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# ON THE SIMULATION OF KINK BANDS IN FIBER REINFORCED COMPOSITES

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## ABSTRACT

Simulations of kink band formation in fiber reinforced composites are carried out using the commercial finite element program ABAQUS. A smeared-out, plane constitutive model for fiber reinforced materials is implemented as a user subroutine, and effects of fiber misalignment on elastic and plastic deformation are studied under plane strain conditions.

Keywords: Kink band, constitutive model, ABAQUS, fiber misalignment

## 1. INTRODUCTION

Failure by kinking in fiber reinforced composites has been the subject of a number of recent investigations. It has been found that the compressive failure strength is considerably lower than the tensile strength, typically in the order of 50 to 60 percent for carbon fiber composites (Kyriakides et al., 1995). It has also been found that the compressive strength is governed by plastic yielding in the matrix (Budiansky, 1983), and futhermore, that small misalignments of the fibers have a large influence on the compressive strength, see Kyriakides et al. (1995). Several investigations of compression of a fiber reinforced material under the assumption of perfectly aligned fibers have predicted bifurcations stresses much higher than results obtained from experiments. Liu, Fleck and Sutcliffe (2004) include the effect of random waviness of the fibers using a Cosserat smeared-out finite element model.

In Kyriakides et al. (1995) the problem of predicting compressive strength for fiber reinforced materials is approached through an idealized model composite, with individual discretization of fiber and matrix material, and this two-dimensional michromechanical model was later (Kyriakides et al.,1998) extended to a three dimensional model. In these investigations the post-buckling respone is also studied, and it is shown that deformation localizes into well-defined bands of bent fibers.

Kink band formation in fiber reinforced materials was investigatated in Christoffersen and Jensen (1996) and in Jensen and Christoffersen (1997), where a plane constitutive model for perfectly bonded layered materials was introduced. It was found that this model contains essentially the same information in one point of the material as a complete finite element discretization of a representative volume element, like the model introduced in Kyriakides et al. (1995), and furthermore, that the critical stress is highly influenced by fiber volume fraction and the constitutive behavior of the constituents. Another conclusion is that the kink stress is reduced by taking non-linearity of the fibers into account compared to the predicted critical stress assuming linear-elastic fibers.

In the present study, a smeared-out plane constitutive model, as formulated by Christoffersen and Jensen (1996), is implemented as a user subroutine in the finite element program ABAQUS. Effects of fiber misalignment for elastic and plastic response are studied, and qualitatively compared to earlier results.

## 2. IMPLEMENTATION OF THE CONSTITUTIVE MODEL

For a detailed description of the constitutive model used in these kinkband simulations, see Jensen and Christoffersen (1997) and Christoffersen and Jensen (1996).

### 2.1 The constitutive model.

The general relations between rates of Cauchy stress and the strain rates in three dimensions are given by

$$\dot{\sigma}_{ij} = L_{ijkl}\epsilon_{kl} \tag{1}$$

where $L_{ijkl}$ are the elastic-plastic tangent moduli. The strain rates are given by

$$\epsilon_{ij} = \tfrac{1}{2}\left(v_{i,j} + v_{j,i}\right) \tag{2}$$

with $v_{i,j}$ denoting the components of the gradient of velocity components $v_i$.

The two-dimensional constitutive equations implemented in ABAQUS are given in the form

$$\dot{s}_{ij} = C_{ijkl}v_{l,k}, \quad i,\ j,\ k,\ l \in \{1,2\} \tag{3}$$

where $\dot{s}_{ij}$ are components of the nominal stress rate, $v_{i,j}$ are components of the gradient of velocity components $v_i$, and $C_{ijkl}$ are components of the tensor of nominal moduli, and from this formulation $L_{ijkl}$ is found by the relation

$$L_{ijkl} = C_{ijkl} + \tfrac{1}{2}\delta_{il}\sigma_{kj} + \tfrac{1}{2}\delta_{ik}\sigma_{lj} + \tfrac{1}{2}\sigma_{il}\delta_{kj} - \tfrac{1}{2}\sigma_{ik}\delta_{lj}, \quad i,\ j,\ k,\ l \in \{1,2\} \tag{4}$$

by futhermore using the assumption $\tau_{ij} \approx \sigma_{ij}$, with $\tau_{ij}$ being components of Kirchoff stress.

