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μ -Synthesis for the Coupled Mass Benchmark Problem¹

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Abstract

A robust controller design for the coupled mass benchmark problem is presented in this paper. The applied design method is based on a modified D - K iteration, i.e. μ -synthesis which take care of mixed real and complex perturbations sets. This μ -synthesis method for mixed perturbation sets is a straightforward extension of the standard D - K iteration for complex perturbation sets.

1. Introduction

A large number of papers deals with design of robust controllers for the ACC-90 and ACC-91 benchmark problem, [WB90], see the papers and their references in the special issue of Journal of Guidance, Control and Dynamics, e.g. [WB92].

The benchmark problem includes 3 real parameter uncertainties, see the description below. As a consequence of this, the standard μ -synthesis which handle only complex perturbations has not been studied widely for the benchmark problem, in fact only in [BM92]. The reason is that controllers designed by using the standard μ -synthesis method will in general be too conservative when some of the perturbations are real. The results in [BM92] confirm this. The conservatism in using complex uncertainties instead of real uncertainties was 67%. Some new μ -synthesis methods have been derived in the last years, [Hel95], [You94] and [TCASN95]. The method derived by Young [You94] is an approximate solution to the mixed μ upper bound problem, denoted D , G - K iteration. D , G - K iteration is, in principle, a straightforward extension of the D - K scheme to mixed perturbations. Unfortunately, the mathematics required to do this becomes very messy and the current version of the MATLAB μ toolbox [BDG⁺93] cannot readily be applied to perform the iteration.

Recently, Helmersen [Hel95] has proposed a different approach to mixed μ synthesis, denoted W - K iteration, where asymmetric multipliers rather than symmetric scalings are used to formulate an upper bound problem. This approach looks promising.

Another approach has been derived by Tøffner-Clausen et al. [TCASN95] which is also a straightforward extension of the D - K iteration. This approach, denoted μ - K iteration, seems to be more easily applied to real problems than the D , G - K iteration. Further, the MATLAB μ toolbox can directly be applied to perform the

iteration. Preliminary results on mixed μ - K iteration can be found in [TCASN94a], [TCASN94b], and in [TCASN95].

The key result in this paper is to show how this μ - K iteration can be applied to the benchmark problem with mixed perturbation sets and compare the results for controllers designed by using the standard D - K iteration for complex perturbations.

The rest of this paper is organized as follows. In Section 2, the benchmark problem is shortly introduced followed by a short description of the modified μ - K iteration method in Section 3. Section 4 includes the design example. A conclusion is given in Section 5.

2. The Benchmark Problem

A complete description of the benchmark problem can be found in [WB92]. The state space description of the two mass/spring system is represented by:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & 0 & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{bmatrix} w$$

$$y = x_2 + v$$

$$z = x_2 \tag{1}$$

where x_1 and x_2 are the position of body 1 and 2, respectively; x_3 and x_4 are the velocities of body 1 and body 2, respectively; u is the control input acting on body 1; y is the measurement signal; w is the disturbance acting on body 2; v is sensor noise; z is the output to be controlled; k is the spring constant; m_1 is the mass of body 1; m_2 is the mass of body 2.

The design problem for the benchmark problem is defined by, [WB92]:

Problem 1 Design a compensator with the following properties:

1. Maximize the stability margin with respect to the three uncertain parameters m_1 , m_2 and k whose nominal values are $m_1 = m_2 = k = 1$
2. For a unit impulse disturbance exerted on body 1 and/or body 2, the controlled output $z = x_2$, has a settling time of about 15s for the nominal system with $m_1 = m_2 = k = 1$.
3. The closed-loop system is insensitive to high-frequency sensor noise.
4. Reasonable performance/stability robustness and reasonable gain/phase margins are achieved with reasonable bandwidth.

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5. Because of finite actuator response time, the controller bandwidth must be ≤ 50 rad/s.
6. The control input $u(t)$ should be reasonable.
7. The number of controller states should be reasonable.

3. Mixed μ Synthesis

A short description of the modified μ - K iteration for mixed perturbation sets is given in this section after a description of μ . The technical details of this μ - K iteration can be found in [TCASN94a], [TCASN94b] and in [TCASN95].

