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Theory and Design of Flow Driven Vehicles Using Rotors for Energy Conversion

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Abstract:

This work investigates the energy conversion mechanisms that vehicles driven by converting energy from two media moving relative to each other are based on. Specifically, focus is on using a horizontal axis wind turbine and/or a propeller on a vehicle to make it move relative to the media. A simple one point optimization method, based on the Blade Element Momentum theory, for both the wind turbine and propeller rotor is presented. Issues related to the practical implementation of making a wind turbine car go as fast as possible into the wind are illuminated, as well as a few related topics. Despite the obvious lack of commercial exploitability of wind turbine cars, the presentation will hopefully illuminate surprise and amuse the readers.

Keywords: Wind turbine car, Unconventional use of wind energy.

1 Introduction

The initial reaction for many asked whether they think it would be possible to put a wind turbine on a car or a boat to make it go directly against the wind, could be doubt. If added, that no upper velocity limit exist, most people would probably disagree. Certainly, if the question was whether it is possible to build a car that can go faster than the free-stream velocity in the downwind direction, using a propeller to provide the propulsive force, generating the power for the propeller via the wheels, the answer would most likely be negative. Many such, seemingly "impossible" machines are in fact possible, and do not violate any physical laws. One of the aims of the present work is to show why and how vehicles such as these are in fact possible.

Even though the idea of using a wind turbine to propel a car against the wind is not new, the theory that describes this, has, to the authors' knowledge, not been expressed explicitly yet to the scientific community, even though it is merely application of previously known basic theory.

Therefore, the present work outlines the explicit theory to describe the energy conversion mechanisms for vehicles driven, or powered, by media (solids or fluids) with a velocity relative to each other. The wind turbine cars, as seen in the recent Aeolus race in the Netherlands [1], are just one of the possible realizations of the vehicles in this category.

Furthermore, a simple method, based on the Blade Element Momentum (BEM) theory [2], to optimize both propellers and wind turbines for these kinds of applications, are formulated.

Due to the recent Aeolus race, special attention is given to cars propelled straight into the wind using wind turbines.

The present work is a condensed version of [3], and will focus on the governing mechanisms/principles for this type of vehicles and highlight some paradoxical results that follow from the theory.

Finally, the present presentation aims at inspiring more institutions or companies to participate in this year's Aeolus race [1].

2 Theory

In this section the basic energy conversion considerations are outlined. From these the application to two interesting cases, the wind turbine car and the propeller car, will be treated subsequently. After this a general one point optimization method, based on the classical BEM theory [2], for the layout of rotors for both the generator case (wind turbine, as for the wind turbine car) and the propulsion case (propeller, as for the propeller car). We begin with the general basic energy conversion concepts.

2.1 General energy conversion concept

In order to keep things simple we consider only vehicles that travel in line with the direction of the velocity difference between two media: In either of the two directions. Both this case and the more general one where the traveling direction is arbitrary are treated in detail in [3]. Furthermore, we only treat the "high speed case" here, in which the relative velocity of both media has the same direction. This is the most interesting case if the objective is to build a vehicle that is capable of high velocities. Any readers interested in the general case are referred to [3] for a complete treatment of the problem.

Figure 1 below show the notation used in the investigation of the general energy conversion processes at play for vehicle "powered" by a velocity difference between two media[1].
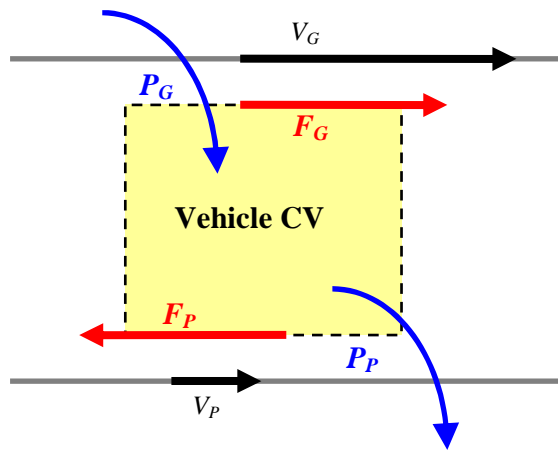

Figure 1: Notation used in investigation of energy conversion mechanisms for vehicles powered by a difference in velocity of two media.

