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Abstract

Real time decision support in railway operations is an area which has so far received limited attention. In this paper we address real time recovery of a rolling stock plan. Given a disturbed rolling stock plan the objective is to return quickly and inexpensively to the original rolling stock plan. Each train unit is hence rerouted through the train network so that each terminal departure is covered sufficiently wrt. seats relative to demand and so that the train unit paths are feasible with respect to connections.

We address the rolling stock recovery problem using a method based on decomposition where first the number and order of train units for each departure are determined. Given this knowledge we find the train path for each train unit. The experimental results show promising solution times and quality indicating applicability in practice.

1 Introduction

During the last years there has been an increased focus on developing tools to aid the planning process in railway transportation. The tools are computer software, which can fully or partially automate parts of the planning process. As in other industries the initial focus has been on strategic, tactical and operational planning. Only lately focus has turned to the area of short term and real time planning. This paper concentrates on the area of rolling stock real time planning. All models are based on the suburban railway network in Copenhagen, Denmark. The railway operator operating the network is DSB S-tog A/S.

The areas of operational, short-term and real time planning, can with respect to rolling stock be described as follow.

Operational The operational planning process is based on the tactical plan, which defines the number of train units and which type is assigned to each defined train task. A train task in this context is defined by a departure from a station and an arrival at another station.The stations most often are rolling stock depots. Rolling stock unit types are assigned to train tasks in such a way that later, train unit routes can be build for physical units that enables each train unit to visit the maintenance center within the predefined safety time and kilometer limit.

Also, in operational planning adjustments are made with respect to infrastructure maintenance works. This happens between the tactical and the operational rolling stock planning.

Short-term Short-term planning in the railway business concerns the routing of the physical train units 1-3 days in advance of operation. Also in this phase small adjustments to the number and type of train units assigned to each train task may be necessary.

Real time The major difference from operational to short-term planning of rolling stock is that for the latter information of the physical train ID's are included. This level of detail is maintained also in real time. Real time planning is conducted during the operation. Real time rolling stock planning is the re-planning or recovery of the plan for physical train units after disruption has occurred. This is also called rolling stock disruption management.

In practice rolling stock dispatchers monitor the operation of the rolling stock plan and the depot plans. In the cases where the operation does not run according to the rolling stock plan, the rolling stock dispatcher makes real time decisions on the re-assignments of train units to train tasks. Often suboptimal decisions are made due to the complexity of the task of manually establishing an integrated solution taking into consideration the recovery of several trains.

There is a substantial cost of re-allocating train units after a disruption in the rolling stock plan. The reallocation is necessary to meet end-of-day depot balance requirements and the maintenance requirements of each individual train unit. Furthermore, if too many train units are allocated to trains ending up at particular depot there may not be sufficient physical space in the depot to park all the train units.

Today, when a disruption has occurred the depot balances are often off implying that the rolling stock plan for the following day is also disrupted. Thus, either some train task must be covered insufficiently or not covered at all resulting in a cancellation of the task.

Next, in Section 2 a review of related literature is given. In Section 3, we give an introduction to the terms of rolling stock planning. Hereafter, in Section 4 we define terms concerning disruption. We introduce the Train position model in Section 5, in Section 6 the Train sequence model is presented and in Section 7 the Train unit routing model is presented. Finally, in Sections 8 and 9 we present the Computational results and give a conclusion.

2 Literature review

The research within the area of rolling stock schedule optimization has up to recently mainly focused on the planning phases prior to the day of operation. Only little emphasis has been on the area of real time rolling stock recovery, see Nielsen [10]. Huisman et al. [8] give a survey on state-of-the-art Operations Research methods for solving passenger railway related planning problems. The real time handling of rolling stock is briefly mentioned and reference is made to the problems of short time planning, which resembles the real-time situation. Short-term rolling stock planning is done on a day-to-day basis, also adjusting the rolling stock plans according to changes in the timetable due to e.g. rail network maintenance work, or adjusting according to passenger flows, which may have changed the need for rolling stock assigned to each train task.

Other recent surveys on rail operation models are given by Cordeau et al. [5], and Törnquist [14].

At S-tog, the depots are physically not very large, and only one workshop is available for maintenance checks. Already in the initial operational rolling stock plan, the paths for the train units lead them pass to the workshop at regular intervals in time and distance.

The problem of planning rolling stock can be divided into two subproblems: Firstly, finding the compositions for each train task in the network and secondly, finding the paths for each virtual train unit ensuring depot feasibility and regular maintenance checks. The compositions indicate the type, number and order of train units assigned to a train task. The paths ensure that all train units are routed to pass the workshop at regular intervals.

The first problem of determining compositions is widely explored. There is a distinction between the problems of allocating rolling stock when the fleet is composed by train units compared to when it is composed by train carriages and train locomotives. Papers concerning the locomotive scheduling problem are Cordeau et al. [4], Lingaya et al. [9] and Brucker et al. [3].

The first paper concerning the problem with self-propelled train units is Schrijver [13]. In this paper a minimum circulation of rolling stock on a single train line running from Amsterdam to Vlissingen and vice versa is determined. The objective is to ensure sufficient seats available for each train task. The model does not take the train unit order within a composition into account. The problem is solved with commercial software for respectively one and two train unit types.

In Ben-Khedher [2] the problem of capacity adjustment is discussed. It is based on the problem of finding railway capacity for high speed trains running in the TGV network of SNCF, France. The model is based on the seat reservation system and the objective is to maximize expected profit.

Alfieri et al. [1] address the problem of constructing circulations of train units. Focus is again on a single line. The model couples and decouples train units from trains as the depots are passed. The order within each composition is taken into consideration. The model is tested for two train types. The solution approach is based on a hierarchical decomposition into sub problems. First, the model, not taking compositions into consideration, is solved. Second, it is checked whether there is a feasible solution for the composition problem.

Peeters and Kroon [11] present a branch-and-price algorithm for solving the allocation of train units to a single line or a set of interacting train lines. The model is tested on several real-life instances of the railway operator, NS Reizigers. Objectives considered are those of minimizing train unit km shortage, minimizing number shunting operations and number of driven train unit km. The model is based on a transition graph as is the model described in Alfieri et al. [1]. The authors apply a Dantzig-Wolfe decomposition, reformulating so that a variable is associated with each path through the transition graph of all trains.

In Fioole et al. [6] a model for finding the compositions of train units on train tasks is presented. Each solution is feasible with respect to composition order in depots and with respect to depot capacities. The model additionally takes into consideration combining and splitting of trains in depot junctions. It is an extension of the model described in Peeters and Kroon [11]. The objective considers minimizing with respect to efficiency, service and robustness. The model is implemented and solved in the commercial integer programming solver CPLEX. This procedure improved the solution used in practice with up to 6 % with respect to number of driven train unit kilometers.

Given that the composition problem is solved at short term or real time level the problem of finding paths resembles the problem of finding work plans (lines of work) for crew. The train tasks form a time and space restricted path. Extensive research within the area of crew planning has been carried out. Within the area of rail we refer to the survey of Huisman et al. [8].

In Nielsen [10] a generic framework for modelling the real time rolling stock re-scheduling problem is described. This is the problem of re-balancing the use of rolling stock on train tasks in real time. Rolling stock is considered at train type level. The modelling is based on the composition model presented in Fioole et al. [6] and expanded to consider the end-of-day balances of rolling stock. The model have the objectives of minimizing number of cancelled trips, changes to the rolling stock depot plans and the end-of-day off balances. The model is solved using CPLEX 10.1. Computation times varies from few seconds up to a minute depending on the problem instances solved. All computational results are based on data from the Dutch railway operator NS Reizigers.

A recent paper, Rezanova and Ryan [12], on the Train Driver Recovery Problem approaches the problem of recovering a train driver plan in real time given that some disturbances have disrupted the plan. The problem is solved using a set partitioning formulation. Fractional solutions for the LP relaxation of the IP problem is solved used constraint branching, however, most solutions are integer due to strong integer properties of the model. Solutions are found within few seconds.

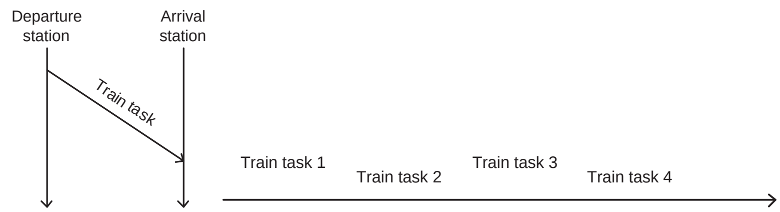
Another interesting paper on railway recovery is Walker et al. [15]. In this paper a model is described for simultaneous recovery of the train timetable and the corresponding crew plan. Promising results are presented for a single line of a New Zealand operator.

The current paper addresses the area of real time rolling stock recovery. No prior research is available on this subject. We introduce a decomposition method for the problem which provides good quality solutions quickly.

3 Basic elements of a rolling stock plan

Train operation runs according to a timetable consisting of terminal departures with predefined stopping patterns. Terminal departures are assembled in *Trains*. Each train is represented by a set of *Train tasks* forming a *Train sequence*, see figures 1(a) and 1(b). The train tasks of a train sequence form a predefined work plan for the train in which each train task, except for the first and the last, have a known predecessor and successor. This means that for two subsequent tasks t_1 and t_2, $ArrivalTime(t_1) < DepartureTime(t_2)$ and $ArrivalDepot(t_1) = DepartureDepot(t_2)$, see figure 2. In the models presented later in this paper we exploit the predecessor/successor relation between the train tasks.

Both rolling stock and crew operate according to plans which are detailed to a daily level i.e. for each train task it is known which specific driver and which specific train units will cover the train task. The rolling stock and crew plans are assumed optimal for the situation without disturbances. Therefore, given a disturbance to either of the plans, we seek to return to



(a) A train task (b) A train sequence

Figure 1: Illustrating train terms

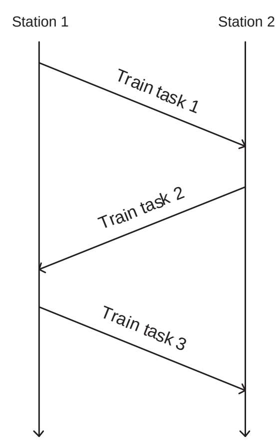


Figure 2: Illustration of a train sequence in a time-space network

the original plan as soon as possible. Returning to the original plan means that each train unit returns to its originally planned path, which eventually will route the train unit to the maintenance center.

A set of trains with the same stopping pattern and a uniform frequency between trains form a *Train line*. The train line concept is first of all used externally for representing the timetable to the customers, but it is also used internally for planning and prioritizing.

A rolling stock schedule consists of a set of *Train unit routes* where each route refer to a specific train unit and covers a path of train tasks. These train tasks may or may not belong to the same train sequence.

When a train unit leaves or is added to a train sequence it is said to be decoupled from or coupled to the train task. The set of train units assigned to a train is called a *composition*. As mentioned earlier, the composition defines the number of each type of train units and the order in which they are coupled. At S-tog there are two different train unit types. These can be coupled in all possible combinations limited by a maximum length of the train.

At S-tog coupling/decoupling always occurs at only one end of the train depending on the depot at which the coupling/decoupling occurs i.e. the train is only open for coupling/decoupling in one end. The route of a train unit must be feasible with respect to the open end of the train. That is, if a train unit is to be decoupled from a train, it must be in the open end of the composition. When coupling a train unit to a train, the train unit must also be assigned to the open end of the train. The open versus the closed end of a composition at a terminal is illustrated in Figure 3.

