Technical University of Denmark

Appendix to Power Dissipation in Division

Liu, Wei; Nannarelli, Alberto

Publication date:
2008

Document Version
Publisher's PDF, also known as Version of record

Appendix to Power Dissipation in Division

IMM Technical Report 2008-15

Wei Liu and Alberto Nannarelli

DTU Informatics, Technical University of Denmark, Kongens Lyngby, Denmark

Abstract

This document is an appendix to the paper: Wei Liu and Alberto Nannarelli, *"Power Dissipation in Division"*, Proc. of 42nd Asilomar Conference on Signals, Systems, and Computers, October 2008.

The purpose of the document is to provide the necessary information for the implementation of the architectures described in the above paper.

1 Newton-Raphson Reciprocal Approximation

The reciprocal $R = 1/d$, can be obtained by approximation with the Newton-Raphson method [1]

$$R[j+1] = R[j](2 - R[j]d) \qquad j = 0, 1, \ldots, m$$

The convergence is quadratic and the number of iterations m needed depends on the initial approximation $R[0]$ (implemented by a look-up table (LUT) in our case). Table 1 reports the values of $R[0]$ that have been computed according to [2].

The look-up table of Table 1 is addressed by the first 8 fractional bits of the divisor $d \in [1.0, 2.0)$, and produces as output the first 8 fractional bits of $R[0] \in [0.5, 1.0)$ (bit of weight 2^{-1} is always 1 and therefore omitted in the LUT). The values in Table 1 are listed as integers obtained by

$$\hat{d} = \lfloor (d - 1.0) \cdot 2^8 \rfloor \quad \text{and} \quad \hat{R}[0] = \text{round-to-integer}(R[0] \cdot 2^8)$$

2 Division by Polynomial Approximation

By using quadratic polynomial approximation, the reciprocal is approximated as follows [3]:

$$f(x) \approx \begin{cases} K_y + K_m(x - x^*) + K_{p1}(x - x^*)^2 & \text{for } x < x^* \\ K_y + K_m(x - x^*) + K_{p2}(x - x^*)^2 & \text{for } x > x^* \end{cases} \tag{1}$$

The coefficients K_p, K_m and K_y are reported in Table 2. The values in the table are listed as integers obtained by

$$\hat{d} = \lfloor (d - 1.0) \cdot 2^7 \rfloor \quad \text{and} \quad \hat{K} = \text{round-to-integer}(K \cdot 2^{15})$$

References

[1] M. Ercegovac and T. Lang, *Digital Arithmetic*. Morgan Kaufmann Publishers, 2004.

[2] D. DasSarma and D. W. Matula, "Measuring the Accuracy of ROM Reciprocal Tables," *IEEE Transactions on Computers*, vol. 43, no. 8, pp. 932–940, Aug. 1994.

[3] D. De Caro, N. Petra, and A. G. M. Strollo, "A high performance floating-point special function unit using constrained piecewise quadratic approximation," *Proc. of IEEE International Symposium on Circuits and Systems (ISCAS 2008)*, pp. 472–475, May 2008.

