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Soliton Compression to Few-cycle Pulses by Cascaded Quadratic Nonlinearities

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For second-harmonic generation (SHG) the phase mismatch is given by $\Delta k = k_2 - 2k_1$. For $\Delta k \gg 0$ a large negative Kerr-like nonlinear phase shift may be induced on the fundamental wave (FW). This self-*defocusing* nonlinearity together with normal dispersion may compress a pulse, and problems normally encountered due to self-focusing in cubic media are avoided. Thus, having no power limit, in bulk media a self-defocusing soliton compressor can create high-energy near single-cycle fs pulses [1]. Here we present a theoretical and numerical investigation of pulse-compression in a nonlinear crystal. We introduce an SHG soliton number and show that compression only takes place when it is larger than the "usual" Kerr soliton number. This can be achieved by adjusting Δk, but only if the quadratic material nonlinearity is sufficiently stronger than the cubic Kerr nonlinearity. Also the group-velocity mismatch (GVM) between the FW and second harmonic (SH), given by the inverse group velocity difference $d_{12} = 1/v_{g,1} - 1/v_{g,2}$, limits the compression regime [1a,1c]. We show that in a typical nonlinear crystal, efficient, good-quality compression down to <2 optical cycles may be observed when optimizing Δk within the compression regime.

The system is modeled by coupled nonlinear Schrödinger equations including quadratic nonlinear terms and self-steepening [1d]. The rescaled equations have a characteristic Kerr soliton number N_{Kerr}, similar to the fiber optics one, and a new SHG one N_{SHG}. Because the material Kerr nonlinearity is self-focusing it counteracts the cascaded self-defocusing nonlinearity. Thus, a requirement for FW soliton compression is $N_{\mathrm{SHG}} > N_{\mathrm{Kerr}}$, posing an upper limit on Δk as

$$\bar{N} = \frac{N_{\mathrm{SHG}}}{N_{\mathrm{Kerr}}} > 1, \quad \Rightarrow \quad |\Delta k| < \Delta k_c = \frac{\omega_1 d_{\mathrm{eff}}^2}{cn_1 n_2 n_{\mathrm{Kerr}}} \qquad N_{\mathrm{Kerr}}^2 = L_{\mathrm{D},1} \frac{2\omega_1 n_{\mathrm{Kerr}} I_{1,\mathrm{in}}}{\varepsilon_0 c^2 n_1}, \qquad N_{\mathrm{SHG}}^2 = L_{\mathrm{D},1} \frac{2\omega_1^2 d_{\mathrm{eff}}^2 I_{1,\mathrm{in}}}{\varepsilon_0 c^3 n_1^2 n_2 |\Delta k|}$$