4 Defining a disruption

Incidents occur in real time that disturb the planned operation. Some of these incidents are of such a size that also the rolling stock plan is disturbed. For a more detailed description of the effect disruptions have on the S-tog timetable see Hofman et al. [7].

To minimize the impact of an incident, network controllers employed by

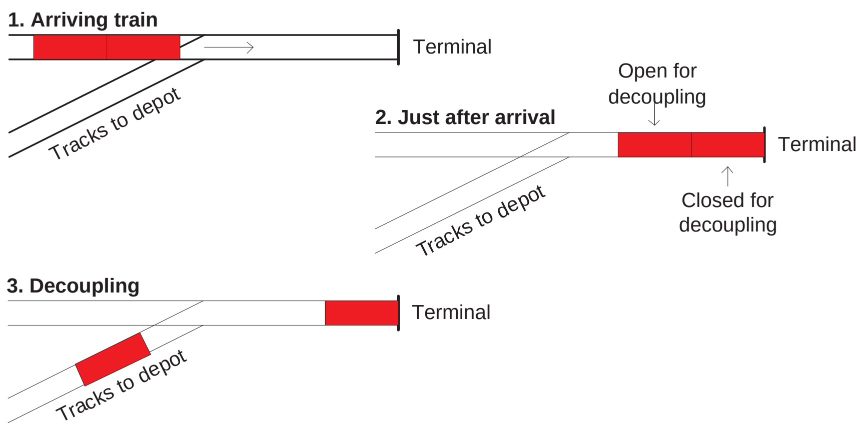


Figure 3: Illustration of the open and closed end of a composition at the terminal station

the infrastructure owner reroute trains to get operation back to normal as quickly as possible.

The delays disturbing the timetable may, as mentioned, be of a size that also disrupts the rolling stock and crew schedules.

A rolling stock schedule is disrupted when train units are not able to cover the train tasks they were expected to cover. The rolling stock schedule is affected by the delays both directly and indirectly. An example of a directly disrupting effect is the break down of a train unit thereby causing the train unit not to be able to cover its scheduled train task. Indirectly, the rolling stock schedule is affected by the actions of the train route dispatchers trying to return the departures to normal.

There are several potential negative consequences of a disruption in the rolling stock schedule. A rolling stock disruption may imply an imbalance in the rolling stock available at the rolling stock depots. This again may lead to train tasks being insufficiently covered according to their expected passenger demand. Another secondary disruptive effect can be that the reallocation of train units to train tasks other than the originally scheduled ones may lead to broken maintenance constraints for individual train units.

The set of train units being assigned extraordinarily to cover another train sequence are not necessarily of the same type and number as the set of train units originally intended for that train sequence. Hence, future couplings/decouplings on the train sequence and other trains running on the same route may also be affected.

4.1 Objectives when minimizing rolling stock disruption

The rolling stock dispatcher does not have the time to take into account several objectives when minimizing the extent of a disruption to the rolling stock plan. He tries to minimize the number of departures not covered and chooses the first feasible solution he discovers in the manual solution process.

Several objectives are interesting to include in a rolling stock recovery model. Fioole et al. [6] mention seat shortage, efficiency and robustness as relevant for the operational planning phase. These are also relevant in real time.

Seat shortage refers to the difference between the number of seats on the train units allocated to a train task and the expected seat demand of the train task. Maximizing the efficiency means that we do not want to operate a train task with more train units assigned than necessary, either considering the number of excess seats or the number of train unit kilometers driven. The two objectives of seat shortage and efficiency can be conflicting and will hence have to be weighted. Robustness in a rolling stock recovery plan is translated directly to the number of couplings and decouplings planned in a recovery plan. A recovery plan with many couplings and decouplings is less robust than one that has fewer. We wish to maximize robustness in a plan given that we still weigh the objectives of seat shortage and efficiency against each other. Robustness is therefore also assigned a weight in the final objective function.

Seat shortage, efficiency and robustness are all objectives concerning the assignment of train unit types to train tasks. Other objectives concern the physical train units. In real time the aim is to recover to the original rolling stock plan. However, it may not be possible within the time window of recovery or even within the same day of operation to route the train units back to their original work plans. Hence, an objective to include in the objective is the difference in end depot balance between the original and the recovered plan.

Type	Length	No Seats
SE	46	150
SA	86	336

Table 1: rolling stock types

4.2 Basic concepts in a disruption

It is likely that several delays call for recovery occur during the day. In the real-time situation time is a critical factor and recovery decisions must be made fast. For each recovery scenario we therefore solve within a specified time window e.g. two hours and include a limited set of train units. The start and end of the time window is the considered start and end time of the disruption.

Typically, all train units, $k \in K$, assigned to the train lines of the affected train units are included in the recovery scenario plus possibly some of the other lines running on the same route and sharing the same depots. Also, all train units located on the affected depots at the start time of disruption will be included in the set of train units to be replanned for.

Each train unit has a kilometer limit, $KmLimit_k$. It indicates the maximum number of kilometers that the distances of the tasks assigned to the train unit during recovery must sum up to. Each train unit has a seat capacity matching its train type. For each train unit the start depot, $\delta_\alpha(k)$, and a preferred end depot, $\delta_\omega(k)$, are given.

At all times two rolling stock types, $m \in M$, are available. These are short and long train units named SE and SA respectively. Sizes of the two rolling stock types are listed in Table 1.

The train tasks, $t \in T$, considered are those left uncovered, those which are insufficiently covered w.r.t. demand and those for which the assigned train units have been included in the recovery scenario.

For each train task, t, the start and end time, $\tau_d(t)$ and $\tau_a(t)$, and start and end depot, $\delta_d(t)$ and $\delta_a(t)$, are known. Each t is associated with a length in kilometers, Km_t, and a duration measured in seconds, $Time_t$. The set of tasks having no predecessors constitutes T_0. The train tasks having no successors constitute T_1. The successor of the train task t is denoted $\nu(t)$. Each train task has a seat demand, $Demand_t$.

The set of depots involved in the recovery scenario, D, is defined by the routes of the train lines included. For each depot, $d \in D$, included in the recovery scenario the start capacity of each type of train unit m is given by $DepotCap_{d,m}$.

Composition	Order	NO seats	Length
0	SE	150	46
1	SE - SE	300	92
2	SA	336	86
3	SE - SA	486	132
4	SA - SE	486	132
5	SA - SA	672	172

Table 2: Compositions

The maximum length of the composition assigned to a train is equivalent to the length of two SA train units. Given this maximum length, in fact a composition consisting of three SE train units or a composition consisting of two SE and one SA train unit are applicable in practice. Even though these train composition consisting of three train units are feasible, we omit them from our model. Seen from a modelling perspective our model is significantly reduced in size when reducing the number of allowed train units from three to two. Seen from a practical perspective, only few train tasks at S-tog will normally be assigned three train units. More specifically, at the tactical planning level no train tasks will be assigned more than two units. In a recovery situation three units on a train task occurs not even on a daily basis.

In the first model we will not permit train exchanges. That is, decoupling of all train units after a train task and coupling of an entirely new set of train units to the train task successor is not possible.

It is an important fact that all depots in the S-tog network are open for coupling/decoupling in only one end of the platform tracks. This enables us to use the position in a train and information of which end of the tracks is open for coupling/decoupling to decide whether a composition change is valid.

5 The train unit position model

In this section we introduce the variables, objective, and constraints of the Train Unit Position model (the Position model). The main variables of the model describe the assignment of train type to train task and position.

$$X_{tp}^{m} = \begin{cases} 1 & \text{If a train unit of type } m \text{ is assigned to task } t \text{ in position } p \\ 0 & \text{Otherwise} \end{cases}$$

From these X-variables the L-variables are derived. The L_t^m variables are inventory variables indicating the number of train units of type m present at the departure depot of t immediately before the departure of t.

Finally, O_t^m and N_t^m are variables indicating whether respectively coupling and decoupling is carried out between the tasks t and $\nu(t)$. Both sets of variables are binary.

L^0 are the start inventory parameters. L_{dm}^0 indicates the number of train units of type m located in depot d at the beginning of the disruption. L_{dm}^1 are the end capacity variables indicating the number of train units of type m present at depot d in the end of the considered recovery period. A desired end depot capacity is given by the parameter $E[cap]_d^m$. The variables E_d^m indicate the shortage of train units of type m in depot d in the end of the recovery period.

L_t^m are calculated from L_{dm}^0 and X_{tp}^m. As both are integers, the L-variables will automatically be integer. Therefore, we only require that $L_t^m \in \Re_+, \quad \forall t \in T, m \in M$.

The relevant aspects we include in the objective of the positioning model are seat shortage, number of composition changes, the cost of covering train tasks with train units and the sum of differences to the originally scheduled capacity on the depots, see Eq. 1.

$$
\begin{aligned}
&Minimize \quad OBJ = \\
&W_1 \cdot \textstyle\sum_{t\in T}(Demand_t - \sum_{m\in M,p\in P} Seats^m \cdot X^m_{tp})+ \\
&W_2 \cdot \textstyle\sum_{t\in T,p\in P,m\in M} Km_t \cdot X^m_{tp}+ \\
&W_3 \cdot \textstyle\sum_{t\in T}(\sum_{m\in M,p\in P} Seats^m - Demand_t \cdot X^m_{tp})+ \\
&W_4 \cdot \textstyle\sum_{t\in T,m\in M} O^m_t + W_5 \cdot \sum_{d\in D,m\in M} E^m_d
\end{aligned} \tag{1}
$$

As a train has a maximum length each train task cannot be covered by more than the maximum number of train units per train. This is guaranteed by Eq. 2.

$$
1 \leq \textstyle\sum_{m\in M,p\in P} X^m_{tp} \leq MaxTrainLength, \qquad \forall \qquad t \in T \tag{2}
$$

Physically at most one train unit can be assigned to each position of a train task. Eq. 3 ensures this.

$$
\textstyle\sum_{m\in M} X^m_{tp} \leq 1, \qquad \forall \quad t \in T, p \in P \tag{3}
$$

We control the incoming and outgoing flow of depots by three sets of inventory constraints, see Eq. 4 to 6.