3.1. The Structured Singular Value

The definition of the structured singular value, μ , is dependent on the underlying block structure of the perturbations which is defined as follows. For simplicity we assume in the following that $M \in \mathcal{C}^{n \times n}$ is square. Given M and three non-negative integers m_r , m_c and m_G with $m = m_r + m_c + m_G \leq n$, the block structure $\mathcal{K}(m_r, m_c, m_G)$ is an m -tuple of positive integers:

$$\mathcal{K}(m_r, m_c, m_G) = (k_1, \dots, k_{m_r}, k_{m_r+1}, \dots, k_{m_r+m_c}, k_{m_r+m_c+1}, \dots, k_m)$$

where we require that $\sum_{i=1}^m k_i = n$ so these dimensions are compatible with M . This now defines the set of allowable perturbations, namely

$$\Delta_{\mathcal{K}} = \left\{ \text{diag} \left(\delta_1^r I_{k_1}, \dots, \delta_{m_r}^r I_{k_{m_r}}, \delta_1^c I_{k_{m_r+1}}, \dots, \delta_{m_c}^c I_{k_{m_r+m_c}}, \Delta_1, \dots, \Delta_{m_G} \right) \mid \delta_i^r \in \mathcal{R}, \delta_i^c \in \mathcal{C}, \Delta_j \in \mathcal{C}^{k_{m_r+m_c+j} \times k_{m_r+m_c+j}} \right\}$$

Note that $\Delta_{\mathcal{K}} \subset \mathcal{C}^{n \times n}$ and that this block structure is sufficiently general to allow for repeated real scalars, repeated complex scalars, and full complex blocks. The purely complex case corresponds to $m_r = 0$ and the purely real case to $m_c = m_G = 0$. Define also the corresponding complex perturbation set $\Delta_{\mathcal{K}}^c$ as:

$$\Delta_{\mathcal{K}}^c = \left\{ \text{diag} \left(\delta_1^c I_{k_1}, \dots, \delta_{m_r+m_c}^c I_{k_{m_r+m_c}}, \Delta_1, \dots, \Delta_{m_G} \right) \mid \delta_i^c \in \mathcal{C}, \Delta_j \in \mathcal{C}^{k_{m_r+m_c+j} \times k_{m_r+m_c+j}} \right\}$$

which we will use in connection with mixed μ synthesis. The structured singular value, μ , is then defined as follows.

Definition 1 (Structured singular value)

The structured singular value, $\mu_{\Delta}(M)$, of a matrix $M \in \mathcal{C}^{n \times n}$ with respect to a block structure $\mathcal{K}(m_r, m_c, m_G)$ is defined as

$$\mu_{\Delta}(M) \triangleq \frac{1}{\min \{ \bar{\sigma}(\Delta) : \Delta \in \Delta_{\mathcal{K}}, \det(I - M\Delta) = 0 \}} \quad (2)$$

unless no $\Delta \in \Delta_{\mathcal{K}}$ makes $I - M\Delta$ singular, in which case $\mu_{\Delta}(M) \triangleq 0$.

Unfortunately, (2) is not suitable for computing μ since the implied optimization problem may have multiple local minima [DP87, FTD91]. However, upper and lower bound for μ may be effectively computed. The upper and lower bound theory for μ relies on the definition

of some sets of block diagonal matrices (which are also dependent on the underlying block structure of the perturbations).

$$\mathbf{Q} = \left\{ Q \in \Delta_{\mathcal{K}} \mid \delta_i^r \in [-1, 1], \delta_i^c \delta_i^c = 1, \Delta_j^* \Delta_j = I_{k_{m_r+m_c+j}} \right\}$$

$$\mathbf{D} = \left\{ \text{diag} (D_1, \dots, D_{m_r+m_c}, d_1 I_{k_{m_r+m_c+1}}, \dots, d_n I_{k_m}) \mid D_i \in \mathcal{C}^{k_i \times k_i}, D_i^* = D_i > 0, d_j \in \mathcal{R}, d_j > 0 \right\}$$

$$\mathbf{G} = \left\{ \text{diag} (G_1, \dots, G_{m_r}, O_{k_{m_r+1}}, \dots, O_{k_m}) \mid G_i \in \mathcal{C}^{k_i \times k_i}, G_i = G_i^* \right\}.$$

Define

$$\beta_{\min} = \inf_{\beta \in \mathcal{R}_+, G \in \mathbf{G}, D \in \mathbf{D}} \left\{ \beta \mid \bar{\sigma} \left[\left(\frac{DMD^{-1}}{\beta} - jG \right) (I + G^2)^{-\frac{1}{2}} \right] \leq 1 \right\}$$

