Technical University of Denmark

# Advances in Topology and Material Optimization - Methods and Industrial Applications

Extended Abstracts

**Stolpe, Mathias**

*Publication date:*
2009

*Document Version*
Publisher's PDF, also known as Version of record

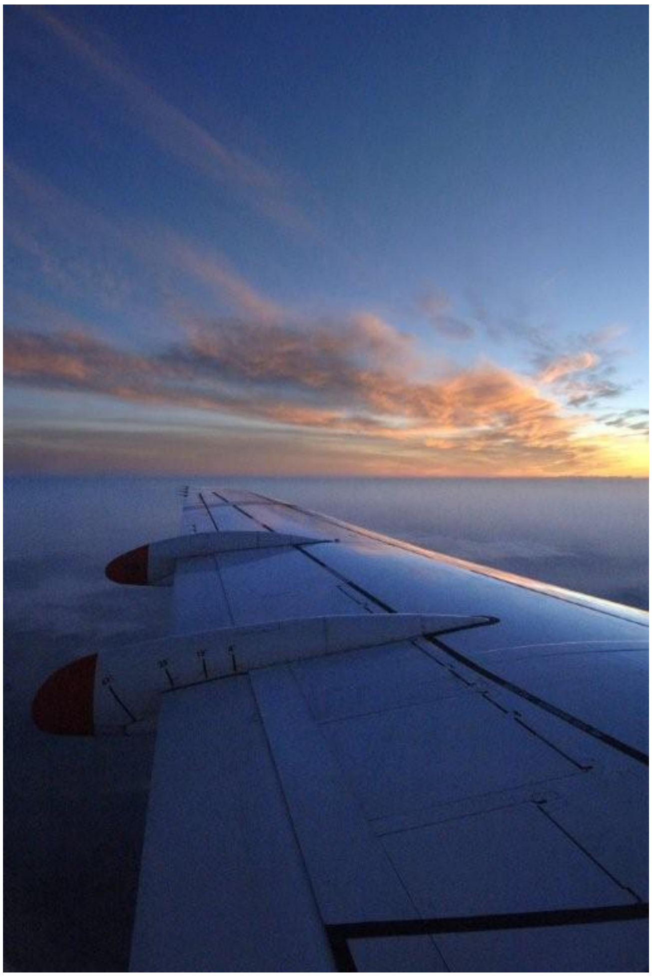


Photo: Colorbox

PLATO-N International Workshop

# Advances in Topology and Material Optimization — Methods and Industrial Applications

23—25 September, 2009

Technical University of Denmark

Kgs. Lyngby, Denmark

## Extended Abstracts

Advances in Topology and Material Optimization —
Methods and Industrial Applications: Extended Abstracts
Edited by M. Stolpe
Technical University of Denmark, 2009
DTU Mathematics Report No. 2009 — 02
ISSN: 0904 — 7611

Welcome to the PLATO-N International Workshop

# Advances in Topology and Material Optimization — Methods and Industrial Applications

23—25 September, 2009

Technical University of Denmark, Denmark

## Plenary Speakers

Dr. Thomas A. Grandine, The Boeing Company, USA
Dr. Lars Krog, Airbus, United Kingdom
Professor Michael Stingl, Friedrich-Alexander Universität Erlangen-Nürnberg, Germany

## Scientific Committee

Professor Michal Kocvara, University of Birmingham, United Kingdom
Professor Klaus Schittkowski, University of Bayreuth, Germany
Professor Aharon Ben-Tal, Technion – Israel Institute of Technology, Israel
Professor Günter Leugering, Friedrich-Alexander University Erlangen-Nuremberg, Germany
Dr. Mathias Stolpe, Technical University of Denmark, Denmark

## Workshop Venue

The workshop is held at Scion DTU on the Lyngby campus of the Technical University of Denmark (DTU). The address of Scion DTU is Diplomvej, Building 372, DK – 2800 Kgs. Lyngby, Denmark.

## Organization

The workshop is organized by the partners in the EU funded project PLATO-N (www.plato-n.org). PLATO-N is an EU project within the 6th Framework Programme (Aeronautics). PLATO-N aims at enabling the operational integration of optimization assistance as a standard procedure in the conceptual design process for the European aerospace industry.

The local organizing committee consists of

Dr. Mathias Stolpe
DTU Mathematics
Matematiktorvet, B. 303S
DK – 2800 Kgs. Lyngby
Denmark

M.Sc Ellen Juel Nielsen
DTU Mathematics
Matematiktorvet, B. 303S
DK – 2800 Kgs. Lyngby
Denmark

| **September 23, Morning session (Scion DTU, Building 373)**<br>**Chair: Mathias Stolpe** | | |
|---|---|---|
| **Time** | **Presenting author** | **Title** |
| 08.30 – 09.30 | | Registration |
| 09.30 – 09.40 | Mathias Stolpe | Welcome and practicalities |
| 09.40 – 10.00 | Martin P. Bendsøe | Introduction to the PLATO-N project<br>*M.P. Bendsøe* |
| 10.00 – 11.00 | Michael Stingl | Free Material Optimization and the PLATO-N software<br>*M. Stingl and M. Kočvara* |
| 11.00 – 11.30 | | Coffee break |
| 11.30 – 12.00 | Gábor Bodnár | FMO Results Interpretation: Supporting the Development of Design Concepts<br>*G. Bodnár* |
| 12.00 – 12.30 | Michael Bogomolny | Laminate Interpretation of Results from Free Material Optimization<br>*M. Bogomolny, M. Stolpe, and M.P. Bendsøe* |

# Free Material Optimization and the Plato-N software

**M. Stingl[1] and M. Kočvara[2]**

[1] Applied Mathematics II, Department Mathematics, University of Erlangen Nägelsbachstrstr. 49b, 91052 Erlangen, Germany, stingl@am.uni-erlangen.de
[2] School of Mathematics, University of Birmingham, Edgbaston Birmingham B15 2TT, UK, kocvara@maths.bham.ac.uk

**Abstract**
Recently, a new class of multidisciplinary free material optimization (MDFMO) models has been introduced. In the framework of the EU FP6 project Plato-N, efficient algorithms for the solution of MDFMO problems have been developed and implemented in the software platform Plato-N. The software platform allows for definition and solution of MDFMO problems as well as visualisation and interpretation of the optimal results.

**Keywords:** Structural optimization, Free material optimization, Optimization software.

## 1. Introduction
Free material optimization (FMO) is a branch of structural optimization that gains more and more interest in the recent years. The underlying FMO model was introduced in [1]. The main advantage of FMO is that the design variable is the full elastic stiffness tensor that can vary from point to point. Compared to more traditional structural optimization approaches such as topology optimization, FMO leads to significantly lighter optimal structures having the same compliance as the TO results.
The basic FMO model has other drawbacks, though. For example, structures may fail due to high stresses, or due to lack of stability of the optimal structure. Moreover, due to the given design freedom freedom, one has to face the question of interpretation of the optimal result. Both issues, the generalization of the basic FMO model as well as the question of practical interpretation of FMO results have been intensively studied in the recent EU FP6 project PLATO-N.

## 2. Free material optimization

### 2.1. A multidisciplinary FMO model

One of the major goals of the Plato-N project was to extend the basic FMO model with respect to multidisciplinary constraints such as displacement constraints, stress- and strain constraints and global stability constraints (see, for instance, [2, 3]). The generalized FMO models lead to very large-scale nonconvex semidefinite programming problems, Accordingly, new mathematical tools as well as new optimization algorithms had to be developed.

### 2.2. FMO algorithms
The major optimization algorithms used in Plato-N extend the sequential convex programming concept to problems with matrix variables and semidefinite constraints (see [4]). The basic idea of the new method is to approximate the original optimization problem by a sequence of subproblems, in which nonlinear functions (defined in matrix variables) are approximated by block separable convex functions. The subproblems are semidefinite programs with a favorable structure, which can be efficiently solved by existing SDP software. Most recently, the method is combined with an exterior penalty approach in order to treat 'difficult' constraints.

### 2.3. The software platform Plato-N
The software system Plato-N comprises of two major application programs: the FMO Kernel (FMK) and the FMO Studio (FMS). The system is capable of communication with third party software via standardized file interfaces, e.g. NASTRAN BDF. FMK is responsible for performing FMO computations, while the role of FMS is to provide a graphical user interface for convenient configuration of the FMO-problem

and to serve as a post-processing tool with various visualization possibilities helping the interpretation of FMO results. A simplified scheme and a snapshot of the Plato-N software system is presented in the following Figure 1.

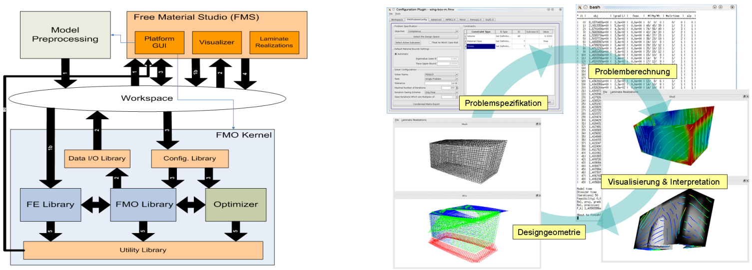



Figure 1: The software system: scheme (left) and snapshot (right)

2.4. Case studies

A few case studies are presented. Among them are FMO problems with displacement, stress and stability constraints. Figure 2 (top left) shows a classic L-shape domain which is clamped at the bottom and loaded in vertical direction on the right hand side. Using the basic FMO model, one obtains optimal material properties as indicated in Figure 2 (top left and right) and a stress concentration in the reentrant corner, Figure 2 (top middle). Constraining the maximal von Mises stress from above, the stress concentration can be avoided Figure 3 (bottom middle). In Figure 3 (bottom right) one can see that the optimal MDFMO result uses an 'arch-like' material orientation in order to avoid the sharp corner.

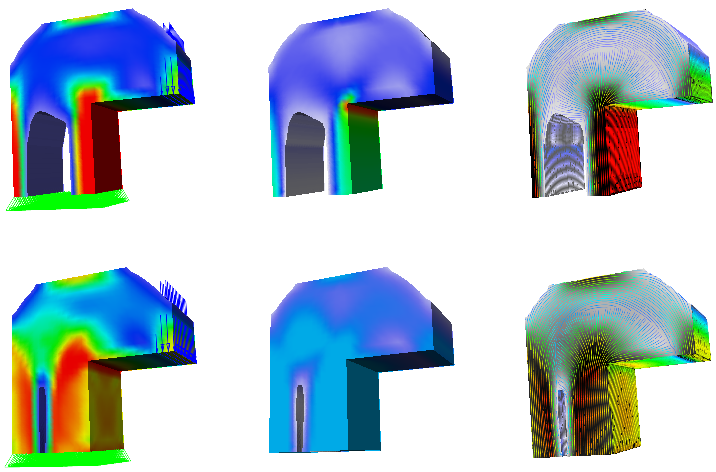


Figure 2: L-shape: loading and material distribution (left) and stress concentration (middle), material directions (right)


[1] M. P. Bendsøe, J. M. Guades, R. B. Haber, P. Pedersen, and J. E. Taylor. An analytical model to predict optimal material properties in the context of optimal structural design. *J. Applied Mechanics*, 61:930–937, 1994.

[2] M. Kočvara and M. Stingl. Solving nonconvex SDP problems of structural optimization with stability control. *Optimization methods and Software*, 19:595–609, 2004.

[3] M. Kočvara and M. Stingl. Free material optimization for stress constraints. *Structural and Multidisciplinary Optimization*, 33:323–335, 2007.

[4] M. Stingl and M. Kočvara. A sequential convex semidefinite programming algorithm with an application to multiple-load free material optimization. *Solving nonconvex SDP problems of structural optimization with stability control*, 20(1):130–155, 2009.

# FMO Results Interpretation: Supporting the Development of Design Concepts

**Gábor Bodnár**

RISC Software GmbH, Hagenberg, Austria, Gabor.Bodnar@risc-software.at

**Abstract**
One achievement of the PLATO-N project is an integrated software system, called Free Material Studio (FMS), for visualization and interpretation of Free Material Optimization (FMO) results. On the one hand, FMS provides means to visually represent data derived from the rank-four material tensors of FMO together with the traditional finite element analysis (FEA) results (stress tensors, displacements, etc). On the other hand, FMS offers a palette of interpretation tools that support the user in obtaining design ideas from the available FMO / FEA data. In this presentation we will provide an overview of the available features of FMS via a set of examples and reflect on possible future developments.

**Keywords:** Free Material Optimization, Scientific Visualization, Material Tensors

## 1. Introduction

Within the PLATO-N project of the 6th Framework Program of the EU the consortium develops mathematical optimization theory, algorithms, and software to bring free material optimization to the level of an industry-ready technology. An important aspect of this work is to provide an integrated software system with advanced visualization capabilities that make the complex FMO / FEA data available for the human mind for inspection and interpretation. While the design concept eventually arises from the engineer, it is also important to offer not only visualization methods but also interpretation tools that allow interactive experimentation, measurements and investigations on the data at hand. Finally, to simplify process of building up of design concepts in a form that can be reused in further analysis or optimization steps of the preliminary design phase, converting and exporting data in industry standard formats should also be supported. The software system Free Material Studio (FMS) tackles all these tasks with an extendible software framework, whose components allow almost completely free combinability. This way the user can realize a wide range of creative interpretation and visualization ideas with FMS, rendering it a useful tool in the conceptual design phase. For more detailed discussions on the topics of this extended abstract we refer to [1,2,3].

## 2. Layout

The main goals behind the philosophy of the architecture of FMS are to provide a flexible but robust, extendible but tightly integrated system. This is achieved by supporting user definable visualization pipelines (instead of hard coded visualization features) on the basis of a plug-in software architecture that allows new pipeline members to augment the system without the need of change in already existing parts. In this framework, geometry and data attributes (e.g. FMO / FEA data) travel from data sources via filter algorithms to actors of the visualization scene. Compatible data interfaces ensure that the directed graph of the pipeline can be set up by the user in a way that its complexity is bounded only by hardware processing power limitations. The somewhat tedious process of setting up the pipeline graphs is alleviated by the save / load feature, which makes the reuse of previously set-up pipelines in contexts of future examples possible.

### 2.1. Visualization Features

FMS provides the standard visualization possibilities for scalar and vector fields. In the case of scalars the traditional approach is color mapping with predefined color schemes, mapping functions and mapping ranges. For vectors the standard approach is to place appropriately oriented small line segments on the originating geometric entities to display the vector field as the forest of glyphs. Beside these approaches FMS supports evenly spaced streamlines both on 2D and 3D models, and ellipsoid based visualization for rank-two tensors. In the latter, the axes of the ellipsoids are aligned with the principal directions of the underlying tensors and the axis-lengths are made proportional to the corresponding eigenvalues. For rank-four tensors, we have to fall back to visualization of derived rank-two tensors, vectors and scalars.

2.2. Interpretation Tools

Interpretation of the FMO data is challenging because the optimal FMO material tensors describe theoretical materials that eventually need to be replaced by ones that are available and allow effective manufacturing. Typically composite materials with advanced manufacturing technologies (draping, tow-placement) are the main candidates to approximate the FMO materials. In the first stages of this process, it is helpful to obtain qualitative information about the FMO materials. Probably the simplest example is the stiffness (the trace of the tensor) that provides a scalar value, usually identified with "material density". Beside this relevant information can be obtained from symmetry based material classification. FMS implements the Cowin-Mehrabadi [4] classification scheme, by defining classification functions for a selected subset of material classes, each measuring how well the input tensors satisfy the criteria of the represented class. The classification scheme provides not only the scalar values of the above functions but also a pair of rank-two derived tensors: the dilatational and the Voigt tensors. Eigenvector analysis on them can deliver valuable information about material symmetry axes in some of the material classes. FMS also features an interactive material tensor evaluator, which, for instance, can be used to display single tensor entries in user defined coordinate systems across the model. Once properly set up, this feature makes directional material strength visualization possible. Moreover, its combination with a simple algorithm that searches the coordinate frames where the selected evaluation function attains maximum (or minimum) can deliver the extrema and the corresponding directions.

Geometric interpretation is another aspect from which FMO results can be analyzed. FMS supports contouring with respect to scalar fields, clipping and mesh smoothing. These and several other filters not belonging to this category were made "geometric scope aware", a feature that enables the user to restrict the domain of the filter algorithm to a subset of the model defined by the user interactively as the union of spatial primitives (boxes, spheres, half-spaces).

2.3 Converters, Exporters

It is important to make the findings of the visualization and interpretation process accessible for further analysis and optimization software. FMS currently provides only a few converting and exporting features that cover the most important aspects, but the framework can be extended in this direction. With the PCOMP exporter the user can build up standard NASTRAN composite property fields for selected subsets of the model, based on available data attributes that define thickness and orientation values. Concerning altered geometries, FMS provides the possibility to export selected subsets of the model in STL format for further processing (smoothing, solid modeling, meshing).

## 3. Conclusions

Free Material Studio provides a solid basis for FMO / FEA data visualization and interpretation on examples with industry grade size and complexity. The freely combinable visualization and interpretation tools provide a wide range of possibilities for creative users to experiment with the FMO data and come up with design concepts. However, there are still several ideas that could not be implemented within the PLATO-N project due to time and resource constraints. One main line of partially implemented features is the palette of conversion and export tools in industry standard formats, but a few not yet implemented geometric interpretation features (e.g. FMO data guided skeletonization) could also find useful applications.

## References


[1] G. Bodnár, Efficient algorithms for visualizing FMO results, *PLATO-N Technical Report*, 2007

[2] G. Bodnár and M. Bogomolny, Methods for computer aided interpretation of FMO results, *PLATO-N Technical Report*, 2008

[3] G. Bodnár, Visualization and Interpretation Tools for Free Material Optimization Results, In Proceedings of *8th World Congress on Structural and Multidisciplinary Optimization*, 2009

[4] S.C. Cowin and M.M. Mehrabadi, On the identification of material symmetry for anisotropic elastic materials, *Quarterly J. of Mechanics and Applied Mathematics*, 40, 451-476, 1987

# Laminate Interpretation of Results from Free Material Optimization

**M. Bogomolny[1], M. Stolpe[2] and M.P. Bendsøe[3]**

[1] Department of Mathematics, Technical University of Denmark (DTU),
Matematiktorvet, Building 303S, DK-2800 Kgs. Lyngby, Denmark
m.bogomolny@mat.dtu.dk.
[2] Department of Mathematics, Technical University of Denmark (DTU)
Matematiktorvet, Building 303S, DK-2800 Kgs. Lyngby, Denmark
m.stolpe@mat.dtu.dk
[3] Department of Mathematics, Technical University of Denmark (DTU)
Matematiktorvet, Building 303S, DK-2800 Kgs. Lyngby, Denmark
m.p.bendsoe@mat.dtu.dk

## Introduction

The objective of this work is to provide tools for interpretation of solution from *Free Material Optimization* (FMO) using laminated composites and to give an engineering sense to optimal material properties. The aim of FMO is not only to determine the optimal distribution of material as in classical *Topology Optimization*, but also the material properties themselves. These material properties are given as a solution of a mathematical problem; however their engineering meaning is not clear.

The basic FMO models are introduced by Bendsøe at el. (1994) and solution algorithms are proposed by Zowe et al. (1997). The design variables in FMO are the elasticity tensors that can vary from point to point in the design domain. Therefore, FMO deals with the larger number of design variables than classical topology optimization. Usually, the FMO deals with minimization of compliance under “weight” constraint or vice versa. However, the meaning of the word “weight” in FMO is abstract, since it is measured in units of stiffness and it introduces the material properties.

Since the solution of the FMO problem is the anisotropic material tensors, the visualization of the FMO is an important issue. It is possible to describe the amount of material by the visualization of the trace of the material tensor. Kocvara and Stingl 2007 visualize FMO solution using the traces and the principle strains, when directions of the principal strains indicate material orientation. However, this kind of visualization does not include enough information for engineering understanding of the FMO design.

Stacking of the layered orthotropic materials, when each layer is rotated by a given angle, allows building anisotropic material with a given constitutive matrix (Reddy 2004).

The objective of this work is to provide tools for interpretation of the FMO using the laminated composites in application to plates and shells for practical design problems. The main idea is to use the laminates and their special properties in order to build a layered structure that has material tensor as close to the FMO tensor as possible. The problem is defined as an inverse problem and solved as an optimization problem. For this purpose two different approaches are proposed: one continuous and one discrete. Using the continuous model the layer thicknesses for every material are treated as the continuous variables, when the stacking sequence is fixed and constant in the design domain. The materials that are used here are the rotated laminate composites by a given angle. In discrete model the thickness of each layer and number of layers are fixed. Using the discrete model we adjust a given material to a given layer, therefore the stacking sequence is constructed during the optimization process. The materials that are used here are the rotated laminate composites and void (no material).

Result of the visualization allows understanding of the characteristic of the FMO solution from engineering point of view, i.e. where given materials should be placed and in which directions.

Several numerical examples of plates and shells illustrate the capabilities of the two proposed approaches.

## Example

Here we introduce a numerical example. A 2-D rectangular plane-stress model, fixed on the left boundary and loaded on the lower right corner has been designed using the FMO. Figure 1 shows the traces of the elasticity matrices for the FMO design and the laminated composites interpretation. Figure 2 shows the directions of the laminated composites.

