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Published in:
Interface design of polymer matrix composites - mechanics, chemistry, modelling and manufacturing. Proceedings

Publication date:
2007

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
Mikkelsen, L. P. (2007). Length-scale dependent crack-growth. In B. F. Sørensen, L. P. Mikkelsen, H. Lilholt, S. Goutianos, & F. S. Abdul-Mahdi (Eds.), Interface design of polymer matrix composites - mechanics, chemistry, modelling and manufacturing. Proceedings (pp. 235-240). Roskilde: Risø National Laboratory.

Proceedings of the 28th Risø International Symposium on Materials Science:
Interface Design of Polymer Matrix Composites – Mechanics, Chemistry, Modelling and Manufacturing
Editors: B. F. Sørensen, L. P. Mikkelsen, H. Lilholt, S. Goutianos, F. S. Abdul-Mahdi
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LENGTH-SCALE DEPENDENT CRACK-GROWTH

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ABSTRACT

The Fleck-Hutchinson strain gradient theory from 2001 are used in the presented work to predict the length-scale dependency of crack growth in the case of small scale yielding at a crack tip loaded in pure mode I. The gradient-dependent plasticity model has been implemented using the user element interface in the commercial finite element code ABAQUS. The crack-tip fields are found to be strongly influenced by the occurrence of an internal material length scale. Compared with a conventional plasticity theory, a large incorporated length scale relatively to the size of the current yield zone is found to give a significant higher stress level at the crack-tip. As a consequence, crack-growth is predicted to be more pronounced under the influence of a large incorporated length scale.

1. INTRODUCTION

A finite element model based on a conventional plasticity theory lack the information of the underlying microstructure of the material. Therefore, finite element predictions will be size independent in spite of the fact that deformations gradients at small scale is known in a number of material to give a scale effects. Such a scale effect has been reported for a number of cases in metals; see e.g. Fleck and Hutchinson (1997) but also other materials show scale dependency. For epoxy, Lam and Chong (2000) has measured a scale dependency using micro indentation test.

The stress field in crack-tip and crack growth simulation is strongly localized. Therefore, if a material length scale is present a large influence on the local stress field is expected. In addition, for strong interfaces (4-5 times the yield stress of the material), conventional plasticity theories can not simulate crack growth, Tvergaard and Hutchinson (1992). This in spite of the fact that crack-growth from an experimental point of view can occurs for such cases.

In fiber reinforced composites, extensive plastic deformation can occurs at the fiber/matrix interface, Wang and van der Giessen (2004) due to the relatively large interface strength compared with the yield stress of the matrix material, Goutianos, Drews, Nielsen, Kingshott, Hvilsted ans Sørensen (2006). Therefore, in order to predict crack-growth in such cases, a length scale dependent model can be important.

In the present work, crack-tip and crack growth is modeled for an elastic-plastic material loading in pure mode I. In addition, the strain gradient dependent material behavior is demonstrated in a simple essential one-dimensional shear problem.

2. GRADIENT DEPENDENT J_2-FLOW PLASTICITY MODEL

In the present work, only the one length scale version of the Fleck and Hutchinson (2001) gradient dependent model is used. More detailed on the theory can be found in Fleck and Hutchinson (2001) and only a short summary is given here. In the model, a gradient dependent effective plastic strain is proposed,

$$\dot{E}^{P^2} = \dot{\varepsilon}^{P^2} + \ell_* \dot{\varepsilon}^P_{,i} \dot{\varepsilon}^P_{,i}, \tag{1}$$

which depends not only on the local effective plastic strain, $\dot{\varepsilon}^P$, but also the spatial gradient thereof, $\dot{\varepsilon}^P_{,j}$. In addition to the higher order plastic strains, $\dot{\varepsilon}^P_{,j}$, the model introduce higher order stresses, τ_i, (work conjugated to $\dot{\varepsilon}^P_{,j}$) and higher order boundary condition

$$\dot{\tau}_i n_i = 0 \ \text{or} \ \dot{\varepsilon}^P = 0 \tag{2}$$

E.g, corresponds the he boundary condition $\dot{\varepsilon}^P = 0$ to a case where to plastic deformation is “frozen” at an elastic-plastic/elastic interface where the elastic material prevent plastic deformation at the interface. The hardening of the material is chosen to be given by a standard power-law hardening such that the materials hardening modulus is given by

$$h[E^P] = \left(\frac{E E_T[E^P]}{E - E_T[E^P]} \right) \text{ where } E_T[E^P] = \frac{E}{n} \left(\frac{E E^P}{\sigma_y} + 1 \right)^{\frac{1}{n}-1} \tag{3}$$

where E denote Young’s modulus, σ_y the initial yield stress and n the hardening exponent. Note that in equation (3), the hardening modulus depends on the gradient dependent effective plastic strain E^P from equation (1) and not the conventional effective plastic strain ε^P_e.

