Numerical modelling of multiple scattering between two elastical particles

Bjørnø, Irina; Jensen, Leif Bjørnø

Published in:
OCEANS '98 Conference Proceedings

Link to article, DOI:
10.1109/OCEANS.1998.724309

Publication date:
1998

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
Bjørnø, I., & Jensen, L. B. (1998). Numerical modelling of multiple scattering between two elastical particles. In OCEANS '98 Conference Proceedings (Vol. 2, pp. 598-602). IEEE. DOI: 10.1109/OCEANS.1998.724309

NUMERICAL MODELLING OF MULTIPLE SCATTERING BETWEEN TWO ELASTICAL PARTICLES

Irina BJØRNØ, Ph.D., and Leif BJØRNØ, professor

Dept. of Industrial Acoustics, Technical University of Denmark, DK-2800, Lyngby, Denmark

Abstract - **Multiple acoustical signal interactions with sediment particles in the vicinity of the seabed may significantly change the course of sediment concentration profiles determined by inversion from acoustical backscattering measurements. The scattering properties of high concentrations of sediments in suspension have been studied extensively since Foldy's formulation of his theory for isotropic scattering by randomly distributed scatterers. However, a number of important problems related to multiple scattering are still far from finding their solutions. A particular, but still unsolved, problem is the question of particle proximity thresholds for influence of multiple scattering in terms of particle properties like volume fraction, average distance between particles or other related parameters. A few available experimental data indicate a significance of multiple scattering in suspensions where the concentration is higher than 20 g/l of sand particles. This paper reports an attempt to illuminate and to solve the proximity threshold question, by an in-depth numerical study of the interaction of ultrasonic signals with two canonically shaped elastical particles. Introductory experimental results seem to create evidence for the applicability of this new numerical model.**

I. INTRODUCTION

The development of practical ultrasound backscattering methods for remote monitoring of suspended materials close to the seabed, as for instance, the measurement of suspended sediment concentrations has been reported elsewhere[1-2]. As the concentration of the suspension becomes high (experiments have shown values of 20 g/l for sand particles), particle interactions become important in the task of inversion of backscattered acoustical data to find the concentration[2]. As a first step in solving the problem of multi-target scattering, the scattering by two metal spheres has been studied both theoretically and experimentally. The main emphasis was put on the understanding of the influence of the relative position of the targets on the backscattered signals. The form function for single particle scattering was used to illuminate the changes in the course of the form function for two spheres due to the multiple scattering. The work has mainly been focused on the following issues:

(i) to develop a numerical model for scattering by two elastical spheres,

(ii) to perform experimental measurements of scattering by two spheres in a water tank, and

(iii) to compare the numerical results with measured data.

II. SCATTERING BY ELASTICAL SPHERES. A SIMPLIFIED THEORY.

Scattering by two elastic spheres has been investigated theoretically by several different approaches, as for instance an exact integral solution starting from the boundary condition definition [3] and a procedure based on the GTD approach[4]. In the present work, a simplified theory for two-sphere scattering is proposed based upon the **Resonance Scattering Theory** (RST) for single sphere scattering.

A. A general configuration of two-sphere scattering

A general configuration of two-sphere scattering is sketched in Fig.1, where the two spheres (A and B) have different sizes. But for simplicity, later in the discussions the two spheres will be assumed to have the same size.

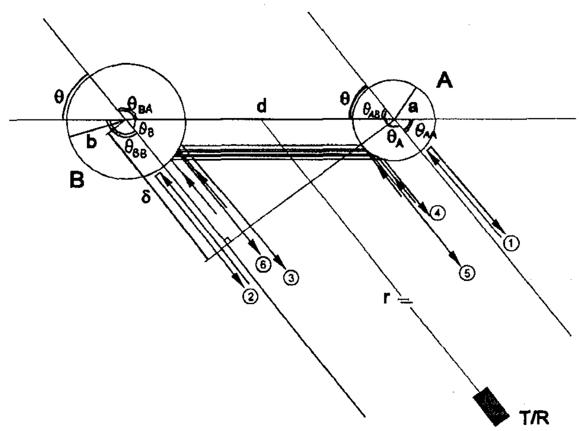

Fig. 1. Geometry of scattering by two spheres

Among the parameters shown in **Fig.1**, the distance between the geometrical centres of the two scattering spheres*d*, the angle of incidence for the scattering system, θ, and the radii of the two spheres, a and b, T/R is the transmitter/receiver. Using the definition of the form-function for the far-field, the scattering form-function for two spheres can be expressed as:

