



## The Field Radiated by a Ring Quasi-Array of an Infinite Number of Tangential or Radial Dipoles

**Knudsen, H. L.**

*Published in:*  
Institute of Electrical and Electronics Engineers. Proceedings

*Link to article, DOI:*  
[10.1109/JRPROC.1953.274261](https://doi.org/10.1109/JRPROC.1953.274261)

*Publication date:*  
1953

*Document Version*  
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*  
Knudsen, H. L. (1953). The Field Radiated by a Ring Quasi-Array of an Infinite Number of Tangential or Radial Dipoles. Institute of Electrical and Electronics Engineers. Proceedings, 41(6), 781-789. DOI: 10.1109/JRPROC.1953.274261

## DTU Library

Technical Information Center of Denmark

---

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

# The Field Radiated by a Ring Quasi-Array of an Infinite Number of Tangential or Radial Dipoles\*

H. LOTTRUP KNUDSEN†

**Summary**—A homogeneous ring array of axial dipoles will radiate a vertically polarized field that concentrates to an increasing degree around the horizontal plane with increasing increment of the current phase per revolution. There is reason to believe that by using a corresponding antenna system with tangential or radial dipoles, a field may be obtained that has a similar useful structure as the above-mentioned ring array, but which in contrast to the latter is essentially horizontally polarized. In this paper a systematic investigation has been made of the field from such an antenna system with tangential or radial dipoles. Recently it was stated in the literature that it is impossible to treat the general case where the increase of the current phase per revolution is arbitrarily large by using ordinary functions. The results obtained in this paper disprove this statement.

A similar investigation has been made of the field from the antenna system with tangential dipoles described above in the case where the current distribution on this system is a standing wave instead of a progressing wave.

When the increment of the current phase per revolution converges towards infinity, the gain of the antenna systems treated here converges towards infinity, too, i.e. supergain occurs. Based on the theory of supergain an approximate expression has been derived for the minimum value of the radius of the antenna system, which it is possible to use in practice.

## INTRODUCTION

AS WAS SHOWN by Chireix,<sup>1</sup> an azimuthally omnidirectional antenna system with a field that is concentrated around the horizontal plane and with vertical polarization may be obtained by placing a number of vertical dipoles equidistantly along a circle in the horizontal plane (see Fig. 1(d)) and by feeding these dipoles with currents having the same numerical value but a phase increasing uniformly along the circle. An antenna system of this type will be called a homogeneous ring array of axial dipoles. In order that this ring array may be completely omnidirectional in the horizontal plane the number of dipoles must be infinitely large (see Fig. 1(a)). The current phase increases a whole number,  $H$ , times  $2\pi$  during one revolution in the positive direction.

Denoting the radius of the circle by  $a$ , the angle between the normal to the plane of the circle and the direction from the ring array to the field point by  $\theta$ , and the azimuth of this point by  $\phi$ , we find—according to Chireix—the following expression for the array characteristic  $G(\theta, \phi)$  of a homogeneous ring array with infinitely many elements:

$$G = K_1 e^{iH\phi} J_H(ka \sin \theta). \quad (1)$$

\* Decimal classification: R120. Original manuscript received by the Institute, November 11, 1951; revised manuscript received December 17, 1952.

† The Microwave Laboratory, The Royal Technical University of Denmark, Copenhagen, Denmark.

<sup>1</sup> H. Chireix, "Antennes a rayonnement zénithal réduit," *L'Onde Élec.*, vol. 15, pp. 440-456; 1936.

<sup>2</sup> As time factor is used  $e^{-i\omega t}$ .

In this expression  $K_1$  is a constant depending on the choice of the reference antenna, whereas  $k$  denotes the intrinsic transmission coefficient,  $k = 2\pi/\lambda$ , of free space, where  $\lambda$  is the wavelength.

If the radius  $a$  of the circle is so small that  $ka \ll 1$ , we find from (1) the following approximate expression for  $|G|$

$$|G| \cong K_2 \sin^H \theta, \quad (2)$$

where  $K_2$  is a constant. This expression shows that the field, which is polarized in the  $\theta$ -direction for any value of the increment,  $H2\pi$ , of the current phase, concentrates more and more around the horizontal plane with increasing values of  $|H|$ . Chireix proposes that this property of the homogeneous ring array might be used for obtaining a fading-reducing antenna system.

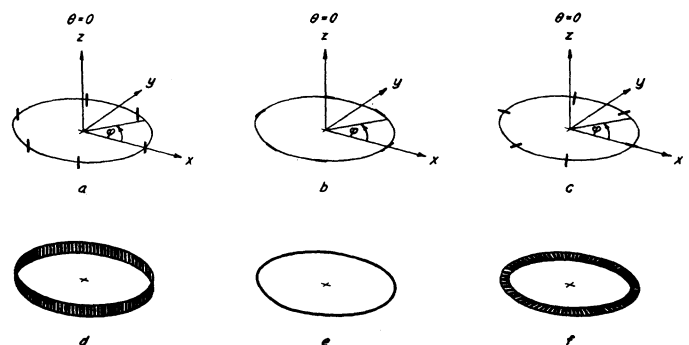


Fig. 1—(a) Ring array of a finite number of axial dipoles. (b) Ring quasi-array of a finite number of tangential dipoles. (c) Ring quasi-array of a finite number of radial dipoles. (d) Ring array of an infinite number of axial dipoles. (e) Ring quasi-array of an infinite number of tangential dipoles. (f) Ring quasi-array of an infinite number of radial dipoles.

Of late years FM and television have aroused interest in horizontally polarized fields. It is an obvious thing, therefore, in analogy with Chireix's investigation of a homogeneous ring array of axial dipoles, to carry out an investigation of the field from corresponding ring-shaped antenna systems with tangential or radial dipoles. The elements in these antenna systems are identical, but not identically oriented. In what follows we shall denote an antenna system composed of identical but not necessarily identically oriented elements as a quasi-array. In this paper we set the problem of calculating the field from a homogeneous ring quasi-array of an infinite number of tangential or radial dipoles. The adjective "homogeneous" denotes here, as in the case of the ring array, that the antennas are placed equidistantly, and that they carry currents with the same numerical value but with a phase that increases uniformly

along the circle. The increment of the phase during one revolution is denoted by  $H2\pi$ . With the notation used in Fig. 1 a ring quasi-array of tangential or radial dipoles may be expected to radiate a field that is polarized essentially in the  $\phi$ -direction, and which concentrates to an increasing degree around the horizontal plane with increasing  $|H|$ .

Figs. 1(b) and (c) show a ring quasi-array of respectively tangential and radial dipoles. In Figs. 1(e) and (f) are shown the corresponding antenna systems with an infinite number of elements. It is observed that a ring quasi-array of tangential dipoles then degenerates into a wire bent in a circle, i.e. a circular frame aerial.

Carter<sup>3</sup> has carried through a calculation of the field from an array of a finite number of axial dipoles and a ring quasi-array of a finite number of tangential or radial dipoles placed concentrically around a conducting cylinder and with  $H=0$  or  $H=1$ . In the case where the radius of the conducting cylinder is equal to zero, and where the number of dipoles converges towards infinity, the antenna systems treated by Carter constitute special cases of the systems investigated in this paper.

