observers, however, the asymptotic procedures will not be effective, since in general the difference between a full-state loop transfer (target design) and the full asymptotic loop transfer remains finite [4,11]. A detailed discussion of the mechanisms behind this fact is given in [4].
Loop transfer recovery is still possible, however, but different methods must be applied. In [4,11] such methods are discussed - and referred to as exact loop transfer recovery. In [4] the conditions for exact recovery for full-order observers were outlined, and some preliminary design considerations for minimum-phase continuous-time systems based on full-order observers were presented in [10]. In this note a more general treatment of exact recovery in discrete-time is provided. Exact recovery for minimum-phase as well as non-minimum phase plants based on full-order observers are discussed. Further results on exact recovery based on minimal-order observers are presented, and it is shown that in certain - common - cases very powerful designs procedures are possible. This is the first treatment of LTR for minimal-order observers in discrete-time. Earlier studies [16,17] were in continuous-time, but due to the same problems as for full-order observers the continuous-time methods cannot be generalized to discrete-time. Hence new methods based on exact recovery must be developed. Notice that the issue of recovery for non-minimum phase is particulary relevant in discrete-time since the sampling proces often produces zeros outside the unit-circle [13]. An advantage of using the exact recovery concepts presented here is that the controllers are of finite gains, whereas the usual continuous-time LQG/LTR method often produces high-gain controllers.

The paper is organized as follows. In § 2-4 the full-order observer case is treated, and in § 5-7 minimal-order observer results are presented follow in § 8 by some examples.

2 EXACT LOOP TRANSFER RECOVERY

In the following square discrete-time minimum phase systems $S(A,B,C)$ are considered. It will be assumed that the model is minimal. The plant transfer matrix $G(z)$ and the model-based compensator $H(z)$ are given

$$\begin{aligned} G(z) &= C\Phi(z)B, & \dim G(z) &= m \times m \\ \Phi(z) &= (zI - A)^{-1} & \dim \Phi(z) &= n \times n \\ H(z) &= K(zI - A + BK + FC)^{-1}F, & \dim H(z) &= m \times m \end{aligned} \qquad (2\text{-}1)$$

Here K is the full-state feedback gain and F is the full-order observer-gain. Let the number of transmission zeros be p. In order to formulate the loop-shape robustness constraints the uncertainties (disturbances, noise and modelling errors) are reflected to the plant input mode [4,14]. The target loop transfer is then the full-state loop transfer $K\Phi B$ and the full loop transfer is HG [3,5]. The difference between these two indicators is defined as the loop recovery error $E_I(z)$:

$$E_I(z) = K\Phi(z)B - H(z)G(z) \qquad (2\text{-}2)$$

In order to have exact recovery it is required that $E_I(z) \equiv 0$ for all z. For square systems Goodman [4] has shown that

$$E_I(z) = M_I(z)(I + M_I(z))^{-1}(I + K\Phi(z)B) \qquad (2\text{-}3)$$

$$M_I(z) = K(zI - A + FC)^{-1}B$$

It is, however, straightforward to derive the same results for non-square systems as well. Now let $M_I(z)$ be rewritten in the residual form:

$$M_I(z) = \sum_{i=1}^{n} \frac{K v_i w_i^T B}{z - \lambda_i} \tag{2-4}$$

where v_i and w_i^T are right and left eigenvectors associated with the eigenvalue λ_i of A - FC. It is easy to show that

$$\begin{aligned} E_I(z) &= 0 \quad \text{iff} \\ M_I(z) &= 0 \quad \text{iff} \qquad\qquad (2\text{-}5a,b,c) \\ Kv_i &= 0 \text{ or } w_i^T B = 0, \quad i = 1,\dots,n \end{aligned}$$

if A - FC is non-defective. The latter formulation of the exact recovery condition is suitable deriving the associated compensators.