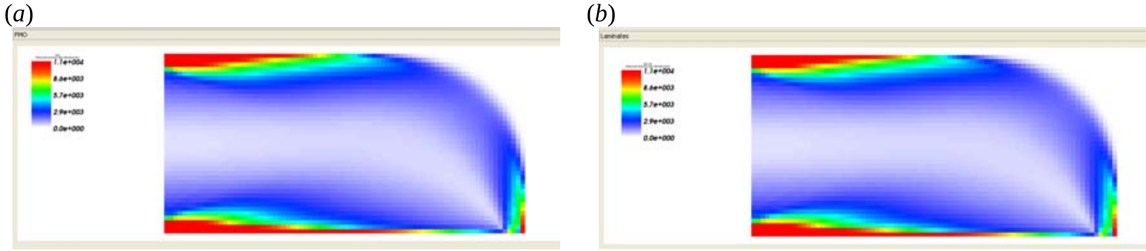


**Figure 1.** Trace of the elasticity matrices: (*a*)FMO (*b*)Laminated Composites

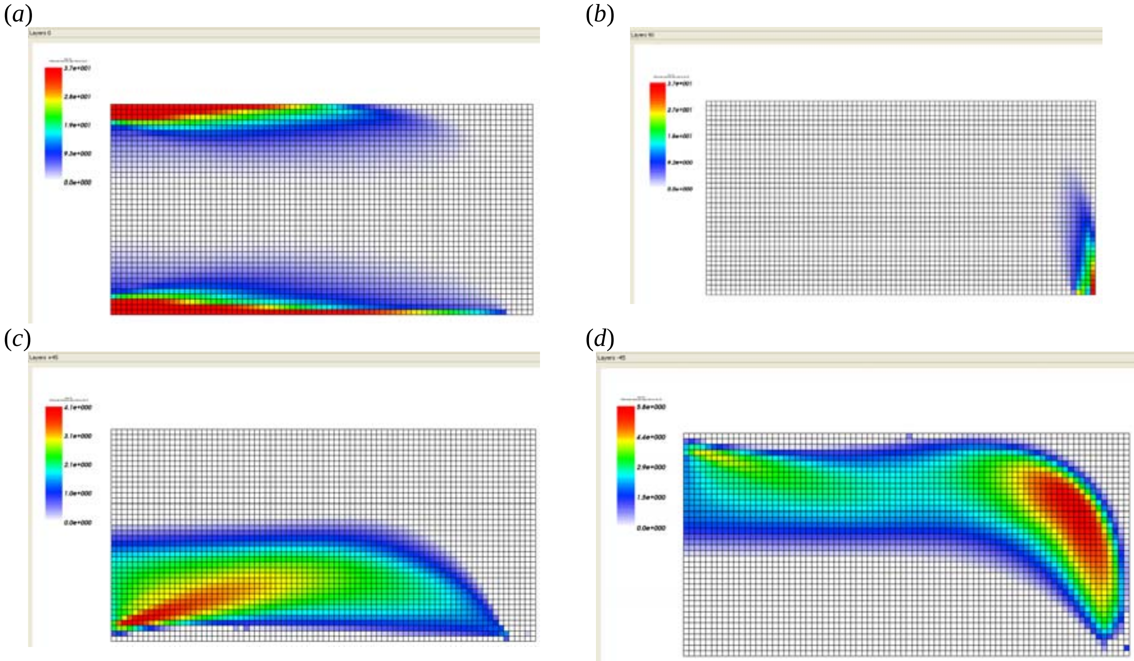


**Figure 2.** Continuous Approach:

- *(a)* Thickness of the layers rotated by angle $0^0$
- *(b)* Thickness of the layers rotated by angle $90^0$
- *(c)* Thickness of the layers rotated by angle $45^0$
- *(d)* Thickness of the layers rotated by angle $-45^0$

**References**


1. Bendsøe M.P. and Sigmund O. (2002) – “Topology optimization. Theory, Methods and Applications”, *Springer*.
2. Bendsøe M.P., Guades J.M., Haber R.B., Pedersen P and Taylor J.E. (1994) – “An analytical model to predict optimal material properties in the context of optimal structural design”, *J. Appl. Mech* **61**, pp. 930-937.
3. Zowe J., Kocvara M. and Bendsøe M.P. (1997) – “Free material optimization via mathematical programming”, *Math. Progr.* **79**, pp. 445-466.
4. Reddy J.N. (2004) –”Mechanics of laminated composite plates and shells. Theory and Analysis”, *Second Edition*, CRC Press.
5. M. Kocvara, M. Stingl, and R. Werner. – “MOPED User's Guide”. Version 1.02. Research Report 262, Institute of Applied Mathematics, University of Erlangen, 2000.

| **September 23, Afternoon session I (Scion DTU, Building 373)**<br>**Chair: Michal Kočvara** | | |
|---|---|---|
| **Time** | **Presenting author** | **Title** |
| 13.30 – 14.00 | Thomas Lehmann | MISQP - An SQP Method for Mixed-Integer Optimization<br>*T. Lehmann and K. Schittkowski* |
| 14.00 – 14.30 | Eduardo Munoz | Global optimization by Benders decomposition in structural optimization<br>*E. Munoz and M. Stolpe* |
| 14.30 – 15.00 | Christian G. Hvejsel | Failure Constraints for Discrete Material Optimization<br>*C.G. Hvejsel and E. Lund* |

| **September 23, Afternoon session II (Scion DTU, Building 373)**<br>**Chair: Michal Kočvara** | | |
|---|---|---|
| **Time** | **Presenting author** | **Title** |
| 15.30 – 16.00 | Peter D. Dunning | Introducing Loading Uncertainty in Topology Optimization<br>*P. Dunning, H.A. Kim, and G. Mullineux* |
| 16.00 – 16.30 | Franz-Joseph Barthold | Applications of singular value decomposition in structural optimization<br>*F-J. Barthold and N. Gerzen* |

# MISQP - An SQP Method for Mixed-Integer Optimization *

**T. Lehmann**[1] and **K. Schittkowski**[2]

[1] Department of Computer Science, University of Bayreuth, Germany, thomas.lehmann@uni-bayreuth.de
[2] Department of Computer Science, University of Bayreuth, Germany, klaus.schittkowski@uni-bayreuth.de

**Abstract**
We present an algorithm called MISQP for solving mixed-integer nonlinear programming (MINLP) problems by a sequence of mixed-integer quadratic programming (MIQP) subproblems. MISQP can be considered as an extension of the well-known SQP methods for solving mixed integer programs. It is designed for solving problems arising, e.g., in industrial engineering that are based on expensive simulations. Extensive numerical tests show that even for non-convex problems, acceptable solutions are efficiently found.

**Keywords:** mixed-integer nonlinear programming, MINLP, mixed-integer quadratic programming, SQP, trust region methods.

## 1 Introduction

Mixed-integer nonlinear programming (MINLP) refers to mathematical programming considering both continuous and discrete variables as well as a nonlinear objective function and nonlinear constraints.

$$x \in \mathbb{R}^{n_c}, y \in \mathbb{Z}^{n_i} : \begin{array}{l} \min f(x,y) \\ g_j(x,y) = 0 \ , \quad j = 1, \ldots, m_e \ , \\ g_j(x,y) \geq 0 \ , \quad j = m_e + 1, \ldots, m \ , \end{array} \tag{1}$$

where $x$ and $y$ denote vectors of $n_c$ continuous and $n_i$ integer variables, respectively. It is assumed that the problem functions $f(x,y)$ and $g_j(x,y)$, $j = 1, \ldots, m$, are continuously differentiable subject to all $x \in \mathbb{R}^{n_c}$. Upper and lower bounds are omitted for simplicity.

MINLP problems can become extremely difficult depending on the number and the structure of the integer variables, e.g., whether they are boolean or not. A large number of applications, e.g., in process industry, finance, engineering and operations research require the solution of MINLP problems, and there is a need for efficient and reliable solution methods.

## 2 The Algorithm

We present an algorithm for solving mixed-integer nonlinear programming problems called MISQP [2]. The idea is to modify a sequential quadratic programming method by taking integer variables into account, and to solve mixed-integer quadratic subproblems of the form

$$\begin{array}{l} d \in \mathbb{R}^{n_c}, \\ e \in \mathbb{Z}^{n_i}, \\ \delta \in \mathbb{R} \end{array} : \begin{array}{l} \min \frac{1}{2} \begin{pmatrix} d \\ e \end{pmatrix}^T B_k \begin{pmatrix} d \\ e \end{pmatrix} + \nabla_x f(x_k, y_k)^T d + \nabla_y f(x_k, y_k)^T e + \sigma_k \delta \\ -\delta \leq \nabla_x g_j(x_k, y_k)^T d + \nabla_y g_j(x_k, y_k)^T e + g_j(x_k, y_k) \leq \delta \ , \ \ j = 1, \ldots, m_e \ , \\ -\delta \leq \nabla_x g_j(x_k, y_k)^T d + \nabla_y g_j(x_k, y_k)^T e + g_j(x_k, y_k) \ , \ \ j = m_e + 1, \ldots, m \ , \\ 0 \leq \delta \ . \end{array} \tag{2}$$

successively including a relaxation variable $\delta$ and a corresponding penalty parameter $\sigma_k$ to prevent infeasibility.

*Sponsored by Shell SIEP, Rijswijk, in form of a Game Changer project

| *code* | $p_{succ}$ | $n_{loc}$ | $n_{err}$ | $n_{its}$ | $n_{func}$ | *time* | *error* |
|---|---|---|---|---|---|---|---|
| MISQP/RX | 99 % | 3 | 3 | 34 | 575 | 0.08 | - |
| MISQP | 99 % | 35 | 1 | 27 | 514 | 0.24 | 3.5 % |
| MISQPOA | 100 % | 20 | 0 | 253 | 6.874 | 2.57 | 2.3 % |
| MINLPBB | 100 % | 5 | 1 | 4,078 | 72,402 | 0.67 | 0.3 % |

Table 1: Numerical Results over a Set of 145 Mixed-Integer Test Problems

The algorithm is stabilized by a trust region method with Yuan's [3] second order corrections. The Hessian of the Lagrangian function is approximated by BFGS updates subject to the continuous and integer variables. Generated MIQP problems are solved by a new branch-and-cut method.

Since global convergence cannot be guaranteed by our approach even for convex problems, our method is extended by a linear outer approximation framework. It is important that typical standard assumptions such as relaxable integer variables or convexity are not required.

# 3 Numerical Results

We introduce our algorithm and present numerical results for a set of 145 academic and industrial test problems, and we compare MISQP with alternative solution methods based on outer approximations and branch-and-bound. The test problems with a more practical background were provided by Shell coming from gas and oil production models.

Some of them are widely used to develop and test new algorithms, see for example Floudas et al. [1]. More details about the test problems, individual numerical results, and also of comparative results for continuous test problems are found in Exler and Schittkowski [2].

To give at least a brief impression of the performance of our code, we report here the following data:

| | | |
|---|---|---|
| $p_{succ}$ | - | percentage of successful terminations |
| $n_{loc}$ | - | number of local solutions obtained |
| $n_{err}$ | - | number of test runs with error messages (IFAIL>0) |
| $n_{its}$ | - | average number of iterations |
| $n_{equ}$ | - | average number of equivalent function calls (with function calls used for gradient approximations) |
| *time* | - | average execution time in seconds |
| *error* | - | error, i.e., average relative deviation from global solution |

The first line shows results for relaxed problems, i.e., integer variables are handled as continuous ones (MISQP/RX). In the subsequent lines, we report results for an outer approximation (MISQPOA) and a branch-and-bound implementation (MISQPBB).

# 4 Conclusions

Extensive numerical tests show that MISQP is very efficient in terms of the number of function evaluations. For industrial applications based on an expensive simulation code, this is the most important performance criterion. However, convergence to a global minimum cannot be guaranteed, and we propose to perform warmstarts with outer approximation and, if necessary, subsequent execution of a branch-and-bound method if an achieved solution is not acceptable.

**References**


[1] C.S. Adjiman W.R. Esposito Z.H. Gumus S.T. Harding J.L. Klepeis C.A. Meyer C.A. Floudas, P.M. Pardalos and C.A. Schweiger. *Handbook of Test Problems in Local and Global Optimization.* Kluwer Academic Publishers, 1999.

[2] O. Exler and K. Schittkowski. A trust region SQP algorithm for mixed-integer nonlinear programming. *Optimization Letters*, 1:269–280, 2007.

[3] Y.-X. Yuan. On the convergence of a new trust region algorithm. *Numerische Mathematik*, 70:515–539, 1995.

# Global optimization by Benders decomposition in structural optimization

**E. Munoz**[1] **and M.Stolpe**[2]

[1](Department of Mathematics, Technical University of Denmark, Matematiktorvet 303, 2800 Lyngby, Denmark. Email: e.munoz@mat.dtu.dk)
[2](Department of Mathematics, Technical University of Denmark, Matematiktorvet 303, 2800 Lyngby, Denmark. Email: m.stolpe@mat.dtu.dk)

**Abstract**
We investigate a spectrum of structural design problems in their mixed 0-1 formulation. We aim to solve these problems to global optimality, or if not possible, obtain a good candidate design, and meaningful estimation for the globally optimal objective value. For this purpose, we use the Benders decomposition method, a well known method for nonlinear mixed integer optimization, developed in the 90's, but not applied extensively to structural optimization yet. Under special conditions, this algorithm guarantees convergence to a global optimum in a finite number of iterations. We show an overview of the performance of this algorithm on several numerical applications in topology design problems, truss topology design problems, and the design of multi–material composite laminates.

**Keywords:** Structural design, Global optimization, Benders decomposition, Topology optimization.

## 1. Problem Formulation and outline of the method

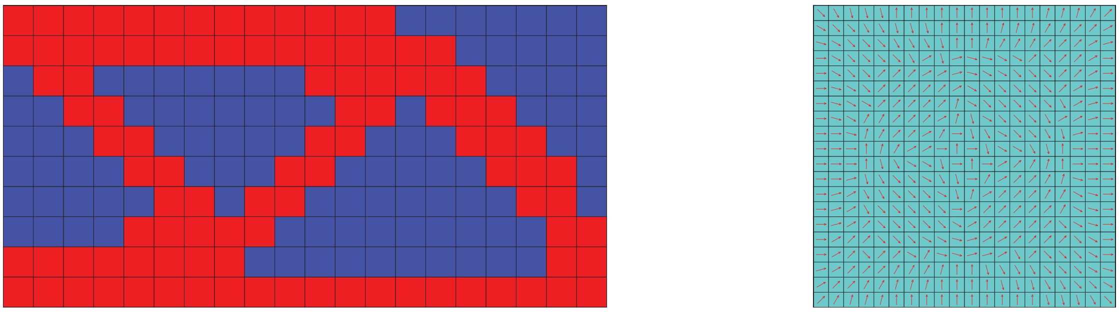


Figure 1: Left: Mbb beam design in a shell element discretization: Right: Optimal angle selection for a clamped plate under a distributed vertical load condition.

Consider a design domain $\Omega \in \mathbb{R}^2$, piecewise-$C^1$, and a finite element discretization of the continuum problem. Suppose that $\Omega$ is independently discretized in $m$ design subdomains. This discretization of the design space may or may not, coincide with the discretization in finite elements. We aim to select, out of a set of candidate materials, with different properties, the material that will be assigned for each of the design subdomains, obtaining a globally optimal configuration of the lay-out and material selection of the structure. The discretization of the design domain implies the introduction of $n = m \cdot s$ discrete design variables ($x \in \{0,1\}^n$). This notation describes a material distribution on the design space. The state variable $u \in \mathbb{R}^d$ represents the displacement of the structure for its $d$ degrees of freedom, when the structure is under $n_l$ multiple load conditions $f_l \in \mathbb{R}^d$, and suitable support conditions. $K(x) \in \mathbb{R}^{d \times d}$ denotes the global stiffness matrix. The discrete formulation of the minimum weight (or cost) problem constrained to a certain maximum compliance is given by

$$\begin{array}{lll} \underset{x\in\mathbb{R}^n, u\in\mathbb{R}^d}{\text{minimize}} & \displaystyle\sum_{i,j}^{n} \rho_{ij} x_{ij} & \\ \text{s.t.} & K(x)u_l = f_l & i,j = 1, ..., n_l \\ & \max\limits_{l}\{f^T u_l\} \leq \overline{C} & \\ & Ax \leq b & \\ & x_{ij} \in \{0,1\} & i,j = 1, ..., n, \end{array} \tag{1}$$

where $\overline{C}$ is the maximum allowed compliance, and the linear constraint $Ax \leq b$ represents any technical additional constraint, or valid inequalities reducing the feasible set and therefore, improving the convergence of the method. We also consider the analogous formulation of the mass/cost constrained minimum compliance problem. These classes of problems have been extensively studied by the introduction of continuous relaxations and interpolation schemes (see for example [2]). We solve these problems applying the Benders decomposition method ([3],[4]). This technique solves (1) iteratively, where at each iteration, a local linear approximation of the compliance function and the feasible set are included in a mixed integer linear program, called the relaxed master problem. These approximations are obtained by solving one analysis problem and one adjoint problem. The process continues, until the optimal value of the relaxed master problem and the best found feasible design get close enough with respect to certain tolerance. Then, it can be claimed that the current best design is a global optimum within this tolerance. The typical convergence of upper and lower bound for the global optimum value in Benders decomposition algorithm is presented in Figure 2. At any stage of the algorithm, a valid lower bound for the global optimum is obtained, and improved along the optimization process.

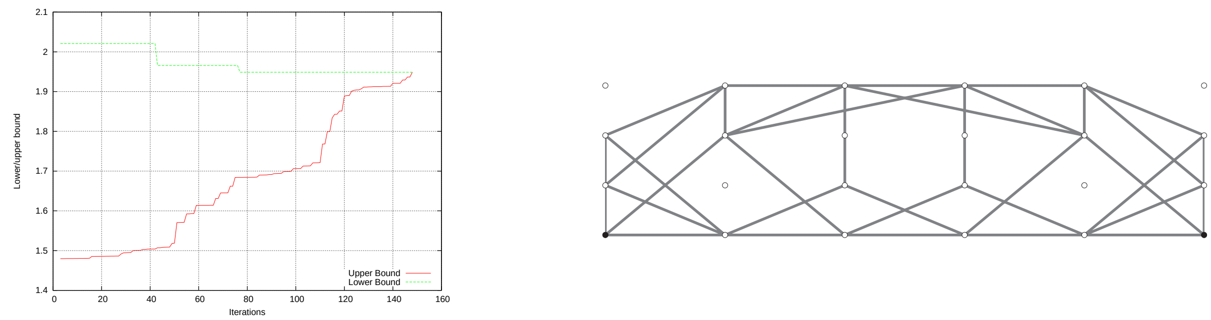



Figure 2: Left: typical convergence history for the Benders decomposition in a structural problem. Right: Example of an optimal design for a truss topology problem.

Benders decomposition shows in general different performance properties, depending on the type of problem we are trying to solve. The convergence properties depend in general on the size of the problem, the number and type of candidate materials avalaible, and the finite element discretization. We present and overview of the behavior of the algorithm for different types of design problems. We consider minimum compliance and minimum weight problems, applied to single and multiple load conditions. This formulation is used to solve general class topology design problems, such as material selection problems, truss design problems and multi-material design of composite laminates (figures 1, 2). A comparison of the results is presented and an assessment of the general skills of the method is then extracted form this analysis. Also a general comparison with other results in the area of global optimization for structural problems is briefly presented ([5], [1]). We end up with proposing a general overview of the future work regarding Benders decomposition in structural optimization.

## References


[1] W. Achtziger and M. Stolpe. Truss topology optimization with discrete design variables - guaranteed global optimality and benchmark examples. *Structural Multidisciplinary Optimization*, 34(1):1–20, 2007.

[2] M.P. Bendsøe and O. Sigmund. *Topology Optimization — Theory, Methods and Applications*. Springer, 2003.

[3] A. M. Geoffrion. Generalized Benders decomposition. *Journal of Optimization Theory and Applications*, 10(4):237–260, 1972.

[4] R. Lazimy. Extension of the generalized Benders decomposition. *Communications on Applied Numerical Methods*, 6:195–203, 1986.

[5] M. Stolpe and J. Stegmann. A Newton method for solving continuous multiple material minimum compliance problems. *Structural and Multidisciplinary Optimization*, 35:93–106, 2007.

# Failure Constraints for Discrete Material Optimization

**C.G. Hvejsel and E. Lund**

Department of Mechanical Engineering, Aalborg University, Denmark, cgh@me.aau.dk and el@me.aau.dk

**Abstract**
In this work we study stress-based failure criteria that are consistent with the stiffness modeling of the material interpolation scheme employed. We use the hybrid Voigt-Reuss scheme for which there is a clear interpretation of the stress and strain state in each phase and with this knowledge we formulate a consistent averaged failure criterion for mixed-phase materials. Analytical as well as numerical examples demonstrate the capabilities of the proposed scheme.

**Keywords:** laminated composites, optimal design, failure criteria, stress constraints, material interpolation schemes.

## 1. Introduction

Optimal design of laminated composite structures is still an open challenge for academia as well as industry. The complexities associated with modeling and analysis of composites makes design of laminate lay-ups an even more complicated task. However, not only modeling poses difficulties, but also non-convexity of the objective (and constraint functions) is a challenge when ply orientations are used as design variables in e.g. compliance minimization. Alternatively, the lay-up design problem may be formulated as a *material selection problem* using a different parameterization, and in this paper the so-called Discrete Material Optimization (DMO) approach is applied. This approach is based on ideas from multi-phase topology optimization where the discrete material selection problem is relaxed to a continuous equivalent problem by expressing intermediate material properties as weighted sums of user-defined candidate material properties allowing for the use of gradient based mathematical programming techniques. This methodology has been applied successfully to problems involving global criteria functions such as compliance, see e.g. [2], but may as well be extended to problems involving local criteria functions such as constraints on the allowable stresses and strains. Including criteria on the local strength leads to designs that perform well not only from a stiffness point of view but also have sufficient strength in terms of satisfaction of some failure criterion of the chosen materials. In this paper we investigate the behavior of different multi-material interpolation schemes. The stiffness and strength of mixtures is deduced from physical interpretations of the mixing rules. The aim is to obtain consistent failure criteria for the mixture rules on basis of the failure behavior of the constituent phases. First, a mechanical interpretation of two interpolation schemes is given, followed by the formulation of consistent first-constituent failure criteria.

## 2. Problem formulation

In a relaxed material selection problem, the concern is not the microstructural realization of intermediate densities as in e.g. [1], but rather the physical behavior that a given mixture rule represents and that it eventually leads to a distinct material selection in order to enable a manufacturable physical interpretation of the final result. Also, interpolations should be reasonably simple and computationally efficient since the resulting optimization problems are very large scale. Various formulations fulfilling these requirements exist and will be presented. One class of interpolations are special SIMP-like weighting functions which have been used with DMO to solve compliance minimization problems, [2]. These weighting functions use penalization of intermediate densities to avoid mixtures. However, the penalization is not enough to drive the design to full convergence (0/1 solutions) due to a necessary scaling that flattens the design space and consequently the optimizer gets stuck for certain problems. Convergence to a distinct material choice turns out to be of utmost importance since a final rounding to 0/1-designs has been shown to severely affect the result when local failure criteria are included, contrary to the situation with only global criteria functions where rounding typically is reasonable. In this work the hybrid Voigt-Reuss scheme proposed by [3, 4] provides a physically based mixing rule that leads to solutions closely fulfilling the 0/1-constraint of the originally discrete problem. This scheme has been adopted for DMO and turns out to be well suited for a number of reasons. The scheme has some desirable properties in terms of an

unambiguous meaning of the stress and strain state within each phase of the mixed material. To assess failure of the mixed phase material we propose a volume-averaged failure criterion, which uses the fact that the failure of each phase may be assessed knowing the stress or strain in the phase and the strength of that particular phase.
This approach is used to select the best of two orthotropic materials with identical properties, oriented at $\theta = 0°$ and $90°$ respectively. This selection problem can be parametrized with one variable. The bi-axial state of stress is illustrated in Figure 1 along with the corresponding interpolated failure index.

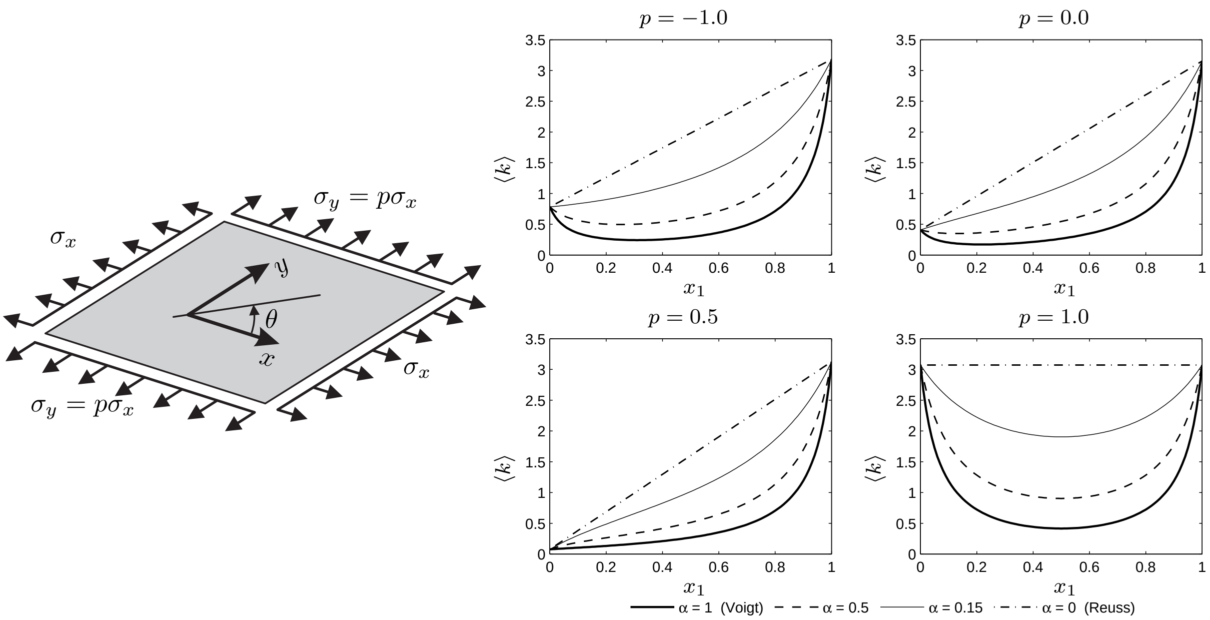



Figure 1: Left: Bi-axial stress states with coordinate system. Right: Averaged inverse reserve factor (wrt. Tsai-Wu failure prediction) with hybrid Voigt–Reuss scheme for different bi-axial stress states. Note that it is strictly positive.

