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Petersen, Christian Leth; Hansen, Ole

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The diagonal and off-diagonal AC conductivity of two-dimensional electron gases with contactless Corbino geometry in the quantum Hall regime

Christian Leth Petersen^{a)}

Mikroelektronik Centret, Technical University of Denmark, Denmark

Ole Per Hansen

The Niels Bohr Institute, Oersted Laboratory, University of Copenhagen, Denmark

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We have investigated the AC conductivity elements in the quantum Hall regime of two-dimensional electron gases coupled capacitively to electrodes with Corbino geometry. The samples are GaAlAs/GaAs single heterostructures, and the measurements are made at low frequencies, up to 20 kHz. The diagonal conductivity is derived from magnetocapacitance measurements. It increases with increasing frequency according to a power law at integer filling factors. The exponent of the power law depends on both temperature and filling factor. Ratios between Hall conductivities at different filling factors are obtained by inductive measurements of the circulating current. They are found to agree with quantization in multiples of e^2/h at the integer filling factors. © 1996 American Institute of Physics. [S0021-8979(96)06320-7]

I. INTRODUCTION

In this article we use a contactless method to investigate the quantum Hall effect of a two-dimensional electron gas (2DEG). Such contactless techniques have recently been applied elsewhere, using the torque magnetometer method¹ and the inductive coil method.²

It is well known that the source-drain current of a Corbino sample with a fixed magnetic field perpendicular to the 2DEG plane depends only on the diagonal part of the conductivity tensor. This symmetry effect has long been used to detect the diagonal conductivity σ_{xx} directly.

The low frequency diagonal AC conductivity can be obtained by examining the magnetocapacitance of Corbino samples with high-impedance (capacitive) contacts. The conversion from capacitance to conductivity was previously accomplished with either a discrete^{3,4} or a distributed^{5,6} impedance model. The distributed model as elaborated by Stern⁷ is one-dimensional, but was used for the Corbino geometry without analysis of the justification.^{5,6}

A two-dimensional distributed model for capacitive coupling to electrodes with Corbino geometry has been used for many years to analyze nondegenerate 2DEGs on cryogenic surfaces.^{8,9} In the present work we shall adopt this so-called Sommer-Tanner model,¹⁰ which, to our knowledge, has not previously been applied to degenerate semiconductor 2DEGs. There is a significant difference between the conductivity results obtained with the Stern and Sommer-Tanner models for certain sample geometries.

It is harder to obtain the Hall conductivity σ_{xy} directly. Usual Hall voltage measurements on Hall bars are not suited since they give the Hall resistivity, and the usual Corbino measurements only give the diagonal conductivity.

However σ_{xy} can be made to appear by modifying the standard transport measurements. For example, the source-drain current of a Corbino sample will contain a contribution

proportional to the Hall conductivity if the magnetic field changes with time (since $\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$). At integer filling factors where $\sigma_{xx} \approx 0$ this is the only contribution, and thus σ_{xy} can be obtained by measuring the transfer of charge between the electrodes.¹¹ This charge transfer effect relies on very strong quantization in order to avoid backflow of charge between the electrodes. Another approach is to short the probes on a Hall bar and measure the Hall current instead of the Hall voltage.¹²

We have detected the circulating current in our Corbino samples with an inductive coupling. This current is basically a product of σ_{xy} and the potential difference across the 2DEG. The potential difference depends on the voltage division between the impedances of the capacitive coupling and of the 2DEG, which in turn is determined by the diagonal conductivity σ_{xx} . In the quantum Hall regime the variations in σ_{xx} are much larger than in σ_{xy} and therefore the circulating current will reflect the diagonal conductivity the strongest. As we shall demonstrate, it is, however, still possible to derive the Hall conductivity at integer filling factors from the current.