3 SOLUTION OF THE EXACT LTR PROBLEM

From eigenstructure assignment it is known that the left eigenvectors w_i^T with the eigenvalue λ_i of A-FC are given by [9]:

$$\begin{bmatrix} w_i^T & z_i^T \end{bmatrix} \begin{bmatrix} \lambda_i I - A \\ -C \end{bmatrix} = 0, i=1,\dots,n$$

$$w_i^T F = -z_i^T \tag{3-1}$$

The condition $w_i^T B = 0$ from (2-5) imply that

$$\begin{bmatrix} w_{i0}^T & z_{i0}^T \end{bmatrix} \begin{bmatrix} \lambda_{i0} I - A & B \\ -C & 0 \end{bmatrix} = 0 \tag{3-2}$$

Maximally p eigenvectors w_{i0}^T can satisfy this condition, if λ_{i0} is selected as a transmission zero of S(A,B,C) [8]. Let these p eigenvalues/-vectors be selected from (3-2), it is then straightforward to see that F is parameterized by:

$$F = -\begin{bmatrix} w_1^T \\ \vdots \\ w_n^T \end{bmatrix}^{-1} \begin{bmatrix} z_1^T \\ \vdots \\ z_n^T \end{bmatrix} \tag{3-3}$$

$$z_i^T = z_{i0}^T, \quad w_i^T = w_{i0}^T \quad i = 1,\dots,p$$

and (λ_i, z_i^T, i=p+1,...,n) are free design parameters, since w_i^T is determined by λ_i and z_i. The remaining n-p conditions in (2-5c) must be satisfied by selecting K suitably. Condition (2-5c) imply

$$K[v_1,\dots\dots,v_n] = [\Omega \;\; 0] \tag{3-4}$$

with dim Ω= m x p but otherwise arbitrary. Now

$$K = [\Omega \;\; 0]V^{-1} = \Omega \begin{bmatrix} w_{10}^T \\ \vdots \\ w_{p0}^T \end{bmatrix} = \Omega\, \Gamma \tag{3-5}$$

with dim $\Gamma = p \times n$. Γ consists of the left eigenvectors w_{i0}^T comstrained in (3-2), and is thus a matrix of fixed elements. Eq (3-3) and (3-5) are therefore simple parameterizations of the controller matrices which achieves exact recovery.

A few important consequences of exact LTR are discussed next:

* The parameterization of the state-feedback imply that K must be selected as an output feedback controller, where Ω is the free parameter output feedback matrix. Γ is the equivalent output matrix with p independent colums. Since $p < n-m$, $\alpha = m+p-1$ ($< n$) eigenvalues can be assigned for such a problem [7]. Consequently all of the close-loop eigenvalues cannot be assigned freely, and no stability guarantees are available. However, in square discrete-time systems the rank[CB] is often maximal. This ensures that G(z) has the maximum possible number of finite zeros. Which in turn will result in maximal freedom in selection of K.
* The selection of F is only constrained by eq. (3-3) and stability can always be achived.
* Good input sensitivity and stability for plant input modelling errors can only be achived if $p \geq m$. If rank[K]< m ($p<m$) the target loop transfer KΦB is rank defective and loop-shaping is not feasible.
* Dual results apply for the plant output loop breaking point.
* The structure of the controller H(z) can be studied by looking at the system matrix for the controller P_H:

$$P_H = \begin{bmatrix} Iz - A + BK + FC & F \\ K & 0 \end{bmatrix} \tag{3-6}$$

By using the transformation matrix T = diag(V^{-1},I), eq.(3-6) can be transformed into:

$$P_H = \begin{bmatrix} Iz - \Lambda_p & 0 & -Z_1 \\ \gamma\Omega & Iz - \Lambda_{n-p} & -Z_2 \\ \Omega & 0 & 0 \end{bmatrix}$$

where γ, Z_1 and Z_2 has full rank. Λ_p are the plant zeros and Λ_{n-p} are the remaining n-p poles of A - FC assigned in eq. (3-3). Notice that Λ_p are the poles of H(z) and Λ_{n-p} are output decoupling zeros of H(z). Hence the resulting loop transfer HG will have n poles.
* It has been assumed that S(A,B,C) is minimal. The results could be extended to non-mimimal systems as well - although this issue is not pursued here.
* Further the treatment is also possible for non-square systems. Since this is strainht-forward no details are given here.
* *Notice* that the exact recovery controllers outlined above are of finite gains, whereas the continuous-time LQG/LTR procedures usually produces a high-gain controller.