The field from an arbitrarily large, circular frame aerial with constant current (i.e.,  $H=0$ ) has previously been investigated by Foster<sup>4</sup> and Moullin.<sup>5</sup> This field calculation is included in the present investigation as a special case. The general case of a circular frame aerial with a standing or a progressing current wave has been tackled by Sherman.<sup>6</sup> However, Sherman arrives at the conclusion that a calculation of the field in an arbitrary direction cannot be carried out. This statement is contradicted by the results obtained in this paper. The present paper deals particularly with quasi-arrays with progressing current waves. It seems as if a satisfactory treatment of a circular frame aerial with a standing current wave has not been given in the literature; this case is therefore also treated in the present paper.

#### A HOMOGENEOUS RING QUASI-ARRAY OF TANGENTIAL DIPOLES

##### Calculation of the Field

In this section we shall calculate the field from a homogeneous ring quasi-array of an infinite number of dipoles placed along a circle with radius  $a$  so that the dipoles are tangent to this circle, as shown in Fig. 1(e). We first formulate the problem of the corresponding ring quasi-array with a finite number,  $s$ , of elements, each one having the length  $L$ , see Fig. 1(b). As sketched in this figure, we introduce a spherical co-ordinate system  $(r, \theta, \phi)$  and a rectangular co-ordinate system  $(x, y, z)$  corresponding to the first system in the usual way. In

the rectangular co-ordinate system the  $j$ 'th dipole has the co-ordinate  $(a \cos u_j, a \sin u_j, 0)$ , where

$$u_j = \frac{2\pi j}{s}, \quad j = 1, 2, \dots, s. \quad (3)$$

The current  $I_j$  in the  $j$ 'th dipole is given by

$$I_j = I e^{iH u_j}, \quad (4)$$

where  $I$  is a complex constant and  $H$  an integral number.

In calculating the radiated field we use the method of radiation vectors described by Schelkunoff.<sup>7</sup> Denoting by  $\zeta$  the intrinsic impedance of free space, by  $\vec{r}$  the vector pointing from the center of the antenna system to the field point, and setting  $|\vec{r}| = r$ , we define a constant  $T$  by

$$T = \frac{iksL\zeta I}{4\pi r} e^{ikr}. \quad (5)$$

By using the radiation vector  $\vec{F}(\theta, \phi)$  we may then express the electric and the magnetic field strength in the radiated field as follows

$$\vec{E} = T\vec{F}, \quad \vec{H} = \zeta^{-1} \frac{\vec{r}}{r} \times \vec{E}, \quad (6, 7)$$

where

$$\begin{aligned} F_r &= 0, \\ F_\theta &= F_x \cos \theta \cos \phi + F_y \cos \theta \sin \phi - F_z \sin \theta, \\ F_\phi &= -F_x \sin \phi + F_y \cos \phi. \end{aligned} \quad (8)$$

The  $x$ -,  $y$ -, and  $z$ -components of the radiation vector  $\vec{F}$  are given by

$$F_x = -S \sin u_j, \quad F_y = S \cos u_j, \quad F_z = 0, \quad (9)$$

where

$$S = \frac{1}{s} \sum_{j=1}^s e^{i[H u_j - z \cos(\phi - u_j)]}, \quad (10)$$

$$z = ka \sin \theta. \quad (11)$$

The parameter  $z$  must not be mistaken for the co-ordinate  $z$ , which does not occur in the calculation. The electric field strength is essentially expressed through the vector  $\vec{F}$ . For this reason  $\vec{F}$  shall be called the normalized electric field strength.

In the present case where the number of elements is finite, the following limiting process must be performed in the above expression

$$\begin{aligned} s &\rightarrow \infty, \\ u_j &\rightarrow u, \\ \Delta u_j &= \frac{2\pi}{s} \rightarrow du. \end{aligned} \quad (12)$$

<sup>3</sup> P. S. Carter, "Antenna arrays around cylinders," *Proc. I.R.E.*, vol. 31, pp. 671-693; 1943.

<sup>4</sup> D. Foster, "Loop antennas with uniform current," paper presented before a joint meeting of the I.R.E. and U.R.S.I.; April, 1937.

<sup>5</sup> E. B. Moullin, "Radiation from large circular loops," *Jour. IEE* (London), pt. III, vol. 93, pp. 345-351; 1946.

<sup>6</sup> J. B. Sherman, "Circular loop antennas at ultrahigh frequencies," *Proc. I.R.E.*, vol. 32, pp. 534-537; 1944.

<sup>7</sup> S. A. Schelkunoff, "Electromagnetic Waves," D. van Nostrand Company, Inc., New York, N. Y., pp. 330-336; 1943.

Setting

$$h = \frac{1}{2\pi} e^{i[Hu - z \cos(\phi - u)]}, \quad (13)$$

we obtain

$$F_x = - \int_0^{2\pi} h \sin u du, \quad F_y = \int_0^{2\pi} h \cos u du, \quad (14)$$

$$F_z = 0.$$

By a further transformation and application of Sommerfeld's integral formula for a Bessel function<sup>8</sup> we find, as shown in the appendix,

$$F_x = \left[ \frac{H}{z} J_H(z) \cos \phi - iJ_H'(z) \sin \phi \right] e^{iH(\phi - \pi/2)},$$

$$F_y = \left[ iJ_H'(z) \cos \phi + \frac{H}{z} J_H(z) \sin \phi \right] e^{iH(\phi - \pi/2)}, \quad (15)$$

$$F_z = 0.$$

By inserting these expressions in (8) we find the following expressions for the  $\theta$ - and  $\phi$ -components of the normalized, electric field strength:

$$F_\theta = \frac{H}{z} J_H(z) \cos \theta e^{iH(\phi - \pi/2)}, \quad (16)$$

$$F_\phi = iJ_H'(z) e^{iH(\phi - \pi/2)}.$$

Discussion of the Result

The homogeneous ring quasi-array in question is seen to be azimuthally omnidirectional. However, the phase of the field depends on  $\phi$  in the way expressed through the factor  $e^{iH(\phi - \pi/2)}$ . The equiphase lines in the horizontal plane thus constitute a system of  $H$  Archimedes's spirals as is the case of the field from a ring array. The field is said to be one that is spiral-phase. As has been detailed by Sandeman,<sup>9</sup> spiral-phase fields are utilized in navigation for the purpose of direction-finding. Granquist<sup>10</sup> has used a spiral-phase field and the corresponding field obtained by reversing the current phases to a simple determination of both distance and direction.

The component  $F_\theta$  is 90° out of phase with  $F_\phi$ . Therefore, the field will generally be elliptically polarized. The polarization will only in special cases degenerate into circular or linear polarization.

For  $H \neq \pm 1$  and for  $\theta = 0$  or  $\pi$  we find  $F_\theta = F_\phi = 0$ . The radiation in the direction of the axis is different from zero only for  $H = \pm 1$ . The corresponding result for a standing current wave was derived already by Sherman<sup>6</sup> through a more direct argumentation.