The methodology is applied to examples of laminate design problems including local failure criteria controlling the strength of the individual plies.

## 3. Conclusions

We have shown an approach to handle failure constraints with multi-material problems. The formulation of the volume averaged criterion is based on physical interpretations of the stress and strain state in each phase as implied by the constitutive mixture rule. The method may be applied to problems involving materials governed by different types of failure criteria, e.g. selection between foam, wood and a fiber reinforced material.

## References


[1] M.P. Bendsøe and O. Sigmund. Material interpolation schemes in topology optimization. *Archive of Applied Mechanics*, 69:635–654, 1999.

[2] J. Stegmann and E. Lund. Discrete material optimization of general composite shell structures. *International Journal for Numerical Methods in Engineering*, 62:2009–2027, 2005.

[3] C.C. Swan and J.S. Arora. Topology design of material layout in structured composites of high stiffness and strength. *Structural and Multidisciplinary Optimization*, 13:45–59, 1997.

[4] C.C. Swan and I. Kosaka. Voigt–Reuss topology optimization for structures with linear elastic material behaviours. *International Journal for Numerical Methods in Engineering*, 40(16):3033–3057, 1997.

# Introducing Loading Uncertainty in Topology Optimization

**P.D. Dunning[1], H.A. Kim[2] and G. Mullineux[3]**

[1] University of Bath, Bath (UK, P.D.Dunning@bath.ac.uk)
[2] University of Bath, Bath (UK, H.A.Kim@bath.ac.uk)
[3] University of Bath, Bath (UK, G.Mullineux@bath.ac.uk)

**Abstract**
Including uncertainty in topology optimization is important in producing robust design solutions. An efficient topology optimization method is developed to minimize expected strain energy under loading with uncertain magnitude. An example demonstrates that by including these uncertainties a more robust solution can be obtained compared to the deterministic case.

**Keywords:** Loading uncertainty, topology optimization, level set method.

## 1. Introduction

Various computational methods have been developed to solve structural topology optimization problems and for continua these methods can be categorized into element based methods, [1] and boundary based methods, such as the level set method [2, 3]. These methods have been well developed to solve deterministic problems. However, little attention has been given to problems with uncertain variables, especially with respect to the level set method. Accounting for uncertainty in structural optimization problems is important to produce robust solutions. Various uncertainties can affect the robustness of a structure, including loading, geometry and material properties [4]. The focus of this work is to develop a method that accounts for loading magnitude uncertainty in structural topology optimization in order to produce a robust solution. The method is then demonstrated by an example using the level set method.

## 2. Formulation

The classic problem often solved by structural topology optimization methods is to minimize compliance or strain energy, subject to a volume constraint:

$$\begin{aligned} \min : J(u) &= \int_\Omega A\varepsilon(u)\varepsilon(u)d\Omega \\ s.t.: \int_\Omega A\varepsilon(u)\varepsilon(v)d\Omega &= \int_\Omega pvd\Omega + \int_\Gamma fvd\Gamma \\ \int_\Omega d\Omega &\le Vol^* \end{aligned} \tag{1}$$

where Ω is the structure domain, Γ the structure boundary, A the material property tensor, ε(u) the strain tensor, p the body forces, f the surface tractions and Vol* the constraint on material volume. Assuming the static equilibrium constraint is satisfied, the remaining constraint for (1) is the volume constraint. A limit state function for (1) is required in order to apply classic structural reliability techniques, such as the First Order Reliability Method (FORM) [4]. A limit state function in this case represents the difference between loading and capacity, such that a negative value signifies a failure. However, uncertain loading is not related to the volume constraint, therefore a meaningful limit state function is not easily constructed and techniques such as FORM are inappropriate to solve (1) under uncertain loading.

There are two main approaches to solve (1) under uncertain loading that do not require a limit state function. The first is to minimize the worse case scenario, or the maximum strain energy, (2). The second is to minimize the expected or average strain energy, (3), [5].

$$\min \quad \max\{J(u)\} \tag{2}$$

$$\min \quad E[J(u)] \tag{3}$$

where E(x) is the expected value of uncertain variable x. The first approach is a min/max problem where the solution is likely to be optimized for a loading case far form the mean, possibly producing a highly non-optimal response for loading cases near the mean. Therefore, it is argued here that the min/max approach is unlikely to produce a desirable solution if the objective is to design an efficient and robust structure under uncertain loading. This is because the structure is likely to be optimized for loading conditions unlikely to be seen in reality with no consideration given to more likely loading conditions nearer the mean. Therefore the second approach (3) is adopted as it accounts for the complete range of uncertain loading conditions and is more likely to produce an efficient and robust structure. Assuming loading magnitude uncertainties are normally distributed an analytical form of (3) is derived. The presentation will show that the expected strain energy is equivalent to a summation of strain energy values derived from deterministic loading cases. This provides a convenient and efficient formulation for strain energy sensitivities required to solve (3).

### 3. Example

The method described in Section 2 for computing expected strain energy was applied to a basic level set method. It was used to minimize expected strain energy of a beam under three independent loads with uncertain loading magnitudes, Figure 1. Young's modulus was 1.0, Poisson's ratio 0.3, 200 x 100 elements were used to discretize the beam and the volume constraint was 50% of the domain volume.

| load | $\mu$ | $\sigma$ |
|---|---|---|
| $f_1$ | 1.0 | 0.5 |
| $f_2$ | 1.0 | 0.1 |
| $f_3$ | 1.0 | 0.2 |

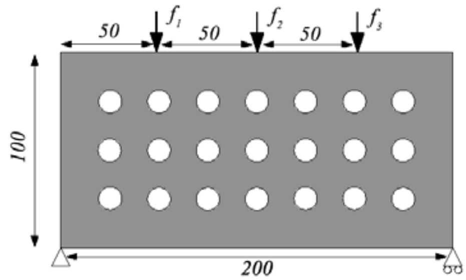



Figure 1: Initial beam structure.

The beam was optimized for deterministic loading ($\sigma = 0$), Figure 2a, and uncertain loading, Figure 2b. Note that a different optimum topology was found for the uncertainty problem compared to the deterministic solution. Expected strain energy under the uncertain loading for the deterministic solution was 44.48, compared to 43.56 for the uncertainty solution. Thus, the preliminary results show that a more robust topology solution can be achieved as the average strain energy is less that for the deterministic solution.

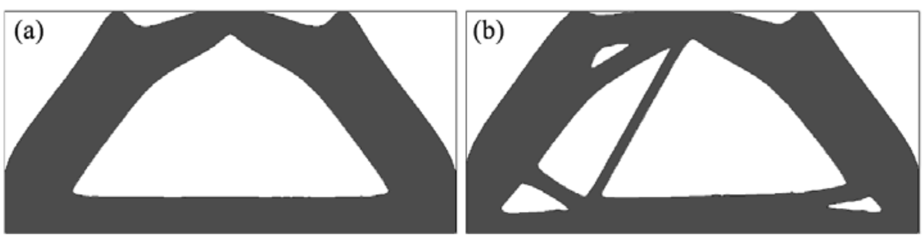



Figure 2: Optimum beam solutions: (a) deterministic, (b) uncertainty

### 4. Conclusions

The paper presents an analytical formulation for expected strain energy under uncertain loading. This method is applied to the level set method. A preliminary topology optimization solution shows that the robust solution has a different topological solution to that of the deterministic solution and the expected strain energy for the uncertainty solution is less than that of the deterministic result. This highlights the importance of including loading uncertainty in topology optimization to produce efficient and robust solutions.

### References


[1] M.P. Bendsøe and O. Sigmund. *Topology Optimization: Theory, Methods and Applications*, Springer-Verlag, Berlin, 2003.

[2] M. Y. Wang, X. Wang, and D. Guo. A level set method for structural topology optimization. *Computer Methods In Applied Mechanics and Engineering*, 192(1-2):227–246, 2003.

[3] G. Allaire, F. Jouve, and A. Toader. Structural optimization using sensitivity analysis and a level-set method. *Journal of Computational Physics*, 194(1):363–393, 2004.

[4] S.-K. Choi, R. V. Grandhi, and R. A. Canfield. *Reliability-based Structural Design.* Springer, London, 2007.

[5] G. C. Calafiore and F. Dabbene. Optimization under uncertainty with applications to design of truss structures. *Structural and Multidisciplinary Optimization*, 35(3):189–200, 2008.

# Applications of singular value decomposition in structural optimisation

**F.-J. Barthold**[1] **and N. Gerzen**[2]

[1] Numerical Methods and Inform. Proc., TU Dortmund, Germany, franz-joseph.barthold@tu-dortmund.de
[2] Numerical Methods and Information Processing, TU Dortmund, Germany, nikolai.gerzen@tu-dortmund.de

**Abstract**
The usage of the singular value decomposition (SVD) in scope of structural optimisation is motivated. We briefly outline several applications of SVD to point out their similarities with structural optimisation. Here, our focus is on model reduction, on numerical defects and on their quantitative description using SVD. In this context we address the singular value decomposition of matrices which appear in an optimisation algorithm, i.e. the pseudo load matrix and the sensitivity matrix. W outline the applicability of SVD on the example of topology optimisation.

**Keywords:** singular value decomposition, sensitivity analysis, topology optimization

## 1. Abstract setting of a topology optimisation problem

We assume that a linear elastic body $\Omega$ is given in its discrete formulation $\Omega = \sum_{e=1}^{n} \Omega_e$. Furthermore, we assume that body loads are not present. Matrix notation is used to lessen the clerical work. The SIMP method (Simple Isotropic Material with Penalization [2],[5]) is used to carry out topology optimisation. In this case we are interested in the solution of the following problem: Find $\boldsymbol{s} \in \mathbb{R}^n$ such that

$$C(\boldsymbol{U}(\boldsymbol{s}), \boldsymbol{s}) \rightarrow \min_{\boldsymbol{s} \in \mathbb{R}^n} \qquad \text{subject to} \quad g(\boldsymbol{s}) = \sum_{e=1}^{n} s_e V_e - \bar{V} \geq 0 \quad \text{and} \quad \boldsymbol{R}(\boldsymbol{U}(\boldsymbol{s}), \boldsymbol{s}) = \bar{\boldsymbol{K}}(\boldsymbol{s})\boldsymbol{U} - \boldsymbol{F} = \boldsymbol{0}, \tag{1}$$

where $C = \boldsymbol{F}^T \boldsymbol{U}$ is the compliance of the given structure. Here, $\bar{\boldsymbol{K}} = \bigcup_e \bar{\boldsymbol{K}}_e = \bigcup_e s_e^p \boldsymbol{K}_e$ denotes the modified global stiffness matrix, $s_e$ are the design variables with $0 < s_e \leq 1$ and $p$ is the penalisation factor. The constraints are the volume constraint $g(\boldsymbol{s}) \geq 0$ and the weak form of equilibrium $\boldsymbol{R}(\boldsymbol{U}(\boldsymbol{s}), \boldsymbol{s}) = \boldsymbol{0}$. In order to solve the problem (1), for example, with the steepest decent method [4], the total derivative of the compliance with respect to the design variables must be calculated. By means of the variational sensitivity analysis [1] we obtain the following relation

$$\frac{dC(\boldsymbol{U}(\boldsymbol{s}), \boldsymbol{s})}{d\boldsymbol{s}} = \frac{\partial C}{\partial \boldsymbol{s}} + \frac{\partial C}{\partial \boldsymbol{U}} \frac{d\boldsymbol{U}}{d\boldsymbol{s}} \quad \text{with} \quad \frac{d\boldsymbol{U}}{d\boldsymbol{s}} = -\left(\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{U}}\right)^{-1} \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{s}} = -\hat{\boldsymbol{K}}^{-1} \boldsymbol{P} = \boldsymbol{S}. \tag{2}$$

The partial derivatives of the compliance can be obtained at once, but the total derivative of the displacement vector field $\boldsymbol{U}$ must be computed by means of the total derivative of the residual $\boldsymbol{R}$ as given in the above equation (see [1] for more details). Here, the matrices $\boldsymbol{P}$ and $\boldsymbol{S}$ are the pseudo load matrix and the sensitivity matrix, which build up the central parts of the sensitivity analysis. Thus the derivative of the compliance is

$$\frac{dC}{d\boldsymbol{s}} = \boldsymbol{F}^T \boldsymbol{S} \quad \text{which corresponds to the classical notation} \quad \frac{dC}{d\boldsymbol{s}} = -\boldsymbol{U}^T \frac{\partial \bar{\boldsymbol{K}}}{\partial \boldsymbol{s}} \boldsymbol{U}. \tag{3}$$

One of the principal ideas of this article is to get essential information about the sensitivity by means of singular value decomposition of the matrix $\boldsymbol{S}$.

**2. Introduction in singular value decomposition (SVD)** Let $\boldsymbol{S}$ be a $m \times n$ real matrix. Then there exists a factorisation of the form

$$\boldsymbol{S} = \boldsymbol{Y} \boldsymbol{\Sigma} \boldsymbol{Z}^T = \sum_{i=1}^{\min(m,n)} \sigma_{ii} \boldsymbol{y}_i \boldsymbol{z}_i^T, \tag{4}$$

where $\boldsymbol{Y}$ is a $m \times m$ matrix, the matrix $\Sigma$ is $m \times n$ diagonal matrix with nonnegative numbers $\sigma_{ii}$ (singular values, which are sorted in decreasing order) on the diagonal and $\boldsymbol{Z}$ a $n \times n$ matrix. The matrices

$\boldsymbol{Y}$ and $\boldsymbol{Z}$ contain a set of orthonormal 'output' and 'input' basis vectors for $\boldsymbol{S}$ (left and right singular vectors) with $\boldsymbol{Y}^{-1} = \boldsymbol{Y}^T$ and $\boldsymbol{Z}^{-1} = \boldsymbol{Z}^T$. SVD is a mathematical powerful tool [3] which is used in different scientific fields. The last term in equation (4) shows that any matrix $\boldsymbol{S}$ can be decomposed in several matrices of rank one weighted with singular values. As far as the singular values are sorted in decreasing order, the main parts of the given matrix (the first $k$ parts) can be determined and used for a matrix approximation. This technique is applied in image compression to reduce storage space. Moreover the transformation $\hat{\boldsymbol{s}} = \boldsymbol{Z}^T\boldsymbol{s}$ provides new variables which are sorted at the extent of their importance. The column vectors of $\boldsymbol{Z}$ make it possible to define the influence of the old variables on the new ones. This technique is often used in statistics and psychology to find out main influences on the given data base.

**3. Transformation of the topology optimisation problem using SVD** We carry out the SVD of the sensitivity matrix $\boldsymbol{S} = \boldsymbol{Y\Sigma Z}^T$ and introduce the transformation $\boldsymbol{s} = \boldsymbol{Z}\hat{\boldsymbol{s}}$ for the design variables. The column vectors of $\boldsymbol{Z}$ are perturbations in design (design modes) which induce the column vectors of $\boldsymbol{Y}$ which are the perturbations in the displacement field (displacement modes). Hence the new design variables are scaling factors for the design modes. As an example we show in figure 1 the design modes for a cantilever beam which is clamped on the left side, is loaded on the right side and is discretized by means of four node bilinear elements. The first modes induce global changes in the displacement field

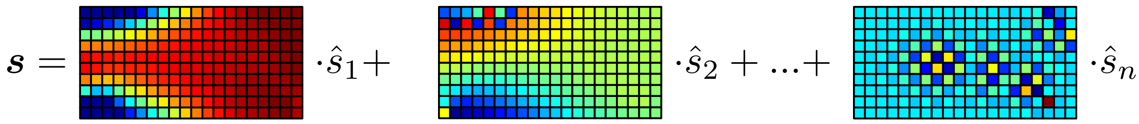



Figure 1: Design modes

and the last modes contain checkerboard modes. The derivative of the compliance with respect to the new design variables is

$$\frac{dC}{d\hat{\boldsymbol{s}}} = \frac{dC}{d\boldsymbol{s}}\frac{d\boldsymbol{s}}{d\hat{\boldsymbol{s}}} = \frac{dC}{d\boldsymbol{s}}\boldsymbol{Z} = \boldsymbol{F}^T\boldsymbol{Y\Sigma}. \tag{5}$$

The components of the new derivative are weighted by singular values. Therefore the first design modes are favored during an optimisation process (e.g. steepest decent). A model reduction can be carried out by using only the first $k$ design modes, the most important modes. The new design modes have to be computed in each iteration and so there is no information from the last iteration which can be used to accelerate the optimisation process (e.g. old gradients). However the Hessian

$$\boldsymbol{H} = \frac{d^2C}{d\boldsymbol{s}d\boldsymbol{s}} \quad \Rightarrow \quad \hat{\boldsymbol{H}} = \frac{d^2C}{d\hat{\boldsymbol{s}}d\hat{\boldsymbol{s}}} = \boldsymbol{Z}^T\boldsymbol{HZ} \tag{6}$$

can be used to obtain a more effective optimisation algorithm. In order to simplify the update formula the current design can be added to the matrix $\boldsymbol{Z}$ as a mode $\boldsymbol{z}_a$. We carried out this optimisation procedure for several examples with only few design modes and obtained reasonable solutions.

**4. Conclusions** We showed that a model reduction by using singular value decomposition is possible and one can substitute classical design variables for a few new ones. We expect smoother solutions to be provided using this transformation of the optimisation problem.

## References


[1] F. J. Barthold. Remarks on variational shape sensitivity analysis based on local coordinates. *Engineering Analysis with Boundary Elements*, 32:971–985, 2008.

[2] M.P. Bendse and O. Sigmund. *Topologiy Optimization*. Springer Verlag Berlin Heidelberg New York, 2003.

[3] J.W. Demmel. *Applied Numerical Linear Algebra*. Society for Industrial and Applied Mathematics, Philadelphia, 1997.

[4] J. Nocedal and S. Wright. *Numerical optimization*. Springer Science and Business Media, 2006.

[5] M. Zhou and G. I. N. Rozvany. The COC algorithm, Part II: Topological, geometry and generalized shape optimization. *Comp. Meth. Appl. Mech. Engng.*, 89:197–224, 1991.

| September 23, Poster session (DTU Mathematics, Building 303S) | | |
|---|---|---|
| **Time** | **Presenting author** | **Title** |
| 16.30 – 18.30 | Jan Fiala | Academic Benchmark Cases in PLATOLib<br>*J. Fiala, R. Boyd, M. Bogomolny, and M. Kočvara* |
| 16.30 – 18.30 | Esben Lindgaard | A Comparison of Linear Versus Nonlinear Buckling Formulation for the Optimization of Composite Structures<br>*E. Lindgaard and E. Lund* |
| 16.30 – 18.30 | Erik Lund | From Topology to Detailed Design of Multi-Material Laminated Composite Structures<br>*E. Lund, C.G. Hvejsel, and E. Lindgaard* |
| 16.30 – 18.30 | Nam Nguyen Canh | A method for solving discrete topology optimization problems<br>*N. Nguyen Canh and M. Stolpe* |
| 16.30 – 18.30 | Peter Nielsen | Iso-geometric analysis and shape optimization in mechanical engineering<br>*P.N. Nielsen and D.M. Nguyen* |

# Academic Benchmark Cases in PLATOLib

**J. Fiala**[1], **R. Boyd**[2], **M. Bogomolny**[3] **and M. Kočvara**[4]

[1] School of Mathematics, University of Birmingham, Birmingham, UK, fialaj@maths.bham.ac.uk
[2] Altair Engineering Ltd, UK, richard.boyd@uk.altair.com
[3] Department of Mathematics, Technical University of Denmark, Lyngby, Denmark, m.bogomolny@mat.dtu.dk
[4] School of Mathematics, University of Birmingham, Birmingham, UK, kocvara@maths.bham.ac.uk

**Abstract**
An example library called PLATOLib is one of the objectives of the PLATO-N Project. Its aim is to collect academic and industry benchmark cases to provide the first comprehensive benchmark library for Free Material Optimization. On the poster, we introduce the academic part of the library and content of problem sheets. To see the exploitation of the library differences between FMO and Topology Optimization are demonstrated together with a collection of included academic test cases. The library will be opened to public and its further development is expected.

**Keywords:** Free Material Optimization, benchmark library, test cases

# A Comparison of Linear Versus Nonlinear Buckling Formulation for the Optimization of Composite Structures

**Esben Lindgaard**[1] **and Erik Lund**[2]

[1] Department of Mechanical Engineering, Aalborg University, Denmark, E-mail: elo@me.aau.dk
[2] Department of Mechanical Engineering, Aalborg University, Denmark, E-mail: el@me.aau.dk

**Abstract**
The paper presents an approach to nonlinear buckling fiber angle optimization of laminated composite shell structures. The approach accounts for the geometrically nonlinear behaviour of the structure by utilizing response analysis up until the critical point. Sensitivity information is obtained efficiently by an estimated critical load factor at a precritical state, though including a number of the lowest buckling factors such that the risk of "mode switching" during optimization is avoided. The optimization is formulated as a mathematical programming problem and solved using gradient-based techniques. The presented optimization formulation is compared to the traditional linear buckling formulation and two numerical examples, including a large laminated composite wind turbine main spar, clearly illustrate the pitfalls of the traditional formulation and the advantage and potential of the presented approach.

**Keywords:** Composite laminate optimization, buckling, design sensitivity analysis, geometrical nonlinearity, composite structures.