4 NON-MINIMUM-PHASE SYSTEMS

Sampling of a continuous-time system will often result in a non-minimum phase discrete-time system [13]. If the LTR results from section 3 are used on a non-minimum phase system $G(z)$, the resulting controller will be unstable. It is, however, still possible to achieve LTR for non-minimum phase systems. In order to facilitate exact recovery for non-minimum phase plants note, that in selecting F only a subset j of the eigenvectors constrained by eq. (3-2) need to be chosen. In doing this, however, the dimension of Ω, the free parameters of K, is reduced to m x j. Consequently such selection are only advisible for non-minimum phase systems. If only the plant's q minimum phase zeros are used in eq. (3-2), the equations for F and K become:

$$F = -\begin{bmatrix} w_1^T \\ \vdots \\ w_n^T \end{bmatrix}^{-1} \begin{bmatrix} z_1^T \\ \vdots \\ z_n^T \end{bmatrix} \quad (4\text{-}1)$$

$$z_i^T = z_{i0}^T, \quad w_i^T = w_{i0}^T, \; i = 1,\ldots,q$$

$$K = \bar{\Omega} \begin{bmatrix} w_{10}^T \\ \vdots \\ w_{q0}^T \end{bmatrix} = \bar{\Omega}\,\bar{\Gamma}$$

where dim $\bar{\Omega}$ = m x q.

Some of the consequences of exact LTR for non-minimum phase plants are:

* The following equation will be satisfied

$$K\Phi(z)B = H(z)G(z) \quad (4\text{-}2)$$

  The non-minimum zeros of G are not cancelled out on the right hand side. Hence HG and $K\Phi B$ are both non-minimum phase. This in turn limits the achievable performance [12], and "good" loop-shapes for $K\Phi B$ are, of course, difficult to achive. Notice how the achievable loop-shapes - under the exact recovery constraint - are parameterized explicitly in eq. (4-1) by the constraints of K. This results is in agreement with the results in [18].
* The freedom in the selection of K will decrease by the number of non-minimum phase zeros in $G(z)$.
* The consequences of exact LTR from section 3 are still valid.

5 MINIMAL ORDER OBSERVERS

In the following the discrete-time system S(A,B,C) will be partitioned as:

$$A = \begin{array}{c} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{array}{l} \updownarrow m \\ \updownarrow n\text{-}m \end{array} \\ \begin{array}{cc} \overset{\longleftrightarrow}{m} & \overset{\longleftrightarrow}{n\text{-}m} \end{array} \end{array} \qquad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{array}{l} \updownarrow m \\ \updownarrow n\text{-}m \end{array}$$

$$C = \begin{array}{c} [\, I_m \quad 0 \,] \\ \begin{array}{cc} \overset{\longleftrightarrow}{m} & \overset{\longleftrightarrow}{n\text{-}m} \end{array} \end{array} \quad (5\text{-}1)$$

There is no loss of generality in assuming that $C = [I_m \; 0]$ since any system can be transformed into this form. The system is assumed to be minimum-phase, with p zeros. The minimal order observer for (5-1) is [15]:

$$z(k+1) = Dz(k) + Gu(k) + Ey(k)$$

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} I & 0_m \\ V_2 & I_{n-m} \end{bmatrix} \begin{bmatrix} y(k) \\ z(k) \end{bmatrix}$$

with (5-2)

$$D = A_{22} - V_2A_{12}$$

$$G = B_2 - V_2B_1$$

$$E = A_{12} - V_2A_{11} + A_{22}V_2 - V_2A_{12}V_2$$

and V_2 is the observer gain matrix.

The feedback law is:

$$u(k) = -Kx(k) = -K_1x_1(k) - K_2x_2(k) \quad (5\text{-}3)$$

It is assumed that (C,A) is observable, which implies that (A_{12},A_{22}) is observable [15].

It is known that the separation principle applies for this feedback system. Hence stability is achieved by making the full-state and the minimal-order observer stable. The condition for LTR for the minimal-order observer based design is [16,17]:

$$V_2(I+A_{12}\Phi_{22}V_2)^{-1}A_{12}\Phi_{22}(B_2 - V_2B_1) = B_2 - V_2B_1 \quad (5\text{-}4)$$

where

$$\Phi_{22} = (Iz - A_{22})^{-1}$$

This condition is similar to the continuous-time version, but the design results from [16,17] can not be generalized, and new methods for utilising (5-4) in discrete-time are derived in § 6.

