

Technical University of Denmark



The Analysis of Insulation Breakdown Probabilities by the Up-And-Down Method

Vibholm (fratrådt), Svend; Thyregod, Poul

Published in:
IEEE Transactions on Electrical Insulation

Link to article, DOI:
[10.1109/TEI.1986.348935](https://doi.org/10.1109/TEI.1986.348935)

Publication date:
1986

Document Version
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

Citation (APA):
Vibholm (fratrådt), S., & Thyregod, P. (1986). The Analysis of Insulation Breakdown Probabilities by the Up-And-Down Method. IEEE Transactions on Electrical Insulation, EI-21(2), 133-136. DOI: 10.1109/TEI.1986.348935

DTU Library

Technical Information Center of Denmark

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

THE ANALYSIS OF INSULATION BREAKDOWN PROBABILITIES BY THE UP-AND-DOWN METHOD

S. Vibholm and P. Thyregod
The Technical University
Lyngby, Denmark

ABSTRACT

This paper discusses the assessment of breakdown probability by means of the up-and-down method. The Dixon and Mood approximation to the maximum-likelihood estimate is compared with the exact maximum-likelihood estimate for a number of response patterns. Estimates of the 50% probability breakdown voltage and the scale parameter of the breakdown probability function are considered, and the estimation bias is investigated.

INTRODUCTION

In order to design insulation systems, it is necessary to assess the breakdown probability of the various air gaps and components of recovery-type insulation systems. The "up-and-down" method is widely used for estimation of the 50% probability breakdown voltage U_{50} . The analysis may be extended to yield estimates of a scaling parameter σ in the breakdown probability function. For most breakdown probability functions, the scaling parameter is simply the standard deviation. There are doubts, however, as to whether these estimates of σ are reliable.

The prediction of fractiles corresponding to extremely low probabilities, which are of particular interest to the design engineer, requires precise knowledge of both U_{50} and σ . A certain error in σ may double the error in the estimation of the 5% fractile, and it may more than triple the error in a predicted 0.1% fractile.

The classical up-and-down method of Dixon and Mood was based on a maximum-likelihood estimation of U_{50} and σ for a normal distribution [1,2]. However, because of the lack of modern computing facilities at that time, Dixon and Mood developed a simple approximation to the maximum-likelihood estimator. It is, therefore, of interest to consider how good this approximation actually is. In the past, such studies have been undertaken by many authors [3-9]. Wetherill performed extensive simulation studies of various strategies for the determination of U_{50} and σ and concluded that the up-and-down method, with the approximation used by Dixon and Mood, leads to biased estimates of σ . In the present paper the maximum likelihood estimates of U_{50} and σ for a normal distribution of breakdown probabilities are investigated.

MAXIMUM-LIKELIHOOD ESTIMATION OF

BREAKDOWN VOLTAGE PARAMETERS

The simple up-and-down method is designed to estimate the U_{50} value. In this test the voltage is applied at various levels A_i selected such that $A_{i+1} = A_i + d$, where

d is a constant voltage increment. Only one shot at a time is applied at a given level. The voltage level is then changed to A_{i-1} if the application resulted in a breakdown, or to A_{i+1} if the result was a withstand. A test sequence consists of a total of N voltage applications at I different voltage levels. The result of such a sequence can, without loss of relevant information, be summarized in a square matrix $\{n_{ij}\}$ with I rows and I columns. Each matrix element n_{ij} indicates the number of times the level has been changed from i to j . Consequently, all elements which are not of the form $n_{i, i-1}$ or $n_{i, i+1}$, are zero. For $I=5$ the matrix will be of the form:

$$\begin{pmatrix} 0 & n_{12} & 0 & 0 & 0 \\ n_{21} & 0 & n_{23} & 0 & 0 \\ 0 & n_{32} & 0 & n_{34} & 0 \\ 0 & 0 & n_{43} & 0 & n_{45} \\ 0 & 0 & 0 & n_{54} & 0 \end{pmatrix} \quad (1)$$

It is seen that

$$N = \sum_{i=1}^I (n_{i, i-1} + n_{i, i+1}) \quad (2)$$

Let $P(A_i, \mu, \sigma)$ denote the breakdown probability at level A_i in a normal distribution with mean $\mu = U_{50}$ and standard deviation σ . The likelihood function corresponding to the observations $\{n_{i,j}\}$ is the probability of obtaining the observations, which can be written as

$$L(\mu, \sigma) = \prod_{i=1}^I \left[\{1 - P(A_i, \mu, \sigma)\}^{n_{i, i+1}} P(A_i, \mu, \sigma)^{n_{i, i-1}} \right] \quad (3)$$

The maximum likelihood estimates of μ and σ are the values of μ and σ maximizing $L(\mu, \sigma)$. In practice it is simpler to maximize $\ln(L)$. Generally no explicit solution exists and hence the maximum has to be determined by numerical methods.