II. EXPERIMENTAL SETUP

Our samples were square (7.5×7.5 mm²) pieces of GaAlAs/GaAs heterostructures glued on top of a set of Corbino electrodes with a low-temperature glue (General Electric No. 7031). This sample preparation technique was developed by Templeton.⁴ The sample holder with electrodes is shown in Fig. 1(a). The radius of the disk electrode is 2.1 mm; the inner and outer radii of the ring electrode are 3.1 and 3.75 mm, respectively. We have used two heterostructures with different characteristics: sample 1 with density of 6.1×10^{15} m⁻² and mobility of 11.2 T⁻¹ at 1.5 K, and sample 2 with density of 1.4×10^{15} m⁻² and mobility of 30 T⁻¹ at 1.5 K.

The magnetocapacitance of the Corbino samples was measured with a Thompson bridge^{13,14} as shown in Fig. 1(b),

^{a)}Electronic mail: clp@mic.dtu.dk

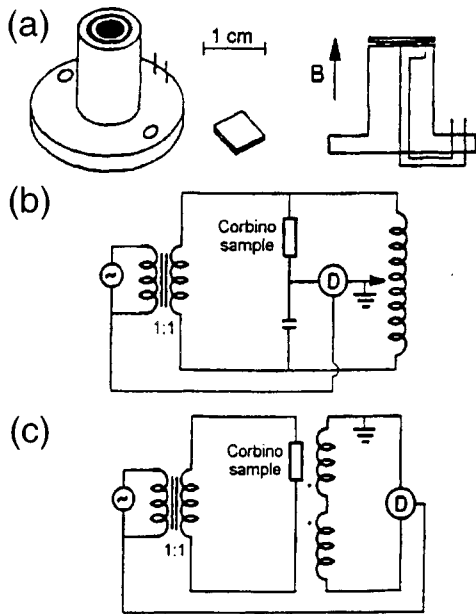


FIG. 1. (a) Corbino electrodes, (b) capacitance bridge, and (c) counterwound coils for inductive measurement of circulating current.

while for the circulating current we have used two counterwound coils placed around the sample, the Corbino being located in the center of one coil. This is illustrated in Fig. 1(c). The major contribution to the voltage across the coils comes from the inductive coupling to the circulating current, and the voltage is simply proportional to the current (apart from a 90° phase shift).

The measurements were performed by a two-phase lock-in detector with frequencies in the range of 2.5–20.0 kHz at liquid He temperatures (1.5 and 4.2 K) and magnetic field strengths up to 7.5 T. The phase setting of the lock-in detector was zero at all frequencies, and the voltage over the 1:1 transformer was 4.0 V root-mean-square.

III. DETECTOR VOLTAGES

Typical outputs from the circuits in Figs. 1(b) and 1(c) are shown in Fig. 2. The peaks in the voltages appear at integer filling factors, i.e. at plateaus in the Hall conductivity. The voltage from the capacitance bridge Fig. 2(a) resembles the data from previous magnetocapacitance measurements.^{4–6} No influence from $\partial \mathbf{B} / \partial t$ was observed.

Only one Shubnikov-de Haas period is observed in Fig. 2(a), and it coincides with previous DC Hall-bar measurements on the same heterostructure.¹⁴ This suggests that the electron density is not significantly perturbed by the electrodes. In Corbino samples with evaporated contacts, the stronger coupling could cause alternating depletion and enhancement of the density under the electrodes.³

From Fig. 2 it is clear that the behavior of the coil voltage is qualitatively identical with that of the bridge voltage apart from the 90° phase change introduced by the inductive coupling. This is due to the division of the Corbino electrode voltage between the capacitive impedance of coupling to the