We shall first investigate the appearance of the components of the normalized, electric field strength for

<sup>8</sup> E. Jahnke and F. Emde, "Funktionentafeln," Dritte Auflage, B. G. Teubner Verlag, Leipzig and Berlin, p. 149; 1938.  
<sup>9</sup> E. K. Sandeman, "Spiral-phase fields," *Wireless Eng.*, vol. 26, pp. 96-105; 1949.  
<sup>10</sup> C. E. Granqvist, "Radiofyr för avstånd och riktning," paper presented before Radiovetenskapliga Konferensen, Stockholm, Sweden; 1949.

small values of the radius of the circle. Assuming that  $(a/\lambda) \ll (1/2\pi)$  we obtain the following results:

$$H = 0, \quad F_\theta = 0, \quad F_\phi \cong -i\alpha_0 \sin \theta. \quad (17)$$

$H \geq 1$ .

$$F_\theta \cong \alpha_H \cos \theta \sin^{H-1} \theta e^{iH(\phi - \pi/2)},$$

$$F_\phi \cong i\alpha_H \sin^{H-1} \theta e^{iH(\phi - \pi/2)}. \quad (18)$$

In these expressions  $\alpha_H$  denotes constants. In Figs. 2(a) and (c-e),  $1/\alpha_H$  times the  $\theta$ - and  $\phi$ -components of the normalized, electric field strength  $\bar{F}$  are plotted as func-

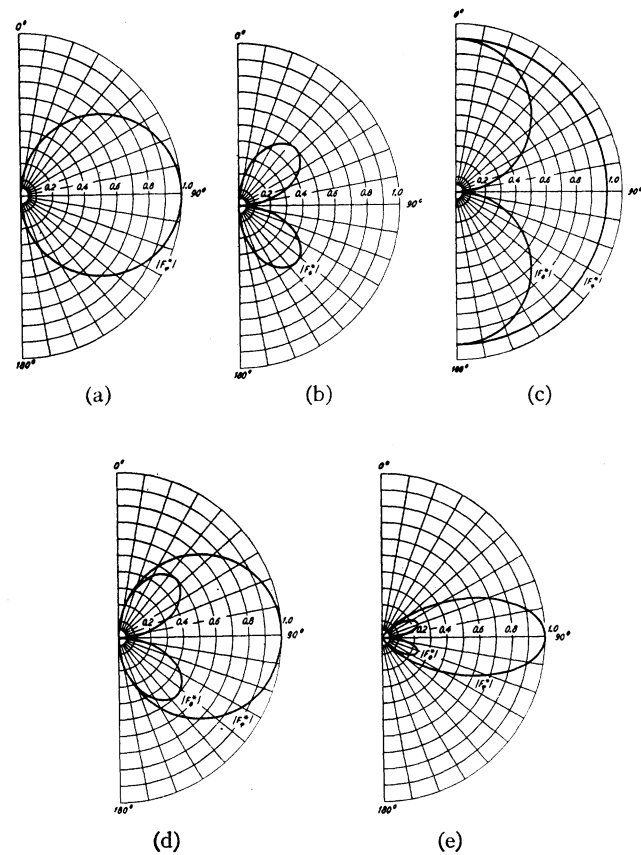


Fig. 2.—Vertical pattern of  $\alpha_H^{-1}$  multiplied by the normalized, electric field strength  $\bar{F}(\theta, \phi)$  for a small, homogeneous ring quasi-array of an infinite number of tangential or radial dipoles. (a) Tang. dipoles,  $H=0$ . (b) Rad. dipoles,  $H=0$ . (c) Tang. and rad. dipoles,  $H=1$ . (d) Tang. and rad. dipoles,  $H=2$ . (e) Tang. and rad. dipoles,  $H=7$ .

tions of  $\theta$ . When  $H \geq 2$ , the field is seen to have the same character for any value of  $H$ . When  $H$  increases, the component  $F_\phi$  will concentrate to an increasing degree around the plane of the circle, whereas  $F_\theta$  will decrease towards zero. In the case of  $H \geq 1$  the maximum value of  $F_\theta$  occurs for  $\theta = \theta'$  and  $\theta = \theta'' = \pi - \theta'$ , where

$$\theta' = \arctg \sqrt{H - 1}. \quad (19)$$

The angle  $\theta' = \pi - \theta''$  is plotted as a function of  $H$  through the firm line curve in Fig. 3. The quotient  $\kappa$ , between the maximum values of  $F_\theta$  and  $F_\phi$ , is expressed

through

$$\kappa = \cos \theta' \sin^{H-1} \theta'. \quad (20)$$

The quotient  $\kappa$  is plotted as a function of  $H$  through the firm line curve in Fig. 4. When  $H$  converges towards infinity, the angle  $\theta'$  will converge towards  $\pi/2$ , i.e., the two loops in  $F_\theta$  will approach unlimitedly the horizontal plane. At the same time the quotient  $\kappa$  will converge towards zero, i.e., the field will in the limit become horizontally polarized. However, as Fig. 4 shows, the horizontal component of the field becomes vanishingly small first when  $H$  becomes very large. As will be discussed in a following section, large fundamental difficulties will arise when the increment of the current phase,  $H2\pi$ , for an antenna with fixed dimensions increases beyond a certain limit.

For very large values of  $H$  the following approximate expressions for the angle  $\theta'$  and the quotient  $\kappa$  may be derived:

$$\theta' \cong \frac{\pi}{2} - \frac{1}{\sqrt{H-1}}, \quad (21)$$

$$\kappa \cong \frac{1}{\sqrt{e(H-1)}} \cong \frac{0.6065}{\sqrt{H-1}}, \quad (22)$$

where  $e$  denotes the basis of the natural logarithms. These approximate expressions are plotted as the dotted curves in Figs. 3 and 4.

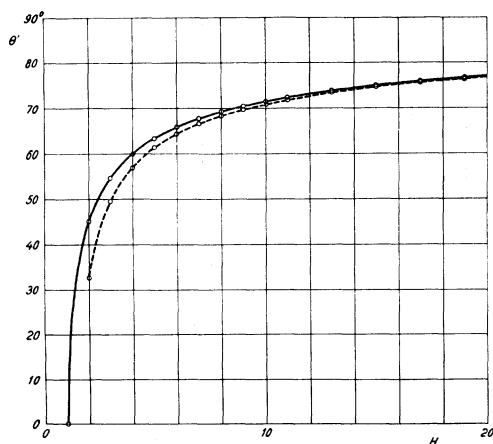


Fig. 3—The angle  $\theta'$  or  $(\pi/2) - \theta'$ ,  $\theta'$  and  $\theta''$  being the pole distances for the directions in which maximum occurs for the  $\theta$ -component of the electric field strength of the field from a small, homogeneous ring quasi-array of an infinite number of tangential or radial dipoles. The firm line curve corresponds to the exact expression (19), whereas the dotted curve corresponds to the approximate expression (21).

