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Method for Measuring Small Nonlinearities of Electric Characteristics

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A method is described for measuring very small deviations from linearity in electric characteristics. The measurement is based on the harmonics generated by the nonlinear element when subjected to a sine wave signal. A special bridge circuit is used to balance out the undesired harmonics of the signal generator together with the first harmonic frequency. The set-up measures the small-signal value and the first and second derivative with respect to voltage. The detailed circuits are given for measuring nonlinearities in Ohmic and capacitive components. In the Ohmic case, a sensitivity in the measurement of the relative second order nonlinearity of about 5×10^{-8} is obtained. In the capacitive case, the sensitivity expressed in terms of the minimum measurable values of the derivatives is $dC/dV=1.5\times10^{-17}$ F/V and $d^2C/dV^2=2\times10^{-16}$ F/V^2.

1. INTRODUCTION

It is important in a number of applications to measure small deviations from linearity in the electric characteristics of devices or systems. One purpose may be to test the desired linearity of resistances, capacitances, contacts, etc. In other cases, the magnitude of the nonlinearity gives direct information about some properties that are under investigation. As an example one may cite the nonlinear capacitance-voltage relationship in p-n junctions which can be used to determine the impurity profile in semiconductor crystals.[1] Another example is the warm electron effect in semiconductors giving rise to small deviations from Ohm's law.[2] In these cases, a point by point measurement of the characteristics requires an extreme accuracy in order to reveal small deviations from linearity. Also in many instances, the characteristics are temperature dependent, which make the point by point measurement even more complicated.

A method has recently been described[3] which can give a direct reading of nonlinearities in the electric conductance with special reference to high resistance values. This method is based on the use of the combination frequencies which are generated when two sine waves are mixed in a nonlinear resistance. The ratio between the frequencies is so chosen that the desired beat note is of zero frequency. The measurement of the nonlinearity is thus converted into a measurement of a small dc component. This method appears to be quite sensitive at high values of the nonlinear resistances, but it will of course exhibit the usual difficulties at low dc levels.

In this paper we shall describe another method for the direct determination of small nonlinearities, which is based on the harmonics generated by the nonlinear element, when subjected to a sine wave signal. This method has proved very sensitive for a wide range of impedance levels. The difficulty arising from the fact that the signal generators usually do not supply a sufficiently pure sine wave has been circumvented by using a circuit which balances out the undesired harmonics of the signal generator together with the first harmonic (fundamental) frequency. In this way one is left with the harmonic signals from the nonlinear element at the input of the detector, and at the same time one avoids overloading of the detector from the first harmonic signal. As another advantage it may be mentioned that the magnitude of the small-signal value is measured concurrently with the nonlinearities.

The application of this principle to the determination of small nonlinearities in capacitances has been briefly mentioned in a previous paper.[1] The experimental results given there were, however, obtained on a simple capacitance bridge which was not designed with the view of measuring higher harmonics. A special bridge has now been constructed which has resulted in an improvement in sensitivity of several orders of magnitude as compared to the previous results.[1] This bridge is described in detail in Sec. 5.

The detailed circuit is also given of a bridge for measuring small deviations from linearity in Ohmic resistances. This bridge has been used to advantage in the investigation of warm electron effects in semiconductor crystals as a function of temperature. The application to warm electron effects will be used in Sec. 2 as an illustration of the general principle.

2. HARMONICS GENERATED BY NONLINEAR ELEMENTS

We shall consider a nonlinear element with the equivalent circuit shown in Fig. 1.

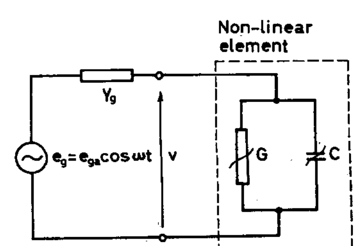

Fig. 1. Nonlinear element connected in circuit with sinewave signal generator.

