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Topology Optimization of Metallic Microwave Devices

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Abstract
This paper presents a novel extension to the topology optimization method, that allows systematic design of metalic/dielectric devices for microwave applications. This is accomplished by interpolating Maxwell's equations and an impedance boundary condition between two material phases, i.e. a dielectric and a conductor. The numerical optimization scheme is demonstrated by the design of resonators for energy harvesting.

Keywords: topology optimization, conductor design, finite elements, Maxwell's equations.

1. Introduction

The motivation for this work originates from the ever increasing usage of small hand-held, or autonomous, electrical devices such as hearing aids, medical implants and communication devices. These devices all require antennas for communication as well as a power supply, usually a battery. Since more and more functionality is incorporated into these devices, standard antenna designs are becoming less and less usable and the requirements to the batteries are increasing. The power supply issue took a new turn in 2007, where a MIT group lead by Prof. M.Soljacic demonstrated that one could obtain efficient mid-range wireless energy transfer (WiTricity) using magnetically resonant coupled copper coils [1].

Common for the design of antennas and devices for WiTricity are that they consist of an elaborate spatial distribution of a conductor, e.g. copper, in a dielectric background, e.g air. This makes the design of such devices an obvious candidate for the topology optimization method [2].

2. Optimization Setting

In order to pose the optimization problem as a mathematical program, one must first include design dependence in the governing equations, i.e. Maxwell's equations. However, for microwave applications the FEM is known to cause computational problems, i.e. extreme mesh refinement, due to the need for resolving the skin depth, when modeling conductors as volumetric entities [4]. Thus, the design parametrization must remedy this bottleneck for the scheme to be efficient.

In the following we will denote the material parameters for the dielectric with ϵ_r^d, μ_r^d and σ^d, and likewise for the conductor ϵ_r^m, μ_r^m and σ^m. Then we define a design variable, $0 \leq \rho^e \leq 1$, for each finite element, e, in the mesh. The design variable can be understood as the density of the conductor. Including design dependence and an element boundary condition to resolve the skin depth problem, Maxwell's vector wave equation can be stated for each design element as

$$
\begin{aligned}
\nabla \times (\tilde{A} \nabla \times \boldsymbol{u}) - k_0^2 \tilde{B} \boldsymbol{u} = 0, & \qquad \text{in } \Omega^e \\
\boldsymbol{n} \times (A \nabla \times \boldsymbol{u}) - (\rho^e)^{p_{BC}} j k_0 \sqrt{AB} \boldsymbol{n} \times (\boldsymbol{n} \times \boldsymbol{u}) = 0, & \quad \text{on } \Gamma^e
\end{aligned}
\tag{1}
$$

where Ω^e and Γ^e refer to the volume and boundaries of element e in the finite element mesh respectively, $\boldsymbol{n}$ is an outward normal for element e and k_0 is the free space wave number. The field $\boldsymbol{u}$ and the parameters A, B, $\tilde{A}$, $\tilde{B}$ and p_{BC} are given in Table 1. The interpolation functions are given as follows

$$
\begin{aligned}
\epsilon_r(\rho^e) &= \epsilon_r^d + \rho^e(\epsilon_r^m - \epsilon_r^d) \\
\mu_r(\rho^e) &= \mu_r^d + \rho^e(\mu_r^m - \mu_r^d) \\
\sigma(\rho) &= 10^{\log 10(\sigma^d) + \rho^e[\log 10(\sigma^m) - \log 10(\sigma^d)]}
\end{aligned}
\tag{2}
$$

Note that for dielectrics with $\sigma^d = 0$ one should use $\sigma^d \approx 10^{-4}$ for numerical stability. The governing equations are solved by FEM and the optimization problem by MMA by K.Svanberg [3]. A more in depth discussion of the above design parametrization can be found in [5].

$\boldsymbol{u}$	A	B	$\tilde{A}$	$\tilde{B}$	p_{BC}
$\boldsymbol{E}$	$(\mu_r^m)^{-1}$	$\epsilon_r^m - j\frac{\sigma^m}{\omega\epsilon_0}$	$\mu_r(\rho^e)^{-1}$	$\epsilon_r(\rho^e) - j\frac{\sigma(\rho^e)}{\omega\epsilon_0}$	≈ 13
$\boldsymbol{H}$	$\left(\epsilon_r^m - j\frac{\sigma^m}{\omega\epsilon_0}\right)^{-1}$	μ_r^m	$\left(\epsilon_r(\rho^e) - j\frac{\sigma(\rho^e)}{\omega\epsilon_0}\right)^{-1}$	$\mu_r(\rho^e)$	≈ 1

Table 1: Field dependent parameters for the design parametrization used for conductor/dielectric based topology optimization, equations (1). The functions $\mu_r(\rho^e)$, $\epsilon_r(\rho^e)$ and $\sigma(\rho^e)$ is given in equation (2).
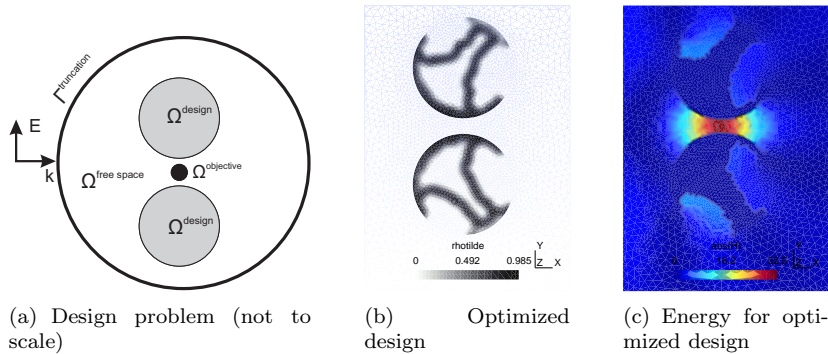

(a) Design problem (not to scale)

(b) Optimized design

(c) Energy for optimized design

Figure 1: Design problem and optimized design for 115 MHz using copper (black) and air (white). The design was obtain after 106 iterations.

2.1. Numerical Example

In this section a simple optimization problem for 2D TE polarization, i.e. $\boldsymbol{H} = H_z$, is solved to demonstrate the potential of the design parametrization. The design task is to maximize the magnetic energy between two cylindrical design domains, each with radius $0.65m$, separated by a gap of $0.2m$, c.f. figure 1(a). The optimization problem can be stated as a standard mathematical program

$$
\begin{aligned}
\max_{\rho \in \mathbb{R}^N} \quad & \log 10(\bar{H}_z^T Q H_z) \\
\text{s.t.,} \quad & \text{Governing eqs.} \\
& \frac{\sum_e^N \rho_e V_e}{V f^*} - 1 < 0 \\
& 0 \le \rho^e \le 1, e = 1, N
\end{aligned}
\tag{3}
$$

where N is the number of design variables and the second constraint is a possible volume constraint imposed to control the amount of used material. The sensitivities are derived using the adjoint method [6], and the implementation is done in Matlab. The materials are chosen to be air, i.e. $\epsilon_r^d = \mu_r^d = 1$, and copper $\epsilon_r^m = \mu_r^m = 1$, $\sigma^m = 5.9 \cdot 10^7 S/m$ [4]. The target frequency is set to 115 MHz. The optimized design and magnetic energy distribution can be seen in figure 1(b,c). Compared to the scenario with both cylinders filled with copper, the objective for the optimized design is improved by more than 300 %. The optimized design is verified by evaluating a post processed design modeled by a perfect electric conductor (PEC) condition. The post processed design is found to be in good agreement with the optimization result.

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