Reflection of sound from finite-size plane and curved surfaces

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Reflection of sound from finite-size plane and curved surfaces

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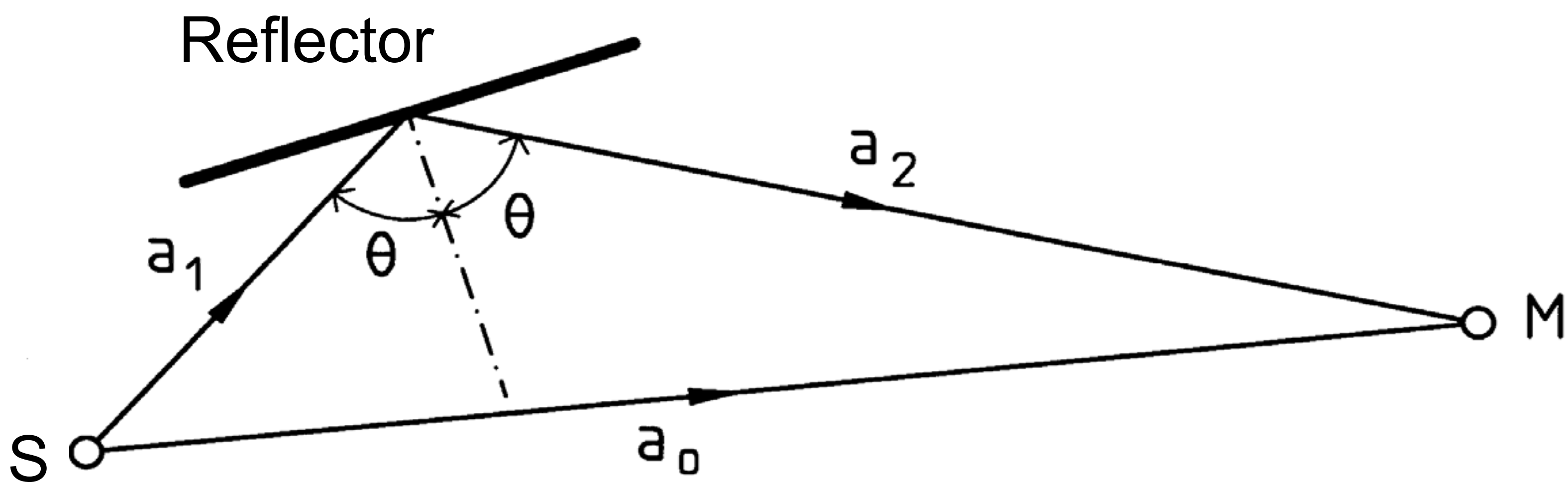
DTU

Outline

- General model
- Curved reflectors
- Finite size single reflectors
- Array of reflectors
- Conclusion
- Example of application

(Based on work done between 1982 and 1991, but not all has been published)

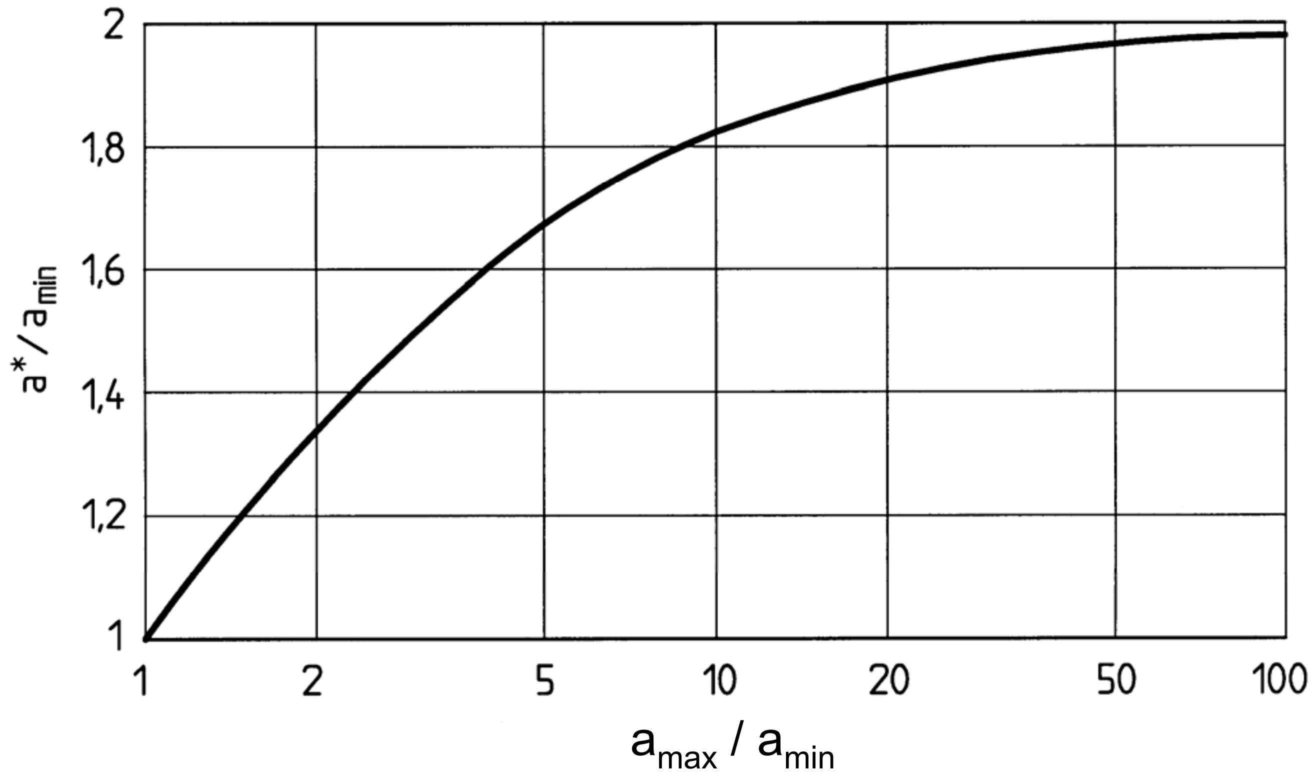
General model of sound reflection



$$\Delta L = L_{refl} - L_{dir} = \Delta L_a + \Delta L_m + \Delta L_k + \Delta L_s \quad \text{(dB)}$$

attenuation from	a:	distance
	m:	absorption (material)
	k:	curvature
	s:	reflector size

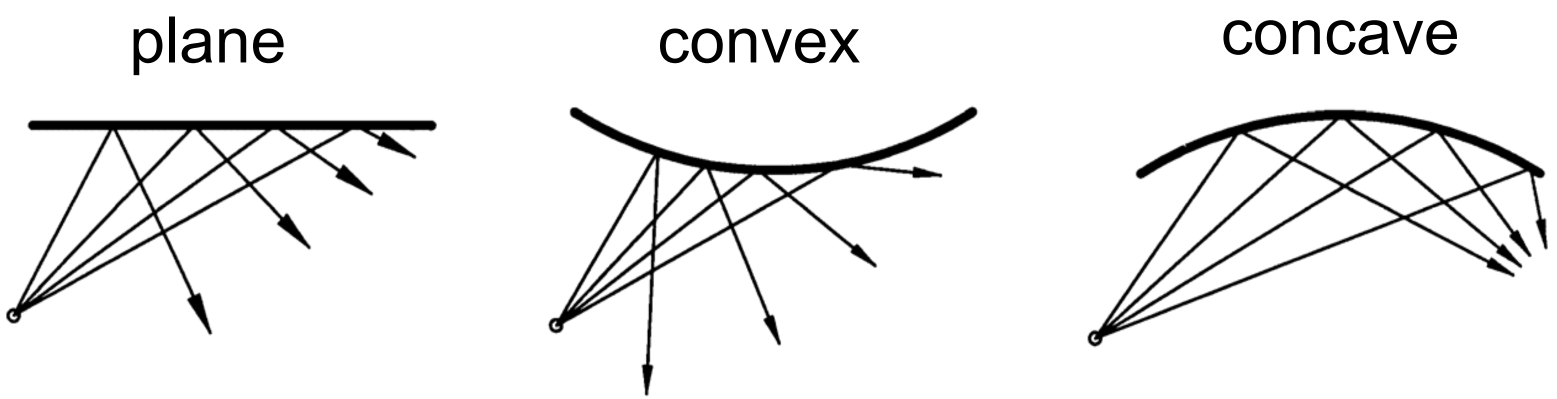
Characteristic distance, a*



$1/a^* = (1/a_1 + 1/a_2) / 2$ (The harmonic average)

$a^* = 2 a_1 a_2 / (a_1 + a_2)$

Curved reflectors

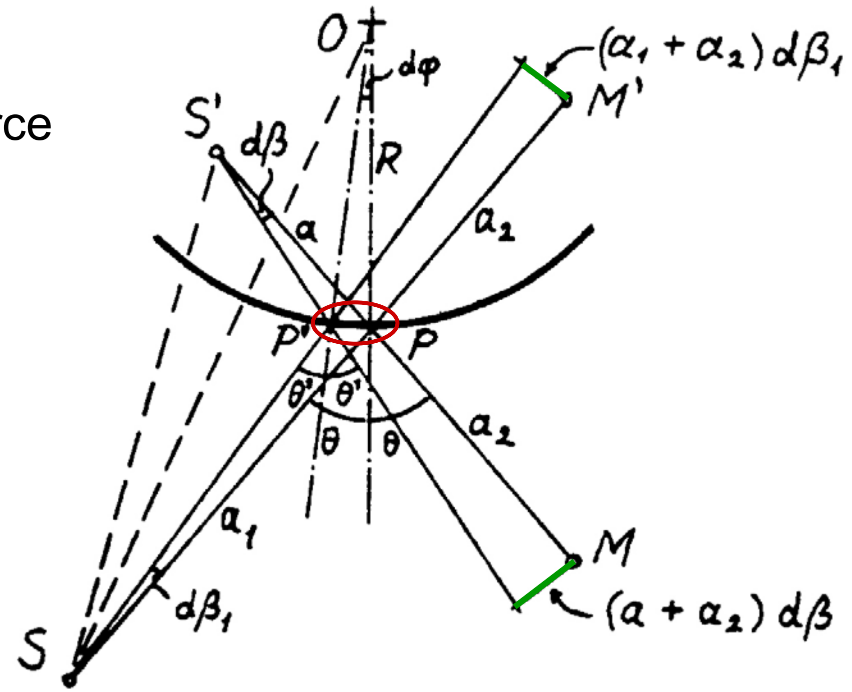


References:
1982: Lydrefleksion fra konvekse og konkave cylinderflader. (In Danish). NAS-82, Stockholm. Proceedings pp. 71-74.
1985: Attenuation of Sound Reflections from Curved Surfaces. 24th Conference on Acoustics, Strbské Pleso. Proceedings pp. 194-197.

Geometrical analysis

Image source

Image Receiver



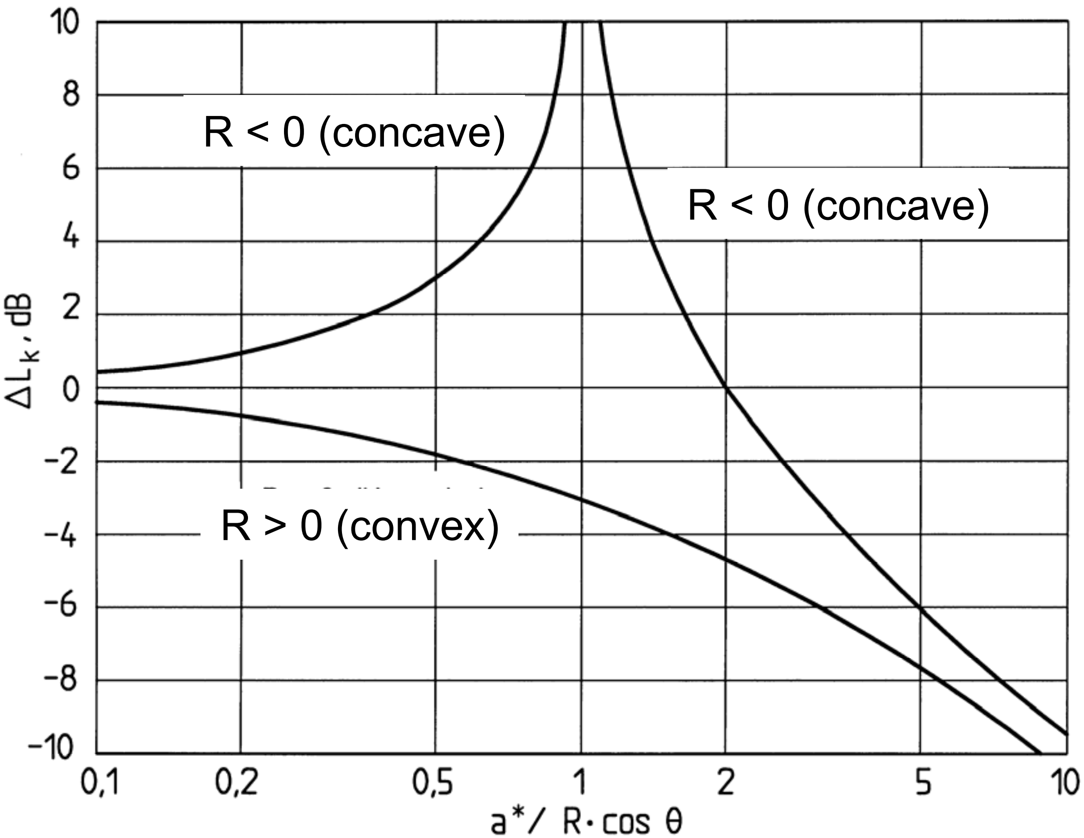
Receiver

Source

$$R \cdot \mathrm{d}\varphi = a \cdot \mathrm{d}\beta / \cos\theta = a_1 \cdot \mathrm{d}\beta_1 / \cos\theta$$

$$\Delta L_k = -10\log\left(\frac{(a + a_2)\,\mathrm{d}\beta}{(a_1 + a_2)\,\mathrm{d}\beta_1}\right)$$

Attenuation due to curvature



$$\Delta L_k = -10\log\left|1+\frac{a^*}{R\cos\theta}\right| \qquad a^* = \frac{2\,a_1\,a_2}{a_1+a_2}$$

