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Krenk, Steen; Brønden, Anders; Kristensen, Anders

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PLACEMENT AND TUNING OF RESONANCE DAMPERS ON FOOTBRIDGES

Steen KRENK
Professor
Dept. of Mechanical Engineering
Technical University of Denmark
Lyngby, Denmark

Anders BRØNDEN
Engineer
Buro Happold
Glasgow, Scotland

Anders KRISTENSEN
Engineer
RAMBØLL
Virum, Denmark

Summary

The placement and tuning of multiple tuned mass dampers is discussed with reference to the newly designed Langelinie Footbridge, presently under construction in Copenhagen. First the principles of resonance dampers are briefly reviewed, and a novel result for the calibration is presented. It has been shown that classical frequency tuning combined with minimization of the relative motion of the damper mass leads to an increase of the damping relative to the classical procedure with about 12 pct. The two design approaches: i) minimum resonance for frequency dependent load, and ii) robustness against resonance by sufficient damping ratio, are discussed. A simple two-step procedure is presented for design of multiple dampers on footbridges: first the properties of tuned mass dampers for the relevant free vibration modes are estimated by generalizing the simple basic equations to modal vibrations, and then the parameters are fine tuned by vibration analyses including the suspended masses from the resonant dampers for the other modes. Usually a single correction is sufficient to obtain equal damping in the combined system.

Keywords: footbridge dynamics; vibrations; damping; tuned mass dampers; multiple dampers.

1. Introduction

Slender structures with variable loads are potentially susceptible to vibration problems [1]. Many modern footbridges belong to this category – slender structures with long spans and pedestrian loads from walking, jumping and running. The loads will typically have main frequency components below about 5 Hz [2,3], and footbridges crossing major roads, rivers or railway terrains often have one or more vibration modes within this range. It is difficult to increase the lowest frequencies beyond this range, because increased stiffness will also require increased mass. Thus, damping becomes an important issue for these bridges. The lateral vibration problems of the London Millennium Bridge have led to great attention to this kind of vibration problem. However, most footbridges of more conventional design have considerably larger lateral than vertical bending stiffness, leaving the direct loading from the footfall as the main source of pedestrian load. Lateral vibrations are of the 'self induced' kind, where the motion of the bridge deck leads to changes in the walking pattern, generating an unfavourable load. Essentially, this is an instability that can be cured by introducing sufficient damping, whereby the loading is eliminated. Conversely, the vertical pedestrian load is only little influenced by the bridge deck motion. In this case the purpose of the damping is to reduce the magnitude of the dynamic response, while the loading is left virtually unchanged. Tuned mass dampers are efficiently used for this purpose. The idea of the tuned mass damper is to suspend a mass in such a way that the relative motion of the mass is governed by resonance. This produces a large relative motion which is then damped by a conventional damper, connecting the mass and the bridge deck. The present paper describes an accurate and systematic procedure for design of a system of tuned mass dampers for damping of several vibration modes of a footbridge. Due to uncertainty regarding the precise loading conditions, the procedure is based on an initial estimate of the necessary damping ratios of the concerned modes. The dampers are then designed by a two step procedure: i) first an initial estimate based on the individual uncoupled modes of vibration, and then ii) an adjustment based on modes including the damper masses of the other modes. The procedure makes use of newly found properties of modal damping ratios for tuned mass damper systems, and leads to a simple, yet accurate, distribution of the damping over the modes. The procedure is illustrated for the Langelinie Footbridge, presently under construction in Copenhagen. First the design procedure is presented and demonstrated, and then some particular load cases representing jumping and running are illustrated.

1.1 The Langelinie Footbridge

The design of the Langelinie Footbridge in Copenhagen was submitted in 2004, and the bridge is now under construction. The bridge is a steel box girder with four spans, crossing the railway terrain behind Østerport Station and connecting the Østerbro part of the city with a park containing part of the old city fortifications. A general view and a cross-section are shown in Fig. 1. The following analyses are based on the initial design of the bridge. In the final design some corrections have been introduced, and details of the results may therefore not be fully representative for the completed bridge.

The bridge is a continuous four-cell box girder in Corten steel with modulus of elasticity 300 GPa and Poisson's ratio of $\nu = 0.3$. The main dimensions of the bridge are shown in Fig.1. The total length is 169.8 m, and the box section is 6.85 m wide and 1.00 m high. The flanges vary from $20-30$ mm in thickness over the full length of the bridge with more material in compressive zones. Longitudinal stiffeners and transverse bulkheads have a thickness of 10.5mm and 12 mm, respectively. The distributed mass is approximately 3200 kg/m corresponding to a total mass including transverse bulkheads of about $5.6 \cdot 10^5$ kg/m.