The figure show the forces and energy flow involved for vehicles of this kind in a schematic way. The media, with velocities V_G and V_P relative to the vehicle Control Volume (CV), are shown above and below the CV. The corresponding forces, F_G and F_P, and powers, P_G and P_P, from interaction with the media are also shown. It may be shown from basic considerations and application of the definition of work [3], that in order to make a vehicle of this type work, the generation of power (power gained *from* the medium) should occur at the fluid with the highest velocity relative to the vehicle, and the propulsion (power delivered *to* the medium) should occur at

the medium with the lowest velocity relative to the vehicle. This is the reason for the direction of the forces and energy flow in Figure 1. Keeping in mind that the relation between power, force and relative velocity in the ideal case is

$$P = FV \qquad (1)$$

we may define efficiencies for the interaction with the generation medium

$$\eta_G = \frac{P_G}{P_{G,ideal}} = \frac{P_G}{F_G V_G} \qquad (2)$$

and the propulsion medium

$$\eta_P = \frac{F_P}{F_{P,ideal}} = \frac{F_P V_P}{P_P} \qquad (3)$$

A transmission loss is introduced to link the power generated and the power available for propulsion

$$P_P = P_G \eta_T \qquad (4)$$

If we are interested in obtaining relations for the (constant) maximum speed, the forces from the two media should balance

$$F_G = F_P \qquad (5)$$

Combining Equations (2) to (5) yields the general relation between the efficiencies and the relative velocities

$$\frac{1}{\eta_G \eta_T \eta_P} = \frac{V_G}{V_P} \qquad (6)$$

Equation (6) is a general expression applicable for any type of vehicle based on energy conversion between two media. In order for us to more easily see what this correspond to we will consider two cases below: A: All velocities measured relative to the propulsive media (as for instance in the wind turbine car case). B: All velocities measured relative to the generation media (as for instance in the propeller car case). Please note that the equations are generally valid, and therefore applicable to any system. This includes also the case where the energy transfer between both media is done using rotors, as would be the case for boats. We now proceed first to the first of the two cases mentioned above.

2.2 Velocities measured relative to the propulsive medium (Example: Wind turbine car case)

If we consider the velocities measured relative to the propulsive medium, and indicate vehicle velocity by V, and velocity difference between the media by V_∞, the velocities relative to the vehicle are

$$V_G = V + V_\infty \qquad (7)$$

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[1] Note that the notation "two media" is only used to set the two regions with a velocity difference apart. The theory to be derived will also be applicable to a situation with two zones of the same type of medium, for instance air, having a velocity difference.

$$V_P = V \qquad (8)$$

Therefore the general result of Equation (6) may be rewritten to

$$\frac{V}{V_\infty} = \frac{1}{\dfrac{1}{\eta_P \eta_T \eta_G} - 1} \qquad (9)$$

This is the general expression for determination of the maximum velocity of the vehicle measured relative to the propulsive medium. Note that this correspond to for instance the wind car case (wind driven car riding against the wind), see Figure 2 below.

Figure 2: Example of a wind car: the WinDTUrbineracer [4] racing along the dike, against the wind at the 2008 Aeolus race in Den Helder, the Netherlands.

It is seen from Equation (9), that the velocity of this type of vehicle tend to infinity for a given velocity difference between the media (= wind speed) as the product of the efficiencies tend to unity. In order to relate the generation efficiency to known wind turbine quantities and other aerodynamic losses (drag on the part of the car that is not rotor), we consider again the equation for the generation efficiencies, Equation (2), noting that the total force from the generation medium (air in the wind car case) is the sum of the thrust from the rotor and the aerodynamic drag on the rest of the car. This results in the following relation

$$\eta_G = \frac{P_G}{F_G V_G} = \frac{C_P}{C_T + \frac{A_V}{A} C_D} \qquad (10)$$

Here, the usual non-dimensional thrust and power coefficients for a wind turbine are

$$C_T = \frac{T}{\frac{1}{2} \rho V_{rel}^2 A} \qquad (11)$$

$$C_P = \frac{P}{\frac{1}{2} \rho V_{rel}^3 A} \qquad (12)$$

Furthermore, the drag coefficient for the car excluding the rotor is

$$C_D = \frac{D_A}{\frac{1}{2} \rho V_{rel}^2 A_V} \qquad (13)$$

where, the subscript V refers to vehicle. Inserting the efficiency in this case (10) into the general equation for vehicle velocity, we get the following expression

$$\frac{V}{V_\infty} = \frac{1}{\dfrac{1}{\eta_P \eta_T} \dfrac{C_T + \frac{A_V}{A} C_D}{C_P} - 1} \qquad (14)$$