If (5-4) is satisfied then:

$$K_2(\Phi_{22}^{-1} + V_2A_{12})^{-1}(B_2 - V_2B_1) = 0 \quad (5\text{-}5)$$

is also satisfied [16,17]. Eq. (5-5) is a necessary and sufficient condition for LTR with minimal-order observers. If (5-4) is satisfied the full-state loop transfer $K\Phi B$ and the minimal-order observer based loop transfer are identical.

Let (5-5) be rewitten in the residual form:

$$0 = \sum_{i=1}^{n-m} \frac{K_2v_iw_i^T(B_2 - V_2B_1)}{z - \lambda_i} \quad (5\text{-}6)$$

where v_i and w_i^T are right and left eigenvectors associated with the eigenvalue λ_i of $A_{22} - V_2A_{12}$ and from eigenstructure assignment [9] it is easily found that

$$w_i^T = z_i^TA_{12}\Phi_{22}(\lambda_i), \quad i = 1,\ldots,n-m \quad (5\text{-}7)$$

and

$$w_i^TV_2 = -z_i^T \quad (5\text{-}8)$$

It is easy to show that eq. (5-6) is satisfied if:

$$K_2 v_i = 0 \text{ or } w_i^T(B_2 - V_2 B_1) = 0, \; i=1,\dots,n-m \tag{5-9}$$

The condition implies 3 different design cases depending on the rank of B_1.

6 LTR SOLUTIONS FOR MINIMAL ORDER OBSERVERS

<u>Case 1</u> $\quad r(B_1) = 0$

The recovery condition (5-9) now becomes:

$$K_2 v_i = 0 \quad \text{or} \quad w_i^T B_2 = 0 \;, i=1,\dots,n-m \tag{6-1}$$

The second condition in (6-1) together with (5-7) result in:

$$z_{i0}^T A_{12}\Phi_{22}(\lambda_{i0})B_2 = 0, \; i = 1,\dots,n-m \tag{6-2}$$

This condition can be satisfied if λ_{i0} is selected as the transmission zeros of $S(A,B,C)$, see [6]. Eq. (6-2) can be satisfied for maximally p eigenvalues λ_{i0} [8]. Let these eigenvalues be selected from (6-2), it is then straightforward to see that V_2 is parameterized by:

$$V_2 = -\begin{bmatrix} w_1^T \\ \vdots \\ w_{n-m}^T \end{bmatrix}^{-1} \begin{bmatrix} z_1^T \\ \vdots \\ z_{n-m}^T \end{bmatrix} = -W^T Z \tag{6-3}$$

$z_i^T = z_{i0}^T, \; w_i^T = z_{i0}^T A_{12}\Phi_{22}(\lambda_{i0}) \;, i = 1,\dots,p$

and λ_i, $z_i^T, i = p+1,\dots,n-m$ are free design parameters. The first equation in (6-1) must be satisfied for the remaining n-m-p conditions by selecting K_2 as:

$$K_2[\tilde{v}_1,\dots,v_{n-m}] = [\,\Omega \;\; 0\,] \tag{6-4}$$

with dim $\Omega = m \times p$ but otherwise arbitrary. Now

$$K_2 = [\,\Omega \;\; 0\,]\, V^{-1}$$

$$= \Omega \begin{bmatrix} w_{10}^T \\ \vdots \\ w_{p0}^T \end{bmatrix} = \Omega\,\Gamma \tag{6-5}$$

with dim $\Gamma = p \times (n-m)$. Γ consists of the left eigenvectors w_{i0}^T constrained in (6-2).

<u>Case 2.</u> $\quad r(B_1) = m$

The condition $r(B_1) = m$ indicate that the system $S(A,B,C)$ has $p=n-m$ zeros. The recovery conditions can now be satisfied only by V_2 and K_2 is free to design.