The requirement for vibration comfort were those given by the Danish Road Directorate [4] consisting of a reference load scenario with two walking persons, represented by an equivalent harmonic load and the maximum acceleration

$$a_{max} \le 0.25 \, f_j^{0.78} \tag{1}$$

where f_j is the frequency of mode No. j in Hz, and a_{max} is the maximum acceleration in m/s^2.
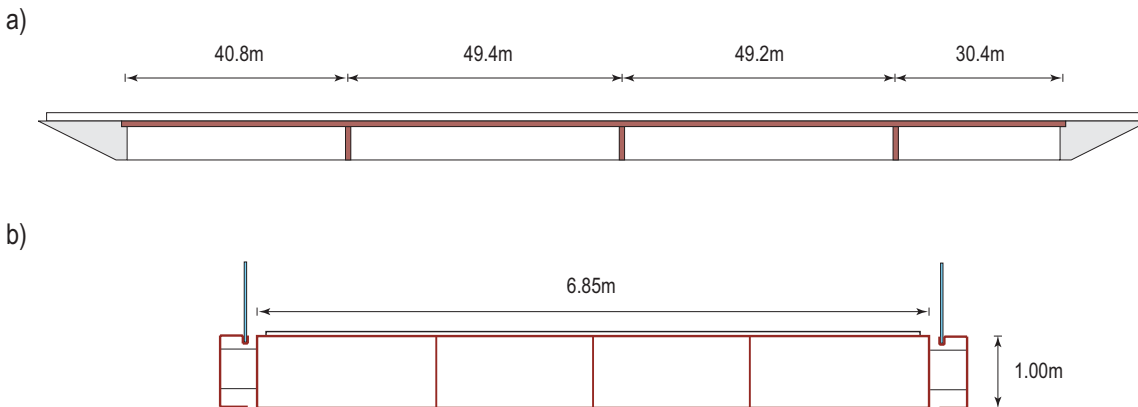
a)

b)

Fig. 1 Langelinie Footbridge, a) general view, b) cross-section.

1.2 Damping of vibrations from pedestrian loads

The initial design of dampers for footbridges can follow either of two approaches: design against dynamic amplification for a harmonic load, or a design based on robustness against dynamic amplification by introducing a sufficient damping ratio ς for each of the relevant modes. The present case is concerned with vertical displacements only, and the displacements of the nodes of the bridge can be written in vector format as $\mathbf{u}^T = [u_1, u_2, \cdots]$. The equations of motion are conveniently expressed in the form of the following matrix equation, usually obtained by a finite element model,

$$\mathbf{M\ddot{u} + C\dot{u} + Ku = Q}(t) \tag{2}$$

where a dot denotes time derivative. $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrix, respectively, while $\mathbf{Q}(t)^T = [Q_1(t), Q_2(t), \cdots]$ denotes the time dependent vector of nodal loads.

An estimate of the dynamic amplification of mode No. j can be obtained from the equations of motion (2) by considering the individual modes one at a time. The response of an individual mode is given as the product of the mode shape vector

$\mathbf{u}_j$ and the modal amplitude $r_j(t)$ in the form

$$\mathbf{u}(t) = r_j(t)\mathbf{u}_j \tag{3}$$

Substitution of this expression into (2), followed by multiplication from the left by the transpose of the mode shape vector leads to the equation for the modal amplitude,

$$\ddot{r}_j(t) + 2\varsigma_j\omega_j\dot{r}_j(t) + \omega_j^2 r_j(t) = \frac{1}{m_j}q_j(t) \tag{4}$$

In this equation ω_j is the angular frequency and ς_j is the damping ratio of mode No. j. The modal load $q_j(t)$ is given as the product of the original nodal load vector $\mathbf{Q}(t)$ and the mode shape vector $\mathbf{u}_j$,

$$q_j(t) = \mathbf{u}_j^T \mathbf{Q}(t) \tag{5}$$

and m_j is the modal mass, defined by

$$m_j = \mathbf{u}_j^T \mathbf{M}\mathbf{u}_j \tag{6}$$

The modal mass gives an impression of how much of the structural mass participates in the motion of this particular mode as discussed for the Langelinie Footbridge in Section 3. The relation (6) can be used to normalize the mode shape vectors. In particular, if the modal mass m_j is selected to be the mass of the bridge deck, the mode shape vector reflects a typical 'unit displacement'.