The first set of constraints controls that the initial inventory level is not violated. This means that for each depot d the tasks departing before the first arriving task can not use more capacity than what is present initially given by L^0_{dm}. The set of departing tasks before the first arrival task on depot d is denoted ϕ_d for all $d \in D$. See Eq. 4.

$$
\textstyle\sum_{p\in P,t\in\phi_d} X^m_{tp} \leq L^0_{dm}, \qquad \forall \qquad d \in D, m \in M \tag{4}
$$

The inventory in a depot of train unit type m immediately after the arrival of a train task t is given by the start capacity on the depot minus the sum of every train unit of type m coupled to train tasks at that depot before and including t and plus the sum of every train unit decoupled from train tasks at that depot before and including t. This is handled by Eq. 5.

$$L_t^m = L^0_{\delta_a(t)m} - \sum_{\substack{p\in P, t'\in T\\ \tau_d(t')\le\tau_a(t)\\ \delta_d(t')=\delta_a(t)}} X^m_{t'p} + \sum_{\substack{p\in P, t'\in T\\ \tau_a(t')\le\tau_a(t)\\ \delta_a(t')=\delta_a(t)}} X^m_{t'p}, \qquad \forall \qquad t\in T, m\in M \tag{5}$$

The last set of inventory constraints concerns the end capacity. The end capacity, L^1_{dm}, of train unit type m in depot d is given by L_t^m for which t is the last train task arriving on d, θ_d. See Eq. 6.

$$L^1_{dm} = L^m_{\theta_d}, \qquad \forall \qquad d\in D, m\in M \tag{6}$$

We wish to control the end depot balance by minimizing in the objective function the shortage of train units defined by variables E_d^m. These are defined in Eq. 7

$$E_d^m \ge E[cap]_d^m - L^1_{dm}, \qquad \forall \quad d\in D, m\in M \tag{7}$$

Each depot has an individual upper capacity on the number of units that can be stored at that depot. The upper capacity is estimated by controlling the length of the rolling stock stored at each depot relative to the length of the depot tracks, $DepotCap_d$. Eq. 8 controls the capacity of each depot right after the departure of each task, that is, $\delta_d(t)$ is the departing depot of t.

$$0 \le \sum_m Length_m \cdot L_t^m \le DepotCap_{d(t)}, \qquad \forall \qquad t\in T \tag{8}$$

The coupling and decoupling variables are determined in Eq. 9 and 10. We use a constant M to find the O_t^m and N_t^m variables. This is potentially very expensive considering computation time when M has a high value, however, M can be limited to the maximum train length plus one and as the maximum train length is 2 units M has a low value.

$$M\cdot O_t^m \ge L^m_{\nu(t)} - L_t^m, \qquad \forall \qquad t\in T\backslash T^1, m\in M \tag{9}$$

$$M\cdot N_t^m \ge L_t^m - L^m_{\nu(t)}, \qquad \forall \qquad t\in T\backslash T^1, m\in M \tag{10}$$

To ensure that no train unit is decoupled from a train if it is positioned in the closed end of the train composition, we one of the set of equations in Eq. 11 depending on the value of the 0-1 parameter $ChangePosition_t$. This parameter indicate whether open position is changed from one end of the train to the other after train task t.

$$ChangePosition_t = \begin{cases} 1 & \text{If closed position of task } t \text{ is different from} \\ & \text{closed position of successor } \nu(t) \\ 0 & \text{Otherwise} \end{cases}$$

$$\begin{array}{ll} \text{If } ChangePosition_t = 1 & \\ & X_{tp}^{m} \leq \sum_{p' \in P, p' != p} X_{\nu(t)p'}^{m} + W_{\nu(t)} \\ & X_{tp}^{m} \leq \sum_{p' \in P, p' = p} X_{\nu(t)p'}^{m} + V_{\nu(t)} \\ \text{else} & \\ & X_{tp}^{m} \leq X_{\nu(t)p}^{m} \end{array} \tag{11}$$

V_t and W_t are length indicator variables. V_t is one if one train unit is assigned to task t and zero otherwise for all $t \in T$. W_t is one if two train units are assigned to task t and zero otherwise for all $t \in T$. They are determined through Eq. 12 and 13.

$$W_t \leq \sum_{p \in P, m \in M} X_{tp}^{m} - 1 \leq 2 \cdot W_t, \qquad \forall \qquad t \in T \tag{12}$$

$$V_t + W_t = 1, \qquad \forall \qquad t \in T \tag{13}$$

The results achieved when solving the TUP model are comparable to the results that are achieved when solving the model described in Fioole et al. [6]. The difference between the two models lie in the handling of the compositions. In the Fioole model the compositions are handle as set of train unit types i.e. a composition is assigned to each train task and binary variables describes specifically the transformation from composition to composition on consecutive train tasks on the same train sequence. In our model we handle the positions in the train's composition specifically. The Position model is a feasible choice due to that the maximum length of compositions on train tasks is limited to 2 train units.

5.1 Size of model

Given a time window of the disruption of two hours and including all train lines intersecting the most dense part of the S-tog network, a total of 550 train tasks result. We have restricted the problem to only consider compositions up to two unit as opposed to the real restriction of three units. The Train unit position model has therefore approx. 6500 variables and approx. 6500 constraints. An expanded model for the problem considering compositions of up to three train units (the S-tog maximum length of composition) will have approx. 8000 variables and approx. 8500 constraints. This is an estimate as Equation 3 must be changed according to the new maximum composition length.

In comparison the Fioole model in comparison has approx. 27000 variables and 12000 constraints when considering compositions up to two train units and approx. 41000 variables and 16000 constraints when considering compositions up to three train units.

5.2 Solution approach for Train unit position model

The model is implemented in C# using Concert Technology from ILOG and solved using Cplex 10.0. Given the size of the problem we expect solutions to be achieved within acceptable computation times.

6 Train sequence model

When a train unit's path consists of one train sequence it is certain that the train unit is not decoupled or coupled at any time. That is, coupling and decoupling refer to train unit flows to and from the train sequence. Both are time demanding and in a periodic timetable there will not necessarily be sufficient time for performing these. It is assumed that if the number of couplings/decouplings are decreased the robustness of the rolling stock plan is increased. That is, the rolling stock plan will be less sensitive towards minor interferences in the operation.

This section describes the Train sequence model (Sequence model). The model is an assignment model, which if possible assigns a single, physical train unit to each train sequence in the disruption scenario, such that the train unit can feasibly cover the entire train sequence. In this way the model mirrors qualities that are part of solutions known to work well in practice.

The consequence of only covering the set of train sequences with one train unit each (if in fact a train unit exists that can make a feasible cover) is that for a set of train tasks the demand will not be fully covered. For some of the train tasks one train unit will be assigned but the demand exceeds the seat capacity of that train unit. For some train tasks no train unit will be assigned and the demand not covered at all. The set of train tasks not covered sufficiently will be addressed in a third model.

There is a preference of which train unit type to assign in the process of assigning train units to train sequences. The preferred train unit type is chosen given the results from the Train unit position model. Recall that this model gives information regarding number and type of train unit types assigned to each train task. For each train sequence the train unit type chosen as the preferable coverage is the type being present on each composition of the train tasks of the train sequence.

The Train sequence model has one set of variables, ϕ_s^k, which assign physical train units to train sequences.

$$\phi_s^k = \begin{cases} 1 & \text{If train unit } k \text{ is assigned to train sequence } s \\ 0 & \text{Otherwise} \end{cases}$$

The objective function of the Train sequence model is to maximize the sum of preferences of train units, k, assigned to train sequences, s, see Eq. 14. As many train sequences are assigned a train unit as possible provided that a train unit exists for the train sequence that contributes to a feasible solution. The preference of assigning train unit k to train sequence s is c_s^k. It takes the value of 1 if train unit k is a possible match for sequence s and -1 if it is not.

$$Maximize \quad \textstyle\sum_{s \in S, k \in K} c_s^k \cdot \phi_s^k \tag{14}$$

Each train can be covered by at most one train unit. The train unit, k, must have the same start and end depot, $\delta_\alpha(k)$ and $\delta_\omega(k)$, as the train sequence.

Start and end depot of the train sequence s are denoted $\delta_\alpha(s)$ and $\delta_\omega(s)$. This is ensured by Eq. 15.

$$\sum_{\substack{k \in K, \\ \delta_\alpha(s)=\delta_\alpha(k) \\ \delta_\omega(s)=\delta_\omega(k)}} \phi_s^k \leq 1, \qquad \forall \quad s \in S \tag{15}$$

For the train unit covering a train sequence maintenance requirements must be respected. This is easily included in the Sequence model, see Eq. 16. $EndRun_s$ is a parameter indicating the number of kilometers which are left after recovery until the depot is reached. This can be derived from the original rolling stock plan. $KmBefore_k$ is a parameter indicating the number of kilometers that the unit has driven before the start of the recovery plan.

$$\sum_{s \in S} (Km_s + EndRun_s) \cdot \phi_s^k \leq KmLimit_k - KmBefore^k, \quad \forall \quad k \in K \tag{16}$$

The 550 train tasks mentioned in the dimensioning of the Train unit position model groups into less than 70 train sequences. Available for covering the problem are at most 130 train units. This results in approximately 9000 variables and less than 350 constraints.

Again the model is not of considerable size and we solve it using Cplex 10.0 and Concert Technology where the model is implemented in C#.

7 The train Routing model

As mentioned in the previous section 6 the Train sequence model will only cover some of the train tasks according to their respective demands. Some will either be left uncovered or covered insufficiently according to demand. These must be covered by valid train task paths using the train units not yet assigned to a train path. This is done by the Train Routing Model, which is an assignment model considering each train task individually.

The main variables of the Train Routing model are, q_t^k. These variables assign train units to train tasks.

$$q_t^k = \begin{cases} 1 & \text{If train unit } k \text{ is assigned to train task } t \\ 0 & \text{Otherwise} \end{cases}$$

To control the solutions of the model a second set of variables is introduced, ρ_t^k. The ρ_t^k variables are used to control the number of couplings/decouplings in the solution.

$$\rho_t^k = \begin{cases} 1 & \text{If train unit } k \text{ is assigned to train task } t \text{ and to the successor of } t,\ \nu(t) \\ 0 & \text{Otherwise} \end{cases}$$

A set of artificial tasks are added to the problem representing the sources, T_{so}, and sinks, T_{si}, of train tasks. There are $|K|$ sources and $|D| \times |K|$ sinks. The set of train tasks are in the set T_{tasks}. The joint set of tasks is $T = T_{tasks} \bigcup T_{so} \bigcup T_{si}$.

The objective function maximizes the total sum of covered demand and the sum of couples of consecutive tasks covered by the same train unit. The use of physical train units also included in the objective by the sum of sources and sinks. All terms are weighted using weights, W_1 to W_4. See Eq. 17.

$$\begin{aligned} Maximize \quad & W_1 \cdot \sum_{t \in T_{tasks}, k \in K} q_t^k + W_2 \cdot \sum_{t \in T_{so}, k \in K} q_t^k + \\ & W_3 \cdot \sum_{t \in T_{si}, k \in K} q_t^k + W_4 \cdot \sum_{t \in T_{tasks}, k \in K} \rho_t^k \end{aligned} \tag{17}$$

Each train task must be covered at most corresponding to the number of each train unit type assigned to the task in the Position model, see Eq. 18 and 19. The parameter $cars_m$ represent the number of cars on train unit type m. constraining the number of cars and the number of train units on a train task to be the same in the Routing model as in the Position model, we are ensured that the right train composition is assigned to the train task.

$$\sum_{k \in K, type_k = m} cars_{m(k)} \cdot q_t^k \leq \sum_{p \in P, m \in M} cars_m \cdot X_t^{m,p}, \qquad \forall \quad t \in T_{tasks} \tag{18}$$

$$\sum_{k \in K, type_k = m} q_t^k \leq \sum_{p \in P, m \in M} \cdot X_t^{m,p}, \qquad \forall \quad t \in T_{tasks} \tag{19}$$

The ρ_t^k variables are defined in Eq. 20.

$$2 \cdot \rho_t^k \leq q_t^k + q_{\nu(t)}^k, \qquad \forall \quad k \in K, t \in T_{tasks} \setminus T^1 \tag{20}$$

The train tasks assigned to a train unit must form a valid train route i.e. a path through the network, which is feasible with respect to time and place of each adjacent pair of train tasks on the route. Also, the train route for each individual train unit must be valid with respect to any required start and end depots of the train unit. We add a set of virtual nodes to the network, one set representing the source nodes, N_{so}, of each individual train unit and one set representing the sink nodes, N_{si}, of each individual train unit. For each train unit there is a sink node for each depot i.e. there are $|Depots| \cdot |Trainunits|$ sinks in total.