[1] N. I. Meyer and T. Guldbrandsen, Proc. IEEE 51, 1632 (1963).
[2] K. J. Schmidt-Tiedemann, Festkörperprobleme (F. Vieweg 1962), Vol. 1, p. 122.
[3] C. E. Skov and E. Pearlstein, Rev. Sci. Instr. 35, 962 (1964).

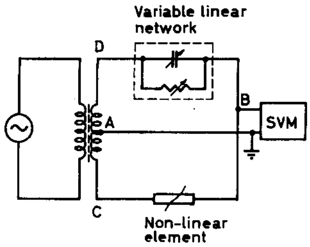

FIG. 2. Schematic diagram of bridge circuit for measuring harmonic components generated by a nonlinear element.

The ac voltage across the nonlinear element in Fig. 1 is written in the following form:

$$v = v_1 + v_2 + v_3,$$

representing the terms of angular frequencies ω, 2ω, and 3ω, while harmonics higher than the third one are neglected. With the assumption

$$|v_1| \gg |v_2|, |v_3|$$

one finds[4]

$$v_2 = \tfrac{1}{4}\{1/[Y_g(2\omega)+Y(2\omega,V_0)]\}[dY(2\omega)/dV]v_1^2 \quad (1)$$

and

$$v_3 = \frac{1}{Y_g(3\omega)+Y(3\omega,V_0)}\left[\frac{1}{24}\frac{d^2Y(3\omega)}{dV^2}v_1^3 + \frac{1}{2}\frac{dY(3\omega)}{dV}v_1v_2\right], \quad (2)$$

where

$$Y(n\omega, V) \equiv G(V) + jn\omega C(V). \quad (3)$$

Here V is the dc voltage, V_0 is the voltage corresponding to the working point in question, and Y_g is the internal generator admittance. All derivatives are taken at $V = V_0$.

The dominating capacitive case has been treated previously[1] and we shall here concentrate on the dominating Ohmic case. The value of ω is thus chosen sufficiently low that the expressions (1) and (2) simplify into

$$v_2 = \tfrac{1}{4}(G_g + G)^{-1}(dG/dV)v_1^2 \quad (4)$$

and

$$v_3 = \tfrac{1}{24}(G_g + G)^{-1}[(d^2G/dV^2)v_1^3 + 12(dG/dV)v_1v_2], \quad (5)$$

where G is the linear (small signal) conductance of the element and G_g is the equivalent generator conductance. By measuring v_1, v_2, v_3, G_g, and G it is thus possible to determine the nonlinearity characteristics (dG/dV) and (d^2G/dV^2).

Taking the warm electron case as a specific example one obtains the following expression for the current-voltage characteristics of a semiconductor subjected to a homogeneous electric field along a crystal direction of high symmetry[2]

$$I = VG_0(1 - BV^2). \quad (6)$$

This expression is valid in the case of silicon and germanium for fields up to about 50 V/cm. The interesting term is the coefficient B which gives quantitative information about the electron-phonon coupling constants.[5] The small-signal conductance $G(V)$ is found from (6) as

$$G(V) = dI/dV = G_0 - 3G_0BV^2. \quad (7)$$

When (7) is inserted into (4) and (5) one finds for the working point $(V_0, I_0) = (0,0)$ that $v_2 = 0$, while v_3 is proportional to B. Thus, in this example the third harmonic component is a direct measure of the physical quantity under investigation, while the disappearance of the second harmonic can be used as a check on the correctness of the measurement (influence of contacts, etc.).

3. GENERAL DESCRIPTION OF CIRCUIT FOR MEASURING HARMONIC COMPONENTS

The schematic diagram of the bridge circuit used for measuring harmonic components generated by a nonlinear element is shown in Fig. 2.

The measuring procedure is as follows: The first harmonic component between A and B is balanced out by an appropriate choice of the components in the variable linear network. Except for higher order effects, one balances out at the same time harmonics generated by the signal generator and by the transformer. When aiming at the extreme sensitivities, however, one should minimize these disturbances. The magnitude of the first harmonic signal over the element is measured between A and C.