We now return to the general case where not necessarily  $a/\lambda \ll 1$ , as was assumed in the preceding paragraph. First we consider the case where the current in the frame aerial has not only a constant numerical value but also a constant phase. For a frame aerial being fed at a single point of the circumference, this current distribution constitutes a good approximation to the actual

distribution, when the perimeter of the frame is small as compared with the wavelength. This has been utilized in the elementary theory of frame aeriels.<sup>11</sup> In frame aeriels, the perimeter of which is large as compared with the wavelength, the current can only be constant if special arrangements are made for feeding the antenna. Foster<sup>12</sup> has proposed to feed the antenna sectionally; this idea has been carried out by Alford and Kandoian.<sup>13,14</sup> Moullin<sup>5</sup> has proposed to realize a constant current distribution by using a large number of tangential dipoles. In the case being considered here,  $H=0$ , the field has only a  $\phi$ -component as an elementary consideration also shows. For  $H=0$  the expressions obtained above for the field components were previously derived and discussed by Foster<sup>4</sup> and by Moullin.<sup>5</sup> For the sake of completeness  $|F_\phi|$  is plotted as a function of  $\theta$  and for various values of  $a/\lambda$  in Figs. 5(a-c).

For  $H=1$  the numerical values of the components of the normalized, electric field strength are plotted in Figs. 5(d-f) for the same values of  $a/\lambda$  as in the case  $H=0$ . For  $\theta=0$  or  $\pi$  we find

$$F_\theta = \mp \frac{i}{2} e^{i\phi}, \quad F_\phi = \frac{1}{2} e^{i\phi}. \quad (23)$$

The field is seen to be circularly polarized in the direction of the axis. For  $a/\lambda=0.1$  to 0.2, i.e., when the perimeter of the circle is about one wavelength, the current distribution on the antenna has the same character

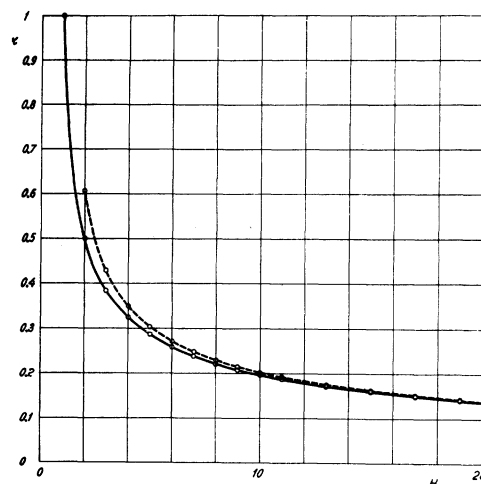


Fig. 4—The quotient  $\kappa$  of the maximum values of  $F_\theta$  and  $F_\phi$  for the field radiated by a small, homogeneous ring quasi-array of tangential or radial dipoles. The firm line curve corresponds to the exact expression (20), whereas the dotted curve corresponds to the approximate expression (22).

as the current distribution on a single turn of the helical antenna developed by Kraus.<sup>15</sup> The fields from the two

<sup>11</sup> J. H. Dellinger, "Principles of transmission and reception with antennas and coil aeriels," U. S. Bur. Stand., vol. 15, pp. 435-495; December, 1919.

<sup>12</sup> D. Foster, "Loop antennas with uniform current," Proc. I.R.E., vol. 32, pp. 603-607; 1944.

<sup>13</sup> A. Alford and A. G. Kandoian, "Ultra-high-frequency loop antennas," Trans. AIEE, vol. 59, pp. 843-848; 1940.

<sup>14</sup> A. G. Kandoian, "Three new antenna types and their applications," Proc. I.R.E., vol. 34, pp. 70 W-75W; 1946.

<sup>15</sup> J. D. Kraus, "The helical antenna," Proc. I.R.E., vol. 37, pp. 263-272; 1949.

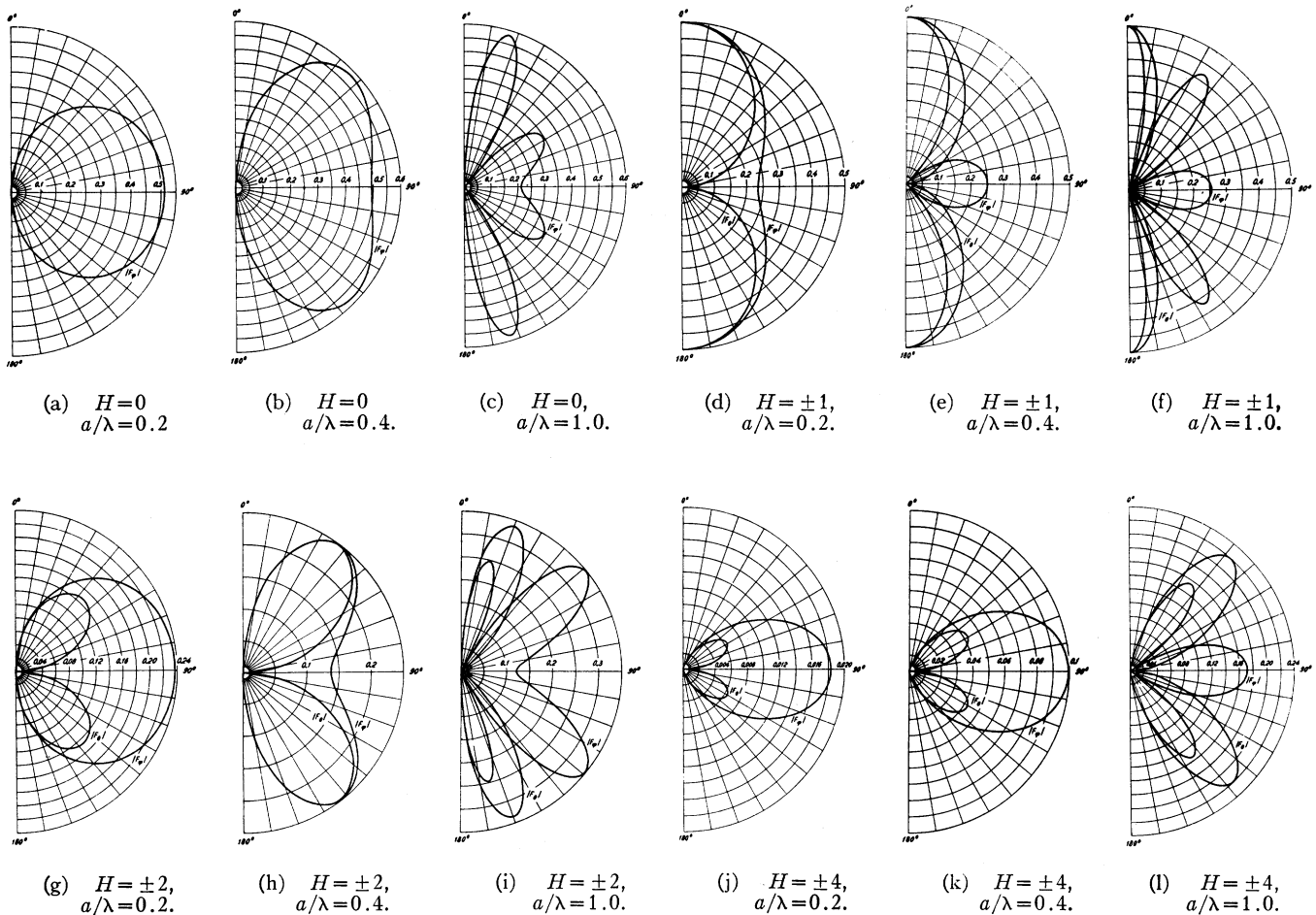


Fig. 5—(Right) Vertical pattern of the  $\theta$ - and  $\phi$ -components of the normalized, electric field strength  $\bar{F}(\theta, \phi)$  of the field from a homogeneous, ring quasi-array of an infinite number of tangential dipoles.

antennas are also similar, and the calculation of the fields may proceed along the same lines.<sup>16,17</sup>

For  $H=-1$ , similar statements regarding the field apply as in the case of  $H=1$ .