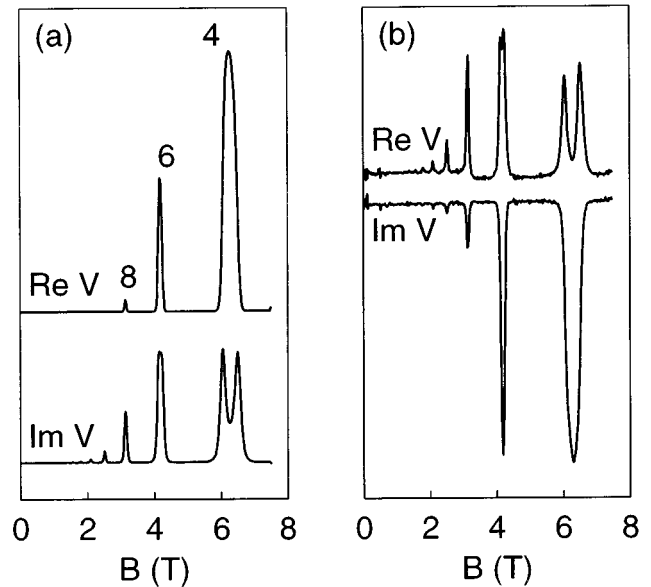


FIG. 2. Real and imaginary parts of detector voltages from (a) capacitance bridge and (b) counterwound coils as functions of the magnetic field obtained with sample 1 at $T = 1.5$ K, with a frequency of 10 kHz.

2DEG and the resistive impedance of the 2DEG. However, as we shall demonstrate, a quantitative analysis allows the extraction of σ_{xy} from the coil voltage.

IV. MAGNETOCAPACITANCE

The magnetocapacitance can be extracted directly from the bridge voltage, but in order to calculate the diagonal conductivity from the capacitance some sort of model is necessary. We have used the distributed impedance model of Sommer and Tanner, which is based on the idea that the 2DEG above the measuring electrodes is capacitively coupled to them with a capacitance per square, C_\square . If an electrode is kept at potential V_e and the potential in the 2DEG is denoted by V , this leads to the partial differential equation

$$\nabla^2 V = \frac{i\omega C_\square}{\sigma_{xx}} (V - V_e). \quad (1)$$

If we assume cylindrical symmetry, $\nabla^2 V$ is given by

$$\nabla^2 V = \frac{\partial^2 V(r)}{\partial r^2} + \frac{1}{r} \frac{\partial V(r)}{\partial r}, \quad (2)$$

and we arrive at a Bessel differential equation. (Here we may note that the first order derivative in Eq. (2) makes the difference as compared to the one-dimensional model of Stern.) In writing down the solutions corresponding to the three Corbino areas, disk, gap and ring, we omit the second kind Bessel term in the disk area, in order to ensure that the solutions remain finite at $r = 0$. This then results in three equations with five integration constants. Continuity requirements on $V(r)$ and its first derivative at $r = r_1$ and $r = r_2$ give four equations for the determination of the constants. As a fifth equation we use the requirement of charge conservation in the flow of current between the disk and the ring.

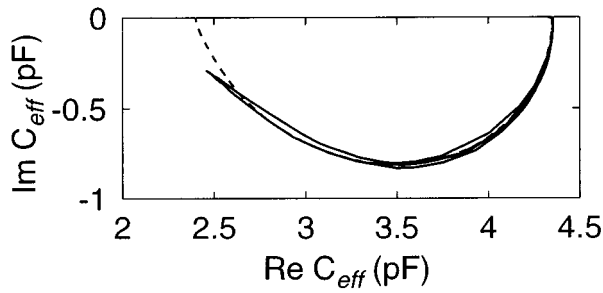


FIG. 3. Parametric plot showing the measured magnetocapacitance (solid line) and a fit with the Sommer-Tanner model (dashed line) obtained using sample 1 at $T=1.5$ K, with a frequency of 10 kHz.

From the potential distribution $V(r)$ the effective (magneto)capacitance can be calculated using the relation:

$$C_{\text{eff}} = \frac{I}{i\omega V_0} = \frac{2\pi C_{\square}}{V_0} \int_{\text{disk}} (V(r) - V_{\text{disk}}) r dr, \quad (3)$$

where I is the current through the Corbino, V_{disk} is the potential of the disk electrode, and V_0 is the potential difference between the disk and ring electrodes.

We have investigated the influence of the square geometry of the samples by additionally solving Eq. (1) numerically in rectangular coordinates. The resulting values of C_{eff} coincide with those obtained analytically assuming cylindrical symmetry.