Measurement set-up using TDS technique

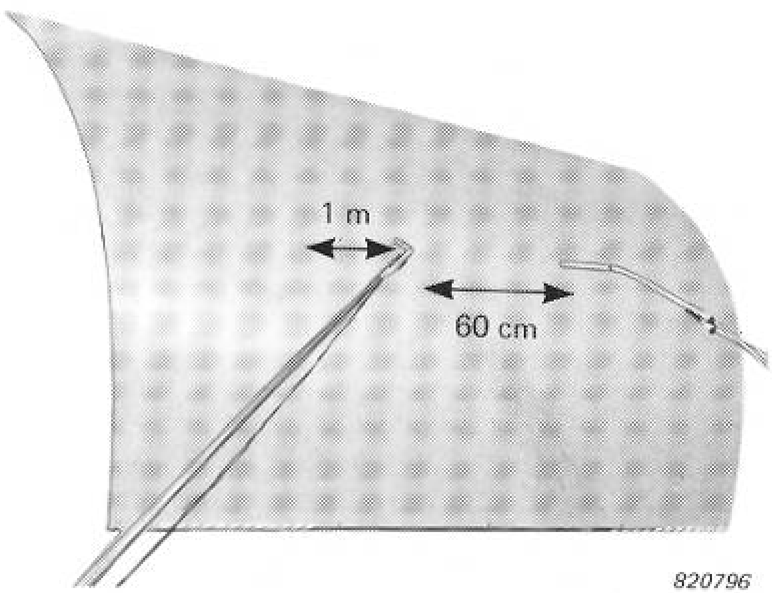


Fig. 6. Convex reflector

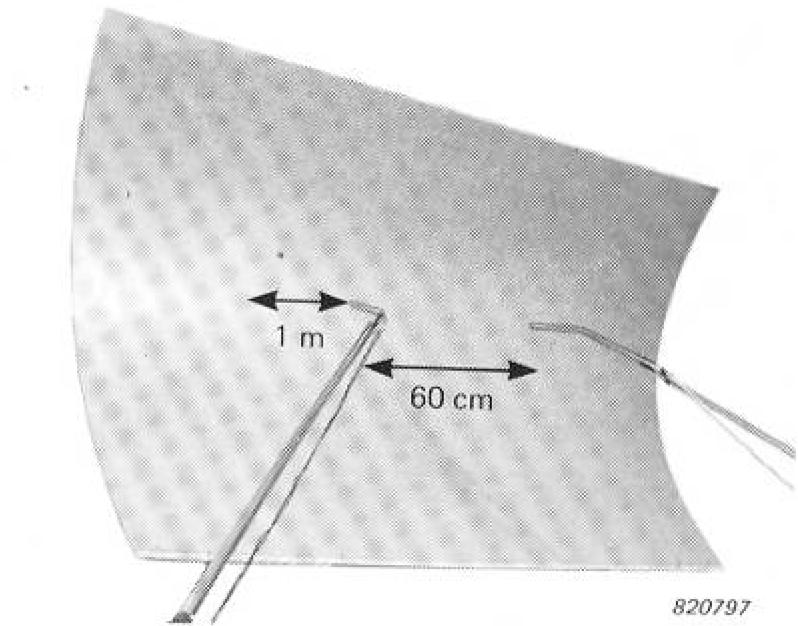
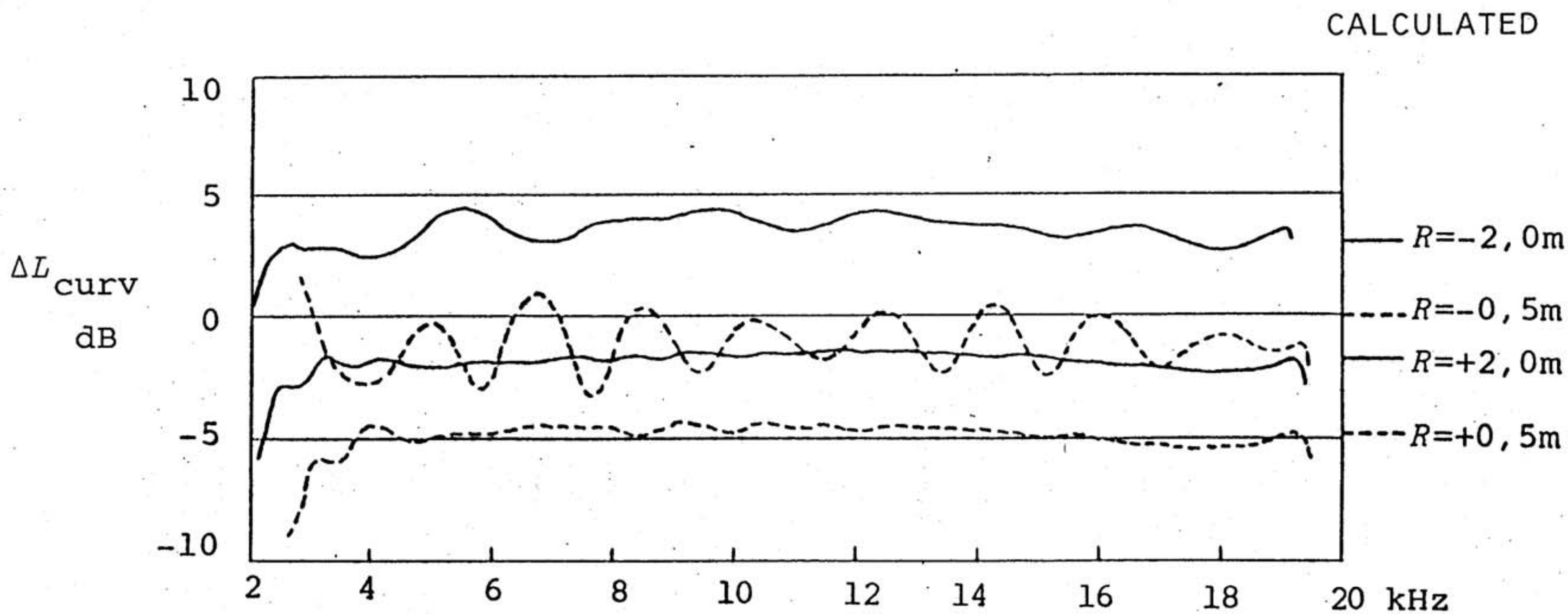
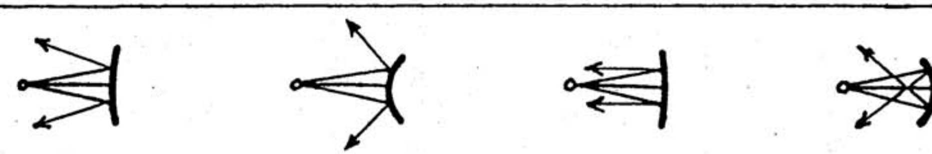


Fig. 7. Concave reflector

Comparing measured and calculated results

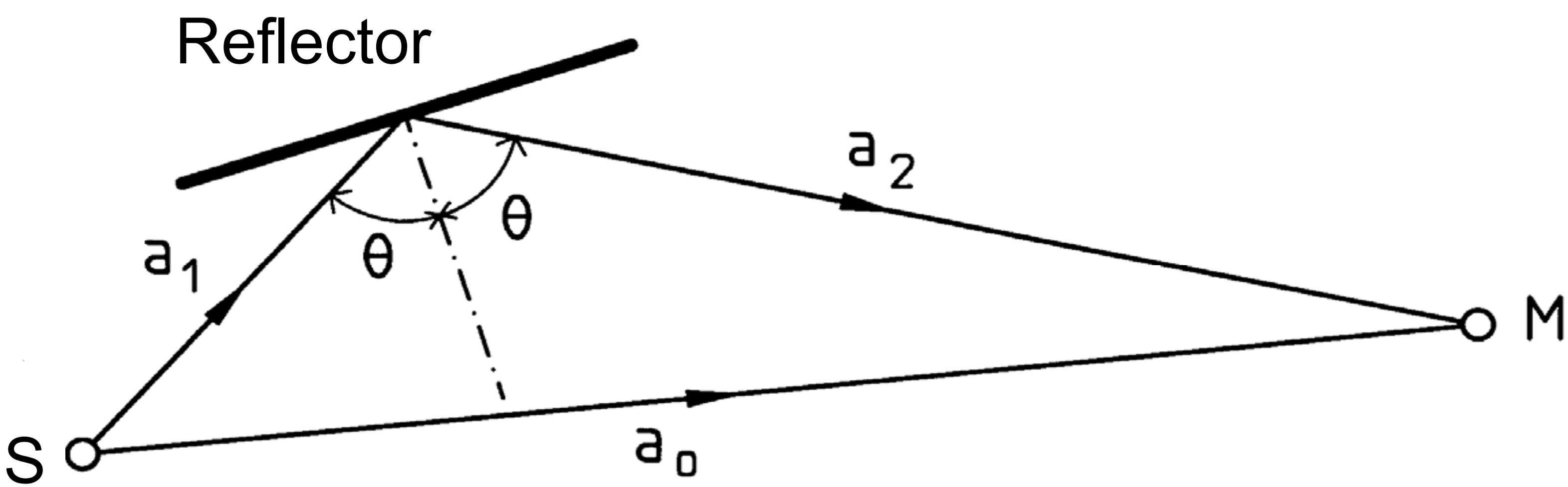


sketch				
radius R	2.0 m	0.5 m	−2.0 m	−0.5 m
$a^*/R \cos\theta$	0.5	2.0	−0.5	−2.0
ΔL_{curv}(calc.)	−1.8 dB	−4.8 dB	3.0 dB	0.0 dB
ΔL_{curv}(meas.)	−2 ± 0.5 dB	−5 ± 1 dB	3 ± 1 dB	−1 ± 2 dB

Comments to results on curved reflectors

- Satisfactory agreement between measurements and theoretical model,
- but diffraction effects are seen as fluctuations in the case of R = - 0.5 m
- so, the finite size should also be taken into account

Finite size single reflectors

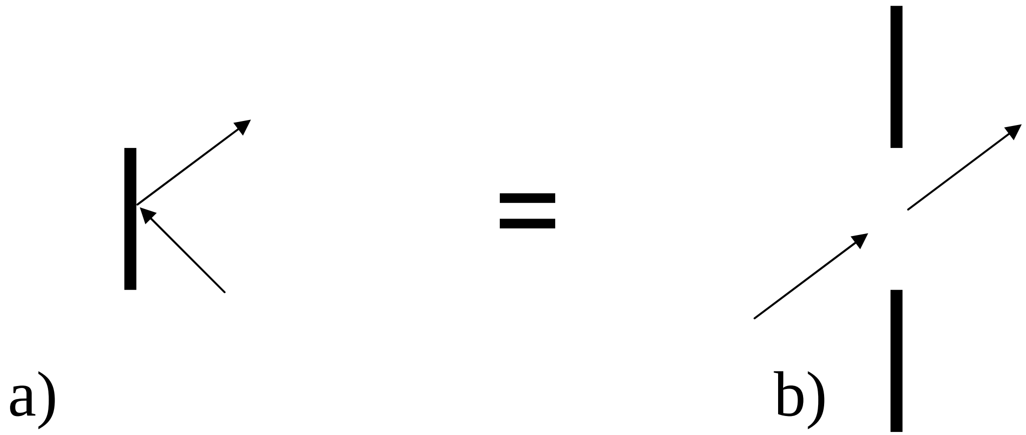


References:

1986: Attenuation of Sound Reflections due to Diffraction. NAM-86, Aalborg. Proceedings pp. 257-260.

1992: Acoustic Design of Reflectors in Auditoria. Institute of Acoustics, Proceedings, Vol. 14: Part 2, pp.119-128.

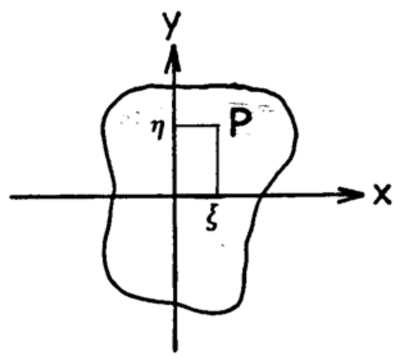
Babinet's Principle

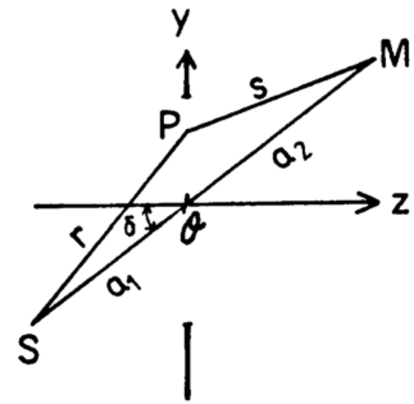


Reflection from a surface (a)
is equivalent to transmission through an aperture (b)
with same size and shape, surrounded by a rigid baffle

Kirchhoff-Fresnel approximation

Coordinate system has Origo in the point of geometrical reflection





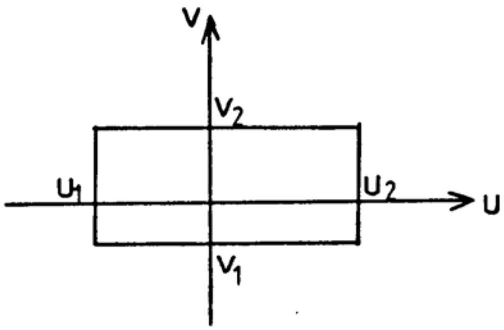
$$\Phi = j\frac{Q}{8\pi\lambda}\int_A \frac{\mathrm{e}^{-jk(r+s)}}{rs}\big(\cos(n,r)-\cos(n,s)\big)\,\mathrm{d}A$$

$$\Phi \cong j\frac{Q}{4\pi\lambda}\frac{\cos\theta}{a_1 a_2}\int_A \mathrm{e}^{-jk(r+s)}\,\mathrm{d}A$$

Dimensions of the surface << a_1 and a_2

Transformation of variables

Rectangular aperture



$$u = \sqrt{\frac{2}{\lambda}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)} \cdot \xi$$

$$v = \sqrt{\frac{2}{\lambda}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)} \cdot \cos\theta \cdot \eta$$

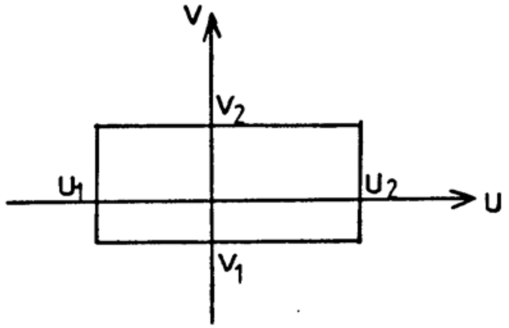
$$\Phi \cong j \frac{Q}{8\pi} \frac{\mathrm{e}^{-jk(a_1+a_2)}}{a_1 + a_2} \left(M - jN\right)$$

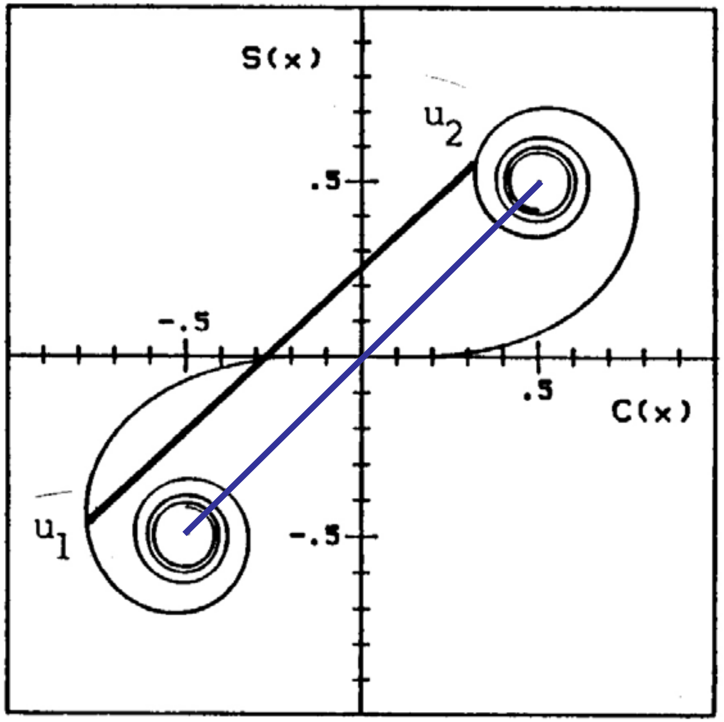
$$M - jN = \int_{u_1}^{u_2} e^{-j\frac{\pi}{2}u^2}\, \mathrm{d}u \int_{v_1}^{v_2} e^{-j\frac{\pi}{2}v^2}\, \mathrm{d}v$$

$$M - jN = \left[C(u_2) - C(u_1) - j\left(S(u_2) - S(u_1)\right)\right] \cdot \left[C(v_2) - C(v_1) - j\left(S(v_2) - S(v_1)\right)\right]$$

Cornu's spiral

The Fresnel integrals: $C(v) = \int_0^v \cos\left(\frac{\pi}{2} z^2\right) \mathrm{d}\, z$ $S(v) = \int_0^v \sin\left(\frac{\pi}{2} z^2\right) \mathrm{d}\, z$

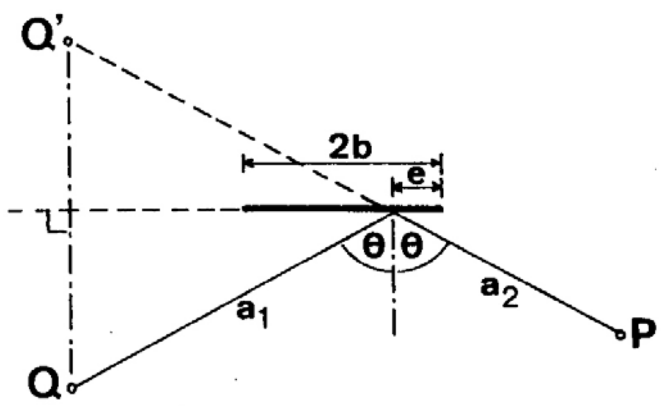




Result for an infinite large surface:

