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On the maximum backscattering cross section of passive linear arrays

Solymar, L.; Appel-Hansen, Jørgen

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puted as

$$\xi(x) = \begin{cases} (k/2a^2)^{1/3}x, & \text{convex parabola, } a > 0 \\ (k/2|a|)^{1/3} \exp(i2\pi/3)x, & \text{concave parabola, } a < 0. \end{cases} \quad (16)$$

III. CONCLUDING REMARKS

By using the concept of a negative radius of curvature, an analytic continuation of a parameter in the well known expression for the Fock currents on a convex parabola leads to a residue series representation for the fields on the surface of a concave parabola. The residue series result is applied to the calculation of the fields on the surface of a specific concave parabola, and the resulting fields are in excellent agreement with the fields calculated using an integral equation solution.

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REFERENCES

- [1] K. Furutsu, "On the field strength over a concave spherical surface," private communication.
- [2] J. R. Wait, "The whispering gallery nature of the earth-ionosphere waveguide at VLF," *IEEE Trans. Antennas Propagat.*, vol. AP-15, pp. 580-581, Jul. 1967.
- [3] R. H. Ott, "An alternative integral equation for propagation over irregular terrain," *Radio Sci.*, vol. 6, pp. 429-435, Apr. 1971.
- [4] R. G. Kouyoumjian, private communication, 1973.
- [5] H. Hochstadt, "Asymptotic formulas for diffraction by parabolic surfaces," *Commun. Pure Appl. Math.*, vol. X, pp. 311-329, 1957.
- [6] J. B. Keller, "Decay exponents and diffraction coefficients for surface waves on surfaces of non-constant curvature," *IRE Trans. Antennas Propagat.*, vol. 7, pp. 552-561, Dec. 1959.
- [7] C. O. Bloom, "Diffraction by a hyperbola," *University Microfilms*, 1964.
- [8] V. A. Fock, *Electromagnetic Diffraction and Propagation Problems*. New York: Pergamon, 1965, especially chapter 11.
- [9] M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions* (Applied Mathematics Series 55). Washington, D. C.: NBS, 1964.
- [10] R. H. Ott, "An alternative integral equation for propagation over irregular terrain," *Radio Sci.*, vol. 5, pp. 767-771, May 1970.

On the Maximum Backscattering Cross Section of Passive Linear Arrays

L. SOLYMAR AND J. APPEL-HANSEN

Abstract—The maximum backscattering cross section of an equispaced linear array connected to a reactive network and consisting of isotropic radiators is calculated for $n = 2, 3$, and 4 elements as a function of the incident angle and of the distance between the elements. On the basis of the results obtained, some conjectures pertaining to the general n -element array are presented.

A general formula for the relative scattering cross section (scattering cross section divided by the scattering cross section of the individual element) of an n -element linear array connected to an n -port matching network was derived in [1]. The approach was made specific in [2], where both theoretical and experimental results were presented for two parallel dipoles. In the present paper we shall give contour plots for the maximum back-scattering cross

section of linear arrays consisting of 2, 3, and 4 equispaced isotropic elements. Based on the results some conjectures are presented for an n -element array.

The relative backscattering cross section may be expressed as follows [1], [2]:

$$A_b(\theta) = |f(\theta)(Z_a + Z_m)^{-1}f(\theta)|^2 \quad (1)$$

where

$$f(\theta) = [\exp jkd \cos \theta, \exp j2kd \cos \theta, \dots, \exp jnk d \cos \theta] \quad (2)$$

and $Z_a = R_a + jX_a$ is the impedance matrix of the array, $Z_m = R_m + jX_m$ is the impedance matrix of the matching network, k is the free space wavenumber, d is the distance between the elements, θ is the incident angle, and n is the number of elements. It was assumed that the self-reactance of the elements was zero.