Investigating the ideal case with no losses in transmission or propulsion systems, zero drag on the remainder of the car, and ideal rotor performance, from 1D momentum theory [5, 3], where the non-dimensional thrust and power coefficients only depend on the axial induction factor, a

$$C_T = 4a(1-a) \qquad (15)$$

$$C_P = 4a(1-a)^2 \qquad (16)$$

We arrive at the following interesting relation

$$\frac{V}{V_\infty} = \frac{1}{a} - 1 \qquad (17)$$

From this is is seen that the axial induction that maximizes vehicle velocity in this case tends to zero. The vehicle velocity in this case tends to infinity. Albeit clearly not close to realizable in real life, it still shows that the rotor design for the these kinds of applications is generally different from the rotor design for maximizaton of power output for a conventional stationary wind turbine which is close to a=1/3 in order to have a high C_P. A general expression for the optimal axial induction as function of $C_D A_V /A$ can be computed from Equation (14). This is given in [3]. As losses ocurring in the real worl is included, the maximum velocity of the vehicle drops to values below infinity, but the intriguing result is that no definitive upper limit exist. As long as efficiencies are increased, the velocities of the vehicle will increase. As the losses increase above zero, the axial induction factor that maximizes the vehicle velocity increases asymptotatically to 1/3 (shown in [3]), so only for the case of a wind turbine car with very high losses should the rotor design be close to a conventional wind turbine rotor.

It is noted that the standard 1D momentum theory results, Equation (15) and (16), are of course valid for these applications because the

equations from which they are derived only require the rotor coordinate system to be an inertial system (that is: non-accelerating). The only thing one should be aware of is of course that the axial induction is defined relative to the free stream velocity as observed from the wind turbine, as given in Equation (7). Another ting to note is that the concept of using kinetic energy to analyze the energy conversion of a rotor stems from the energy equation (see basic textbooks on fluid mechanics, for instance [6]) applied in an inertial system (=non-accelerating) in which the rotor is stationary. Therefore argumentation using the concept of kinetic energy in the wake of the wind car as observed from the ground in case of a wind turbine car is nonsense. Using the energy equation in a ground-fixed coordinate system in this case introduces additional unsteady terms in the energy equation which must be taken into account when analyzing the performance of the rotor. When done correctly, the results are of course still identical to the results obtained using the much simpler analysis in the moving reference frame, as shown in [3].

2.3 Velocities measured relative to the generation medium (Example: Propeller car case)

Viewing the velocities as seen from the generation medium, the velocities are

$$V_G = V \tag{18}$$

$$V_P = V - V_\infty \tag{19}$$

Inserting these in the general relation between relative velocities and efficiencies, Equation (6), we arrive at

$$\frac{V}{V_\infty} = \frac{1}{1 - \eta_P \eta_T \eta_G} \tag{20}$$

Analogous to the previous case, it is seen that the velocity of the vehicle also in this case tends to infinity as the product of the efficiencies tend to one. In case of a land-based vehicle using this way of harvesting the energy available due to the velocity difference between the two media, we note that this would correspond to driving along with the wind (faster than the wind), generating the power at the wheels and producing the thrust with a propeller in the air. This may at first sound impossible, but the energy conversion processes are in fact possible, and the mechanism is no different from the wind car case, where the same is in fact happening when taking the viewpoint of the air. Analogous to the generation case, we may introduce rotor (propeller) performance non-dimensional numbers to express the efficiency of everything connected to the propulsive medium (in this case the air) as

$$\eta_P = \frac{F_P V_P}{P_P} = \frac{C_{T,P} - \frac{A_V}{A} C_D}{C_{T,P}} \tag{21}$$

In the derivation of this we have used that the total force from the propulsive medium (air for the propeller car case) is the thrust from the propeller minus the drag on the rest of the vehicle. The propeller thrust and power coefficients are defined as in Equations (11) and (12), noting that the positive direction for the power and thrust is opposite to that of the wind turbine case. Inserting the propulsive efficiency (21) in (20) results in

$$\frac{V}{V_\infty} = \frac{1}{1 - \eta_G \eta_T \dfrac{C_{T,P} - \frac{A_V}{A} C_D}{C_{P,P}}} \tag{22}$$

This is the general equation for the top speed of a propeller car. Simulations including realistic values for car drag and rotor performance show that this type of vehicle may in fact be faster than a corresponding wind turbine car [3]. The issue with such a vehicle is of course that it needs to be pushed up to velocities above the wind velocity for the energy conversion process to take over since a propeller does not work when the flow comes from "the wrong side". This corresponds to a helicopter descending too fast. In analogy with the generation case we may consider an ideal vehicle of this type. Drag is zero and generation and transmission effocoencies are one, and 1D momentum theory is applied for the rotor aerodynamics. This corresponds in this case to

$$C_{T,P} = 4 a_P (1 + a_P) \tag{23}$$

$$C_{P,P} = 4 a_P (1 + a_P)^2 \tag{24}$$

Note that the positive direction of the axial induction for the propeller case is changed to reflect the usual working state for this application. Inserting these values in the relation for the velocity, Equation (22) results in

$$\frac{V}{V_\infty} = \frac{1}{a_P} + 1 \tag{25}$$

As in the wind car case it is seen that this is maximised for a_P tending to zero, for which the velocity ratio tends to infinity. So the conclusion in this case is as the prevoius case. The obtainable velocity is unbounded. As long as efficiencies are increased the velocity ratio increases. In analogy with the previous case a general expression for the optimal axial induction as function of $C_D A_V / A$ can be computed from Equation (22). This expression can be found in [3].