PROCEDURE

In the following analysis the total number of shots in a test sequence has been fixed to $N=20$, which makes it practicable to identify the possible array of the matrix $\{n_{ij}\}$ associated with a number of test sequences. Furthermore we shall restrict the analysis to sequences containing either 4 or 5 levels, which for $N=20$ ensures that meaningful information can be obtained. We consider only sequences where the highest voltage level applied always resulted in breakdowns and the lowest level always in withstands. All other levels must possess at least one breakdown. The sequence is always started at the lowest level.

All possible sequences with 4 voltage levels and $N=20$ with the above mentioned restrictions lead to 64 different 4th order square matrices representing 9403 different sequences. With 5 levels there are 175 different 5th order square matrices representing 19105 sequences. Each individual matrix thus contains information from many possible test sequences, each of which having the same elementary probability of occurrence. The following matrix

$$\{n_{ij}\} = \begin{pmatrix} 0 & 4 & 0 & 0 \\ 3 & 0 & 5 & 0 \\ 0 & 4 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix} \quad (4)$$

thus represents 525 different sequences. Three of these, marked (a), (b), and (c), are shown schematically in Fig. 1, in which o denotes withstand and x breakdown.

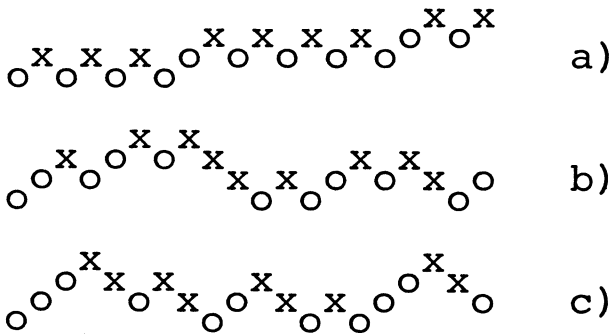


Fig. 1: 3 out of 525 possible sequences represented by matrix (4).

Sequence (a) is rather special, while (b) and (c) represent a large number of possible sequences. It must be emphasized, however, that all individual sequences forming that particular matrix have the same elementary probability and hence, the same likelihood function.

The maximum likelihood estimates of U_{50} and σ are calculated by maximizing the likelihood function with respect to U_{50} and σ . In order to facilitate the analysis the following dimensionless quantities m and s are introduced

$$m = \frac{U_{50} - A_1}{d} \quad (5)$$

$$s = d/\sigma,$$

in which A_1 is the lowest voltage level in the sequence, and d the step size.

Tables 1 and 2 show for selected matrices $\{n_{ij}\}$ the exact values of the maximum likelihood estimates \hat{m} and \hat{s} , and the corresponding values obtained from Dixon and Mood's approximation.

Table 1

Exact maximum likelihood estimates of $m=(U_{50}-A_1)/d$ and $s=d/\sigma$ compared with Dixon and Mood's approximation for selected responses in an up-and-down test with $N=20$ shots over 4 voltage levels. A normal probability distribution is assumed.