Backscattering for $\theta = 90^\circ$ is a special case of specular scattering, so we may quote the general result of [2] that the maximum occurs when

$$(Z_m)_{ii} = 0 \quad (Z_m)_{ik} = -j(X_a)_{ik} \quad (3)$$

and the value of the maximum is

$$A_b(90^\circ) = G_{\max}^2(90^\circ) \quad (4)$$

where $G_{\max}(90^\circ)$ is the maximum gain an array can have in the broadside direction. No such simple relation exists for other angles, and we are faced with the general problem of finding the $n(n+1)/2$ unknown elements of the matrix Z_m (assumed to be purely reactive).

The maximum backscattering cross section is found partly analytically (when the expressions are not too complex) and partly numerically, with the aid of a computer. Plots of $[A_b(\theta)]_{\max} = \text{constant}$ are shown in Fig. 1 for $n = 2, 3$, and 4 as functions of the incident angle θ and the spacing in wavelengths d/λ . In view of (4), the $\theta = 90^\circ$ sections of the diagrams should (and do) agree with the curves of Tai [3], calculated for maximum broadside gain.

The general shape of all three diagrams is the same. We feel justified, therefore, to extend our observations to n elements in general (they can be taken as having been proved for $n \leq 4$, and as conjectures for $n > 4$).

i) The optimum matching network (not necessarily unique—there may be other solutions as well) satisfies the following conditions:

$$(X_m)_{ii} = -(X_m)_{n-i+1, n-i+1} \quad (X_m)_{ik} = -(X_a)_{ik} \quad i \neq k \quad (5)$$

and

$$(X_m)_{(n+1)/2, (n+1)/2} = 0, \quad \text{for } n \text{ odd.} \quad (6)$$

ii) For $d/\lambda = 0.5$ and 1, $[A_b(\theta)]_{\max} = n^2$ in the directions $\theta = 0^\circ$ and 90° , and does not exceed n^2 in any other direction.

iii) For $d/\lambda < 0.5$, the maximum of $[A_b(\theta)]_{\max}$ as a function of θ occurs at $\theta = 0^\circ$.

iv) For $0.5 < d/\lambda < 1$ the maximum of $[A_b(\theta)]_{\max}$ as a function of θ occurs at $\theta = 90^\circ$.

v) The absolute maximum of $[A_b(\theta)]_{\max}$ for a given value of n occurs at $\theta = 0^\circ$ when $d/\lambda \rightarrow 0$ and its value is $\frac{1}{4}(n+n^2)^2$.

The last conclusion is particularly interesting because it shows the "supergain" effect ($[A_b(\theta)]_{\max}$ is proportional to n^4 when $n \gg 1$) and states, at the same time, that the absolute maximum of backscattering is always below the absolute maximum of forward scattering

$$\frac{1}{4}(n+n^2)^2 < n^4, \quad \text{for } n > 1 \quad (7)$$

as may be easily proven.

Taking real elements, the conclusions are not expected to alter appreciably. We found, for example, that the main change for half-wave dipoles in parallel is that the relative backscattering cross section is somewhat increased in the direction $\theta = 90^\circ$ and reduced for $\theta = 0^\circ$.

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The authors are with the Laboratory of Electromagnetic Theory, Technical University of Denmark, Lyngby, Denmark.