0-7803-5045-6/98/$10.00 ©1998 IEEE

$$f_\infty = f_\infty^A(\theta_A)\exp(i\varphi_A) + f_\infty^B(\theta_B)\exp(i\varphi_B) + f_\infty^A(\theta_{AB})\cdot f_\infty^B(\theta_{BB})\exp(i\varphi_{AB} + i\varphi_{BB}) + f_\infty^B(\theta_{BA})\cdot f_\infty^A(\theta_{AA})\exp(i\varphi_{BA} + i\varphi_{AA}), \quad (1)$$

where, $f_\infty^A(\theta_A)$ represents the backscattering form-function of sphere A, $\exp(i\varphi_A)$ is the phase shift corresponding to the backscattering from sphere A; $f_\infty^B(\theta_B)$ represents the backscattering form function of sphere B, $\exp(i\varphi_B)$ is the phase shift corresponding to the backscattering from sphere B; $f_\infty^A(\theta_{AB})$ represents the scattering form-function of sphere A in the direction towards the center of sphere B, $\exp(i\varphi_{AB})$ is the corresponding phase shift; $f_\infty^B(\theta_{BB})$ represents the scattering form-function of sphere B towards the *T/R* with an incidence from the center of sphere A, $\exp(i\varphi_{BB})$ is the corresponding phase shift; $f_\infty^B(\theta_{BA})$ represents the scattering form-function of sphere B in the direction towards the center of sphere A, $\exp(i\varphi_{BA})$ is the corresponding phase shift; and $f_\infty^A(\theta_{AA})$ represents the scattering form-function of sphere A towards the *T/R* with an incidence from the center of sphere B, $\exp(i\varphi_{AA})$ is the corresponding phase shift. The phase shifts are calculated using the following expressions:

$$\varphi_A = 2kr_A, \quad \varphi_B = 2kr_B, \quad \varphi_{AB} = k(d + r_A), \quad \varphi_{BB} = kr_B, \quad \varphi_{BA} = k(d + r_B), \quad \varphi_{AA} = kr_A, \quad (2)$$

where k is the wavenumber of an incident plane wave.

B. Contributions from the different spheres to the total form function

Physical interpretation of Eq.(1) can easily be carried out using the following methodology:

(1) When a plane wave insonifies the two sphere with an angle of incidence, θ, the major contribution of the two-sphere scatterer to the total backscattering form-function is the sum of the backscattering form-functions of the two individual spheres, which are given in Eq.(1) by $f_\infty^A(\theta_A)\exp(i\varphi_A) + f_\infty^B(\theta_B)\exp(i\varphi_B)$;

(2) The sphere A also scatters energy in other directions, apart from the backward direction. The scattering of the incident energy by sphere A to sphere B is given by $f_\infty^A(\theta_{AB})\exp(i\varphi_{AB})$ in Eq.(1), and this scattered energy is then used as the incident energy for sphere B which, in turn, scatters it to all directions including the direction towards the *T/R* which is given in Eq.(1) by $f_\infty^B(\theta_{BB})\exp(i\varphi_{BB})$. The total effect of this two-step scattering leads to a contribution $f_\infty^A(\theta_{AB})\cdot f_\infty^B(\theta_{BB})\exp(i\varphi_{AB} + i\varphi_{BB})$ to the total scattering form-function;

(3) The sphere B also scatters energy in other directions, apart from the backward direction. The scattering of incident energy by sphere B towards sphere A is given by $f_\infty^B(\theta_{BA})\exp(i\varphi_{BA})$ in Eq.(1), and this scattered energy is then forming the incident energy for sphere A which in turn scatters it in all directions including the direction towards the *T/R*. This contribution is given in Eq.(1) by $f_\infty^A(\theta_{AA})\exp(i\varphi_{AA})$. The total effect of this two-step scattering leads to a contribution $f_\infty^B(\theta_{BA})\cdot f_\infty^A(\theta_{AA})\exp(i\varphi_{BA} + i\varphi_{AA})$ to the total scattering form-function.