Figure 3 shows a parametric plot of a typical magnetocapacitance result along with a fit with Eq. (3). As fitting parameter we have used the radius r_1 of the disk electrode. The fitted value is 2.6 mm, to be compared with the geometrical value of 2.1 mm. The fitted value does not have a direct physical significance, since all three radii are fitting parameters. We keep two of the effective radii fixed at the geometrical values in order to simplify the fitting procedure. The difference between the geometric and fitted radii is likely caused by a slight misalignment of the sample with respect to the electrodes, and by the fact that the dielectric constant in the spacing between sample and electrodes is not that of vacuum, as assumed in the model.

The peaks in the capacitance data are not revealed in Fig. 3. This is so because the only changing parameter in the capacitance is the diagonal conductivity, and therefore all peaks lie on the same curve. Away from the integer filling factors the diagonal conductivity is large, corresponding to an almost pure capacitive signal (around 4.35 pF in this example). At the integer filling factors the diagonal conductivity becomes significantly smaller, and the capacitance starts to move out along a curve which is determined by the geometry of the sample.

Previously, magnetocapacitance measurements on degenerate 2DEGs with Corbino geometry were analyzed^{5,6} with the one-dimensional Stern model, that was developed for use on slim rectangular metal-oxide-semiconductor field effect transistor (MOSFET) structures. The Stern model can successfully be applied to thin ring electrodes with large radii. For an electrode with inner radius r_1 and outer radius

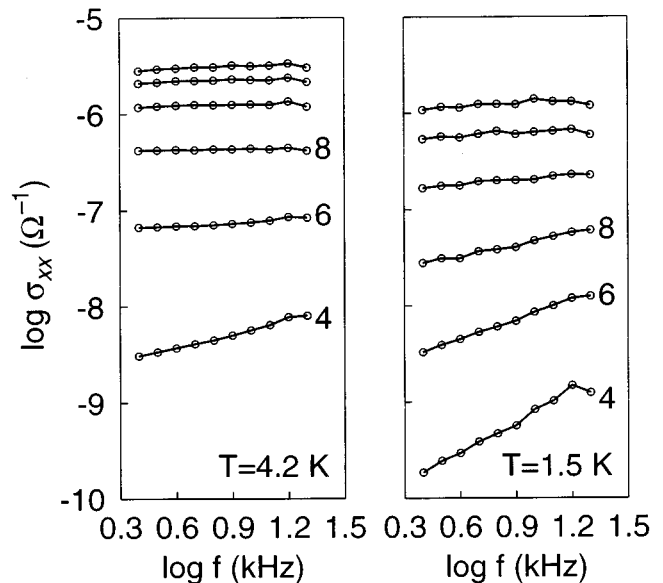


FIG. 4. The frequency dependence of the diagonal conductivity at integer filling factors. The data were obtained using sample 1.

r_2 the requirement, in order that $r^{-1}\partial V/\partial r$ can be neglected in Eq. (2), is $r_2/(r_2-r_1) \gg 1$. If the Stern model is used to derive the diagonal conductivity of a system with a cylindrical electrode that does not fulfill this requirement, the result becomes inaccurate. In particular for the ultimate limit $r_1=0$ (a point disk electrode) the error in the calculated conductivity is around 300%. With the radii used by Goodall et al.,⁵ we find the error to be only a few percent.

V. DIAGONAL CONDUCTIVITY

The diagonal conductivity can easily be derived from parametric plots like Fig. 3. This is so because the parameter of the experimental data (Fig. 2) is the magnetic field, whereas the Sommer-Tanner fit is parameterized by the diagonal conductivity. The procedure for obtaining the magnetocapacitance is as follows. Choose a magnetic field value B , and look at the corresponding experimental capacitance data point on the parametric plot (Fig. 3). Now determine the closest model capacitance data point in the plot. This data point corresponds to a value of the diagonal conductivity σ_{xx} . Thus $\sigma_{xx}(B)$ is obtained. Fig. 4 shows the result of plotting the extracted values of the diagonal conductivity at integer filling factors as a function of frequency. It is seen how the conductivity increases with increasing frequency according to a power law. This has been observed previously.^{3,6} The exponents of the power laws are found to increase as temperature falls, and are largest for the lowest filling factors. Thus, the stronger the quantization, the stronger frequency dependence observed. There seems to be a small departure from the power law dependence at the 20 kHz data points. However, it is not obvious whether this is a true effect or an instrument artifact. Further measurements at higher frequencies are needed to clarify this.