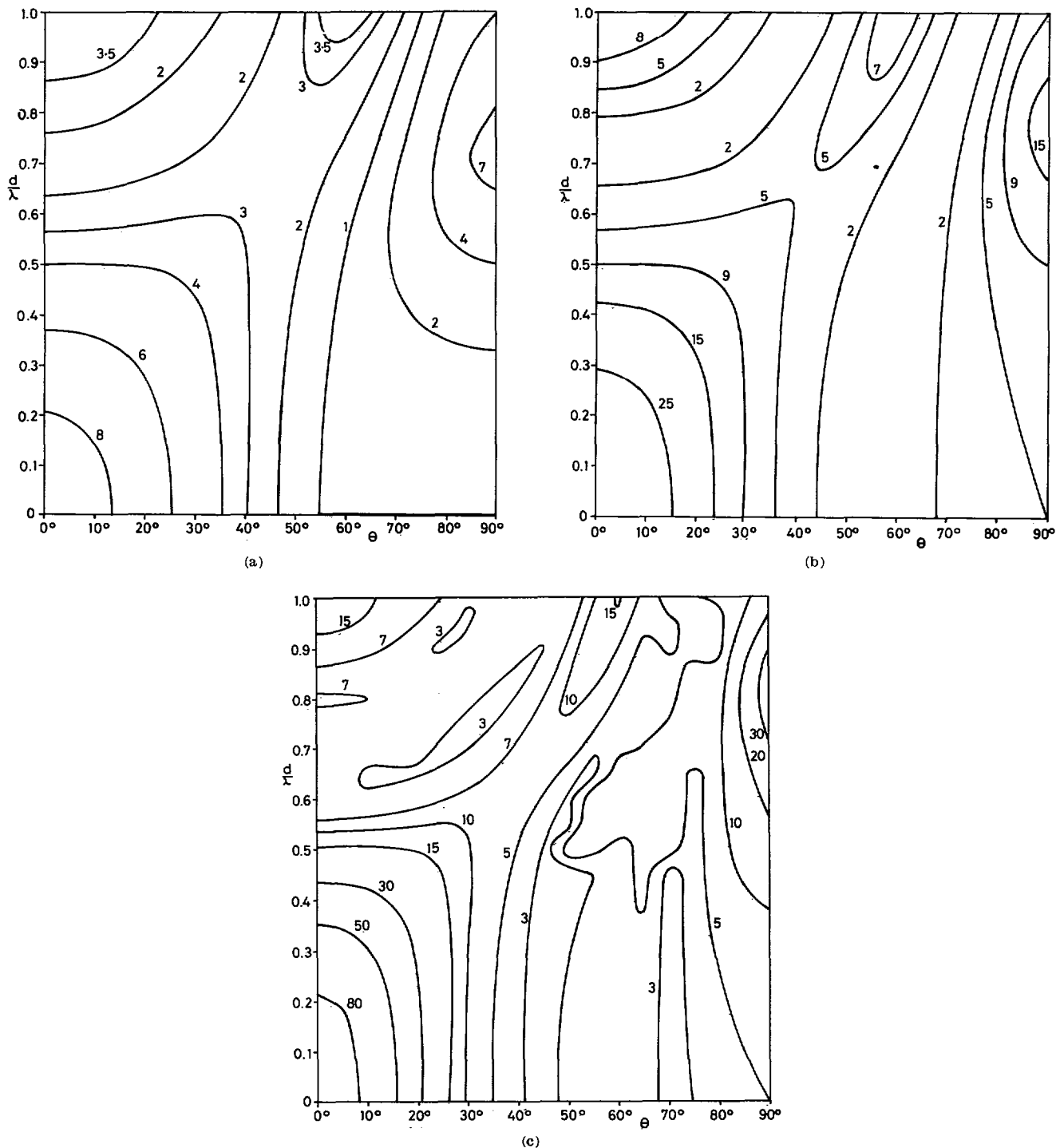


Fig. 1. Contour plots of $[A_b(\theta)]_{\max}$. (a) $n = 2$. (b) $n = 3$. (c) $n = 4$.

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REFERENCES

- [1] L. Solymar, "On the scattering cross section of passive linear arrays," *IEEE Trans. Antennas Propagat.* (Commun.), vol. AP-21, pp. 391-392, May 1973.
- [2] J. Appel-Hansen and L. Solymar, "Theoretical and experimental determination of the maximum scattering cross-section of passive linear arrays," to be published in *Proc. European Microwave Conference*, Brussels, Belgium, Sept. 1973.
- [3] C. T. Tai, "The optimum directivity of uniformly spaced broadside arrays of dipoles," *IEEE Trans. Antennas Propagat.*, vol. AP-11, pp. 447-454, Jul. 1964.

Backscattering by Cylinders Enclosed in Wire Cages

C. M. TANG AND H. S. TAN

Abstract—Calculations of the backscattering width of various metal cylinders enclosed in wire cages are compared with measured values obtained from a standing-wave-ratio method. The calculations are done by assuming that the wire cage simulates a thin continuous current sheath with suitable boundary conditions. The

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 The authors are with the University of Malaya, Kuala Lumpur, Malaysia.