2.4 Net power out case

Another interesting result pop out if we consider the case where only a part of the power produced from interaction with the generation medium is used for propulsion. Specifically, interest is on how the net production can be affected compared to the stationary reference case. In order to avoid unnecessarily complicated derivations and analysis, we consider motion of the vehicle in line with the relative velocity as was done in the previous sections. Here, in soft terms, we want to trade some speed in the previous sections for power production. The relations we derive also applies in the case extra power is put into the system by an engine. In that case the sign of the produced power is just negative. Consider the power equation in the case where an amount of mechanical energy, P_{OUT}, is taken out before the rest of the power is used for propulsion

$$(P_G - P_{OUT})\eta_T - P_P = 0 \qquad (26)$$

Using the definition of the efficiencies, Equations (2) and (3), and equilibrium of the forces at top velocity, Equation (5) and measure the velocities relative to the propulsive media, Equations (7) and (8) we arrive at

$$P_{OUT} = F_G V_\infty \eta_G \left(1 + \frac{V}{V_\infty}\left(1 - \frac{1}{\eta_P \eta_T \eta_G}\right)\right)(27)$$

The usual non-dimensionalization used in the stationary generation case (wind turbine) uses V_∞ as the reference velocity. The upper limit in the stationary case is given by 1D momentum theory, and is the well known Betz limit of $C_{Pmax,stat}=16/27$. Therefore, if the stationary case is to be used for comparison it will be convenient to non-dimensionalize the net power, P_{OUT}, the same way. This result, after some rearrangement (see [3] for details) in

$$C_{P,OUT} = \frac{P_{OUT}}{\frac{1}{2}\rho_G V_\infty^3 A}$$

$$= C_P \left(1 + \frac{V}{V_\infty}\right)^2 \left(1 + \frac{V}{V_\infty}\left(1 - \frac{1}{\eta_P \eta_T \eta_G}\right)\right) \qquad (28)$$

Here, it is seen, that the equation holds the expected characteristics (Note that the C_P value is defined in the vehicle reference frame, using the relative free-stream velocity for non-dimensionalization). For zero vehicle velocity, $C_{P,OUT}$ and C_P are identical because this is the stationary wind turbine case. For zero power out, Equation (28) can be reduced to the result in Equation (9). An interesting result pops out if the partial derivative of $C_{P,OUT}$ with respect to the

velocity ratio V/V_∞ is evaluated for $V/V_\infty =0$. The result is

$$\left[\frac{\partial C_{P,OUT}}{\partial \left(\frac{V}{V_\infty}\right)}\right]_{\frac{V}{V_\infty}=0} = C_P \left(3 - \frac{1}{\eta_P \eta_T \eta_G}\right)(29)$$

This shows that for non-zero C_P the net power, $C_{P,OUT}$, can be increased by moving the turbine in the upstream direction if the total efficiency is above 1/3. Further, it is possible to find the relative velocity that maximizes the output for a given total efficiency and C_P. These derivations can be found in [3]. Employing again 1D momentum theory to determine the upper bound of what is obtainable, we obtain the results shown in Figure 3 below.
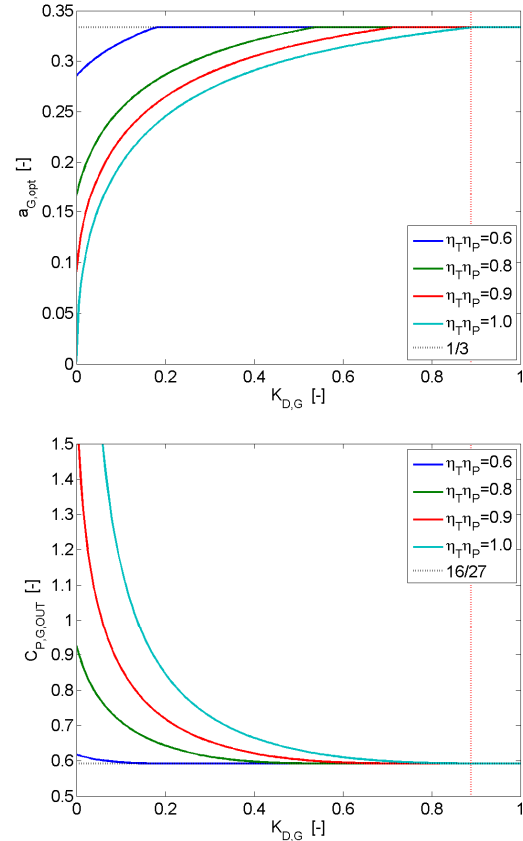
Figure 3: Optimal axial induction (upper graph) and corresponding net output power coefficient (lower graph) as function of generator medium drag loss coefficient $K_D = C_D A_V/A$ for different values of the product of transmission and propulsion efficiencies.