## 1. Introduction
Nowadays, multilayered composite structures are popular in the fields that require low weight and high performance. In order to continually improve the performance of these structures, it is a necessity that the material utilization is pushed to the limit. A consequence hereof is that the structures are becoming thin-walled and local buckling becomes an issue in compressively loaded regions.
Stability is one of the most important objectives/constraints in structural optimization and this also holds for many laminated composite structures, e.g. a wind turbine blade. In stability analysis the buckling load is often approximated by linearized eigenvalue analysis at an initial prebuckling point (linear buckling analysis) and the buckling load is generally overestimated. In the case where nonlinear effects cannot be ignored nonlinear path tracing analysis is necessary. For limit point instability, several standard finite element procedures allow the nonlinear equilibrium path to be traced until a point just before the limit point. The traditional Newton like methods will probably fail in the vicinity of the limit point and the post-critical path cannot be traced. More sophisticated techniques, e.g. the arc-length methods, are among some of the techniques available today for path tracing analysis in the post-buckling regime.
A more accurate estimate of the buckling load, than that obtainable with linear buckling, can be obtained by performing a nonlinear response analysis and determine the buckling load by an eigenvalue analysis on the deformed configuration. Various eigenvalue problems have been suggested for the stability analysis of nonlinear structures. [1] formulated linear eigenvalue problems with information at one load step on the nonlinear prebuckling path. This formulation is often in literature referred as the "one-point" approach, where stiffness information is extrapolated until a singular tangent stiffness is obtained. Also a "two-point" approach has been proposed where a linear eigenvalue problem is formulated utilizing tangent information at two successive load steps on the nonlinear prebuckling path.
Optimization with stability constraints have been studied extensively in the past. Traditionally linear formulations are applied in buckling optimization for a general type of stability, see e.g. the works by [2]. In the last decades a number of nonlinear formulations for the buckling problem, taking geometrical nonlinearity into account, have been proposed, see e.g. [6], [3], [5], and [4].
This paper presents a benchmark study between the traditional linear formulation for the buckling problem and the newly proposed method described in [4]. The traditional linear formulation for buckling analysis and sensitivity analysis is presented together with the method in [4] and benchmarked upon several engineering examples of composite structures.

## 2. Linear and Nonlinear Buckling Analysis
The finite element method is used for determining the buckling load factor of the laminated composite

structure. The linear buckling problem is formulated on basis of the solution to the linear static equilibrium equation. From the static solution the initial stress stiffness matrix is evaluated and the traditional linear buckling problem is formulated as an eigenvalue problem. The nonlinear buckling load is obtained by including the nonlinear response by a path tracing analysis, after the arc-length method. The nonlinear path tracing analysis is stopped when a limit point is encountered and the critical load is estimated at a neighbouring precritical load step according to the "one-point" approach. In case of bifurcation buckling imperfections may be applied to the numerical model in order to convert the bifurcation point into a limit point.

**3. DSA of the Linear and Nonlinear Buckling Problem**

The objective of the optimization is, by use of gradient-based techniques, to maximize the lowest buckling load factors, and thus the buckling load factor sensitivities should be computed in an efficient way. Design sensitivities of the critical load factor for the linear and nonlinear problems are obtained semi-analytically by the direct differentiation approach on the approximate eigenvalue problem described by discretized finite element matrix equations. A number of the lowest buckling load factors are considered in order to avoid problems related to "mode switching". In [4], an optimization procedure was proposed that ensures the prebuckling approximation point is updated at each iteration such that it always lies in the neighbourhood of the real buckling load whereby precise estimates of buckling load factors and their sensitivities are obtained. The optimization problem of maximizing the lowest of the buckling load factors is formulated using a bound formulation and solved using the Method of Moving Asymptotes.

**4. Results**

The two formulations are benchmarked on two engineering examples of laminated composite structures. These examples which are to be presented in the PLATO-N workshop clearly demonstrate the importance of nonlinearity in structural design optimization w.r.t. stability and that the traditional linear formulation may lead to misleading unreliable design results.

**5. Conclusions**

Buckling behaviour of arbitrary composite structures can reliably be improved by the method described in [4]. The method includes accurate nonlinear path tracing analysis and the buckling load is estimated at a precritical point on the deformed configuration whereby a more precise estimate is obtained than that obtainable by traditional linear buckling analysis. General sensitivity formulas for the nonlinear buckling load, described by discretized finite element matrix equations, have been derived and the design sensitivities are approximated at the precritical point. Using this approach structures can reliably be optimized with respect to a general type stability, i.e. either bifurcation and limit point stability, and especially in cases where geometrically nonlinear effects cannot be ignored. This allows the material utilization of buckling critical laminated structures to be pushed to the limit in an efficient way yet allowing lighter and stronger structures.

**References**


[1] B. Brendel and E. Ramm. Linear and nonlinear stability analysis of cylindrical shells. *Computers and Structures*, 12(4):549–558, 1980.

[2] N.S. Khot, V.B. Venkayya, and L. Berke. Optimum structural design with stability constraints. *International Journal for Numerical Methods in Engineering*, 10(5):1097–1114, 1976.

[3] T.S. Kwon, B.C. Lee, and W.J. Lee. An approximation technique for design sensitivity analysis of the critical load in non-linear structures. *International Journal for Numerical Methods in Engineering*, 45(12):1727–1736, 1999.

[4] E. Lindgaard and E. Lund. Nonlinear buckling optimization of composite structures. *Submitted*, 2009.

[5] J.S. Park and K.K. Choi. Design sensitivity analysis of critical load factor for nonlinear structural systems. *Computers and Structures*, 36(5):823–838, 1990.

[6] C.C. Wu and J.S. Arora. Design sensitivity analysis of non-linear buckling load. *Computational Mechanics*, 3(2):129–140, 1988.

# From Topology to Detailed Design of Multi-Material Laminated Composite Structures

**E. Lund, C.G. Hvejsel and E. Lindgaard**

Department of Mechanical Engineering, Aalborg University, Denmark. Email: {el,cgh,elo}@me.aau.dk

**Abstract**
In this work the potential of using the so-called Discrete Material Optimization (DMO) approach as a starting point for multi-material design optimization of laminated composite structures is studied for multi-criteria problems with conflicting design criteria such as mass, compliance, buckling load factors and local failure criteria. Furthermore, several methods for automatic conversion of the initial topology design to detailed laminate optimization problems are described and illustrated by examples.

**Keywords:** Laminated composite structures, multi-material topology optimization, multi-criteria optimization.

## 1. Introduction

The use of laminated composite structures with Glass and Carbon Fiber Reinforced Polymers (GFRP and CFRP) is popular for lightweight constructions due to their superior strength and stiffness characteristics. In order to fully exploit the weight saving potential of these multilayered structures, it is necessary to tailor the laminate layup and behavior to the given structural needs. In some design situations it is most cost effective to combine several different materials, such that a multi-material design problem is considered. An example of such a design problem is wind turbine blades where sandwich structures are used in many parts of the structure, and where many different materials are combined, for example GFRP, CFRP, birch wood, wood-carbon/epoxy, balsa wood, and different foam materials. In order to obtain a cost effective design, it is desirable to have a general computer aided tool that can generate a high performance topology in the initial design phase, together with tools for automatic conversion to more detailed laminate design models.
Thus, the design problem considered in this work consists of optimal distribution of different materials in multi-layered composite shell structures, taking different structural performance criteria into account.

## 2. Multi-Material Topology Design

The design parametrization method applied is denoted Discrete Material Optimization (DMO), see [6, 7, 4, 3]. The basic idea in the DMO approach is to formulate an optimization problem using a parametrization that allows for efficient gradient based optimization on real-life problems while reducing the risk of obtaining a local optimum solution when solving the discrete material distribution problem. The approach is related to the mixed materials strategy suggested by Sigmund and co-workers [5, 2] for multi-phase topology optimization, where the total material stiffness is computed as a weighted sum of candidate materials. By introducing differentiable weighting functions for the material interpolation, the topology optimization problem is converted to a continuous problem that can be solved using standard gradient based optimization techniques.
The analysis is based on shell finite element models, and the criteria functions that have been implemented for linear and geometrically non-linear problems include mass and cost (price based on cost per material unit), compliance, eigenfrequencies, buckling load factors based on linear analysis, limit load factors based on geometrically nonlinear analysis, and local stress/strain criteria, which is ongoing challenging work. The DMO method generally generates many design variables, such that the adjoint method in most cases is the most efficient method for DSA. The optimization problems are solved using MMA [8], and in case of min-max optimization problems the bound formulation, see [1], is used.

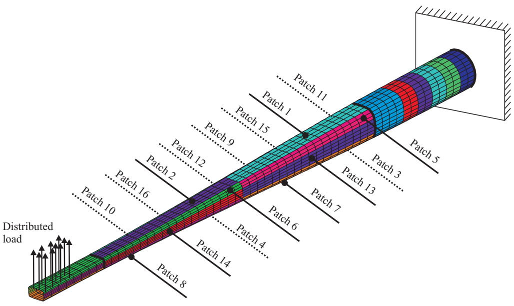
## 4. Subsequent Detailed Laminate Design

Postprocessing procedures have been developed that automatically convert the solution of the initial multi-material topology design problem to either a final layup description of the laminated structure or to

a refined laminate optimization model where design variables may be continuous layer thicknesses together with fiber angles of the chosen fiber reinforced materials, depending on the specified parametrization wanted for the different materials. In this way the obtained topology design may automatically be fine tuned by subsequent detailed optimization, such that the outcome is a detailed layup description of the laminated composite structure.
In case of having an initial DMO solution where non-distinct choices of material are obtained in some layers of the laminates structure, automated procedures have been developed that subdivides non-converged layers into new layers where the new layer thicknesses are set according to the ratios of the chosen candidate materials.

## 5. Examples and Concluding Remarks

The design optimization approach will be illustrated on several examples including results from a generic main spar from a wind turbine blade, see illustration on Figure 1. The examples illustrate that the DMO approach is good at obtaining distinct choices of material if the candidate materials are well suited for the design problem considered, and a single objective optimization problem is solved or the multi-criteria optimization problem does not involve completely contradictive criteria.
However, even in cases of non-distinct choices of material, the automatic conversion of the DMO model to a refined model where non-converged layers have been subdivided according to the chosen candidate materials yields high performance design solutions that typically fulfill the structural constraints specified.




| Patch \ Layer | L1 | L2 | L3 | L4 | L5 | L6 | L7 | L8 | L9 | L10 |
|---|---|---|---|---|---|---|---|---|---|---|
| P1 | -45°/45° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | -45°/45° |
| P2 | -45°/45° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | -45°/45° |
| P3 | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° |
| P4 | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° |
| P5 | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° |
| P6 | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° |
| P7 | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° |
| P8 | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° |
| P9 | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° |
| P10 | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° |
| P11 | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° |
| P12 | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° | 0° |
| P13 | biax | foam | foam | foam | foam | foam | foam | foam | foam | biax |
| P14 | biax45 | foam | foam | foam | foam | foam | foam | foam | foam | biax45 |
| P15 | biax | foam | foam | foam | foam | foam | foam | foam | foam | biax |
| P16 | biax45 | foam | foam | foam | foam | foam | foam | foam | foam | biax45 |

Figure 1: Left: Simplified parametrization of main spar example. Right: Example of initial DMO solution for minimum compliance design with buckling and mass constraints.

## References


[1] M.P. Bendsøe, N. Olhoff, and J.E. Taylor. A variational formulation for multicriteria structural optimization. *Journal of Structural Mechanics*, 11:523–544, 1983.

[2] L.V. Gibiansky and O. Sigmund. Multiphase composites with extremal bulk modulus. *Journal of the Mechanics and Physics of Solids*, 48:461–498, 2000.

[3] E. Lund. Buckling topology optimization of laminated multi-material composite shell structures. *Composite Structures*, 91:158–167, 2009.

[4] E. Lund and J. Stegmann. On structural optimization of composite shell structures using a discrete constitutive parametrization. *Wind Energy*, 8(1):109–124, 2005.

[5] O. Sigmund and S. Torquato. Design of materials with extreme thermal expansion using a three-phase topology optimization method. *Journal of the Mechanics and Physics of Solids*, 45:1037–1067, 1997.

[6] J. Stegmann. *Analysis and optimization of laminated composite shell structures*. Ph.D. Thesis, Institute of Mechanical Engineering, Aalborg University, Denmark, 2004. Special report no. 54, available at www.ime.aau.dk/˜js.

[7] J. Stegmann and E. Lund. Discrete material optimization of general composite shell structures. *Int. Journal for Numerical Methods in Engineering*, 62(14):2009–2027, 2005.

[8] K. Svanberg. The method of moving asymptotes - a new method for structural optimization. *Numerical Methods in Engineering*, 24:359–373, 1987.

# A method for solving discrete topology optimization problems

**Nam Nguyen Canh**[1] **and Mathias Stolpe**[2]

[1] Department of Mathematics, Technical University of Denmark (DTU),
Matematiktorvet, Building 303S, DK-2800 Kgs. Lyngby, Denmark
n.n.canh@mat.dtu.dk.
[2] Department of Mathematics, Technical University of Denmark (DTU)
Matematiktorvet, Building 303S, DK-2800 Kgs. Lyngby, Denmark
m.stolpe@mat.dtu.dk

## Introduction

We consider multiple load structural topology optimization problems in which the design variables are chosen from a finite set of given values. In particular, our interest is in minimum weight problems with constraints on the global stiffness of the structure, i.e., the compliance, and on local stress properties, such as the von Mises stress. The problems involve a large number of discrete design variables which can represent areas in truss structures, thicknesses in the two-dimensional case, and materials in the three dimensional case. From an optimization modeling point of view, these problems are non-convex mixed 0-1 programs on the form [3, 4]

$$\begin{aligned}
&\underset{x,u_1,\dots,u_M}{\text{minimize}} \sum_{j=1}^{n} x_j \rho_j \\
&\text{subject to} \quad K(x)u_k = f_k \qquad \forall k \\
&\qquad\qquad\quad f_k^T u_k \le \bar{\gamma}_k \qquad \forall k \\
&\qquad\qquad\quad u_k^T W_j u_k \le \bar{\sigma}^2 \qquad \forall k, \forall j : x_j = 1 \\
&\qquad\qquad\quad x \in \{0,1\}^n
\end{aligned}$$

We assume that the stiffness matrix $K(x) \in \mathbb{R}^{d\times d}$ depends affine on the design variables $x$, i.e.,

$$K(x) = K_0 + \sum_{j=1}^{n} x_j K_j$$

where $x_j K_j$ is the symmetric and positive semi-definite stiffness matrix of the *j*-th element and $K_0$ is a given symmetric positive semi-definite matrix (possibly equal to zero). The static external loads are given by the vectors $f_k \in \mathbb{R}^d \setminus \{0\}, k = 1,2,\dots,M$. The density $\rho_j$ for the *j*-th element is assumed to be strictly positive. The scalar $\bar{\gamma}_k > 0$ denotes upper bounds on the compliances. The stress constraints are described by the symmetric and positive semi-definite matrix $W_j \in \mathbb{R}^{d\times d}$ and the stress bound $\bar{\sigma} > 0$. We study both the theoretical and practical aspects of a global optimization method for solving the above class of problems.

The method is based on the concept of branch-and-cut in which a large number of continuous relaxations, i.e. continuous optimization problems that give lower bounds on the objective function of the considered problem, are solved. These relaxations are ideally manageable and give good approximations of the considered discrete problems.

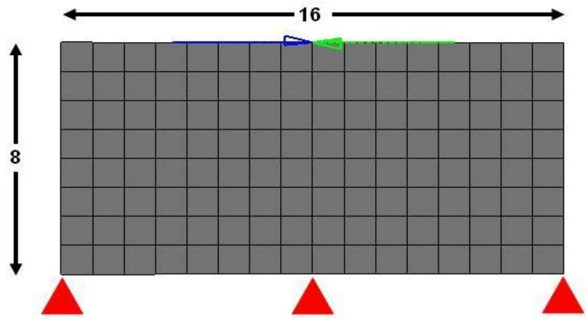
A new reformulation of the resulting relaxation which is suitable for a practical implementation in a nonlinear branch and cut method is also presented. Here our work generalizes and extends the available theory from the literature. Although the reformulation is non-convex, it is possible to avoid the non-convexity and to obtain global optimizers [3]. The relaxations and their reformulations provide the foundation for several heuristics which are used to find good feasible designs and to improve on already found feasible designs [4].

We also present an algorithm for generating additional constraints to strengthen the quality of the relaxations, to eliminate candidate designs that are not optimal, and to accelerate the branch-and-bound method. This algorithm exploits the mathematical structure and the mechanical assumptions of the problem, in particular the disjunctive nature of the constraints and the variables [4].

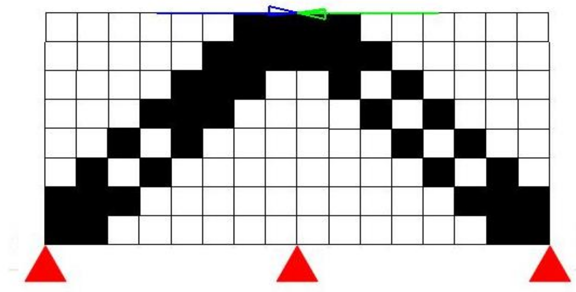
The branch-and-cut method gives correctly, after a finite number of iterations, a global minimize, i.e., a structure whose optimality is guaranteed or determines the problem as infeasible. The method is developed and implemented within the PLATO-N project (www.plato-n.org) and is used to solve benchmark examples which are used to validate other methods and heuristics. An example is given in the next section. The main aspects of the object oriented implementation of the method as well as numerical examples will be presented.

**Example**

We give a numerical example with the branch-and-cut method. The method is applied to solve the minimum weight problem with compliance constraints as illustrate in Figure 1 (a).




(a) Design domain, boundary conditions and the external loads



(b) Optimal design structure

Figure 1: Example

The construction material is isotropic and Young's modulus is equal to 1 ($E \coloneqq 1$) and Poisson's ratio is equal to zero ($\vartheta \coloneqq 0$). The design variable $x_j$ represents the thickness of the $j$-th element and must only attaint the values unity or zero. We have two external loads both with value 10. The finite elements used in the computations are bilinear iso-parametric elements in plane stress. The design domain is divided into square elements of equal size. Figure 1(b) illustrates the global optimal design obtained while the numerical results are reported in the Table 1.

| Elements | DoF | Loads | $\sum_k \gamma_k$ | Optimal Value | Iterations | Cuts |
|---|---|---|---|---|---|---|
| 98 | 596 | 2 | 1700 | 237.5 | 32430 | 1 |

Table 1: Numerical results

**References**

[1] M.P. Bendsøe and O. Sigmund. *Topology Optimization: Theory, Methods and Applications*, Springer-Verlag, Berlin, 2003.

[2] R. Horst and H. Tuy. *Global Optimization : Deterministic Approaches*. Springer-Verlag, 1993.

[3] M. Stolpe and R. Stainko and M. Kocvara. *Lower bounding problem for stress constrained discrete structural topology optimization problems*. EU project FP-6 STREP 30717 PLATO-N (Aeronautics), PLATO-N Public Report PU-R-5-2007, 2007.

[4] N. Nguyen Canh and M. Stolpe. *Efficient methods for solving mixed integer stress constrained problems*. EU project FP-6 STREP 30717 PLATO-N (Aeronautics), PLATO-N Public Report 2008.

# Iso-geometric analysis and shape optimization in mechanical engineering

**P.N. Nielsen**[1], **D.M. Nguyen**[2]

Department of Mathematics, Technical University of Denmark,
Matematiktorvet, Building 303S, 2800 Kgs. Lyngby, Denmark.
[1] E-mail: p.n.nielsen@mat.dtu.dk
[2] E-mail: d.m.nguyen@mat.dtu.dk

## Abstract

Iso-geometric analysis is a new computational methodology for engineering problems that offers a new way to exactly represent shapes, to easily refine the mesh in the numerical model, and to use the same basis to describe both the geometry and the physics in the problem [2]. Since geometry refinements are very easily made within iso-geometric analysis, it is an ideal tool for shape optimization purposes. In this poster the fundamentals of iso-geometric analysis are briefly outlined, and results from an application of the method to a simple 1-dimensional optimization problem of structural vibrations are presented, clearly pointing towards the great potential of the method [1]. Finally some future challenges for iso-geometric analysis and its applications are identified within structural vibrations, with focus on the vibrating membrane, and fluid mechanics, with focus on Stokes flow around a pipe bend.

**Keywords:** Iso-geometric analysis, shape optimization, structural vibrations, fluid mechanics.

## References

[1] J. A. Cottrell, A. Reali, Y. Bazilevs, and T. J. R. Hughes. Isogeometric analysis of structural vibrations. *Comput. Methods Appl. Mech. Engrg.*, 195:5257–5296, 2006.

[2] T. J. R. Hughes, J. A. Cottrell, and Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Comput. Methods Appl. Mech. Engrg.*, 194:4135–4195, 2005.

| **September 24, Morning session (Scion DTU, Building 373)**<br>**Chair: Erik Lund** | | |
|---|---|---|
| **Time** | **Presenting author** | **Title** |
| 09.30 – 10.30 | Thomas A. Grandine | Geometry Generation for Design Optimization<br>*T.A. Grandine* |
| 10.30 – 11.00 | | Coffee break |
| 11.00 – 11.30 | Markus J.D. Wagner | PLATO-N in an industrial context<br>*M.J.D. Wagner and J. Keller* |
| 11.30 – 12.00 | Claus B.W. Pedersen | Practical industrial topology optimization applications including non-linear modelling and automated interpretations for verification modeling<br>*C.B.W. Pedersen* |
| 12.00 – 12.30 | Stefanie Gaile | Free Material Optimization for Naghdi Shells<br>*S. Gaile, G. Leugering, and M. Stingl* |

# Geometry Generation for Design Optimization

**Thomas A. Grandine**[1]

[1] The Boeing Company, USA

**Abstract**
Many engineering discipline simulation and analysis tools require access to three-dimensional geometric models of proposed designs, often to varying levels of detail and fidelity. One approach to design optimization has been to put parametric geometry models inside an optimization loop along with these simulation and analysis codes and surrounding the whole with an optimization procedure. Doing so, however, is fraught with challenges that impose stringent requirements on the parametric geometry generation tools. This talk will explore many of these difficulties, discuss the challenges in trying to meet them with commercial CAD tools, and describe what needs to be done to rise to meet them.

# PLATO-N in an industrial context

**Markus JD Wagner**[1] **and Julian Keller**[2]

[1] EADS-MAS
Germany
markus.jd.wagner@eads.com

[2] EADS-MAS
Germany
julian.keller@eads.com

**Abstract**
Industrial needs require a new software for concept design incorporating more freedom in terms of material properties and more restrictions regarding justifications aims. PLATO-N, a EU framework 6 project, provides such a tool which is about to be finished as operational by end of the year. First results give an overview of what kind of optimisation problems can be solved and what can be expected for future applications.

**Keywords:** conceptual design, free-material-optimisation, industrial requirements, multi-disciplinary constraints

## 1. Introduction

PLATO-N is a STREP within the 6. EU-framework comprising ten partners from academia and industry. Major aim is to develop a software platform enabling an engineer to use Free Material Optimisation FMO [1] for conceptual design studies. This includes large scale capabilities in terms of number of design variables and very efficient algorithms to solve the optimisation problem. Close collaboration between the partners offers the possibility to on the one hand define industrial requirements and on the other hand to investigate different problem formulations to end up in mathematical tractable solutions for the desired user needs. Finally the platform should be able to solve very large scale problems using the free material approach in a context of a variety constraints i.e. displacements, stress, buckling and vibrations. Once material tensors are obtained from the optimisation the visualisation and interpretation leads into a structural concept. For this issue a powerful user interface is developed.

## 2. Industrial requirements

There are several circumstances pointing to the necessity of a new tool. Beneath others the most important are presented here.

### 2.1. Exploitation of material properties

Regarding state-of-the-art materials, i.e. fibre reinforced plastics, within aircraft industries the question arose how to make best usage of the material properties. Up to now only the stiffness-to-density ratio and the strength-to-density ratio is exploited to come up in some lighter structures than with traditional metallic alloys. In fact composite materials comprise more than just these scalar values but rather more the opportunity to place orthotropic layers to certain directions. In a single load scenario it is well known practice to follow main principle stress/strain directions as a result of the theory of mechanics. But considering a multi load case, for each single one of them a different stress/strain tensor is applied. A pure linear combination of these loads ends up in a too conservative structure cause there is no possibility of exploitation of any kind of internal redundancy.