Note that this four-terms scattering procedure describes the contributions of two sphere scattering to the total backscattering form-function up to first-order interactions between the two spheres. The advantages of this procedure are several: (1) This method can easily be extended to three- or more sphere scattering ; (2) Each individual sphere is considered separately, therefore, the introduction of spheres with different sizes does not add any additional difficulties to the calculations; (3) Due to the fact that the scattering pattern is used in the calculation of the scattered field, different shapes of individual scatterers may be considered; (4) Higher order interactions between the two spheres may be included accordingly.

The disadvantage of this procedure is that far-field is always assumed in the calculation which makes the procedure inaccurate for small distances between the scattering objects and T/R.

III. NUMERICAL RESULTS ON TWO-SPHERE SCATTERING. COMPARISON WITH THE EXPERIMENTS.

Based upon the results of simplified theory for scattering by two elastical spheres, numerical calculations were performed for different angles of incidence(Θ = 0°, 45°, and 90°) and different separation distances (d=2a, 4a, 8a, and 16a). The main parameters used in the numerical calculations for water and stainless steel sphere are: ρ_f=998 kg/m^3, c_f=1475 m/s, ρ_s=7930 kg/m^3, c_p=5980 m/s, and c_s=3300 m/s. As an example for numerical calculations, **Fig. 2** a-c show the results of numerical calculations for Θ=0°, d=4a (**Fig. 2a**), 8a (**Fig. 2b**), 16a (**Fig. 2c**) - solid line. The short dashed line shows the form-function for a single sphere; the long dashed line shows the double form-function of a single sphere.

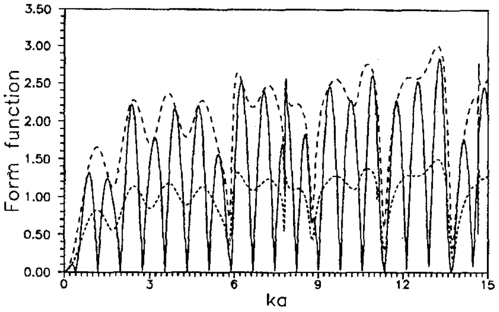

Fig. 2 a.

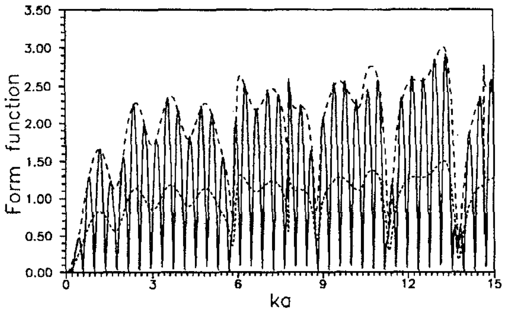

Fig. 2b.

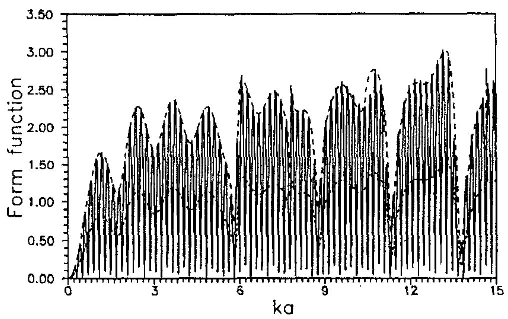

Fig. 2c.