The numerical values of the components  $F_\theta$  and  $F_\phi$  are plotted in Figs. 5(g-i) for  $H=\pm 2$  and in Figs. 5(j-l) for  $H=\pm 4$ . These figures show that the field concentrates, to an increasing degree, around the horizontal plane with increasing  $|H|$ , but that the structure of the field changes considerably when the radius increases beyond a certain value dependent on  $|H|$ , extra loops then beginning to appear in the radiation pattern. If it is desired to utilize the properties of the field that are characteristic of small values of the radius, and which were discussed in detail above, it is therefore necessary to choose a radius that does not exceed the just-mentioned value dependent on  $|H|$ . On the other hand, other considerations set a lower limit for the radius as will be discussed in a following section.

<sup>16</sup> H. L. Knudsen, "The field from a circular and a square helical beam antenna." Trans. Danish Acad. Tech. Sci. (Copenhagen, Denmark) 1950, No. 8, 55 pp.

<sup>17</sup> H. L. Knudsen, "Radiation field of a square, helical beam antenna," Jour. Appl. Phys., vol. 23, pp. 483-491; 1952.

### RING QUASI-ARRAY OF TANGENTIAL DIPOLES SINE-SHAPED CURRENT DISTRIBUTION

#### Calculation of the Field

We desire to calculate the field from a ring quasi-array of an infinite number of tangential dipoles, i.e., a circular frame aerial, as shown in Fig. 1(e), and with a sine-shaped current distribution. We start from a corresponding ring quasi-array of a finite number of elements (see Fig. 1(b)) calling the number of the elements  $s$  and their length  $L$ . The position of the  $j$ 'th dipole being given by the angle,  $u_j$ , defined in (3), the current,  $I_j$ , in the  $j$ 'th dipole is in the present case expressed through

$$I_j = I \cos H u_j, \tag{24}$$

where  $I$  is a complex constant and  $H$  an integral number.

The standing current wave given in (24) may be decomposed into two current waves progressing in opposite directions

$$I_j = \frac{1}{2}(e^{iH u_j} + e^{-iH u_j}). \tag{25}$$

The field from the sine-shaped current distribution considered here may now be calculated as  $\frac{1}{2}$  times the field calculated in the last section for a progressing current wave plus  $\frac{1}{2}$  times this field with  $H$  replaced by  $-H$ .

With the factor  $T$  expressed by (5) the electric field strength is given by the normalized field strength  $\bar{F}$ . From the expressions obtained earlier for the components  $F_\theta$  and  $F_\phi$  for a homogeneous ring quasi-array of an infinite number of tangential dipoles, we find for the sine-shaped current distribution in question

$$F_\theta = -(-i)^{H+1} \frac{H}{z} J_H(z) \cos \theta \sin H\phi, \quad (26)$$

$$F_\phi = -(-i)^{H+1} J_H'(z) \cos H\phi.$$

### Discussion of the Result

The  $\theta$ - and  $\phi$ - components of the normalized, electric field strength are seen to be in phase; the radiated field is therefore linearly polarized. Denoting by  $\alpha$  the angle which the electric field vector in a point of the far zone makes with a circle of latitude through this point, we find

$$\alpha = \text{arctg} \frac{F_\theta}{F_\phi} = \text{arctg} \left[ \frac{\frac{H}{z} J_H(z)}{J_H'(z)} \cos \theta \text{tg} H\phi \right]. \quad (27)$$

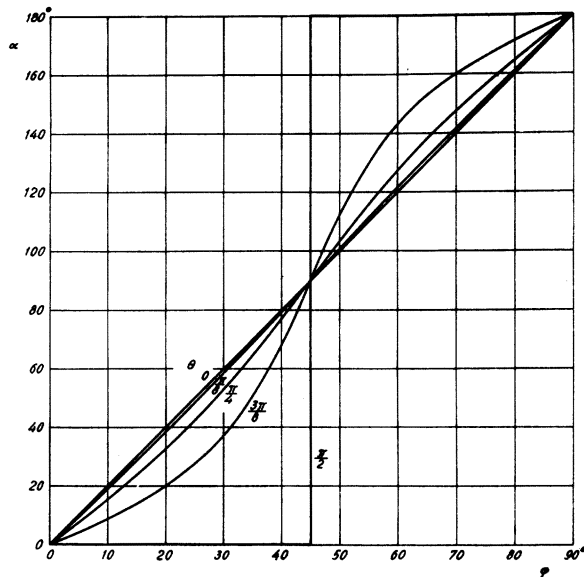


Fig. 6—The direction,  $\alpha$ , of the polarization as a function of azimuth  $\phi$  for various values of the pole distance  $\theta$  for a ring quasi-array with a sine-shaped current distribution, of an infinite number of tangential dipoles, characterized by  $H=2$  and  $a/\lambda=0.2$ . The parts of the curves falling outside those shown here may be generated from these by a parallel translation in  $\pi/2$  in the direction of the  $\phi$ -axis and  $n\pi$  in the direction of the  $\alpha$ -axis ( $n$  an integral number).

For  $H=0$  the field is horizontally polarized in any direction. During one revolution along a circle of latitude  $\theta=\text{constant}$ ,  $\alpha$  will increase by  $H2\pi$ . In Fig. 6 the angle  $\alpha$  is plotted as a function of  $\phi$  for  $a/\lambda=0.2$ ,  $H=2$ , and  $\theta=0, \pi/8, \pi/4, 3\pi/8$ , and  $\pi/2$ .

The functions  $|F_\theta|$  and  $|F_\phi|$  occurring here are obtained from the corresponding functions for the field produced by a progressing current wave with the same value of  $H$  through multiplication, respectively, by  $|\sin H\phi|$  and by  $|\cos H\phi|$ . For  $H=\pm 2$  these functions

are plotted in Fig. 7. By using these curves or similar curves for other values of  $H$  and the curves in Fig. 5 showing  $|F_\theta|$  and  $|F_\phi|$  for a circular frame aerial with a progressing current wave, one may easily form a mental picture or make a numerical calculation of the field from a circular frame aerial with a sine-shaped current distribution.