For a periodic load the most severe situation is most often resonance. Resonance is characterized by a harmonic response with angular frequency ω_j. At resonance the inertial term represented by $\ddot{r}_j(t)$ precisely cancels the stiffness term $\omega_j^2 r_j(t)$. In terms of magnitudes the equation can therefore be written as

$$\omega_j^2 r_j = \frac{q_j}{2\varsigma_j m_j} \tag{7}$$

This relation implies that the contribution to the acceleration of the vertical motion of the bridge for a mode in resonance is given by

$$\ddot{\mathbf{u}} = \omega_j^2 r_j \mathbf{u}_j = \frac{q_j}{2\varsigma_j m_j}\mathbf{u}_j \tag{8}$$

Thus, the amplitude of the acceleration is given as the modal load divided by twice the damping ratio times the modal mass, $q_j / 2\varsigma_j m_j$. It is noted that this formula for the acceleration does not contain the factor ω_j^2 sometimes seen in hand calculation type estimates. This factor would be introduced, if the modal mass m_j were replaced with the modal stiffness k_j. However, while the modal mass and the corresponding mode shape vectors are easily normalized in an intuitively clear way, the modal stiffness increases rapidly with the mode number.

The limit on vertical dynamic response from pedestrian loads is usually considered as a comfort criterion based on acceleration. It is seen from (8) that if the modal mass is chosen equal to the mass of e.g. the deck the response depends on the product of the load with the mode shape vector, divided by the damping ratio. Thus local modes can lead to severe response, if the load is also local. On the other hand distributed loads lead to smaller modal load q_j thereby limiting the response. For distributed loading this suggests the alternative design principle, in which all the relevant modes, e.g. modes with natural frequencies below 5 Hz, are given a sufficient damping ratio. If this principle is adopted, the final design should be checked for local loads, e.g. from a small group deliberately trying to excite the bridge.

2.   Tuned Mass Dampers

Traditionally the principle of the tuned mass damper is developed for a single damper mass m_d attached to a moving structural mass m_0 by a spring with stiffness k_d and a viscous damper with parameter c_d. The undamped frequency of the structure is characterized via the structural stiffness k_0. The idealized tuned mass damper is illustrated in Fig. 2a. Figure 2b illustrates a case in which two dampers are used in parallel. This is typically used for flexible structures to

distribute the action of the dampers over the structure. A three step design procedure for tuned mass dampers on flexible structures can consist of: i) selection of optimal parameters based on the idealized systems shown in Fig.2, ii) evaluation of representative structural parameters for the modes to be damped, and finally iii) adjustment of the parameters to account for the presence of multiple dampers. First the optimal parameters for the idealized system are identified, and then the reinterpretation for flexible structures is dealt with in Section 3.
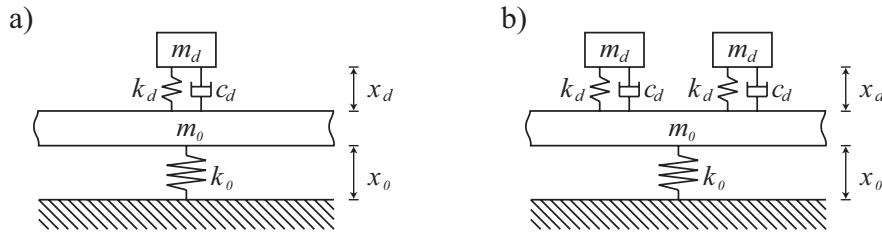
a)

b)

Fig. 2 Tuned mass damper(s), a) single resonant mass, b) double masses.

2.1   Optimal calibration

The masses of the idealized system in Fig.2a are characterized by the mass ratio $\mu = m_d / m_0$, while the stiffness of structure and damper are characterized by their angular frequencies,

$$\omega_0^2 = k_0 / m_0 \quad , \quad \omega_d^2 = k_d / m_d \tag{9}$$

The natural angular frequency of the structure ω_0 is given, and the frequency of the damper is selected to provide optimal properties of the combined system. The classical method is to consider a frequency diagram of the dynamic amplification [5]. Two points are independent of the magnitude of the damping, and the frequency tuning of the damper is determined to

$$\omega_d = \frac{\omega_0}{1 + \mu} \tag{10}$$

The damper is characterized by its damping ratio. When an additional mass from the damper is introduced, the original structural vibration mode splits into two modes with closely spaced frequencies. It has recently been demonstrated that for moderate mass ratio the frequency tuning (10) leads to identical damping ratio for both these modes.