The constraints ensuring valid paths are in Eq. 21 to 25. Eq. 21 ensures that if the source of a train unit is not covered, the train unit is not covering any of the train tasks. Eq. 22 ensures that if the source is covered for a train unit, then so is exactly one of the sinks of the train unit. Eq. 23 and 24 are equivalent to the flow constraints of a multi commodity flow model. They ensure that if train unit k is covering train task t then at least one of the predecessors, $pred(t)$, respectively successors, $succ(t)$ are covered. Finally, Eq. 25 ensure that if train unit k is assigned to t then it can cover none of the train tasks parallel in time to t. Time parallelism is illustrated in Fig. 4. The four tasks t_1 to t_4 are all time parallel to t because they intersect the time interval between departure time and arrival time of t. The parameter n in Eq. 25 indicates the maximum number of train tasks present within the time interval of t on any other sequence in the relevant problem instance. See Fig. 5.

$$q_t^k \leq q_{t'}^k, \qquad \forall \quad k \in K, t' \in T_{source(k)}, t \in T_{tasks} \setminus T_{source(k)} \tag{21}$$

$$\textstyle\sum_{t \in T_{sinks(k)}} q_t^k - q_{t'}^k = 0, \qquad \forall \quad k \in K, t' \in T_{source(k)} \tag{22}$$

$$\textstyle\sum_{t' \in T_{pred(t)}} q_{t'}^k \geq q_t^k, \qquad \forall \quad k \in K, t \in T_{tasks} \tag{23}$$

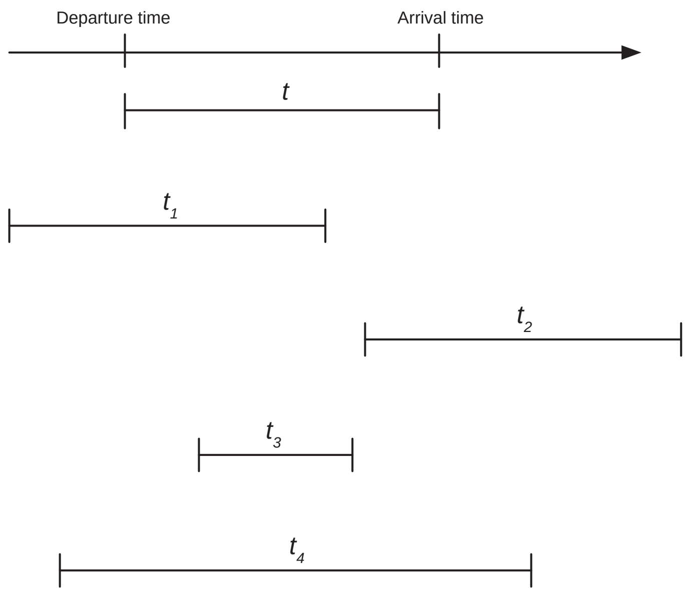


Figure 4: Illustrating time parallelism: $t_1, ..., t_4$ are all time parallel with t

$$\sum_{t' \in T_{succ(t)}} q_{t'}^k \geq q_t^k, \qquad \forall \quad k \in K, t \in T_{tasks} \tag{24}$$

$$\sum_{t' \in T_{parallel(t)}} q_{t'}^k \leq n - (n-1) \cdot q_t^k, \qquad \forall \quad k \in K, t \in T_{tasks} \tag{25}$$

Note that the Train position model and the Train ID model can function without the Train Sequence model. The Train sequence gives us two advan-

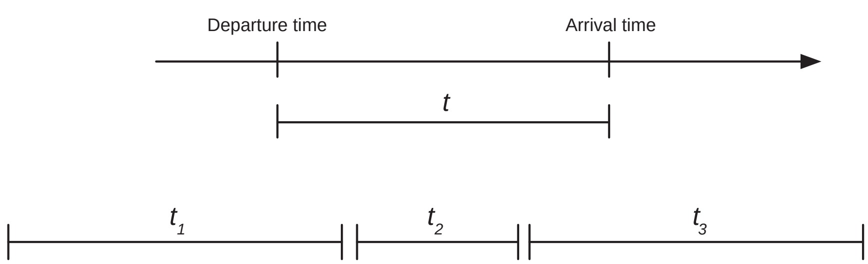


Figure 5: Illustrating the meaning of parameter n: Three train tasks are present in the train sequence during the time span of t

tages when included in the solution process. First, it heavily reduces the number of variables that must be taken into account in the Train routing model. Second, we decrease the number of broken composition constraints. The disadvantage is that decomposing into three models instead of two may give a solution farther from optimal. However, the train sequence model imitates features of solutions working well in practice. When the train sequence model is included in the solution process the constraints in Eq. 26 are included in the train routing model ensuring that train unit k is assigned to train task t if t is in train sequence s and s has been covered by k in the Sequence model.

$$\Phi_s^k = 1 \Rightarrow q_t^k = 1, \qquad \forall \qquad k \in K, t \in T_{tasks} \tag{26}$$

We implement the model using Concert Technology and solve the model with Cplex. There are, however, potentially more than 75,000 variables and solving the model with Cplex is expected to be too time consuming. The large number of variables stem mainly from the q_t^k variables, which account for 60,500 of the total. The rest are the auxiliary variables.

8 Computational results

Extensive experiments have been carried out for the decomposed approach. We first discuss experiments with the main purpose of choosing a setting of the weights in the objective function of the Position model. The weights must provide a sufficiently good solution quality and a sufficiently short computation time. The second set of experiments aims at determining a weight setting for the Routing model objective function. Finally, we present experiments illustrating the different results achieved when respectively including and excluding the Sequence model in the solution approach.

8.1 Experimental results for Position model

A set of experiments on various weight settings for the objective function of the Position model form the basis for further experiments. The aim of the

Factor	Type
A	Standing passengers
B	Train unit kilometers
C	Excess seat
D	Composition Changes
E	End capacity difference
F	Instance size

Table 3: The factors with varying weights.

Level	A	B	C	D	E	F
0	0	0	0	0	0	C
1	1	1	1	1	10^4	A, A+
2	10		10	1000	10^5	
3	100		100			

Table 4: The different levels used for factor experiments.

experiments is to derive a set of weights for which solution quality and computational time are both acceptable. The experiments will be constructed as a statistical design of experiments (DOE), see **?**].

Two sets of factor experiments is conducted each having a statistical DOE. In the first set of experiments six factors of varying levels are included, see Tab. 3. The second set of experiments includes factors A to E. Factor A to E represent the weights of the objective function of the Position model as described in Tab. 3.

We have used the values presented in Tab. 4 for each weight.

A full design of experiments contains 288 instances without factor F and 576 including factor F. We have used a design limiting the number of experiments to 72 for all experiments where each of the 72 experiments is equivalent to a specific weight setting of the objective function.

The disruptions are based on real-life data from the timetable in 2006. A data set is chosen with low punctuality and in which train units ended up in wrong locations according to their individually planned end station. A disruption is limited within a time window. The train tasks included in the disruption intersect the time window and are included in the train sequences of a set of train lines given as input. The train units included in the disruption are those being assigned to the included train tasks plus the train units being located at the depots of the train lines at the start of the disruption time window.

We run two types of experiments. In the first type, factor F is included at the levels shown in Tab. 4. In the second type we exclude factor F. We have run 4 different sets of lines, $A\&A+$, C, E and $H\&H+$ for the type 2 experiments[1]. The experiments were run with an upper limit on the solution time on 300 seconds.

We have used the method described in **?**], to develop the DOE. A statistical function for a general linear model is derived using information from the 72 experiments. The function can be used for estimating the contribution of each factor to the objective for some parameter setting. We assume that the contributions from third order correlations and higher are negligible. For a DOE with three factors, the statistical function is shown in Eq. 27. A function for 6 factors A to F follows the same structure.

$$F_{OBJ} = A + B + C + AB + AC + BC + \varepsilon \qquad (27)$$

The basic idea of using DOE is to reduce the number of experiments necessary to gain information on the contribution and importance of each term in the objective. By calculating the value of the statistical objective function and comparing it to the values observed in the results, we get an impression of how well the chosen experiments describe the effect from each factor. If the average error, ε, is low the chosen experiments are assumed representable for choosing a weight set for the objective function of the mathematical model that can be used in further experiments. We also evaluate the contributions from each factor on each of the terms in the objective. If the contribution is as expected, we assume the experiments representable and thereby a sufficient basis for choosing a weight set for the objective function of the mathematical model that can be used in further experiments.

We use the statistical function to calculate the contribution of factor A to E, the computational time and the joint objective function. In Tab.

