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## Elastic Instability Phenomena

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**Boundary Element Techniques. Theory and Applications in Engineering.** By C. A. Brebbia, J. C. F. Telles, and L. C. Wrobel. Springer Verlag, New York, 1984. 464 pages. Price \$78.

REVIEWED BY S. MUKHERJEE<sup>1</sup>

Although the boundary element method (BEM) is rooted in classical integral equation formulation of problems, numerical implementation of the method, with applications to various branches of engineering, is of relatively recent vintage. The past decade, however, has witnessed a great deal of activity in this area of research as evidenced by many publications in journals and several recent books on the subject. The method has now been established as a strong competitor to the more widely used finite element method, both in terms of range of applicability to different kinds of problems and with regard to computational efficiency.

This book by Brebbia, Telles, and Wrobel is a very welcome addition to the literature on BEM applications in the general areas of solid and fluid mechanics and heat transfer. It is a comprehensive text on the subject which covers the entire field from the fundamental mathematical formulations to the latest research results. The subject matter is presented clearly throughout the text.

The book is divided into 14 chapters. It starts with a discussion of an unifying approach to approximate methods of analysis (such as finite difference, finite elements, and boundary elements) based on weighted residual methods. This is followed by a discussion of potential problems. Next comes a discussion of different kinds of interpolation functions that are necessary in order to reduce the governing integral equations to a system of algebraic ones.

The book then addresses itself to diffusion and then several topics in solids mechanics such as elastostatics, elastoplasticity, viscoplasticity, and plate bending. This is followed by chapters on elastodynamics, vibrations, and a brief discussion on applications in fluid mechanics. The concluding chapters present a discussion of coupling of the BEM with other methods such as the finite element method (FEM), followed by a computer program for two-dimensional elastostatics. Three appendices complete the book.

The fact that the authors of this book are well-recognized researchers in the field is clearly in evidence throughout the

book. I very much liked the attention to mathematical detail and the discussions of numerical implementations of the various integral equations. While, in my opinion, all the different topics are adequately treated here, the chapters on solid mechanics are particularly strong. Careful attention to the differentiated versions of the displacement integral equations—those that deliver the strains and stresses pointwise inside a solid body—is particularly useful. These equations often have strongly singular kernels and one must be very careful in dealing with them. Another attractive feature of the book is the discussion of exterior problems in infinite and semi-infinite domains. It is in these problems that the BEM is often more efficient than domain type techniques. Finally, the authors do a good job in pointing out how in certain problems a coupling of the BEM and FEM can lead to the two methods complementing each other so that the particular strengths of each can be exploited to advantage.

In summary, I feel that this text by Brebbia, Telles, and Wrobel will be very useful to researchers in computational mechanics. It can be used as an introductory text by readers unfamiliar with boundary element methods. It is also an excellent reference volume for others who are already familiar with this powerful, general purpose analytical/computational technique.

**Elastic Instability Phenomena.** By J. M. T. Thompson and G. W. Hunt. Wiley-Interscience, New York, 1984. 209 Pages. Price \$32.95.

REVIEWED BY V. TVERGAARD<sup>2</sup>

This book treats the elastic buckling of structures with a strong emphasis on interpretations in terms of the mathematical topological theory, known as catastrophe theory. The book is restricted to conservative systems, and it can be seen as a complement to the earlier book *A general Theory of Elastic Stability*, Wiley (1973), by the same authors. The book is intended for undergraduate and post-graduate courses in civil, mechanical, marine, and aerospace engineering.

In Chapter 1 fundamental principles of equilibrium and stability are introduced for an  $n$ -degree-of-freedom conservative mechanical system. Discretizations by a modal expansion or a finite element method, leading to  $n$ -degree-of-freedom approximations, are illustrated in Chapter 2 for vibrations and buckling of beams and struts.

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Loads and imperfections are introduced as control parameters in the potential energy, in Chapter 3. Limit points, asymmetric and symmetric bifurcation points are discussed, leading to an introduction of expressions for the seven "elementary" catastrophes which much of the book centers on. Four of these expressions relate to a single critical mode, and the remaining three relate to two simultaneous modes.

Chapter 4 treats single-mode buckling phenomena at distinct critical points. Simple rigid bar-spring models are used to illustrate the "fold catastrophe," which corresponds to snap-buckling at a limit point or to an asymmetric bifurcation point. The "cusp catastrophe" corresponds to symmetric postbifurcation behavior. Furthermore, the two higher-order unimodal singularities, the "swallowtail catastrophe" and the "butterfly catastrophe" are briefly discussed. As an example of the cusp singularity, a two-mode Rayleigh-Ritz analysis of a shallow arch is carried out in Chapter 5. The symmetric postbifurcation behavior corresponding to an asymmetric mode, the imperfection-sensitivity, and the influence of prestress are illustrated in this example, and the results are compared with experiments.