A possible objection to the present (and previous) results for $\sigma_{xx}(\omega)$ in the quantum Hall regime is that the measure-

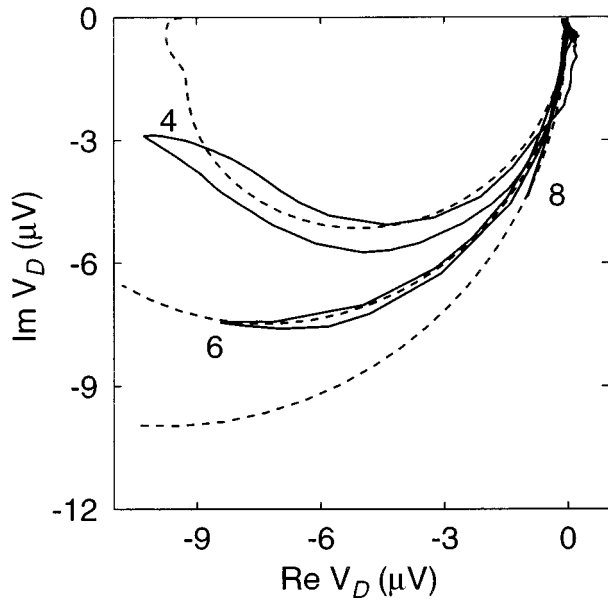


FIG. 5. Parametric plot showing the inductive coil voltage (solid line) and fits with the Sommer-Tanner model (dashed lines) from sample 1 at $T=1.5$ K, with a frequency of 7.9 kHz.

ments were all performed on electrons embedded in semiconducting material. At strong quantizations the conductivity of the 2DEGs is so small that currents in the semiconductor might influence the signal from the samples. Such a contribution would have a frequency dependence similar to the one observed. Therefore, part of the frequency dependence could originate from the semiconductor surroundings.

Theoretical work on the frequency dependence has used the concepts of percolation¹⁵ and localization.^{16,17} However, a comparison with the experiments is not straightforward.

VI. CIRCULATING CURRENT

The circulating current in the Corbino samples was detected by means of the counterwound coils. The voltage from the coils should be simply proportional to the current, apart from a phase turn of 90° . In reality there is an extra constant contribution to the signal that comes from wires in the vicinity of the coils. After having corrected for this contribution, and rotated the data to compensate for the inductive phase turn, the result is as shown in Fig. 5. This graph is directly proportional to the circulating current. The constant of proportionality depends on the strength of the inductive coupling.

The extra information present in the voltage from the coils as compared with the voltage from the capacitance bridge becomes apparent by comparing Fig. 3 to Fig. 5. All the peaks in the data depicted in the parametric plot of Fig. 3 lie on a single curve. In Fig. 5 on the other hand, each peak has an individual curvature. This is direct evidence of another changing parameter besides σ_{xx} . This extra parameter is the Hall conductivity. It can also be seen that, for a given peak in the coil voltage, the curvature remains approximately the same for the entire extent of the peak, i.e. a given peak moves back and forth along one single curve. This implies

that σ_{xy} is constant over the extent of the peak, and thus the plateau formation is also directly observable in the parametric plot of Fig. 5. It is possible to obtain quantitative results for the Hall conductivity at the smallest integer filling factors by using the Sommer-Tanner model.