This is taken as the reference for the attenuation due to size

Rectangular reflector



Deviation from geometrical acoustics:

$$\Delta L_s = 10 \log\left(K_1 K_2\right)$$

$$K_1 = \tfrac{1}{2}\left(\left(C(u_2) - C(u_1)\right)^2 + \left(S(u_2) - S(u_1)\right)^2\right)$$

$$K_2 = \tfrac{1}{2}\left(\left(C(v_2) - C(v_1)\right)^2 + \left(S(v_2) - S(v_1)\right)^2\right)$$

(Two orthogonal sections)

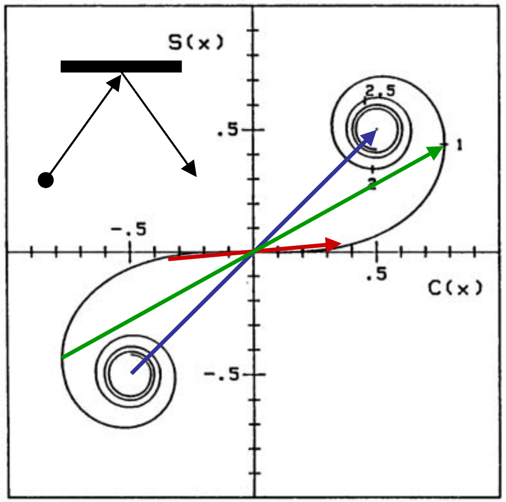
$$v_{1,i} = \frac{2}{\sqrt{\lambda a^*}}\left(e - 2b\right)\cos\theta$$

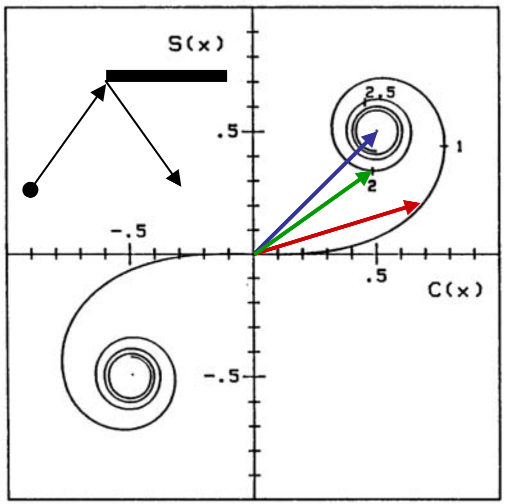
(corresponds to left edge of plate)

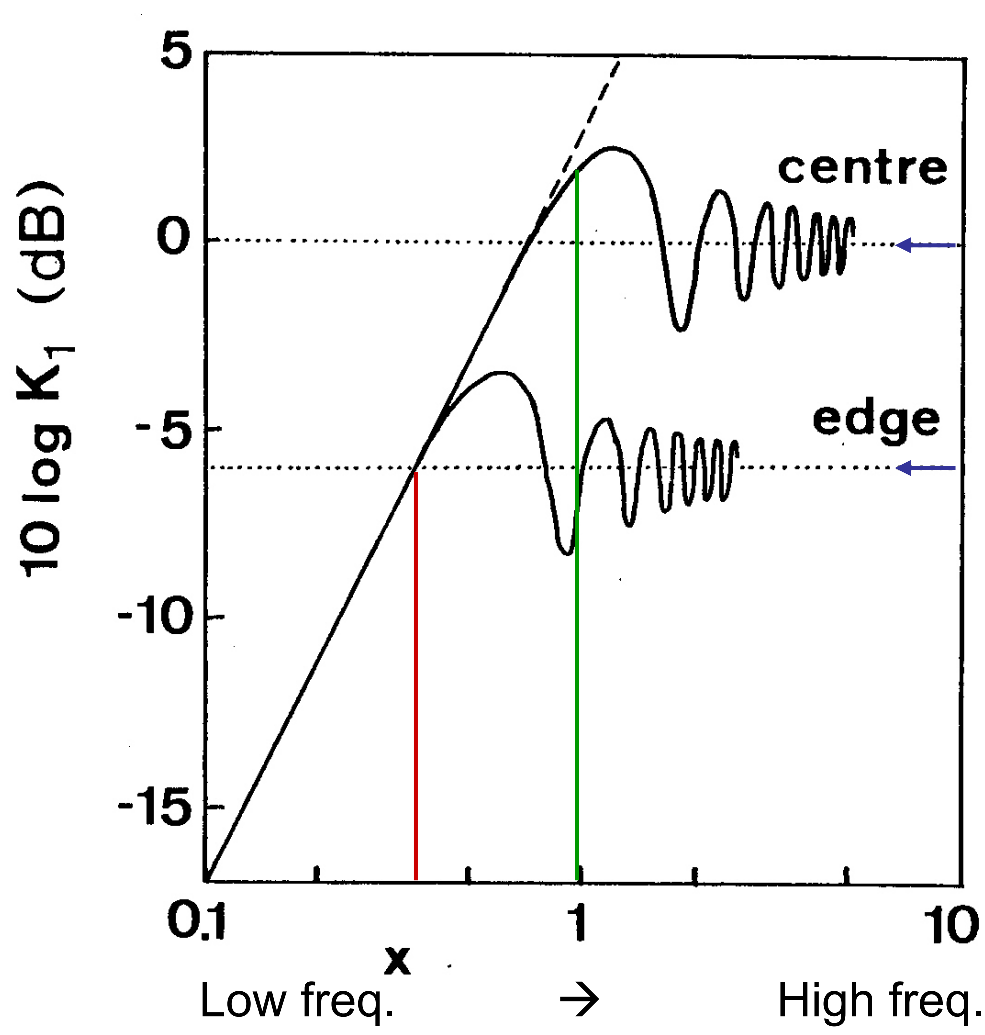
$$v_{2,i} = \frac{2}{\sqrt{\lambda a^*}} e \cos\theta$$

(corresponds to right edge of plate)

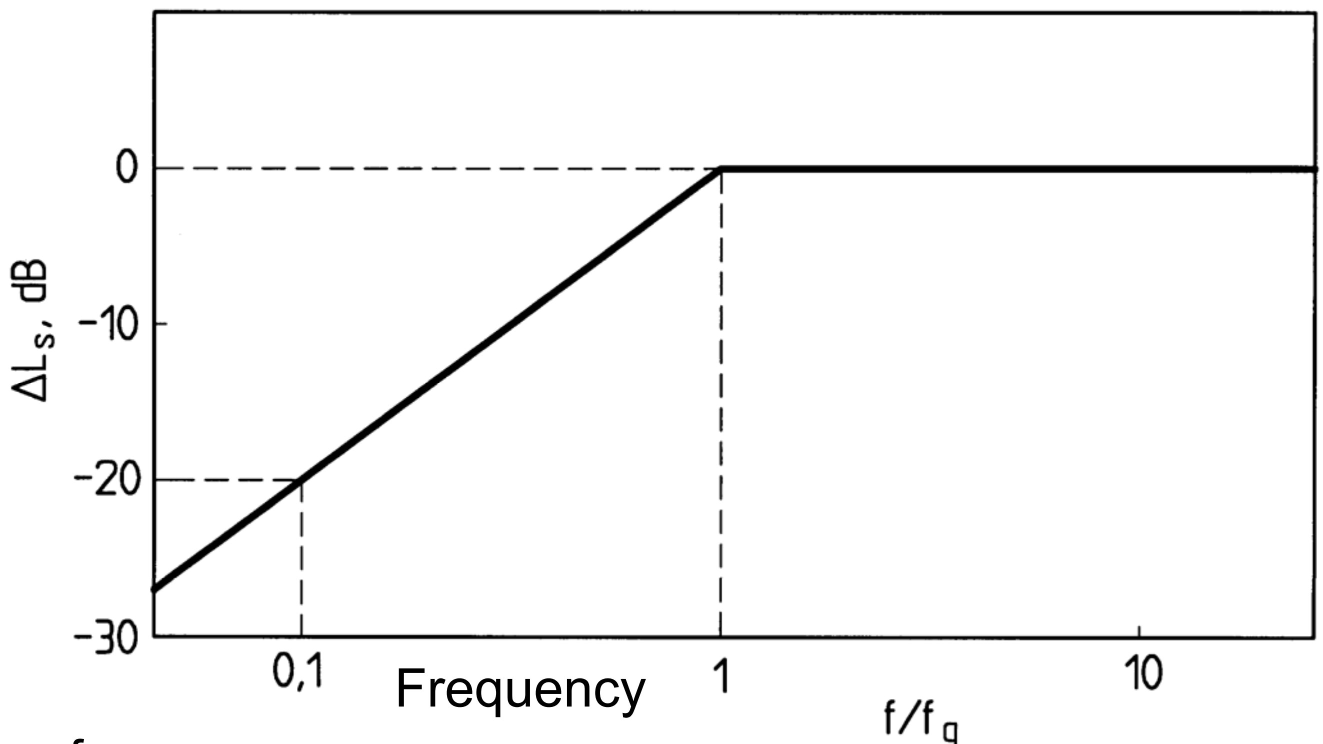
Rectangular reflector







Attenuation due to size – simplified model



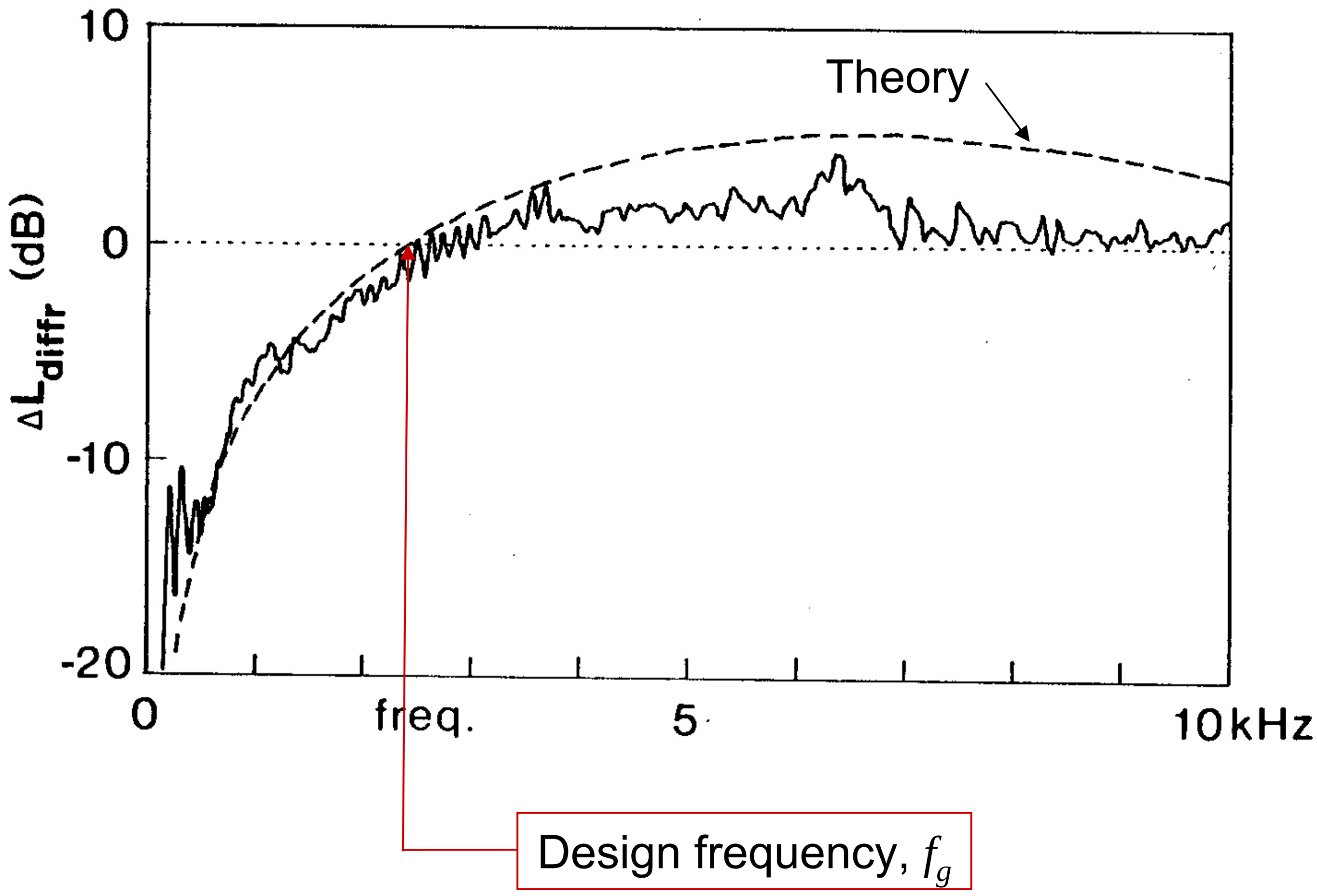
Design frequency:

$$f_g = \frac{c\,a^*}{2\,S\cos\theta}$$

c = 344 m/s is speed of sound
a* is characteristic distance
S is area of reflector
θ is angle of incidence