The recovery condition is:

$$w_i^T(B_2 - V_2 B_1) = 0, \; i = 1,\dots,n-m \tag{6-6}$$

This equation can be rewritten as (by using eqs. (5-7) and (5-8)):

$$z_{i0}^T(A_{12}\Phi_{22}(\lambda_{i0})B_2 + B_1) = 0 \tag{6-7}$$

The n-m equations can be satisfied by selecting λ_{i0} as the zeros of the system $S(A,B,C)$, see [6]. The solution is:

$$V_2 = -\begin{bmatrix} w_1^T \\ \vdots \\ w_{n-m}^T \end{bmatrix}^{-1} \begin{bmatrix} z_1^T \\ \vdots \\ z_{n-m}^T \end{bmatrix} = -W^{-1}Z \tag{6-8}$$

with $w_i^T = z_{i0}^T A_{12}\Phi_{22}(\lambda_{i0})$ and $z_i^T = z_{i0}^T$, $i=1,\dots,n-m$

Notice that V_2 is uniquely determined. A different expression can be derived from (6-6):

$$V_2 = B_2 B_1^{-1} \tag{6-9}$$

<u>Case 3.</u> $\quad 0 < r(B_1) < m$

The recovery condition (5-9) is:

$$K_2 v_i = 0 \quad \text{or} \quad w_i^T(B_2 - V_2 B_1) = 0 \;, i=1,\dots,n-m \tag{6-10}$$

The second recovery condition can again be rewritten as:

$$z_{i0}^T(A_{12}\Phi_{22}(\lambda_{i0})B_2 + B_1) = 0 \tag{6-11}$$

Maximally p, p < n-m, eigenvectors w_{i0}^T satisfy this condition by selecting the eigenvalues $\lambda_i = \lambda_{i0}$ as the zeros of the system $S(A,B,C)$, and $z_i^T = z_{i0}^T$ as the corresponding zerodirections (see [6]). The first equation in (6-10) must then satisfy the remaining n-m-p conditions by suitably selecting K_2.

The solution in this case is similar to case 1.

$$V_2 = -\begin{bmatrix} w_1^T \\ \vdots \\ w_{n-m}^T \end{bmatrix}^{-1} \begin{bmatrix} z_1^T \\ \vdots \\ z_{n-m}^T \end{bmatrix} = -W^{-1}Z \tag{6-12}$$

with $z_i^T = z_{i0}^T$, $\; w_i^T = z_{i0}^T A_{12}\Phi_{22}(\lambda_{i0})$, $i=1,\dots,p$

$$K_2 = \Omega \begin{bmatrix} w_{10}^T \\ \vdots \\ w_{p0}^T \end{bmatrix} = \Omega\,\Gamma \tag{6-13}$$

with dim $\Gamma = p \times (n-m)$, dim $\Omega = m \times p$ but otherwise arbitrary.

V_2 can be rewritten into a form which emphasizes the fact that case 3 is inbetween case 1 and case 2. To see this, we assume that B_1 is transformed into:

$$B_1 = \begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix}$$

where Λ is a diagonal matrix.
Let A_{12}, B_2 and V_2 be partitioned as:

$$A_{12} = \begin{bmatrix} A_{121} \\ A_{122} \end{bmatrix} \begin{matrix} \updownarrow\ r(B_1) \\ \updownarrow\ m - r(B_1) \end{matrix} \qquad (6\text{-}14)$$

$$B_2 = \begin{bmatrix} \underset{\longleftrightarrow}{B_{21}} & \underset{\longleftrightarrow}{B_{22}} \end{bmatrix} \; n - m$$
$$\qquad\quad r(B_1) \quad m-r(B_1)$$

$$V_2 = \begin{bmatrix} \underset{\longleftrightarrow}{V_{21}} & \underset{\longleftrightarrow}{V_{22}} \end{bmatrix} \; n - m$$
$$\qquad\quad r(B_1) \quad m-r(B_1)$$

The second condition in (6-10) can now be written as:

$$w_i^T[(B_{21} \quad B_{22}) - (V_{21} \quad V_{22}) \begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix}] = 0$$

$$i = 1,\ldots\ldots,n\text{-}m \qquad (6\text{-}15)$$

By *selecting* V_{21} as:

$$V_{21} = B_{21}\Lambda^{-1} \qquad (6\text{-}16)$$

(6-15) will be reduced to:

$$w_i^T B_{22} = 0 \;,\; i=1,\ldots,n\text{-}m \qquad (6\text{-}17)$$

where the left eigenvectors w_i^T in (6-17) are given by:

$$w_i^T = z_i^T A_{122} \bar{\Phi}_{22}(\lambda_i) \qquad (6\text{-}18)$$

where $\bar{\Phi}_{22}(\lambda_i) = (Iz - \bar{A}_{22})^{-1}$

and $\bar{A}_{22} = A_{22} - B_{21}\Lambda^{-1}A_{121}$

$w_i^T B_{22} = 0$ can now be satisfied for maximally p eigenvectors w_i^T if $\lambda_i = \lambda_{i0}$ in (6-18) are selected as the transmission zeros of $S(A_{22},B_{22},A_{122})$, which are equal to the transmission zeros of S(A,B,C) [6].