[1] The S-tog lines are illustrated in the S-tog network in Fig. 6

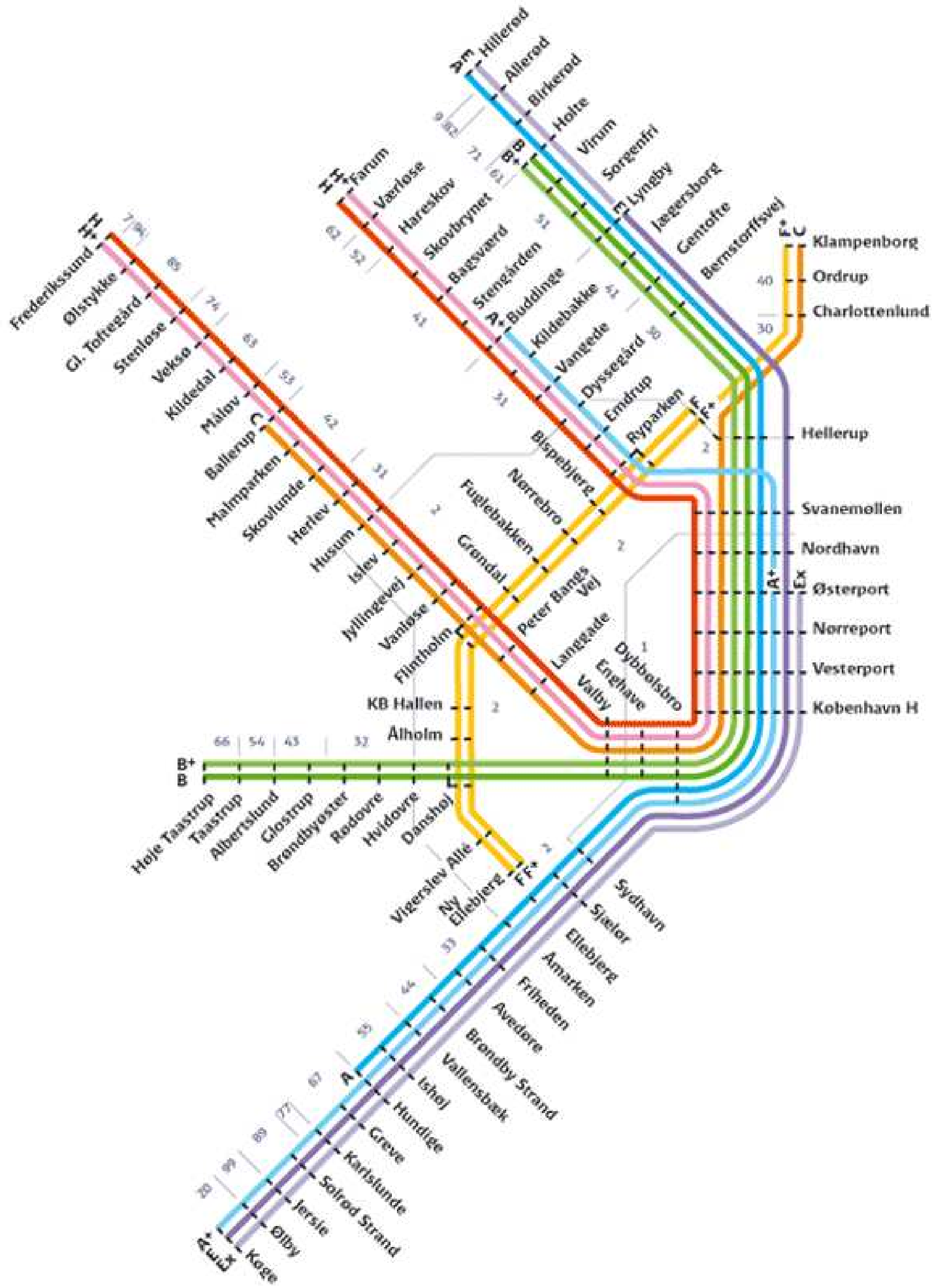


Figure 6: The S-tog network

Factor	ε_{Type_1}	$\varepsilon^{A\&A+}_{Type_2}$	$\varepsilon^{C}_{Type_2}$	$\varepsilon^{E}_{Type_2}$	$\varepsilon^{H\&H+}_{Type_2}$
A	7.93	6.05	6.05	8.75	4.50
B	1.50	3.71	3.09	2.47	1.46
C	1.29	11.90	10.44	8.61	4.90
D	2.23	9.32	4.48	4.41	5.16
E	1.32	27.91	64.08	14.40	9.82

Table 5: The average error contribution for the different terms in the objectives given the estimated objective function.

5 the average error contributions measured in percentage of the average observation are listed for all experiments.

$Type_1$ **experiments**: For experiments of $Type_1$ we see that the average error contributions for all terms are lower than those of $Type_2$ except for factor A. The low error contributions indicate that the $Type_1$ experiments are representative, however, evaluating the contributions from each factor on all terms in the objective we observe that the contributions cannot be reasonably explained. For example, factor C at the high levels contributes to the kilometer term of the objective function of the Position model, see Appendix E.A. This is a contradiction as factor C relates to the standing passengers. If the number of standing passengers are decreased by the model more train units are used and hence the number of train unit kilometers is increased. Another example of a contradiction is that factor C punish itself at all levels. The first order factor contributions are enclosed in Appendix E.A.

Because of the lack of consistency between expected and actual contributions we conclude that $Type_1$ experiments are not representative.

$Type_2$ **experiments**: Considering the $Type_2$ experiments we make the following observations on the error contributions and the first order factor contributions[2]:

$A\&A+$: The error contributions are especially high for factor E and C. Also, if we consider the different contributions that the factors make

[2]The first order factor contributions are enclosed in Appendix E.B

to each term in the objective there are contradictions similar to the ones observed for $Type_1$ results.

C : The error contributions are especially high for factor E and C. The contributions from each factor on each term in the objective are all as expected.

E**and**$H\&H+$: For both the experiments on E and $H\&H+$ the error contributions are especially high for factor E. The contributions from each factor on each term in the objective are all as expected.

For all line combinations but $A\&A+$ the instances solve to optimality within the computation time limit of 300 seconds. A large part of the $A\&A+$ instances do not find the optimal solution within the 300 seconds. The error contribution and the lack of ability to describe the contributions of the $A\&A+$ instances indicate that these are not representative. As they are not representative, we will not use them for determining the weight set used for further experiments.

Given the average error contributions in Tab. 5 and the evaluation of the expected versus the actual contributions commented above, we base our choice of a weight setting for further experiments on the instances of C, E and $H\&H+$.

We also investigated whether one can trace dependency between the computational time and the weight setting used for the objective. However, results show that there is no connection. When we use the statistical function for estimating the computational time the average error contribution varies from 20 to 75 %.

Choice and validation of weight setting

We have chosen the set of weights by filtrating the experimental results with respect to the criteria listed below.

1. Choose a subset of instances with lowest end capacity difference.

2. Choose a subset of instances where the maximum number of standing passengers is low. Preferably the maximum number of standing passenger should not exceed 36. This is 10 percent of the seat capacity in an SA train unit.

3. Choose the experiments which has the lowest average values of excess seats.

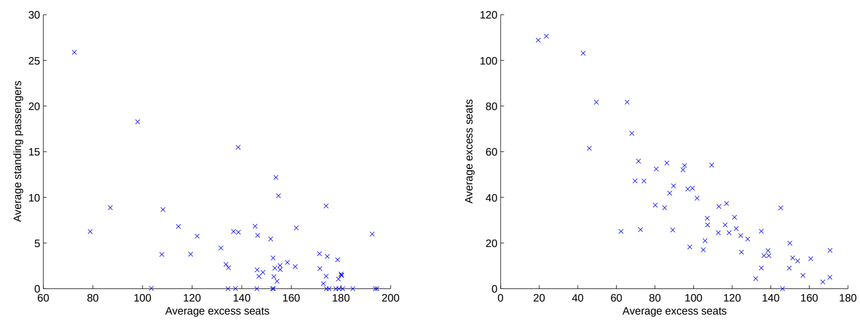
4. Choose the set of instances with lowest number of driven train unit kilometers.

Given a selection of instances, which are based on the criteria above, we assume that results are satisfactory with respect to all terms in the objective function. Based on the sorting and filtration we have chosen the instance that has a short computation time. The final choice of weight setting used for all further experiments is the combination $Weights_1 = (100, 1, 10, 0, 10^4)$. Furthermore we have chosen one more weight set, $Weights_2 = (1, 1, 1, 100, 10^5)$, for comparison. $Weights_1 = (W_1, W_2, W_3, W_4, W_5)_1$ and $Weights_2 = (W_1, W_2, W_3, W_4, W_5)_2$ are parameters used for the objective function in the Position model where W_i is the weight on the ith term in the objective function. We expect that $Weights_1$ emphasizes specifically the number of standing passenger whereas we expect that $Weights_2$ puts a higher emphasis on number of driven kilometers and the amount of excess seats, though standing passengers are still given some importance.

We have run a set of experiments on each of the two weight sets. The purpose of the experiments is to verify the expected difference of objectives for each of the two weight settings and to see if $Weights_1$ are more likely to have a short computation time than $Weights_2$. Each experiment is defined by a set of lines and recovery time window. The line sets are represented in Tab. 6. The recovery windows are respectively 1, 2 and 3 hours in the morning peak hour starting from 7 o'clock. The line combinations listed in Tab. 6 combined with the three different time periods gives 63 experimental instances. As explained these instances are run for two weight sets which gives a total of 126 experiments. The upper limit on computational time for each instance is 3600 seconds.

Lines
A
C
E
H
A+
H, H+
A, A+
C, H+
H, C
C, A+
E, A+
E, A
E, A, A+
C, H, H+
E, H+, C
E, A+, C
E, H+, A
E, C, A+, A
E, C, H, H+
H, H+, C, A+, A
H, H+, C, A+, A, E

Table 6: Lines included in experiments, see Fig. 6.



(a) $Weights_1 = (100, 1, 10, 0, 10^4)$ (b) $Weights_2 = (1, 1, 1, 100, 10^5)$

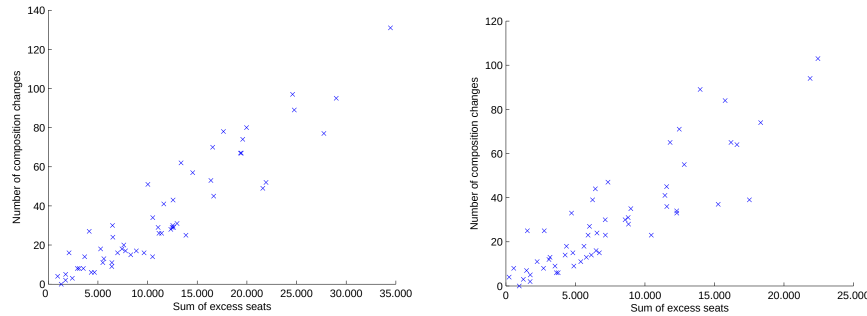
Figure 7: Each point in the plots is the average for a solution over all its train tasks of number of respectively standing passenger and excess seats

In both plots in Fig. 7 the average excess seats versus the average standing passengers for each of the 63 experimental instances are illustrated. Each point in the plots relates to a solution. Notice that it is possible in a solution to have both an average number of excess seats and an average number of standing passengers larger than zero as we average over all train tasks in that solution. For a single train task the number of respectively standing passengers and excess seats cannot both exceed zero.

If we inspect the two figures in 7 we see that the instances illustrated in 7(a) as expected in general have much fewer standing passengers on average than the instances in 7(b). The average numbers of excess seats in the $Weights_1$ solutions are not much higher than numbers of excess seats in the $Weights_2$ solutions. For both figures the relationship between the average number of standing passengers and the average number of excess seats seems approximately linear.

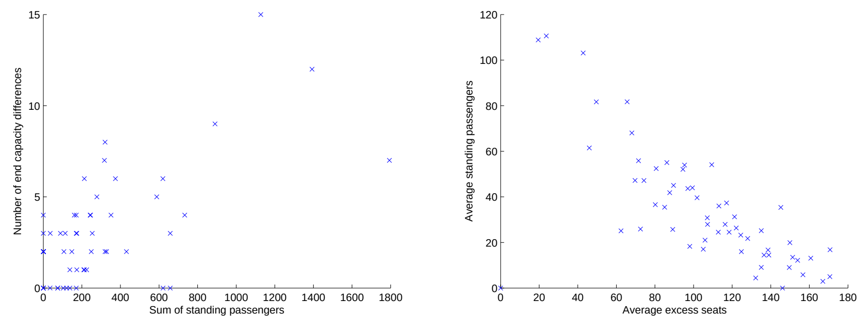
In Fig. 8 the two plots show the sum of excess seats versus the number of composition changes for each experimental instance. The numbers of composition changes only vary little from the $Weights_1$ solutions to the $Weights_2$ solutions. Both Fig. 8(a) and 8(b) indicate a linear relationship between composition changes and excess seats.

The two plots in Fig. 9 shows the sum of standing passengers versus the number of end capacity differences for each experimental instance. There



(a) $Weights_1 = (100, 1, 10, 0, 10^4)$ (b) $Weights_2 = (1, 1, 1, 100, 10^5)$

Figure 8: Sum of excess seats versus number of composition changes.



(a) $Weights_1 = (100, 1, 10, 0, 10^4)$ (b) $Weights_2 = (1, 1, 1, 100, 10^5)$

Figure 9: Sum of standing passengers versus number of end capacity differences.

are much fewer standing passengers in the $Weights_1$ solutions, see Fig. 9(a) than in the $Weights_2$ solutions, see Fig. 9(b). The number of depot end capacity differences only vary little, however, a tendency shows that a high emphasis on few standing passengers results in relatively more end capacity differences. The number of end capacity differences do not increase much in the $Weights_1$ solutions.