The literature has various definitions, depending on normalizing frequency, but here the damping ratio of the damper is defined by the free vibrations of the damper if the structure remains stationary, i.e. $\varsigma_d = c_d / 2m_d \omega_d$. The magnitude of the damping ratio ς_d must be selected large enough to produce damping, but not so large that it limits the relative motion of the damper. It can be shown that the largest damping ratio that can be used without increasing the relative motion of the damper is given by, [6],

$$\varsigma_d = \sqrt{\frac{1}{2} \frac{\mu}{1 + \mu}} \tag{11}$$

This is different from the classic result [5], which has the factor $\tfrac{1}{2}$ as $\tfrac{3}{8}$. The dynamic amplification of the structural mass is illustrated in Fig. 3a, and the relative motion of the damper mass in Fig. 3b for a mass ratio of $\mu = 0.05$. It is seen that the relation (11) leads to a flat maximum of the relative motion of the damper mass, and also that it essentially removes the double peaks in the motion of the structural mass. It has been be shown that the damping of the combined structure-damper modes is approximately equal to half the damping ratio of the damper $\varsigma_{struc} \simeq \tfrac{1}{2} \varsigma_d$. Thus, the additional 12 pct. damping introduced by (11) relative to the classical formula also leads to an increase of 12 pct. in the damping of the structural modes.
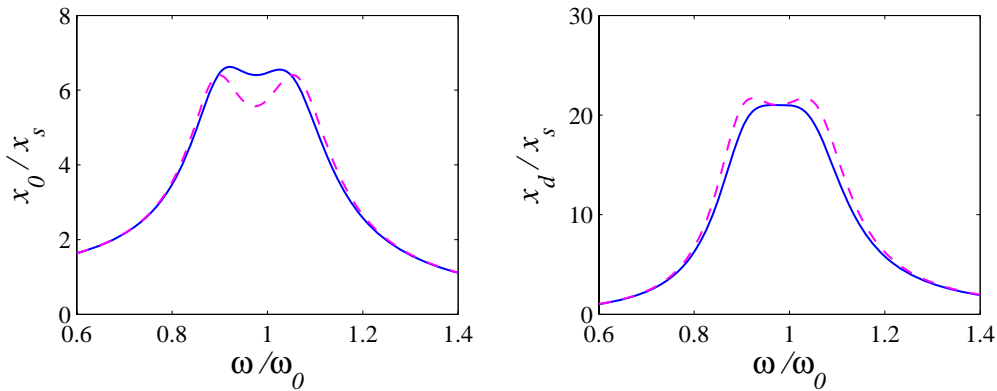
Fig. 3 Dynamic amplification for harmonic load, a) motion of structural mass, b) relative motion of damper mass. The full line corresponds to damping given by (11), while the dashed line is the classical result [5].

2.2   Double dampers

When mounting dampers on flexible structures like footbridges it may be desirable to use two or more dampers for the same mode in order to limit the size of the individual damper and to distribute the effect of the dampers. The idealized model with two dampers is shown in Fig. 2b. This is a system with three degrees of freedom, and therefore three modes of vibration. If the two dampers are identical there are two modes, where the dampers exhibit identical motion. These modes correspond to those described above. When the dampers are correctly tuned these modes will have a damping ratio equal to half the damping ratio of the dampers, $\varsigma_{struc} \simeq \frac{1}{2}\varsigma_d$. In the third mode the two dampers oscillate in opposite motion, leaving the structural mass at rest. This mode has the damping ratio of the dampers, $\varsigma_{struc} \simeq \varsigma_d$. For flexible structures with 'double dampers', these will typically be balanced to provide equal participation, and the additional mode will therefore typically involve even larger motion of the dampers relative to the motion of the structure, and accordingly the damping is also larger than the ordinary modes – typically about double. This additional mode is therefore not a design problem.