Note that for each of the points on the lines in Figure 3 the optimal velocity ratio is used. Further details on this may be found in [3]. From the

figure we see that in theory it is possible to increase the net power output from a wind turbine substantially by using some of the produced power to propel it against the wind. In fact it can be shown that in the ideal case with 1D momentum theory results for rotor aerodynamics and also the rest of the system ideal, there is no upper bound for the obtainable net power coefficient. Please refer to details on this in [3]. The physical explanation for the increased power output is that energy is extracted from a longer stream-tube than for the corresponding stationary turbine in the same amount of time.

Since this analysis is general, the same is of course possible also seen from the other medium, and therefore also theoretically possible to extract net output power from for instance the propeller car. The details of this can be shown in [3], where it is also shown that the power in this case can tend to infinity if the losses tend to zero.

2.5 One point rotor design algorithm based on BEM theory

So far, we have investigated several applications of wind turbines and propellers on vehicles moving relative to two media with a velocity difference. So far we have treated the problems using general relations, or, in some places, the rather crude 1D momentum theory for the rotor aerodynamics in order to provide a rough upper estimate. In this section a simple one point rotor optimization method, based on the Blade Element Momentum (BEM) method [2] will be derived for the generation (wind turbine) case for horizontal-axis rotors. The corresponding optimization for the layout of the propulsion case is analogous to the generation case, and is given in [3]. Furthermore, a general optimization algorithm optimizing all involved rotors (up to two: one generation and one propulsion), is given in [3].

The most obvious starting point for rotor optimization in the generator case could be a simple maximization of the generator efficiency. For a given, fixed drag loss this can be done two ways. Either to maximize C_P for each of a number of given C_T values and choose the one that maximizes the efficiency. The other way could be to minimize C_T for each of a number of given C_P and choose the one that maximizes the efficiency. This approach, however, requires that an integral quantity is kept constant at a given value while another integral quantity is either maximized or minimized. Albeit not impossible, this approach requires some extra effort to keep the integral quantity constant while optimizing the other.

In order to circumvent this we now derive an alternative method based on one of the most basic assumptions in BEM theory, the interdependence of the annular stream-tubes, which makes the optimization of rotors for the generation side for the type of vehicles treated in the present work straightforward and very efficient. The method is based on expressing the drag loss, $C_D A_V/A$, in terms of the rotor quantities. Isolating the drag loss factor from the generation efficiency, we get

$$K_D = \frac{A_V}{A} C_D = \frac{1}{\eta_G} C_P - C_T \qquad (30)$$

Reorganizing this using Equation (9) results in

$$K_D = \eta_P \eta_T \left(1 + \frac{1}{V/V_\infty} \right) C_P - C_T \qquad (31)$$

The right hand side of this relation can be viewed as the non-dimensional (using V_G) total effective propulsive force stemming from the isolated rotor, and could also have been derived using that line of thought. This quantity is of course what we want to maximize, given values for the efficiencies[2] and non-dimensional velocity. During the course of the optimization of a rotor for a vehicle, we may not know the final non-dimensional velocity or even the correct values of the specific efficiencies if these depend on the solution of the generator side rotor. Therefore these will have to be updated in an iterative manner in such cases. Introducing the local generation thrust and power coefficients also used in the review of the BEM method in Appendix B of [3]

$$C_{T,loc} = \frac{dT}{\frac{1}{2}\rho V_G^2 dA} = \frac{dT}{\frac{1}{2}\rho V_G^2 2\pi r dr} = \frac{\left(\frac{dT}{dr}\right)}{\pi \rho V_G^2 r} \quad (32)$$

$$C_{P,loc} = \frac{dP}{\frac{1}{2}\rho V_G^3 dA} = \frac{dP}{\frac{1}{2}\rho V_G^3 2\pi r dr} = \frac{\left(\frac{dP}{dr}\right)}{\pi \rho V_G^3 r} \quad (33)$$

From these the local coefficients the corresponding integral coefficients can be integrated

$$C_T = 2\int C_{T,loc} \cdot \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) \quad (34)$$

$$C_P = 2\int C_{P,loc} \cdot \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) \qquad (34)$$

---

[2] Note that the product of the efficiencies in Equation (31) is the effective transmission efficiency for the part of the mechanical power produced on the generator rotor that is used ideally for propulsion.