Measurements using pulse gating technique

22 mm hardboard, 0.6 m * 0.6 m, a_1 = 6.0 m, a_2 = 4.0 m, θ = 0°

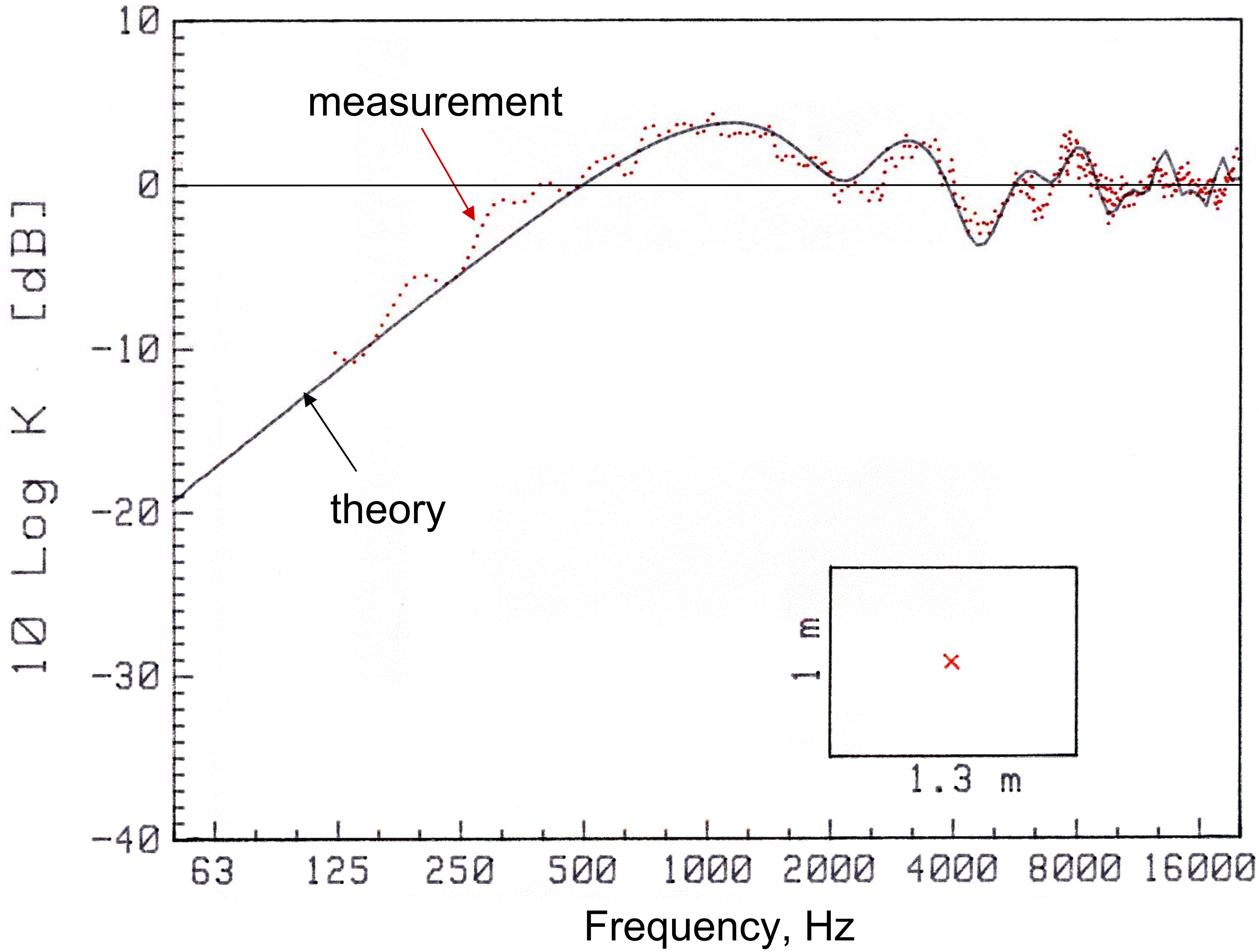


Comment to first results on single reflectors

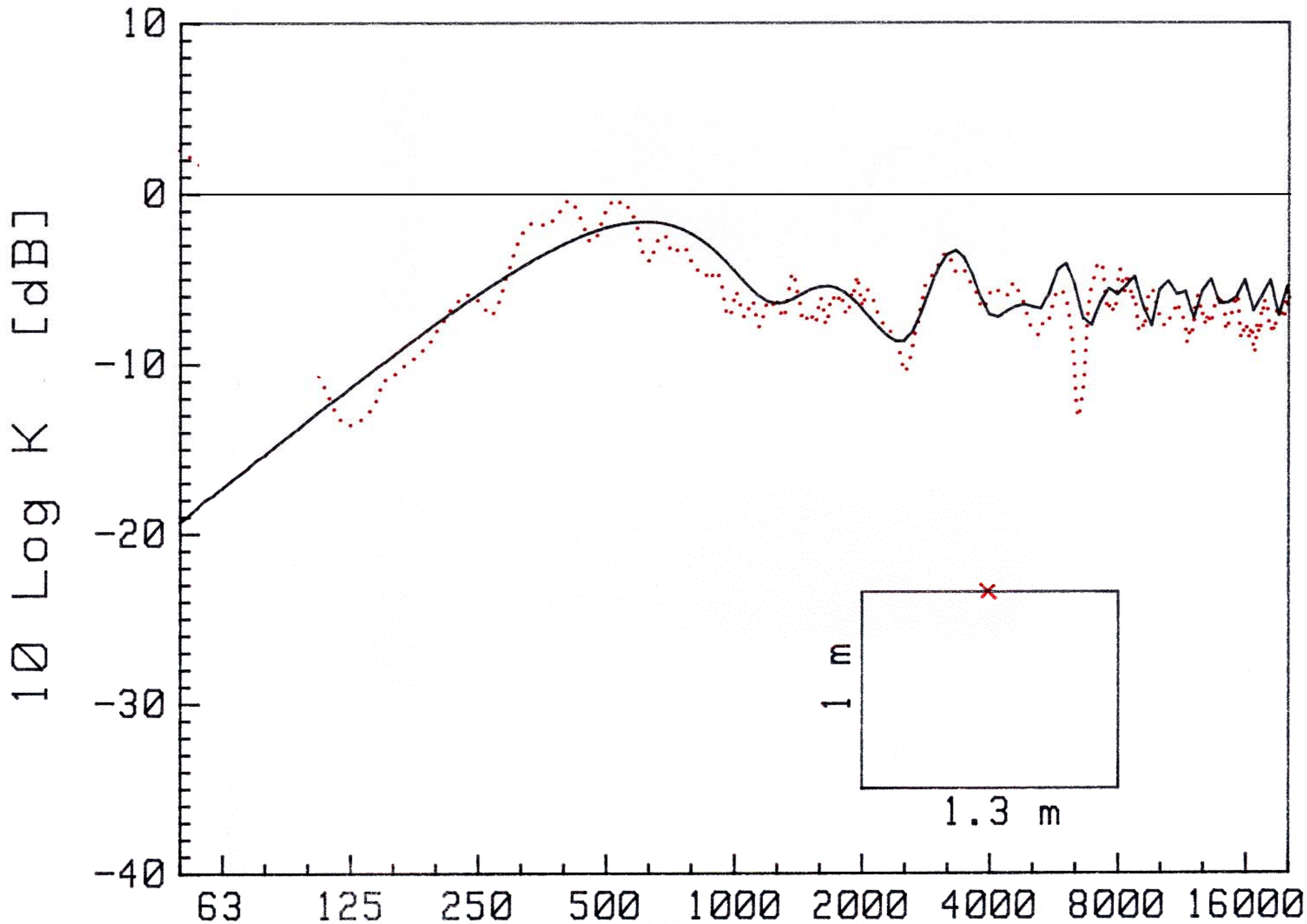
- A design frequency was derived from the theory and confirmed by the measurements
- However, this new design frequency is one octave lower than that previously suggested by L. Cremer (1953)
- Cremer agreed that the new design frequency is correct in his last conference paper at ICA 1989