Then

$$\max_{Q \in \mathbf{Q}} \rho(QM) \leq \mu_{\Delta}(M) \leq \beta_{\min}. \quad (3)$$

For purely complex perturbation sets ($m_r = 0$), the bounds in (3) reduce to

$$\max_{Q \in \mathbf{Q}} \rho(QM) \leq \mu_{\Delta}(M) \leq \inf_{D \in \mathbf{D}} \bar{\sigma}(DMD^{-1}) \quad (4)$$

3.2. μ Synthesis

We may now formulate an optimal robust performance problem in terms of μ :

$$K(s) = \arg \min_{K(s) \in \mathbf{K}} \|\mu_{\Delta}(F_l(N(s), K(s)))\|_{\infty} \quad (5)$$

where \mathbf{K} denotes the set of all nominally stabilizing controllers (there might not exist an admissible controller achieving the minimum, but we make this abuse of notation for convenience). Note that $F_l(N(s), K(s)) = M(s)$, see also Figure 1. Unfortunately (5) is not tractable since μ cannot be directly computed. Rather the upper bound β_{\min} is used to formulate the control problem:

$$K(s) = \arg \min_{K \in \mathbf{K}} \sup_{\omega} \inf_{D(\omega) \in \mathbf{D}, G(\omega) \in \mathbf{G}} \inf_{\beta(\omega) \in \mathcal{R}_+} \left\{ \beta(\omega) \mid \bar{\sigma}(\Sigma(\omega)) \leq 1 \right\} \quad (6)$$

where

$$\Sigma(\omega) = \left(\frac{D(\omega) F_l(N(j\omega), K(j\omega)) D^{-1}(\omega)}{\beta(\omega)} - jG(\omega) \right) (I + G^2(\omega))^{-\frac{1}{2}}.$$

For purely complex perturbations, the control problem reduce to

$$K(s) = \arg \min_{K \in \mathbf{K}} \sup_{\omega} \inf_{D \in \mathbf{D}} \left\{ \bar{\sigma}(D(\omega) F_l(N(j\omega), K(j\omega)) D^{-1}(\omega)) \right\}. \quad (7)$$

The control problems (6) and (7) are both *scaled \mathcal{H}_{∞} optimization problems*. Scaled \mathcal{H}_{∞} optimizations have recently been an area of intensive research within the automatic control community. However, no solution to (6) or (7) has yet been found. Rather iterative approximate solution procedures have been developed for both purely complex and mixed perturbation sets.

3.2.1 Complex μ Synthesis / D - K Iteration:

An approximation to complex μ synthesis can be made by the following iterative scheme, usually denoted D - K iteration. For a fixed controller $K(s)$, the problem of finding $D(\omega)$ at a set of chosen frequency points ω is just the complex μ upper bound problem which is a convex problem with known solution. Having found these scalings we may fit a real rational stable minimum phase transfer function matrix $D(s)$ to $D(\omega)$ by fitting each element of $D(\omega)$ with a real rational stable minimum phase SISO transfer function. We may impose the extra constraint that the approximations $D(s)$ should be minimum phase (so that $D^{-1}(s)$ is stable too) since any phase in $D(s)$ is absorbed into the complex perturbations. For a given magnitude of $D(\omega)$, the phase corresponding to a minimum phase transfer function system may be computed using complex cepstrum techniques. Accurate transfer function estimates may then be generated using standard frequency domain least squares techniques.

For given scalings $D(s)$, the problem of finding a controller $K(s)$ which minimizes the norm $\|F_\ell(D(s)N(s)D^{-1}(s), K(s))\|_{\mathcal{H}_\infty}$ will be reduced to a standard \mathcal{H}_∞ problem. Repeating this procedure several times will yield the complex μ upper bound optimal controller provided the algorithm converges. Even though the computation of the D scalings and the optimal \mathcal{H}_∞ controller are both convex problems, the D - K iteration procedure is *not jointly convex* in $D(s)$ and $K(s)$ and counter examples of convergence has been given [Doy85]. However, D - K iteration seems to work quite well in practice and has been successfully applied to a large number of applications. Furthermore, with the release of the MATLAB μ -Analysis and Synthesis Toolbox, commercially available software now exists to support complex μ synthesis using D - K iteration.