### 2.2. State-of-the-art conceptual design method SIMP [2]

As the SIMP- method is limited to isotropic material there is no chance to obtain any information about stiffness directions. Even more the penalisation (a well working mathematical trick) forces a structure to a truss-like concept, which is not always suitable, e.g. for aerodynamic surfaces which need to be closed. In this case the question is not where to put the wholes anymore but where to place the directed stiffness of the composite material. This means penalisation stands in contradiction to the basic problem formulation. Using SIMP without penalty exponent results in intermediate material states which are hard to be interpreted with isotropic materials, but having more information available i.e. stiffness directions these intermediate states get more meaningful.

2.3. Multi-disciplinary requirements [3]
In terms of simulation of real physical phenomena lots of requirements have to be considered. However during the conceptual design phase only a small choice needs to be satisfied such that the main global design driving requirements are met. Regarding the structural concept a first approach is to consider firstly static loads against the a material allowable and against initial buckling and secondly the fundamental frequency. This means the static equilibrium needs to be solved simultaneously to an Eigenvalue problem resulting from the linear buckling or normal modes analysis.

**3. Small industrial example**
Providing a small industrial test example all (possibly) implicitly given academic assumption have to be overcome. All of them are of course in the nature of the finite element modeling. Here exists a long list of desired features but a reasonable choice needed to be done not to overload the project. However some mandatory requirements were not considered in detail such that some corrective actions are involved in the ongoing testing phase.
The example comprises a wing box segment cut out somewhere in the middle of a full wing, see figure 1. The

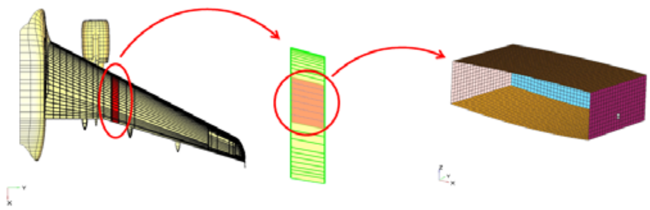


Figure 1: Wing box test example: modeling

geometric surfaces were discretised by three- and four- noded elements and the interior filled up with solids. To obtain a multi-load scenario, two linear independent load cases were created, one with pure lift and one with pure torsion on the wing, the drag was neglected. However the loading is just a rough estimation without any relation to an aircraft in service. Incorporating a load generation process is desirable but not part of the PLATO-N- project.
First results were achieved so far using the standard FMO-problem formulation, see figure 2 .

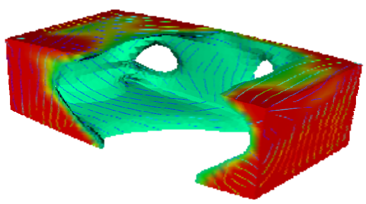


Figure 2: Results (density distribution and stiffness directions)

FMO-configurations with additional constraints are in a test status.

**4. Conclusions**
To face current and future tasks in the conceptual design phase the necessity of the new platform PLATO-N incorporating several constraints of the subsequent preliminary design phase is pointed out. Its development within the project itself is finished and the test and tuning phase is ongoing. A few but mandatory features were caught up and first industrial test results are available. The tests are to be finished by the end of the year aiming towards a fully operational software. Enabling engineers to investigate a material distribution and stiffness directions using the free material approach new more composite suitable concepts are expected.

**References**


[1] M. Kocvara and M. Stingl. Free material optimization for stress constraints. *Structural and Multidisciplinary Optimization*, 33:323-335, 2007.

[2] M.P. Bendsøe and O. Sigmund. *Topology Optimization: Theory, Methods and Applications*, Springer-Verlag, Berlin, 2003.

[3] M. Wagner and L. Krog. *Use Case Definition, Deliverable D2 – PLATO-N*, http://www.plato-n.org/upload/institutter/mat/plato-n/reports/plato-n%20pu-r-1-2007.pdf, 2007

# Practical industrial topology optimization applications including non-linear modelling and automated interpretations for verification modelling

**Claus B.W. Pedersen**[1]

[1]FE-DESIGN, Haid-und-Neu-Straße 7, D-76137 Karlsruhe, Germany, www.fe-design.com , claus.pedersen@fe-design.com

**Abstract**
The present work deals with two aspects of industrial structural topology optimization [1]. Initially, the work shows how topology optimization algorithms and software can be applied for highly non-linear and large scale finite element models. Secondly, the work shows how automated interpreted topology optimization results can be automatically remodeled and remeshed in an existing CAE workflow including the original boundary and loading conditions.
**Keywords:** Industrial applications, structural optimization software, non-linear modelling, design interpretations.

## 1. Practical industrial topology optimization applications for non-linear modelling

One or several of the following: contact modelling, non-linear materials and large deformation modelling [2,3] are frequently included in the finite element models for obtaining realistic modelling of industrial applications. Consequently, the topology optimization algorithms and software should also deal with these non-linear modelling issues. The present topology optimization algorithm [4] dealing with the previous mention modelling issues derives the sensitivities using the adjoint method [1,4,5] and the relative densities are updated using mathematical programming [6]. An interpolation law for non-linear material properties using the density approach in topology optimization is implemented according to [7].

## 2. Topology optimization example including contact modelling and large displacements

The present example includes contact modelling and large displacement modelling as shown in figure 1a. The contact is modeled using a friction coefficient of 0.2. Initially, the structure is optimized for stiffness by minimizing the compliance applying a mass constraint equal to 30% of the initial mass. The result of the stiffness optimization is shown in figure 1b. As expected the result obtained from the topology optimization is basically a bar in compression. However, the contact zones are highly concentrated at a small region initiating the contact forces to be extremely high in this small region. Consequently, a third optimization is executed where additional constraints for the contact forces are applied enforcing a more even distribution of the contact forces, see figure 1c. The obtained topology optimization result almost has a complete even distribution of the contact forces.

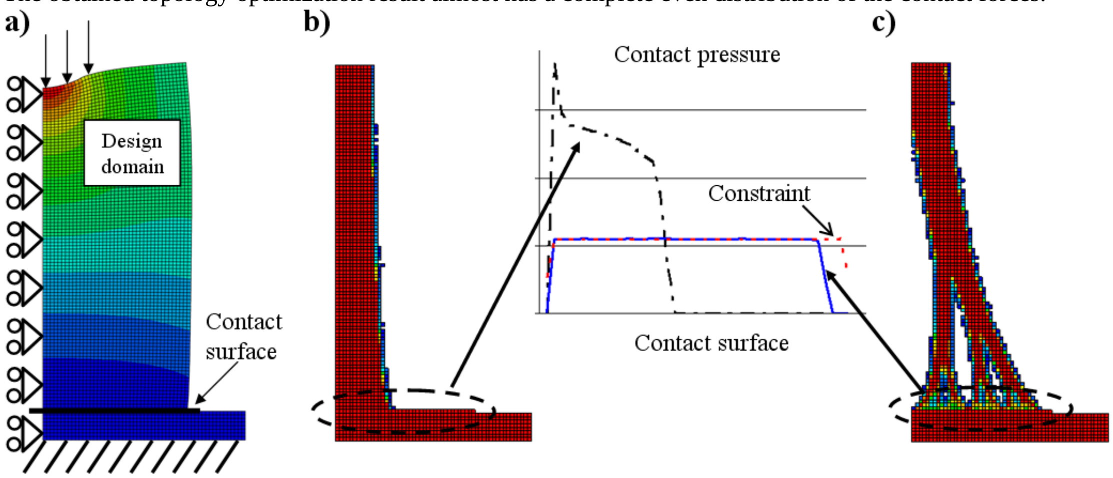



Figure 1 (a) Displacements of initial model including contact between the two parts where one part contains the design domain. (b) Design optimized for stiffness. (c) Design optimized for stiffness having additional constraints on the contact forces ensuring an even distribution of the contact forces.

Several other non-linear and large scale optimization results will also be shown at the conference.

## 3. Topology optimization interpretations and remodelling in an industrial workflow

A very important step in an automated industrial design workflow is to export the topology optimized designs into new finite element models for verification analysis or for exporting the designs to CAD systems. A final topology optimized result consists of a large amount of elements being solid or void and some elements having intermediate densities at the design borders, see figure 2b. Therefore, an important issue is that the topology optimization data from the optimizations are converted (TOSCA.smooth) using smoothing causing a data reduction. The calculation of the surfaces is shown in figure 2c. Thereby, all void and intermediate density elements are eliminated in the models. The smoothed designs can be exported in CAD-compatible formats (STL or IGES) containing isosurfaces or/and splines.

Frequently after the smoothed designs have been obtained the designer wants quickly to verify and validate if the structural responses of the smoothed designs are still feasible [4,8]. Finite element models generated for validating can be created using a fully automated process. The surfaces from smoothed structures are automatically remeshed using the TOSCA ANSA environment [4], see figure 2d. The smoothed surfaces in the design domain are automatically filled with generated tetrahedron elements. The new mesh is compatible with the original mesh as well as the original positions of the loading and boundary conditions are kept. Afterwards, the designer can then do a structural finite element validation of the smoothed design as shown in figure 2e.

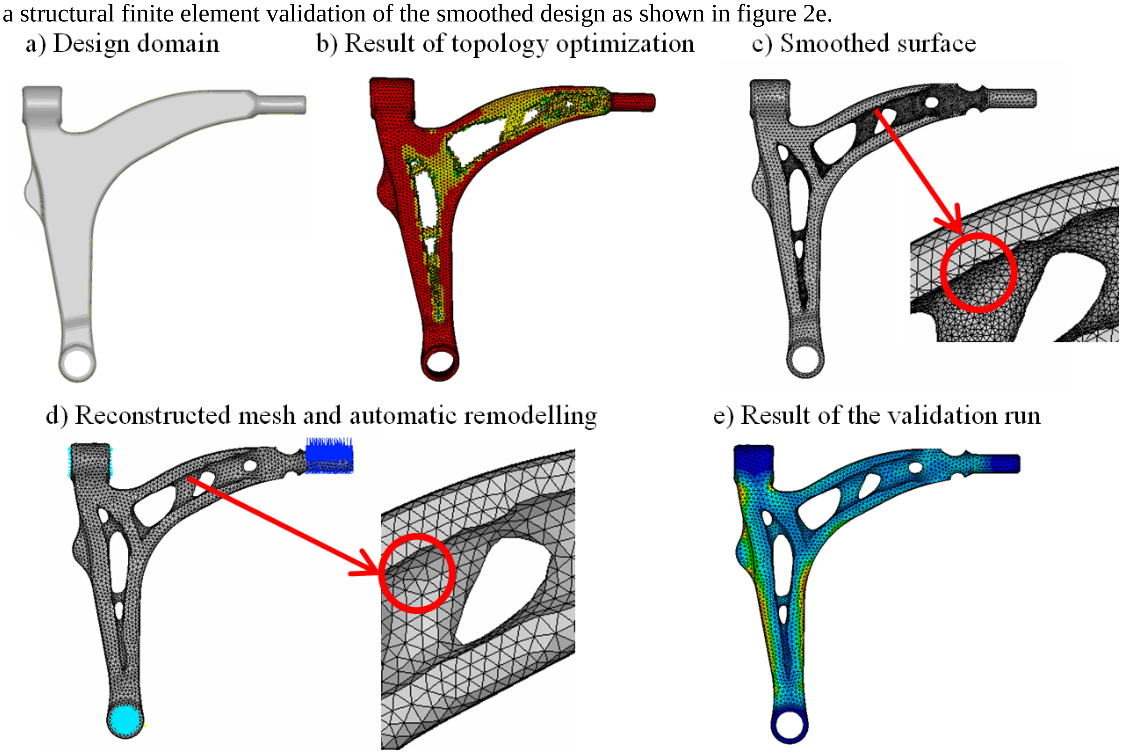



Figure 2. (a) Design domain, (b) optimization result, (c) smoothed surface and (d) validation model automatically meshed using the smoothed surfaces. The generated mesh has the original loading and boundary conditions (TOSCA ANSA environment). (e) Structural response of the validation model.

**References**

[1] Bendsøe, M. P. and Sigmund, O. (2003), Topology Optimization: Theory, Methods and Applications, Springer-Verlag, Berlin.

[2] Ansys, by ANSYS Inc, www.ansys.com

[3] Abaqus, by Dassault Systèmes, www.simulia.com

[4] TOSCA Structure 7.0, by FE-Design GmbH, www.fe-design.com

[5] Buhl, T., Pedersen, C. B. W. and Sigmund, O. (2000), Stiffness design of geometrically non-linear structures using topology optimization, Structural and Multidisciplinary Optimization, 19(2), pp.93-104.

[6] Svanberg, K. (1987), The Method of Moving Asymptotes - A New Method for Structural Optimization. International Journal for Numerical Methods in Engineering, 24, 359-373.

[7] Pedersen, C. B. W., Revisiting Topology Optimization of Continuum Structures with Elastoplastic Response, 15th Nordic Seminar on Computational Mechanics, 18-19 October, 2002, Aalborg, Denmark, Eds.: E. Lund, N. Olhoff, J. Stegmann.

[8] Clausen, P.M. and Pedersen, C. B. W., Non-parametric NVH optimization of industrial designs, 8th World Congress on Structural and Multidisciplinary Optimization, June 1 - 5, 2009, Lisbon, Portugal.

# Free Material Optimization for Naghdi Shells

**S. Gaile, G. Leugering and M. Stingl**

Department of Mathematics, Applied Mathematics II, University of Erlangen-Nuremberg, Germany
stefanie.gaile@am.uni-erlangen.de

**Abstract**
Free Material Optimization (FMO) deals with the problem of finding the ultimately best material distribution and properties for an elastic structure subjected to a given set of loads. Its original formalism has been extended to treat multidisciplinary optimization constraints such as displacement, stress or vibration constraints. Here we investigate FMO with multidisciplinary optimization constraints based on the elastic theory for Naghdi shells.

**Keywords:** Design Optimization, Free Material Optimization, Naghdi Shells, Thin-walled Structures, Semidefinite Programming.

## 1. Introduction

Free Material Optimization, as introduced by [1], is a subbranch of structural optimization, where the full elasticity tensor is considered as design variable providing not only the optimal material distribution, but also the optimal material properties at each point $x \in \omega$. Due to this freedom in the design space FMO has gained special interest in the optimization of airplane components, where the use of advanced materials is widespread. The desire to find the optimal material for thin-walled structures as e.g. the fuselage has given rise to augment the FMO problem formulation by using Naghdi's shell model.

## 2. Naghdi's Shell Model

Naghdi shells employ the theory of Cosserat continua by describing the shell as a midsurface $\omega$ with a director vector attached to each point [3, 6]. Hence the displacements can be split into translations of the points on the surface $u \in [H^1(\omega)]^3$ and rotations of the director vectors $\theta \in [H^1(\omega)]^2$. As rotations of the director vectors around their own axis are neglected the displacements take the form

$$U(\xi^1, \xi^2, \xi^3) = u(\xi^1, \xi^2) + \xi^3 \theta_\lambda(\xi^1, \xi^2) a^\lambda(\xi^1, \xi^2)\,, \tag{1}$$

where $\xi_i$, $i = 1, 2, 3$ represent the curvilinear coordinates and $a^\lambda$, $\lambda = 1, 2$ is the contravariant basis of the midsurface $\omega$. By segmenting the boundary $\partial\omega$ into two disjoint sets $\partial\omega_0$ and $\partial\omega_1$ the set of admissible displacements can be written as

$$\begin{aligned} \mathcal{U} \;:=\; \Big\{ (u,\theta) \in \left[H^1(\omega)\right]^5 \;\Big|\; & \exists\, i \in \{1,2,3\} \text{ and/or } \alpha \in \{1,2\} \text{ such that} \\ & u_i = 0 \text{ and/or } \theta_\alpha = 0 \text{ on } \partial\omega_0 \Big\}\,. \end{aligned} \tag{2}$$

An equilibrium condition for Naghdi shells can be found by minimizing the shell's potential energy

$$\Pi(u,\theta) = \frac{1}{2}\int_\omega t\gamma^\top C\gamma + \frac{t^3}{12}\chi^\top C\chi + tk\zeta^\top D\zeta \, d\mathcal{S} - \int_\omega t f^\top u \, d\mathcal{S} - \int_{\partial\omega_1} g_u{}^\top u + g_\theta{}^\top \theta \, dl\,, \tag{3}$$

where $t$ is the shell's thickness, $f \in [L^2(\omega)]^3$, $g_u \in [L^2(\partial\omega_1)]^3$ and $g_\theta \in [L^2(\partial\omega_1)]^2$ are the applied loads and moments and $k$ is the shear correction factor. Due to their symmetry the material tensor $C$ accounting for membrane and bending behaviour and the material tensor $D$ determining the shear energy can be written as matrices. The same is true for the membrane strains $\gamma$, the bending strains $\chi$ and the shear strains $\zeta$. As all these strains depend linearly on the displacements Naghdi's shell model is a linear elastic model.

## 3. Finite Element Discretization

A Finite Element method is used to discretize the equilibrium condition [2]. To this end the midsurface $\omega$ is partitioned into $M$ quadrangular elements $\omega_m$. The material matrices $C$ and $D$ are approximated by element-wise constant matrices $C_1, \ldots, C_M$ and $D_1, \ldots, D_M$.

A piecewise linear approximation containing the bilinear 2D Lagrange shape functions $\vartheta_i(r,s)$ is used for the displacements leading to the global stiffness matrix $K_{\text{shell}}(C,D)$ combining the membrane, bending and shear terms. Furthermore the forces and moments $f$, $g_u$ and $g_\theta$ are replaced by their discrete counterparts.

**4. Free Material Optimization for Shells**

To set up the minimum weight formulation of the FMO problem for shells the summed traces of the material tensors are taken as a measure for the used amount of material and serve as objective for the optimization problem. Besides the lower and upper bounds on the material matrices and the equilibrium condition a constraint on the compliance $f_h{}^\top(u,\theta)$ is added to the problem formulation to restrict the deformation of the structure under the given loads.

$$\begin{aligned} \min_{\substack{(u,\theta)\in\mathcal{U}_h\\(C,D)\in\mathcal{C}_h}} \sum_{m=1}^{\text{EltNr}} t_m\cdot \mathrm{tr}C_m + \frac{t_m}{2}\cdot\mathrm{tr}D_m \qquad (4)\\ \text{subject to} \qquad C_m\succeq 0\,;\ D_m &\succeq 0 \quad \forall\, m=1,\dots,M\\ \begin{pmatrix} t_mC_m & 0\\ 0 & \frac{t_m}{2}D_m\end{pmatrix} - \varepsilon_m \mathbb{1}_5 &\succeq 0 \quad \forall\, m=1,\dots,M\\ t_m\cdot\mathrm{tr}C_m + \frac{t_m}{2}\cdot\mathrm{tr}D_m &\leq \rho^+ \ \ \forall\, m=1,\dots,M\\ \sum_{m=1}^{\text{M}} K_{\text{shell}}(C_m,D_m)(u,\theta) &= tf+g_u+g_\theta = f_h\\ f_h{}^\top(u,\theta) &\leq c \end{aligned}$$

This problem formulation can be extended by several multidisciplinary optimization constraints. Stress constraints for example have been introduced in [5]. The following constraint limits the in-plane stresses of the optimized shell

$$\|C_m\gamma_m + t_mC_m\chi_m\|^2 \leq s_\sigma^{\text{ip}}|\omega_m|\,. \qquad (5)$$

Another alternative is to include constraints on the eigenfrequency [7]. This is achieved by adding a linear semidefinite constraint containing the mass matrix $M(C,D)$, which bounds the minimal eigenfrequency of the optimal structure from below by $\hat\lambda$

$$K_{\text{shell}}(C,D) - \hat\lambda M(C,D)\succeq 0\,. \qquad (6)$$

By adding these constraints to the original problem formulation it is possible to obtain an optimal structure that is much better suited to realistic engineering needs. The resulting nonlinear semidefinite programs can e.g. be solved by using the nonlinear SDP code PENNON [4].

**References**


[1] M. P. Bendsøe, J. M. Guedes, R. B. Haber, P. Pedersen, and J. E. Taylor. An analytical model to predict optimal material properties in the context of optimal structural design. *J. Appl. Mech. Trans. ASME*, 61:930–937, 1994.

[2] D. Chapelle and K. J. Bathe. *The Finite Element Analysis of Shells – Fundamentals.* Springer, Heidelberg, 2003.

[3] E. Cosserat and F. Cosserat. *Théorie des Corps Déformables.* Hermann, Paris, 1909.

[4] M. Kočvara and M. Stingl. PENNON - a code for convex nonlinear and semidefinite programming. *Optimization Methods and Software*, 18(3):317–333, 2003.

[5] M. Kočvara and M. Stingl. Free material optimization: Towards the stress constraints. *Structural and Multidisciplinary Optimization*, 2006. DOI: 10.1007/s00158-007-0095-5.

[6] P. M. Naghdi. The theory of shells and plates. In *Handbuch der Physik VIa/2*, pages 425–640. Springer, 1972.

[7] M. Stingl, M. Kočvara, and G. Leugering. Free material optimization with fundamental eigenfrequency constraints. *SIAM J Optimization*, 2008. accepted.

| **September 24, Afternoon session I (Scion DTU, Building 373)**<br>**Chair: Christian Zillober** | | |
|---|---|---|
| **Time** | **Presenting author** | **Title** |
| 14.00 – 14.30 | Sonja Lehmann | A Feasible Sequential Convex Programming Method for Free Material Optimization<br>*S. Lehmann, K. Schittkowski, and C. Zillober* |
| 14.30 – 15.00 | Luba Tetruashvili | A Sequential Parametric Convex Approximation Method with Applications to Nonconvex Truss Topology Design Problems<br>*A. Beck, A. Ben-Tal, and L. Tetruashvili* |
| 15.00 – 15.30 | Michal Kočvara | Algorithms of Free Material Optimization<br>*M. Kočvara, C. Bogani, J. Fiala, M. Stingl, and Y. Xia* |

| **September 24, Afternoon session II (Scion DTU, Building 373)**<br>**Chair: Alicia Kim** | | |
|---|---|---|
| **Time** | **Presenting author** | **Title** |
| 16.00 – 16.30 | Jakob Andkjær | Design of grating couplers using topology optimization for the efficient excitation of surface plasmons<br>*J. Andkjær, O. Sigmund, S. Nishiwaki, and T. Nomura* |
| 16.30 – 17.00 | Niels Aage | Topology Optimization of Metallic Microwave Devices<br>*N. Aage and O.Sigmund* |
| 17.00 – 17.30 | René Matzen | Design of Optical Circuit Devices Using Transient Topology Optimization<br>*R. Matzen, J.S. Jensen, and O. Sigmund* |
| 17.30 – 18.00 | Etienne Lemaire | Influence of the material model on local pull-in in electromechanical microdevices topology optimization<br>*E. Lemaire, L. van Miegroet, P. Duysinx, and V. Rochus* |

# A Feasible Sequential Convex Programming Method for Free Material Optimization*

**S. Lehmann**[1], **K. Schittkowski** [2] **and C. Zillober** [3]

[1] Department of Computer Science, University of Bayreuth, D-95440 Bayreuth, sonja.lehmann@uni-bayreuth.de
[2] Department of Computer Science, University of Bayreuth, D-95440 Bayreuth, klaus.schittkowski@uni-bayreuth.de
[3] Department of Mathematics, University of Würzburg, D-97074 Würzburg, christian.zillober@uni-wuerzburg.de

**Abstract**
In Free Material Optimization (FMO), we try to find the *best* mechanical structure in the sense of maximizing its stiffness with respect to given load cases. Design variables are the elementary material matrices, based on a FE discretization. These matrices must be positive definite to guarantee a non-singular global stiffness matrix and to be able to evaluate other constraints and the objective function. Thus, it is required that all iterates of the optimization process must be strictly feasible subject to these constraints. Any other constraint may be violated at intermediate iterates. Based on sequential convex programming (SCP) we propose a modification such that selected constraints retain feasible. Separable and strictly convex nonlinear subproblems of the SCP method are extended by nonlinear constraints, which guarantee positive definite elementary matrices.