Measurement results

Angle of incidence = 30°, a_1 = 3.0 m, a_2 = 3.0 m

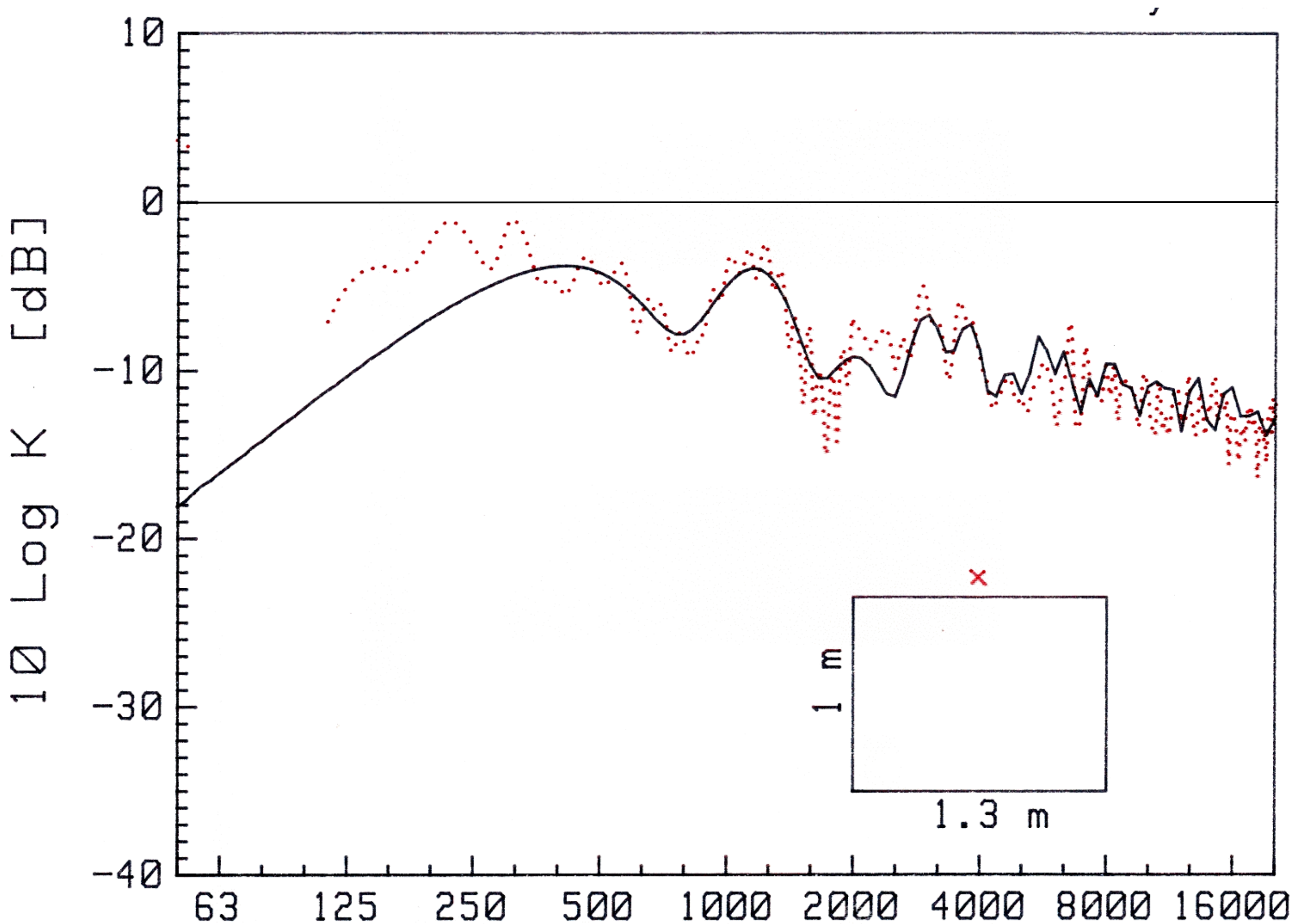


Angle of incidence = 30°, a_1 = 3.0 m, a_2 = 3.0 m



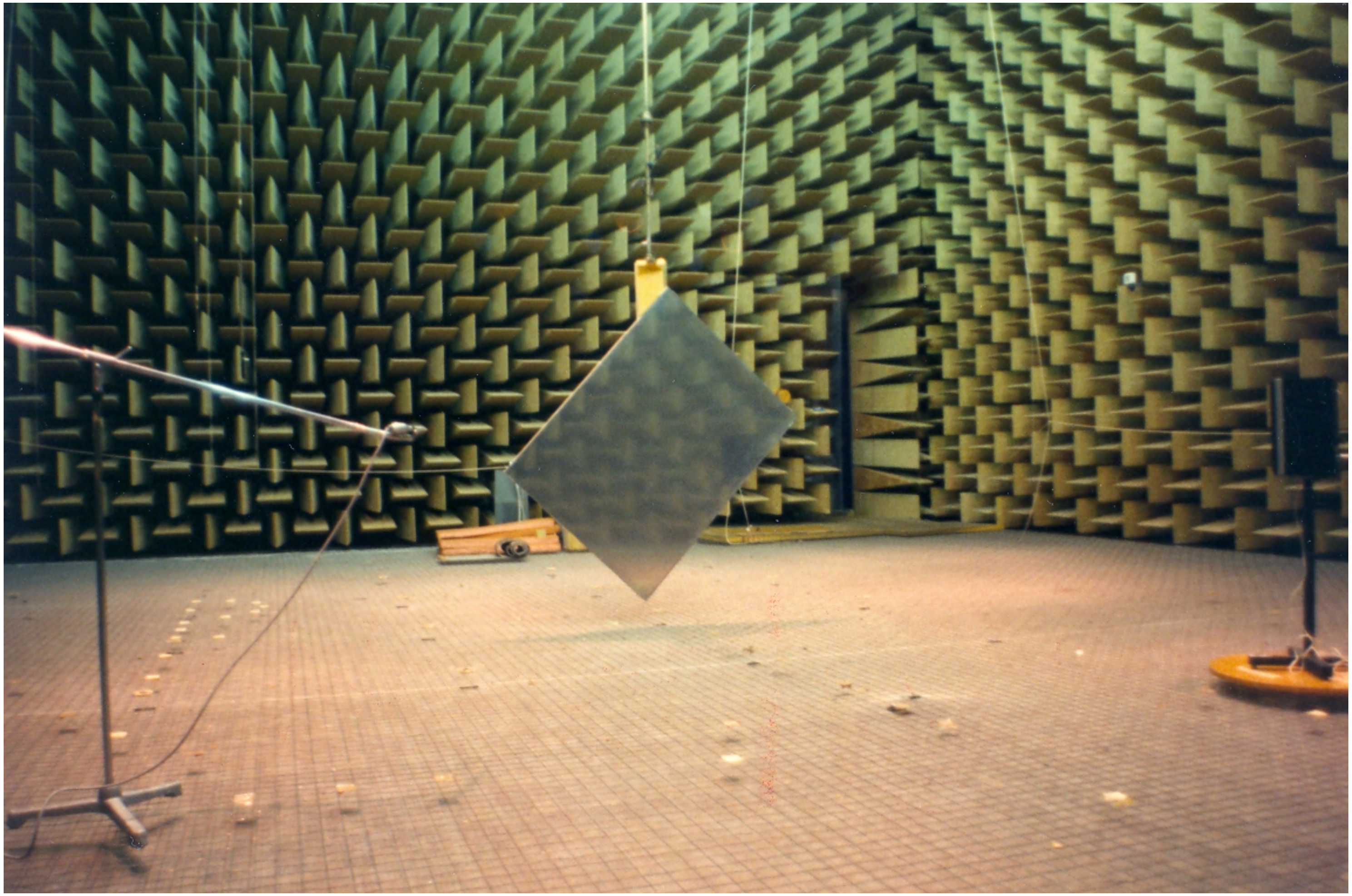
Frequency, Hz

Angle of incidence = 0°, a_1 = 4.0 m, a_2 = 2.4 m



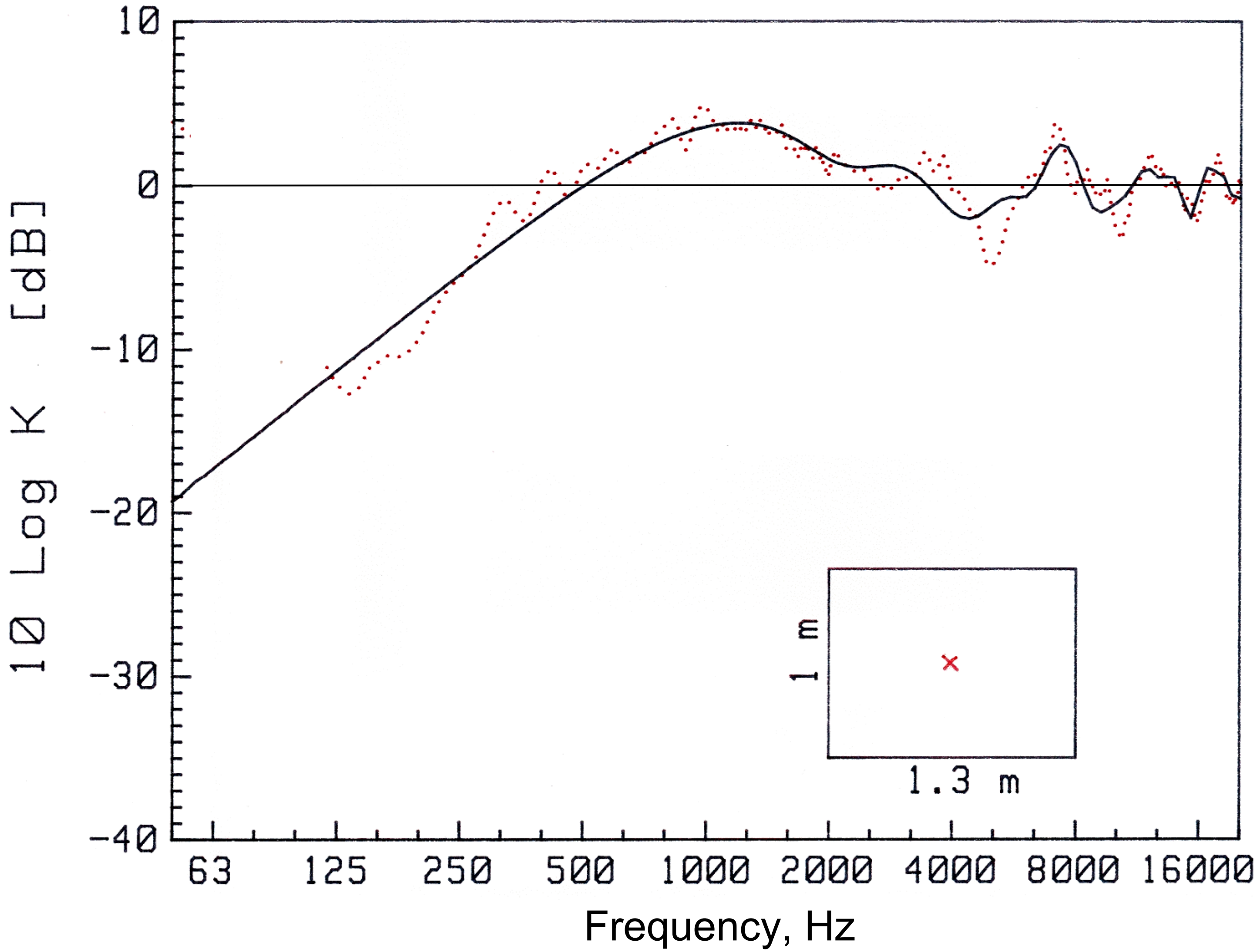
Frequency, Hz

Rotation of reflector
- to demonstrate that the theoretical model is orthogonal

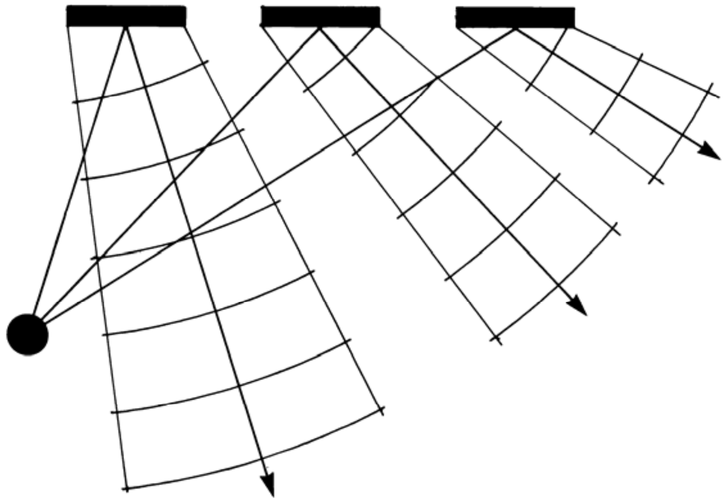


Rotation 30°

Angle of incidence = 30°, a_1 = 3.0 m, a_2 = 3.0 m

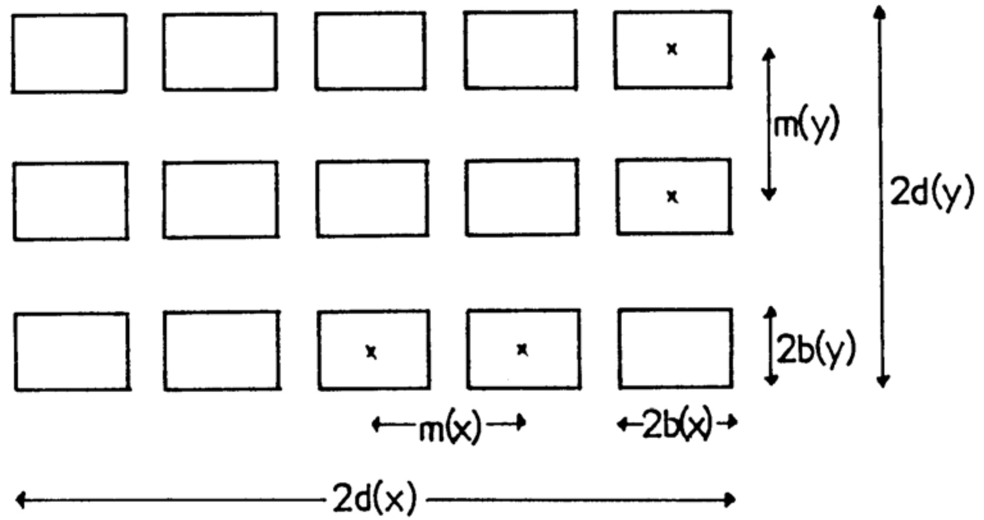


Reflector array



References:
1988: Bølgeteoretiske metoder. (In Danish). Romakustisk prosjektering, NIF kursus, Geilo. (17 p).
1990: The Design of an Array of Reflectors for Improved Ensemble on a Concert Hall Platform. NAM-90, Luleå. Proceedings pp. 129-134.
1990: Attenuation of Sound Reflections from an Array of Reflectors. 29th Conference on Acoustics, Strbské Pleso. Proceedings pp. 231-234.
1991: Design of New Ceiling Reflectors for Improved Ensemble in a Concert Hall. Applied Acoustics 34, pp. 7-17.

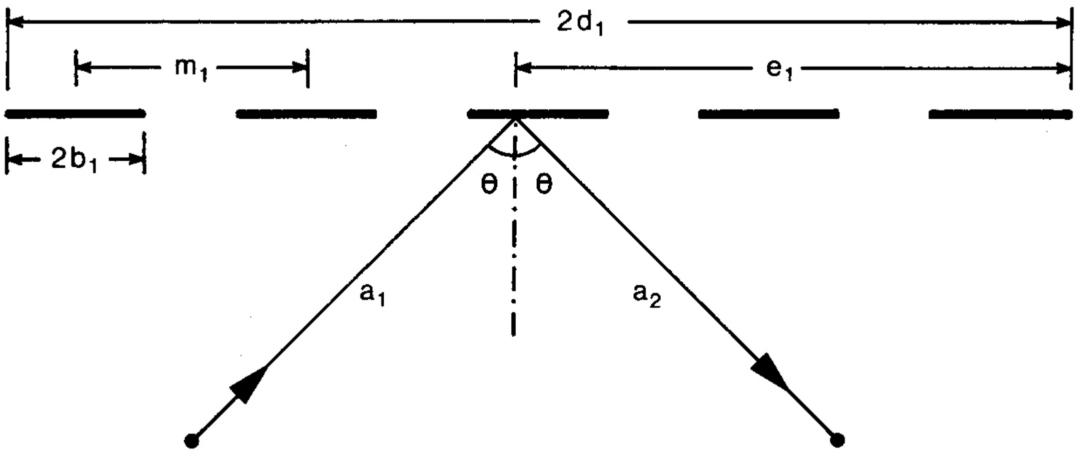
Reflector array

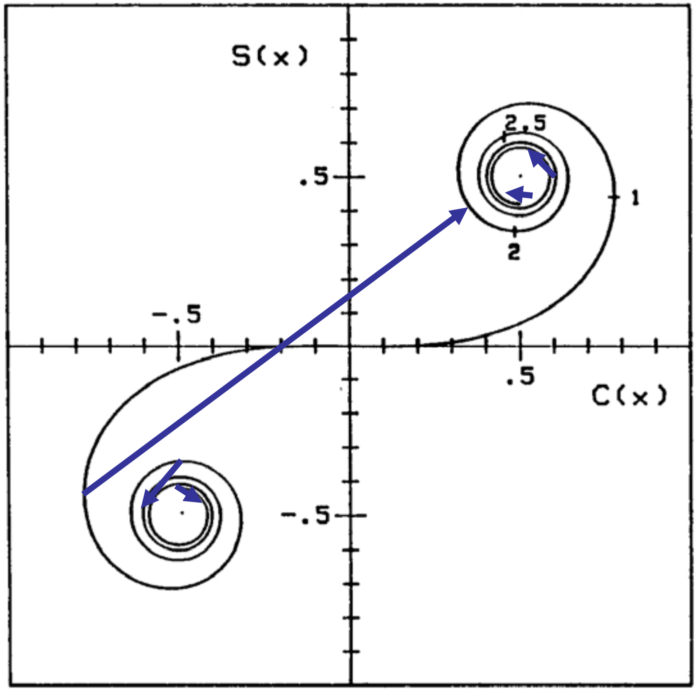


$$K_1 = \tfrac{1}{2}\left[\left(\sum_{i=1}^{n}\big(C(v_{2,i}) - C(v_{1,i})\big)\right)^2 + \left(\sum_{i=1}^{n}\big(S(v_{2,i}) - S(v_{1,i})\big)\right)^2\right]$$

$$v_{1,i} = \frac{2}{\sqrt{\lambda\, a^*}}\big(e_1 - 2b_i - (i-1)m_1\big)\cos\theta$$ (left edges of plates)

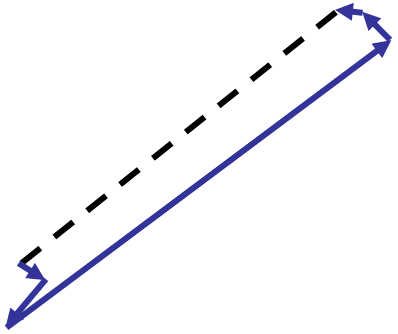
$$v_{2,i} = \frac{2}{\sqrt{\lambda\, a^*}}\big(e_1 - (i-1)m_1\big)\cos\theta$$ (right edges of plates)

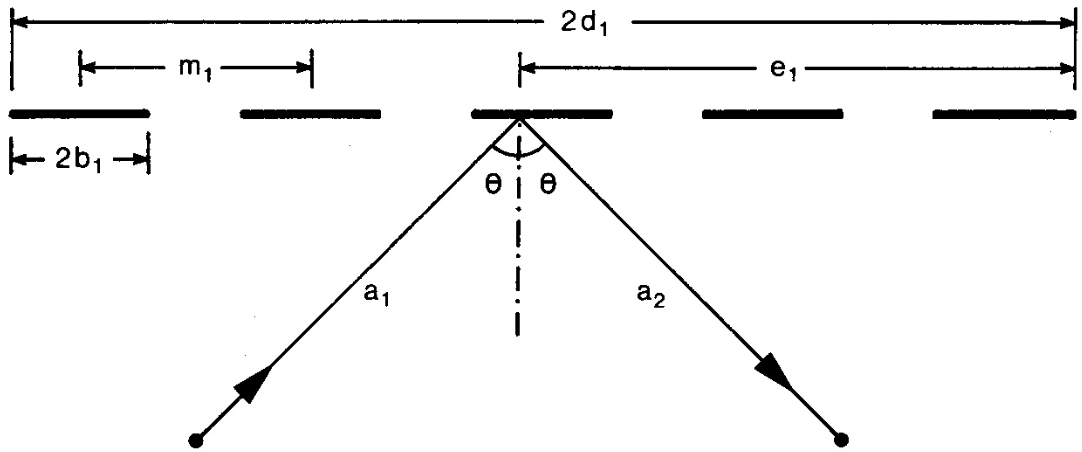


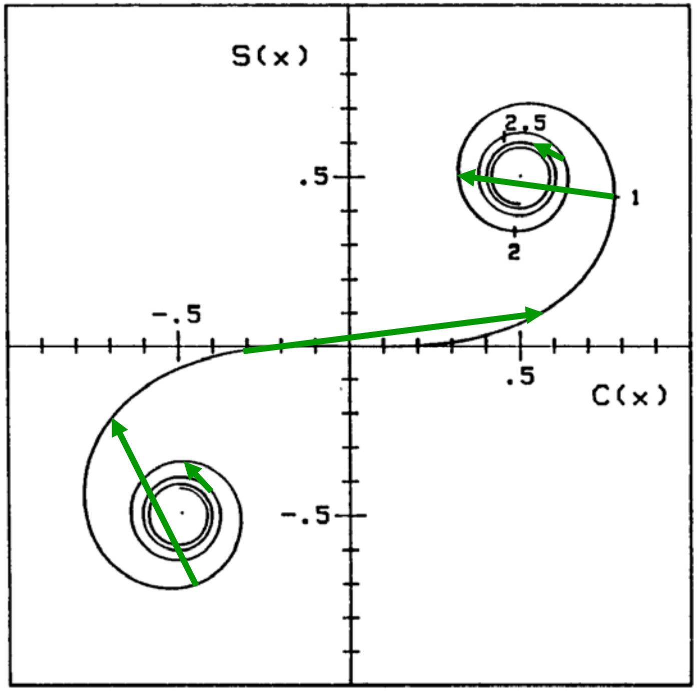


High frequencies:

One plate dominates

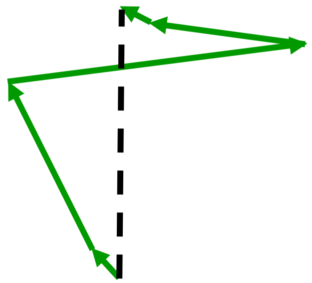


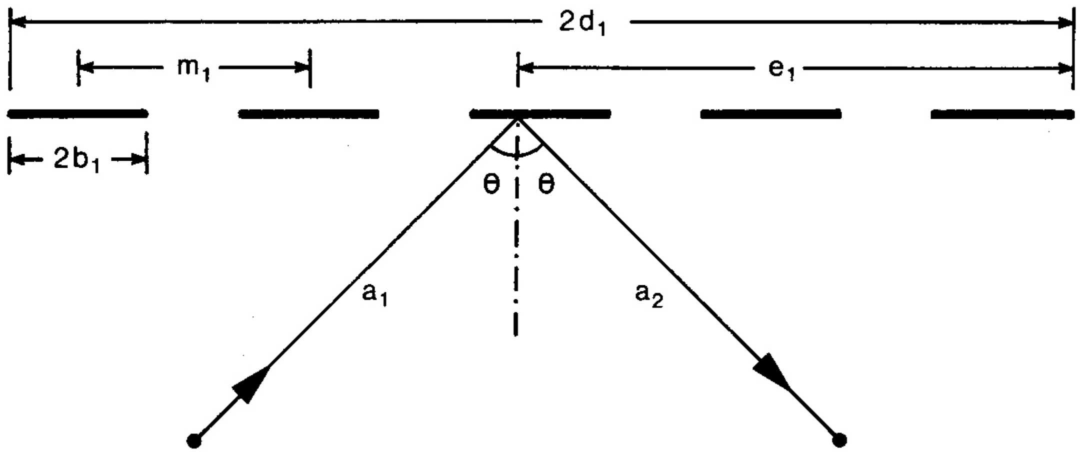


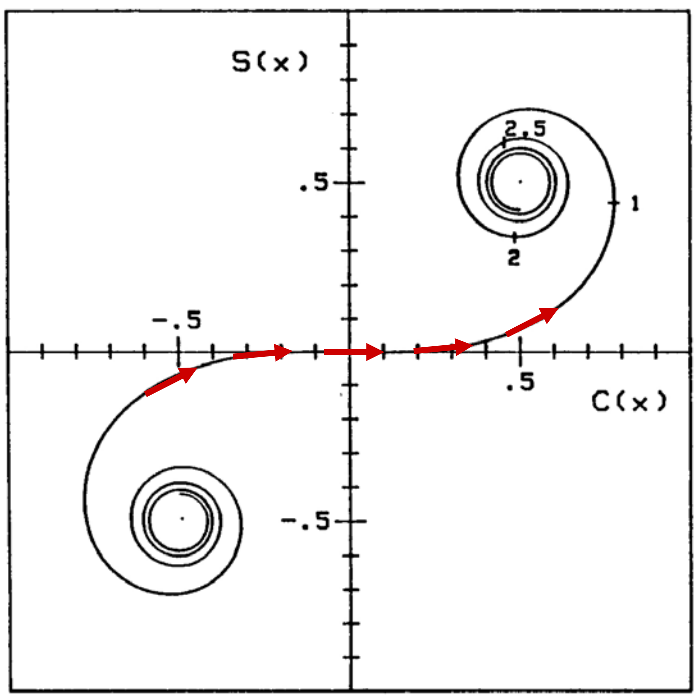


Middle frequencies:

Several plates contribute, but out of phase

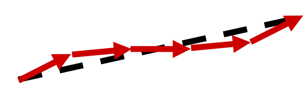






Low frequencies:

All plates contribute, and add almost in phase



Measurement results

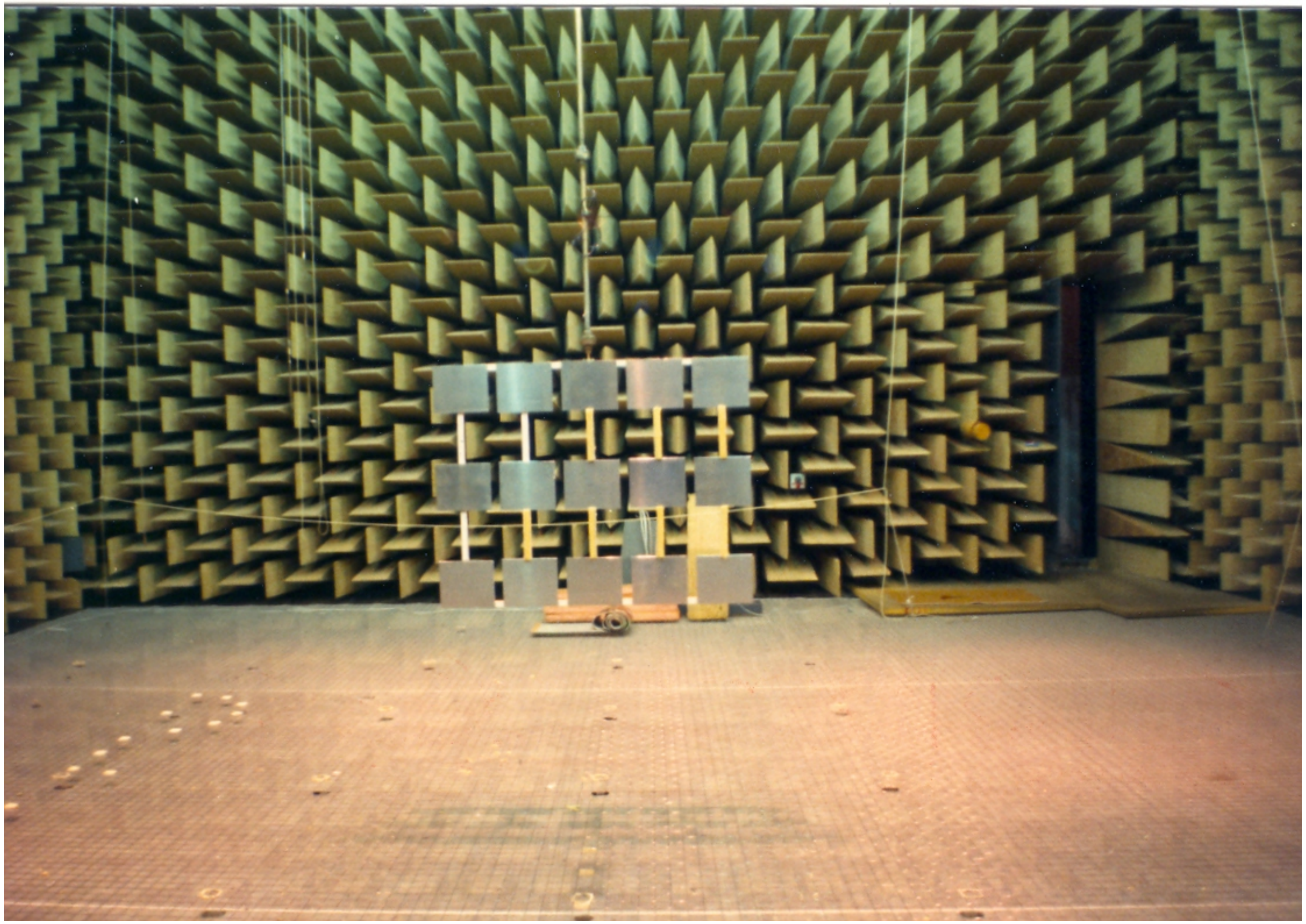


Plate dimensions: 0.23 m * 0.20 m, θ = 30°, a_1 = 3.0 m, a_2 = 2.0 m

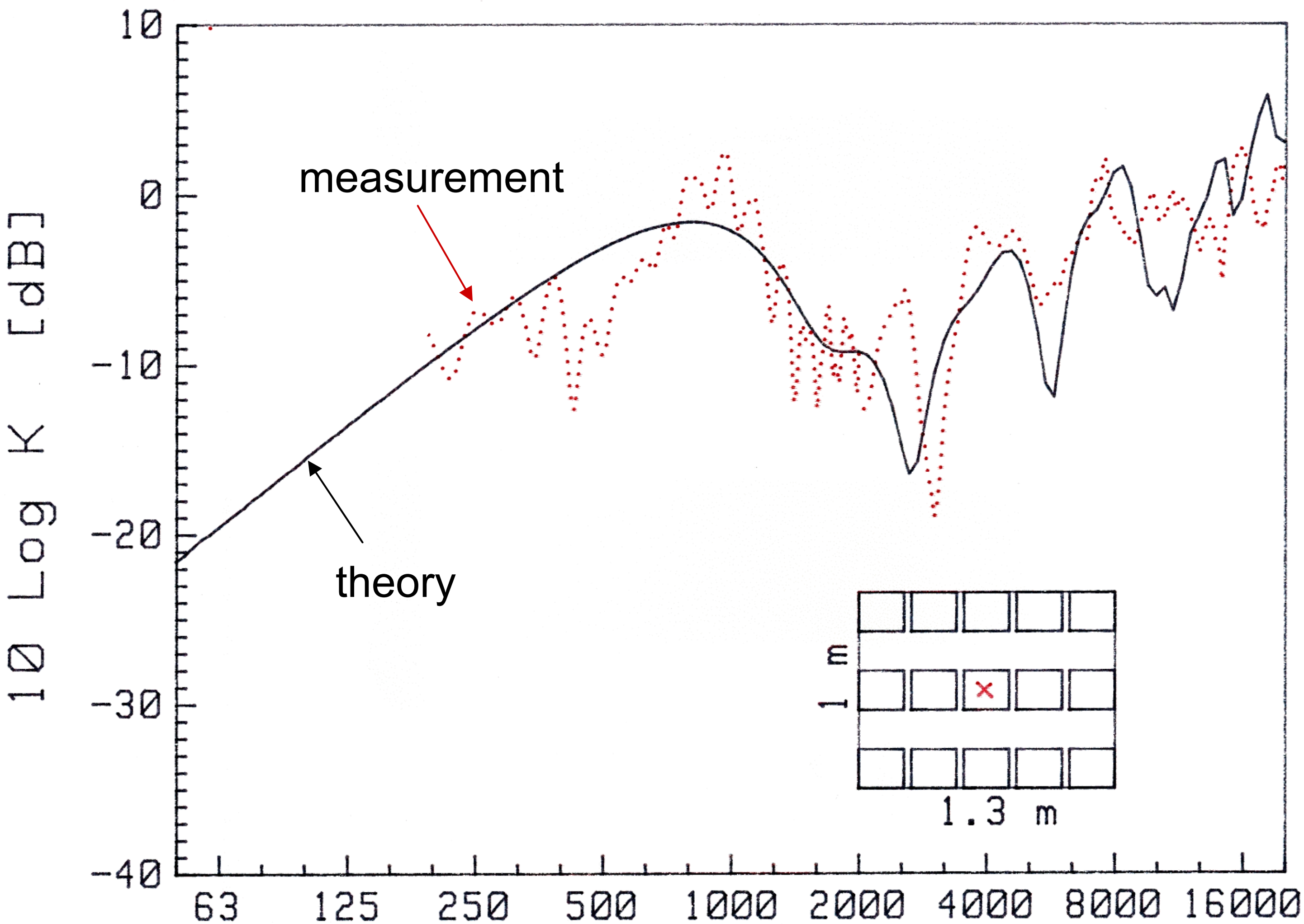


Plate dimensions: 0.23 m * 0.20 m, θ = 30°, a_1 = 3.0 m, a_2 = 2.0 m

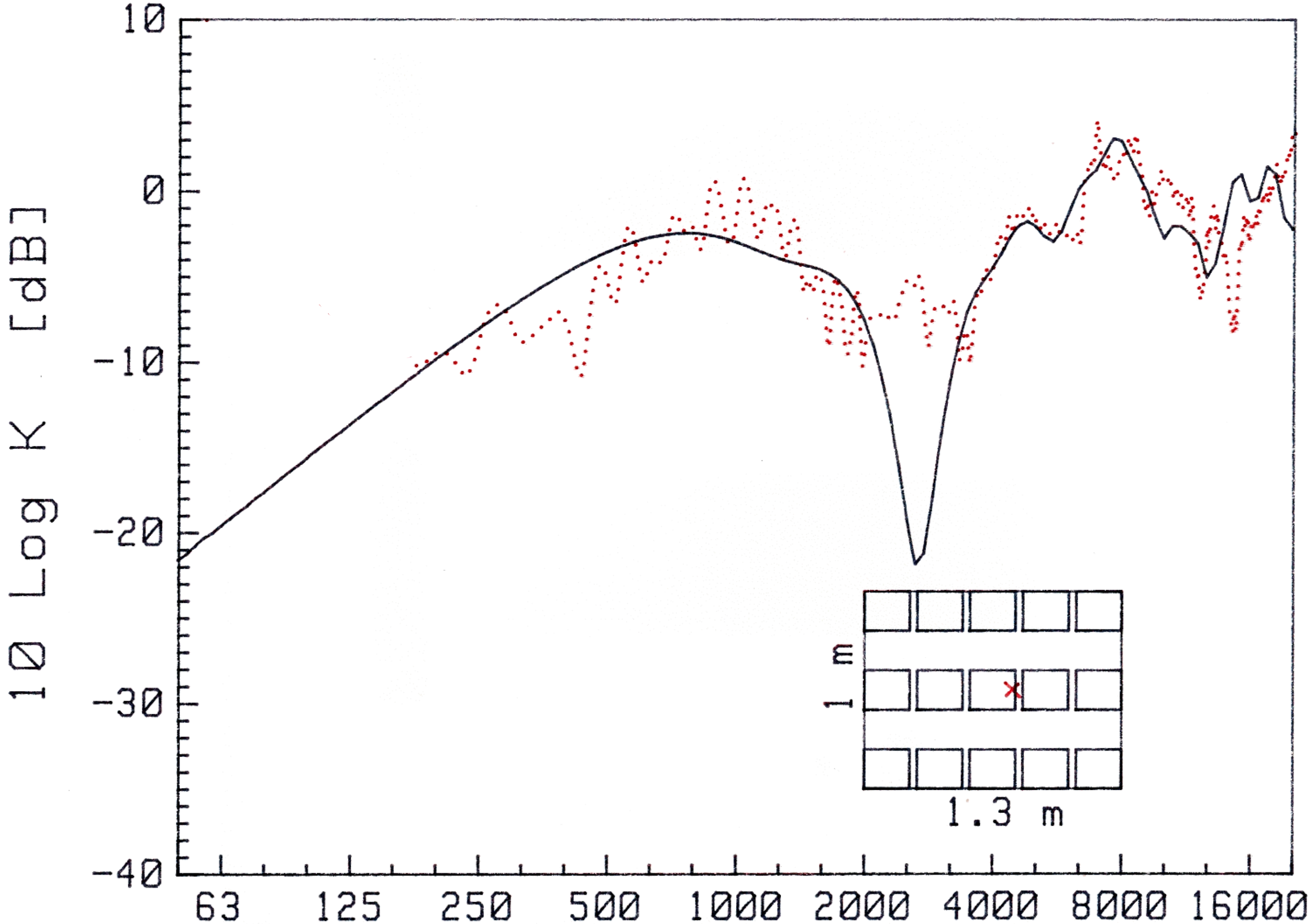


Plate dimensions: 0.23 m * 0.20 m, θ = 30°, a_1 = 3.0 m, a_2 = 2.0 m

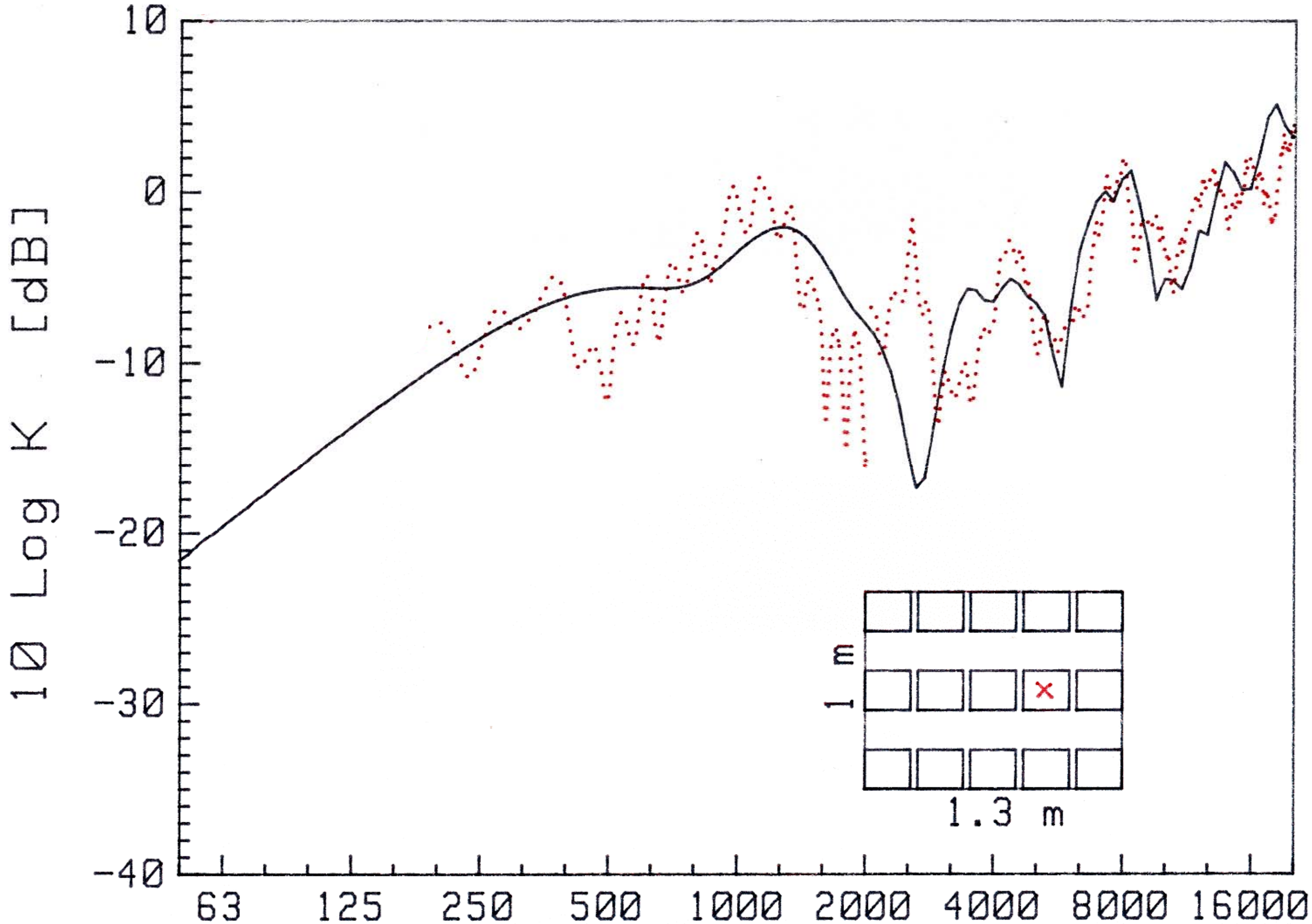


Plate dimensions: 0.23 m * 0.20 m, θ = 30°, a_1 = 3.0 m, a_2 = 2.0 m

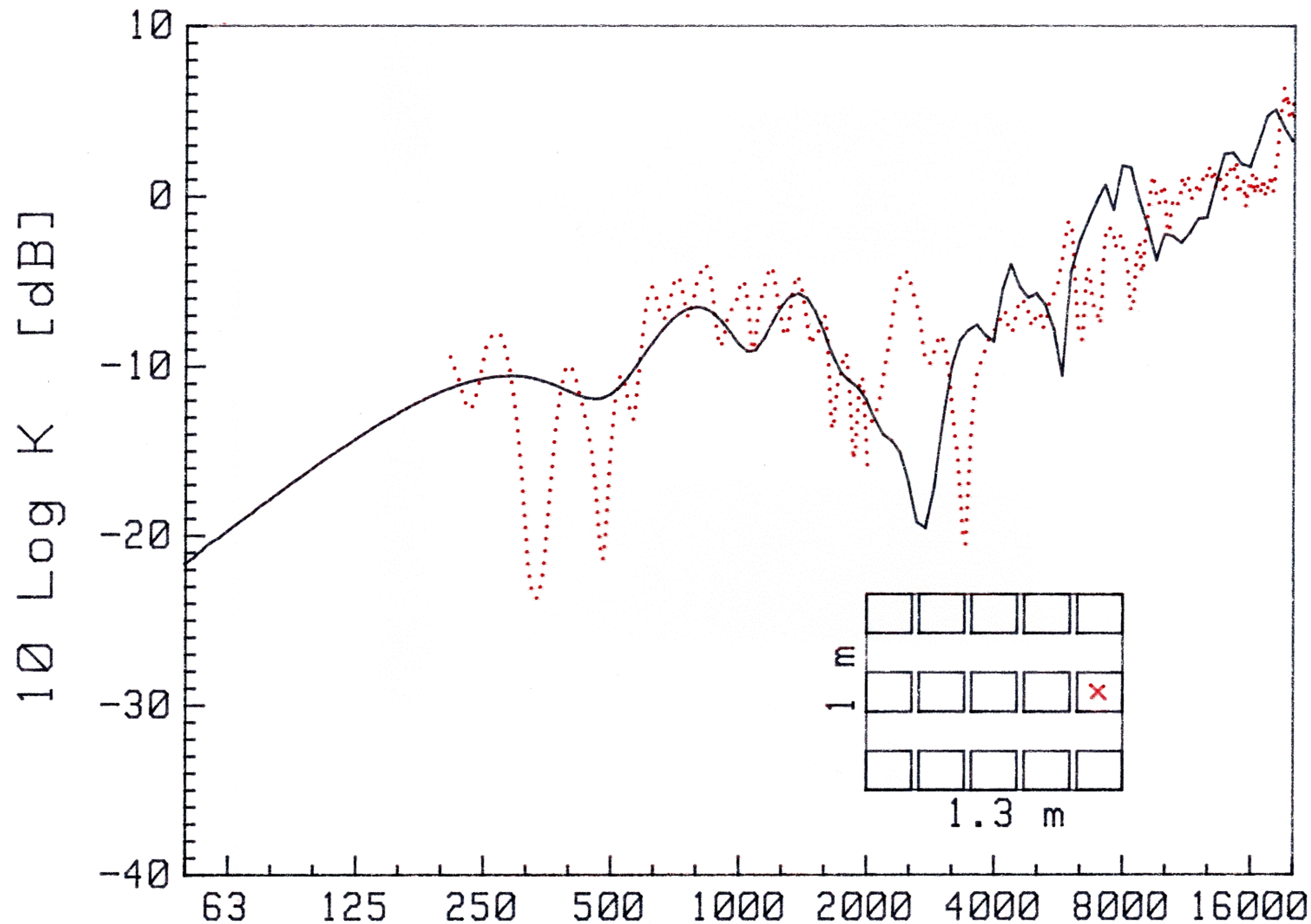
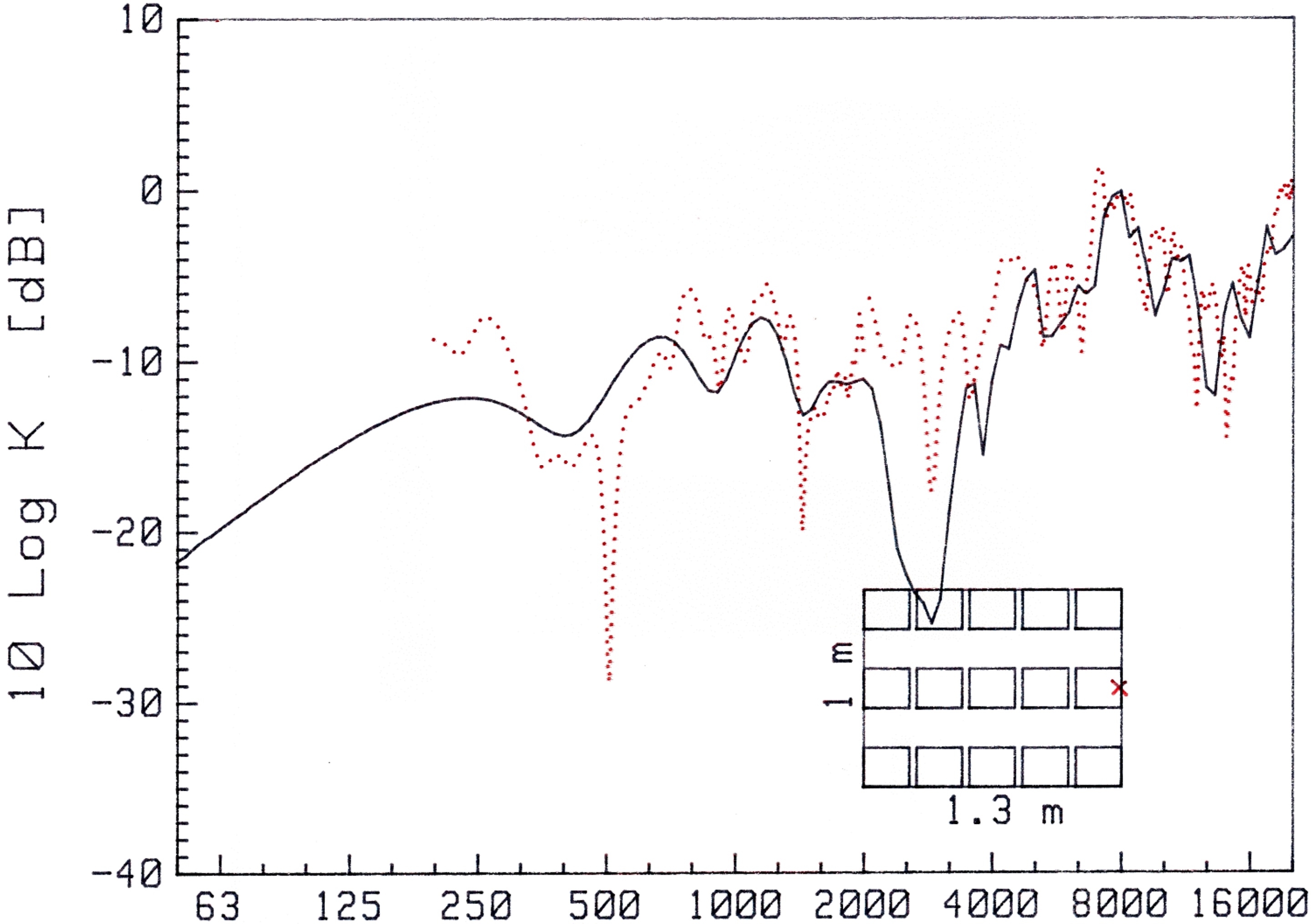
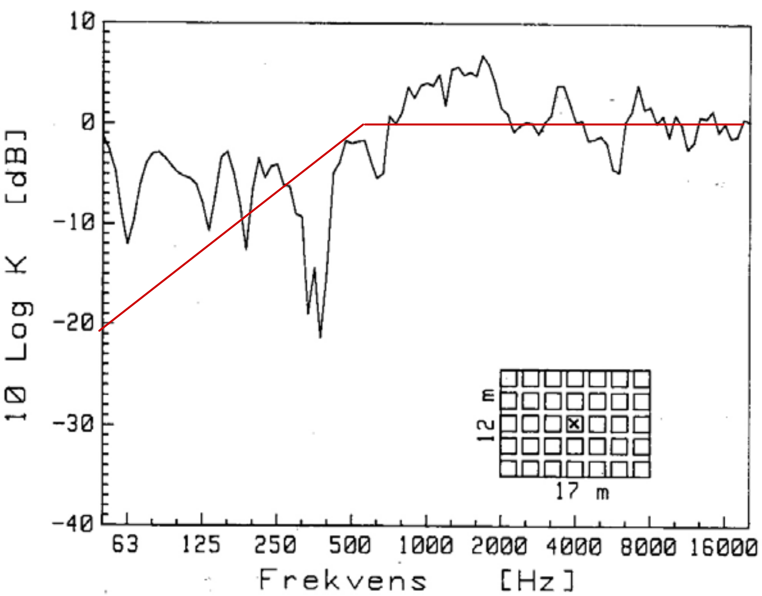