We have chosen $Weights_1$ partly because these weights lead to low computation time. We are interested in whether the low computation time observed in the initial experiments is low in general. We therefore compare computation times of $Weights_1$ results with those of $Weights_2$. In Fig.

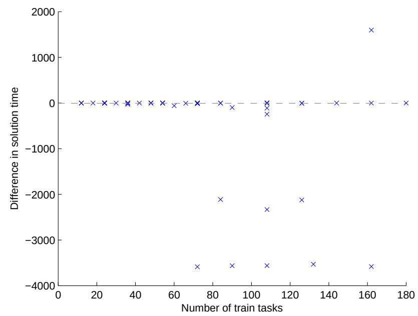


Figure 10: The difference in computation time for each instance versus the number of train tasks.

10 the differences in solution time between the $Weights_1$ solutions and the $Weights_2$ solutions are illustrated. Generally there is little difference between the solution time for the two weight sets, however, there is a set of 8 to 10 problems that solve in much shorter time for $Weights_1$. Solution times for $Weights_1$ are approximately 90% faster than those of $Weights_2$. There is only one instance where $Weights_2$ is much faster than $Weights_1$.

General comments on Position model results

There is a large variation in the results of the experiments with respect to depot end capacities. The quality with respect to standing passengers and excess seats varies independently of the depot end capacities. Finding a good balance between standing passengers and excess seats may effect the depot end capacities. A high weight on depot end capacity will often increase both the number of standing passengers and the excess seats. When we assign a low weight to the number of standing passengers we experience an increase in excess seats.

In practice it is subjective whether emphasis must be on e.g. low number of standing passengers or low end capacity differences on depots. How the weights are set will affect the computation time.

8.2 Experimental results for Routing model objective function weights

As for the Position model we have for the Routing model run experimental instances for a set of different weight sets. We have used three setups, see Tab. 7. The first setup includes the Sequence model solutions in each run of an instance. The second setup discards the Sequence model solution. The third setup varies the use of the Sequence model over including the model, excluding the model or including the model as preferences in the objective function. In the latter case, preferences are generated from the result of the Sequence model. That is, if a train unit has been assigned to a train sequence in the Sequence model, there is in the Routing model a high preference for assigning the same train unit to the train tasks of the train sequence in the Routing model.

For each of the setups presented in Tab. 7 we have used a factorial design to perform a set of experiments. The factors are the weights in the objective function. The weight of covering a task is named factor A, the weight assigned to sources is named factor B, sinks are named C and the weight of the binary variables telling whether two subsequent train tasks are assigned to the same train unit is named D. In the instances following the third setup the varying use of the Sequence model is included as a factor E. The value levels of each factor used are listed in Tab. 8.

We run instances based on the $A\&A+$ and C train lines described in Section 8.1. A full DOE contains $3^5 = 243$ experiments for $Setup_3$ and 81 for $Setup_1$ and $Setup_2$. By using the DOE the number of runs has been reduced for each Setup according to Tab. 7.

As we are interested in a reasonable solution quality within a short computation time we put an upper limit on the computation time of each run. Prior to each run of the Routing model an execution of the Position model finds the number of train units of each type to assign to each train task. Hereafter, the Sequence model is run. The upper limits on the computation time of the Position model is 600 seconds. The Sequence model is solved at an aggregated level and needs no upper limit as it always solves to optimality in less than 1 second for the instances chosen in our test setups. Finally, we have set the upper limit on the Routing model computation time to 3600

Number	Setup	Number of runs
$Setup_1$	Including the Sequence model in each experiment	53
$Setup_2$	Excluding the Sequence model in each experiment	53
$Setup_3$	Varying the use of Sequence model as a factor	72

Table 7: Experimental setups used for the routing parameter choice experiments.

Level	A	B	C	D	E
0	0	0	0	0	Incl. Seq.
1	100	-10	-10	1000	Pref. from Seq.
2	1000	-50	-50	10000	Excl. Seq.

Table 8: The different levels used for factor experiments of the Routing model.

seconds.

In Tab. 9 the error contributions in percentage of the average objective are listed for five different measures for each of the experimental setups[3]. The highest average error contributions are of the tests on $Setup_3$ where factor E is included. For $Setup_2$ the error contributions are higher for instances on train lines $A\&A+$ than those on C. All average error contributions are high when estimating computation time.

The results suggest that an estimate has a high error if many of the runs in the experiment cannot be solved to optimality. Also, if the use of the sequence model is varied, the error contribution will be high. Even though the average error contributions are low on various objectives of the experimental setup, the average error contribution on the estimate of computation time is high indicating that computation time cannot be predicted with the statistical function. Given these observations we have chosen to use the experiments based on setup 1 from Tab. 7 to base the choice of weight set used for further experiments.

Given the experiments corresponding to $setup_1$ we have filtered the solution data relative to the maximum difference in depot end capacity, the maximum number of standing passengers and the maximum average num-

[3]For information on factor contributions see Appendix E.C

Objective	$Setup_3$		$Setup_2$		$Setup_1$	
	C	A, A+	C	A, A+	C	A, A+
Standing passengers	45.78	23.85	0.27	5.14	$1.98 \cdot 10^{-13}$	0.93
Excess seats	6.84	4.12	0.38	5.32	$2.49 \cdot 10^{-12}$	0.22
Driven kilometers	7.22	5.00	0.13	3.37	0	0.07
End capacity diff.	4.60	4.85	0.88	4.02	$2.36 \cdot 10^{-12}$	1.59
Computation time	16.38	14.05	16.44	0.01	18.96	15.01

Table 9: The average error contribution on different objectives given the estimated objective function.

ber of standing passengers. Based on the filtering for train lines $A\&A+$, we have chosen the weight set, $Weights_1 = (100, 0, -50, 1000)$, for further experiments. We have chosen not to use the results for line C on $setup_1$ as there is too little difference in the solutions i.e. it is very easy to achieve a good solution. For comparisons we have chosen the weight set, $Weights_2 = (100, -10, -10, 1000)$. We expect that $Weights_2$ will provide the same quality in results as $Weights_1$ as they give similar weights to sinks and sources and the same weights on train tasks and subsequent covers. We want to verify this and to see if there is any difference in computational time.

$Weights_1$ and $Weights_2$ have been used in two separate experiments of 36 runs counting 12 line combinations and three time periods. The line combinations are listed in Tab. 10. The time periods are all starting at 7 o'clock and are of respectively 1, 2 and 3 hours of duration. In the 36 runs the Sequence model is included in the solution process.

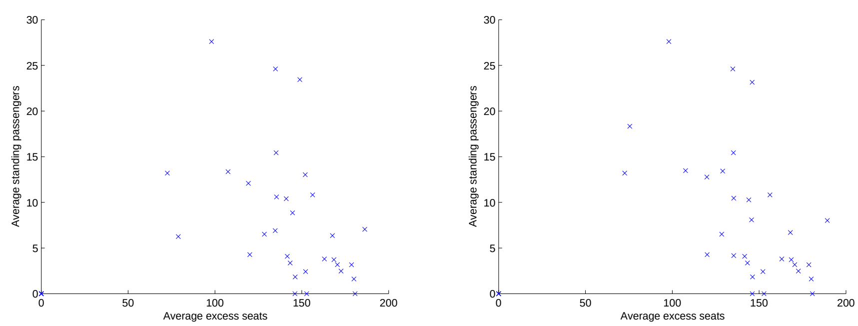
In 5 instances out of the 36 instances a solution for the underlying Position problem could not be found within 600 seconds. We will discard these when evaluating the quality of the Routing model.

In Fig. 11 two plots are given of the average excess seats versus the average standing passengers. There is only little difference between the $Weights_1$ solutions in Fig. 11(a) and the $Weights_2$ solutions in Fig. 11(b).

Fig. 12 shows two plots of the sum of standing passengers versus the difference in depot end capacities. As for the plots in Fig. 11 there is only little difference between the $Weights_1$ solutions in Fig. 12(a) and the $Weights_2$

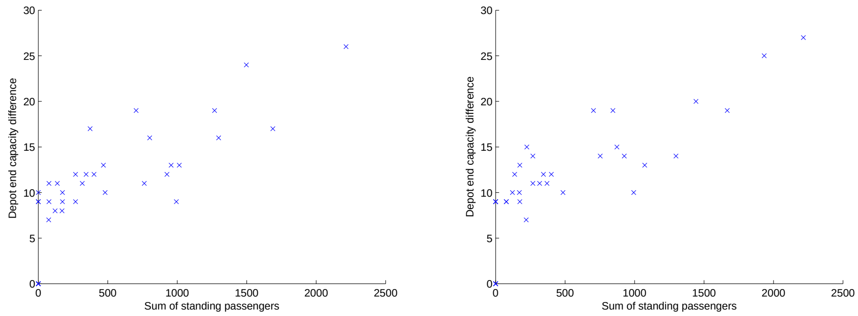
Nr.	Lines
1	A
2	C
3	E
4	A, A+
5	C, A+
6	E, A+
7	E, A+, A
8	C, H+, H
9	E, H+, C
10	A+, H, H+, C
11	H+, H, C, A+, A
12	H+, H, C, A+, A, E

Table 10: Lines included in experiments, see Fig. 6.



(a) $Weights_1 = (100, 0, -50, 1000)$ (b) $Weights_2 = (100, -10, -10, 1000)$

Figure 11: Average excess seats versus average standing passengers.



(a) $Weights_1 = (100, 0, -50, 1000)$ (b) $Weights_2 = (100, -10, -10, 1000)$

Figure 12: Sum of standing passengers vs. difference in end capacity.

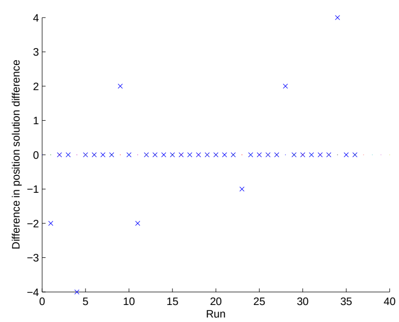


Figure 13: The difference between $Weights_1$-solutions and $Weights_2$-solutions for the Position solution differences.

solutions in Fig. 12(b).

The difference between $Weights_1$ solutions and $Weights_2$ solutions for the Position solution differences is illustrated in Fig. 13. We see that there is a difference to the Position solution when either the type of train unit assigned in the Routing model does not match the type assigned in the Position model or the number of train units assigned in the Position model does not match the number of train units assigned in the Routing model. We see that there is only little difference in the differences from $Weights_1$ solutions to $Weights_2$ solutions. The average difference over all runs in difference to Position solution is $\Delta Pos_1 = 3.0556$ for $Weights_1$ solutions and $\Delta Pos_2 = 3.0833$ for $Weights_2$ solutions.

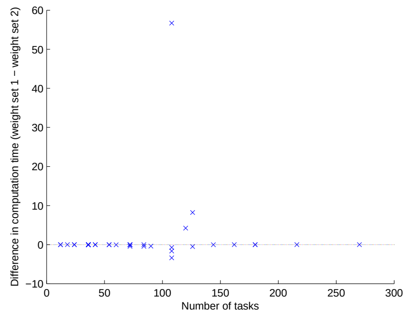


Figure 14: The number of train tasks versus the difference in solution time for each instance.