where $L_{\mathrm{D},1}$ is the FW dispersion length, $I_{1,\mathrm{in}}$ the FW input intensity, n_{Kerr} the Kerr nonlinear refractive index, d_{eff} the effective quadratic nonlinearity, n_j the refractive indices. Notice that $\bar{N}$ is independent on $I_{1,\mathrm{in}}$. GVM poses another, lower, compression limit: $\Delta k > \Delta k_{\mathrm{sr}} = 4\pi |d_{12}|/T_{1,\mathrm{in}}$, where $T_{1,\mathrm{in}}$ is the FW input pulse duration. In this *stationary regime* the nonlinear phase shift builds up before GVM separates the fields [1a]. These limits define a Δk-compression window, which opens ($\Delta k_c > \Delta k > \Delta k_{\mathrm{sr}}$) when GVM effects are small and $d_{\mathrm{eff}}^2/n_{\mathrm{Kerr}}$ is large. An excellent material for compression is β-barium-borate (BBO) because it has a high $d_{\mathrm{eff}}^2/n_{\mathrm{Kerr}}$-ratio. Performing realistic numerical simulations of a BBO for $\lambda_1 = 1060$ nm, Fig. 1(a) shows that the transition to compression happens exactly at $\bar{N} = 1$. As $\bar{N}$ is increased by reducing Δk, the output pulse intensity increases until it peaks at position (2) giving a ~12 fs pulse (compression factor $f_{\mathrm{comp}} \sim 17$) containing 45% of the input energy, see (2) in Fig. 1(b). For larger $\bar{N}$, the pulse splits up, see (3) in Fig. 1(b). In fact, (2) represents the optimal compression point z_{max} for this input pulse and crystal length $L = 50$ mm. z_{max} depends strongly on the input pulse parameters as shown in Fig. 1(c). A characteristic hyperbolic behavior is observed as $\bar{N}$ is varied, with z_{max} diverging at the transition point $\bar{N} = 1$. The corresponding pulse duration at $z = z_{\mathrm{max}}$ drops as $\bar{N}$ is increased, and is minimum at $\bar{N} = 1.65$ ($\Delta k = 50$ mm⁻¹), where as the inset shows a clean 6.6 fs pulse is seen (~1.9 optical cycles, $f_{\mathrm{comp}} \sim 30$, containing 31 % of the input energy). Beyond this point the pulse duration increases again, firstly because such large $\bar{N}$-values require Δk small so $\Delta k \sim \Delta k_{\mathrm{sr}}$, and secondly because GVM induces a Raman-like effect with characteristic time $T_{\mathrm{R,SHG}} = 2|d_{12}|/\Delta k$ [1c] resulting in an asymmetric pulse with a reduced pulse compression quality and factor.
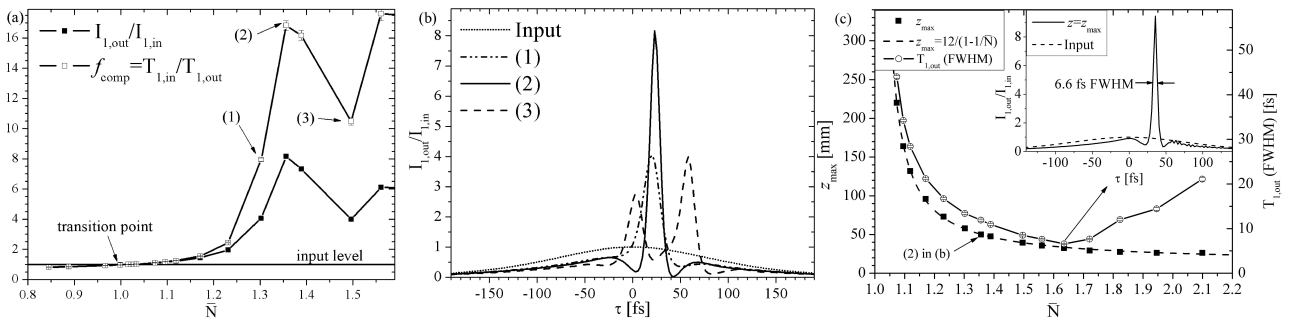
Fig. 1 (a) $I_{1,\mathrm{out}}/I_{1,\mathrm{in}}$ and $T_{1,\mathrm{in}}/T_{1,\mathrm{out}}$ for a $L = 50$ mm BBO crystal. (b) Selected output pulses from (a). (c) z_{max} and corresponding pulse duration vs. $\bar{N}$. Input pulse: 200 fs FWHM, $I_{1,\mathrm{in}} = 50$ GW/cm². $\Delta k_{\mathrm{sr}} = 10$ mm⁻¹ and $\Delta k_c = 137$ mm⁻¹.

To conclude, pulse compression with cascaded quadratic nonlinearities requires that the ratio of the SHG and Kerr soliton numbers $\bar{N} > 1$. A moderate $\bar{N}$ optimizes the compression. Similar results hold in the fiber case, where we may use the dispersion control offered by photonic-crystal fibers [2] to investigate compression when GVM is reduced.

References
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