The components of the tensor of nominal moduli $C_{ijkl}$ are found from a mixture of fiber and matrix properties.

The constitutive model is based on three assumptions:

- Material lines parallel with the fibers are subject to a common stretching and rotation

- Planes parallel with the fibers transmit identical tractions
- The material of the constituents is elastic or elastic-plastic

The first of these assumptions correspond to a Voigt estimate for effective material properties whereas the second corresponds to a Reuss estimate. The third assumption is a standard specification of time independant materials.

The expressions for $C_{ijkl}$, the components of the tensor of nominal moduli, can be found in Christoffersen and Jensen (1996).

2.2 The behavior of the constituents.

For both constituents we have the relations

$$\dot{s}^c_{ij} = C^c_{ijkl} v^c_{l,k}, \tag{5}$$

similar to equation 3. Furthermore, both materials are assumed to be characterized by a power-law hardening with isotropical hardening. The power-law is given by

$$\epsilon = \begin{cases} \frac{\sigma}{E} & , \ \sigma \leq \sigma_y \\ \frac{\sigma_y}{E} \left[ \frac{1}{n} \left( \frac{\sigma}{\sigma_y} \right)^n - \frac{1}{n} + 1 \right] & , \ \sigma > \sigma_y \end{cases} \tag{6}$$

where $\sigma_y$ is the yield stress of the material and $n$ the hardening parameter. From this relation the tangentmodulus $E_t$ is found, and subsequently the values of $L^c_{ijkl}$.

In the kink-band simulations in the present paper, yielding only occurs in the matrix material, and thus the fibers are treated as linear elastic.

2.3 ABAQUS implementation.

The user subroutine UMAT in ABAQUS is used to implement the material behavior of the composite. In this routine, stresses in fibers and matrix are updated within each increment, and the elastic-plastic moduli are calculated by mixture of the properties of the constituents. The moduli $L_{ijkl}$ are determined from equation 4 and this value is returned to ABAQUS from UMAT. ABAQUS uses an updated Lagrangian formulation.

The model is implemented as a plane strain model, but can also be formulated for plane stress situations. In the present study, the fiber volume fraction is assumed to remain constant throughout the deformation.

Further details regarding the implementation of a constitutive model in ABAQUS can be found in Dunne and Petrinic (2005).

## 3. RESULTS

In this section, results for simulation of kink-band formation, and the response of compression of a single element, are presented. In the simulations, a rectangular block of fiber reinforced material, as shown in Fig. 1, is analyzed. The block is loaded under plane strain conditions, and has the dimensions $L = 10$ and $H = 3$ with the fiber direction outside the

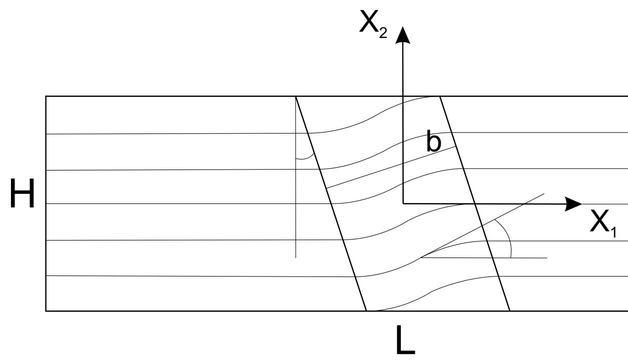



Fig. 1: Kink band geometry

kink band coinciding with the $x_1$-axis. Furthermore, the displacements $u_1$ and $u_2$ satisfy the boundary conditions

$$u_1 = 0 \; on \; x_1 = -\tfrac{L}{2} \tag{7}$$

$$u_2 = 0 \; on \; (x_1, x_2) = \left(-\tfrac{L}{2}, -\tfrac{H}{2}\right) \tag{8}$$

The width of the imperfection is $b$ and $\beta$ is the angle of the kink band. Inside the kink band the fibers are assumed to be at an angle $\phi$, and this imperfection is given by the expression:

$$\phi\left(x_1, x_2\right) = \tfrac{1}{2}\phi_m \left[\cos\left(\frac{2\pi\cos\beta}{b}\left(x_1 + x_2\tan\beta\right)\right) + 1\right] \tag{9}$$

where $\phi_m$ is the value of the angle in the middle of the kink-band. In all simulations, the values $b = 2$ and $\beta = 5°$ are used.