3.   Multiple Dampers on Flexible Structures

When the simple results for frequency tuning (10) and calibration of damping (11) are to be used for multiple dampers on flexible structures three questions have to be addressed: i) definition of suitable generalized parameters to be used in the design formulae, ii) interaction effects between the dampers, and iii) possible changes in the mode shapes due to the additional mass of the dampers. Equivalent parameters can be defined in terms of the undamped mode shapes of the structure and used for preliminary design of the damper system as described in Section 3.1. A subsequent detailed design can then be obtained from a free vibration analysis making use of complex mode shapes and frequencies, including the effect of the dampers. This procedure is briefly indicated in Section 3.2 and illustrated for the Langelinie bridge in Section 4.

3.1   Equivalent modal parameters

The preliminary design analysis is based on the modes of the structure without any dampers mounted. For each of these modes the displacement of the bridge deck is given by the representation (3) in terms of the modal amplitude $r_j(t)$ and the corresponding mode shape vector $\mathbf{u}_j$. One or more dampers are installed for mode j. They are all tuned to the same frequency $\omega_{d,j}$, and the design is based on the situation, in which they move in phase. The motion of this group of dampers can then be represented by the mode shape vector $\mathbf{u}_j$ in a form similar to (3) for the bridge deck,

$$\mathbf{u}_d(t) = r_{d,j}(t)\mathbf{u}_j \tag{12}$$

As seen from this representation the motion of each damper in the group is proportional to the local motion of the bridge deck. The effective mass of the group of dampers can therefore be calculated by an expression similar to (6) for the modal mass of the bridge,

$$m_{d,j} = \mathbf{u}_j^T \mathbf{M}_{d,j} \mathbf{u}_j \qquad (13)$$

where $\mathbf{M}_{d,j}$ is a diagonal matrix containing the masses of the dampers for mode j in the appropriate diagonal positions. The equivalent mass ratio for mode j is then given by $\mu_j = m_{d,j} / m_{0,j}$. The frequency tuning then follows from (10), and the calibration of the dampers from (11).

When mounting a tuned mass damper, the original mode of vibration splits into two modes with closely spaced frequencies. In the optimal damper configuration each pair of modes has identical damper ratio. Typically the preliminary design leads to somewhat different damping ratios, when dampers are introduced for several modes. This is due to sensitivity to the additional mass of the other dampers. The importance of these effects depends on the number of modes to be damped, and thereby the damper mass added to the structure, and on the stiffness of the bridge deck. The interaction effects can be estimated and the preliminary design improved by recalculating the natural frequencies for each mode with all the damper masses *for the other modes* included with spring stiffness, but without dampers. This gives a good estimate of the interaction effect, and in practice constitutes a suitable basis for the design of the damper system.

3.2 Analysis by complex modes and frequencies

The design procedure outlined above provides an estimate of the necessary masses, the stiffness of their support, and the necessary damping, including an estimate of the interaction effect of multiple dampers, but neglecting any change in mode shapes introduced by the damping. The behavior of the bridge with dampers can be investigated by performing a free vibration analysis of the bridge, including the tuned mass dampers in the model. In the free vibrations the motion of the dampers is not in phase with the motion of the bridge, and the vibrations can therefore not be described by a conventional real-valued modal analysis. In order to describe vibrations that are not in phase a complex-valued modal analysis is required. The idea – originally presented by Foss [7] - is to represent the motion as the real part of a complex-valued harmonic function,

$$\mathbf{u}(t) = \mathrm{Re}[\exp(i\omega t)\mathbf{u}_j] \qquad (14)$$

The free vibrations can then be found from a linear eigenvalue problem of the double size of the original system of equations,

$$\left(\begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & -\mathbf{M} \end{bmatrix} + i\omega \begin{bmatrix} \mathbf{C} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{bmatrix} \right) \begin{bmatrix} \mathbf{u} \\ i\omega\mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \qquad (15)$$

The matrix $\mathbf{C}$ contains the damping coefficients. When the system includes local dampers, as in the present case, the angular frequency ω_j, determined from the eigenvalue problem, becomes complex. The damping ratio ς_j of the corresponding mode is determined from the imaginary part of the complex frequency,

$$\varsigma_j = \frac{\mathrm{Im}[\omega_j]}{|\omega_j|} \qquad (16)$$

Thus, the damping ratio of any mode of the bridge with dampers can be determined by solving the eigenvalue problem (15). This can be done by standard numerical software. The solution includes the full damper system and therefore accounts for interaction effects as well as any change of mode shapes. The full complex analysis in the following to demonstrate the accuracy of the two-step design procedure outlined in Section 3.1 consisting of a preliminary design based on the undamped mode shapes of the structure without dampers, followed by an adjustment based on recalculated natural frequencies including the other damper masses. Thus, a full complex modal analysis will most often not be needed in the actual design procedure.