A. General remarks on the backscattering form function for two spheres

Based upon the numerical calculations of backscattering form-function for two spheres with different separations between the spheres and different angles of incidence, a few conclusion may be appropriate: (1) Due to the coherent addition of the two signals backscattered from the two spheres, the curve of the form-function for two spheres is a modulation of that for a single sphere, which gives peaks and troughs in the form-function curve; (2) The envelope of the two sphere scattering form-function is about the same as twice the single sphere scattering form-function, and the form-function curve for a single sphere looks like the "mean" curve of that for two spheres. Due to the interaction between the two spheres, the envelope of the form-function for two sphere scattering differs from the curve corresponding to twice the single sphere scattering form-function; (3) The period of change of peaks and troughs is dependent on both **the separation** between the two spheres and **the angle of incidence**; (4) At 90° incidence, the two backscattered signals from the two spheres are in phase, therefore, the form-function in this case is almost the same as twice of the single sphere scattering form-function. The difference is due to the interaction between the two spheres.

B. Experimental set-up.

The measurements of scattering by two stainless steel spheres (both of radius a) were performed in a water tank of dimensions $1.5\times0.5\times0.5$ m^3. The spheres were insonified by a short pulse of approximately 5 μs duration, which was transmitted/received by a transceiver (T/R) operated at a central frequency of 546 kHz (Panametrics A301S). The distance between the (T/R) and the geometrical center of the two spheres was chosen to 34 cm, which is clearly in the far-field of the transceiver (Rayleigh distance: 18 cm). The 3-dB opening angle of the main beam of the transducer was 4°, and the 3-dB width of the beam at the distance of 34 cm was about 4,5 cm. During the measurements both spheres must be placed within the main lobe of the transceiver and, therefore, the separation between the two spheres in the experiment was chosen not to be larger than 4.5 cm. The backscattered signals were received by the T/R, amplified, filtered by the high-pass filter, digitized via the IEEE-488 interface, and recorded by a PC.

The following parameters were changed during the measurements. The separation (d) between the two spheres; the angle of incidence (θ) which was defined as the angle between the line connecting the two spheres and the beam axis of the transceiver, and the distances ($r_{1,2}$) between the geometrical centers of the two spheres (A and B) and the center of the transceiver (T/R).

C. Experimental results.

First, the dependence on the angle of ultrasound incident on to the two spheres was examined experimentally for θ equal to 0°, 45° and 90°. The form-function of a single sphere was used as a reference to indicate the changes in the course of the form-function of two spheres. The two spheres were having the same radius of 2.5 mm. The distance between the two spheres was 20 mm. Second, the dependence of the multiple scattering amplitude on the separation between the two sphere centers (d) was studied. Scattering form function curves for two spheres (radius a = 2.5 mm) at 0° incidence, with different separations (d) between the spheres (d = 4a, 8a, 12a) were experimentally investigated. For comparison, the scattering form-function for a single sphere of the same size (a = 2.5 mm) was also measured (See experimental results on **Fig. 3 a-c**).

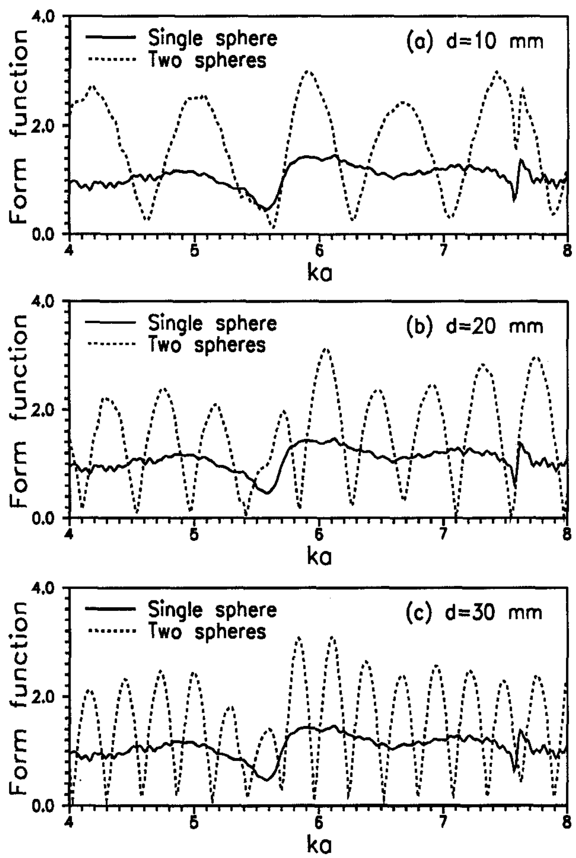

Fig. 3. Scattering form-function for θ=0^0, a=b=2,5 mm.