VII. OFF-DIAGONAL CONDUCTIVITY

The Sommer-Tanner model can be used to analyze the circulating current in the Corbino samples.¹⁸ Generally it can be written

$$I_\theta = \int_0^{r_3} J_\theta(r) dr = \sigma_{xy} [V(0) - V(r_3)], \quad (4)$$

where J_θ is the circulating current density. Thus I_θ is the product of the Hall conductivity and the potential difference in the 2DEG between the center of the Corbino sample and the outer edge of the ring electrode. This potential difference depends only on the diagonal conductivity.

The circulating current corresponding to filling factor one ($\sigma_{xy} = e^2/h$) can be calculated with the Sommer-Tanner model using the parameters obtained from the fits to the capacitance data. This current can then be scaled to fit each of the observed peaks in the coil signal. The scaling factor will be different for each of the peaks, and since the current was calculated for filling factor one, the scaling factor should be given by

$$\kappa = \nu \chi, \quad (5)$$

where ν is the filling factor and χ the inductive coupling. In order to get the absolute values of ν (i.e. σ_{xy}), it is necessary to know the coupling constant χ . Unfortunately, χ is not known with sufficient accuracy in the present experiment. It is however possible to determine ratios between Hall conductivities at different integer filling factors without knowing χ . This is done simply by taking the ratios of the scaling

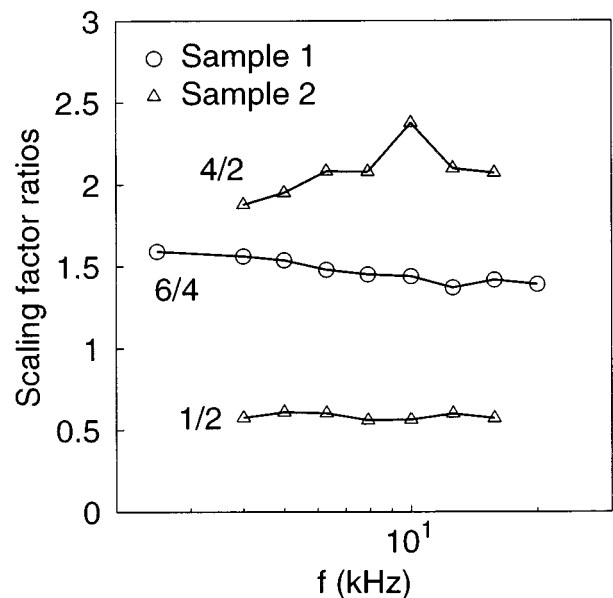


FIG. 6. Ratios between scaling factors at various integer filling factors. The temperature is 1.5 K.

factors. These ratios turn out to agree well with the expected filling factor fractions. This is shown in Fig. 6. The results are within $\pm 10\%$ of that expected. It is not possible on the basis of the present experiment to observe any frequency dependence of σ_{xy} .

The circulating current becomes very large at the lowest integer filling factors, typically around 0.5 mA. Assuming a uniform radial distribution, this corresponds to a current density of $J_{\theta} \approx 0.15$ A/m. The actual current density will be larger than this value, since the distribution is not uniform. For the particular Corbino geometry considered here, the current density is expected to have a maximum value of approximately 0.6 A/m at the rim of the inner electrode.¹⁸ The breakdown of the quantum Hall effect in a GaAlAs/GaAs Corbino sample with ohmic contacts has been reported to occur at a circulating current density of $J_{\theta,c} = 1.16$ A/m.¹⁹

VIII. CONCLUSION

We have used a two-dimensional distributed impedance model to analyze the capacitive coupling between a 2DEG and a pair of electrodes with Corbino geometry. From capacitance measurements we have determined the frequency dependence of the diagonal AC conductivity σ_{xx} at integer filling factors.

We have demonstrated that the circulating current in a Corbino sample can be detected by inductive coupling to a set of counterwound coils. Using the results for σ_{xx} it has been possible to observe the quantization of σ_{xy} , and especially the ratios of σ_{xy} 's at different Hall plateaus.

The method presented for the detection of the circulating current might also be used for purposes other than extracting σ_{xy} . One particular object of further investigation would be the influence of the magnitude of the circulating current at integer filling factors.

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