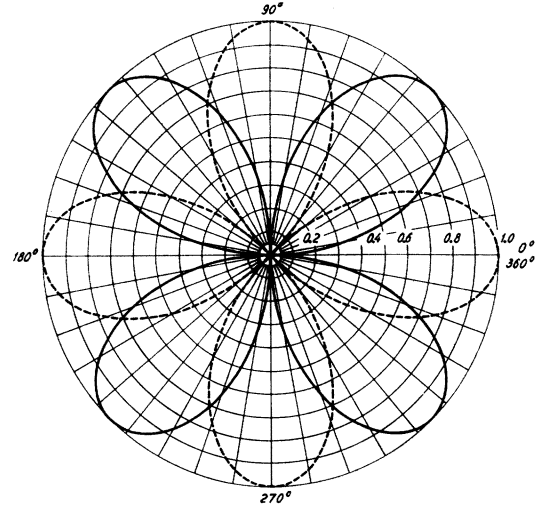


Fig. 7—The functions  $|\sin H\phi|$  (firm line curve) and  $|\cos H\phi|$  (dotted curve) for use in calculating the field from a ring quasi-array, with a sine-shaped current distribution, of an infinite number of tangential dipoles for  $H=\pm 2$ .

## HOMOGENEOUS RING QUASI-ARRAY OF RADIAL DIPOLES

### Calculation of the Field

In this section we shall calculate the field from the homogeneous ring quasi-array of an infinite number of radial dipoles shown in Fig. 1(f). We start with the corresponding quasi-array of a finite number of elements shown in Fig. 1(c). The number of elements is denoted by  $s$  and their length by  $L$ . With the notation used in Fig. 1(c) the  $j$ 'th dipole will have the co-ordinates  $(a \cos u_j, a \sin u_j, 0)$  where  $u_j$  is the angle defined in (3). Current  $I_j$  in  $j$ 'th dipole is supposed to be given by (4).

Using the value of the factor  $T$  given in (5) we find, through calculations similar to those in the previous sections, the following expressions for the components of the normalized, electric field strength  $\bar{F}$ :

$$F_x = S \cos u_j, \quad F_y = S \sin u_j, \quad F_z = 0, \quad (28)$$

where  $S$  is given by (10). Now introducing the notation  $\bar{F}^*$  for the normalized, electric field strength for the corresponding homogeneous ring quasi-array of tangential dipoles we find, by comparing the equations derived above with (9),

$$F_x = F_y^*, \quad F_y = -F_x^*, \quad F_z = 0. \quad (29)$$

By substituting these expressions in (8) we find the following expressions for the  $\theta$ - and  $\phi$ -components of the normalized, electric field strength:

$$\begin{aligned} F_\theta &= F_y^* \cos \theta \cos \phi - F_x^* \cos \theta \sin \phi, \\ F_\phi &= -F_y^* \sin \phi - F_x^* \cos \phi. \end{aligned} \quad (30)$$

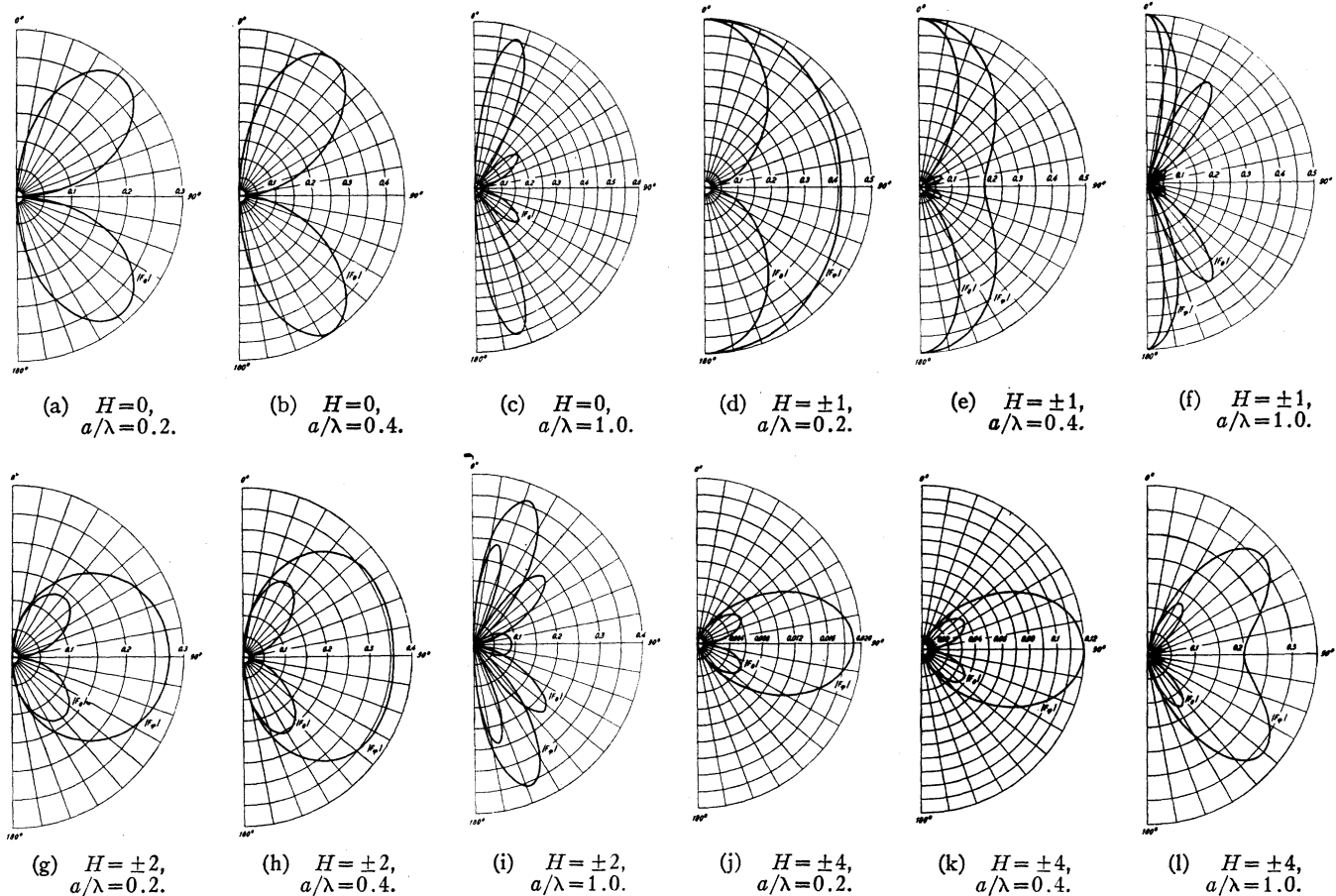


Fig. 8—Vertical pattern of the  $\theta$ - and  $\phi$ -components of the normalized, electric field strength  $\bar{F}(\theta, \phi)$  of the field from a homogeneous ring quasi-array of an infinite number of radial dipoles.