n_{12}	n_{21}	n_{23}	n_{32}	n_{34}	n_{43}	Exact maximum likelihood		Dixon & Mood Approximation	
						\hat{m}	\hat{s}	\hat{m}	\hat{s}
6	5	4	3	1	1	1.47	1.30	1.31	1.24
6	6	3	3	1	1	1.28	1.29	1.29	1.29
6	5	3	2	2	2	1.20	0.95	1.04	0.89
5	4	5	4	1	1	1.70	1.39	1.52	1.30
5	5	4	4	1	1	1.45	1.32	1.45	1.32
5	4	4	3	2	2	1.39	1.03	1.22	0.96
5	5	3	3	2	2	1.15	0.96	1.16	0.97
5	4	3	2	3	3	1.25	0.84	1.08	0.78
4	3	6	5	1	1	2.06	1.56	1.86	1.46
4	4	5	5	1	1	1.70	1.41	1.69	1.41
4	3	5	4	2	2	1.68	1.16	1.50	1.08
4	4	4	4	2	2	1.35	1.04	1.36	1.05
4	3	4	3	3	3	1.51	0.96	1.33	0.89
4	4	3	3	3	3	1.19	0.85	1.20	0.86
4	3	3	2	4	4	1.44	0.85	1.25	0.78
3	2	7	6	1	1	2.65	1.87	2.45	1.76
3	3	6	6	1	1	2.09	1.61	2.06	1.59
3	2	6	5	2	2	2.16	1.40	1.96	1.30
3	3	5	5	2	2	1.67	1.19	1.67	1.19
3	2	5	4	3	3	1.93	1.16	1.74	1.08
3	3	4	4	3	3	1.46	0.97	1.47	0.98
3	2	4	3	4	4	1.83	1.03	1.64	0.96
3	3	3	3	4	4	1.36	0.85	1.37	0.86
3	2	3	2	5	5	1.82	0.96	1.63	0.89
2	1	7	6	2	2	3.02	1.86	2.84	1.76
2	2	6	6	2	2	2.18	1.45	2.16	1.44
2	1	6	5	3	3	2.69	1.55	2.51	1.46
2	2	5	5	3	3	1.91	1.19	1.90	1.19
2	1	5	4	4	4	2.56	1.38	2.39	1.30
2	2	4	4	4	4	1.78	1.04	1.78	1.05
2	1	4	3	5	5	2.57	1.31	2.41	1.24

The data in Table 1 are limited to those 4th order square matrices which each represent not less than 100 individual sequences, and Table 2 is limited to those 5th order square matrices each of which contain more than 200 individual sequences. The limitation of the number of selected matrices has made the analysis as comprehensible as possible without losing significant information. If, in a practical test situation, the pattern of responses is extreme, the test is generally discarded and the testing conditions further investigated. We believe that the matrices given in Tables 1 and 2 represent such response patterns that normally would be accepted as originating from satisfactory testing conditions.

Table 2

Exact maximum likelihood estimates of $m=(U_{50}-A_1)/d$ and $s=d/\sigma$ compared with Dixon and Mood's approximation for selected responses in an up-and-down test with $N=20$ shots over 5 voltage levels. A normal probability distribution is assumed.

n ₁₂	n ₂₁	n ₂₃	n ₃₂	n ₃₄	n ₄₃	n ₄₅	n ₅₄	Exact maximum likelihood		Dixon & Mood Approximation	
								\hat{m}	\hat{s}	\hat{m}	\hat{s}
4	3	4	3	2	2	1	1	1.17	0.68	0.98	0.61
4	3	3	2	3	3	1	1	1.17	0.64	0.97	0.57
4	4	3	3	2	2	1	1	0.89	0.60	0.90	0.60
3	2	5	4	2	2	1	1	1.42	0.79	1.22	0.71
3	2	4	3	3	3	1	1	1.43	0.75	1.23	0.67
3	2	4	3	3	2	2	1	1.22	0.54	1.12	0.64
3	3	4	4	2	2	1	1	1.07	0.67	1.07	0.67
3	2	4	3	2	2	2	2	1.24	0.60	1.03	0.53
3	2	3	2	4	4	1	1	1.48	0.73	1.27	0.65
3	2	3	2	4	3	2	1	1.29	0.55	1.14	0.61
3	3	3	3	3	3	1	1	1.05	0.62	1.06	0.62
3	2	3	2	3	3	2	2	1.30	0.60	1.09	0.53
2	2	3	2	3	2	3	2	1.20	0.48	0.97	0.48
2	1	6	5	2	2	1	1	1.83	0.97	1.63	0.89
2	1	5	4	3	3	1	1	1.86	0.93	1.67	0.86
2	1	5	4	3	2	2	2	1.54	0.68	1.52	0.81
2	1	5	4	2	2	2	2	1.55	0.73	1.35	0.65
2	1	4	3	4	4	1	1	1.95	0.93	1.76	0.86
2	1	4	3	4	3	2	1	1.66	0.70	1.58	0.79
2	2	4	4	3	3	1	1	1.32	0.73	1.32	0.74
2	1	4	3	3	3	2	2	1.66	0.75	1.46	0.67
2	1	4	3	3	2	3	2	1.50	0.60	1.29	0.61
2	1	3	2	5	5	1	1	2.11	0.96	1.92	0.89
2	1	3	2	5	4	2	1	1.83	0.75	1.72	0.81
2	2	3	3	4	4	1	1	1.34	0.71	1.35	0.71
2	1	3	2	4	4	2	2	1.82	0.78	1.62	0.71
2	1	3	2	4	3	3	2	1.67	0.64	1.44	0.64
2	2	3	3	3	3	2	2	1.14	0.57	1.14	0.57
2	1	3	2	3	3	3	3	1.66	0.68	1.45	0.61
2	1	2	1	4	4	3	3	1.89	0.74	1.68	0.67

EXAMPLE

The data in Tables 1 and 2 show that the approximations suggested by Dixon and Mood lead to results which agree very well with results obtained from a maximum-likelihood estimation, provided that the associated matrices are symmetrical along the diagonal, i.e. $n_{ij}=n_{ji}$.