**Keywords:** SCP, sequential convex programming, free material optimization, feasibility restoration.

## 1. Introduction

We consider a nonlinear optimization problem with an objective function $f(x)$, which is minimized with respect to two classes of nonlinear inequality constraints, $c_j(x) \leq 0$, $j = 1, \ldots, m_e$ and $e_j(x) \leq 0$, $j = 1, \ldots, m_f$. The paradigm is that $f(x)$ and $c_j(x)$, $j = 1, \ldots, m_f$ can only be evaluated at $x \in \mathbb{R}^n$ satisfying the constraints $e_j(x) \leq 0$, $j = 1, \ldots, m_f$, and therefore $e_j(x) \leq 0$, $j = 1, \ldots, m_f$ must be feasible during the whole iterative process. Thus, we proceed from the nonlinear program

$$\begin{array}{lll} \min\limits_{x \in \mathbb{R}^n} & f(x) & \\ \text{s.t.} & c_j(x) \leq 0, & j = 1, \ldots, m_e, \\ & e_j(x) \leq 0, & j = 1, \ldots, m_f. \end{array} \tag{1}$$

It is assumed that the feasibility functions $e_1(x), \ldots, e_{m_f}(x)$ are twice continuously differentiable on $\mathbb{R}^n$, and that the objective function $f(x)$ as well as the constraint functions $c_1(x), \ldots, c_{m_e}(x)$ are twice continuously differentiable on the convex subset

$$F := \{x \in \mathbb{R}^n : e_j(x) \leq 0,\ j = 1, \ldots, m_f\} \subset \mathbb{R}^n. \tag{2}$$

## 2. The Algorithm

The idea of the feasible sequential convex programming is to generate a sequence of convex and separable subproblems by approximating the nonlinear constraints $c_j(x)$, $j = 1, \ldots, m_e$ and the objective function $f(x)$. This goes back on the method of moving asymptotes (MMA), see Svanberg [3]. To guarantee strict feasibility of the nonlinear constraints $e_j(x)$, $j = 1, \ldots, m_f$ they are passed to the subproblem directly. As the subset $F \subset \mathbb{R}^n$ is convex these nonlinear subproblems possess an unique solution.
To stabilize the algorithm, we apply a line search strategy, see Zillober [5]. One of the most important features is the introduction of two flexible asymptotes, $L_i$ and $U_i$, $i = 1, \ldots, n$, which restrain the feasible region and prevent the algorithm from performing long steps into the infeasible area.

In each iteration $k$ the objective function $f(x)$ and the nonlinear inequality constraints $c_1(x), \ldots, c_{m_e}(x)$ are linearized subject to reciprocal variables $(U_i^{(k)} - x_i)^{-1}$ and $(x_i - L_i^{(k)})^{-1}$, $i = 1, \ldots, n$. The resulting

*The research was sponsored by FP-6 STREP 30717 PLATO-N (Aeronautics and Space),
PLATO-N - A PLAtform for Topology Optimisation incorporating Novel, Large-Scale, Fee-Material Optimisation and Mixed Integer Programming Methods

approximation of the constraint $c_j(x)$ at an iterate $x^{(k)} \in \mathbb{R}^n$ is

$$\begin{aligned} c_j^{(k)}(x) \;:=\; c_j\left(x^{(k)}\right) \;&+\; \sum_{I_+^{(k)}} \left[\frac{\partial c_j\left(x^{(k)}\right)}{\partial x_i}\left(U_i^{(k)} - x_i^{(k)}\right)^2 \left(\frac{1}{U_i^{(k)} - x_i} - \frac{1}{U_i^{(k)} - x_i^{(k)}}\right)\right] \\ &-\; \sum_{I_-^{(k)}} \left[\frac{\partial c_j\left(x^{(k)}\right)}{\partial x_i}\left(x_i^{(k)} - L_i^{(k)}\right)^2 \left(\frac{1}{x_i - L_i^{(k)}} - \frac{1}{x_i^{(k)} - L_i^{(k)}}\right)\right] \end{aligned} \tag{3}$$

with $I_+^{(k)} := \left\{ i \middle| \frac{\partial c_j\left(x^{(k)}\right)}{\partial x_i} \geq 0 \right\}, \quad I_-^{(k)} := \left\{ i \middle| \frac{\partial c_j\left(x^{(k)}\right)}{\partial x_i} < 0 \right\}, \quad L_i^{(k)} < x_i < U_i^{(k)}.$

An additional positive parameter $\tau_i,\ i = 1, \ldots, n$ is introduced and added to the approximation of $f(x)$, to ensure a strict convex objective function, see Zillober [5].

### 3. Numerical Results

The presented algorithm is used to solve free material optimization problems. The stiffness of a structure is described by compliance functions $f_j^T K^{-1}(E) f_j$ for loads $f_1, \ldots, f_l$, where $l$ is the number of load cases and $K(E)$ the global stiffness matrix. A more detailed description is found in Kočvara and Zowe [2]. The linear system $K(E)u_j(E) = f_j$ can only be solved if $E_i$ is positive definite for $i = 1, \ldots, m$. Test cases are provided by the PLATO-N test case library, see Bogomolny [1]. Problems with up to 20.000 elements, leading to 120.000 variables have been solved up to now. The graphical presentation in Figure 1 is given by plotting the sum of the trace in each element.

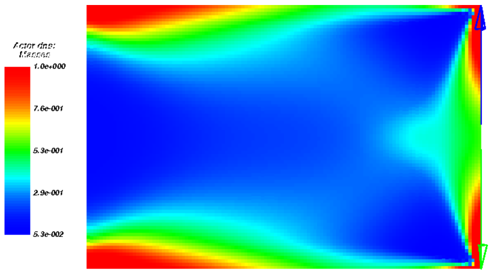



Figure 1: Multiple load case with 28.800 variables.

### 4. Conclusions

We developed a strictly feasible optimization algorithm which guarantees feasibility of a subset of the original constraints in each iteration step. The algorithm is used to solve free material optimization problems. Currently we use IPOPT, see Wächter and Biegler [4], for solving the convex and separable subproblems. We expect that computation time can be reduced significantly by using an active set strategy. A corresponding SQP-IPM code is implemented at present. Furthermore, we plan to add more complicated constraints such as buckling, stress, vibration and displacements corresponding to more realistic FMO applications.

### References

[1] Bogomolny, M. D34 - Test case library PLATOlib. Technical report, PLATO-N public report PU-R-6-2008, 2008.

[2] Kočvara, M., Zowe, J. *Free Material Optimization: an overview.* In A.H. Siddiqi and M. Kočvara, eds., Trends in Industrial and Applied Mathematics, Kluwer Academic Publishers, 2002.

[3] Svanberg, K. The method of moving asymptotes - a new method for structural optimization. *International Journal for Numerical Methods in Engineering*, 24:359–373.

[4] Wächter, A., Biegler, L. T. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Math. Program.*, 106(1):25–57, 2006.

[5] Zillober, C. Software manual for SCPIP 3.0. Technical report, 2004.

# A Sequential Parametric Convex Approximation Method with Applications to Nonconvex Truss Topology Design Problems

**Amir Beck**[1] **Aharon Ben-Tal**[2] **Luba Tetruashvili**[3]

[1] Department of Industrial Engineering and Management, Technion—Israel Institute of Technology, Haifa 32000, Israel. E-mail: becka@ie.technion.ac.il
[2] Department of Industrial Engineering and Management, Technion—Israel Institute of Technology, Haifa 32000, Israel. E-mail: abental@ie.technion.ac.il
[3] Department of Industrial Engineering and Management, Technion—Israel Institute of Technology, Haifa 32000, Israel. E-mail: lubate@tx.technion.ac.il

**Abstract**
We describe a general scheme for solving nonconvex optimization problems, where in each iteration the nonconvex feasible set is approximated by an inner convex approximation. The latter is defined using an upper bound on the nonconvex constraint functions. Under appropriate conditions on this upper bounding convex function, a monotone convergence to a KKT point is established. The scheme is applied to Truss Topology Design (TTD) problems, where the nonconvex constraints are associated with bounds on displacements and stresses. It is shown that the approximate convex problems solved at each inner iteration can be cast as a conic quadratic program, hence large scale TTD problems can be efficiently solved by the proposed method.

**Keywords:** convexifying noncovex constraints, Truss Topology Design (TTD) problem, conic quadratic representation of TTD problem.

## 1. Introduction

Consider the following generic optimization problem:

$$(P)\quad \begin{array}{ll} \min & f(x) \\ \text{s.t.} & g_i(x) \le 0, i = 1, \ldots, m, \\ & x \in \mathbb{R}^n, \end{array}$$

where $f, g_i$ $(i = 1, \ldots, m)$ are all continuously differentiable functions over $\mathbb{R}^n$. In addition, we assume that the functions $f, g_{p+1}, \ldots, g_m$, are convex over $\mathbb{R}^n$. Therefore, the "nonconvex part" of the problem is due to the nonconvexity of the first p constraint functions $g_1, \ldots, g_p$. Suppose that for every $i = 1, \ldots, p$, $g_i$ has a convex upper estimate function, specifically, assume that there exists a set $Y \subseteq \mathbb{R}^r$ and a continuous function $G_i : \mathbb{R}^n \times Y \mapsto \mathbb{R}$ such that

$$g_i(x) \le G_i(x, y) \quad \text{for every } x \in \mathbb{R}^n, y \in Y,$$

where, for a fixed $y$, the function $G_i(\cdot, y)$ is convex and continuously differentiable. The vector $y$ plays the role of a *parameter vector* and correspondingly $Y$ is called the *admissible parameters set*. In this paper we introduce and analyze a method for solving problem (P) via a sequence of convex problems. The basic idea of the method is that at each iteration we replace each of the nonconvex functions $g_i(x)$ $(i = 1, \ldots, p)$ by the upper convex approximation function $x \mapsto G_i(x, y)$ for some appropriately chosen parameter vector $y$. Thus, at step $k$ $(k \ge 1)$ of the method it is required to solve a convex problem of the following form:

$$(P_k)\quad \begin{array}{ll} \min & f(x) \\ \text{s.t.} & G_i(x, y_k) \le 0, i = 1, \ldots, p \\ & g_j(x) \le 0, j = p + 1, \ldots, m, \\ & x \in \mathbb{R}^n. \end{array}$$

The vector $y_k$ is a fixed parameter vector depending on the solution of the problem $(P_{k-1})$. The method will be called *sequential parametric convex approximation (SPCA)* method. Specific details on the underlying assumptions and the SPCA method will be given in the next section.

The idea of iteratively replacing nonconvex functions by convex upper estimates is not new. One well known example is the gradient method as applied to an unconstrained minimization problem

$$\min\{f(x) : x \in \mathbb{R}^n\}.$$

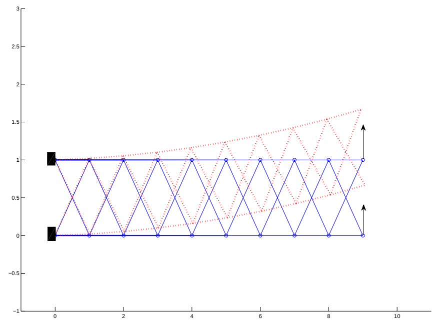


Figure 1: Maximal displacement equals to 1.18 and log(weight)=1.89

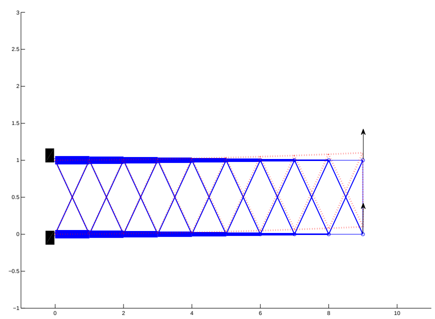


Figure 2: Maximal displacement equals to 0.1 and log(weight)=4.31

Here $f$ is a (possibly) nonconvex function assumed to be continuously differentiable whose gradient satisfies a Lipschitz condition with constant $L$. The gradient method is usually written as (see e.g., [2])

$$x_k = x_{k-1} - \frac{1}{L}\nabla f(x_{k-1}), \quad k \geq 1.$$

An equivalent presentation of the method is

$$x_k = \operatorname*{argmin}_x F(x, x_{k-1}),$$

where

$$F(x,y) = f(y) + \nabla f(y)^T(x-y) + \frac{L}{2}\|x-y\|^2 \tag{1}$$

is an upper approximation of $f(x)$. The fact that $f(x) \leq F(x,y)$ follows from the well known descent lemma (see [2, Proposition A.24]). Thus, at each iteration we replace the function $f(x)$ by its upper approximation $F(x, x_{k-1})$ with the parameter chosen to be the result of the previous iterate ($y = x_{k-1}$). We see that the gradient method is indeed an SPCA-type method.

In the context of structural design problems, a specific convex approximation scheme was used in [1] to convexify global *buckling constraints* and gave rise to an SPCA-type method. In this paper we prove the convergence of the general SPCA method to a KKT point under certain mild conditions. We apply this method to structural design problems with two other nonconvex constraints: *displacement* and *stress constraints.* These type of constraints are in fact an essential part of any realistic structural design specification.

## References


[1] A. Ben-Tal, F. Jarre, M. Kocvara, A. Nemirovski, and J. Zowe. Optimal design of trusses under a nonconvex global buckling constraint. *Optimization and Engineering*, 1:189–213, 2000.

[2] D. P. Bertsekas. *Nonlinear Programming*. Belmont MA: Athena Scientific, second edition, 1999.

# Algorithms of Free Material Optimization

**M. Kocvara**[1] **and C. Bogani**[1]**, J. Fiala**[1]**, M. Stingl**[2] **and Y. Xia**[1]

[1] School of Mathematics, University of Birmingham, UK
[2] Applied Mathematics II, Department of Mathematics, University of Erlangen, Germany

**Abstract**
We will present numerical algorithms for large-scale nonlinear semidefinite programs arising from the primal formulation of the free material optimisation problem. Due to the large number of variables, the algorithms are based on first-order information. We will present the most efficient variants and discuss their applicability to generalized FMO problems with multidisciplinary constraints.

**Keywords:** Structural optimization, Free material optimization.

# Design of grating couplers using topology optimization for the efficient excitation of surface plasmons

**J. Andkjær**[1], **O. Sigmund**[1], **S. Nishiwaki**[2] **and T. Nomura**[3]

[1] Department of Mechanical Engineering, Technical University of Denmark
Nils Koppels Alle, Building 404, 2800 Kgs. Lyngby, Denmark
email: jban@mek.dtu.dk, phone: +45 4525 1369
[2] Department of Aeronautics and Astronautics, Kyoto University
Yosida Honmachi Sakyo, Kyoto 606-8501, Japan
[3] Toyota Central R&D Laboratories Inc., Aichi 480-1192, Japan

**Abstract**
A 2D topology optimization problem is formulated based on Helmholtzs scalar equation and im- plemented in order to design grating couplers for the efficient excitation of surface plasmons at a Ag-SiO2 interface. In this work a coupling efficiency of 66.1% for an optimized grating coupler is achieved.

**Keywords:** Topology optimization, surface plasmon, grating coupler..

## 1. Introduction

Surface plasmons (SP) are electromagnetic waves trapped at the interface between a material with negative real part of the permittivity and a dielectric with positive real part of the permittivity[1]. SP have many promising utilizations, which introduce problem of coupling to surface mode. For some applications a prism coupler (e.g. in Kretschmann and Otto configuration) is too bulky, this motivates the use of grating couplers (GC). Using a generic optimization algorithm, varying the width and position of each groove, a research group [2] has achieved an excitation efficiency of 50%, which is the highest efficiency reported in the literature for a GC. Topology optimization [3] is based on free distribution of material in a design domain in consideration of an objective. Thus topology optimization is a suitable method to use in order to design GC for the efficient excitation of SP.

## 2. Topology optimization of two-dimensional surface plasmon grating coupler

The implementation of the 2D topology optimization problem is based on the finite-element method (FEM) using a combination of the commercial programs COMSOL and MATLAB. We use the settings from [2]. The computational model (see Fig. 1) consist of a $SiO_2$-domain ($\Omega_{SiO_2}$), a Ag-domain ($\Omega_{Ag}$) and a design domain ($\Omega_{des}$). The incident beam excited at $\Gamma_{inp}$ will induced a SP at the Ag-$SiO_2$ interface when the wave-vectors of the incident beam and SP match due to the GC. Reflections from the boundaries are eliminated using perfectly matched layers (PML)[4]. Absorbing boundary condition $\Gamma_{abs}$ have been introduced on all outer boundaries. All interior boundaries have continuous boundary condition.

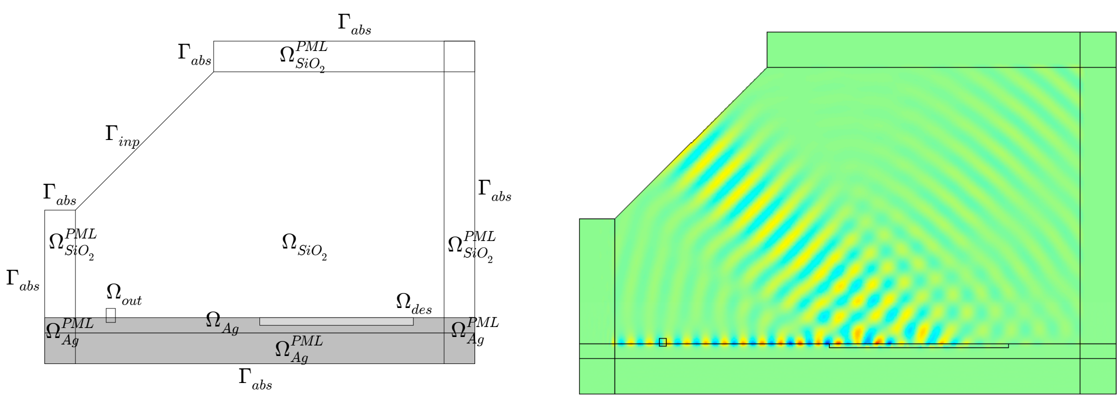



Fig. 1: Computational model and magnetic field for the optimized grating coupler.

| | Fig. | Inital design | iter. | Vol. constr. | Vol. frac. | Efficiency |
|---|---|---|---|---|---|---|
| Initial guess: GC from [2] | 2a | - | 1 | - | 0.264 | 49.7% |
| Example 1 | 2c | GC from [2] | 371 | 0.5 | 0.234 | 66.1% |
| Initial guess: Wave design | 2b | - | 1 | - | 0.5 | 0.1% |
| Example 2 | 2d | Wave design | 367 | 0.5 | 0.293 | 61.6% |

Table 1: Results for optimized designs given in Fig. 2.

(a) GC from [2], efficiency: 49.7%, vol. frac.: 0.26

(b) Wave design , efficiency: 0.1%, vol. frac.: 0.5

(c) Example 1, efficiency: 66.1%, vol. frac.: 0.23

(d) Example 2, efficiency: 61.6%, vol. frac.: 0.29

Fig. 2: Initial and optimized GC designs.

The problem is governed by the H-polarized scalar Helmholtz equation.

$$\nabla \cdot \left(\epsilon_r^{-1} \nabla H_z(\mathbf{r})\right) - \omega^2 c^{-2} H_z(\mathbf{r}) = 0 \quad \text{in} \quad \Omega \tag{1}$$

where $\omega$ is the wave frequency, $H_z(\mathbf{r})$ is the magnetic field in the plane $\mathbf{r} = (x, y)$ and $c$ is the speed of light. The material in the design domain, $\Omega_{des}$, can change numerical by varying the permittivy continuously between the material values for Ag and $SiO_2$ by introducing a design variable $\rho \in [0; 1]$ for each element.

$$\epsilon_r(\rho) = \epsilon'_{Ag} + \rho\big(\epsilon'_{SiO_2} - \epsilon'_{Ag}\big) - i\big(\epsilon''_{Ag} + \rho\big(\epsilon''_{SiO_2} - \epsilon''_{Ag}\big)\big) + i4d(\rho^2 - \rho) \tag{2}$$

where $'$ and $''$ presents the real and the imaginary parts of the permittivity for the given material, $i = \sqrt{-1}$ and $d = 2$(in this problem) is a factor introducing artificial damping for intermediate values of $\rho$. The objective of the optimization problem is to maximize the complex Poynting vector in $\Omega_{out}$ in the negative direction of the $x$-axes.

$$\mathbf{\Phi}(H_z(\rho)) = \frac{1}{L_x} \int_{\Omega_{out}} \frac{\mathbf{n}}{2\epsilon_0 \omega} \Re\Big(i\epsilon_r^{-1} H_z^* \nabla H_z\Big) d\mathbf{r} \tag{3}$$

where $^*$ denotes complex conjugate, $L_x$ is the width of $\Omega_{des}$, $\mathbf{n}$ gives the direction of the power flux and $\Re()$ denotes the real part. A volume constraint is introduced to make the optimization problem well-conditioned. The decay length of the SP and its skin-depth in the two materials are used to evaluate the power flux of the SP at the first grating. The efficiency of the coupling is found as the power flux of the SP at the first grating divided by the power flux at the input boundary, $\Gamma_{inp}$. The gradient-based optimization routine MMA [5] is applied to update the design in an iterative approach until convergence.

Two initial guesses have been tested: the optimized design from [2] (Fig. 2a) and a wave-like design (Fig. 2b). The two optimized designs are shown in Fig. 2c and 2d and the real part of the magnetic field for the optimized design in example 1 is presented in Fig. 1. The results is also presented in Table 1. We obtain the same efficiency of 50% for the optimized design from [2] as they have reported. However with the topology optimization method efficiencies of 66.1% and 61.6% are obtained using the design from [2] and the wave design as initial guess, respectively. The higher efficiencies comes on the cost of a more complex designs.

**References**


[1] H. Raether, Surface Plasmons on smooth and Rough Surfaces and on Grattings, Springer-Verlag Berlin Heidelberg New York 1988.

[2] J. Lu, C. Petre, E. Yablonovitch and J. Conway, Numerical optimization of a grating coupler for the efficient excitation of surface plasmons at an Ag-$SiO_2$ interface, *Opt. Soc. Am.,* vol. 24, no. 9, pp. 2268-2272, 2007.

[3] M.P. Bendsoee and O. Sigmund, Topology Optimization; Theory, Methods and Applications, Springer-Verlag Berlin Heidelberg New York, 2nd edition, 2004.

[4] M. Koshiba, Y. Tsuji and S. Sasaki, High-performance absorbing boundary conditions for photonic crystal waveguide simulations, *IEEE Microw. Wirele. Compon. Lett.,* vol. 11, pp. 152-154, 2001.