3. NUMERICAL IMPLEMENTATION

The implementation of the strain gradient dependent plasticity model is inspired by Niordson and Hutchinson (2003) and many details regarding the implementation can be found there. Contrary to Niordson and Hutchinson (2003), the presented work implement the model in the user element subroutine interface in the commercial finite element code ABAQUS. In the finite element implementation, both the nodal displacement increments $\dot{U}^n_i$ and the nodal increment of the effective plastic strain $\dot{\varepsilon}_n{}^P$ are taken as fundamental unknowns.

$$\dot{u}_i = \sum_{n=1}^{2k} N_i^n \dot{U}^n \quad \text{and} \quad \dot{\varepsilon}^p = \sum_{n=1}^{l} M^n \dot{\varepsilon}_n^p \tag{4}$$

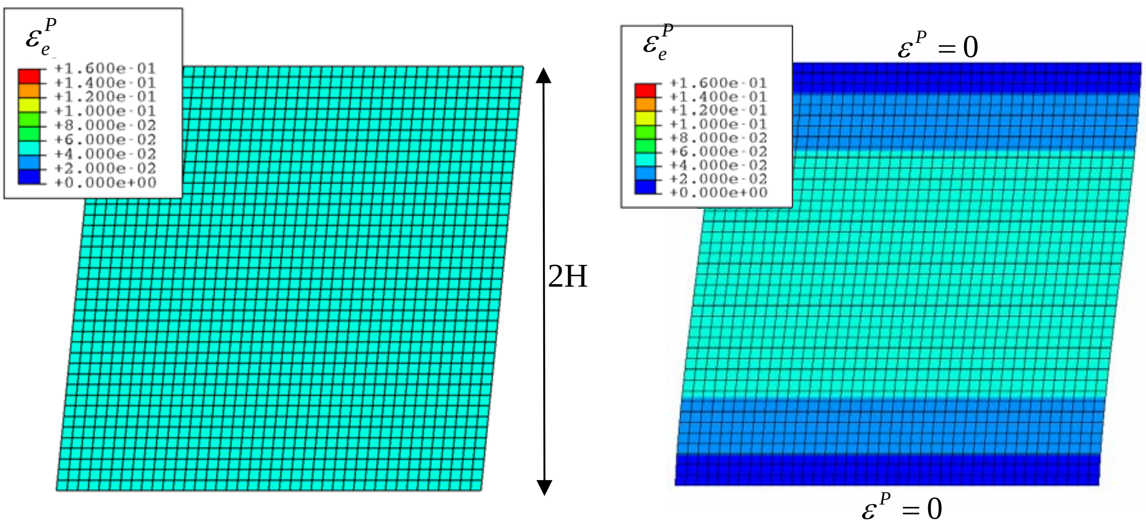
where $N_i^n(x_j)$ and $M_i^n(x_j)$ are shape functions defined such that equation (4) gives their respectively values in the point x_j inside the element. In the presented implementation, both the increment of the displacement and the increment of the effective plastic strain have been modeled using standard isoparametric shape functions. Nevertheless, the isoparametric element used to discretize the displacements increments and the effective plastic strain do not necessary include the same number of nodes. Later the best performance is found for an element with $k = 8$ and $l = 4$. Substitute (4) into the virtual work on incremental form, see Niordson and Hutchinson (2003) results in the system of linear equitation shown in (5)

$$\begin{bmatrix} K_e & K_{ep} \\ K_{ep}{}^T & K_p \end{bmatrix} \begin{bmatrix} \dot{U} \\ \dot{\varepsilon}^P \end{bmatrix} = \begin{bmatrix} \dot{F}_1 \\ \dot{F}_2 \end{bmatrix} + \begin{bmatrix} C_1 \\ 0 \end{bmatrix} \tag{5}$$

where $\dot{U}$ and $\dot{\varepsilon}^P$ are the fundamental unknowns in the finite element model. The term C_1 is the equilibrium term from where the unbalanced nodal forces can be extracted and from where ABAQUS found equilibrium when other element from ABAQUS is included in the model. The model is implemented in a plane strain version.