Plate dimensions: 0.23 m * 0.20 m, θ = 30°, a_1 = 3.0 m, a_2 = 2.0 m

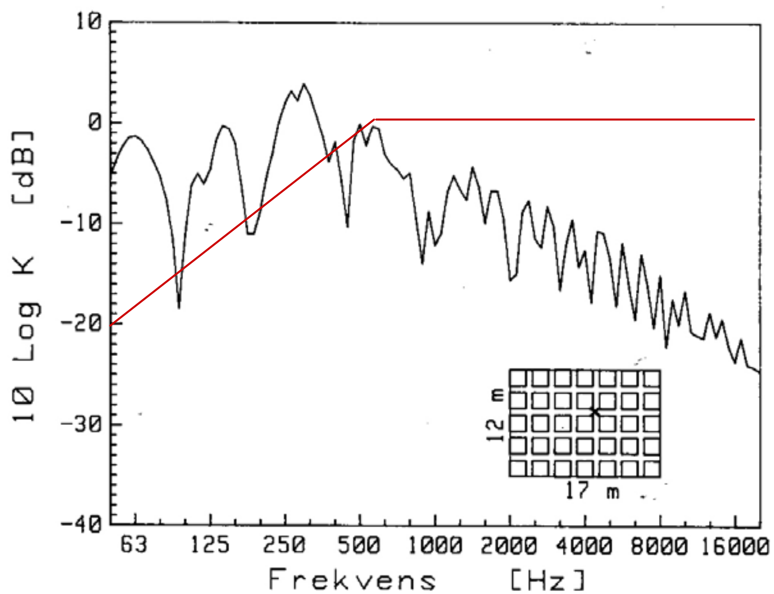


Reflector array - Parameter study

Best position



Worst position (near centre)

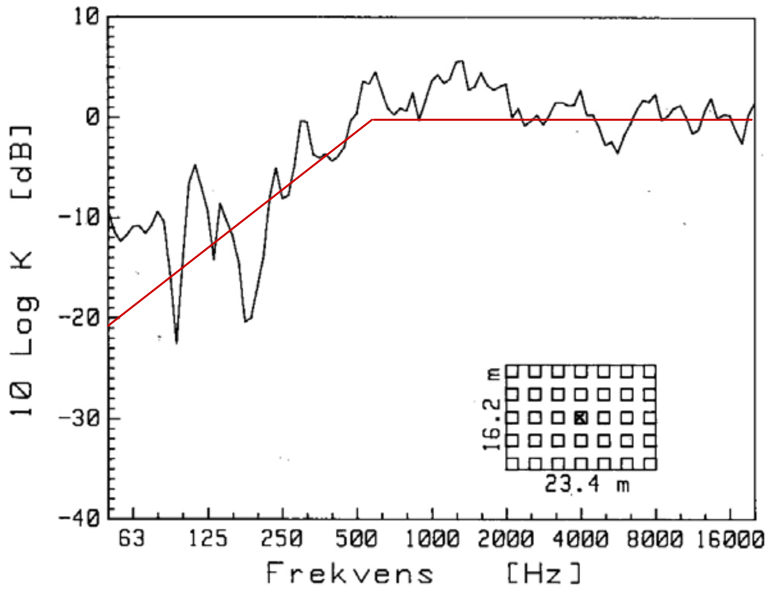


Panel size: 1.8 m * 1.8 m, angle of incidence = 45°, $a_1 = a_2 = 7.1$ m

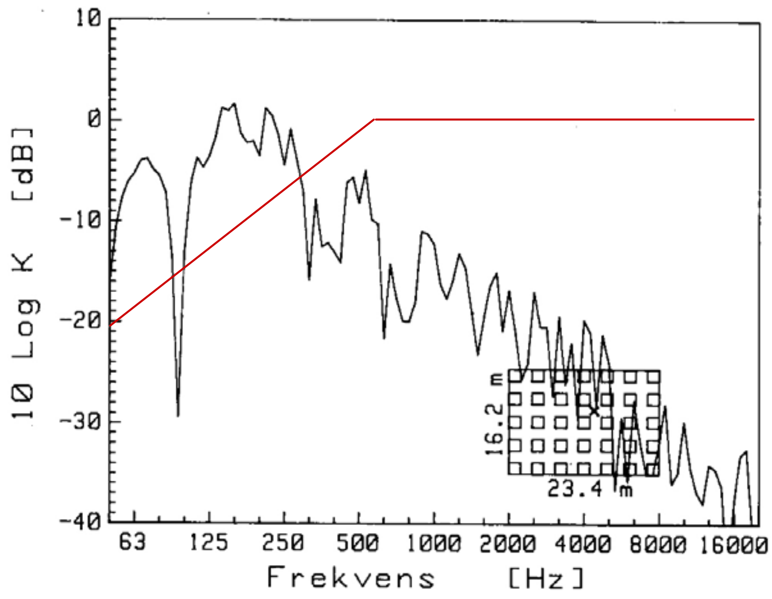
Density of array: μ = 50%

Reflector array - Parameter study

Best position



Worst position (near centre)

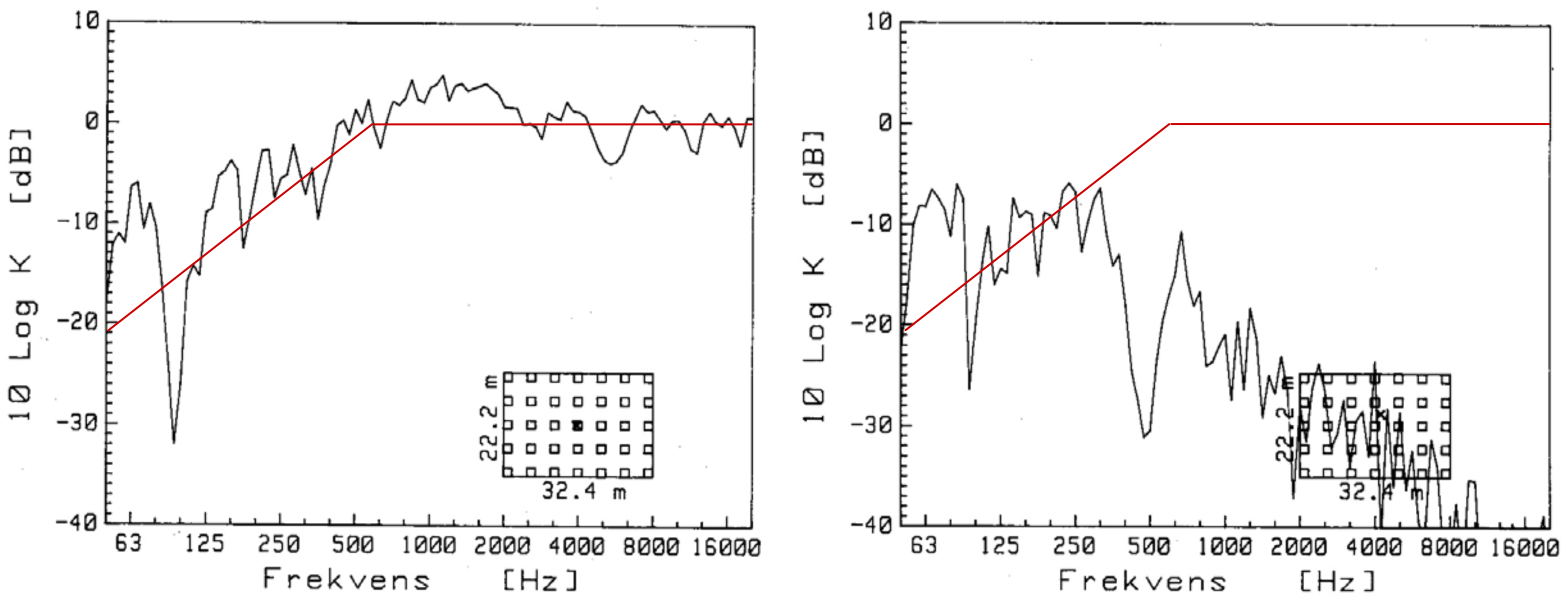


Panel size: 1.8 m * 1.8 m, angle of incidence = 45°, $a_1 = a_2 = 7.1$ m

Density of array: μ = 25%

Reflector array - Parameter study

Best position | Worst position (near centre)