[5] K. Svanberg, A class of globally convergent optimization methods based on conservative convex separable approximations, *SIAM J. OPTM.,* vol. 12, pp. 555-573, 2002.

# Topology Optimization of Metallic Microwave Devices

**N. Aage**[1] **and O. Sigmund**[2]

[1] Technical University of Denmark, Department of Solid Mechanics, Denmark, naa@mek.dtu.dk
[2] Technical University of Denmark, Department of Solid Mechanics, Denmark, sigmund@mek.dtu.dk

**Abstract**
This paper presents a novel extension to the topology optimization method, that allows systematic design of metalic/dielectric devices for microwave applications. This is accomplished by interpolating Maxwell's equations and an impedance boundary condition between two material phases, i.e. a dielectric and a conductor. The numerical optimization scheme is demonstrated by the design of resonators for energy harvesting.

**Keywords:** topology optimization, conductor design, finite elements, Maxwell's equations.

## 1. Introduction

The motivation for this work originates from the ever increasing usage of small hand-held, or autonomous, electrical devices such as hearing aids, medical implants and communication devices. These devices all require antennas for communication as well as a power supply, usually a battery. Since more and more functionality is incorporated into these devices, standard antenna designs are becoming less and less usable and the requirements to the batteries are increasing. The power supply issue took a new turn in 2007, where a MIT group lead by Prof. M.Soljacic demonstrated that one could obtain efficient mid-range wireless energy transfer (WiTricity) using magnetically resonant coupled copper coils [1].

Common for the design of antennas and devices for WiTricity are that they consist of an elaborate spatial distribution of a conductor, e.g. copper, in a dielectric background, e.g air. This makes the design of such devices an obvious candidate for the topology optimization method [2].

## 2. Optimization Setting

In order to pose the optimization problem as a mathematical program, one must first include design dependence in the governing equations, i.e. Maxwell's equations. However, for microwave applications the FEM is known to cause computational problems, i.e. extreme mesh refinement, due to the need for resolving the skin depth, when modeling conductors as volumetric entities [4]. Thus, the design parametrization must remedy this bottleneck for the scheme to be efficient.

In the following we will denote the material parameters for the dielectric with $\epsilon_r^d$, $\mu_r^d$ and $\sigma^d$, and likewise for the conductor $\epsilon_r^m$, $\mu_r^m$ and $\sigma^m$. Then we define a design variable, $0 \leq \rho^e \leq 1$, for each finite element, $e$, in the mesh. The design variable can be understood as the density of the conductor. Including design dependence and an element boundary condition to resolve the skin depth problem, Maxwell's vector wave equation can be stated for each design element as

$$\begin{aligned} &\nabla \times (\tilde{A}\nabla \times \boldsymbol{u}) - k_0^2\tilde{B}\boldsymbol{u} = 0, && \text{in } \Omega^e \\ &\boldsymbol{n} \times (A\nabla \times \boldsymbol{u}) - (\rho^e)^{p_{BC}} jk_0\sqrt{AB}\boldsymbol{n} \times (\boldsymbol{n} \times \boldsymbol{u}) = 0, && \text{on } \Gamma^e \end{aligned} \tag{1}$$

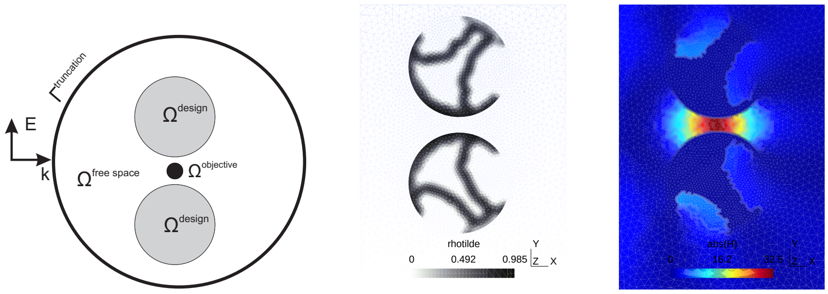
where $\Omega^e$ and $\Gamma^e$ refer to the volume and boundaries of element $e$ in the finite element mesh respectively, $\boldsymbol{n}$ is an outward normal for element $e$ and $k_0$ is the free space wave number. The field $\boldsymbol{u}$ and the parameters $A$, $B$, $\tilde{A}$, $\tilde{B}$ and $p_{BC}$ are given in Table 1. The interpolation functions are given as follows

$$\begin{aligned} \epsilon_r(\rho^e) &= \epsilon_r^d + \rho^e(\epsilon_r^m - \epsilon_r^d) \\ \mu_r(\rho^e) &= \mu_r^d + \rho^e(\mu_r^m - \mu_r^d) \\ \sigma(\rho) &= 10^{\log 10(\sigma^d) + \rho^e[\log 10(\sigma^m) - \log 10(\sigma^d)]} \end{aligned} \tag{2}$$

Note that for dielectrics with $\sigma^d = 0$ one should use $\sigma^d \approx 10^{-4}$ for numerical stability. The governing equations are solved by FEM and the optimization problem by MMA by K.Svanberg [3]. A more in depth discussion of the above design parametrization can be found in [5].

| $\boldsymbol{u}$ | $A$ | $B$ | $\tilde{A}$ | $\tilde{B}$ | $p_{BC}$ |
|---|---|---|---|---|---|
| $\boldsymbol{E}$ | $(\mu_r^m)^{-1}$ | $\epsilon_r^m - j\frac{\sigma^m}{\omega\epsilon_0}$ | $\mu_r(\rho^e)^{-1}$ | $\epsilon_r(\rho^e) - j\frac{\sigma(\rho^e)}{\omega\epsilon_0}$ | $\approx 13$ |
| $\boldsymbol{H}$ | $\left(\epsilon_r^m - j\frac{\sigma^m}{\omega\epsilon_0}\right)^{-1}$ | $\mu_r^m$ | $\left(\epsilon_r(\rho^e) - j\frac{\sigma(\rho^e)}{\omega\epsilon_0}\right)^{-1}$ | $\mu_r(\rho^e)$ | $\approx 1$ |

Table 1: Field dependent parameters for the design parametrization used for conductor/dielectric based topology optimization, equations (1). The functions $\mu_r(\rho^e)$, $\epsilon_r(\rho^e)$ and $\sigma(\rho^e)$ is given in equation (2).




(a) Design problem (not to scale)

(b) Optimized design

(c) Energy for optimized design

Figure 1: Design problem and optimized design for 115 MHz using copper (black) and air (white). The design was obtain after 106 iterations.

### 2.1. Numerical Example

In this section a simple optimization problem for 2D TE polarization, i.e. $\boldsymbol{H} = H_z$, is solved to demonstrate the potential of the design parametrization. The design task is to maximize the magnetic energy between two cylindrical design domains, each with radius $0.65m$, separated by a gap of $0.2m$, c.f. figure 1(a). The optimization problem can be stated as a standard mathematical program

$$\begin{aligned} \max_{\rho\in\mathbb{R}^N} \quad & \log 10(\bar{H}_z^T Q H_z) \\ \text{s.t.,} \quad & \text{Governing eqs.} \\ & \frac{\sum_e^N \rho_e V_e}{V f^*} - 1 < 0 \\ & 0 \leq \rho^e \leq 1, e = 1, N \end{aligned} \tag{3}$$

where $N$ is the number of design variables and the second constraint is a possible volume constraint imposed to control the amount of used material. The sensitivities are derived using the adjoint method [6], and the implementation is done in Matlab. The materials are chosen to be air, i.e. $\epsilon_r^d = \mu_r^d = 1$, and copper $\epsilon_r^m = \mu_r^m = 1$, $\sigma^m = 5.9 \cdot 10^7 S/m$ [4]. The target frequency is set to 115 MHz. The optimized design and magnetic energy distribution can be seen in figure 1(b,c). Compared to the scenario with both cylinders filled with copper, the objective for the optimized design is improved by more than 300 %. The optimized design is verified by evaluating a post processed design modeled by a perfect electric conductor (PEC) condition. The post processed design is found to be in good agreement with the optimization result.

### References

[1] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher and M. Soljacic, Wireless Power Transfer via Strongly Coupled Magnetic Resonances, *Science*, vol. 317, no. 5834, p. 83-86, 2007

[2] M.P. Bendse and O. Sigmund, *Topology Optimization; Theory, Methods and Applications*, 2nd edition, Springer Verlag Berlin Heidelberg New York, 2004.

[3] K. Svanberg, The method of moving asymptotes - a new method for structural optimization, *International Journal for Numerical Methods in Engineering*, vol. 25, 1987.

[4] J. Jin, *The Finite Element Method in Electromagnetics* 2nd edition, John Wiley & Sons, 2002

[5] Aage N., Mortensen N.A. and Sigmund O., Topology optimization of metallic devices for microwave applications, *International Journal for Numerical Methods in Engineering* 2009, SUBMITTED

[6] J. Jensen and O. Sigmund, Topology optimization of photonic crystal structures: a high-bandwidth low-loss T-junction waveguide, *Journal of the Optical Society of America*, vol. 22, no. 6, 2005

# Design of Optical Circuit Devices Using Transient Topology Optimization

**René Matzen, Jakob S. Jensen and Ole Sigmund**

Department of Mechanical Engineering, Technical University of Denmark
Nils Koppels Allé, Building 404, 2800 Kgs. Lyngby, Denmark
Corresponding author: rmat@mek.dtu.dk

**Abstract**
The design of channel drop filter is considered using transient topology optimization. The filter is based on a 2D PhC for TE modes with a point defect cavity. The optimization method is used to predict the material layout in the vicinity of the cavity such that field energy is maximized there.

**Keywords:** topology optimization, transient analysis, waveguide, Helmholtz equation, photonic crystal, wave packet, density filter.

## 1. Introduction

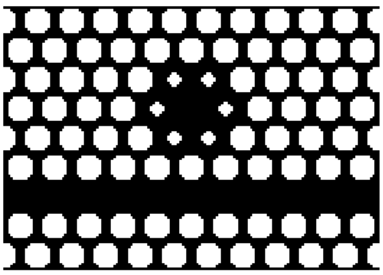
The design of ultra-compact optical devices based on two-dimensional photonic crystal (PhC) structures has recently received much attention. Some of the optical components of concern are *filters* [1], *bends* and *splitters* [9]. In the two latter cases the method of topology optimization [2] based on steady-state analysis have been used to obtain structures exhibiting optimized wave propagation behavior. A great interest in applying topology optimization to problems dealing with full transient behavior has recently emerged [6]. This is due to the fact that optical structures can be optimized for broader frequency ranges by using modulated Gaussian wave pulses. Additionally, optimization of pulse shaping structures is a possibility, as carried out for one-dimensional structures in [3].

This work considers the design of optical circuit devices using transient topology optimization. The method is aimed towards the design of tunable devices using mechanical actuation. A first step towards this goal is to design a channel drop filter with high Q-factor and thereby improving its dropping efficiency. This device allows the transmission of light for a specified frequency (band).

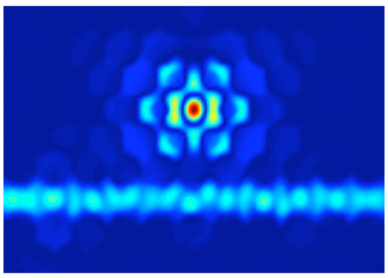
## 2. Method and results

The filter is based on a two-dimensional PhC in which a transverse-electric (TE) single-mode waveguide is coupled to a point defect cavity, see Fig. 1a. The TE-mode in Fig. 1b is modeled by the scalar Helmholtz equation. This open-region problem is solved by the finite-element time-domain (FETD) method for spatial and temporal discretization. The perfectly matched layer (PML) combined with standard absorbing boundary conditions (ABCs) is used as efficient means to truncate the computational domain [5]. The transient problem is integrated by a modified integration scheme [10] yielding fourth order spatial and second order temporal accuracy.

The gradient-based optimization technique is applied to predict the material distribution in the vicinity of the cavity such that the cavity field energy is maximized. The procedure is facilitated by adjoint

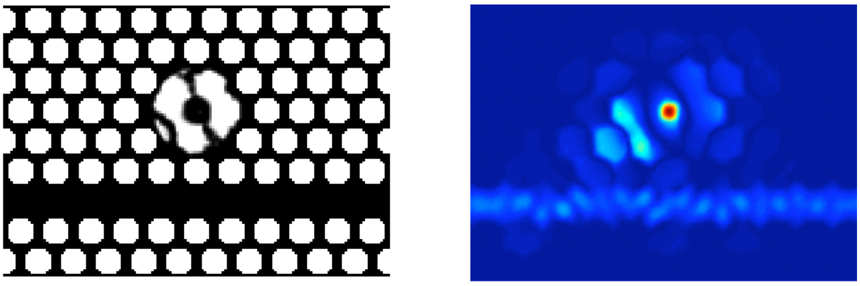


(a) Channel drop filter



(b) Field plot

Figure 1: **Left:** Initial design that is subject to improvement. **Right:** The corresponding field of monopole excited in the cavity at the normalized frequency $\omega = 0.3015 \times 2\pi c/a$.



(a) Optimized filter (b) Field plot of optimized filter

Figure 2: **Left:** Optimized design. **Right:** The corresponding field, and the Q-factor in the cavity is improved by a factor two compared to the original cavity.

sensitivity analysis, in which the adjoint field is found by using the same setup for boundary conditions and time integration scheme as for the open-region problem. Once the sensitivities are established a new design is found by mathematical programming using MMA [8]. To obtain the optimized filter in Fig. 2 a density filtering technique is applied [4, 7]. The Q-factor of the new design is improved by a factor two compared to the original cavity. This is due to the fact that the new design minimizes the transmission of the electromagnetic field through the cavity walls and thereby to the sorroundings.

To facilitate the adjoint analysis in transient optimization it requires the storage of the truncated open-region solution at each time step. This constitutes a major computational bottleneck. To circumvent this issue the state problem solver in the optimization algorithm is parallellized by using MPI for all interprocessor communication in the programming environment provided by FORTRAN 90.

### References


[1] T. Asano, W. Kunishi, M. Nakamura, B. S. Song, and S. Noda. Dynamic wavelength tuning of channel-drop device in two-dimensional photonic crystal slab. *Electronics Letters*, 41(1):37–38, January 2005.

[2] M.P. Bendsø e and O. Sigmund. *Topology Optimization; Theory, Methods, and Applications*. Springer Verlag Berlin Heiddelberg New York, 2nd edition edition, 2004.

[3] J. Dahl, J. S. Jensen, and O. Sigmund. Topology optimization for transient wave propagation problems in one dimension design of filters and pulse modulators. *Structural and Multidisciplinary Optimization*, 36(6):585–595, November 2008.

[4] J. Guest. Imposing maximum length scale in topology optimization. *Structural and Multidisciplinary Optimization*, 37(5):463–473, February 2009.

[5] Jianming Jin. *The Finite Element Method in Electromagnetics*. John Wiley & Sons Inc, 2002.

[6] T. Nomura, K. Sato, K. Taguchi, T. Kashiwa, and S. Nishiwaki. Structural topology optimization for the design of broadband dielectric resonator antennas using the finite difference time domain technique. *International Journal For Numerical Methods In Engineering*, 71(11):1261–1296, September 2007.

[7] O. Sigmund. Morphology-based black and white filters for topology optimization. *Structural and Multidisciplinary Optimization*, 33(4-5):401–424, April 2007.

[8] K. SVANBERG. The method of moving asymptotes - a new method for structural optimization. *International Journal For Numerical Methods In Engineering*, 24(2):359–373, February 1987.

[9] A. Tetu, M. Kristensen, L. H. Frandsen, A. Harpoth, P. I. Borel, J. S. Jensen, and O. Sigmund. Broadband topology-optimized photonic crystal components for both te and tm polarizations. *Optics Express*, 13(21):8606–8611, October 2005.

[10] B. Yue and M. N. Guddati. Dispersion-reducing finite elements for transient acoustics. *Journal of the Acoustical Society of America*, 118(4):2132–2141, October 2005.

# Influence of the material model on local pull-in in electromechanical microdevices topology optimization

**E. Lemaire, L. Van Miegroet, P. Duysinx and V. Rochus**

{E.Lemaire, L.VanMiegroet, P.Duysinx, V.Rochus}@ulg.ac.be
University of Liège (Belgium), Aerospace and Mechanical Engineering Department

**Abstract**
The appearance of local pull-in modes has been noticed during electromechanical microdevices topology optimization. The goal of the present research is to study the influence of material properties (mechanical and electrical) modeling for intermediate densities to see if an appropriate choice allows avoiding such local modes. At first, a simple 1D model is developed to study the influence of the material properties interpolation. Finally, we evaluate if the conclusions from the 1D model can help to prevent the appearance of local modes for a 2D topology optimization problem.

**Keywords:** Topology optimization, electromechanical coupling, local pull-in effect

## 1. Introduction

Electrostatic actuation is often used in MEMS since it allows a fast response and is easily implemented. However, these actuators possess a limit voltage called pull-in voltage (see Ref. [3]) beyond which they are unstable. Above this voltage, elastic forces of suspension system cannot balance electrostatic forces and electrodes stick together. The pull-in effect, can eventually damage the device since it can be impossible to separate the electrodes afterwards. This effect should therefore be considered during the design of such actuators.

The application of topology optimization to MEMS design is still under development but has already provided very interesting and promising results (see for instance Ref. [2, 4, 5]). That's why the authors are developing a topology optimization procedure able to control pull-in voltage during the design of the device.

Unfortunately, the authors show in Ref. [1] that using topology optimization to maximize the pull-in voltage of a microdevice is not straightforward. Indeed as shown in Figure 1, local pull-in modes leading to a strong distortion of the mesh arise at some point of the optimization process. These local modes prevent the completion of the optimization process because they hide the actual global pull-in mode.

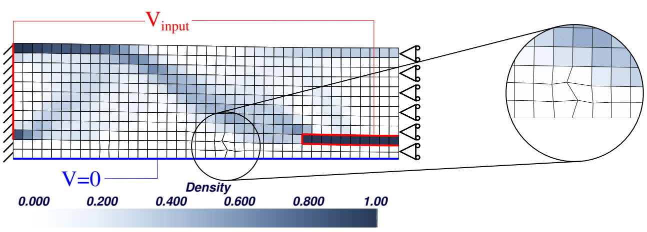
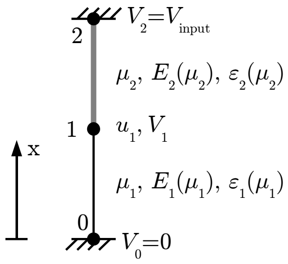



Figure 1: Local pull-in mode appearing during topology optimization problem

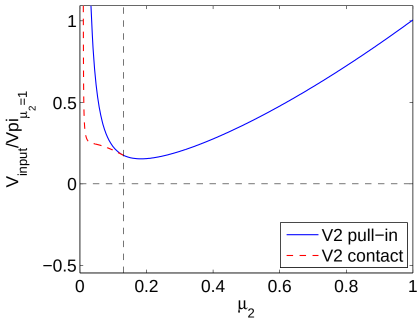
Past research has shown that numerical inaccuracy of the finite element used may be at the origin of the local pull-in modes. Nevertheless in the present study, we wanted to check if these local modes could not be also related to the material model used to represent the properties of intermediate densities. Indeed local modes problems also arise in structural topology optimization considering dynamic behavior or self-weight because of a too low stiffness to mass ratio.

## 2. Simplified 2 dofs system

In the multiphysic problem we are studying, an interpolation of the dielectric permittivity ($\varepsilon$) is added to the Young Modulus ($E$) and density ($\rho$) interpolation. The choice of the Young Modulus and the dielectric interpolation is important since it will determine the ratio between electrostatic forces and mechanical forces. To study the effect of the material models on local instability, we consider the simplified academic actuator drawn in figure 2(a). The system is composed of two elements, the lower one corresponds to the air gap and is void ($\mu_1$ is minimum) while the second is the mobile electrode and its density $\mu_2$ varies between $\mu_{min}$ and 1. Both ends of the system are clamped and $V_0$ is imposed to 0 while $V_2$ corresponds to the driving voltage. Consequently, the system has two degrees of freedom, $V_1$ and $u_1$.




(a) 2 Dofs system




(b) SIMP model

Figure 2: Study of a simplified problem

When it is solid ($\mu_2 = 1$) the upper element should behaves as a perfect conductor. This is possible by associating a high permittivity value to solid material while keeping the permittivity of the void material equal to its actual value. However, the permittivity value has to be limited, Indeed, further study shows that a very high permittivity value for the solid material leads to a rapid increase of the electrostatic force for low density which is far from being ideal since at that point material has a very low stiffness.

With given material models, it is then possible to plot the evolution of the pull-in voltage against the density $\mu_2$ of the upper element. Figure 2(b) presents one example of curve considering that both Young Modulus and permittivity are interpolated using a SIMP law of parameter 3. The study of the curves obtained shows that firstly it is possible that a local pull-in mode appears for an element of intermediate density. However, it is more unlikely that the system becomes unstable if the density of the element is very close or equal to the minimum density.

Moreover, the study shows that an improvement of the pull-in voltage behavior is only possible if different models are chosen for the permittivity and the Young Modulus. This leads to a more complex optimization procedure tuning since at least one additional parameter is introduced. Moreover, the physical relevance of such hybrid model is questionable.

## 3. Effect on a 2D topology optimization problem

Results deduced from the 1D system are then compared with the local modes seen in 2D topology optimization problems. The objective here is to see first if the modes observed in 2D topology problems can be explained on the basis of the 2 dofs model and then if the conclusions of the simplified actuator study can be applied to avoid local modes during topology optimization.

## References


[1] E. Lemaire, P. Duysinx, and V. Rochus. Multiphysic topology optimization of electromechanical microdevices considering pull-in voltage using an electrostatic force filter. In *Proceedings of the 8th World Congress on Structural and Multidisciplinary Optimization*, June 2009.

[2] M. Raulli and K. Maute. Topology optimization of electrostatically actuated microsystems. *Struct. & Mult. Opt.*, 30(5):342–359, November 2005.

[3] V. Rochus, D. J. Rixen, and J.-C. Golinval. Monolithic modelling of electro-mechanical coupling in micro-structures. *Int. J. Numer. Meth. Engng.*, 65(4):461–493, 2006.

[4] O. Sigmund. Design of multiphysic actuators using topology optimization - Part I : One material structures. - Part II : Two-material structures. *Comput. Methods Appl. Mech. Engrg.*, 190(49–50):6577–6627, 2001.