3.2.2 Mixed μ Synthesis / μ - K Iteration:

The main idea of the proposed μ - K iteration scheme is to perform a scaled D - K iteration where the difference between mixed and complex μ is taken into account through an additional scaling matrix $\Gamma(s)$. Given the augmented system $N(s)$, a stabilizing controller $K_1(s)$ (e.g. an \mathcal{H}_∞ optimal controller) we may compute upper bounds for μ across frequency given both the “true” mixed perturbation set $\Delta_{\mathcal{K}}$ and the fully complex approximation $\Delta_{\mathcal{K}}^c$. In order to “trick” the \mathcal{H}_∞ optimization in the next iteration to concentrate more on mixed μ , we will construct an open loop system $N_{D\Gamma}(s)$ which, when closed with the previous controller, has frequency response equal to the mixed μ upper bound just computed. This is equivalent to D , G - K iteration. In μ - K iteration, however, the structure of the approximation is different. $N_{D\Gamma} = \Gamma D N D^{-1}$ is constructed by applying two scalings to the original system $N(s)$. A D scaling such that $\bar{\sigma}(F_\ell(D N D, K))$ approximates the complex μ upper bound and a Γ scaling to shift from complex to mixed μ . In each iteration, Γ can be computed as

$$\Gamma_i(s) = \begin{bmatrix} \gamma_i(s) I_{n_{ze}} & 0 \\ 0 & I_{n_y} \end{bmatrix}$$

where

$$\gamma_i(j\omega) = (1 - \alpha_i) |\gamma_{i-1}(j\omega)| + \alpha_i \frac{\hat{\mu}_\Delta(F_\ell(N(j\omega), K_i(j\omega)))}{\hat{\mu}_{\Delta^c}(F_\ell(N(j\omega), K_i(j\omega)))}$$

α_i is a certain filtering variable, see below, n_{ze} denotes the number of measurement outputs and n_{ze} denotes

the number of external outputs. For perfect realizations of the scalings we will have

$$\bar{\sigma}(F_\ell(N_{D\Gamma_1}(j\omega), K_1(j\omega))) = \hat{\mu}_\Delta(F_\ell(N(j\omega), K_1(j\omega)))$$

where $\hat{\mu}_\Delta$ denoted the upper bound for μ . The controller $K_2(s)$ then will minimize the \mathcal{H}_∞ -norm of an augmented system which closed with the previous controller $K_1(s)$ has maximum singular value approximating mixed μ . New mixed and complex μ bounds may then be computed and the procedure may be repeated. The procedure outlined so far is described in the preliminary paper [TCASN94a]. However, it is easy to construct simple control problems, e.g. the double integrator in [You94], where the iteration does not converge. The problem is that we may suffer from “pop-up” type phenomena.

However, in [TCASN94a] it was demonstrated that by filtering Γ through a stable first order filter, the “pop-up” type phenomena could be avoided with proper choice of filter constant.

4. Design Example

A controller design for the benchmark problem based on the modified μ - K iteration is presented in this section. First, let us consider the design setup based on the design problem given in Section 2.

4.1. Design Setup

We will use the same design setup as used in [BM92]. A short description follows below. The spring constant and the two masses are assumed to be uncertain and are given by:

$$\begin{aligned} k &= k_0 + W_k \delta_k \\ m_1 &= m_{10} + W_1 \delta_1 \\ m_2 &= m_{20} + W_2 \delta_2 \end{aligned} \quad (8)$$

where k_0 , m_{10} and m_{20} are the nominal values and the weights W_k , W_1 and W_2 are used to normalize the uncertainties δ_i so that $|\delta_i| \leq 1$. Simultaneous perturbations in the δ_i are allowed, as long as $|\delta| \leq 1$ for each uncertainties i .

Weighted versions of the noise, disturbance, control input and performance variable are given by

$$\begin{aligned} v &= W_v v' \\ w &= W_w w' \\ u' &= W_u u \\ z' &= W_z z \end{aligned} \quad (9)$$

where in general the input weights W_v and W_w weight the frequencies to be rejected and the relative importance of the noise and disturbance. The W_z is the performance weight and W_u is used to limit the magnitude of the control input. As in [BM92], a diagonal structure is applied for the performance specifications.

We will use the same constant weight matrices as used in [BM92]. Now, consider the block diagram in Fig. 1 with the general interconnection structure for systems with structured perturbations.

