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EXPLICIT SOLUTION TO THE PROBLEM OF EXACT LOOP TRANSFER RECOVERY

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ABSTRACT

The full-state feedback and observer gains which achieves exact recovery of the loop transfer are parameterized in terms of the system zeros and the corresponding zero-directions. The implications of exact recovery are discussed.

INTRODUCTION

The method of loop transfer recovery (LTR) for robust model-based compensation has received much attention in recent years (see e.g. [1-6]).

The fundamental idea in LTR-design is to recover a target feedback loop design with a suitable asymptotic design. The target feedback loop could be a Kalman-filter design which satisfies certain robustness constraints, and the recovery design could be a certain "cheap" LQ full-state design. The loop transfer of the entire loop will approximate the target loop as the control weight tends to zero, and in the limit recovery is achieved completely.

Unfortunately this recovery-procedure results in control gains that can become very large, and the control system may be impractical [11].

To circumvent this potential problem one might instead search for controllers that achieve exact recovery of the loop transfer for controllers with finite gains. Goodman [4] recently derived conditions for exact recovery. However, the observers and full-state controllers which achieve exact recovery were not found. In this short communication an explicit solution to the problem of exact recovery is outlined. The solution provides a parameterization of the controllers which achieve exact recovery. Further the implications of this parameterization is discussed.

LOOP TRANSFER RECOVERY

In this note minimal and square systems S(A,B,C) are considered. Let the number of inputs/outputs be m, the number of states n, and the number of transmission zeros p. Such systems can be controlled by model-based controllers such that:

$$G(s) = C\Phi(s)B \qquad (1)$$
$$\Phi(s) = (sI-A)^{-1}$$
$$H(s) = K(sI-A+BK+FC)^{-1}F$$

G(s) is the plant transfer and H(s) is the controller. K is the full-state feedback gain and F is the observer gain.

The robustness of a feedback system can be expressed in terms of a suitable loop transfer matrix [1]. Here the loop transfer for the plant output node will be chosen. If the robustness objectives are formulated as loop-shape constraints [1,8] on the loop transfer, the LQG/LTR methodology [1,3] provides a mean for systematic robust design. In this procedure the selection of F ensures that the loop-shape is acceptable. This target-design [5] is then recovered asymptotically with a "cheap" full-state design.

The asymptotic design ensures that the difference between the target loop transfer and the actual loop transfer is reduced. To see this consider the loop recovery error [4]:

$$E_o(s) = C\Phi(s)F - G(s)H(s) \qquad (2)$$

Goodman [4] has shown that:

$$E_o(s) = [I+C\Phi(s)F][I+M_o(s)]^{-1}M_o(s) = 0 \text{ iff} \qquad (3)$$
$$M_o(s) = C(sI-A+BK)^{-1}F = 0$$

and that:

$$M_o(s) = \sum_{i=1}^{n} \frac{Cv_i w_i^T F}{s-\lambda_i} \qquad (4)$$

where v_i and w_i^T are right and left eigenvectors associated with the eigenvalue λ_i of A-BK (A-BK is non-defective).

Hence M(s) = 0 if:

$$Cv_i = 0 \quad \text{or} \quad w_i^T F = 0 \quad i=1,\dots,n \qquad (5)$$

Since F is designed to satisfy the loop-shape constraints the second condition in (5) is generically not satisfied. Hence K must be selected so that $Cv_i = 0$. However in [9] it is shown that max p eigenvectors can satisfy this condition if the associated eigenvalues are equal to the transmission zeros.

The remaining n-p conditions in (4,5) can therefore only be satisfied asymptotically by moving n-p eigenvalues λ_i towards infinity. If such a design is not acceptable, the designer can instead search for controllers that satisfy (4,5) apriori.

EXACT LOOP TRANSFER RECOVERY

The condition $Cv_i = 0$ in (5) can be satisfied by p vectors v_i. From eigenstructure assignment (10) it is known that the eigenvectors v_i are given by

$$v_i = \Phi(\lambda_i)Bt_i \quad , \quad i=1,\dots,n \tag{6}$$

$$T = [t_1 \dots t_n] \quad , \; V = [v_1 \dots v_n]$$

and $\lambda_i \neq \lambda(A)$. Here t_i are free parameter vectors. $Cv_i = 0$ imply that

$$C\Phi(\lambda_{io})Bt_{io} = 0 \tag{7}$$

This condition can be satisfied if λ_{io} is a zero of S(A,B,C) and $t_{io} \in Ker(G(\lambda_{io}))$. Hence K can be parameterized as [6,10]

$$K = -TV^{-1} \tag{8}$$

$$t_i = t_{io} \quad , \quad v_i = v_{io} \; , \quad i=1, \dots ,p$$

where λ_i , t_i (i=p+1, ... ,n) are free design parameters. If S(A,B,C) is minimum-phase the full-state design can therefore be stabilizing.