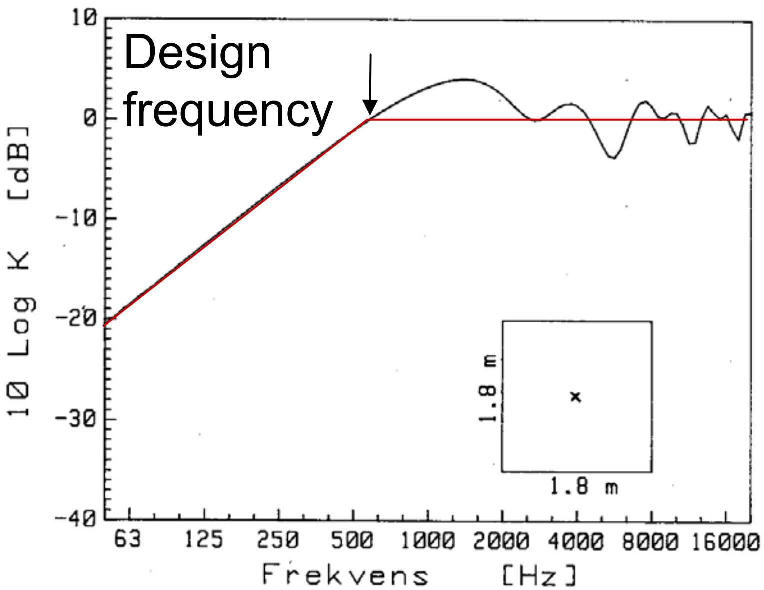


Panel size: 1.8 m * 1.8 m, angle of incidence = 45°, $a_1 = a_2 = 7.1$ m

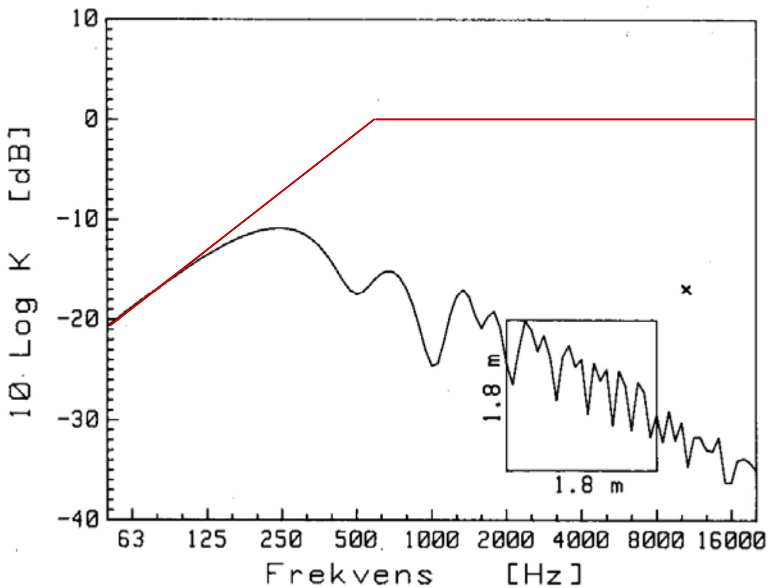
Density of array: μ = 12.5%

Reflector array - Parameter study

Best position



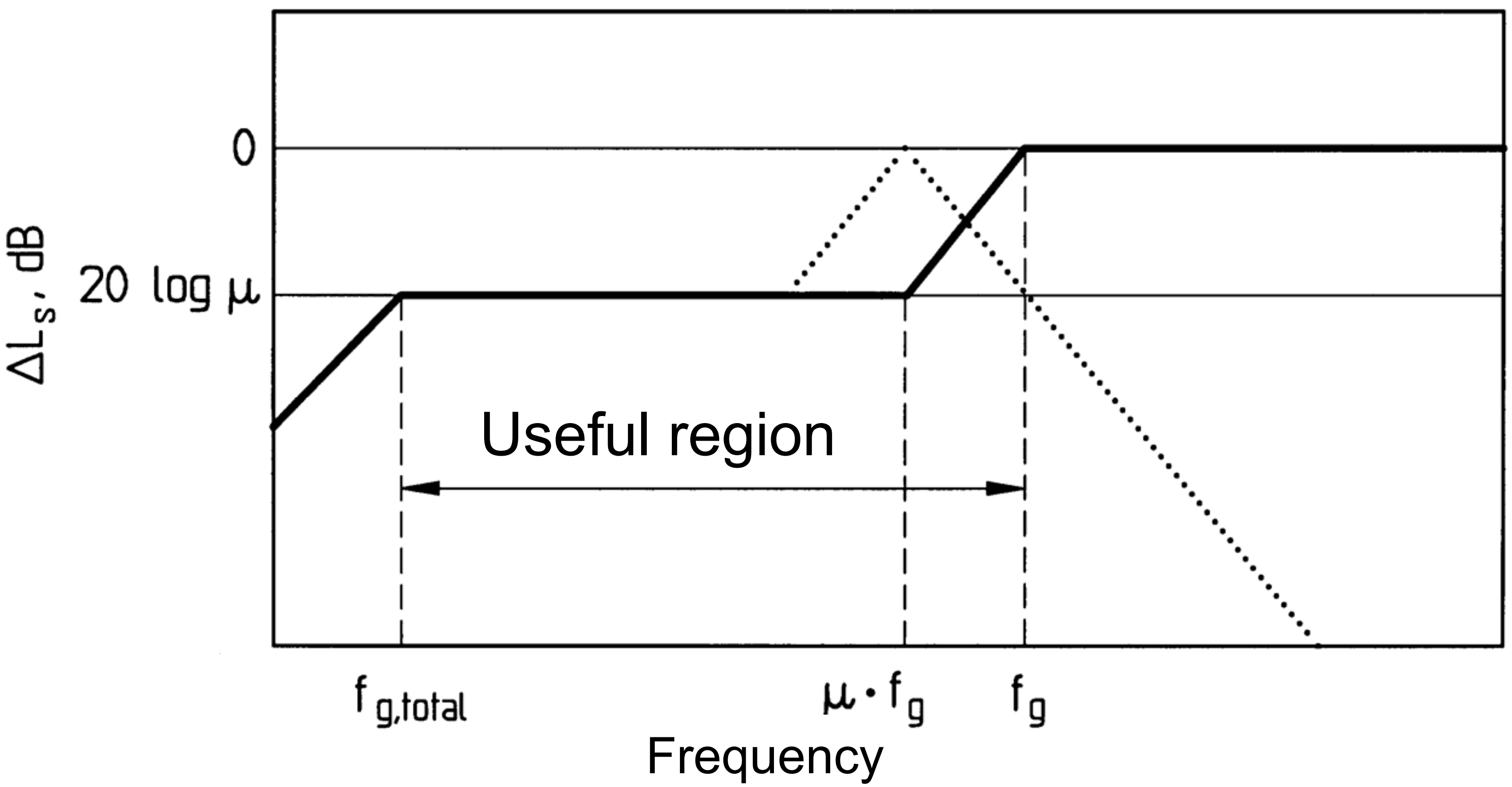
Worst position (near centre)



Panel size: 1.8 m * 1.8 m, angle of incidence = 45°, $a_1 = a_2 = 7.1$ m

One single plate, only

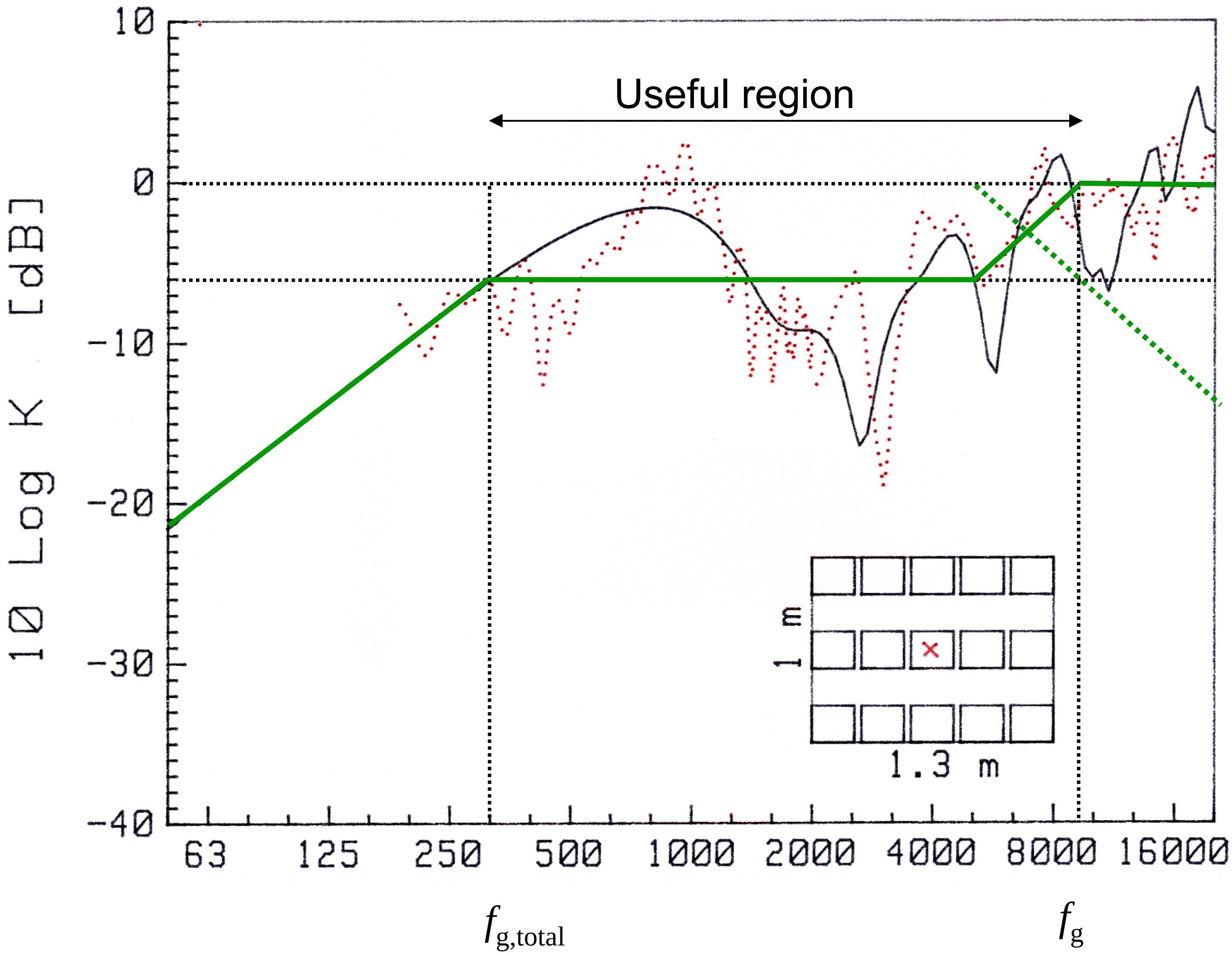
Reflector array – A design guide



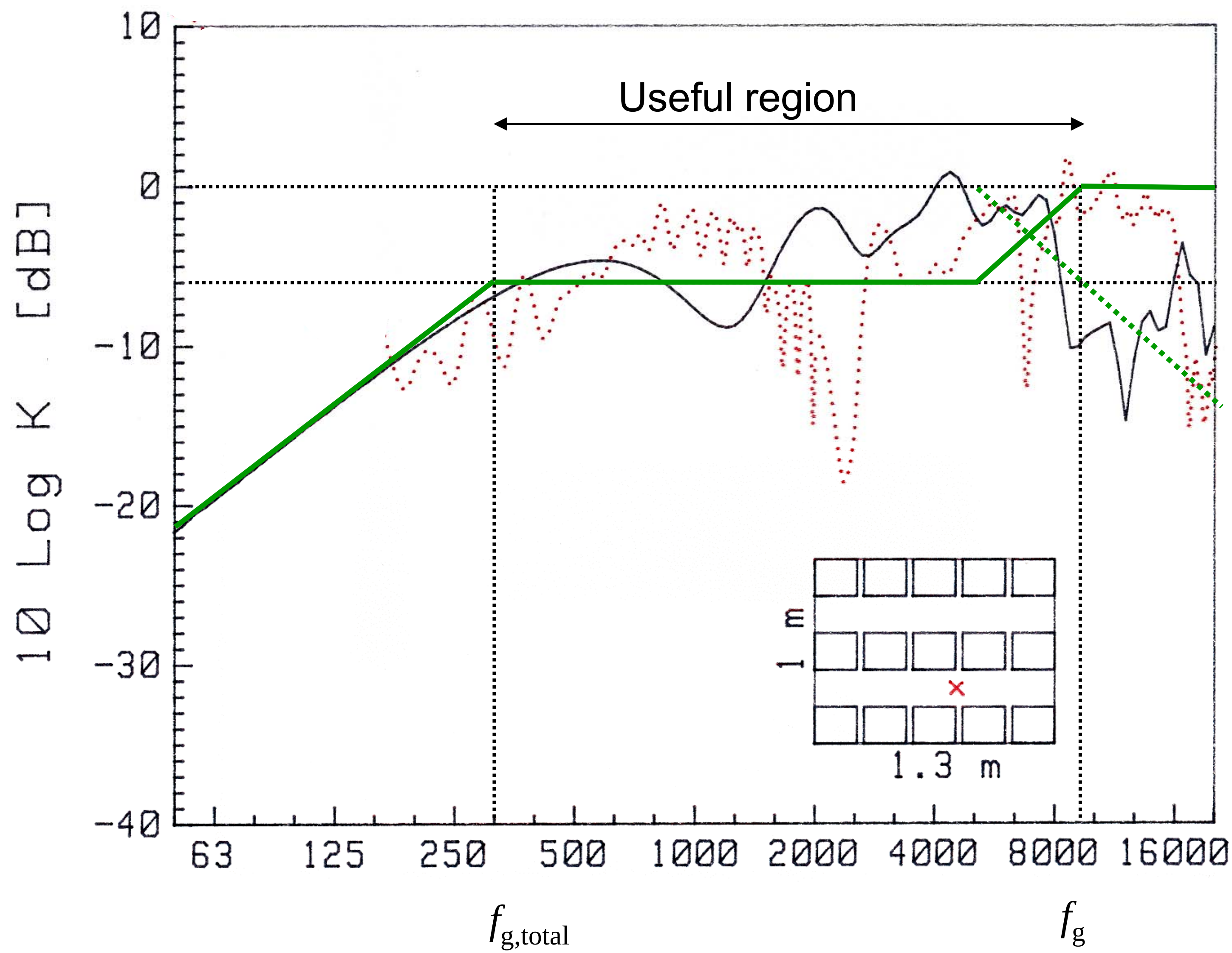
$\Delta L_s = 20 \log \mu \quad \text{(dB)}$

$$f_g = \frac{c\,a^*}{2\,S\cos\theta}$$

Design guide compared to *best* position



Design guide compared to *worst* position



Conclusion

- The *same design frequency* can be used for single reflectors and reflector arrays, but with opposite meaning:
- The useful range for single a reflector is *above* the design frequency
  - i.e. reflectors should be large
- The useful range for a reflector array is *below* the design frequency
  - i.e. reflectors in the array should be small

Example of application

- Danish Radio Concert Hall
  - Originally from 1945, but refurbished 1989 in order to improve the acoustic conditions for the musicians on stage
- Large reflectors introduced on the side walls of the stage
- New suspended reflector array with many small plates, slightly bent to avoid gaps between rows of reflectors

1991: Design of New Ceiling Reflectors for Improved Ensemble in a Concert Hall. Applied Acoustics 34, pp. 7-17.