The second and third harmonic components generated by the nonlinear element is measured between A and B with the selective voltmeter. The equivalent generator impedance (see Fig. 1) is equal to the transformed internal impedance of the signal generator plus the impedance of the variable linear network. When measuring small capacitances or large resistances one may have to correct for the input impedance of the voltmeter. It is preferable to construct the bridge such that the transformed generator impedance is small compared to the impedance of the variable linear network with the bridge in balance.

In this case the sum $(Y + Y_g)$ which enters the expressions (1) and (2) may be measured directly by the following method. A known voltage v_g is applied to the balanced bridge between A and C. The bridge is subsequently brought slightly out of balance by changing the linear network by a known small amount ΔY giving rise to an output signal v_d. The desired quantity taken at the angular frequency ω of v_g is then

$$|Y + Y_g| = |\Delta Y|(v_g/v_d).$$

The following practical limitations on the sensitivity should be mentioned: (1) Nonlinearities in the variable (assumed) linear network. (2) Higher harmonics in the transformer magnetization current combined with a nonideal symmetry of the transformer. (3) Incomplete

[4] N. I. Meyer, Proc. Inst. Elec. Engrs. (London) **106B**, 481 (1959).

[5] M. H. Jørgensen, N. I. Meyer, and K. J. Schmidt-Tiedemann, Solid State Commun. **1**, 226 (1963).

balancing of generator harmonics due to the different frequency dependence of assumed frequency-independent components, or due to drift in component characteristics during the measurement (e.g., temperature effects). (4) Creation of harmonics in the detector due to incomplete balancing of the first harmonic frequency. (5) Noise limit.

The following precautions have been taken in order to minimize these effects: (1) All resistors are metal-film types, potentiometers are wire wound, and condensers are air or polystyrene types. (2) The transformer is wound on a ring core with a secondary consisting of two bifilar windings to keep the leakage low. The axial stray field is reduced by means of extra single turns wound on the circumference of the core for both secondary halves and for the primary winding. The signal generator has a low output impedance in order to reduce the effect of transformer nonlinearities. (3) The signal generator has a relatively low distortion factor. The frequency dependence of the balance is minimized by using a variable linear network which as far as possible is equivalent to the small-signal impedance of the nonlinear element including connecting cables etc. The components which take up appreciable power dissipation are low temperature coefficient types. (4) Special detectors with good linearity are used. (5) A low noise differential amplifier is used in connection with the selective voltmeter.

4. BRIDGE FOR OHMIC MEASUREMENTS

The bridge is designed for measuring resistance values and the first and second derivatives of the resistance with respect to voltage at zero dc bias. The normal range is 10 Ω to 100 kΩ, and the frequency is 2 kc. The range can be extended to higher and lower resistance values simply by inserting an appropriate type of transformer. The detailed measuring circuit is shown in Fig. 3.

The sensitivity of the bridge has been examined in the following way. A 1 kΩ metal-film resistor is inserted instead of the nonlinear element, and the residual third harmonic is measured at the maximum permissible voltage (7.5 V across the nonlinear element). The corresponding third harmonic output signal is below the noise level which is about 1.5×10^{-8} V.

As a measure of the sensitivity one may use the minimum detectable value of the relative second order nonlinearity β_2 defined as

$$\beta_2 \equiv (1/G)(d^2G/dV^2)v_{1,\mathrm{rms}}^2, \tag{8}$$

where $v_{1,\mathrm{rms}}$ is the rms value of v_1. From the expression (5) with $G_g = G$, one finds when neglecting the v_1v_2-term

$$\beta_2 = 24(v_{3,\,\mathrm{rms}}/v_{1,\,\mathrm{rms}}). \tag{9}$$

The minimum detectable value of $v_{3,\,\mathrm{rms}}/v_{1,\,\mathrm{rms}}$ (noise limit) is about 2×10^{-9} corresponding to a value of