Equation (31) presents a convenient basis for optimization using the BEM method, because the introduction of Equations (34) and (35) results in

$$K_D = 2\int \left(\eta_P \eta_T \left(1 + \frac{1}{V/V_\infty} \right) C_{P,loc} - C_{T,loc} \right) \cdot \left(\frac{r}{R} \right) d\left(\frac{r}{R} \right)$$

(35)

Since the cornerstone in BEM theory is the interdependence of the annular stream-tubes, it is seen, that the optimization of the rotor for the specified conditions in this case corresponds to maximizing the expression in the parenthesis in the equation above for each radial element of the rotor. Therefore:

The BEM optimization of the generation rotor for a specified set of propulsion efficiency transmission efficiency and non-dimesional vehicle velocity is obtained by maximizing the radially local effective propulsive force coefficient

$$C_{PROPF,loc} = \eta_P \eta_T \left(1 + \frac{1}{V/V_\infty} \right) C_{P,loc} - C_{T,loc}$$

(36)

for each of the radial elements on the rotor separately.

Please note that the method above could have been applied to Equation (30), but the formulation shown above (Equation (36)) is preferred, since this involves also the performance of the other sub-parts of the wind vehicle. This makes an iterative optimization of the full vehicle including efficiencies that depend on parameters from a different part of the vehicle possible.

It is seen from Equation (36) that as the velocity ratio tends to zero, the optimal solution for the rotor tends to the solution for a conventional wind turbine, since in this case the C_P term is increasingly dominant. As the velocity ratio is increased, the design moves further away from a conventional wind turbine design, as the importance of the C_T term is increased.

The maximization of Equation (36) for each radial section is further simplified by realizing that in the BEM world, drag is always counterproductive for our purposes. Since the actual lift needed at any radial station on the rotor can be obtained by a simple scaling of the chord-length of the airfoil, the measure of the aerodynamic "goodness" of an airfoil section is the lift-to-drag ratio, which is identical to the ratio between the lift and drag coefficients, C_L/C_D. Therefore, if the goal is aerodynamic optimization of the rotor in one single design point, the airfoils should operate at the angle of attack where the maximum value of C_L/C_D occurs. This way, the quantity that should be adjusted so as to maximize $C_{PROPF,loc}$ for a given tip speed ratio and number of blades is only the non-dimensional chord-length, c/R.

For a given number of blades, N_B, tip speed ratio, and non-dimensional vehicle velocity, the aerodynamically optimal layout of the generation rotor is obtained by designing the blade such that the local angles of attack is where the lift to drag ratio, C_L/C_D, has it's maximum value and scaling the nondimensional chord-length c/R to maximize the local propulsive force coefficient, $C_{PROPF,loc}$, given by Equation (36).

In [3] it is shown analytically that the optimization of a horizontal axis power generator (wind turbine) using Equation (36) corresponds to maximizing C_P for a given C_T, or conversely, that for the obtained C_P, the corresponding C_T has the minimally obtainable value. This has interesting applications in other areas as well, since this is the goal in for instance applications within emergency hydraulic or electricity generators from wind for emergency situations in airplanes. Another field where such designs are applicable in the tip rotor concept for wind turbines investigated at Risø DTU [7].

An analogous one point optimization for the propulsion rotor (propeller) is possible, and in this case it can be shown that when optimizing the propulsion rotor using that method we have: For the obtained $C_{P,P}$, the corresponding $C_{T,P}$ has the maximally obtainable value. Or conversely: For the obtained $C_{T,P}$, the corresponding $C_{P,P}$ has the minimally obtainable value. The details of this, including the derivation of the algorithm and the results can be found in [3].

It should be noted that the results obtained using the BEM based algorithm includes the usual assumptions in BEM including neglecting the pressure difference due to rotation in the wake, and the corresponding added thrust. If, however, the tip speed ratios do not get too low (below approx 3), the assumption of negligibility is usually justified.

3. Results

In order to investigate the bounds of what is obtainable with realistic blade aerodynamic data for performance of a wind car rotor, we will here perform a general analysis of the trends of generation rotors optimized using the algorithm described in section 2.5 and investigate the consequences on . Subsequently, the maximum velocity obtainable for a well designed wind turbine car for the Aeolus race [1] is discussed. Finally, the obtainable

3.1. Obtainable generator rotor performance

As described earlier, the performance indicator of an airfoil section for use on a horizontal axis rotor is the maximum lift to drag ratio, which in real life to a first order depends on profile shape and Reynolds number. The general performance of rotor aerodynamics (for a horizontal axis rotor with a given airfoil in uniform, unyawed operation) is dictated by the tip speed ratio, number of blades and blade planform and twist. Since the last two quantities is what the outcome of the optimization, we will therefore present the integral performance of the obtained rotors for different variations of the following key numbers[3].