4. Dampers for Langelinie Footbridge

The Langelinie Footbridge has four vibration modes with resonant frequencies below 5 Hz. The vibration modes are normalized such that the modal mass of each mode equals the physical mass of the bridge (560 tons) and are presented in Fig. 4. These vibration modes are prone to human loading and it is shown that even a small pedestrian group violates the comfort criteria if only the structural damping is regarded. The structural damping for steel footbridges is usually

assumed to be 0.4% and the structural response is evaluated by (7) for the four vibration modes. This indicates that the implementation of additional damping is essential for the serviceability of the footbridge.

The dampers for the Langelinie Footbridge are designed on the basis of a mass ratio of $\mu = 0.05$ and are positioned at maximum crests within each vibration mode indicated with blue boxes in Fig. 4. In the following the dampers are configured by a two step procedure, and the effectiveness of the dampers is evaluated by the resulting damping ratio in the different modes found by solving the eigenvalue problem (15).
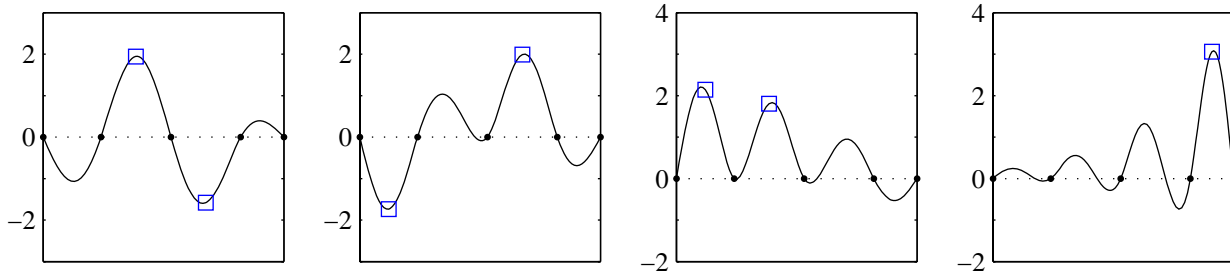

Fig.4 The first four vibration modes of the Langelinie Footbridge with damper positions indicated.

4.1   Preliminary design of dampers from undamped structural modes

The physical mass of the dampers in the individual modes are determined on the modal mass of the bridge alone. Thus the weight of the other dampers is disregarded. The equivalent mass properties of the dampers within each mode are determined by (13) such that the mass ratio $\mu = m_{d,j} / m_j$ is fulfilled. The remaining physical parameters of the dampers are determined by (10) and (11).The parameters of the dampers for the relevant modes are presented in Table 2. The damping in the individual modes are presented by ζ_{struc} . It is noticed that implementation of double dampers yield an extra mode characterized by a high level of damping as mentioned in section 2.2. Equal modal damping is not achieved in any of the modes but the fourth mode. This is most severe for the first mode where only 6% of critical damping is achieved. This is somewhat smaller than the expected level of damping around 7.5% of critical.

Table 2. Damper properties and effect based on undamped mode shapes of structure without dampers.

Mode	1	2	3	4
Damper in span(s)	2 / 3	1 / 3	1 / 2	4
Damper mass (kg)	4450 / 4450	4000 / 4000	3550 / 3550	3000
μ_{eff}	0.0500	0.0499	0.0498	0.0502
$f_{0,j}$ (Hz)	1.67	2.31	3.04	4.59
$f_{d,j}$ (Hz)	1.59	2.20	2.90	4.37
ζ_{damper}	0.154	0.154	0.154	0.155
ζ_{struc}	0.060 / 0.101	0.081 / 0.096	0.095 / 0.081	0.078 / 0.080
	0.141	0.137	0.149	

4.2   Improved design by including damper masses in the modes

The second step in the design procedure includes the mass and stiffness properties of the dampers for the other vibration modes. Thus, these properties are included in a real-valued eigenvalue analysis of the global system matrices. In principle an iterative process must be carried out in order to implement the correct mass of the other dampers as they are modified in the design of the damper(s), but usually only one step is needed. The new design of the full damper configuration is presented in Table 3 The total weight of the dampers is increased app. 5% corresponding to the extra mass introduced in each vibration mode. This design approach is noticed to implement a higher level of damping in the modes and is most visible in the first mode, where the damping is increased by 30%.

Table 3. Damper properties and effect, when including damper mass and stiffness in the modal analysis.