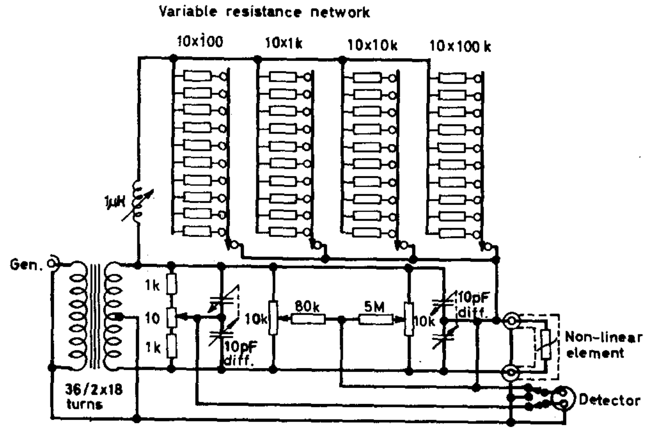

FIG. 3. Circuit diagram of bridge for measuring nonlinearity of Ohmic components.

$v_{1,\,\mathrm{rms}} = 7.5$ V. This gives

$$\beta_{\mathrm{min}} \approx 5\times 10^{-8}. \tag{10}$$

The corresponding quantity in the paper by Skov and Pearlstein[3] is in their notation

$$\beta_2 = 8(I_{\mathrm{dc}}/I_{\mathrm{tot}}). \tag{11}$$

They found a minimum value of 10^{-8} for $I_{\mathrm{dc}}/I_{\mathrm{tot}}$ at resistance values of about 10^6 Ω. This shows that the sensitivity of the two methods for measuring small nonlinearities in Ohmic components is comparable when the latter is applied to high resistance values.

In the set-up shown in Fig. 3, the main measuring uncertainty has been in the determination of the third harmonic output signal. This uncertainty is about 6% for signals down to 0.2 μV. At lower signals the uncertainty is increasing due to noise.

5. BRIDGE FOR CAPACITIVE MEASUREMENTS

The bridge is designed for measurements of capacitances and the first and second derivatives of the capacitance with respect to voltage at dc biases up to 1000 V. In this way small kinks in an otherwise smooth characteristic can be detected independent of a possible drift in capacitance values, e.g., those caused by temperature effects. The capacitance range is up to 1100 pF with an uncertainty of 0.01 pF or 0.03%, whichever is the greatest. The measuring frequency normally used is 100 kc (for the output signals). The detailed measuring circuit is shown in Fig. 4.

In order to increase the sensitivity, a variable LC circuit has been shunted over the input terminals of the detector (L_1C_1 in Fig. 4). In the measurements, the values of L_1 and C_1 are varied to obtain maximum harmonic signals at the detector input. As can be seen from the expressions (1) and (2), this corresponds to minimizing