By comparing these formulas with (8) and using (16), we find the following expressions for the  $\theta$ - and  $\phi$ -components of the normalized, electric field strength  $\bar{F}$ :

$$\begin{aligned}
 F_\theta &= F_\phi^* \cos \theta = iJ_H'(z) \cos \theta e^{iH(\phi-\pi/2)}, \\
 F_\phi &= -F_\theta^* \frac{1}{\cos \theta} = -\frac{H}{z} J_H(z) e^{iH(\phi-\pi/2)}. \quad (31)
 \end{aligned}$$

Discussion of the Result

The expressions derived for the field components show that the field is a spiral-phase field, the phase increasing  $H2\pi$  during one revolution as indicated by the factor  $e^{iH[\phi-(\pi/2)]}$ . The field will generally be elliptically polarized. As was the case for a ring quasi-array of tangential dipoles, the field in the direction of the axis from the quasi-array in question will be different from zero only in the case of  $H = \pm 1$ .

We shall first consider the special case where the radius of the circle is so small that  $(a/\lambda) \ll (1/2\pi)$ . We may then replace the functions occurring in the expressions for the field components by the first term in their series expansions. Hereby, we obtain the expressions:  
 $H = 0$ ,

$$F_\theta \cong -i\alpha_0 \cos \theta \sin \theta, \quad F_\phi = 0. \quad (32)$$

$H \geq 1$ .

$$\begin{aligned}
 F_\theta &\cong i\alpha_H \cos \theta \sin^{H-1} \theta e^{iH(\phi-\pi/2)}, \\
 F_\phi &\cong -\alpha_H \sin^{H-1} \theta e^{iH(\phi-\pi/2)}, \quad (33)
 \end{aligned}$$

where  $\alpha_H$  denotes the same constants as in (17) and (18). Again denoting by  $\bar{F}^*$  the normalized, electric field strength of the field from the corresponding homogeneous ring quasi-array of tangential dipoles, we obtain therefore in the case of  $(a/\lambda) \ll (1/2\pi)$  and  $H \geq 1$ .

$$F_\theta \cong iF_\theta^*, \quad F_\phi \cong iF_\phi^*. \quad (34)$$

For a small radius and  $H \geq 1$  the field radiated from the antenna system in question is therefore—apart from a factor  $e^{i(\pi/2)}$ —identical with the field from the corresponding ring quasi-array of tangential dipoles. In Figs. 2(b-e) are plotted  $1/\alpha_H$  times the components  $F_\theta$  and  $F_\phi$  for various values of  $H$ . For  $(a/\lambda) \ll (1/2\pi)$  and  $H \geq 1$  the quotient  $\kappa$  between the maximum values of the  $\theta$ - and the  $\phi$ -components of the electric field strength is approximately given by the same expression (20), and therefore by the same curve (Fig. 4) as in the case of a ring quasi-array of tangential dipoles.

We now proceed to the general case, in which no assumption is made regarding the size of the antenna system. Whereas in the case of  $(a/\lambda) \ll (1/2\pi)$  and  $|H| \geq 1$ , the field from the quasi-array of radial dipoles considered here is approximately identical with the field from the corresponding quasi-array of tangential dipoles, an increasing inconsistency will set in with increasing radius. The numerical values of the components  $F_\theta$  and  $F_\phi$  of the normalized, electric field strength for the field from a homogeneous ring quasi-array of radial dipoles are plotted in Figs. 8(a-l) for  $H = 0, \pm 1, \pm 2$ , and  $\pm 4$



and for various values of  $a/\lambda$ . In the case of  $|H| \geq 1$  the comments given above to Fig. 5 regarding the field from a homogeneous ring quasi-array of tangential dipoles also apply to the present antenna system with radial dipoles.

For  $H=0$  the quasi-array considered here is of the same type as the disk antenna introduced by Böhm.<sup>18</sup> Furthermore, a homogeneous ring quasi-array of radial dipoles with  $H = \pm 1$  is similar to a turnstile antenna in which the number of arms is infinite and not four as in the usual type.<sup>19</sup>

SUPERGAIN

As previously mentioned, the array characteristic  $G$  of a homogeneous ring array, the radius  $a$  of which is small as compared with the wavelength, and the current phase of which increases  $H2\pi$  during one revolution, may approximately be expressed through  $G = K \sin^H \theta$ . With constant dimensions of the antenna and with increasing  $H$  the radiation pattern will consequently approach the disk-shape, i.e., the gain in horizontal directions will converge towards infinity for  $H \rightarrow \infty$ . However, this contradicts practical experience, according to which the gain of an antenna with fixed dimensions cannot exceed a certain value depending on these dimensions. During the last decade this paradox has been extensively discussed in the literature under the name of supergain. A bibliography of this subject is given in a paper by the author;<sup>20</sup> its connection with the communication theory is discussed in another paper by the author.<sup>21</sup> The explanation of the said paradox is that it is certainly possible to calculate a current distribution giving an arbitrarily large gain; but if this gain increases beyond the value characteristic of the size of the antenna in question, the  $Q$  of the antenna increases so violently that the antenna rapidly becomes inapplicable.

A homogeneous ring array seems to be the simplest example of an antenna having supergain. Other simple examples of supergain antennas are homogeneous ring quasi-arrays of tangential or radial dipoles as shown by (18) and (33). We shall investigate the limitation in the use of these ring quasi-arrays which may be deduced from the theory of supergain. Of the various points of view on supergain contained in the literature we shall base our considerations on Woodward and Lawson's<sup>22</sup> approach to the problem. Woodward and Lawson,

whose paper deals with a linear array, utilize the fact that the horizontal pattern of the array characteristic  $G$ , as expressed as a function of  $t = \sin \phi$ , where  $\phi$  is the angular deviation from the direction normal to the group, as shown in Fig. 9, is the Fourier transform of the

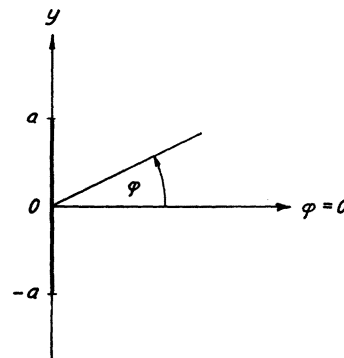


Fig. 9—Co-ordinates used in describing the current distribution and the array characteristic of a linear array with the length 2a.

current distribution  $I$  along the array expressed as a function of the co-ordinate  $y$ . Consequently, the current distribution  $I(y)$  will be equal to the inverse Fourier transform of  $G(t)$ . This inverse transform being expressed through an integration with respect to  $t$  over the interval  $-\infty < t < \infty$ , not only the values of the array characteristic  $G(t)$  corresponding to  $-1 < t < 1$ , i.e., to real values of the azimuth  $\phi$ , but also the values of  $G(t)$  corresponding to  $-\infty < t < -1$  and  $1 < t < \infty$ , will contribute to the current distribution  $I(y)$ .

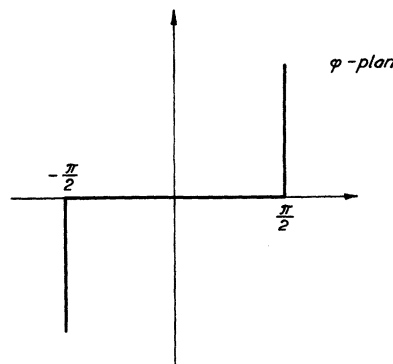


Fig. 10—Path of integration for the contour integral expressing  $I(y)$  as a function of the array characteristic  $G(t)$ .