Mode	1	2	3	4
Damper in span(s)	2 / 3	1 / 3	1 / 2	4
Damper mass (kg)	4775 / 4775	4275 / 4275	3750 / 3750	2925
μ_{eff}	0.0499	0.0499	0.0502	0.0500
$f_{0,j}$ (Hz)	1.58	2.26	3.11	4.60
$f_{d,j}$ (Hz)	1.51	2.16	2.96	4.38
ς_{damper}	0.154	0.154	0.155	0.154
ς_{struc}	0.078 / 0.084	0.086 / 0.089	0.085 / 0.091	0.078 / 0.079
	0.143	0.137	0.151	

The dampers can be fine tuned to equal damping ratio by use of the complex modal analysis technique. Equal modal damping can be obtained by changing the damper stiffness (or damper mass) in small steps. In the present case fine tuning by change of the damper stiffness leads to damping ratios for the four modes of 0.081, 0.087, 0.088 and 0.078, respectively. The extra highly damped modes, associated with the duplicate dampers for the first three modes are not affected by the fine tuning. The following load cases are based on the damper configuration presented in Table 3.

5. Local Loads from Jumping and Running

Local loads can be produced by either jumping or running groups of people. When jumping or running the contact duration t_p constitutes only part of the footfall period T. The time variation of the loading is described by series of half-sine impulses with maximum value $k_p F_0$,

$$F(t) = \begin{cases} k_p F_0 \sin\left(\pi \cdot t / t_p\right) & \text{for} \quad t \leq t_p \\ 0 & \text{for} \quad t_p < t \leq T \end{cases} \tag{17}$$

F_0 is the equivalent load for full contact, and the peak factor $k_p = \pi T / 2t_p$ is determined by the ratio of the contact duration and the footfall period T. This load pattern is now used to illustrate the effect of a group of 5 people jumping at one location, and a person running across the bridge. Additional load cases and damper configurations have been considered in [8].

5.1 Jumping group of people

The response to a group of 5 people in fully synchronized jumping at the same location with frequency $f = 1/T = 2.5$ Hz is considered. This frequency lies in the middle of the range suggested by Bachmann et al. [1]. The contact duration is taken as $t_p = 0.5T = 0.2$ s, yielding a peak factor of $k_p = \pi$. The static weight of one person is set to 0.8 kN, leading to a total static load of the group of 4 kN. Thus, the peak load attains the magnitude of $F_{peak} \approx 13$ kN.

Table 4. Maximum response in load cases 1 and 2.

Load case	1	2
a_{max} [m / s²]	0.376	0.349

In load case 1 the periodic load (17) is placed in the middle of span 2, and in load case 2 the load is placed in the middle of span 4. The response is found by direct numerical time-integration. The maximum acceleration obtained in the two load cases is presented in Table 4, and the corresponding response curves for the nodes with maximum acceleration are presented in Fig. 5. None of the loads violates the comfort criterion (1). In load case 1 the first harmonic component of the load is near resonance with one of the split modes originating from the second vibration mode and is seen from

Fig.5a to be governed by this vibration mode. Thus, a fairly large response results around the excitation frequency $f = 2.5$ Hz. In load case 2 the second harmonic component of the load is near resonance with one of the split modes originating from the fourth vibration mode. The corresponding acceleration record is shown in Fig. 5b, clearly illustrating a response frequency around 5 Hz. A Fourier expansion of the load (17) reveals the first and second force amplitude to be 1.57 and 0.67 respectively. The importance of the local modes is illustrated by the fact that the response of load case 2 is of the same magnitude as that of load case 1, even though the force is less than half the magnitude. All things equal the force amplitudes can change within the group, if the assumption of total synchronization is not fulfilled. This would lead to a smaller response.
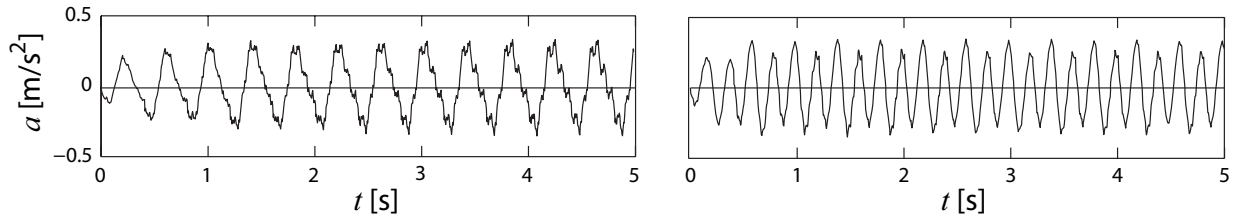

Fig.5 Acceleration records in middle of loaded span due to jumping group,a) span 2, b) span 4.