This reveals the fact that Dixon and Mood's approximation does not take the effect of the initial part of a sequence into account, although this effect cannot be avoided with a sample of limited size.

As an example, the data for the maximum-likelihood estimates for a normal distribution given in the two tables can be applied to an actual up-and-down test with $N=20$ in the following way:

Table 3

Matrix elements obtained in an up-and-down test with $N=20$.

n ₁₂	n ₂₁	n ₂₃	n ₃₂	n ₃₄	n ₄₃
4	4	3	3	3	3

Let the matrix element obtained from the test be as given in Table 3, and let the lowest step voltage $A_1=10.3$ kV and $d=0.2$ kV. With these data it follows from Table 1 that the most likely values for m and s are $\hat{m}=1.18$ and $\hat{s}=0.85$, respectively. The exact maximum-likelihood estimates of U_{50} and σ are then obtained from

$$\hat{U}_{50} = A_1 + \hat{m}\hat{d} \tag{6}$$

$$\hat{\sigma} = d/\hat{s}$$

leading to the estimates $\hat{U}_{50}=10.54$ kV and $\hat{\sigma}=0.24$ kV. An approximate confidence ellipsoid for U_{50} and σ may be obtained from the second derivatives of the likelihood-function as described in [3].

DISCUSSION

In order to study the variation of the exact maximum-likelihood estimates corresponding to various true values of U_{50} and σ one might perform an extensive simulation experiment. Instead of performing such a simulation study we have, however, chosen a more direct approach, i.e., for various combinations of the true values U_{50} and σ we have determined the probability distribution of the corresponding maximum-likelihood estimates \hat{U}_{50} and $\hat{\sigma}$.

The probability of obtaining a specific matrix depends on the true values of U_{50} and σ and on the values of A_1 and d . This probability is thus the probability of obtaining a specific set of values (\hat{m}, \hat{s}) for a maximum-likelihood estimation. For a specified set of matrices this probability, and the corresponding set of numbers of different sequences which will lead to each specific matrix in the set, can be utilized to weigh the maximum-likelihood estimates (\hat{m}, \hat{s}) corresponding to the individual matrices. This weighted average of the various possible values of the maximum-likelihood estimates will be denoted (m_e, s_e) . For fixed values of N , A_1 and d the sum of all the probabilities for all possible matrices, including the ones that we have omitted, will be unity.

In addition to the limitations already imposed on the sets of data under consideration we shall in the following analysis restrict ourselves to consider only such combinations of the true parameters m and s which associate a total probability larger than 5% with the set of matrices under investigation.

Each set m_t and s_t of the true values of m and s specify probabilities of occurrence for the individual matrices and for the corresponding values of the maximum-likelihood estimates. The weighted averages m_e and s_e of the maximum-likelihood estimates thus depend on the true values m_t and s_t . This deviation can be described by the parameters

$$\Delta m \equiv m_e - m_t \tag{7}$$

for the position and

$$a \equiv \frac{s_e - s_t}{s_t} \tag{8}$$

for the scale.

The average estimation bias Δm and a will be functions of both true values m_t and s_t . These functions are illustrated in Figs. 2 and 3 with m_t as abscissa and s_t as parameter. The hatched areas in Figs. 2 and 3 indicate the interval corresponding to plus minus one standard deviation of the estimates for $s_t=1.5$. Figs. 2 and 3 show that both the starting point and the step size are critical for obtaining a small error in the estimated U_{50} and σ . For $N=20$ the best step size is $d=1.5\sigma$ and the starting point a little more than two steps below U_{50} .