Fig. 14 illustrates the number of train tasks on each run versus the difference in computation time between the $Weights_1$ solutions and the $Weights_2$ solutions. The difference in computation times in the two sets of solutions is with but one exception less than 8 seconds. Considering the computation times of the problem instances only three of these have a computation time higher than 15 seconds. The immediate reason for the deviating results is probably that only few assignments were made in the intermediate step of the Sequence model which of course decreased the number of preassigned variables in the Routing model.

There is a a marginal difference in excess seats and standing passengers. In the $Weights_1$ solutions there are slightly fewer standing passengers than the $Weights_2$ solutions. In the $Weights_2$ solutions there are slightly fewer excess seats than the $Weights_1$ solutions. This can all be traced to the difference in deviations from the Position model.

There is only little difference in computation time for the two weight sets. The total average computation time is 215.81 for $Weights_1$ and 214.10 for $Weights_2$. Only three of the instances considered have exceedingly high computation times. One of these solve to optimality in 531.25 seconds for $Weights_1$ and 474.56 seconds for $Weights_2$. Discarding these three instances which results in a high computation time the mean computation time is 1.34 seconds for $Weights_1$ and 1.18 seconds for $Weights_2$.

Running the two instances that do not solve to optimality within an hour

Parameter set	Line set	Length of time interval	Opt. gap 60 sec. (%)	Opt. gap 3600 sec. (%)	Opt. gap 28800 sec. (%)
$Weights_1$	9	2	0.88	0.70	0.61
$Weights_1$	10	1	0.82	0.62	0.38
$Weights_2$	9	2	0.90	0.68	0.59
$Weights_2$	10	1	2.01	1.09	0.02

Table 11: Computational time and optimality gap for 8 hour runs.

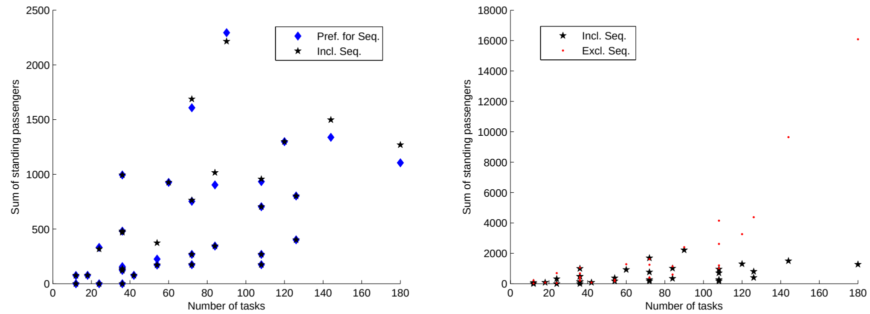
for a longer period of 8 hours for both $Weights_1$ and $Weights_2$ we get the results in Tab. 11. We see that increasing the upper time limit on running time does not result in optimal solutions. In fact, the solution quality only improves very little in the 7 hours increased solution time. Hence, a solution close to the optimal solution is obtained within the first 60 seconds for both instances. This indicate that the Routing model even for these instances is practical applicable.

8.3 Effect of including the Sequence model

In this section we analyze the $3 \cdot 36$ experiments run for $Weights_1$. Test instances are constructed given the three time windows of 1, 2 and 3 hours starting from 7 o'clock and the line combinations in Tab. 10. Each of the 36 instances are solved using three different approaches, excluding the Sequence model, including the Sequence model and including the Sequence model as preference in the objective function.

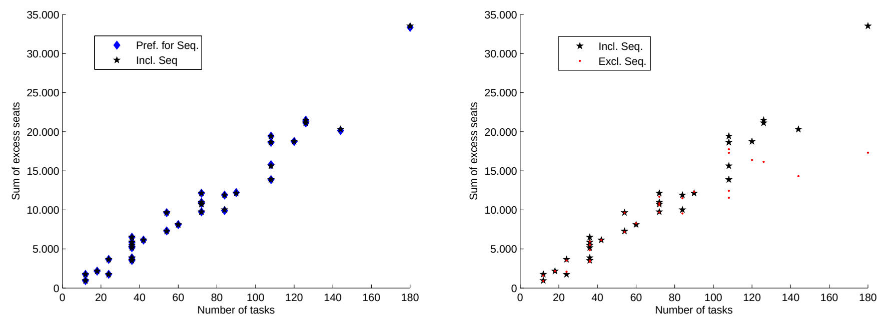
We will in the following refer to the solution approach where the Sequence model solution is included in the Routing solution procedure as $A_{Incl.}$. The solution approach where the Sequence model used as preferences in the Routing solution procedure we refer to as $A_{Pref.}$. Last, the solution approach where we exclude the Sequence model solution we refer to as $A_{Excl.}$.

Fig. 15 shows respectively the sum of standing passengers for each run for $A_{Incl.}$ & $A_{Pref.}$ and for $A_{Incl.}$ & $A_{Excl.}$. For $A_{Incl.}$ and $A_{Pref.}$, see Fig. 15(a), the sum of standing passengers are quite close. For $A_{Excl.}$ the sum of standing passengers is in general much higher. Note the difference in scale on the y-axis.



(a) $A_{Incl.}$ and $A_{Pref.}$ (b) $A_{Incl.}$ and $A_{Excl.}$

Figure 15: Sum of standing passengers for each run.



(a) $A_{Incl.}$ and $A_{Pref.}$ (b) $A_{Incl.}$ and $A_{Excl.}$

Figure 16: Sum of excess seats for each run.

Fig. 16 illustrates the sum of excess seats for each run for respectively $A_{Incl.}$ & $A_{Pref.}$ and for $A_{Incl.}$ & $A_{Excl.}$. Again the sum of excess seats for $A_{Incl.}$ and $A_{Pref.}$ are close, see Fig. 16(a). As the sum of excess seats is often a conflicting objective to the sum of standing passengers it is expected that $A_{Excl.}$ has the same or fewer excess seats than $A_{Incl.}$. This is also what we observe in Fig. 16(b).

The difference in end capacity is illustrated in Fig. 17. Again, we see that there is a little difference in quality regarding $A_{Incl.}$ and $A_{Pref.}$. When regarding $A_{Excl.}$ the quality decreases.

Fig. 18 shows the distribution of the results with respect to computation time. $A_{Incl.}$ has the most short running times and only few very high running

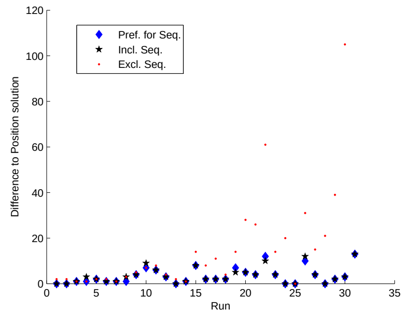


Figure 17: The difference to the position model solution for each run.

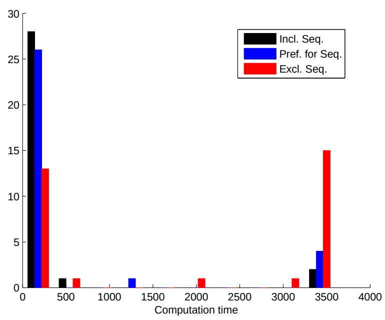


Figure 18: The distribution of runs with respect to computation time.

times. The general mean computation time for $A_{Incl.}$, $A_{Pref.}$ and $A_{Excl.}$ is respectively $\mu_{Incl.} = 250.61$, $\mu_{Pref.} = 510.34$ and $\mu_{Excl.} = 1865.06$.

When for $A_{Incl.}$ disregarding the two runs where an optimal solution can not be found with in the 1 hour time limit the mean computation time is $\mu_{Incl.}^{Modified} = 19.62$ seconds. For $A_{Pref.}$ there are three runs where an optimal solution can not be found within the 1 hour time limit. A modified mean time limit is $\mu_{Pref.}^{Modified} = 179.29$.

Summing up the observations we have presented in this section it seems that the solution quality for respectively $A_{Incl.}$ and $A_{Pref.}$ are comparable. The solution quality of $A_{Excl.}$ is lower, most likely because the optimal solution cannot be found within the Routing model running time limit. Even though the $A_{Pref.}$ renders the same solution quality as $A_{Incl.}$ its computation time is on average more than 50% higher. Hence, if we want to obtain an acceptable solution time and quality in real time we must include the Sequence model in the solution process.

In the $A_{Pref.}$ instances there is a higher degree of freedom for assigning values to variables than in the $A_{Incl.}$ instances. However, it is observed that even when $A_{Pref.}$ solves to optimality, the solution quality is at most marginally better on the chosen measures. This indicates that the inclusion of the Sequence model decreases the solution quality only marginally.

9 Conclusion

In this paper we have addressed the RSRP. We have formulated a solution approach based on decomposition and consisting of three models to be solved iteratively. The models are implemented with commercial software and initial computational results indicate that the models provide a feasible approach for practical problems up to at least 100 train tasks.

The sequence model is an important step in the solution approach. The average solution time when leaving out the sequence model is 1865.06 seconds. When the Sequence model is included and the variables are locked accordingly the average solution time is 250.61. Furthermore, when the Sequence model is left out the solution quality deteriorates, that is, fewer problem instances solve to optimality within the upper time limit set on computation time.

The quality of solutions when using the Sequence model as preferences compared to locking the variables in the Routing model is the same or only marginally better. We therefore conclude that the Sequence model can be included without deteriorating the solution quality more than marginally. This is desirable as the computation time is decreased 50 % when locking the Routing model variables.

Further research concerns other solution methods for the RSRP. An integrated solution approach may be a heuristic approach solving the Position and Routing problem in one. Also, replacing the Sequence and Routing problems with a column generation approach is interesting.

Appendix E.A Factor contributions, Position model: 6 factors included

In Tab. 12 the first order factor contributions are listed for the factor experiments on the position model having 6 factors. The factors, A to F, are listed below.

A Weight on the term of the sum of excess seats in the Position model objective function.

B Weight on the term of the sum train unit kilometers in the Position model objective function.

C Weight on the term of the sum of standing passengers in the Position model objective function.

D Weight on the term of the sum of couplings in the Position model objective function.

E Weight on the term of the sum of depot end capacities in the Position model objective function.

F Experimental instance input.

First order contributions are calculated for 6 different measures. For each measure the contribution from a factor is listed for each level higher than 0 e.g. the contribution of A1 to the sum of standing passengers indicates that factor A on the first level higher than zero has a high decreasing effect on the sum of standing passengers, see Tab. 4.

Standing passengers The sum of standing passengers on all train tasks.

Train unit kilometers The sum of train unit kilometers on all train tasks.

Excess seats The sum of excess seats on all train tasks.

Composition changes The sum of couplings on all train tasks.

End capacity differences The sum of differences to the scheduled end capacity on all depots by the end of recovery.