$$z_{i0}^T A_{122}\bar{\Phi}_{22}(\lambda_{i0})B_{22} = 0 \;,\; i=1,\ldots,p \qquad (6\text{-}19)$$

Now it is straightforward to see that V_{22} is parameterized by:

$$V_{22} = - \begin{bmatrix} w_1^T \\ \vdots \\ w_{n-m}^T \end{bmatrix}^{-1} \begin{bmatrix} z_1^T \\ \vdots \\ z_{n-m}^T \end{bmatrix} = -W^{-1}Z \qquad (6\text{-}20)$$

with $z_i^T = z_{i0}^T$

$$w_i^T = z_{i0}^T A_{122}\bar{\Phi}_{22}(\lambda_{i0}) \;,\; i=1,\ldots,p$$

The resulting V_2 is:

$$V_2 = [\, B_{21}\Lambda^{-1} \;,\; -W^{-1} Z \,] \qquad (6\text{-}21)$$

where $-W^{-1}Z$ is given in (6-20).

The remaining n-m-p conditions in (6-10) must be satisfied by selecting K_2 as before:

$$K_2 = \Omega \begin{bmatrix} w_{10}^T \\ \vdots \\ w_{p0}^T \end{bmatrix} = \Omega\, \Gamma \qquad (6\text{-}22)$$

with dim Γ = p x (n-m).

A few important consequences of exact LTR for minimal-order observers are now discussed here:

* If the system does not have any zeros, exact recovery is still possible with the solution $K_2 = 0$, i.e. no feedback from the state estimates. However in square discrete-time system r(CB) is often maximal, which ensure that $r(B_1)$ is maximal and that G(z) has the maximum possible number of finite zeros,p=n-m. In this special case exact LTR is possible only by selecting the observer gain V_2. The feedback gain K is free to choose, and it is possible to use systematic design rules (e.g. LQG-design) for the K selection for stability and loop-shape reqirements. This is a very useful result for LTR design in discrete-time systems because a full-state target design can *be recovered, without affecting this original* design,simply by choosing the minimal-order observer gain, whereas it is not possible with a full-order observer. Here the full-state design is constrained. Note that by using a minimal-order observer in compensators will require that the processing time of computing the control signal is negligible in compario-son to the sampling interval. The processing time in this case will, however, be reduced compared with the processing time when a filtering observer is used and therefore the minimal-order observer is more attractive than the filtering observer.
* The result in case 3 (6-21) is the general result for exact recovery with minimal-order observer,since the solution constrains case 1 and 2 as special cases.
* Good input sensitivity and stability robust-*ness for plant input modelling errors can* always be achived if the target loop $K\Phi B$ has full rank. This is only guaranteed if $p \geq m$ in case 1 and 3. In case 2 $K\Phi B$ has generi-cally full rank, and therefore good feedback properties can be achieved.
* Finally note that dual results for the plant output cannot be invoked, due to the missing duality of minimal-order observers.

7 NON-MINIMUM-PHASE SYSTEMS

The results for LTR with minimal-order observers of § 6 were based on a minimum-phase assumption. If this assumption is not valid some new results can be obtained. In the following the tree usual cases will be discussed independently, but a basic prerequisite will be the recovery conditions.

$$K_2(Iz - A_{22} + V_2A_{12})^{-1}(B_1 - V_2B_2) = 0 \qquad (7\text{-}1)$$

$$\sum_{i=1}^{n-m} \frac{K_2 v_i w_i^T (B_1 - V_2 B_2)}{z - \lambda_i} = 0$$

where the symbols are defined in § 5,6.

Further let the number of plant zeros be p and the number of minimum-phase zeros be q.