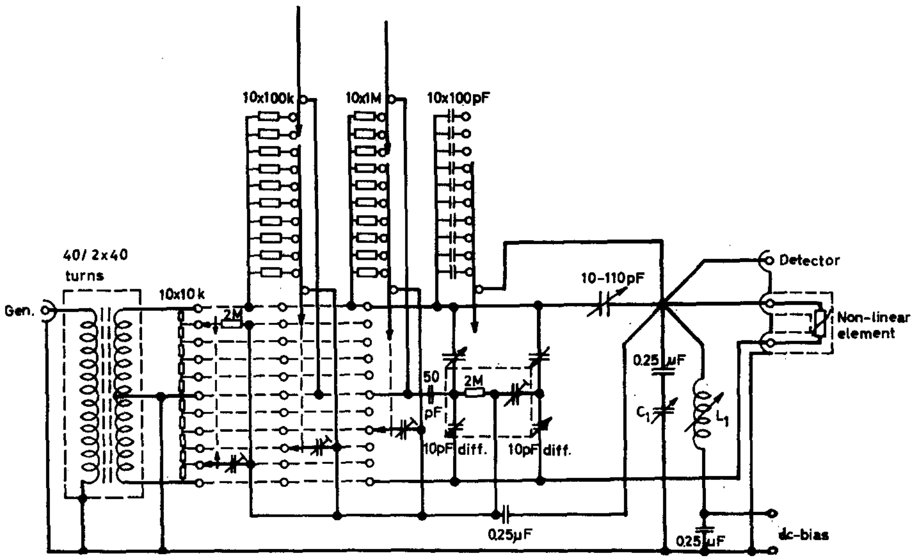

FIG. 4. Circuit diagram of bridge for measuring nonlinearity of capacitive components.

the numerical value of the admittance $(Y+Y_g)$. This admittance is thus tuned into resonance and we are left with a loss conductance G_b which is a characteristic of the bridge.

The intrinsic distortion of the bridge was investigated by using a linear capacitance C_1 in place of the nonlinear element. It was found that the quantities v_2/v_1^2 and v_3/v_1^3 were independent of the magnitude of v_1 and C_1. This intrinsic bridge distortion may be expressed in terms of an equivalent nonlinear capacitance as can be seen when the expressions (1) and (2) are applied to the capacitive case. Neglecting the v_1v_2 term in (2) and approximating $(Y+Y_g)$ by the characteristic loss conductance G_b, one obtains

$$dC/dV=1.4G_b\omega^{-1}(v_{2,\,\mathrm{rms}}/v_{1,\,\mathrm{rms}}^2) \qquad (12)$$

and

$$d^2C/dV^2=4G_b\omega^{-1}(v_{3,\,\mathrm{rms}}/v_{1,\,\mathrm{rms}}^3). \qquad (13)$$

The minimum values of the voltage ratios in (12) and (13) were found to be

$$(v_{2,\,\mathrm{rms}}^2/v_{1,\,\mathrm{rms}}^2)_{\min}=3\times10^{-7}\ \mathrm{V}^{-1},$$

$$(v_{3,\,\mathrm{rms}}/v_{1,\,\mathrm{rms}}^3)_{\min}=10^{-6}\ \mathrm{V}^{-2}.$$

When these values are inserted into (12) and (13) together with the measured value of $G_b\approx10^{-5}\ \Omega^{-1}$, one obtains the following sensitivity limits for the bridge with the frequency of the harmonics equal to 100 kc:

$$(dC/dV)_{\min}=1.5\times10^{-17}\ \mathrm{F/V}$$

and

$$(d^2C/dV^2)_{\min}=2\times10^{-16}\ \mathrm{F/V^2}.$$

The sensitivity of the capacitance bridge has been illustrated by measuring the capacitance and the first and second derivative with respect to voltage as a function of dc bias for a silicon high voltage diode (Philips BYX 11). The results are shown in Fig. 5. As a check, the first derivative dC/dV has been calculated from the capacitance vs voltage measurements. Comparison with the direct results obtained from the second harmonic measurements shows deviations of less than 10% in the region where $C>1$ pF.

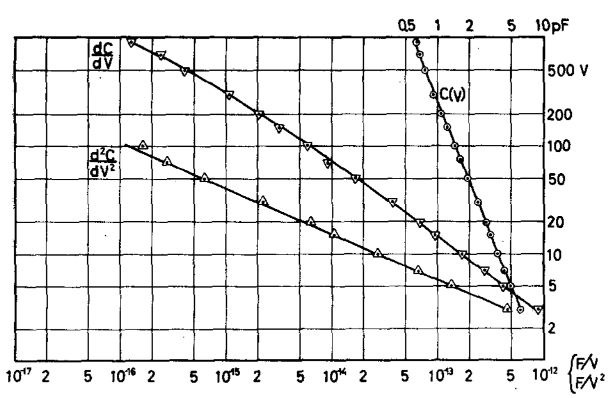

FIG. 5. Experimental results for C, dC/dV, and d^2C/dV^2 of a high voltage silicon diode as a function of bias voltage (reverse direction).