Computation time

	Standing pass.	Train unit Km	Excess seats	Composition changes	End capacity differences	Comp. time
A1	-1507.3	-199.3	-2981.6	9.1	-0.4	-85.3
A2	-590.4	-292.7	-3638.7	4.3	0.3	76.4
A3	943.3	-340.4	-3621.4	9.9	-2.2	-106.3
B1	3809.3	-795.7	-9364.4	10.4	2.1	41.1
C1	-934.1	19.0	928.7	5.9	5.5	2.8
C2	1304.5	-345.5	-3529.1	-4.4	5.9	168.7
C3	811.4	-374.8	-4224.8	0.5	1.4	116.9
D1	-164.1	-347.6	-2688.1	-1.9	1.4	187.1
D2	919.2	-436.9	-2838.2	-4.8	5.7	178.0
E1	1610.6	-305.7	-1929.3	9.3	-3.4	120.6
E2	2463.5	-520.6	-3896.3	7.6	-1.9	158.5
F1	3042.9	1580.0	5958.7	15.0	6.7	130.2

Table 12: 1. order factor contributions, 6 factor experiments, Position model

Appendix E.B Factor contributions, Position model: 5 factors included

In Tab. 13 to 16 the first order factor contributions are listed for the factor experiments on the position model having 5 factors. The factors, A to E, and the measures are described in Appendix 9.

	Composition changes	End cap. differences	Excess seats	Train unit Km	Standing pass.	Comp. time
A1	5.7	-1.6	-4500.2	-216.1	33.2	-0.9
A2	4.8	-2.3	-5191.8	-265.1	67.1	-0.8
A3	4.2	-3.3	-4468.2	-234.2	80.1	-0.6
B1	4.6	-0.8	-5068.3	-260.2	61.0	-0.4
C1	1.6	-0.9	-86.7	13.8	-7.5	0.0
C2	2.0	1.5	-802.7	-40.2	-71.9	0.3
C3	4.0	0.8	-1735.3	-83.6	-38.0	-0.3
D1	-1.8	-0.8	-1432.4	-60.6	56.5	0.1
D2	-4.2	1.3	-698.0	-13.5	6.5	0.3
E1	2.2	-2.3	-1504.4	-69.8	14.9	-0.6
E2	1.3	-2.1	-1851.5	-79.1	32.5	-0.7

Table 13: 1. order factor contributions, 5 factor experiments, Position model, Line C instances

	Composition changes	End cap. differences	Excess seats	Train unit Km	Standing pass.	Comp. time
A1	-2.2	-7.9	1332.6	-62.4	142.6	-71.3
A2	-17.3	-2.5	871.4	44.6	-197.2	-202.0
A3	6.2	-9.7	1171.8	134.0	-807.9	-47.3
B1	2.7	1.2	-242.7	315.0	-6286.1	-102.7
C1	3.3	-8.1	448.2	46.7	-39.2	-204.1
C2	-2.9	-3.7	-1405.9	88.4	-1105.6	-149.4
C3	-9.6	-6.8	2411.2	212.6	-1576.8	-21.5
D1	8.4	0.0	-1798.3	18.0	-1714.9	-158.5
D2	-11.3	4.3	-944.4	17.8	-536.4	-22.6
E1	7.3	-5.8	2109.9	1.9	737.9	52.5
E2	-0.5	-7.4	2239.4	187.2	-1236.2	-177.2

Table 14: 1. order factor contributions, 5 factor experiments, Position model, line A and A+ instances

	Composition changes	End cap. differences	Excess seats	Train unit Km	Standing pass.	Comp. time
A1	3.7	1.3	-4757.1	-481.4	-75.4	-1.6
A2	5.0	1.8	-6011.4	-662.6	327.2	-2.2
A3	5.5	2.6	-6442.8	-648.4	-27.1	0.5
B1	0.8	0.3	-4159.1	-475.8	254.8	-4.1
C1	0.8	-0.2	-1492.9	-135.2	-378.9	-2.5
C2	-1.5	-0.7	-103.9	36.6	-723.1	-3.6
C3	-1.3	-0.8	870.9	133.0	-823.2	-5.0
D1	1.0	0.3	-1813.3	-232.7	162.9	-3.8
D2	-3.2	-0.2	-1663.3	-214.9	-5.3	-2.2
E1	-3.3	-3.0	-937.4	-87.2	-116.2	5.1
E2	-1.3	-3.3	-1806.6	-234.1	97.8	0.6

Table 15: 1. order factor contributions, 5 factor experiments, Position model, Line H and H+ instances

	Composition changes	End cap. differences	Excess seats	Train unit Km	Standing pass.	Comp. time
A1	2.7	-0.2	-7436.6	-661.5	498.2	-0.1
A2	1.8	-0.6	-8354.2	-780.0	935.1	-0.1
A3	2.3	-0.4	-7983.8	-734.3	929.9	-0.1
B1	0.5	-0.3	-7642.5	-703.4	852.1	-0.1
C1	1.7	-0.3	-148.5	-6.3	-16.2	0.0
C2	1.5	0.1	1485.6	176.6	-669.8	-0.1
C3	-0.2	-0.6	1638.9	188.9	-510.8	-0.1
D1	-1.9	-0.5	-2598.8	-269.6	574.5	0.0
D2	-2.1	-0.7	-2666.3	-291.4	568.5	0.0
E1	2.4	-2.6	-849.2	-67.5	-36.3	0.0
E2	1.1	-2.4	-1863.5	-162.4	218.9	-0.1

Table 16: 1. order factor contributions, 5 factor experiments, Position model

Appendix E.C Factor contributions, Routing model

In Tab. 17 to 22 the first order factor contributions are listed for the factor experiments on the Routing model. The factor experiments relative to Tab. 17 to 18 have 5 factors A to E. The factor experiments relative to Tab. 19 to 22 have 4 factors A to D. The factors are described below. Levels of the factors are described in Tab. 8.

A Weight on the term of the tasks in the Routing model objective function.

B Weight on the term of the sources in the Routing model objective function.

C Weight on the term of the sinks in the Routing model objective function.

D Weight on the term of the consecutive covered tasks in the Routing model objective function.

E Use of the train Sequence model.

The first order contributions are calculated for 6 different measures.

Difference to Position solution The sum of assignments made in the Routing solution which differs from the assignments in the Position solution.

Difference to end capacity The sum of differences to the scheduled end capacity on all depots by the end of recovery.

Excess seats The sum of excess seats on all train tasks.

Train unit kilometers The sum of train unit kilometers on all train tasks.

Standing passengers The sum of standing passengers on all train tasks.

Computation time

	Difference to Pos. solution	Diff. to end cap.	Excess seats	Train unit Km	Standing pass.	Comp. time
A1	1,4	0,4	-222,9	-27,4	293,2	0,0
A2	11,6	3,7	-1735,6	-205,1	2175,9	0,1
B1	-9,3	-2,5	1384,4	153,6	-1306,8	0,1
B2	-8,9	-2,4	1315,1	145,2	-1217,2	0,0
C1	-3,0	-0,8	520,6	66,6	-716,3	0,0
C2	-1,8	-0,4	327,6	43,0	-464,2	0,0
D1	-3,0	-0,8	499,1	63,1	-677,4	0,0
D2	-2,4	-0,6	415,5	53,7	-578,5	0,0
E1	-8,1	-2,1	1188,5	129,6	-1051,1	0,0
E2	-7,9	-2,1	1148,9	124,7	-998,2	0,0

Table 17: 1. order factor contributions, factor experiments, Routing model, Line C instances, Choice of Sequence model

	Difference to Pos. solution	Diff. to end cap.	Excess seats	Train unit Km	Standing pass.	Comp. time
A1	3,3	0,5	-308,9	-89,9	1021,7	-22,3
A2	41,1	12,4	-4370,6	-822,7	10348,4	144,1
B1	-46,6	-6,5	4835,9	950,1	-8807,7	89,9
B2	-44,7	-5,1	4810,2	919,8	-8575,5	81,8
C1	-4,1	-1,3	364,3	154,6	-1822,4	51,0
C2	-2,3	-0,6	108,8	83,6	-1263,3	33,9
D1	-7,9	-0,3	738,7	173,4	-2553,3	50,4
D2	-1,5	-1,0	8,1	76,4	-1009,2	42,9
E1	-44,0	-3,8	4459,4	893,2	-8058,4	70,0
E2	-43,7	-5,2	4533,3	873,6	-8062,8	65,8

Table 18: 1. order factor contributions, factor experiments, Routing model, Line A and A+ instances, Choice of Sequence model

	Difference to Pos. solution	Diff. to end cap.	Excess seats	Train unit Km	Standing pass.	Comp. time
A1	0,2	-6,4	3579,0	426,4	-4227,3	0,2
A2	0,2	-6,4	3587,0	426,8	-4229,7	0,2
B1	0,2	-0,4	64,0	2,6	-18,7	0,2
B2	0,1	-0,4	56,0	2,3	-16,3	0,1
C1	0,0	0,0	0,0	0,0	0,0	0,0
C2	-0,1	0,2	-32,0	-1,3	9,3	-0,1
D1	0,2	-6,4	3579,0	426,4	-4227,3	0,2
D2	0,2	-6,4	3587,0	426,8	-4229,7	0,2

Table 19: 1. order factor contributions, factor experiments, Routing model, Line C instances, No use of Sequence model

	Difference to Pos. solution	Diff. to end cap.	Excess seats	Train unit Km	Standing pass.	Comp. time
A1	-68,3	-6,3	6909,6	1616,3	-18028,4	299,6
A2	-66,3	-8,4	7119,2	1691,5	-18514,8	299,5
B1	-2,3	-2,4	76,1	21,1	113,1	0,0
B2	1,0	-2,3	-30,2	16,2	0,8	0,0
C1	3,4	-0,7	-271,1	-59,9	459,3	0,0
C2	2,3	0,6	-189,9	-46,7	277,1	0,1
D1	-71,8	-9,4	7387,9	1718,0	-18662,1	299,5
D2	-72,0	-10,4	7465,6	1665,5	-18739,4	299,5

Table 20: 1. order factor contributions, factor experiments, Routing model, Line A and A+ instances, No use of Sequence model

	Difference to Pos. solution	Diff. to end cap.	Excess seats	Train unit Km	Standing pass.	Comp. time
A1	-4,0	-1,0	494,0	41,6	-106,0	0,0
A2	-4,0	-1,0	494,0	41,6	-106,0	0,0
B1	0,0	0,0	0,0	0,0	0,0	0,0
B2	0,0	0,0	0,0	0,0	0,0	0,0
C1	0,0	0,0	0,0	0,0	0,0	0,0
C2	0,0	0,0	0,0	0,0	0,0	0,0
D1	-4,0	-1,0	494,0	41,6	-106,0	0,0
D2	-4,0	-1,0	494,0	41,6	-106,0	0,0

Table 21: 1. order factor contributions, factor experiments, Routing model, Line C instances, Use of Sequence model

	Difference to Pos. solution	Diff. to end cap.	Excess seats	Train unit Km	Standing pass.	Comp. time
A1	-38,0	-5,1	4079,4	681,9	-5340,6	0,6
A2	-38,0	-3,4	4062,2	674,0	-5357,8	0,6
B1	0,0	0,1	53,4	-4,0	53,4	0,4
B2	0,0	0,1	20,1	-3,2	20,1	0,3
C1	0,0	-0,2	-32,7	-1,6	-32,7	0,1
C2	0,0	-0,2	-40,8	0,8	-40,8	-0,1
D1	-38,0	-2,8	3972,8	670,0	-5447,2	0,7
D2	-38,0	-3,7	3973,6	669,2	-5446,4	0,5

Table 22: 1. order factor contributions, factor experiments, Routing model, Line A and A+ instances, Use of Sequence model

References

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