Case 1. $r(B_1) = 0$

In this case the recovery condition becomes:

$$K_2 v_i = 0 \quad \text{or} \quad w_i^T B_2 = 0 \ , \ i = 1, \ldots, n\text{-}m \qquad (7\text{-}2)$$

Due to the stability requirements only a subset q of the possible solutions to the condition $w_i^T B_i = 0$ can be selected, i.e. the q solutions:

$$z_{i0}^T A_{12} \Phi_{22}(\lambda_{i0}) B_2 = 0 \qquad (7\text{-}3)$$

$$|\lambda_{i0}| < 1 \ , \ i = 1, \ldots, q$$

where λ_{i0} are the zeros of S(A,B,C) - see [6].

The remaining n-m-q conditions constrains K.

As in § 4 the solution becomes:

$$V_2 = \begin{bmatrix} w_1^T \\ \vdots \\ w_{n-m}^T \end{bmatrix}^{-1} \begin{bmatrix} z_1^T \\ \vdots \\ z_{n-m}^T \end{bmatrix} \qquad (7\text{-}4)$$

$$w_i^T = z_{i0}^T A_{12} \Phi_{22}(\lambda_{i0}) \ , \quad z_i^T = z_{i0}^T \ , i=1, \ldots, q$$

$$K_2 = \bar{\Omega} \begin{bmatrix} w_{10}^T \\ \vdots \\ w_{q0}^T \end{bmatrix} = \bar{\Omega} \, \bar{\Gamma}$$

$\dim \bar{\Omega} = m \times q$

$\bar{\Omega}$ is a matrix of free parameters. The remaining n-m-q pairs (λ_i , z_i^T) are free parameters.

Case 2. $r(B_1) = m$

Now the recovery condition becomes

$$K_2 v_i = 0 \text{ or } w_i^T (B_2 - V_2 B_1) = 0, i = 1, \ldots, n\text{-}m \qquad (7\text{-}5)$$

As before only q solutions to the conditions $w_i^T (B_2^T - V_2 B_2) = 0$ can be used, i.e.

$$z_{i0}^T (B_1 + A_{12} \Phi_{22}(\lambda_{i0}) B_2) = 0, i = 1, \ldots, q \qquad (7\text{-}6)$$

$$|\lambda_{i0}| < 1$$

and λ_{i0} is a zero of S(A,B,C) (see [6] for details).

The remaining n-m-q conditions must be satisfied by selecting K_2 appropriately. The expressions for V_2 and K_2 are similar to (7-4) with eq. (7-6) substituted for eq. (7-3).

Case 3. $0 < r(B_1) < m$

In this case the recovery condition are as (7-5). The q possible stable solutions to $w_i^T (B_2 - V_2 B_1)$ are given by eq. (7-6). The last n-m-q constrains K_2 - and the expressions for V_2 and K_2 are similar to eq. (7-4), with eq. (7-6) substituted for eq. (7-3).

A general comment for these results concerns the selection of K_2. In all three cases the matrix K is not free to assign, hence stability-design and loop-shape design are not as straightforward as one would desire. Otherwise the comments from § 4 are also valid here. Notice again that the achievable loop-shapes - subject to the exact recovery constraint - are parameterized explicitly in terms of K, i.e. the free parameters Ω and the left eigenvectors $w_{i0}^T(K_2)$ and K_1.

8 EXAMPLES

Consider the plant

$$G(s) = \frac{1 + 2s}{1 + s} \; \frac{4}{s^2 + 0.8s + 4} \; \frac{1}{s}$$

Let the sampling time be 0.25 sec. The discrete-time version G(z) then has zeros at:

$$z_1 = 0.8825, \quad z_2 = -0.2502, \quad z_3 = 3.3968$$

and G(z) is non-minimum phase. By applying the exact recovery procedure for full-order observers of § 3 the compensator becomes

$$H(z) = \frac{w_1(z - z_2) + w_2(z - z_1)}{(z - z_1)(z - z_2)}$$

Where w_1 and w_2 are the 2 elements of Ω. The resulting loop transfer is then:

$$K \Phi B = G(z)H(z)$$
$$= 0.019(z+3.3968)((w_1 + w_2)z - (w_1 z_2 + w_2 z_1))/d$$

Here d denotes the characteristic polynonium of A.

As expected the non-minimum phase zero shows up in $K\Phi B$. w_1 and w_2 are free design parameters which determines the shape of $K\Phi B$ and stability of the closed-loop system. Notice how the performance for the non-minimum phase control-loop is characterized directly by w_1 and w_2.