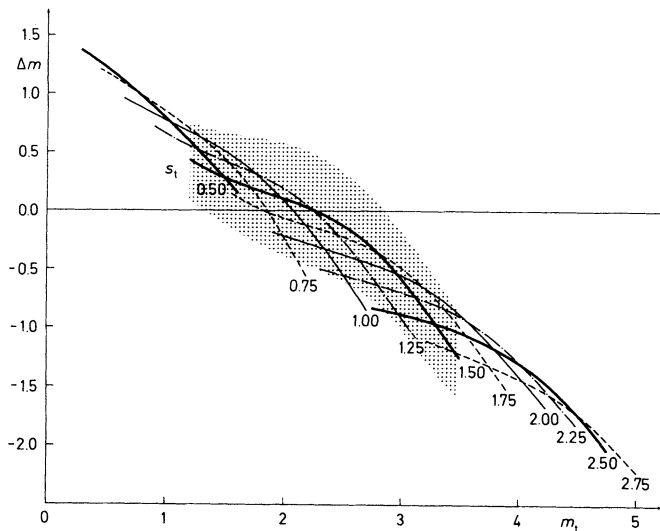


Fig. 2: The average estimation bias $\Delta m = m_e - m_t$ for the position as function of m_t and with s_t as parameter.

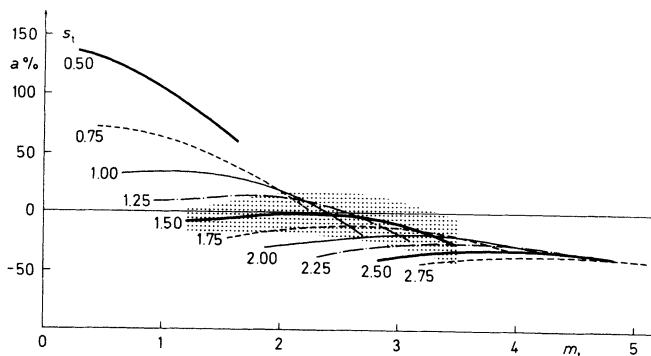


Fig. 3: The relative average estimation bias $a = (s_e - s_t) / s_t$ for the scale as function of m_t and with s_t as parameter.

It should be noted, however, that the larger values of the estimation bias correspond to combinations of true parameter values that assign a total probability of only slightly more than 5% to the investigated patterns. Thus, in general these combinations of true parameters lead to response patterns that would be considered unacceptable.

CONCLUSIONS

With digital computers readily available, a statistical analysis of a set of data obtained from an up-and-down test may just as well be obtained directly from a maximum-likelihood estimation instead of applying the approximate method used by Dixon and Mood which does not take the effect of the initial part of a sequence into account. In cases for which the number of shots in a sequence is $N=20$, estimates of U_{50} and σ can be obtained directly from Tables 1 and 2.

In order to determine the optimal values of N , A_1 and d which would yield the most reliable values of U_{50} and σ for $N>20$, it becomes necessary to apply more involved methods than those referred to in the present study.

REFERENCES

- [1] A. W. F. Edwards, "Likelihood", Cambridge University Press 1976.
- [2] G. W. Brown, "Method of Maximum Likelihood Applied to the Analysis of Flashover Data", IEEE Trans. PAS, Vol. PAS-88, No. 12, pp. 1823-1830, 1969.
- [3] G. Carrara and S. Yakov, "Statistical Evaluation of Dielectric Test Methods", L'Energia Elettrica No. 1, pp. 12-19, 1983.
- [4] K. A. Brownlee, J. L. Hodges, Jr., and Murray Rosenblatt, "The Up-and-Down Method with Small Samples", Journal of the American Statistical Association, Vol. 48, pp. 262-277, 1953.
- [5] W. J. Dixon, "The Up-and-Down Method for Small Samples", Journal of the American Statistical Association, Vol. 60, pp. 967-978, 1965.
- [6] W. J. Dixon and A. M. Mood, "A Method for Obtaining and Analysing Sensitivity Data", Journal of the American Statistical Association, Vol. 43, pp. 109-126, 1948.
- [7] R. E. Little, "The Up-and-Down Method for Small Samples with Extreme Value Response Distributions", Journal of the American Statistical Association, Vol. 69, pp. 803-806, 1974.
- [8] G. B. Wetherill, "Sequential Estimation of Quantal Response Curves", Journal of the Royal Statistical Society, Ser. B, Vol. 25, No. 1, pp. 1-48, 1963.
- [9] N. Hyltén-Cavallius and F. A. Chagus, "Possible Precision of Statistical Insulation Test Methods", IEEE PAS, Vol. PAS-102, No. 8, pp. 2372-2378, 1983.

Manuscript was received 13 February 1985 in revised form 1 May 1985.