These last-mentioned intervals of  $t$  correspond to the two vertical branches of the integration path  $C$  in the complex  $\phi$ -plane shown in Fig. 10, i.e., to complex values of  $\phi$ . Large values of  $G(t)$  for values of  $t$  outside the interval  $-1 < t < 1$  consequently contribute to the current  $I(y)$  without leaving any contribution to the radiation field, i.e., the  $Q$  of the antenna will be large. Hence it follows that for an antenna array to be applicable in practice,

<sup>18</sup> O. Böhm, "Langwellen-Rundfunkantennen mit Unterdrückung der Steilstrahlung," *Telefunkenztg.*, vol. 13, pp. 21-26; March, 1932.

<sup>19</sup> G. H. Brown, "Turnstile aerials," *Electronics*, vol. 9, pp. 14-17 and 48; April, 1936.

<sup>20</sup> H. L. Knudsen, "Superforstærkning hos antenner," *Elektroteknisk Tidsskrift* (Oslo, Norway), vol. 64, pp. 213-221; 1951.

<sup>21</sup> H. L. Knudsen, "Shannons tidsopdelingssætning og superforstærkning hos antenner," *Teknisk Tidsskrift* (Stockholm, Sweden); vol. 82, pp. 1023-1030; 1952.

<sup>22</sup> P. M. Woodward and J. D. Lawson, "The theoretical precision with which an arbitrary radiation-pattern may be obtained from a source of finite size," *Jour. IEE* (London), Pt. III, vol. 95, pp. 363-369; 1948.

array characteristic  $G(t)$  must converge rapidly towards zero when we move outside the interval  $-1 < t < 1$ .

For a homogeneous ring quasi-array of tangential dipoles the components of the normalized, electric field strength  $\bar{F}$  are expressed by the current distribution  $I e^{iHu}$  through the transforms (14). The components of the field from a ring quasi-array of radial antennas are expressed by the current distribution through similar transforms. An argument similar to the one applied above may be used here, and also in the present case we arrive at the conclusion that in order to obtain an antenna that is applicable in practice we must demand that the essential part of the radiation from the antenna takes place for real values of the pole distance  $\theta$ . Defining  $t = \sin \theta$  this means for values of  $t$  in the interval  $0 < t < 1$ . It appears from (15) and (29) that the field components  $F_x$  and  $F_y$  are linear combinations of the functions  $(H/x)J_H(x)$  and  $J_H'(x)$  where  $x = kat$ . The smallest value of  $ka$  for which the essential part of either function comes within the interval  $0 < t < 1$  is roughly given by

$$ka_{\min} \cong H \quad \text{or} \quad a_{\min} \cong \frac{H\lambda}{2\pi}. \quad (35)$$

The increment of the current phase per unit length in a homogeneous ring quasi-array with the minimum applicable radius is seen to be equal to the increment per unit length in a conventional, linear end-fire array. As is well known, the increment of the current phase per unit length in an array of this kind is approximately the largest admissible one. Even in case of such a small extra increment of the current phase along the array as  $\pi$ , as proposed by Hansen and Woodyard,<sup>23</sup> inconveniences begin to arise. This was mentioned by these authors themselves and further stressed by Schelkunoff<sup>24</sup> and Reid.<sup>25</sup> It appears from Figs. 5 and 8 that the minimum radii are not large enough to cause the field from quasi-arrays with these radii to differ essentially from the field of a ring quasi-array with a vanishingly small radius.

#### CONCLUSION

A calculation has been made of the field from a homogeneous ring quasi-array of an infinite number of tangential or radial dipoles, in which the current phase increases  $H2\pi$  during one revolution. It is shown that apart from the case  $H = \pm 1$  the field will, to an increasing degree, concentrate around the horizontal plane with increasing  $H$ . At the same time the field will approach more and more horizontal polarization. A corresponding investigation has been made of the field from a ring quasi-array of an infinite number of tangential

dipoles in the case where the current distribution along the quasi-array is a standing current wave. A circular frame aerial that is fed at a single point of its circumference, approximates such an antenna system. The field radiated from this quasi-array is linearly polarized.

For  $H \rightarrow \infty$  the gain of the antenna systems investigated here converge towards infinity. By applying the theory of supergain an approximate expression has been derived for the minimum radius that the quasi-array may have for a given value of  $H$ , without the antenna having a  $Q$  too large to be applicable in practice.

The present article is the first of a series of papers dealing with homogeneous ring quasi-arrays.

#### ACKNOWLEDGMENT

The investigation presented here was carried out at the Microwave Laboratory of the Royal Technical University of Denmark, with a grant from the Research Council of the Technical Sciences in Denmark. I am indebted to the head of the Microwave Laboratory, Professor J. Oskar Nielsen, for the interest with which he has followed this work, and to Mr. Preben Gudmandsen for careful reading and criticism of the manuscript. The English translation has been revised by Mr. Oscar Kasch, Sworn Translator and Interpreter.

#### APPENDIX

##### Calculation of $F_x$ and $F_y$ for a Homogeneous Ring Quasi-Array of Tangential Dipoles

By transforming the expression for  $F_x$  given in (14) we find

$$\begin{aligned} F_x &= -\frac{1}{2\pi} e^{iH\phi} \int_{-\phi}^{2\pi-\phi} e^{i(Hv-z \cos v)} \sin(\phi + v) dv \\ &= -\frac{1}{2\pi} e^{iH\phi} \left\{ \frac{1}{2i} \left[ \int_0^{2\pi} e^{i[(H+1)v-z \cos v]} dv \right. \right. \\ &\quad \left. \left. - \int_0^{2\pi} e^{i[(H-1)v-z \cos v]} dv \right] \cos \phi \right. \\ &\quad \left. + \frac{1}{2} \left[ \int_0^{2\pi} e^{i[(H+1)v-z \cos v]} dv \right. \right. \\ &\quad \left. \left. + \int_0^{2\pi} e^{i[(H-1)v-z \cos v]} dv \right] \sin \phi \right\} \\ &= \frac{1}{2} \left\{ [J_{H-1}(z) + J_{H+1}(z)] \cos \phi \right. \\ &\quad \left. - i[J_{H-1}(z) - J_{H+1}(z)] \sin \phi \right\} e^{iH(\phi-\pi/2)} \\ &= \left[ \frac{H}{z} J_H(z) \cos \phi - iJ_H'(z) \sin \phi \right] e^{iH(\phi-\pi/2)}. \quad (36) \end{aligned}$$

In a similar way, the expression for  $F_y$  given in (14) is transformed into the expression for  $F_y$  given in (15).

<sup>23</sup> W. W. Hansen and J. R. Woodyard, "A new principle in directional antenna design," *Proc. I.R.E.*, vol. 26, pp. 333-345; 1938.

<sup>24</sup> S. A. Schelkunoff, "A mathematical theory of linear arrays," *Bell Sys. Tech. Jour.*, vol. 22, pp. 80-107; 1943.

<sup>25</sup> D. G. Reid, "The gain of an idealized Yagi array," *Jour. IEE* (London), pt. IIIA, vol. 93, pp. 564-566; 1946.