The material parameters used in these simulations are found in Kyriakides et al. (1995), with superscripts $f$ and $m$ denoting fiber and matrix respectively:

$$\nu^f = 0.263, \; \nu^m = 0.356, \; \frac{\sigma_y^f}{E^f} = 0.019, \; \frac{\sigma_y^m}{E^m} = 0.013 \tag{10}$$

In all simulations, a fiber volume fraction of $c^f = 0.6$ is being used. The hardening parameters are chosen to be $n^m = 4.5$ and $n^f = 2.5$, so the power-law curve closely resembles the Ramberg-Osgood curve used in Jensen and Christoffersen (1997), and results are obtained for $E^f/E^m = 35$ and $E^f/E^m = 100$.

The dotted green curve in Fig. 2 shows the material response under compression of an element of the fiber reinforced material. In Fig. 2, the plastic deformation of the matrix material is suppressed. The material response is modeled using one 4-node element with $2 \times 2$ Gauss integration points. The fibers are given a small initial homogenous inclination of $1°$. During the compressive deformation the fiber inclination will increase, resulting in a lower overall stiffness of the composite material. Nevertheless, for a stiffness ratio of the fiber/matrix system given by $E^f/E^m = 35$ the behavior is rather linear, whereas a stiffness ratio of $E^f/E^m = 100$ will result in a much more pronounced non-linear material response. However, even for this stiffness ratio the material will not experience a material softening behavior. Therefore, a localized deformation state in the rectangular block (Fig. 1) is not expected and is not found. Instead of a localized deformation state, an imperfection insensitive overall Euler buckling mode is developed as shown in Fig. 3. The red curve in Fig. 2

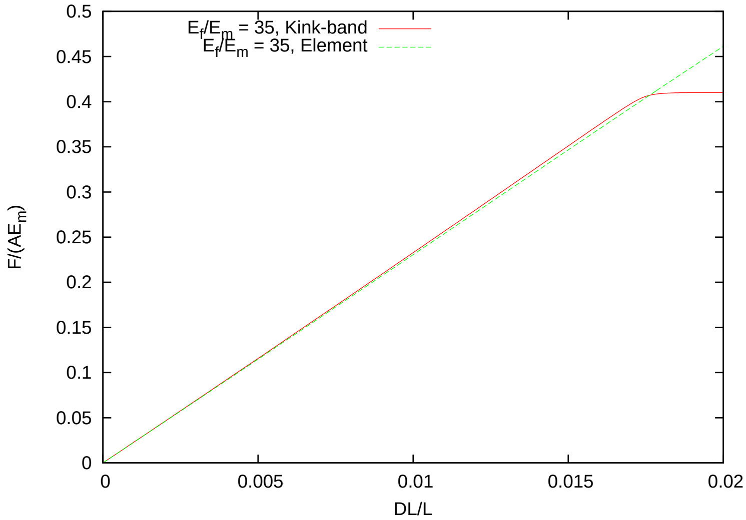



Fig. 2: Elastic deformation

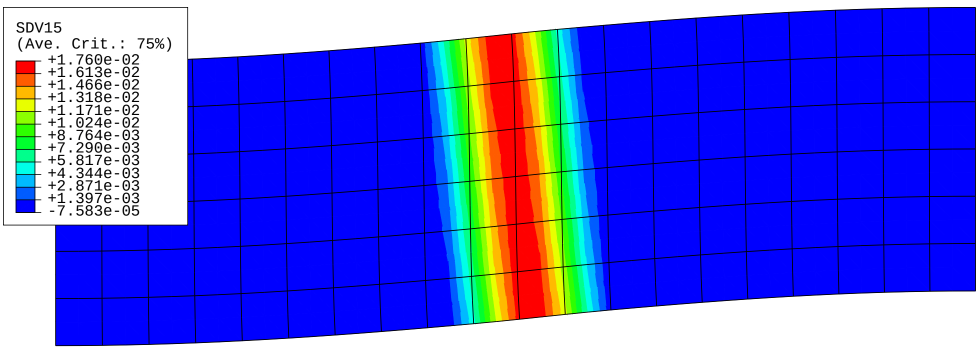



Fig. 3: Contourplot of imperfection

shows the corresponding load versus shortening curve for a rectangular block, indicating a maximum load carrying capacity of approximately $F/(AE^m) = 0.42$. In addition to the deformed mesh, Fig. 3 shows a contourplot of the initial prescribed misalignment of the fibers, with a maximum value of $\phi_m = 0.0176\ rad$ at the center of the kink band, corresponding to an imperfection of $\phi_m = 1°$.