It was clearly found that for all angles of incidences and for all separation distances, the form functions of the two sphere set-up have similar periodical structure of the envelope as the form-function of a single sphere having the same size , but the amplitude of the form-function for two spheres was almost doubled compared with the single sphere form-function. The period of the amplitude variation of form-function for two spheres was a function of the position of the spheres relative to the transceiver. The change of the amplitude was clearly periodical, and the period was found to be a function of the phase delay between two signals arriving from different scatterers. This phase difference can easily be calculated as (see for instance[5,6]):

$$\delta = 2(r_2 - r_1) \approx 2d\cos\theta \quad (r >> d), \tag{3}$$

and the period in the frequency domain in this case will then be:

$$\Delta f = c/\delta = c/2d\cos\theta \;\Rightarrow\; \Delta ka = \pi/D\cos\theta,\; D = d/a. \tag{4}$$

The measured and calculated values for periods of the form-function for the two sphere scattering set-up was in excellent agreement for d > 8a.

IV.CONCLUSIONS

The detailed numerical and experimental investigations of scattering from two spheres allow us to conclude, that the backscattering by two spheres of the same size can be described as a superposition of two types of interactions: (1) the interference between two backscattered signals one from each individual sphere considered as separate scatterers, and (2) the contribution from real multiple scattering. When the multiple scattering term can be neglected (which is the case for d > 8a, approximately, as indicated by our experiments), the interaction between the two spheres can be calculated using the phase delay approach in agreement with equations (3-4). In practice it means that a model consisting of many particles of equal size with an average separation distance of d = 4a represents a suspension of sediments with a concentration of 7% by volume, while the same model, but with a particle separation of d = 10a is equivalent to a suspension with 0.1% concentration by volume, or 3 - 4 g/l by weight. During the sea experiments it was clearly shown[1,2] that a volume concentration of sediments of 10%, in fact, forms the real limit for multiple scattering in suspensions even in the presence of maximum flow near the seabed. This means that the traditional model of sediment suspensions consisting of a number of individual particles with simple corrections for phase shift in the backscattering calculation still forms the most realistic model when the average separation distance between the particles d > 8a. Then multiple scattering need not to be considered.

ACKNOWLEGMENT

This work was supported by European Community, under contract MAS3-CT95-0017 (TRIDISMA), which is gratefully acknowledged. Thanks are also due to Shaobo Sun who helped to collect and manage the experimental data.

REFERENCES

[1]. Hay,A.E., Merseder, D.G., *JASA*, **78**, 1761-1771 (1985).
[2]. Thorne, P.D., Campbell, S.C., *JASA*, **92**, 978-986 (1992).
[3] G. C. Gaunaurd and H. Huang, "Acoustic scattering by a pair of spheres," *J. Acoust. Soc. Am.*, **98**, 495-507 (1995).

[4]A. Folacci, J.-L- Rossi, and J.-P. Sessarego, "GTD Analysis of scattering by two spheres," *ACUSTICA · acta acustica*, **83**, 93-104 (1997).

[5] Guo, G., Bjørnø, I.K. and Bjørnø, L. "Single particle scattering: Effects of the shape and geometry. Experiment," in *Second Scientific and Technical Progress Report SMART/ISUSAT*, Sub-report 5 (1994).

[6]. Sun, S., Thorne, P.D., Bjørnø, I. and Mazoyer, T. "Observations of acoustic backscattering by elastic cubes," in Proceedings of the *3rd European Conference on Underwater Acoustics*, edited by J. S. Papadakis , Crete University Press, Heraklion, 1996, pp. 51-56