As the second example consider the plant:

$$A = \begin{bmatrix} 1.0044 & -5.2447E-3 & 1.4029E-3 & 1.4436E-2 \\ 5.1372E-5 & 1.0001 & 2.3995E-8 & -5.6845E-1 \\ -5.2161E-5 & 5.5818E-3 & 9.9980E-1 & 2.2215E-2 \\ -1.7897E-4 & -2.0729E-4 & -1.2551E-7 & 9.8419E-1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3.5825E-3 & -8.6189E-2 \\ 9.9749E-4 & 2.4174E-5 \\ -1.4399E-3 & 1.2011E-3 \\ -3.4725E-3 & -8.1575E-5 \end{bmatrix} \quad C^T = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

This is an example from [4] transformed into form required for minimal-order observer design. In [4] it was attempted to design a discrete LQG/LTR regulator, but a finite recovery error was obtained for all frequences. Here a minimal-order observer will be applied. The system is minimum-phase with zeros at (+0.99982, -0.99468). The sampling-time is 0.01 sec.

A target feedback design is given by:

$$K = \begin{bmatrix} 3.3072E+2 & 1.8503E+3 & 2.2942E+4 & -9.2927E+3 \\ -1.0656E+3 & -4.2362E+3 & -7.3194E+4 & 2.8251E+4 \end{bmatrix}$$

A nominal observer is designed as $V_{20} = -W^{-1}Z$ with eigenvalues at (5.32E-3, -1.8E-4).
A recovery trajectory is defined from V_{20} to the exact LTR value $V_2 = B_2B_1^{-1}$ by moving the eigenvalues λ_i and zero-directions z_i^T from the nominal to the LTR-values as functions of q, so that

$$\lambda_i(q=0) = \lambda_{i0}\ ,\ \lambda_i(q\rightarrow\infty) = \lambda_{i,LTR}$$

and equally for z_i^T. And $V_2(q\rightarrow\infty) = B_2B_1^{-1}$.

The plot of the singular values of the full loop transfer is shown in fig. 1 and 2 for different values of q. Clearly recovery is achieved. The final value of V_2 which achives exact recovery is:

$$V_2 = B_2B_1^{-1} = \begin{bmatrix} -1.4326E-2 & -1.3920 \\ -2.9920E-5 & -3.4812 \end{bmatrix}$$

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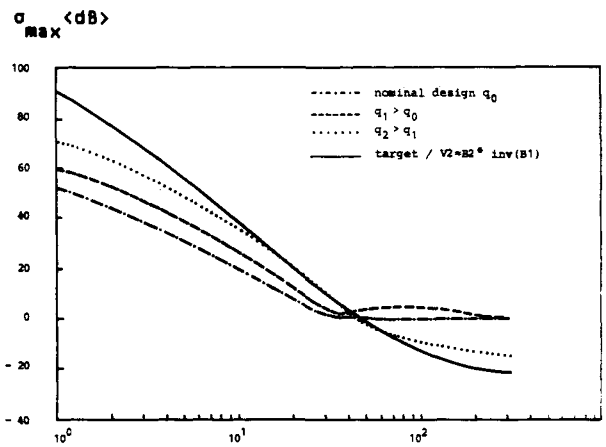

Figure 1 log(w) <r/s>

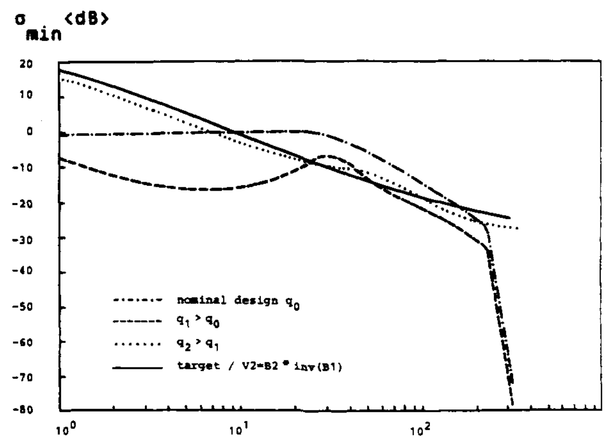

Figure 2 log(w) <r/s>