[5] G.H. Yoon and O. Sigmund. A monolithic approach for topology optimization of electrostatically actuated devices. *Comput. Methods Appl. Mech. Engrg.*, 194:4062–4075, 2008.

| **September 25, Morning session (Scion DTU, Building 373)**<br>**Chair: Martin P. Bendsøe** | | |
|---|---|---|
| **Time** | **Presenting author** | **Title** |
| 09.30 – 10.30 | Lars Krog | Integration of Structural Optimization in an Airframe Design Process<br>*L. Krog* |
| 11.00 – 11.30 | U. Schramm | A Three-Step Method for Designing Composite Layups<br>*U. Schramm, M. Zhou, R. Fleury, and R. Boyd* |
| 11.30 – 12.00 | Alexandra A. Gomes | Topology Optimization of Non-Composite Bistable Winglets<br>*A.A. Gomes* |
| 12.00 – 12.30 | Vassili V. Toropov | Optimization of Blended Composite Wing Panels Using Smeared Stiffness Technique and Lamination Parameters<br>*V.V. Toropov, D. Liu, D.C. Barton, O.M. Querin* |

**Advances in Topology and Material Optimization – Methods and Industrial Applications**
September 23 - 25, 2009, Copenhagen, Denmark

# Integration of Structural Optimization in an Airframe Design Process

**Lars Krog**

[1] Airbus, New Filton House, Filton, Bristol, BS99 7AR, UK; lars.krog@airbus.com

**Abstract**

The application of structure optimization methods in airframe design must be able to support the design process, providing answers to specific questions being asked at different maturity levels for the design. Typically the airframe design process is broken into three separate stages, often know as: conceptual design, preliminary design and detailed design. Following the design through these stages the emphasis of the design process changes from one of global aircraft definition, to definition of major components and finally to consider detailed part definition. Corresponding to the specific task being considered specific optimization methods and processes are generally required. Supporting the aircraft development process therefore requires a range of methods such as for example methods for topology optimization, methods for sizing and laminate optimization, data mining methods for pitch optimization and discrete optimization methods for composite stacking sequence optimization. Along with the need for different methods there is also a need for methods to work at different fidelity. The purpose of this paper is not to provide a detailed description of any particular method but rather to provide an overview of how different optimization methods are being integrated and used throughout an airframe design process.

**Keywords:** Structural Optimization, Aerospace Structures

## 1. Introduction

Numerical optimization methods are increasingly being used as part of the Airbus design process, with methods such as topology optimization being used to determine novel lightweight architectures and part designs, with data mining techniques being used to determine optimum stiffener arrangements and with traditional continuous variable sizing optimization being used extensively in trade studies to determine optimum materials, sizing and laminates. Furthermore discrete optimization techniques for detailed composite layup and stacking sequence optimization are in rapid development. The increased and more systematic use of numerical optimization in the design process is driven by requirements for ever more efficient structures and by requirements to develop faster and is helped along by cheap and accessible numerical computing.

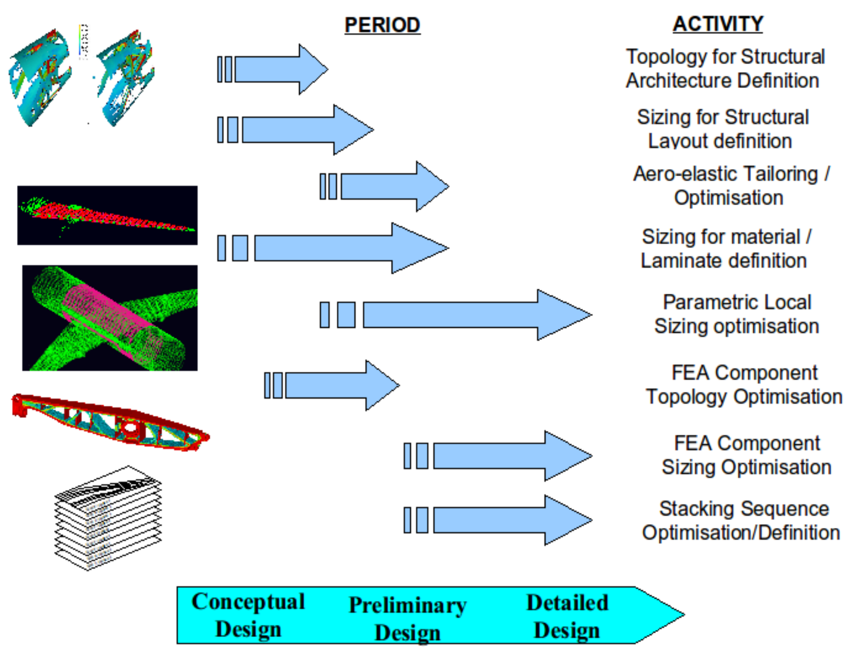
To ensure integration of structural optimization methods into the design process Airbus has created a dedicated service organization responsible for the systematic usage of numerical optimization methods to the A350 XWB program. The A350 XWB optimization service center utilizes a number of different optimization techniques including: use of purpose developed tools for optimization of large composite parts, use of generic optimization frameworks to build local optimization processes around specific stress processes and finally use of commercial finite element based optimization software. Alongside such service activities significant efforts are being made to improve existing methods and to establish new and improved methods for deployment onto next aircraft programs. In this frame specific research efforts are being made both to develop sufficiently efficient optimization techniques to handle aircraft level optimization for global trade studies and at the other end of the scale to develop optimization processes and methods to handle detailed composite optimization. Also research is ongoing considering use of topology optimization for architecture studies and considering development of topology optimization methods for composite optimization. The development of topology optimization methods for composites is part of a European project PLATO-N.

Considering the range of different methods being used / developed, it is important to form a vision of how these different methods support a traditional aircraft development process and to challenge if new optimization methods can be developed that will allow us to improve our current way of working. Typically the airframe design process is broken into three separate stages, often known as: conceptual design, preliminary design and detailed design. Following the design through these stages the emphasis of the aircraft design process changes from one of global aircraft definition, to definition of major components and finally to consider detailed part definition. The following provides a simplistic view of the role of structural optimization during conceptual design, preliminary design and detailed design:

- **Conceptual Design:** In conceptual design the aircraft is optimized in terms of its global performance and configuration and structural optimization provides support to determine optimum choice of materials, optimum stiffening concepts and optimum structural architectures. Typically many of the studies made during conceptual design are multi-disciplinary, spanning simultaneously aero-dynamics, systems, structural and manufacturing considerations.
- **Preliminary Design:** Having frozen the global aircraft definition mainly in terms of external geometry, material choices and major structural architecture the design moves into preliminary design where structural optimization supports structural trade-offs to fully define the structural layout and to determine the sizing of major components to quite mature level.
- **Detailed Design:** The third and final phase of the design process comes after a good overall definition of the airframe and will deal with the detailed design of each structural part down to the level of detailed sizing and shape optimization of for example metallic brackets.

An important driver for the preliminary design phase is to be able to accurately determine the overall stiffness of each major component and therefore the global aircraft such as to be able to perform an accurate calculation of external/internal loads, before entering detailed design where loads must be fixed in order to allow design packages to be outsourced. This requirement directly reflect back to the fidelity of methods being used. Whilst during conceptual design it is acceptable to perform optimization using rough buckling methods and stress/strain allowables, during preliminary design optimization processes must use stressing methods that are in agreement with the detailed sizing. Such requirements clearly influence tool development, and has let to development of complex integrated structural optimization processes, such as COMBOX for optimization of composite box covers.

Without going into below figure provides an illustration of how different optimization techniques are used relative to the aircraft development cycle. The final presentation will provide a more detailed explanation of this picture and also enter into some level of detail describing the specific optimization methods and processes being used at each stage of the development process.




## 2. Conclusions

Integration of structural optimization into an airframe design process requires different methods in order to answer to questions at specific timescales of the development process, where optimization methods must be available to answer questions with a suitable fidelity considering the maturity of the design.

# A Three-Step Method for Designing Composite Layups

**U. Schramm[1], M. Zhou, R. Fleury, and R. Boyd[2]**

[1] Altair Engineering, USA, schramm@altair.com
[2] Altair Engineering, UK, boyd@altair.com

**Keywords:** Composite design, free-size optimization, topology optimization.

In recent years major aerospace companies have started designing airplanes made from composite materials. Previous designs were mostly manufactured from aluminum, which is isotropic. The new materials require new design techniques.

One technique introduced by Altair uses a three step process. The first step is to use free-sizing optimization with fixed fiber angles in predefined plies to determine a concept of ply bundles. After that a step of sizing optimization for the ply bundles is performed to determine the true thickness of these plies. As a third step the sequence of ply-stacking is determined thru an optimization algorithm. All throughout the process manufacturing constraints are considered.

This new process has been developed with a cooperation of industry with software development [1, 2]. The process will be illustrated with several examples.

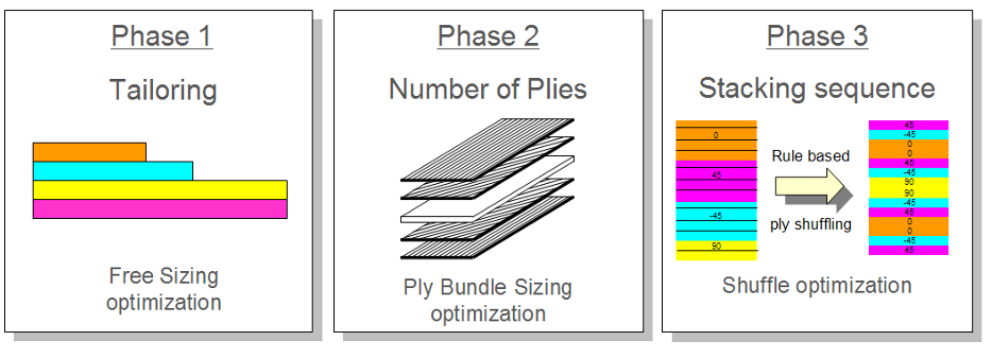



Figure 1: Composite design process

[1] Zhou, M, Fleury, R., and Diaz, W. A Comprehensive Process for Composite Design Optimization. *8th World Congress on Structural and Multidisciplinary Optimization*, Lisbon, Portugal, 2009.
[2] Cervellera, P., Cabrele, S., and Machunze, W., Benefits of Structural Optimization in the Aerospace Design Process: Recent Industrial Application on Composites Structures, *NAFEMS World Congress*, Crete, Greece, 2009.

# Topology Optimization of Non-Composite Bistable Winglets

Alexandra A. Gomes[1]

[1] IDMEC, Instituto Superior Técnico, Portugal, magoncal@dem.ist.utl.pt

**Abstract**
In this work we consider the optimal design of non-composite bistable structures for aeronautical applications. The goal is to determine the layout of a minimum compliance internal structure of an adaptive winglet with two prescribed stable states. The interesting feature of a bistable structure is to remain in a deformed state after the load has been removed. To solve this problem we follow a topology optimization approach using the spectral level set methodology. Preliminary tests show the difficulty in providing a quantitative measure that asserts the structure exhibits bistable behavior.

**Keywords:** Topology optimization, level set methods, adaptive winglet, bistable structure.

## 1. Introduction: Bistable Adaptive Winglet

The benefits of adding a winglet to an aircraft wing are many. These devices reduce fuel consumption, increase range and cruise altitude, reduce take-off noise emission and augment the climb performance [6]. The weight penalty associated with an increased root bending moment, due to the added lift at the wingtip, is traded off by the reduced drag, reducing both fuel burn and overall aircraft weight.

The idea of using composites to design multistable internal mechanisms of adaptive winglets is not new [2, 3]. An adaptive winglet provides the tip of the wing with morphing capabilities so that it can perform optimally at each stage of the aircraft mission. The major advantage of a bistable plate consists in maintaining a deformed state after unloading because the deformation potential energy of a bistable structure is a double well. Therefore, there is no need for permanent actuation. However, bistable composite plates can easily switch between stable states with a small perturbation load. Moreover, because their bistability arises from competing strains in fibres laid in different directions, the amount of out-of-plane deformation is small in comparison to their in-plane dimensions.

The novelty of this work is to produced an optimal bistable winglet from a non-composite plate using topology optimization. In particular, we determine the distribution of one isotropic material over a two-dimensional domain, such that we minimize the compliance of the final structure and achieve a prescribed out-of-plane deformation. Because weight is a critical parameter in aeronautical applications, we also add a solid volume fraction restriction to the statement of the optimization problem. To solve this problem we follow a topology optimization strategy within the framework of the spectral level set methodology, which we briefly discuss in the next section.

## 2. Spectral Level Set Methodology

The level set methods [7] are a mature approach to the formulation of topology optimization problems. The boundary of the relevant structure is defined as the zero level set of a function called the level set function. According to classical level set methodologies, the nodal values of this function are the design variables of the optimization problem. During the search for an optimum these variables evolve and so does the set.

The spectral level set methodology [4] expands the level set function into a truncated Fourier series and considers the Fourier coefficients as the new design variables. One advantage of the proposed methodology is to provide, asymptotically in the number of degrees of freedom and for a sufficiently regular boundary, a lower error bound than non-adaptive classical approaches to structural topology optimization. This implies that for a sufficiently smooth interface, the spectral level set methodology uses significantly less design variables (in some benchmark structural examples, 4% to 8% [4]) than traditional techniques in topology optimization, including the regular level set methods.

## 3. Work-in-progress

The implementation of the topology optimization of the bistable winglet comprises two main steps. The first step consists in coupling the spectral level set formulation with an optimization algorithm, which, in the present paper, is the method of moving asymptotes, [8]. For this we need to calculate the gradients of the objective and constraints functionals of the optimization problem with respect to the design variables, which according to the proposed formulation, are the real and imaginary parts of the Fourier coefficients. Because the functionals involve the indicator or Heaviside function and the Dirac delta distribution, their gradients are calculated using results from the theory of distributions as detailed in [5].

The second main step consists in adequately solve the elasticity problem along static equilibrium paths. To do this we use Riks algorithm [1], which is basically a Newton method able to progress along those paths and to accurately predict snap-through, i.e., the switching between stable states.

The first sets of results shows two major challenges. The first is the following: when the load is removed from a stable state, the structure should retain its deformation. However, it tends to adapt to the new conditions by slightly decreasing the deformation field. It is this alteration that is hard to predict and to to measure quantitatively. Moreover, it is difficult to provide Riks algorithm with a starting loading value given that the optimal structural topology is unknown. The figure below depicts going on analyses of the optimal design of a bistable adaptive winglet.

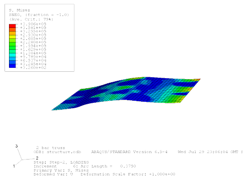


Figure 1: Work-in-progress: intermediate Abaqus layouts of the optimal design of a bistable winglet.

## 4. Conclusions

Our purpose is to design an optimal layout for a bistable internal structure of an aircraft winglet device. We require the structure to have minimum compliance and prescribed values for both out-of-plane deformation and solid volume fraction. To this end, we pursue a structural topology optimization approach, following the framework of the spectral level set methodology, and using the method of moving asymptotes as the optimization algorithm and Riks algorithm to progress along static equilibrium paths. Preliminary results show that asserting quantitatively the bistable potential of a structure can be a challenging task.

## References


[1] M.A. Crisfield. A fast incremental/iteration solution procedure that handles snap-through. *Computers and Structures*, 13:55–62, 1981.

[2] Luís Falcao, Alexandra A. Gomes, and Afzal Suleman. Morphing wing tip devices based on multistable composites. In *Morphing Aircraft Symposium AVT-168 Technical Evaluation Report*, Évora, Potugal, April 2009. NATO.

[3] A. Gatto, F. Mattioni, and M.I. Friswell. Experimental investigation of bistable winglets to enhance aircraft wing lift takeoff capability. *Journal of Aircraft*, 46(2):647–655, 2009.

[4] A. A. Gomes and A. Sulemam. Application of the spectral level set methodology to topology optimization. *Structural and Multidisciplinary Optimization*, 31(6):430–443, June 2006.

[5] Alexandra A. Gomes. Sensitivity analysis for the spectral level methodology. In *Proceedings of the 8th World Congress on Structural and Multidisciplinary Optimization*, Lisbon, Potugal, 1-5 June 2009.

[6] D. McLean. Wingtip devices: What they do and how they do it. In *2005 Boeing Performance and Flight Operations Engineering Conference*, Seattle , Washington, USA, 2005.

[7] J.A. Sethian. *Level Set Methods and First Marching Methods, Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Material Science.* Cambridge Monographs on Applied and Computational Mathematics. Cambridge University Press, Cambridge, UK, 1999.

[8] K. Svanberg. The method of moving asymptotes - a new method for structural optimization. *International Journal for Numerical Methods in Engineering*, 24:359–373, 1987.

# Optimization of Blended Composite Wing Panels Using Smeared Stiffness Technique and Lamination Parameters

**Dianzi Liu[1], Vassili V. Toropov[1,2], David C. Barton[1], Osvaldo M. Querin[1]**

[1] School of Mechanical Engineering, University of Leeds, Leeds, LS2 9JT, UK
(mndl@leeds.ac.uk, d.c.barton@leeds.ac.uk, o.m.qurein@leeds.ac.uk)
[2] School of Civil Engineering, University of Leeds, Leeds, LS2 9JT, UK (v.v.toropov@leeds.ac.uk)

**Abstract**
Smeared stiffness-based method is examined for finding the best stacking sequence of laminated composite wing structures with blending and manufacturing constraints. At the global level of the bi-level optimization procedure the numbers of plies of the pre-defined angles (0, 90, 45 and -45 degrees) are the design variables, buckling, strength and ply percentage are treated as constraints and the material volume is the objective function. The stack shuffling to satisfy global blending and manufacturing constraints is performed at the local level to match zero values of lamination parameters. The latter requirement is due to the ply angle homogeneity through the stack that is achieved by the top level optimization.

**Key Words:** smeared stiffness method, stacking sequence, blending, lamination parameters.

## 1. Introduction

Ply compatibility (also referred to as blending) between adjacent panels is a very important consideration in the design of composite structures [1]. Liu and Haftka [2] defined the composition continuity and the stacking sequence continuity measures. Soremekun *et al*. [3] used multi-step optimization to determine the blended stacking sequence of the laminates. Based on the individual optimized panels, sub-laminates for the blended panel design are redefined by the optimization process for each panel that is referred to as a design variable zone. Seresta *et al*. [4] developed two blending methods, inward and outward blending, to improve the ply continuity between adjacent panels using a guide based GA. Liu and Krog [5] developed an approach to shuffle a set of global ply layout cards and identified a laminate stacking sequence in individual wing panels satisfying inter-panel continuity constraints. Recently, two bi-level composite optimization procedures were investigated by Liu *et al.* [6] to seek the best stacking sequence of laminated composite wing structures with blending and manufacturing constraints. The examined approaches are: a smeared stiffness-based method, that aims at neutralizing the stacking sequence effects on the buckling performance, and a lamination parameter-based method that uses lamination parameters as design variables to formulate the membrane stiffness matrix **A** and bending stiffness matrix **D**. The advantage of the smeared stiffness-based method is that it avoids a stack optimization at the local level by performing a quicker post-processing function of ply shuffling. The advantage of the lamination parameter-based approach is that there is no need to check satisfaction of the strength and buckling constraints as long as the lamination parameters obtained after the local level optimization match the given lamination parameter values that came from the top level optimization. Later, Liu et al. [7] developed a multi-objective approach for the local level optimization, which utilizes three criteria: non-dimensional lamination parameters match index, stack homogeneity index, and $90^{\circ}$ ply angle jump index (the number of occurrences of 90 degree change between two adjacent plies in the stack) to satisfy the rules of the laminated composite design.

## 2. Composite Optimization Strategy

In this paper, smeared stiffness-based approach is used for the optimization of stacking sequence of laminated composite wing structures. The numbers of plies of 0, 90, 45 and -45 degree fibre orientations are design variables and buckling, strength and ply percentage are constraints. The material volume is the objective function at the global level. Then, a permutation GA is used to shuffle the layers at the local level to minimize the difference between the values of computed lamination parameters for a current stack and the ones coming from the top level optimization, which are zero due to the stack homogeneity through the thickness. This is embedded into a blending procedure applied at this level to achieve the global ply continuity. The optimization software OptiStruct [9] is used for the top level optimization of the laminated composite wing structure.

## 3. Smeared Stiffness-Based Method

Smeared stiffness-based method [8] is an approach that aims at neutralizing the stacking sequence effects on the buckling performance by considering homogeneous sections with quasi-isotropic layups. This method is used to calculate the matrices **A** and **D** of laminates without determining a stacking sequence in the top level optimization. The elements of the matrices **A** and **D** of laminates can be expressed as

$$A_{ij} = \int_{-h/2}^{h/2} \overline{Q}_{ij} dz, \quad i = j = 1,2,6, \qquad D_{ij} = \int_{-h/2}^{h/2} \overline{Q}_{ij} z^2 dz, \quad i = j = 1,2,6. \tag{1}$$

According to the smeared stiffness-based method, for a homogeneous material the relationship between **A** and **D** can be formulated as **D** = **A**h$^2$/12.

## 4. Shared Layers Blending

Shared layers blending process [6] is applied to satisfy the global blending requirement as well as the general layup design rules. First, ranking of all panels in terms of the numbers of plies of each angle is performed. Then, for each ply angle, out of all panels the minimum number of plies is selected. This set of three ply numbers defines the first set of shared layers among all panels. The thinnest panel that includes the first shared layers is identified. The first shared layers will be placed outermost in the stacks for all panels. The remaining layers in the thinnest panel are placed after the first shared layers. Next, after this first stage, for the remaining layers of all the panels, except the thinnest panel, the same procedure is applied as at the first stage. This is repeated until the last panel is considered. Finally, for the adjacent panels, the local blending between them is performed for the remaining layers in the adjacent panels. Thus, the stacks for all the panels will become inwardly blended (outer blending), where the outer layers of all the panels are continuous. If the shared layers are placed at the position next to the mid plane instead of the outermost position, the inner blending (outwardly blended composite) will be created. Here, the outer blending procedure is adapted due to the damage tolerance requirements resulting in $\pm 45^\circ$ plies places on the outside of the stack.

A root part of a composite aircraft wing structure has been examined to demonstrate the bi-level optimization with the blending scheme.

## 5. Conclusions

The software implementation of this approach can be considered as an add-on to an OptiStruct [9] run where smeared stiffness-based approach is used for the top level optimization. Given the results from the top level optimization, the stack shuffling to satisfy global blending and manufacturing constraints is performed at the local level to match zero values of lamination parameters related to the bending stiffness matrix. This local level optimization can be treated as a postprocessing phase for determining the detailed ply-book of the laminate while guaranteeing the satisfaction of strain and buckling constraints.

## References


[1] Z. Gurdal, R.T. Haftka and P. Hajela. *Design and Optimization of Laminated Composite Materials*, John Wiley & Sons, 1999.

[2] B. Liu and R.T. Haftka. Composite wing structure design optimization with continuity constraints. *42th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Material Conference*, Seattle, WA, 16-19 April 2001.

[3] G.A. Soremekun, Z. Gurdal, C. Kassapoglou and D. Toni. Stacking sequence blending of multiple composite laminates using genetic algorithm. *Composite Structures*, Vol. 56, pp. 53-62, 2002.

[4] O. Seresta, Z. Gurdal, D.B. Adams and L.T. Watson. Optimal design of composite wing structures with blended laminates. *Composite Part B: Engineering*, 38:469-480,2007.

[5] W. Liu and L. Krog. A method for composite ply layout design and stacking sequence optimization. *ASMO UK-7*, Bath, 7-8 July 2008.

[6] D. Liu, V.V. Toropov, O.M. Querin and D. C. Barton. Bi-level optimization of blended composite panels. *5th AIAA MDO Specialist Conference*, Palm Springs, CA, 4-7 May 2009.

[7] D. Liu, V.V. Toropov, O.M. Querin and D. C. Barton. Stacking sequence optimization of composite panels for blending characteristics using lamination parameters. *8th WCSMO*, Lisbon, Portugal, 1-5 June 2009.

[8] M. Zhou, and R. Fleury. Optimization of composite structures - understanding and meeting the challenges. *Altair Composite Seminar*, Royal Lemington Spa, UK, 2007.

[9] Altair OptiStruct Version 9.0, Altair Engineering Inc., 2008.