5.2 Running across the bridge

The Langelinie Footbridge is intended to be used for public events like the Copenhagen Marathon. Running may involve groups of different sizes and different running styles, involving different degrees of synchronization. For full synchronization the total load will be N times that of a single runner – conditions being otherwise equal – while for complete lack of synchronization the effective load will only be $\sqrt{N}$ times that of a single runner. The following scenario therefore involves only a single runner, and the total load must be obtained by appropriate scaling.

The load of each footfall is again modeled by half-sine pulses given by (17) with the same frequency f and contact duration t_p. This time however the force is not stationary but moving along the length of the bridge giving the load a transient character. The stride length of the runner is set to $l_s = 1.3$ m. The load of one person is set to 0.8 kN. The establishment of the global vector $\mathbf{Q}(t)$ in (2) is schematically presented in Fig. 6. The length of each element corresponds to the stride length of the runner such that a time difference for which the load is applied at adjacent nodes corresponds to the period T.
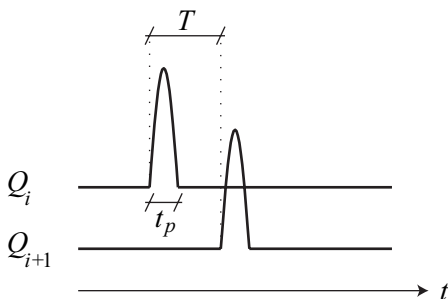
Fig.6 Time history of load components at node i and $i+1$ for running.

The maximum acceleration is found to be $a_{max} = 0.077$ m/s² and occurs at the middle of the fourth span, when the runner passes this point. The response of this node is presented in Fig. 7. The vertical dotted lines indicate when the runner passes a support, thus the runner has left the bridge after the last line at $t = 51.1$s. This corresponds to a forward speed of 3.3m/s. With reference to Fig. 4 it is evident that the response is governed by the fourth vibration mode which is activated through the second harmonic of the footfall frequency.
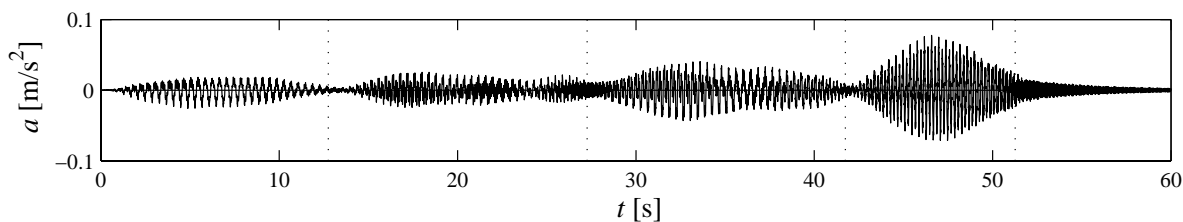
Fig.7 *Response at the middle of span 4 due to load case with one runner crossing the bridge.*

6.   Conclusions

A two-step design procedure for tuned mass dampers on footbridges has been presented and illustrated with application to the new Langelinie Footbridge in Copenhagen. It is based on the close relation between the modal response to a harmonic load and the damping ratio for that mode. The design is based on providing the necessary modal damping ratio for vibration modes with frequency below say 5 Hz. In the present analyses a fairly conservative value of $\varsigma_{struct} \simeq 0.08$ has been used. More detailed values can be estimated from the modal loads by use of the relation (7). The actual design of the damper system has two steps: i) an initial estimate of the dampers for any particular mode is obtained from the frequency and damper formulae (10) and (11) using the generalized mass of the dampers given by (13), ii) the damper parameters are adjusted by re-calculating each mode including the damper masses and springs from the dampers for all *the other* modes. In practice this leads to nearly optimal tuning of the coupled vibration problem of structure and dampers.

Equal damping ratio for all relevant modes corresponds to a loading scenario, where the modal loads are roughly proportional to the modal masses. Local modes are typically associated with smaller modal masses, and it is therefore advisable to check the design for local loads. The effect of local loads has been illustrated by the response to regular jumping and the passage of an individual runner.

7.   References

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