In Fig. 4, the load-displacement curves for one single element, Fig. 4a, and for a block of material, Fig. 4b, are given for the stiffness ratio of the fiber/matrix system $E^f/E^m = 35$. In contrast to Fig. 2, plastic yielding occurs in the following simulation in the matrix material. During compressive loading it can be seen from Fig. 4a, that the smeared out model show extensive material softening and actually also snap-back behavior. Consequently the deformation state in the rectangular block localizes into a kink-band as shown in Fig. 5. The load-displacement curves are linear until a kink stress is reached and for sufficiently small angles of fiber misalignment, the phenomenon of snap-back occurs. From Fig. 4 it can be seen that the critical stress is very sensitive to the initial fiber-misalignment. For instance, for the kink-band formation in the block of material the critical stress almost doubles when the maximum angle of fiber-misalignment $\phi_m$ is reduced from $5°$ to $2°$.

A contour plot of the effective plastic strains in the matrix material during kink band

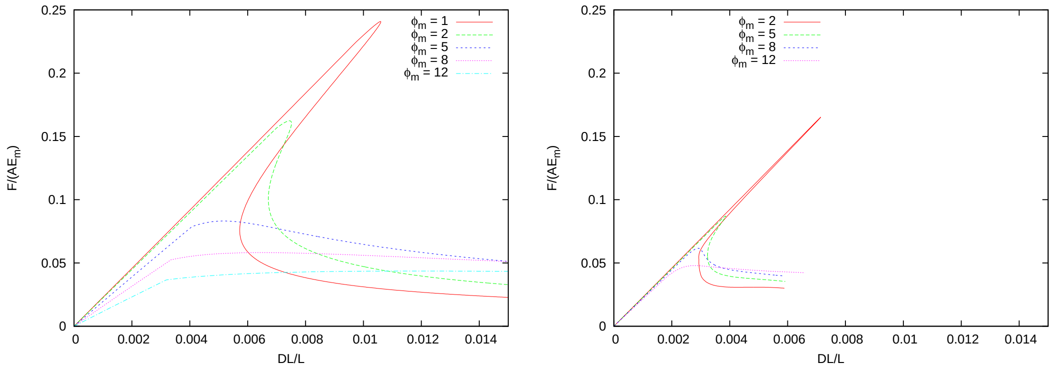



Fig. 4: Plastic deformation $E^f/E^m = 35$

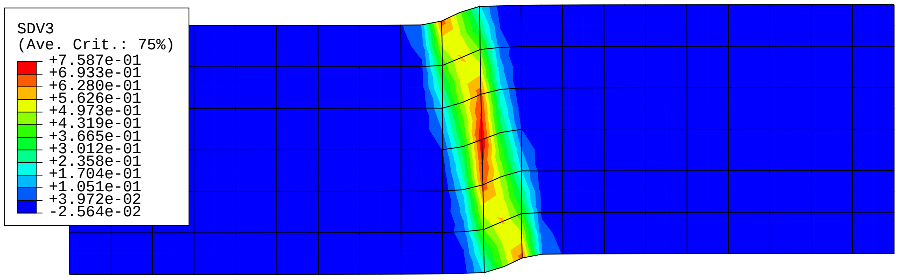



Fig. 5: Contour plot of effective plastic strain in the matrix material

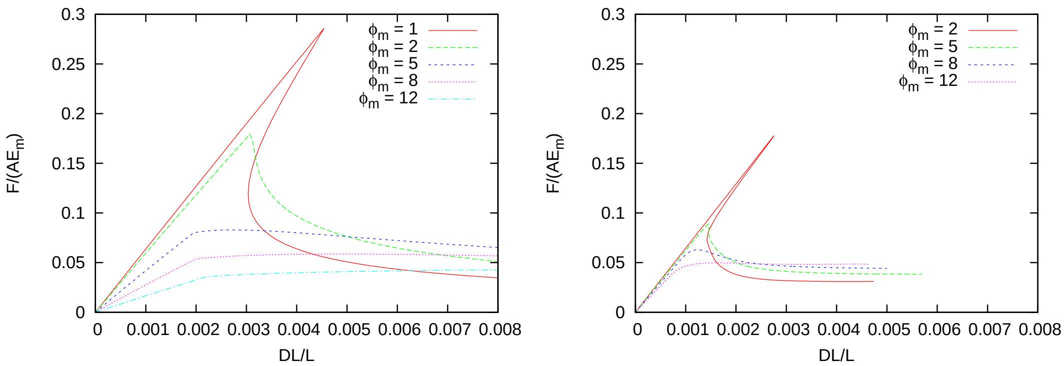
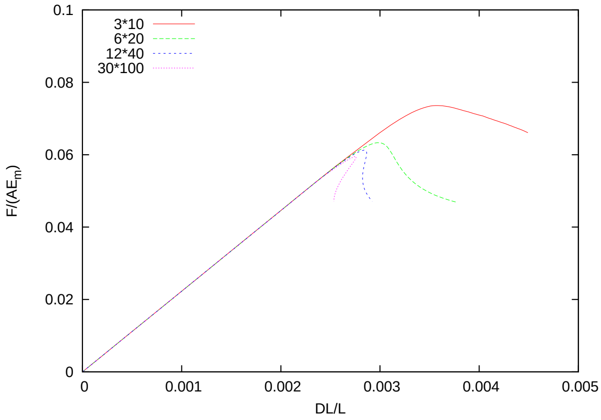



Fig. 6: Plastic deformation $E^f/E^m = 100$

formation is shown in Fig. 5.