$\hat{d}$	$\hat{R}[0]$	$\hat{d}$	$\hat{R}[0]$	$\hat{d}$	$\hat{R}[0]$	$\hat{d}$	$\hat{R}[0]$
0	255	64	204	128	170	192	146
1	255	65	204	129	170	193	146
2	254	66	203	130	170	194	145
3	253	67	203	131	169	195	145
4	252	68	202	132	169	196	145
5	251	69	201	133	168	197	145
6	250	70	201	134	168	198	144
7	249	71	200	135	167	199	144
8	248	72	200	136	167	200	144
9	247	73	199	137	167	201	143
10	246	74	198	138	166	202	143
11	245	75	198	139	166	203	143
12	244	76	197	140	165	204	142
13	243	77	197	141	165	205	142
14	242	78	196	142	164	206	142
15	241	79	195	143	164	207	141
16	240	80	195	144	164	208	141
17	240	81	194	145	163	209	141
18	239	82	194	146	163	210	140
19	238	83	193	147	162	211	140
20	237	84	192	148	162	212	140
21	236	85	192	149	162	213	140
22	235	86	191	150	161	214	139
23	234	87	191	151	161	215	139
24	234	88	190	152	160	216	139
25	233	89	190	153	160	217	138
26	232	90	189	154	160	218	138
27	231	91	189	155	159	219	138
28	230	92	188	156	159	220	138
29	230	93	188	157	158	221	137
30	229	94	187	158	158	222	137
31	228	95	186	159	158	223	137
32	227	96	186	160	157	224	136
33	226	97	185	161	157	225	136
34	226	98	185	162	157	226	136
35	225	99	184	163	156	227	136
36	224	100	184	164	156	228	135
37	223	101	183	165	155	229	135
38	223	102	183	166	155	230	135
39	222	103	182	167	155	231	134
40	221	104	182	168	154	232	134
41	220	105	181	169	154	233	134
42	220	106	181	170	154	234	134
43	219	107	180	171	153	235	133
44	218	108	180	172	153	236	133
45	217	109	179	173	153	237	133
46	217	110	179	174	152	238	133
47	216	111	178	175	152	239	132
48	215	112	178	176	152	240	132
49	215	113	177	177	151	241	132
50	214	114	177	178	151	242	131
51	213	115	176	179	150	243	131
52	212	116	176	180	150	244	131
53	212	117	175	181	150	245	131
54	211	118	175	182	149	246	130
55	210	119	175	183	149	247	130
56	210	120	174	184	149	248	130
57	209	121	174	185	148	249	130
58	208	122	173	186	148	250	129
59	208	123	173	187	148	251	129
60	207	124	172	188	147	252	129
61	206	125	172	189	147	253	129
62	206	126	171	190	147	254	128
63	205	127	171	191	146	255	128

Table 1: Approximation of $R[0]$.

$\hat{d}$	$\hat{K}_p$	$\hat{K}_m$	$\hat{K}_y$	$\hat{d}$	$\hat{K}_p$	$\hat{K}_m$	$\hat{K}_y$
0	31960	31764	32263	64	9548	14264	21620
1	30605			65	9276		
2	30519	30809	31775	66	9257	13974	21399
3	29245			67	8996		
4	29163	29896	31300	68	8978	13693	21183
5	27964			69	8728		
6	27887	29023	30840	70	8710	13421	20971
7	26757			71	8469		
8	26684	28188	30393	72	8453	13156	20763
9	25618			73	8222		
10	25549	27388	29959	74	8206	12900	20560
11	24543			75	7983		
12	24478	26622	29537	76	7968	12650	20360
13	23527			77	7754		
14	23465	25888	29127	78	7739	12408	20164
15	22567			79	7533		
16	22508	25184	28728	80	7519	12173	19972
17	21658			81	7321		
18	21602	24508	28339	82	7308	11944	19784
19	20797			83	7117		
20	20744	23859	27962	84	7104	11722	19599
21	19981			85	6920		
22	19931	23235	27594	86	6907	11506	19418
23	19208			87	6730		
24	19160	22635	27235	88	6718	11296	19239
25	18474			89	6548		
26	18428	22059	26886	90	6536	11091	19065
27	17776			91	6371		
28	17733	21504	26546	92	6360	10893	18893
29	17114			93	6201		
30	17073	20970	26214	94	6190	10699	18724
31	16484			95	6037		
32	16445	20455	25890	96	6027	10510	18558
33	15884			97	5879		
34	15847	19959	25575	98	5868	10327	18396
35	15313			99	5727		
36	15278	19481	25266	100	5716	10148	18236
37	14770			101	5580		
38	14736	19020	24966	102	5569	9974	18078
39	14251			103	5436		
40	14219	18575	24672	104	5426	9804	17924
41	13757			105	5300		
42	13726	18146	24385	106	5290	9639	17772
43	13285			107	5166		
44	13256	17731	24105	108	5156	9477	17623
45	12835			109	5038		
46	12806	17330	23831	110	5029	9320	17476
47	12404			111	4913		
48	12377	16943	23563	112	4905	9166	17331
49	11993			113	4792		
50	11967	16569	23301	114	4785	9017	17189
51	11600			115	4676		
52	11575	16207	23045	116	4668	8871	17050
53	11224			117	4563		
54	11200	15856	22795	118	4556	8728	16912
55	10863			119	4454		
56	10841	15517	22550	120	4447	8589	16777
57	10518			121	4348		
58	10497	15189	22310	122	4342	8453	16644
59	10188			123	4246		
60	10167	14871	22075	124	4239	8321	16513
61	9871			125	4147		
62	9851	14562	21845	126	4140	8191	16383
63	9567			127	4051		

Table 2: Coefficients of approximating polynomial of (1).