- $CLCD_{MAX}=max(C_L/C_D)$: Maximum lift to drag ratio for airfoil section
- $\lambda=\Omega R/V_\infty$: Tip speed ratio
- N_B : Number of blades

In stead of presenting the output of the optimization algorithm (here C_P, C_T and C_{PROPF}) as function of the efficiencies, velocity ratios or optimization controller term (the term on $C_{P,loc}$ in Equation (36)), we condense results by showing C_P as function of C_T for different values of the above key numbers. For any given C_T (which has a corresponding unambiguous value of the optimization controller term) it is the goal of this optimization to maximize C_P, so using the described type of plot, it is easy to see what effect the different combinations of key numbers has on the performance of the rotor. For all shown cases the rotor extends from r/R=0.2 to r/R=1.0.
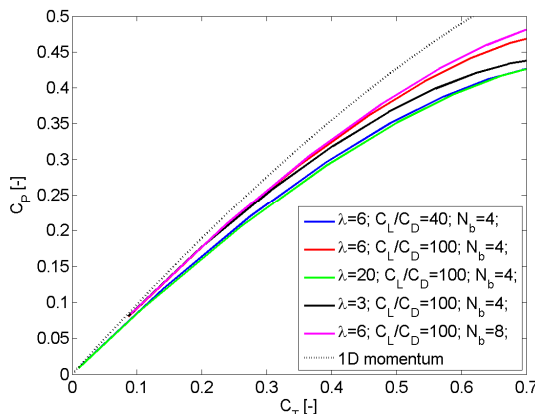

Figure 4: C_P versus C_T for different values of the aerdynamic key numbers.

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[3] In fact the optimization of the generation rotor can be given from $CLCD_{MAX},\lambda,N_B$, and the factor appearing on CP in Equation (36). However, the data are easier to interpret when presented as suggested.

Figure 4 show that increases in lift to drag ratio and number of blades are beneficial for the rotor aerodynamic. Furthermore, an example of the lift-to-drag dependent optimum of the tip speed ratio is shown.

The maximum lift to drag value depends on the airfoil section shape and the Reynolds number. In general thick airfoil sections and a reduction in Reynolds number decrease the maximum lift to drag value. These effects combined with structural strength considerations limit the number of blades and, in some cases, the tip speed ratio which otherwise is indicated by the basic result having a positive effect on the performance of generation rotors for these applications.

3.2. Example of maximum velocity for a wind car

Using the generator rotor design algorithm, it is possible to estimate the maximum velocity for a well designed wind turbine car for the Aeolus race (A=3m^2) on a plane track. The assumptions behind this estimate is car drag coefficient C_D=0.25 and car area A_V= 1m^2. Rolling resistance is based on a rolling resistance factor f_R=0.02 and a total vehicle mass of M=300 kg giving the rolling resistance force F_R= f_R Mg=59N. This is included in the computations in the propulsive efficiency as the only loss in that case

$$\eta_P = \frac{F_P}{F_{P,ideal}} = 1 - \frac{F_R V}{P_P} \qquad (37)$$

Here the power from the propulsion is obtained using Equation (4). The solution of this system requires iteration but is otherwise straightforward. The result of this with four blades, N_B=4, $CLCD_{MAX}$=100 and λ=5 is shown in Figure 5 below.
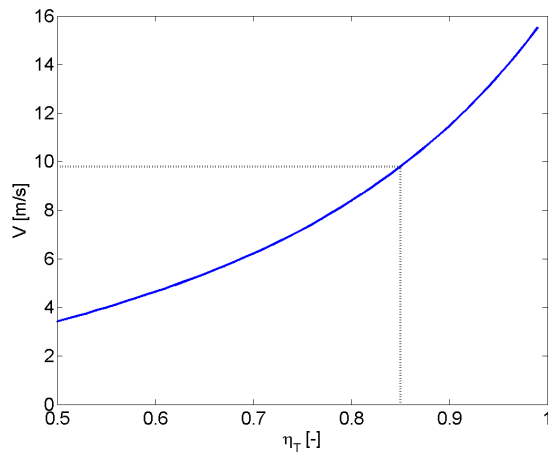
Figure 5: Vehicle velocity with V_∞=10m/s free wind. Vehicle velocity is shown as a function of transmission efficiency. Note that each particular point on the curve corresponds to a specific generator rotor design given by the optimization algorithm.

It is seen that it is imperative to limit transmission losses in order to get an otherwise well designed wind turbine car to co fast. For a transmission loss of 15%, which may be obtainable with careful design of the transmission system, it is seen that if the rotor is designed for these conditions, the velocity ratio of that vehicle is in excess of 98% of the free wind speed. Last years vehicles had top speeds just above 50% of the wind speed, so according to these estimates, there is still room for improvement.

As a comment to the result could be mentioned that the solution for a transmission efficiency of 85% correspond to a propulsive efficiency (losses due to rolling resistance only) of 87%. This figure increases with free wind speed, since in that case the transmitted power increases.