The load-displacement curves for one single element and for a block of material, are shown in Fig. 6 for the stiffness ratio of the fiber/matrix system $E^f/E^m = 100$. These curves demonstrate a behavior similar to the one where $E^f/E^m = 35$. The implementation of the constitutive model does not include a material length-scale, and consequently the solutions show strong mesh dependency. This is illustrated in Fig. 7 which shows the load-displacement curves using four different meshes. All curves are found using the fiber misalignment variation (9) with $\phi_m = 8°$. Similar to what is obtained by e.g. (Pamin, 1994) a more brittle post-localization behavior are obtained when the mesh is refined. From Fig. 8 it can be seen that the kink-band formation occurs in one row of elements. Not only the post-necking behavior, but also the load carrying capacity is seen to be influenced by the mesh size.

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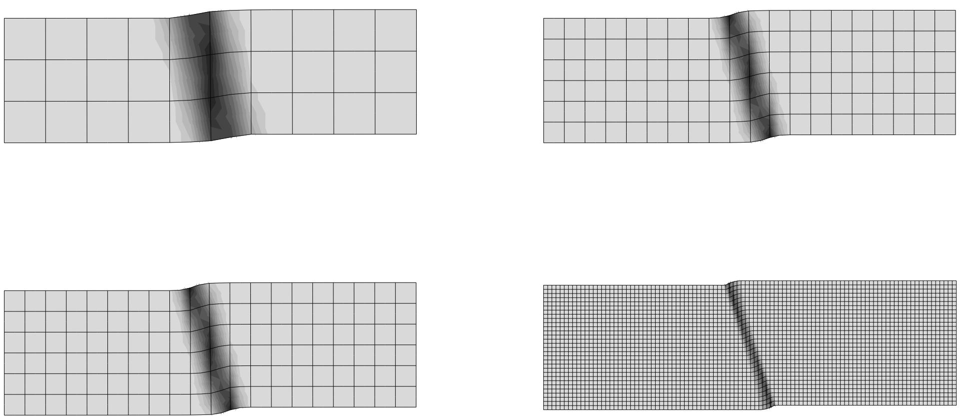

Fig. 8: Kink band mesh dependency

## 4. DISCUSSION

A plane constitutive model for fiber reinforced composites is implemented in ABAQUS as a user subroutine. It is demonstrated that the model qualitatively produces similar behavior as a micro-mechanical model, with regards to kink band development.The implementation of the model in ABAQUS does not include a material length-scale and therefore the kink-band will show strong mesh dependency. The implementation of the smeared-out composite material model in a finite element model has some immediately future applications, for instance kink-band development in more complex structures, such as a plate with a hole subject to compression. Other possible application is simulation of competing compressive failure mechanisms such as buckling and kink-band development in large fiber composite structures.

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