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Published in:
Proceedings. International Symposium on Information Theory, 2004. ISIT 2004.

Link to article, DOI:
10.1109/ISIT.2004.1365199

Publication date:
2004

Document Version
Publisher's PDF, also known as Version of record

Analysis of Bit-Stuffing Codes and Lower Bounds on Capacity for 2-D Constrained Arrays using Quasi-Stationary Measures

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Abstract — **A method for designing quasi-stationary probability measures for two-dimensional (2-D) constraints is presented. This measure is derived from a modified bit-stuff coding scheme and it gives the capacity of the coding scheme. This provides a constructive lower bound on the capacity of the 2-D constraint. The main examples are checkerboard codes with binary elements. The capacity for one instance of the modified bit-stuffing for the 2-D runlength-limited RLL$(2, \infty)$ constraint is calculated to be 0.4414 bits/symbol. For the constraint given by a minimum (1-norm) distance of 3 between 1s a code with capacity 0.3497 bits/symbol is given.**

I. Introduction

We present a method for designing two-dimensional (2-D) constrained codes based on bit-stuffing. We consider 2-D arrays with elements taken from a finite alphabet. The constraint is specified by the set of admissible configurations on an N by M rectangle. For 2-D RLL$(1, \infty)$, bit-stuffing provides efficient coding [1]. In [2] bit-stuffing for 2-D RLL$(d, \infty), d \geq 2$ were considered. We shall take a slightly different approach to bit-stuffing in order to facilitate analysis e.g. providing a constructive lower bound on the capacity of the constraint. The method is generally applicable to checkerboard constraints, where a 1 must be surrounded be a certain pattern of 0s, meaning that a 0 is always admissible.

II. Quasi-stationary measures

A quasi stationary measure may be introduced by concatenating arrays. Given a constraint, let $\mathbf{W}$ denote a stochastic variable defined on an n by m array, which may take on any configuration admissible by the constraint. Let $\mathbf{X}$ and $\mathbf{Z}$ denote variables representing the first and last $M-1$ columns (with n elements). Let $\mathbf{Y}$ denote a variable representing the middle $m - 2M + 2$ columns. We assume that the measures on the boundaries, $\mathbf{X}$ and $\mathbf{Z}$ are identical for the measures, $\mathbf{W}$ to be considered. Thus, starting with $\mathbf{X}_0\mathbf{Y}_0\mathbf{Z}_0$, arrays $\mathbf{Y}_i\mathbf{Z}_i$ may repeatedly be added to form $\mathbf{X}_0\{\mathbf{Y}_j\mathbf{Z}_j\}_0^K$, such that $\mathbf{Z}_{i-1}\mathbf{Y}_i\mathbf{Z}_i$ has the same measure as $\mathbf{W}$. The entropy (per symbol) is given by the conditional entropy of $\mathbf{Y}_i\mathbf{Z}_i$ given $\mathbf{Z}_{i-1}$ which is

$$\frac{H_W(m) - H_X(M-1)}{m - M + 1}. \quad (1)$$

where $H_W(m)$ is the entropy of $\mathbf{W}$ (per row) and $H_X(M-1)$ is the entropy of $\mathbf{X}$ (per row). A simple way to specify $\mathbf{W}$ in (1) is to assign probabilities to the bit-stuffing scheme below. The boundaries $\mathbf{X}$ and $\mathbf{Z}$ are specified by identical but independent bit-stuffing schemes. The middle columns $\mathbf{Y}$ are specified by bit-stuffing conditional on the boundaries $\mathbf{X}$ and $\mathbf{Z}$.

III. Numerical results

Two examples with binary elements and constraint size $N = M = 3$ are considered. For the RLL$(2, \infty)$ constraint, analysis of the modified bit-stuffing was carried out calcultating capacities, C for $m = 12$. The transition probabilities for a new row of $\mathbf{W}$ were determined by the products of (conditional) probabilities adressing and bit-stuffing the elements of the new line of $\mathbf{X}$ and $\mathbf{Z}$ before $\mathbf{Y}$ and using the same conditional probabilities for the corresponding elements of $\mathbf{X}$ and $\mathbf{Z}$. Thus the prerequisites for (1) is satisfied. Let p_1 denote the probabilities of writing a 1 when this is admissible. Simple bit-stuffing writing an unbiased sequence with $p_1 = 1/2$ gave $C = 0.388$. Using a single biased sequence gave $C = 0.437$ for optimal choice of p_1. Finally the values of p_1 may be chosen independently for each column of $\mathbf{X}$ and $\mathbf{Y}$. (The p_1 values of $\mathbf{Z}$ are given by $\mathbf{X}$.) This gave a best value of $C = 0.44149$, also providing a lower bound for the constraint. This is a fair improvement on the lower bound of 0.4267 on the capacity of (diagonal) bit-stuffing in [2].

Capacities were also calculated for applying the modified bit-stuffing scheme to the constraint given by a min. (1-norm) distance of 3 between 1s. The results obtained for $m = 15$ were $C = 0.276$ when writing an unbiased sequence, $C = 0.344$ for a single biased sequence and $C = 0.3477$ choosing different biased sequences for each column of $\mathbf{X}$ and $\mathbf{Y}$. For this constraint the boundaries, $\mathbf{Z}_{i-1}$ and $\mathbf{Z}_i$ must be at least an additional row ahead in order to bit-stuff the elements of $\mathbf{X}$ and $\mathbf{Z}$ independently of past elements of $\mathbf{Y}$. A more elaborate scheme for specifying $\mathbf{W}$ in (1) was also devised. The probabilities p_1 were made dependent on the other elements on the $(N-1 =)$ 2 previous rows. The next row of $\mathbf{X}$ (and $\mathbf{Z}$) is specified by probabilities conditioned on the two previous rows. The new row of $\mathbf{Y}$ is specified by probabilities conditioned on 3 rows of $\mathbf{X}$ and $\mathbf{Z}$ and 2 rows of $\mathbf{Y}$. These conditional probabilities were obtained from the maxentropic solution [3] for $\mathbf{W}$ (with two rows forming the states). This gave a capacity of $C = 0.3497$, which also provides a new lower bound for the constraint.

References

[1] R.M. Roth, P.H. Siegel and J.K. Wolf, "Efficient coding schemes for the hard-square model," *IEEE Trans. Inform. Theory*, vol. 47, pp. 1166-76, March, 2001.

[2] S. Halevy, J. Chen, R.M. Roth, P.H. Siegel, J.K. Wolf, "Improved bit-stuffing bounds on two-dimensional constraints," *Proc. ISIT'02*, Lausanne, Switzerland, p. 385, 2002.

[3] S. Forchhammer and J. Justesen, "Entropy bounds for constrained two-dimensional random fields," *IEEE Trans. Inform. Theory*, vol. 45, no. 1, pp. 118–127, Jan. 1999.