It should also be mentioned that enhancement of the performance using ducts or shrouds around the rotors is possible. The art of estimating the effects of a shroud on the rotor performance is not trivial. This issue was investigated in [8] using a combination of BEM and CFD modeling. The conclusion was that a significant increase in the performance of the rotor/shroud combination may be achieved if it is well designed. There are, however, still many open ends to the effect of this, since for instance the effect on the shroud on the tip effect is not trivial to estimate.

3.3. Obtainable maximum power output for a wind car turbine

It can be shown that the optimization of a rotor using Equation (36) for a given set of efficiencies and velocity ratio also maximizes power output in the net power out case described in section 2.4, see [3] for details. Therefore the results shown in Section 3.1 also apply to this case. By considering the same setup (η_T=0.85, rolling resistance, free stream velocity, drag loss coefficient, areas, mass) as in the previous section, we obtain for a four bladed rotor, again with $CLCD_{max}$=100 and λ=5 the result shown in Figure 6 below using Equation (28).
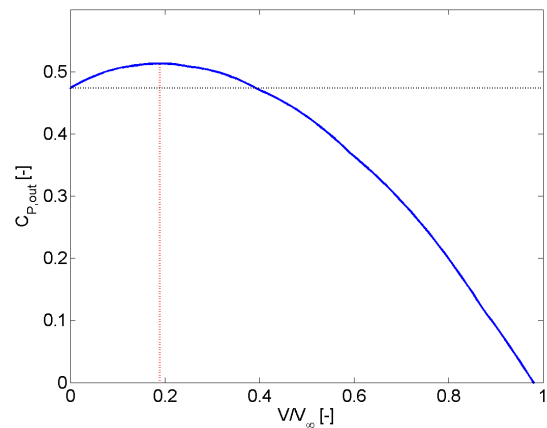

Figure 6: Net output power coefficient $C_{P,NET}$ as function of vehicle velocity ratio for a vehicle with the key numbers of the vehicle in Section 3.2 and rotor key numbers $CLCD_{max}$=100 and λ=5.

It is seen that for this setup, the best velocity ratio at these conditions is V/V_∞=0.19, where the corresponding net output power coefficient is $C_{P,OUT}$=0.513, an increase of 8.3% compared to the corresponding stationary best design of C_P=0.474. Even though probably not commercially exploitable, this result is at least intriguing from an academic point of view.

It is noted that it is also possible to yield net power output from the propeller type wind car. Results from application of realistic rotor data in that case can be found in [3].

4. Conclusions

By applying basic concepts from mechanics and fluid mechanics we have derived the equations applicable for the performance of vehicles "powered" by a difference in velocity between two media. A myriad of intriguing results emerge from these relations.

- It is theoretically and practically possible to build a wind driven car that can go directly upwind (using a generator/wind turbine in the air).
- The rotor design for this type of vehicle is generally substantially different than an ordinary wind turbine design. Only in the case of a car with a very low top speed will the rotor design tend to the usual wind turbine design case.
- It is theoretically possible to build a wind driven car that can go in the downwind direction faster than the free stream wind speed (using a propeller in the air)
- It is theoretically possible to build boats that will do the same as the two above vehicles.
- There does not exist a definitive upper limit for vehicles of this kind. As long as efficiencies are improved, the velocities will increase unasymptotically.
- Using a part of the power produced on a wind turbine to propel itself against the wind, it is theoretically possible to increase the net power output. Also in this case without an asymptotic upper limit.

Furthermore a simple one-point design optimization algorithm for the design of horizontal-axis rotors for these kinds of applications is shown, and examples of the obtainable performance of a wind turbine car and wind turbine car for power production using realistic airfoil data are given.

For a more in-depth treatment of everything treated in this paper including also several additional investigations and considerations, interested readers are referred to [3].

Acknowledgements

References

[1] http://www.windenergyevents.com/downloads/flyerRacingAeolus-druk.pdf

[2] *Glauert, H.*, in Durand, W.F. "Aerodynamic Theory" Division L: Airplane propellers.

[3] *Gaunaa, M.; Øye, S.; Mikkelsen, R.*, "Theory and Design of Flow Driven Vehicles using Streamwise-Axis Rotors for Energy Conversion" Risø Technical Report. Risø-R-1685(EN). To be published.

[4] www.winDTUrbineracer.com

[5] *Rankine*, Trans. naval architect. 1865.

[6] *Fox & McDonald* "An introduction to Fluid Mechanics", Texbook

[7] *Ari Bronstein.* "Aeroelastic modeling of tip-rotors on wind turbines". MSc Thesis. Risø DTU/DTU MEK.

[8] *Witold Skrzypinski.* "Rotor Design for Wind Powered Vehicle". MSc Thesis. Risø DTU/DTU MEK