In Chapter 6 some simple rigid bar-spring models, and a structural model of an eccentrically stiffened panel, all with simultaneous (or nearly simultaneous) bifurcation in two modes, are used to illustrate the "elliptic, hyperbolic, or parabolic umbilic catastrophes." Routes through these umbilic catastrophes are explored. Chapter 7, on the other hand, is not directly oriented toward the catastrophe theory viewpoint, but gives a systematic treatment of bifurcation problems with many simultaneous bifurcation modes.

The last chapter gives a discussion of some engineering buckling problems, including plates, axially compressed cylindrical shells, and spherical shells under external pressure. Results are discussed, without showing many equations, and some relations to catastrophe theory are emphasized.

From the point of view of structural buckling this book treats a special selection of topics. The continuum mechanics approach to elastic postbuckling theory developed by W. T. Koiter (1945), and many later contributions in this area by B. Budiansky, J. W. Hutchinson, and others, is not given much prominence here. Nor are the many numerical investigations of elastic buckling phenomena; see *Buckling of Bars, Plates and Shells* by D. O. Brush and B. O. Almroth, McGraw-Hill (1975).

For the audience interested in the connexion between the elastic stability theory and catastrophe theory the present book will provide a good, broad introduction to the area.

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**Problems of Mixed Mode Crack Propagation.** By E. E. Gdoutos. Martinus Nijhoff, The Hague, 1984. 204 Pages.

**REVIEWED BY L. M. BROCK<sup>3</sup>**

This book is the second in a new series entitled *Engineering Applications of Fracture Mechanics*, edited by G. C. Sih. Sih also edits the long-standing series *Mechanics of Fracture*, and there are similarities between the two: both series highlight the strain-energy density criterion of linear-elastic fracture mechanics, and both give compendia of often previously published fracture analyses. However, as exemplified by this book, the new series is aimed at the applications-oriented researcher and engineer.

In keeping with the theme of mixed-mode fracture, many of

the problems treated in the book involve asymmetries in the alignments of crack path, geometry, and loading. After an introductory chapter on the strain density criterion, these problems are categorized into eight separate chapters and discussed in view of this criterion. The categories are: general two-dimensional crack problems; branched, interacting, and arc-shaped cracks; cracks radiating from holes or inclusions; cracks in composites and plates and shells; and three-dimensional crack problems. Crack propagation in the book does not refer to actual crack motion, so that all the problems treated are quasistatic. Moreover, except in the last chapter, they are all essentially two-dimensional in nature.

Each chapter includes expressions for crack-edge stress fields, strain-energy densities, predicted critical stresses, and parameters of particular importance in the given category, e.g., branching angles in the branched crack problems. Little developmental material is given, but the results are well (if not exhaustively) referenced.

The problem geometries are also clearly illustrated, and numerous parameter graphs are included, as well as, in some cases, illustrations of theoretical-experimental data comparisons.

Mixed-mode fracture is a somewhat controversial topic, since some commonly used fracture criteria, such as the maximum normal stress criterion, see a symmetry between a crack plane and the applied loading. In this book, the author meets the controversy head-on by pointing out situations in which mixed-mode fracture could occur, and by treating idealizations of such situations in view of a criterion that does not require such symmetry.

Indeed, he states that the book's main purpose is to illustrate the utility of the strain-energy density criterion, not to present large numbers of problems. Nevertheless, the number of problems, along with the consideration of crack path-geometry-loading alignment asymmetry and the large number of references given, is sufficient in itself to make the book a useful contribution. Researchers in fracture mechanics as well as the author's targeted audience will find the book a good information source.

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**Numerical Simulation of Non-Newtonian Flow.** By M. J. Crochet, A. R. Davies, and K. Walters. Elsevier, New York, 1984, 352 Pages. Price \$65.50

**REVIEWED BY R. I. TANNER<sup>4</sup>**

The volume under review is one of the first in this rapidly growing area. Due to the nonlinear history-dependent relations between stress and kinematic quantities, the field of non-Newtonian fluid mechanics needs good numerical methods urgently, and the present book is therefore timely.

The first two chapters review physical motivation and describe the equations to be solved. Several rival schools have developed in the description of materials with memory and the present treatment is conciliatory, fair, and ultimately pragmatic in this respect. The description of numerical methods deals both with the finite-difference approach and the finite-element approach. As the former has its greatest number of adherents among fluid dynamicists and the latter

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