The remaining n-p conditions in (5) must be satisfied by selecting F according to

$$w_i^T F = 0 \quad , \quad i=p+1, \dots ,n \tag{9}$$

If the eigenvector-matrix of A-BK is V, and $W = V^{-1}$ eq. (9) imply

$$F^T[w_1 \dots w_p \; w_{p+1} \dots w_n] = [\Omega^T \; 0] \tag{10}$$

With dim Ω = p x m but otherwise arbitrary. Now

$$F = V \begin{bmatrix} \Omega \\ 0 \end{bmatrix} = [v_1 \dots v_p]\Omega = \Gamma\Omega \tag{11}$$

With dimΓ = n x p. The vectors v_1, ... ,v_p are the p eigenvectors of A-BK which is constrained by (5). Hence Γ is a matrix of fixed elements. Therefore eq.(11) parameterizes the matrices F which achieve exact recovery provided that K is selected according to eq.(8).

IMPLICATIONS OF EXACT LOOP TRANSFER RECOVERY

In this section some consequences of exact recovery is discussed.

- The parameterization of the observer-gain implies that F must be selected as an output feedback controller, where Ω is the free parameter parameter output feedback matrix. Γ is the equivalent input matrix with p independent rows, and C is the output matrix. For such a problem α = min(n,m+p-1) eigenvalues can be assigned [7]. Since $p \leq n-m$, $\alpha = m+p-1 < n$. Consequently all of the observer-eigenvalues cannot be assigned freely, and no stability guarantees are available as with the usual design-techniques.
  Still further the simple loop-shape design rules [5] cannot be invoked, since the LQG design concepts do not apply here. Also note that CF = 0. Therefore the "high" frequency loop-shape is

$$C\Phi F \sim -CAF/\omega^2 \tag{12}$$

  In summary no systematic design rules for the F selection for stability and loop-shape requirements is available.
- The selection of K is only constrained by eq.(8), and the p fixed modes will become unobservable. For minimum-phase systems stability can always be achieved. The extra freedom in the K-selection can be applied to satisfy secondary options.
- The applicability of exact recovery in loop-shaping depends on p:

  p = 0 - If p=0 if follows that Γ=F=0.

  0<p<m - Since rank(F)$\leq$p<m it follows that the target loop-transfer is rank-defective. Hence:

$$\underline{\sigma}[T_o(j\omega)]=0, \quad \bar{\sigma}[S_o(j\omega)]\geq 1 \tag{13}$$

  Here S_o is the output sensitivity matrix and T_o is the complementary output sensitivity matrix. Obviously poor output sensitivity and stability robustness for plant output modelling errors is achieved

  p $\geq$ m - Here rank(F) $\geq$ m and loop-shape procedures can be applied without the limitations in (13).
- Dual results apply for the plant-input loop breaking point.
- When F and K are selected as in eqs.(9,11) the loop recovery error $E_o(j\omega) = 0$ for all ω. The resulting compensator therefore depends on F as $H(s) = G(s)^{-1}C\Phi(s)F$. The F and K selection implies a compensator that inverts the plant in cascade with the loop-shape.
  A further study of the structure of H(s) reveals that the poles of H(s) are equal to the eigenvalues of A-BK, and that the n-p free eigenvalues of A-BK are decoupling zeros of H(s) [4].
  The results given here are derived for continuous time systems. Similar results apply to discrete-time systems. For such systems, however, asymptotic recovery (for finite sampling rate) is not possible [4] when prediction estimators are applied, and consequently arbitrary loop-shaping is not feasible. Therefore the exact recovery results given here may be of some interest in discrete-time loop-shaping.

SUMMARIZING REMARKS

It is shown that exact recovery is possible when the number of plant transmissions zeros is nonzero. The price paid for this guarantee is that the nice stability and loop-shape properties of the asymptotic recovery designs are lost. Instead new methods for loop-shaping must be derived. Since asymptotic recovery is not possible in discrete-time exact recovery seems to be most relevant in discrete-time feedback synthesis.

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