3. NUMERICAL RESULTS

Simple gradient dependent case. A simple case showing strain gradient dependent behavior is a thin slab loaded in shear. If the transverse displacement for all nodes is prescribed to vanish, it is essentially a one-dimensional problem; an infinity wide thin layer of material mounted between two rigid boundaries. Fig 1a show the predictions based on a conventional plasticity theory.

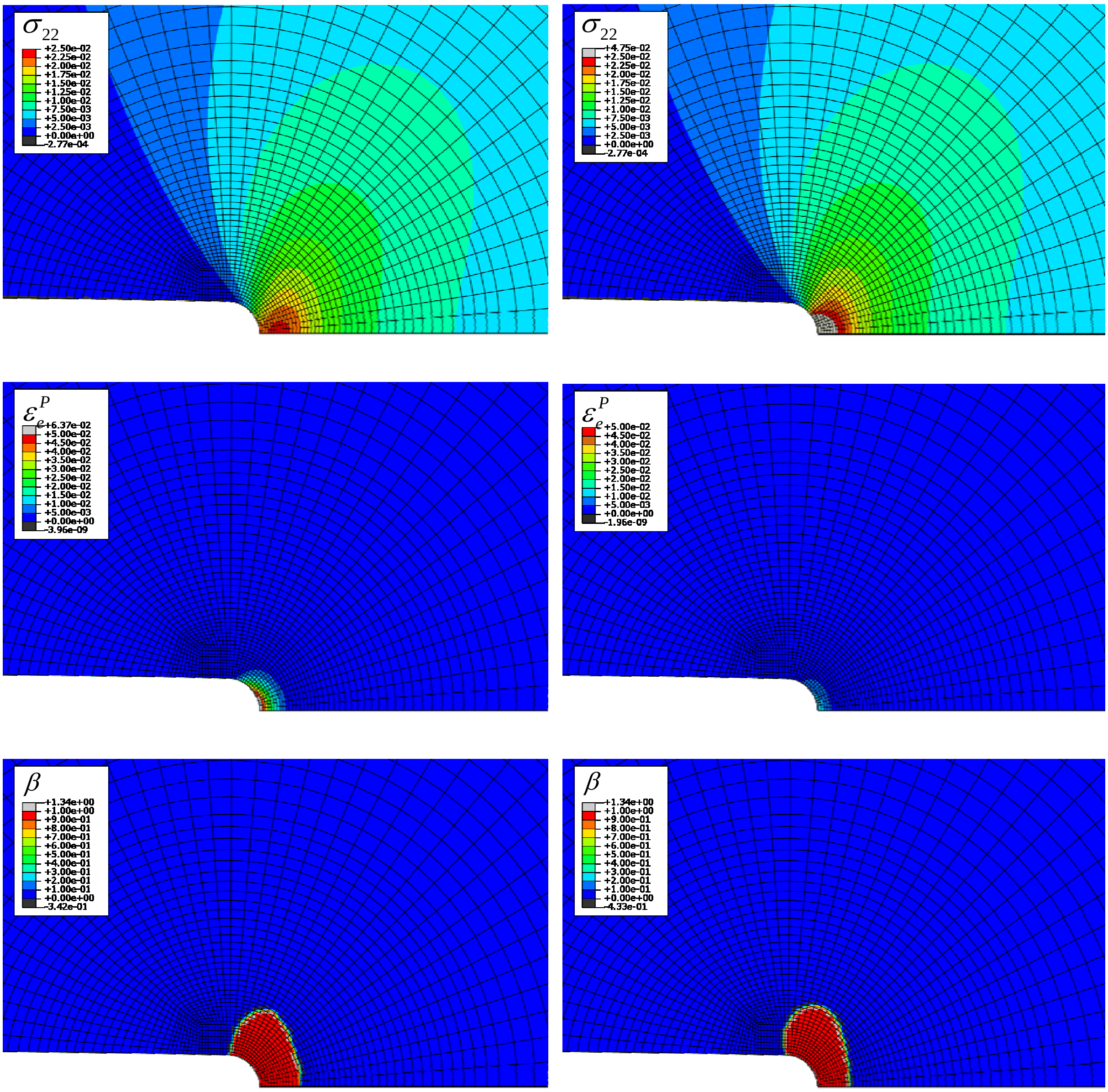


(a) $\ell_* = 0$ (b) $\ell_* = H$

Fig. 1. The variation of the local effective plastic strain of an infinity wide thin slab mounted between two rigid boundaries loaded in pure shear.