In [BM92], all weights has been selected as constant weights, which make it quite simple to setup a state space description of N . Here we will also use constant weights apart from the weight on the measurement noise, W_v , which has been selected as a first order

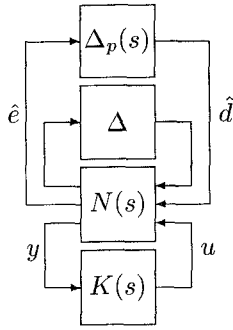


Figure 1: General interconnection structure for systems with structured perturbations

weight. With relation to Fig. 1, let the external input and output vectors be given by:

$$\begin{aligned} \hat{e} &= \begin{bmatrix} u' \\ z' \end{bmatrix} \\ \hat{d} &= \begin{bmatrix} v' \\ w' \end{bmatrix} \end{aligned} \quad (10)$$

and the Δ block is given by:

$$\Delta = \begin{bmatrix} \delta_k & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & \delta_2 \end{bmatrix} \quad (11)$$

and let W_v be represented by the following state space description:

$$W_v(s) = \left[\begin{array}{c|c} A_{W_v} & B_{W_v} \\ \hline C_{W_v} & D_{W_v} \end{array} \right] \quad (12)$$

With these definitions of \hat{d} , \hat{e} and Δ , a state space description of the complete system with perturbations is given by:

$$N(s) = \left(\begin{array}{c|c} A_N & B_N \\ \hline C_N & D_N \end{array} \right) \quad (13)$$

where

$$\begin{aligned} A_N &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\frac{k}{m_{10}} & \frac{k}{m_{10}} & 0 & 0 & 0 \\ \frac{k}{m_{20}} & -\frac{k}{m_{20}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{W_v} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ B_N &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{m_{10}} & \frac{1}{m_{10}} & 0 & 0 & 0 & \frac{1}{m_{10}} \\ \frac{1}{m_{20}} & 0 & \frac{1}{m_{20}} & 0 & \frac{W_w}{m_{20}} & 0 \\ 0 & 0 & 0 & B_{W_v} & 0 & 0 \end{bmatrix} \\ C_N &= \begin{bmatrix} W_k & -W_k & 0 & 0 & 0 & 0 \\ \frac{kW_1}{m_{10}} & -\frac{kW_1}{m_{10}} & 0 & 0 & 0 & 0 \\ -\frac{kW_2}{m_{20}} & \frac{kW_2}{m_{20}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & W_z & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & C_{W_v} & 0 \end{bmatrix} \end{aligned}$$

$$D_N = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{W_1}{m_{10}} & -\frac{W_1}{m_{10}} & 0 & 0 & 0 & -\frac{W_1}{m_{10}} \\ -\frac{W_2}{m_{20}} & 0 & -\frac{W_2}{m_{20}} & 0 & -\frac{W_2 W_w}{m_{20}} & 0 \\ 0 & 0 & 0 & 0 & 0 & W_u \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{W_v} & 0 & 0 \end{bmatrix}$$

4.2. Controller Design

The weight matrices for the robust design problem has been selected as follows:

$$\begin{aligned} W_k &= W_1 = W_2 = 0.3 \\ W_w &= 1.0 \\ W_u &= 15.0 \\ W_z &= 0.06 \\ W_v &= 0.01 \times \frac{0.25s+1}{0.0025s+1} \end{aligned}$$

Note that the weights on the three real uncertain parameters, k , m_1 and m_2 is 0.3 instead of 0.2 as used in [BM92].

Controllers designed both by using the standard D - K iteration for complex perturbations and the μ - K iteration for mixed perturbations are derived with the weight matrices given above. The results of the design is shown in the table below, Figure 2 and in Figure 3.

The D - K iteration result in an upper bound for both complex μ and mixed μ at 1.50 for a controller reduced to order 7. The closed loop is marginally stable with 30% independent variation in the three uncertain parameters. Further, the response of an impulse at w is reduced to less than 10% of maximal output after 16 sec. These values do not meet the original specifications.

For the μ - K iteration, we get the following upper bounds for mixed μ :

Iter. no.	d1	d2	d3	d4	γ	reg. ord.	α_{max}	κ	μ bound
1	3	3	1	3	8	65	1	0.5	16.4602
2	3	3	1	3	6	55	1	0.5	1.1064
3	2	3	1	3	6	55	1	0.5	0.9888
4	2	3	1	3	5	48	1	0.5	0.9927
5	3	3	1	3	5	50	0.8974	0.4	0.9582
6	3	3	1	3	5	50	0.421	0.3	0.9507
6 (red.)						9			0.9504

After iteration no. 6, the controller has been reduced from order 50 to order 9. The reduced order controller designed with the μ - K iteration guarantees robust performance, i.e. $\mu(M) < 1$ (the upper bound for μ when all perturbations are considered as complex is close to 2.5). (Note that the reduced order controller surprisingly does slightly better. This phenomenon, though, was not encountered in other situations.) The closed loop system is robustly stable for up to 40% independent variation in the three uncertain parameters. Further, the response from an impulse response is reduced to less than 10% of max. output in 12 sec, see Figure 3. The upper bounds for some of the μ - K iterations are shown in Figure 2. Note also that the maximal input signal $u(t)$ is less than 0.6, see Figure 3, which is quite reasonable. Further, the selection of the weight matrix for the measurement noise, W_v , makes the controller roll off at high frequencies. The bandwidth for the controller is less than 50 rad/s. This means that all specifications have been met by the controller computed by μ - K iterations.

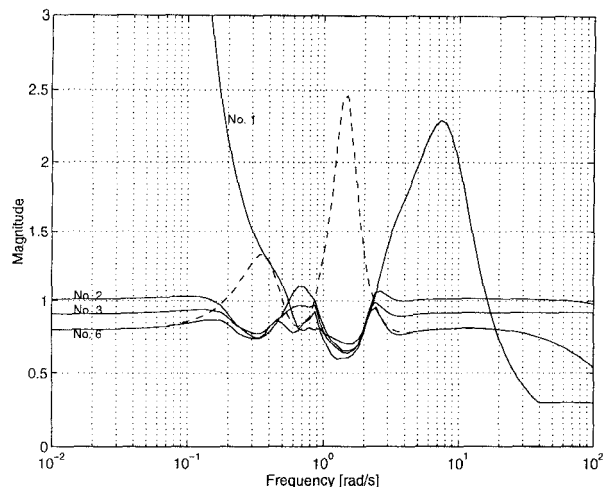


Figure 2: Upper μ bound for step 1 (the optimal H_∞ controller), 2, 3 and 6 in the μ - K iteration for the benchmark problem. The dashed curve shows the complex μ bound.

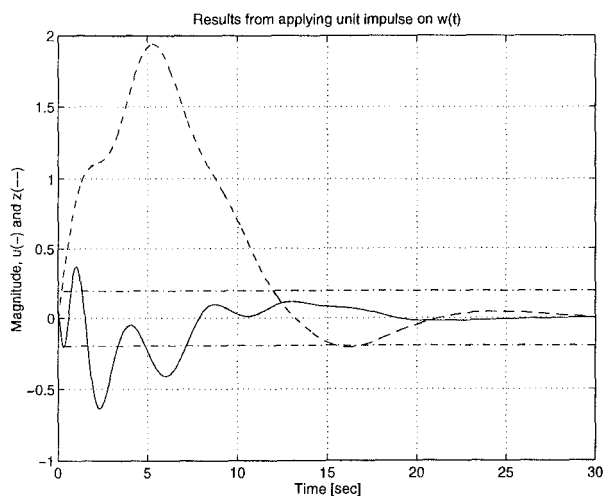


Figure 3: The output response (solid) from an impulse input at w and the associated control signal (dashed). Dash-dotted lines show 10% interval limits of the maximal value of the control signal.

5. Conclusion

The μ - K iteration for mixed perturbation sets has shortly been introduced in this paper. The design method has then been applied on the coupled mass benchmark problem and compared with the standard D - K iteration for complex perturbation sets.

The benchmark example indicates that the presented μ - K iteration can be advantageously applied to the benchmark design problem. Compared to using the standard D - K iteration, where all perturbations are considered as complex perturbations, the μ - K iteration result in much better results. This indicates also that we in general need to take care of real parameter uncertainties to avoid conservatism in the design. The same conclusion has been found in [BM92], where the designed controllers has been analyzed by mixed μ analysis.

As described in Section 3, the rationale of the μ - K iteration is just to use the standard D - K iteration scheme and add an additional scaling to take care of the gap between the mixed and the complex upper bound for

μ . This method is computationally simpler than the μ synthesis method for mixed perturbation sets derived by Young, see e.g. [You94], which is directly based on the upper bound for the mixed μ . The two methods have not been explicitly compared here, but in [TCASN95], a double integrator system taken from [YÅ94] has been considered. The result is that the μ - K iteration approach results in a considerably lower μ upper bound than the controller designed by using the approach by Young. The μ - K iteration approach has been applied to other examples with good results, see the examples given in [TCASN94a, TCASN94b, TCASN95].

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