The material is given by the parameters $\sigma_y / E = 0.01$, $\nu = 0.3$ and $n = 5$. Shearing the thin layer to a given shear deformation, $U = 10\sigma_y H / E$, results for the conventional case in a uniform shear deformation stage. On the other hand, including an internal length scale as shown in Fig. 1b together with an additional boundary condition $\varepsilon^P = 0$, will results in the same strain state in the middle of the specimen, but with a boundary layer with a vanishing plastic strain at the boundary fulfilling the prescribed boundary condition. The overall shear traction required to give this deformation is approximately 50% higher corresponding to the corresponding overall shear traction for the conventional case, see Niordson and Hutchinson (2003) and Mikkelsen (2007).

<u>Crack-tip simulation.</u>



(a) $\ell_* / R_p = 0.05$; $\ell_* / R_{tip} = 0.1$ (b) $\ell_* / R_p = 0.5$; $\ell_* / R_{tip} = 1.0$

Fig. 2. Crack-tip field at a blunted crack-tip loaded in pure mode I at level, K_I, corresponding to $R_p / R_{tip} = 2.1$.

Fig. 2 shows a crack tip simulation based on the enhanced strain gradient dependent plasticity model incorporating an internal length scale. The model represents a blunted crack-tip in an elastic-plastic material. The material is given by the parameters, $\sigma_y / E = 0.01$, $\nu = 0.3$ and $n = 5$. Fig. 2 shows a zoom-in on the crack tip. The distance to the boundary where the prescribed displacement field corresponding to a pure mode I crack opening K_I -field is applied, see Tvergaard and Hutchinson (1992) is $L_\infty = 500 R_{tip}$ which was found to be sufficiently to represent a small scale yielding case. The crack tip is loaded to a level such that

$$R_p = \frac{1}{3\pi}\left(\frac{K_I}{\sigma_y}\right)^2 = 2.1 R_{tip} \tag{6}$$

which represent a measurement of the size of the current yielding zone at the crack-tip. Compared with the contours for the β-value (which is 0 for elastic and 1 for plastic deformation) the size of the yielding zone is found to be of this order. Despite, the yielding zone is quite similar for the two length scale modeled, the contours of the effective plastic strain and the stress state is found to be quite difference. Increasing the incorporated length scale from 5% to 50% of the reference size of the plastic zone, R_p, is found to result in a responds going from a conventional (local) plasticity solution to a solution found for a pure elastic case, see Mikkelsen (2006). The maximum stress level is found to be increased with nearly a factor of 2 indicating the possibility for crack growth at a lower applied K-field.

Crack-growth simulation. Fig. 3 shows the crack resistant curve for such a crack growth simulation using cohesive elements in combination with the enhanced plasticity theory. It can be seen from Fig. 3 that a large incorporated length scale results in crack-growth at a lower steady state level.

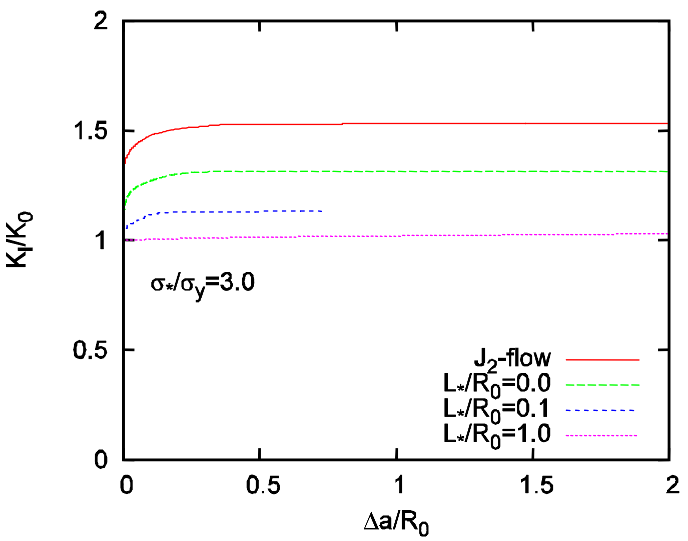


Fig. 3. Crack resistance curve for a the material $\sigma_y / E = 0.003$, $n = 10$, $\nu = 0.3$ and with a interface strength given by $\hat{\sigma} / \sigma_y = 3.0$ (from Mikkelsen, 2006).

ACKNOWLEDGEMENTS

The work has been financial supported by the Danish Technical Research Counsil through the project ’Interface Design of Composite Materials’.

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